Analyzing

Cosmic Structure Formation

by DTFE.

Analyzing Cosmic Structure Formation by DTFE.

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Analyzing Cosmic Structure Formation by DTFE.

MGA Workshop IV:

Multiscale Geometric Methods in Astronomical Data Analysis

November, 2004; IPAM, UCLA

Delaunay Tessellation Field Estimator

Contributors & Collaborators:

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- Willem Schaap
- Emilio Romano
- Pablo Araya Melo
- Erwin Platen

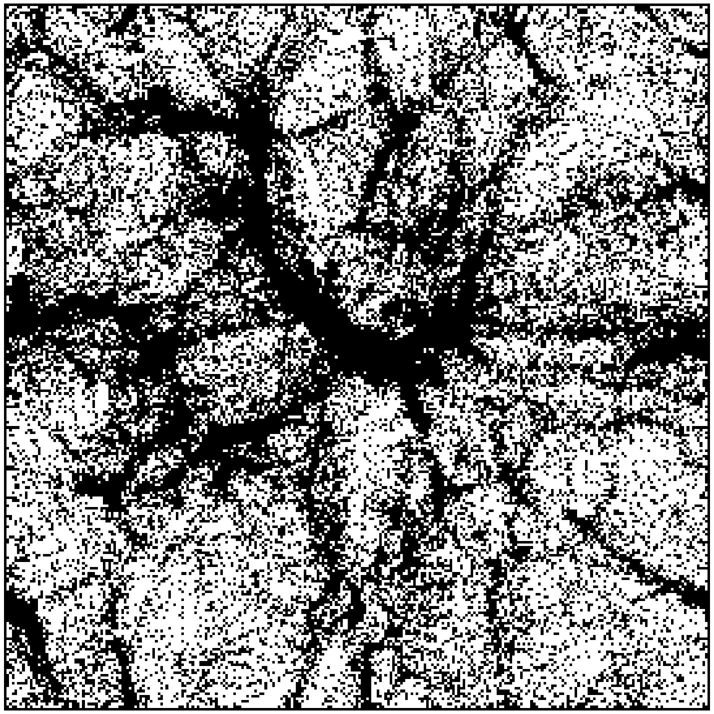
The Issue:

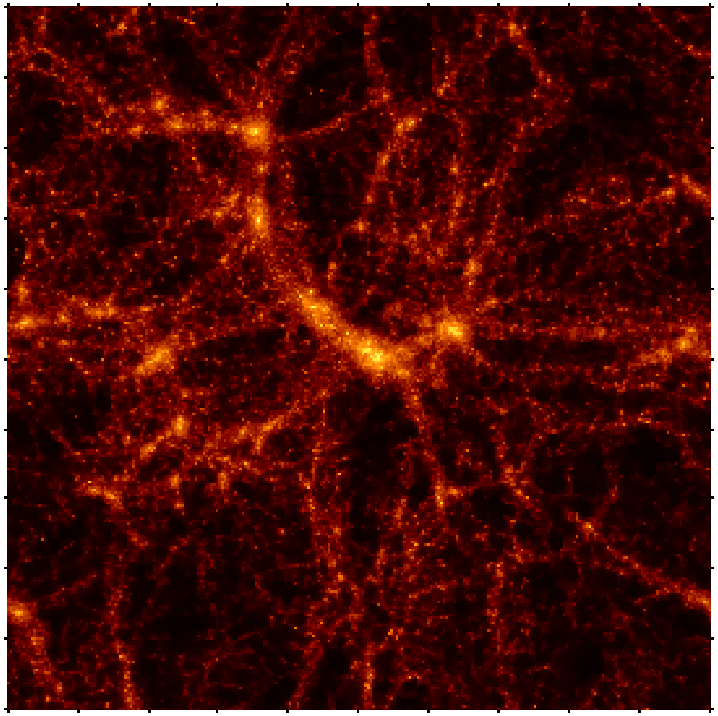
Sampling & Reconstruction

Point Sample ---→ Continuous Field

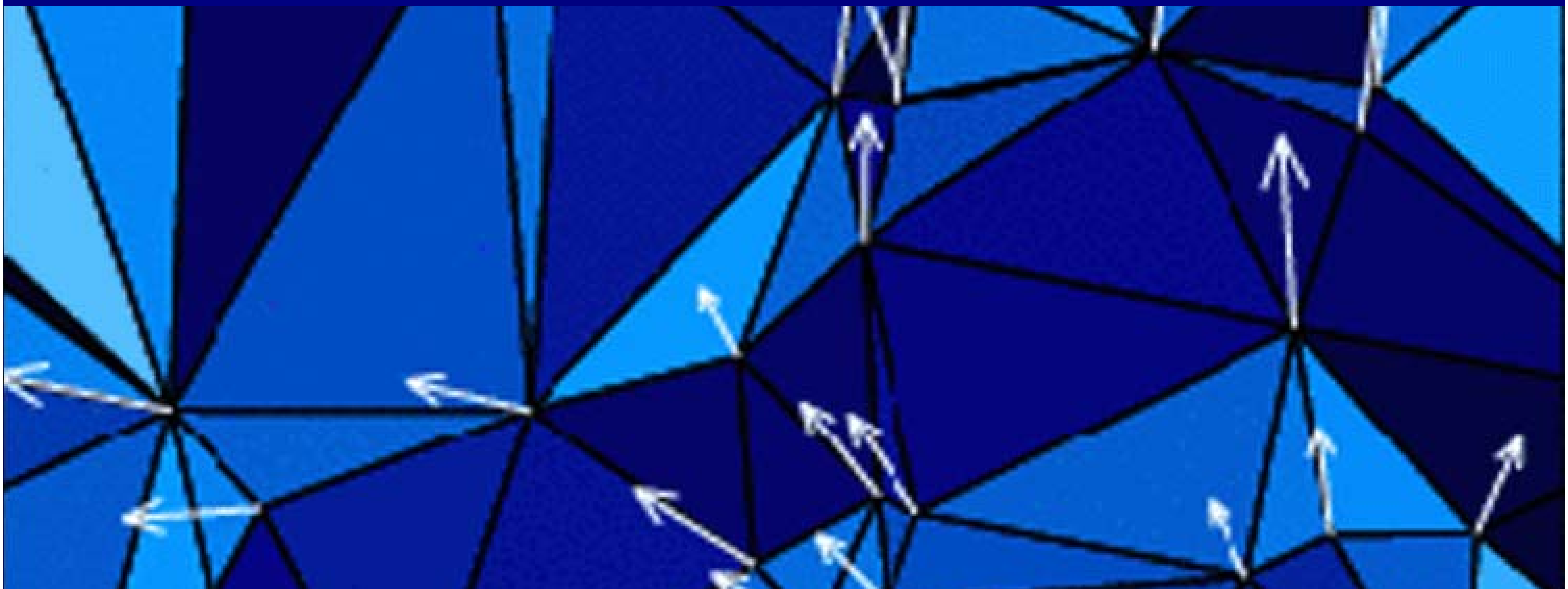
Issues:

- Anisotropy of Structural Features
- Hierarchical Infrastructure
- Inhomogeneous Sampling





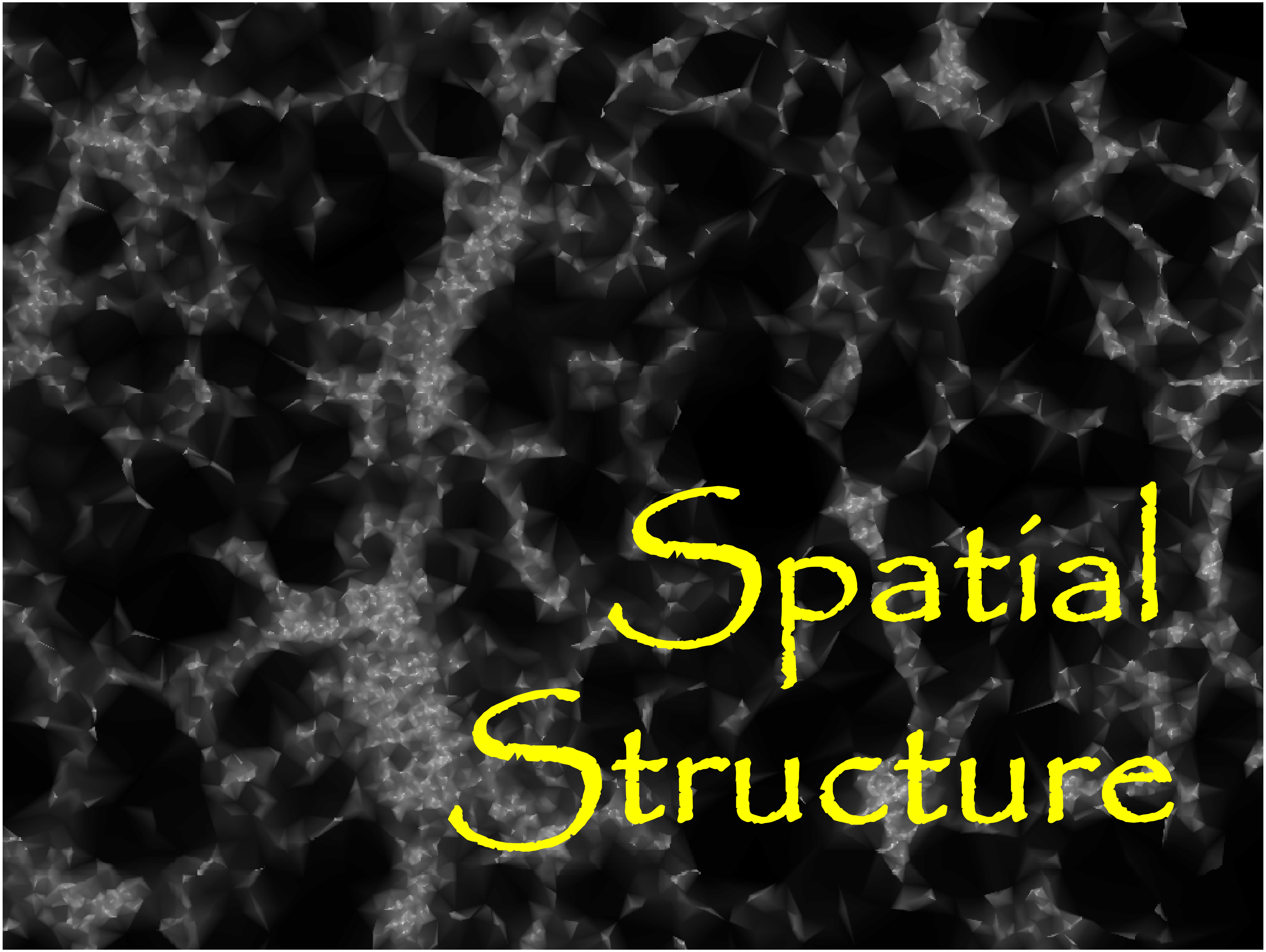
Point Sample → Continuous Field



Inferring information on
Continuous and Volume-covering Fields,
sampled – uniformly or irregularly -
at a finite set of discrete locations

Examples:

Cosmic Foam & Cosmic Flow


Spatial
Structure

Cosmic Foam Reconstruction

How to determine,

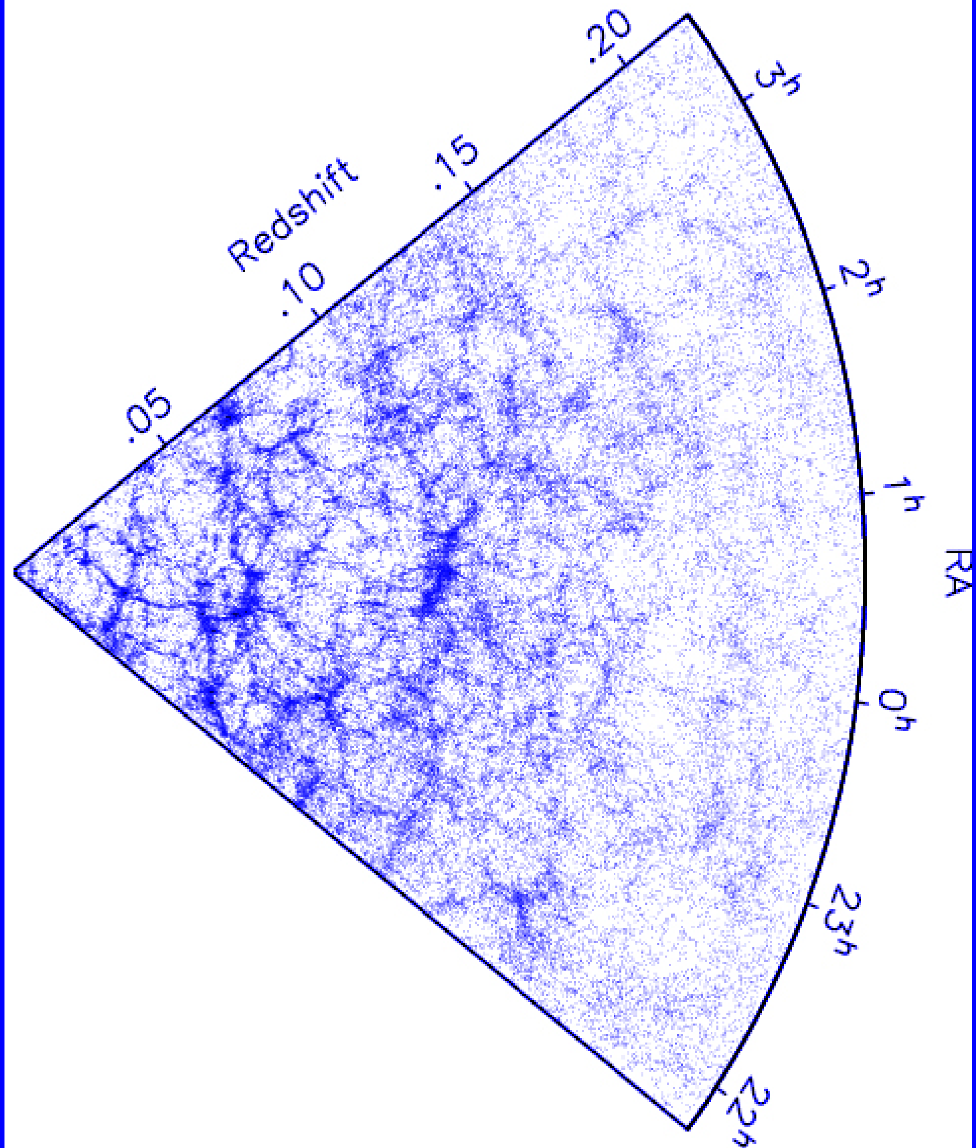
throughout the survey volume,

the corresponding

continuous **Density Field** ???

Such that …

- All structural features are optimally retained & reconstructed
- Internal substructure recovered at all levels



2dFGRS, complete, south slice

Cosmic Foam Reconstruction

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Such that …

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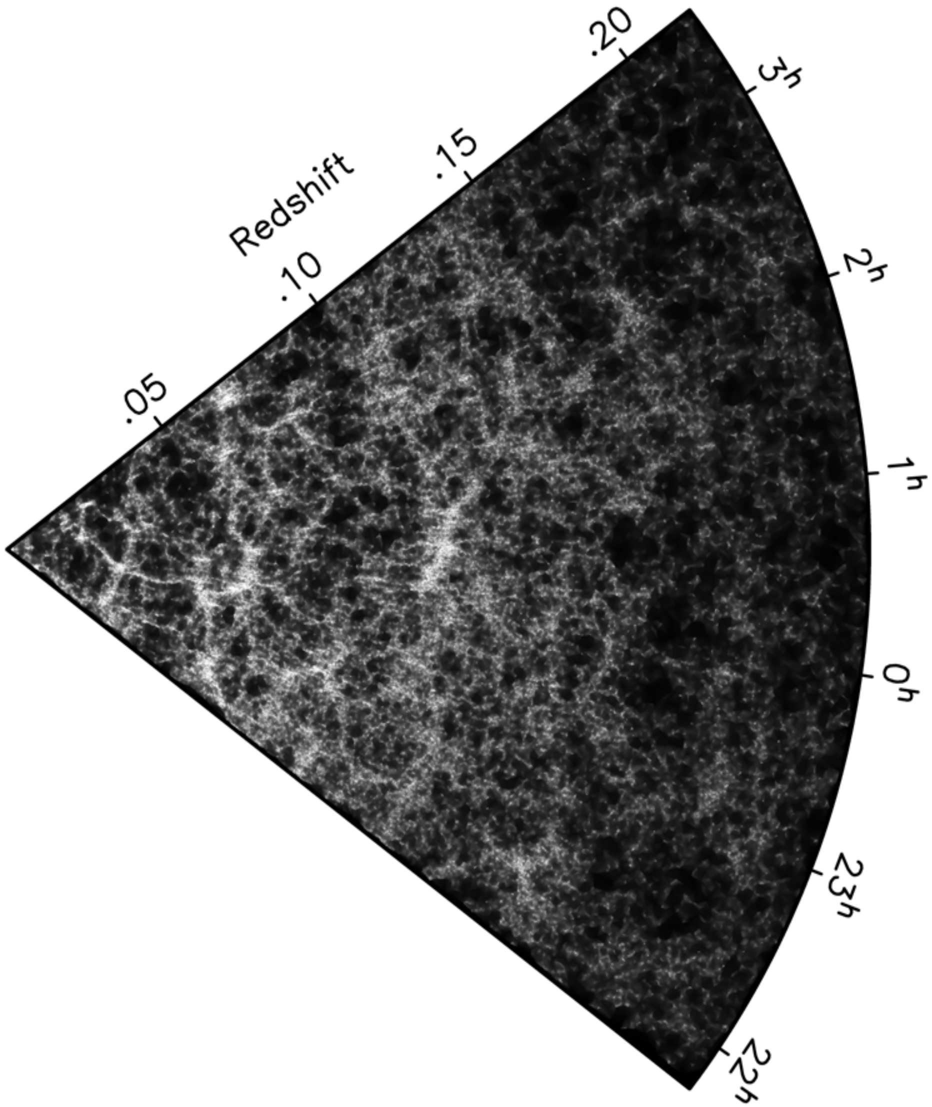
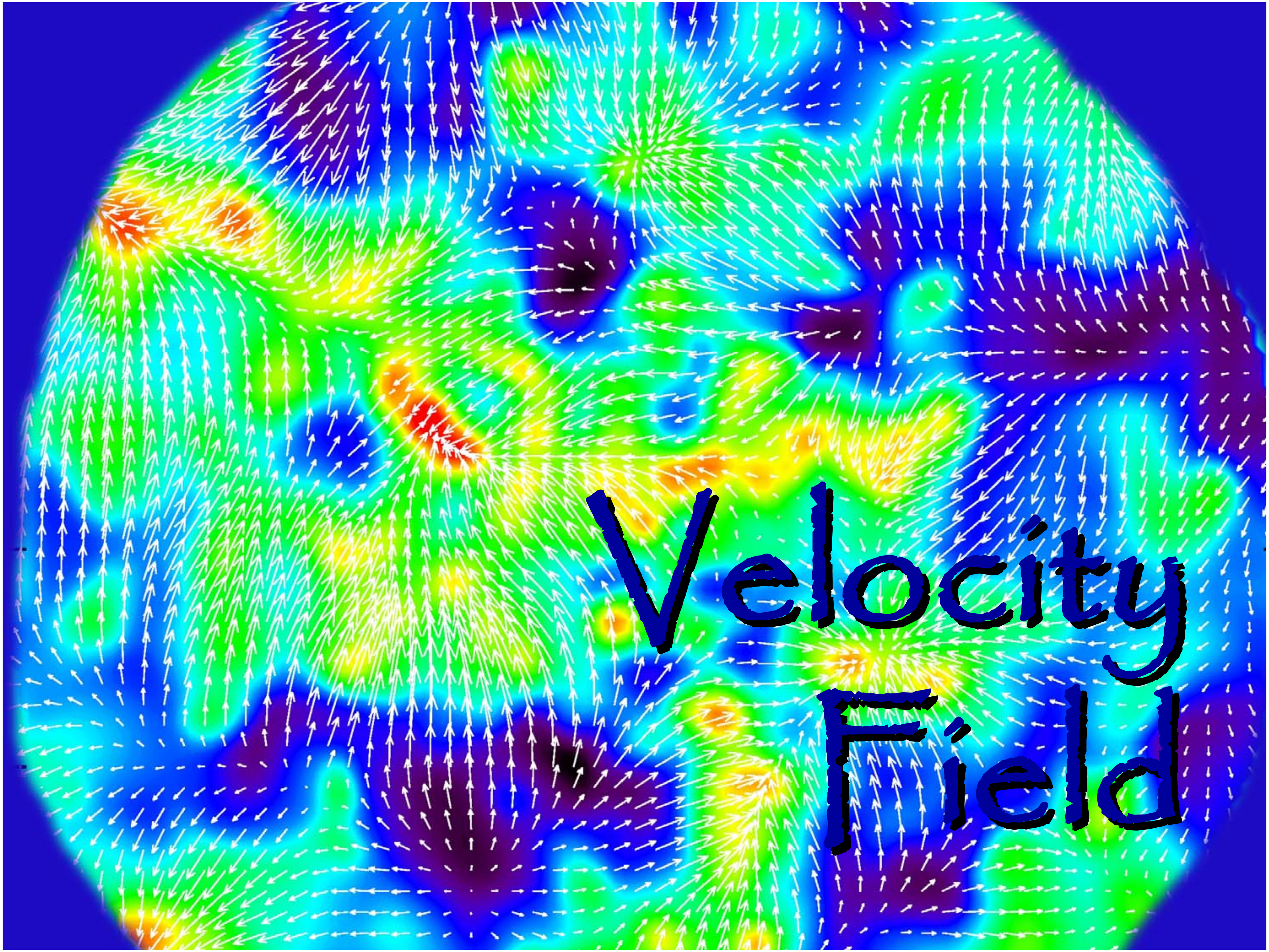


2dFGRS, complete, south slice


Velocity
Field

Cosmic Flow Reconstruction

How to determine,

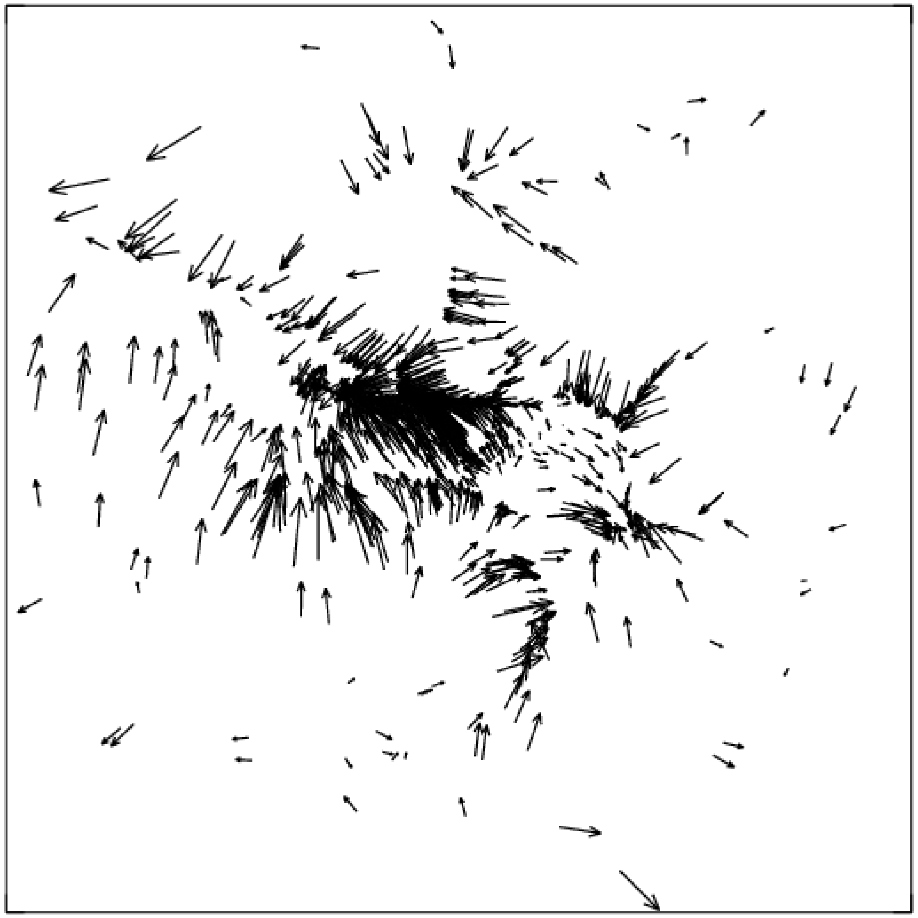
throughout the survey volume,

the corresponding

continuous Velocity Field ???

Such that …

- **Flow throughout the volume restored (assuming no shell-crossing)**
- **Restored velocities volume-weighted**
- **Shot-noise suppressed**



PSCz, (linearized) galaxy vel.

Cosmic Flow Reconstruction

How to determine,

throughout the survey volume,

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Such that …

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- Restored velocities volume-weighted
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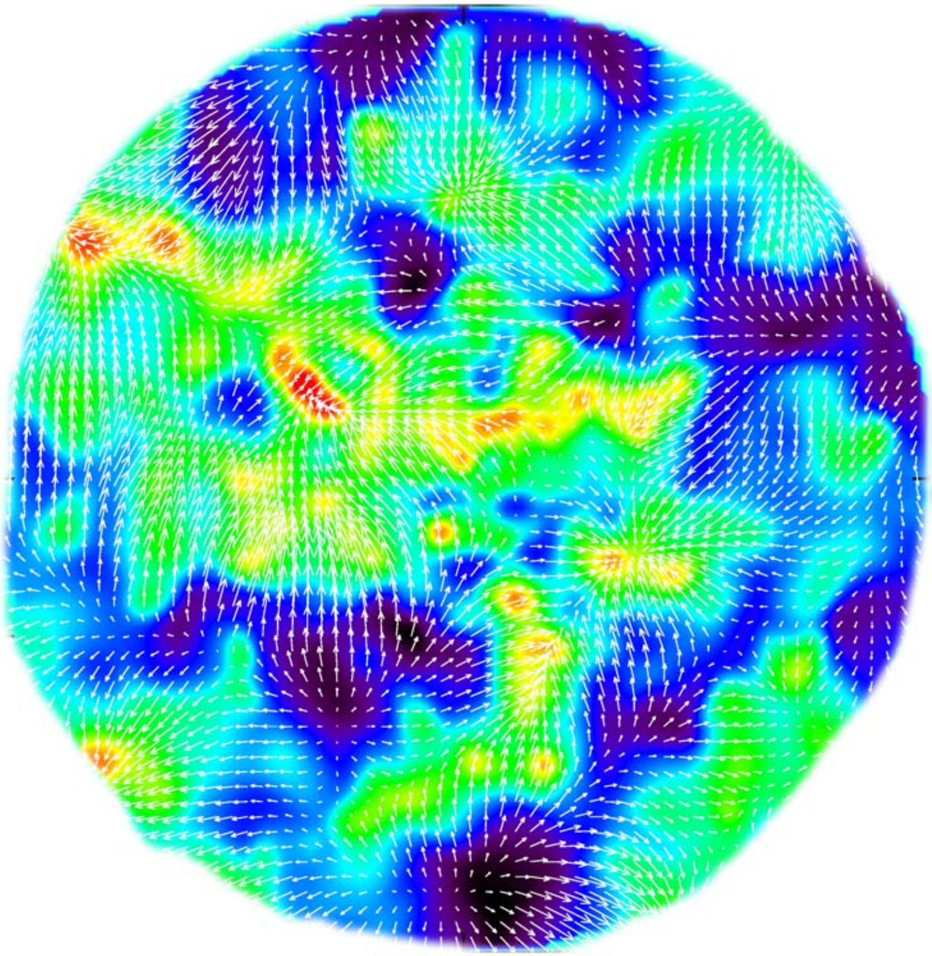


PSCz, velocity+density field

Motivation:

Cosmic Structure Formation

Cosmic Structure Formation

Structure Formation proceeds through Gravitational Instability:

- Small primordial (Gaussian) density and velocity perturbations amplify through their combined gravitational interaction.

$$\mathbf{g}(\mathbf{r},t) = -\frac{1}{a}\nabla\phi = \frac{3\Omega H^2}{8\pi}\int \mathrm{d}\mathbf{x}'\,\delta(\mathbf{x}',t)\,\frac{(\mathbf{x}'-\mathbf{x})}{|\mathbf{x}'-\mathbf{x}|^3}$$

- Following early linear evolution, genuine structures start to emerge in the subsequent phase of nonlinear evolution
- Two of the most salient characteristics of this evolution:

  - hierarchical structure formation
  - anisotropic collapse

Hierarchical Structure Formation

Gaussian Density Perturbations:

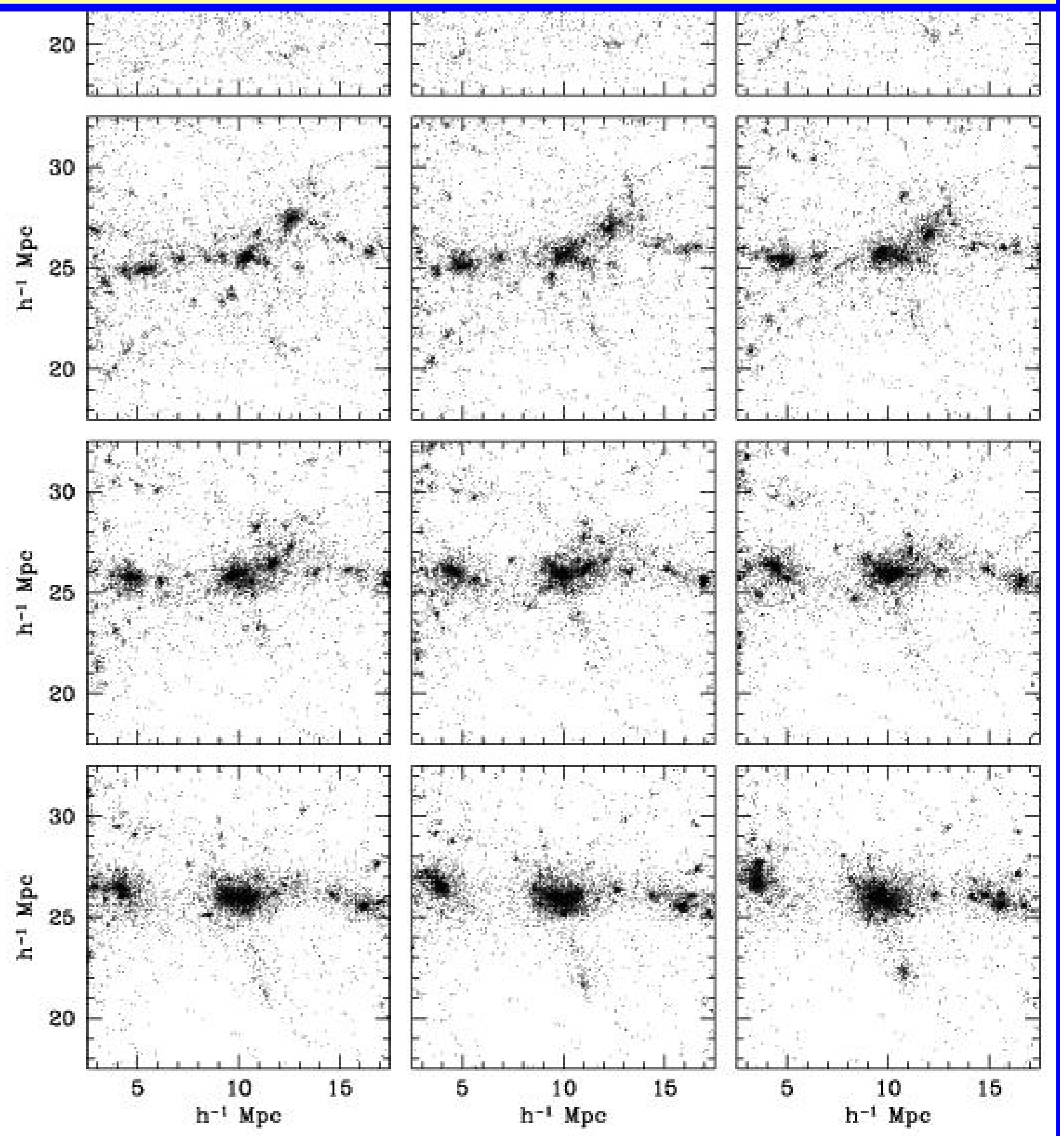
- Power spectrum P(k):

$$n(k) \equiv \frac{d \log P(k)}{d \log k}$$

$$-3 < n < 1$$

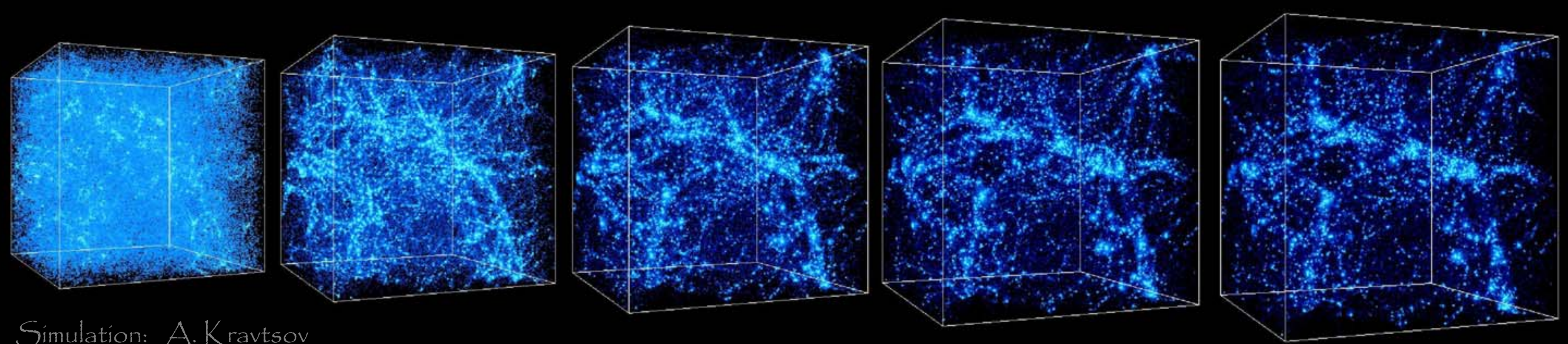
Gradual Hierarchical Assembly:

- small objects emerge & collapse first
- merge with other clumps while forming larger object in hierarchy



Anisotropic Collapse

- collapse along smallest axis → planar geometry ↔ wall
- collapse medium axis → elongated ↔ filament
- full 3-D collapse → clump ↔ clump/halo



Simulation: A. Kravtsov

Sampling the Megaparsec Universe

- Megaparsec large scale structure in the Universe:
  - maps spatial structure:
    Galaxy Redshift Surveys
- Gravitational Structure formation in the Universe:
  - nonlinear clustering processes studied by means of
    N-body computer simulations
  - kinematics & dynamics:
    peculiar velocities of galaxies

Sampling
the Megaparsec Universe

In Non-linear Clustering Regime:

- Statistics unknown and/or highly contrived,
  i.e. "non-Gaussian" !!!
- A priori ignorance of
  level & complexity internal structure
- Complex features, geometries & patterns

The Issue:

Sampling & Reconstruction

Discrete (Point) Sample → Continuous Field

Conventional astronomical "way of things":

- Smoothing procedure, through "user-defined" Filter Process,
  - frequently rigid filter kernel, occasionally adaptive
- Definition of Filter implicitly includes
  - intentions or preferences designer
  - highlighting the desired information
  - usually (unintentionally) suppressing possibly relevant information

Discrete (Point) Sample

→ Continuous Field

In search of Parameterization and Interpolation method:

- to Estimate Field Values,
  throughout D-dimensional sample volume
- Local,
  only dependent on field values direct neighbourhood
- Fully Self-adaptive and entirely Objective
- Defined only, and nothing else but, by the

Point Process itself

Method:

Natural Neighbour Interpolation

Discrete (Point) Sample

→ Continuous Field

A variety of multi-dimensional methods were developed:

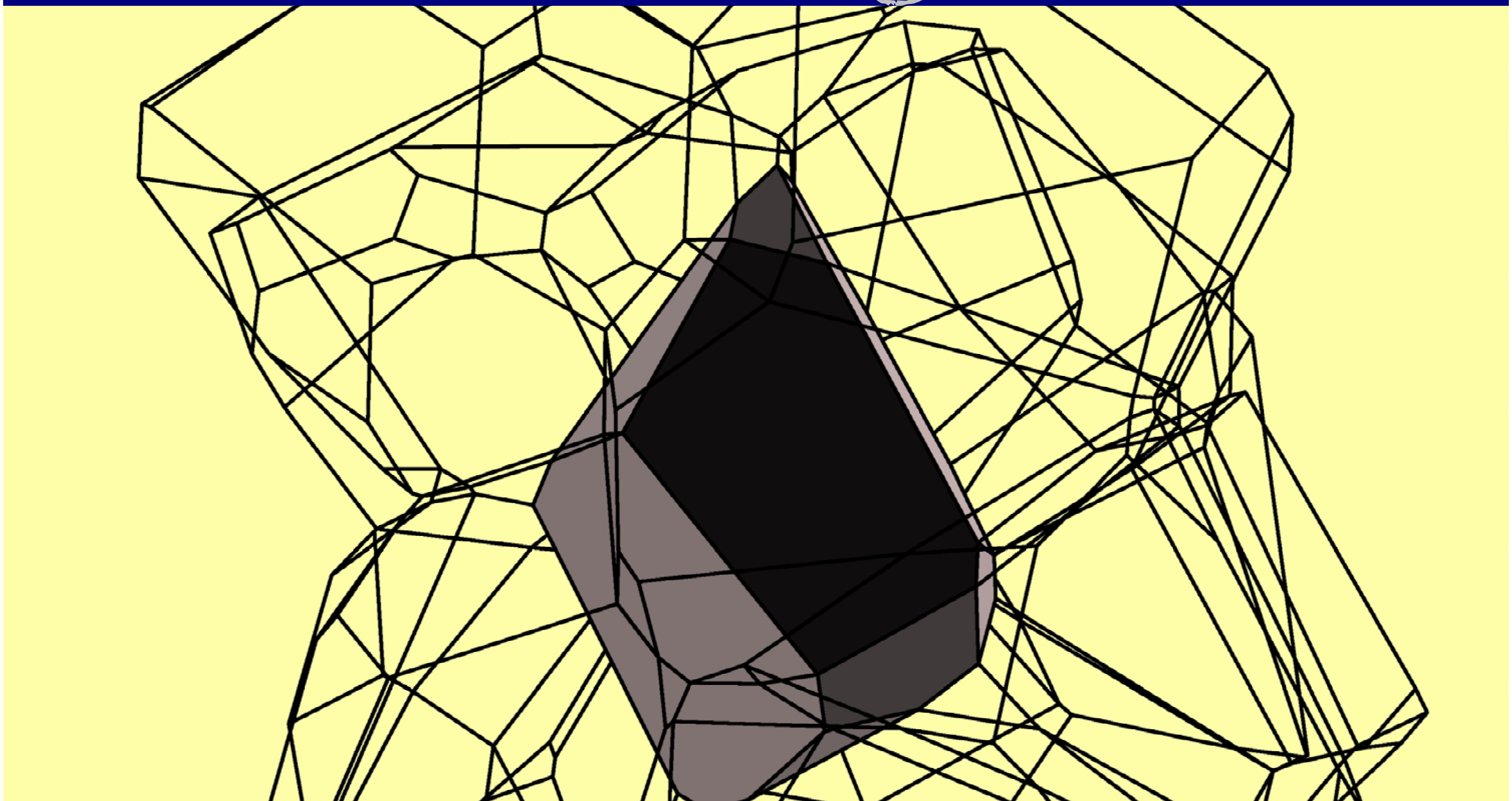
- Multidimensional Spline Interpolation
- Kriging (Matheron 1973)

Here we develop and elaborate on a linear version of a class of methods emerging from the field of computational geometry:

- Natural Neighbour Interpolation

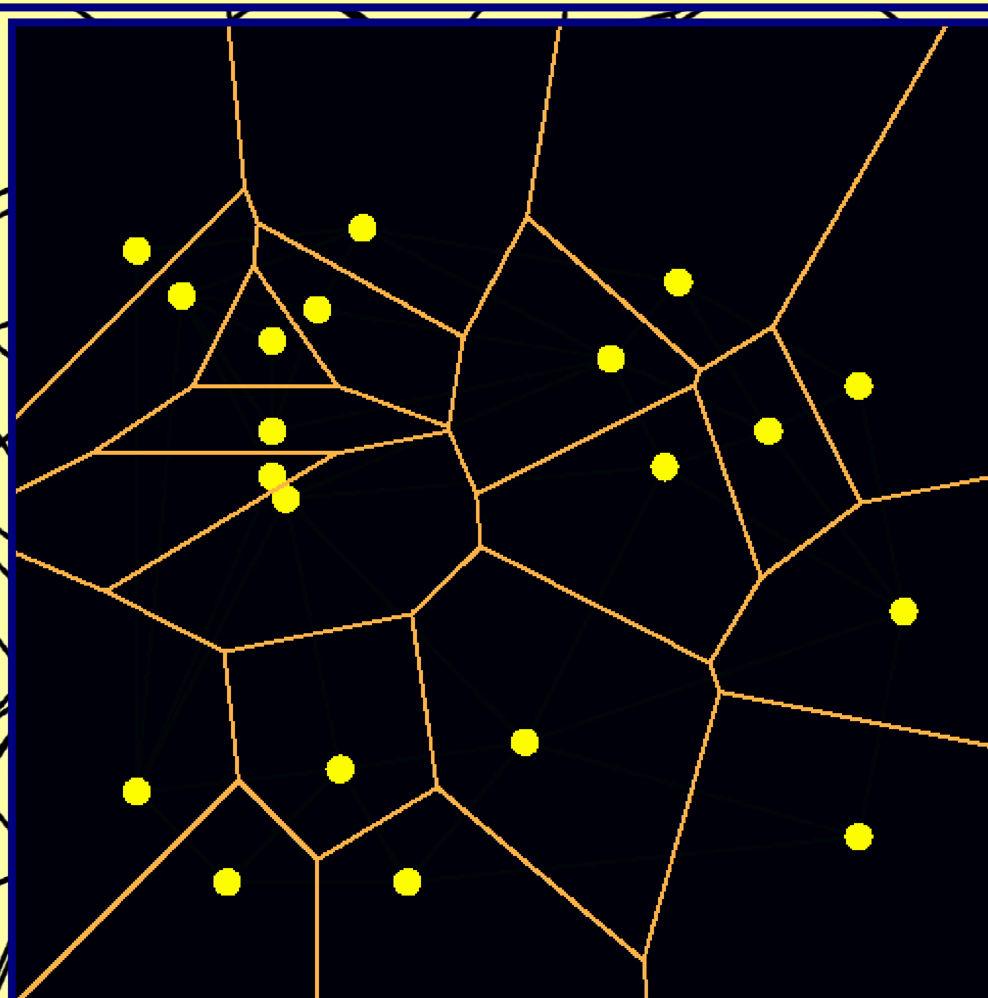
(Sibson 1980, 1981; Watson 1992)

Natural Neighbours



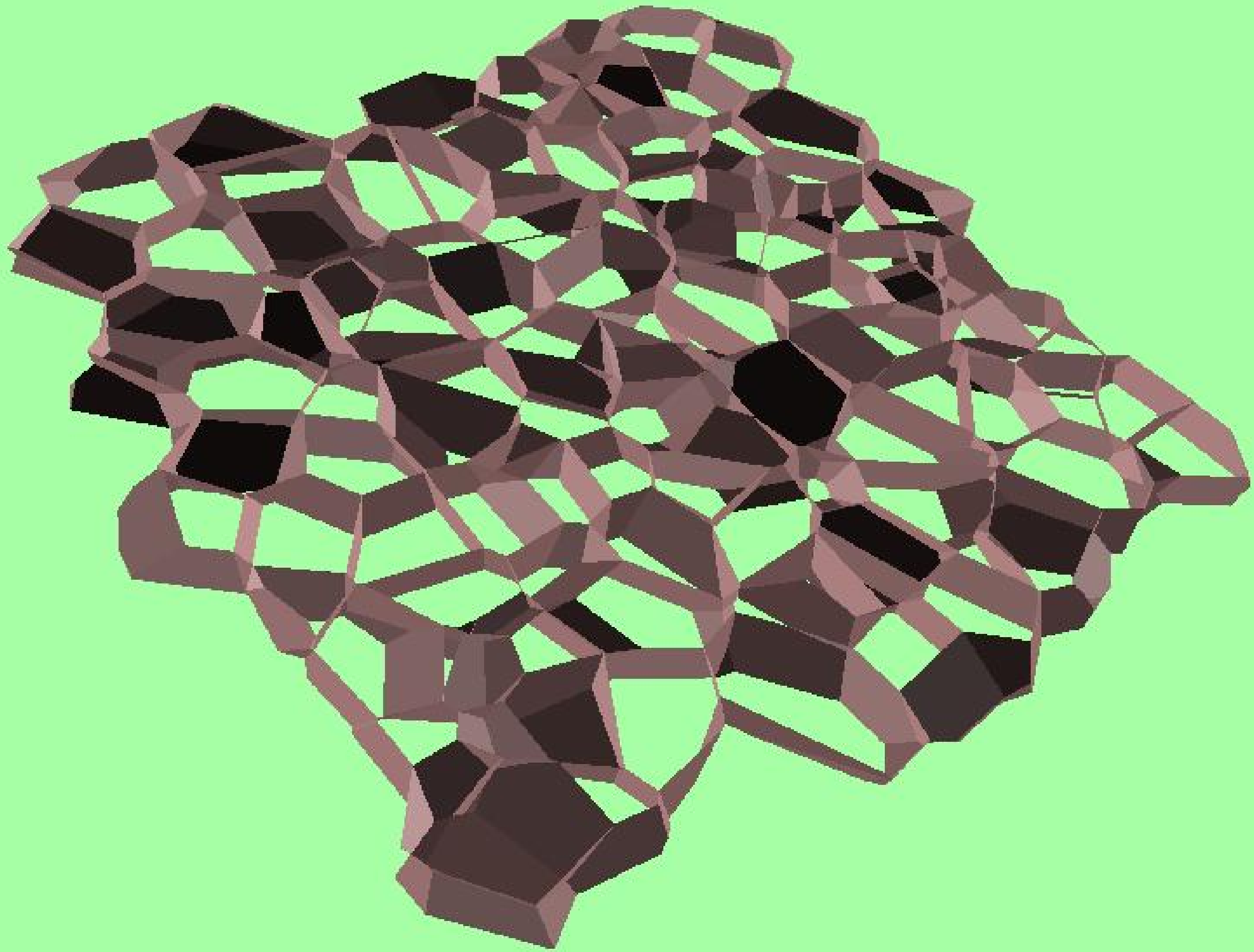
Voronoi Tessellations

Voronoi Tessellations

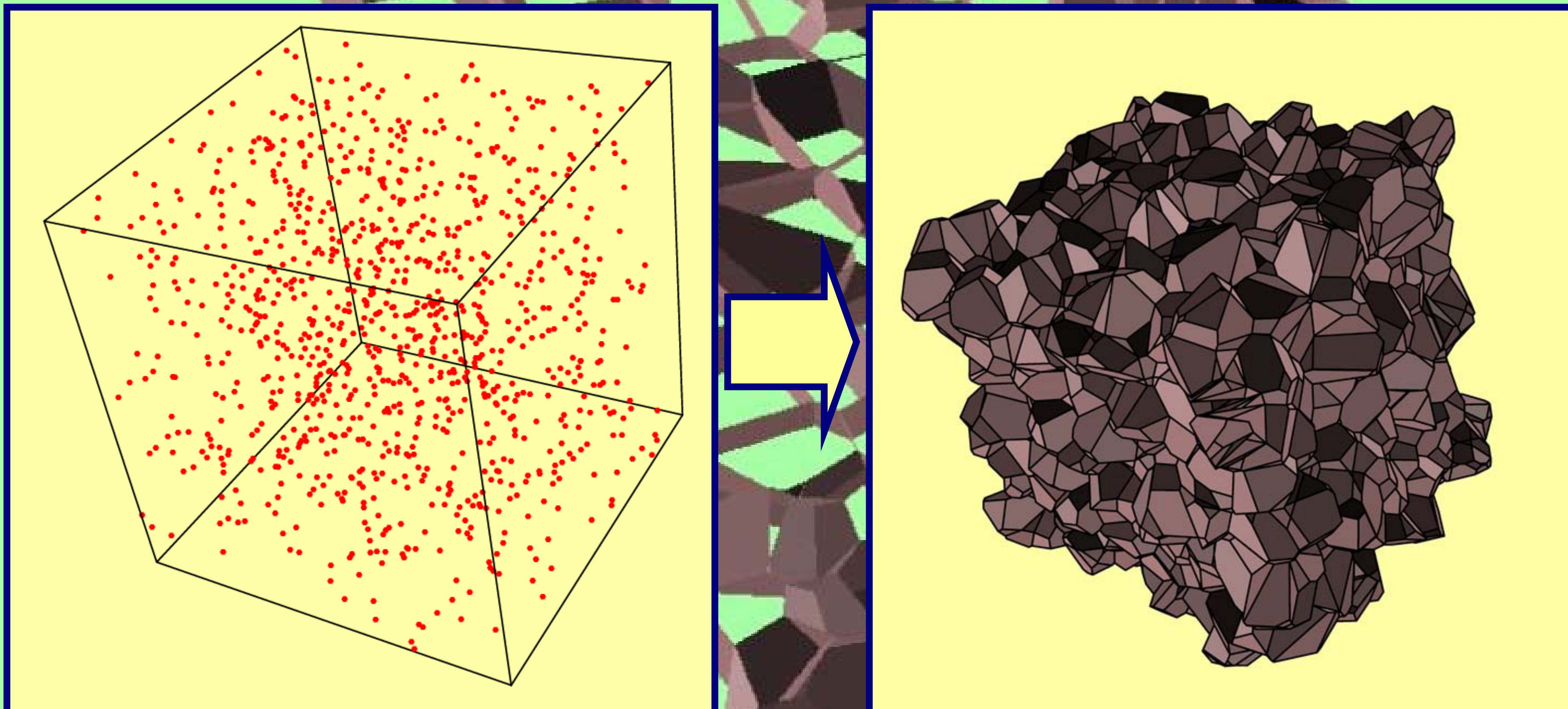


$$\Pi_i = \{\vec{x} | d(\vec{x}, \vec{x}_i) < d(\vec{x}, \vec{x}_j) \quad \text{for all } j \neq i\}$$

Voronoi Tessellations



Voronoi Tessellations



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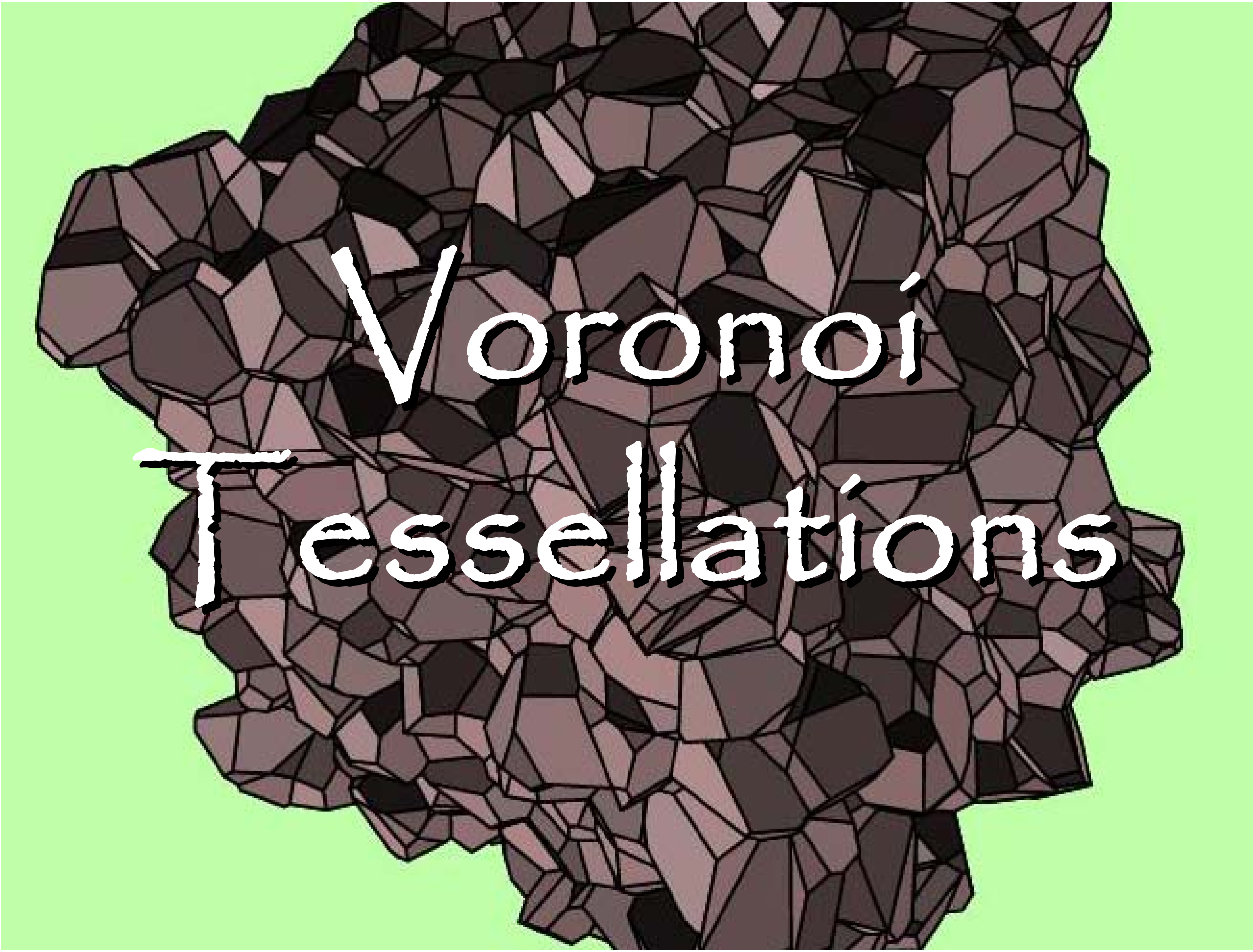
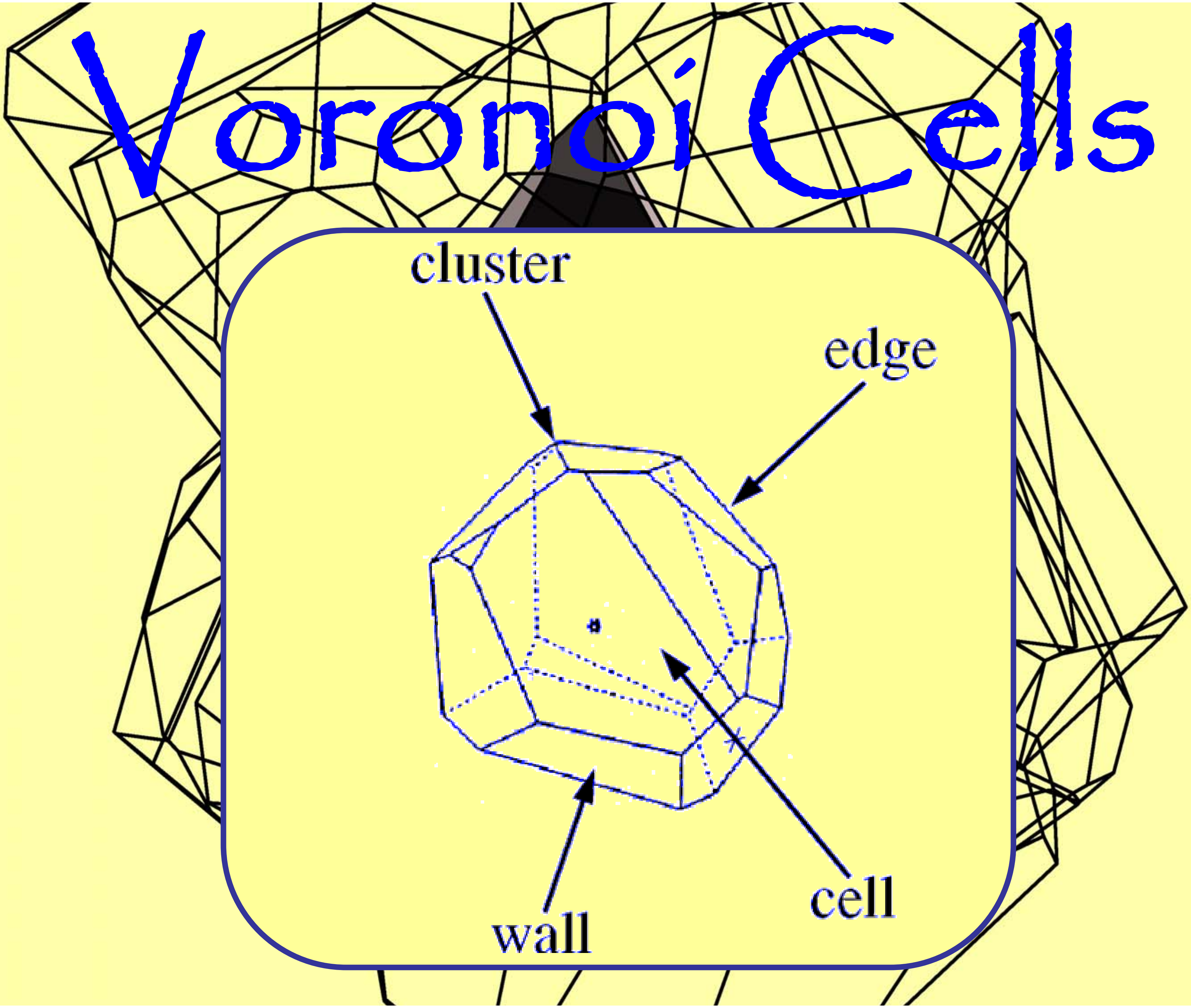
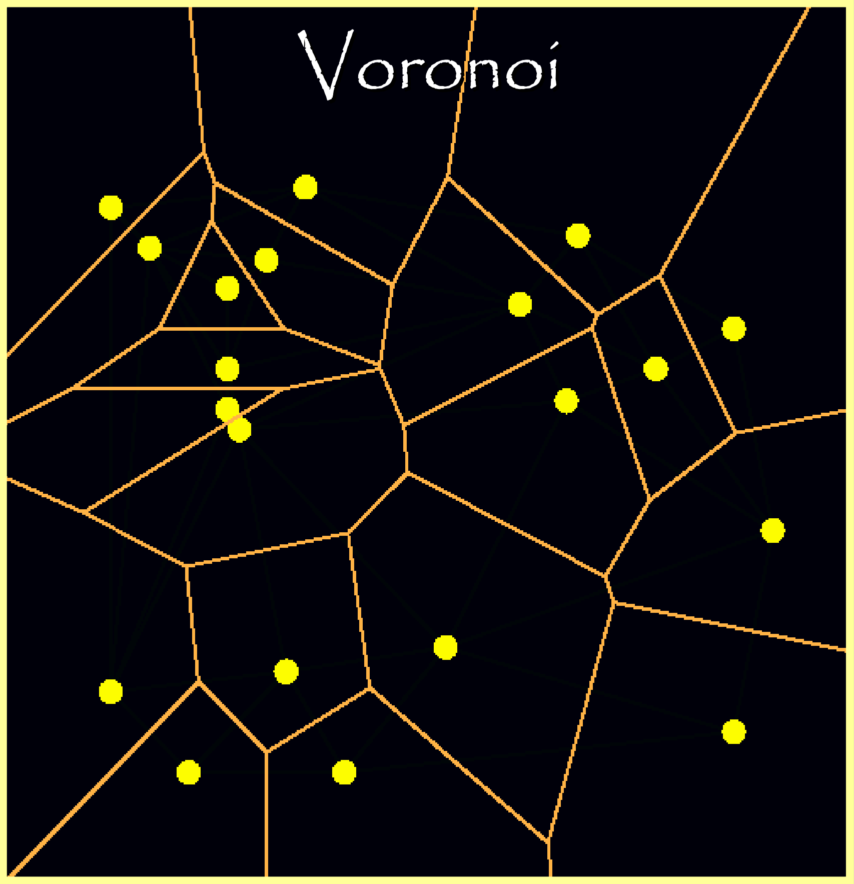
Voronoi Tessellations

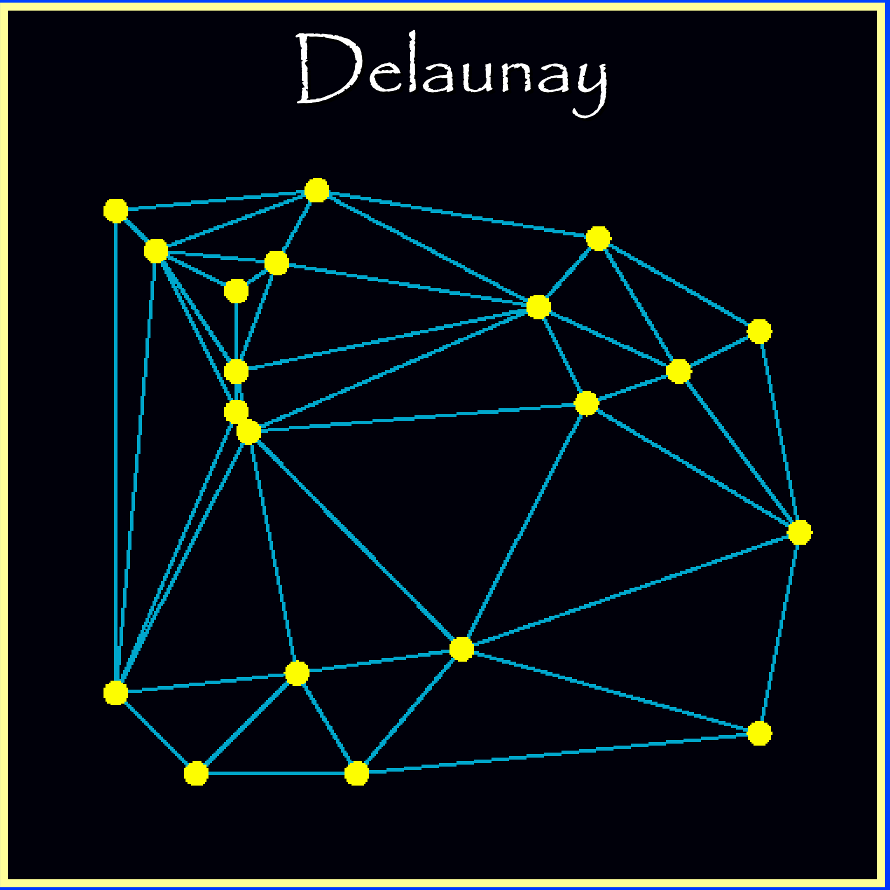


Voronoi Cells



Dual Tessellations





Voronoi Vertices: **Centers Circumscribing Spheres 4 nuclei**

→ **Delaunay Tetrahedron**

Delaunay Tessellation

Delaunay Tetrahedron:

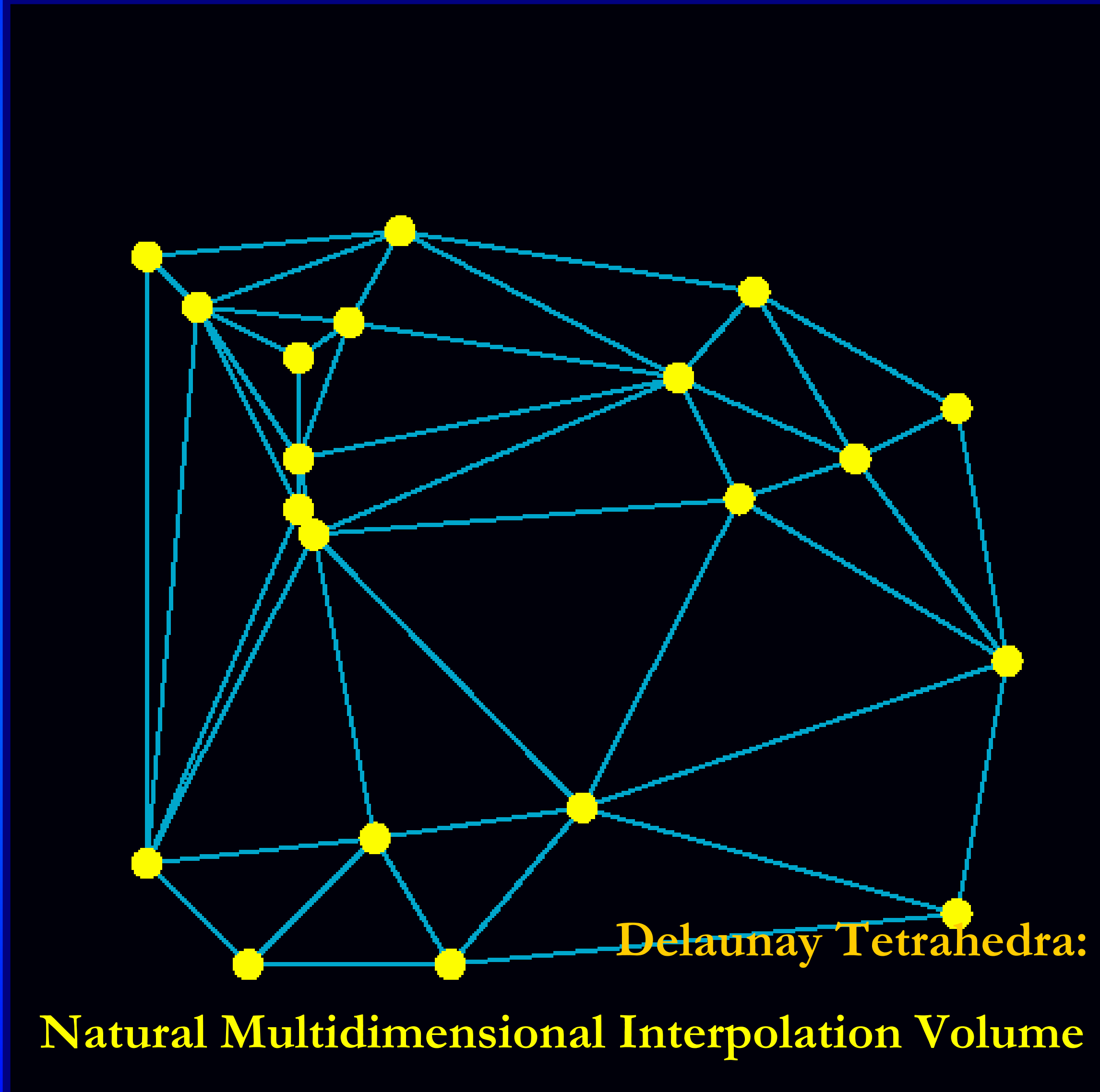
Set of 4 nuclei,
circumscribing sphere
not containing
any other nuclei

Space-Covering Complete Set
Delaunay Tetrahedra

→

Delaunay Tessellation

Voronoi Tessellation &
Delaunay Tessellation:
Duals



Discrete (Point) Sample → Continuous Field

Natural Neighbour Interpolation

(Sibson 1980, 1981; Watson 1992)

- Natural multidimensional interpolation interval:

Delaunay Tetrahedra

Discrete (Point) Sample → Continuous Field

Natural Neighbour Interpolation

(Sibson 1980, 1981; Watson 1992)

- Arrange interpolation kernel such that it is locally determined:

  Natural Neighbours

- Natural Neighbour: point sharing Voronoi wall
- Interpolation kernel set by Voronoi cells around each point:

$$f(x) = \sum_i \varphi(x, x_i)\, f_i$$

Natural Neighbour Interpolation

Definition: Sibson 1980, 1981; Watson 1992

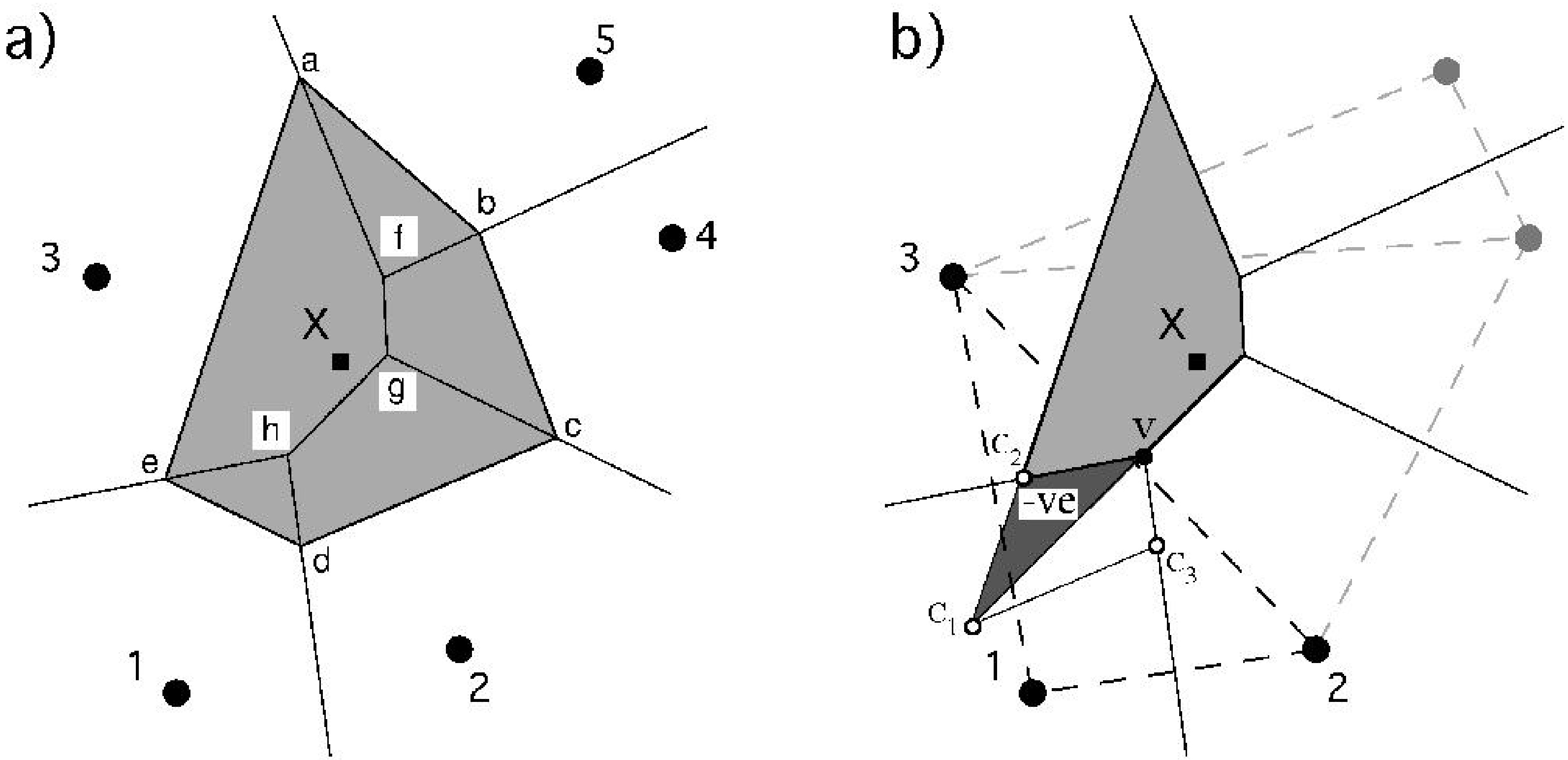
$$\varphi(x, x_i) = \mathcal{V}(x) \cap \mathcal{V}(x_i)$$

Applications:

- geophysics: Sambridge, Braun & McQueen 1999
- finite element: Braun & Sambridge 1995
- solid state physics: Sukumar 1999

NN-neighbour

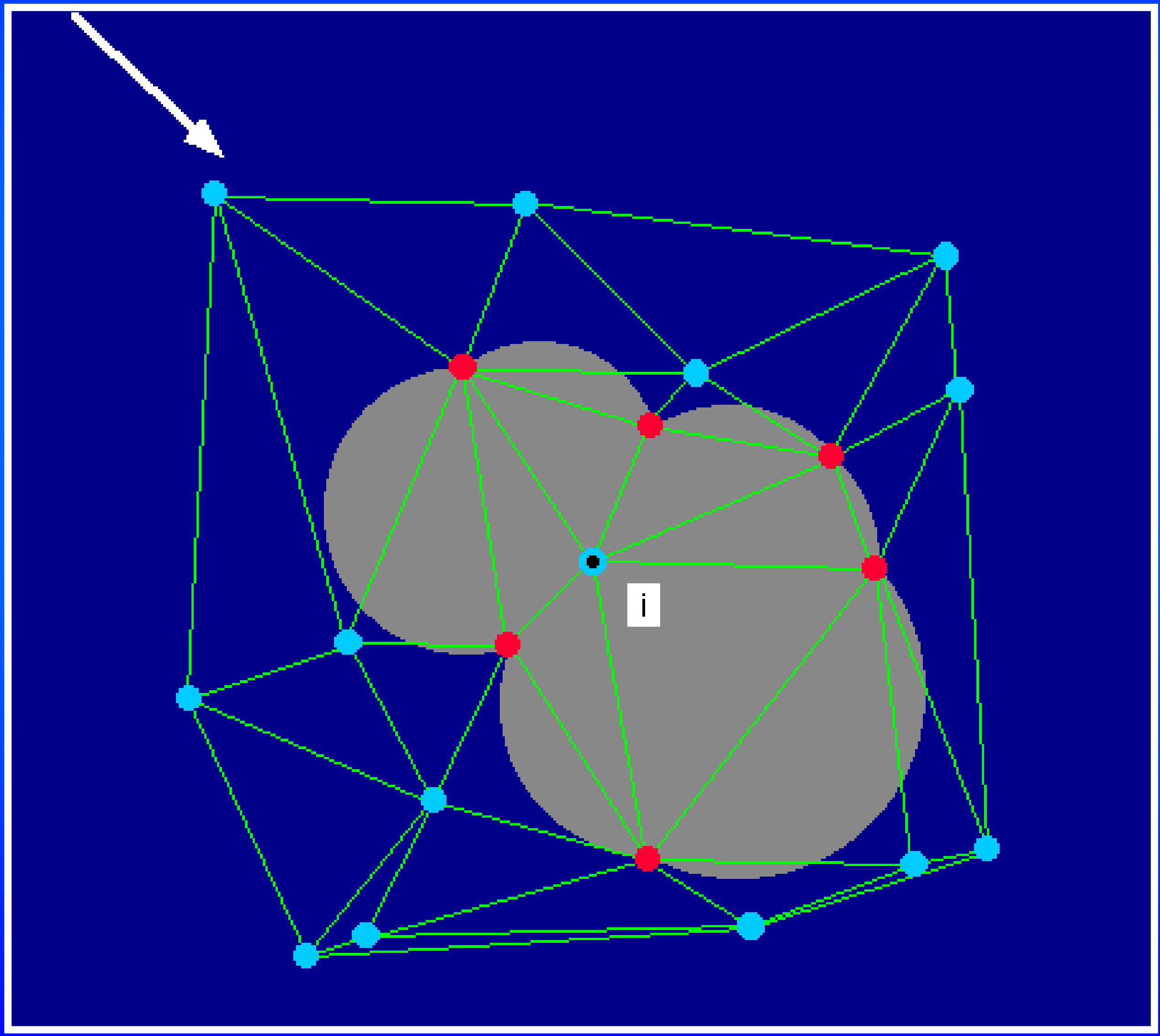
Natural neighbour co-ordinates 1



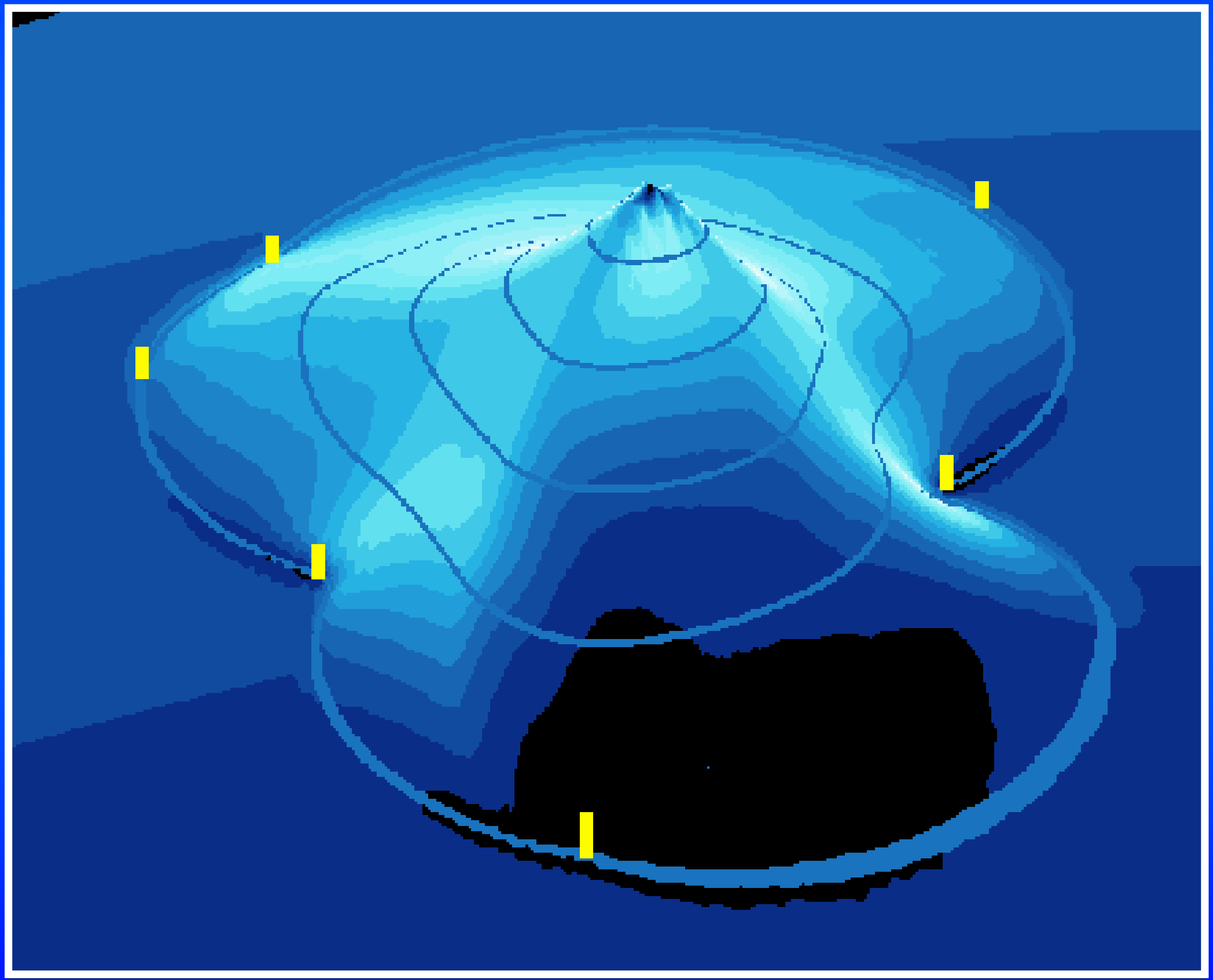
New Voronoi cell about x

Contribution to node 3 from triangle 123

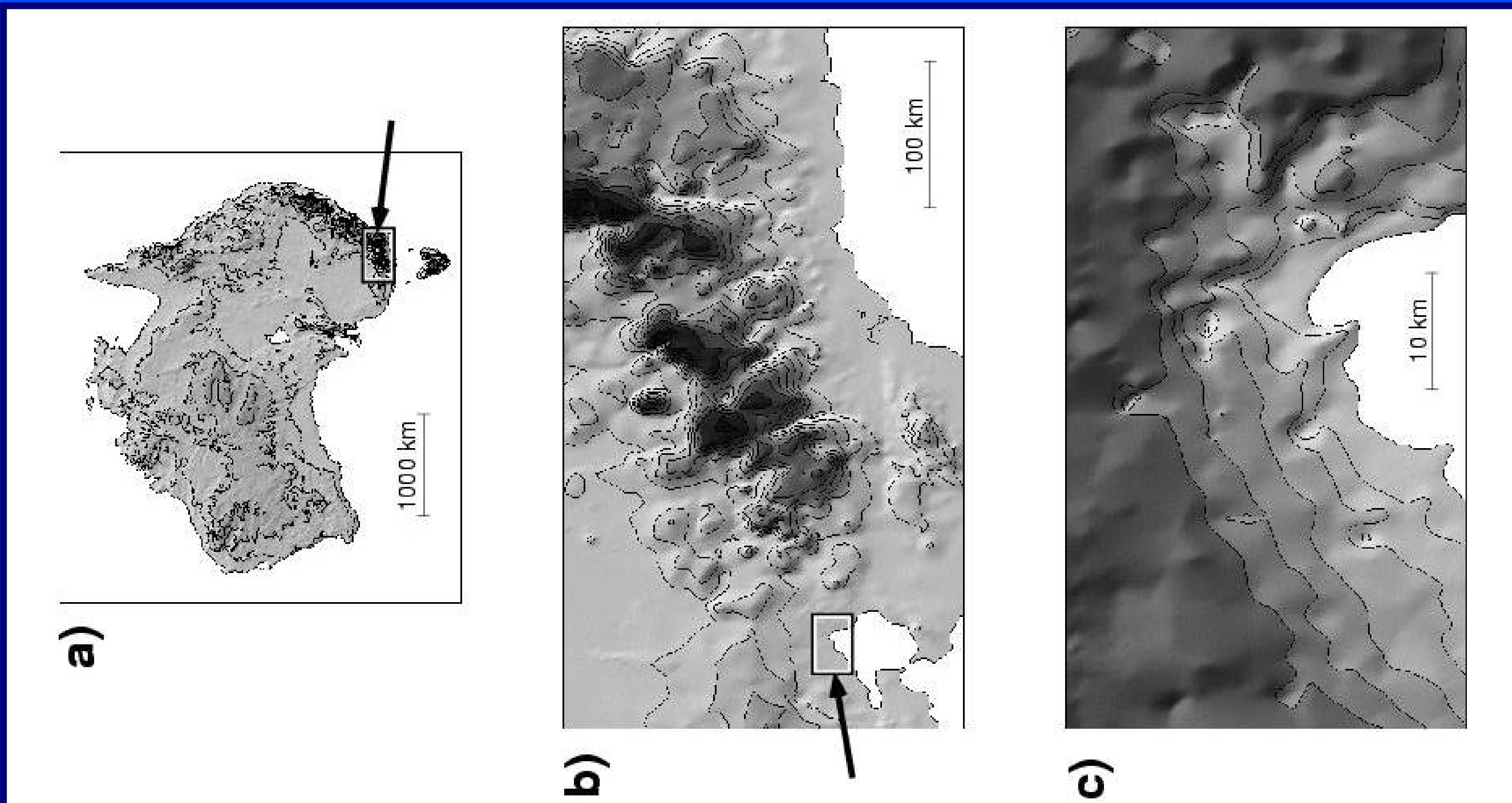
NN-neighbour



NN-neighbour



NN-neighbour



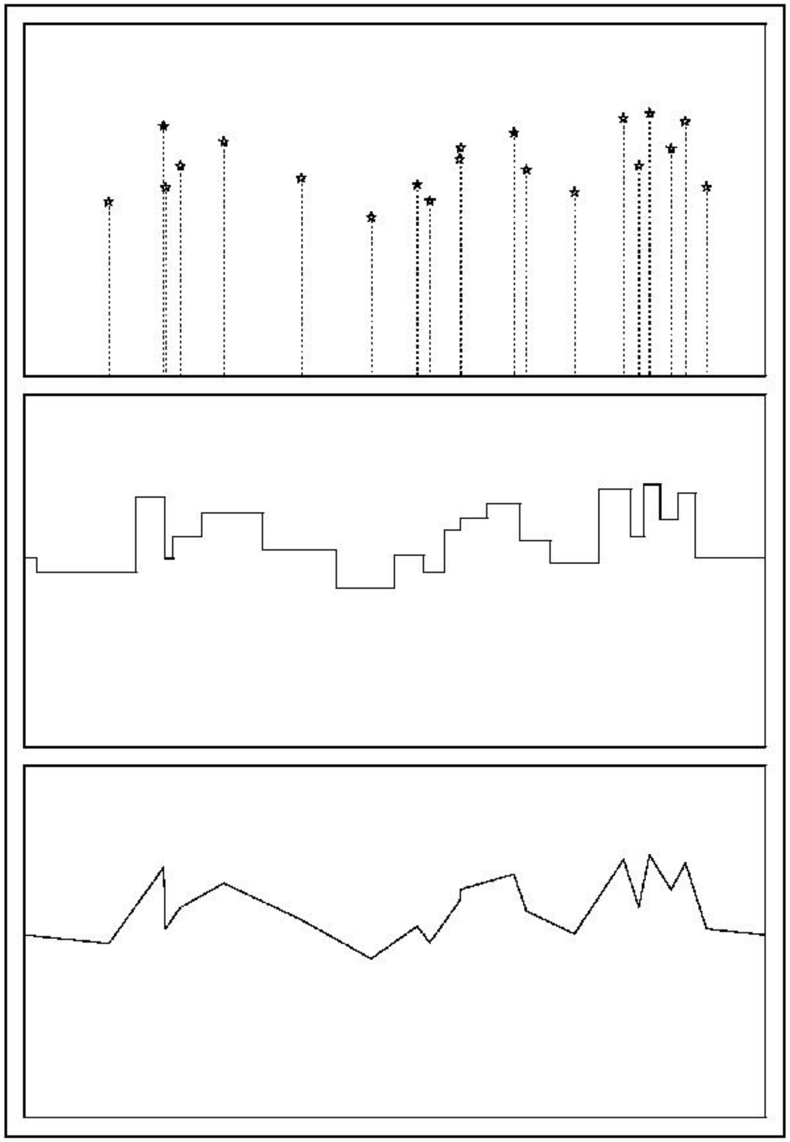
VTFE & DTFE

0^{th} order interpolation corresponds to value interpolation on the basis of constant field value inside: Voronoi Cells

$$f(\mathbf{x}) = \sum_{i=1}^{N} \alpha_i(\mathbf{x})\, f(\mathbf{x}_i)$$

$$\sum_{i=1}^{N} \alpha_i(\mathbf{x}) = 1\,; \qquad \alpha_i(\mathbf{x}_j) = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases}$$

1^{st} order interpolation corresponds to linear interpolation inside natural multi-D interpolation intervals: Delaunay Cells



Delaunay Tessellation Field Estimator

Definition: Bernardeau & van de Weygaert 1996

$$\varphi(x, x_i): \quad = 1 \quad \text{if} \quad x \in \mathcal{V}(x_i)$$
$$\quad = 0 \quad \text{if} \quad x \notin \mathcal{V}(x_i)$$

Applications:

- velocity flow: Bernardeau & van de Weygaert 1996
- N-body: Neyrinck et al. 2004
- X-ray image: Ebeling & Wiedemeyer 1992

Delaunay Tessellation Field Estimator

Definition: Schaap & van de Weygaert 2000

Bernardeau & van de Weygaert 1996

$$\varphi(x, x_i):$$ Linear Interpolant within Contiguous Voronoi Cell

Applications:

- N-body: Schaap & van de Weygaert 2000, 2005
  Schaap, van de Weygaert, Araya 2005
  Platen, van de Weygaert, Araya 2005
- phase space: Arad, Dekel & Klypin 2004
- velocity flow: Romano-Diaz, Schaap & van de Weygaert 2004
- redshift survey: Schaap & van de Weygaert 2004, 2005

Delaunay Tessellation Field Estimator

Additional Virtues use of Tessellations:

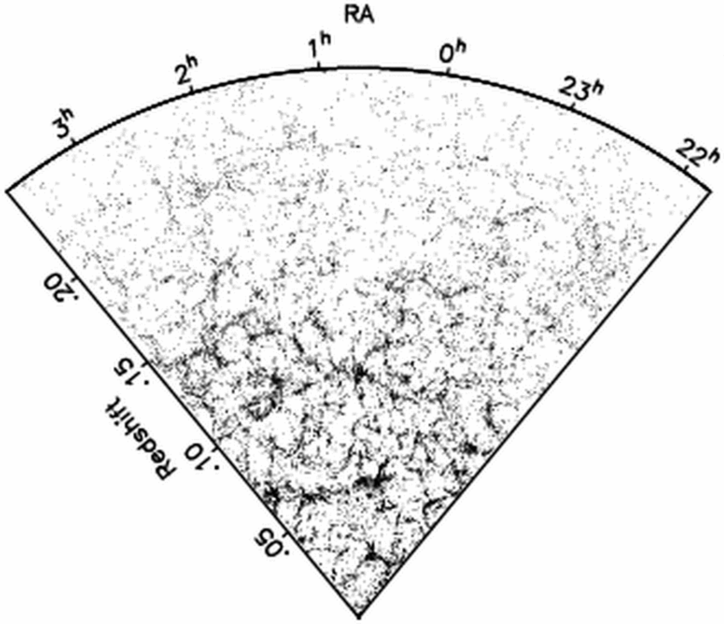
- Optimally Adaptive to
  density point process
- Optimally Adaptive to
  local geometry point process
- Weighting by Voronoi tessellation
  implies volume-weighted estimates

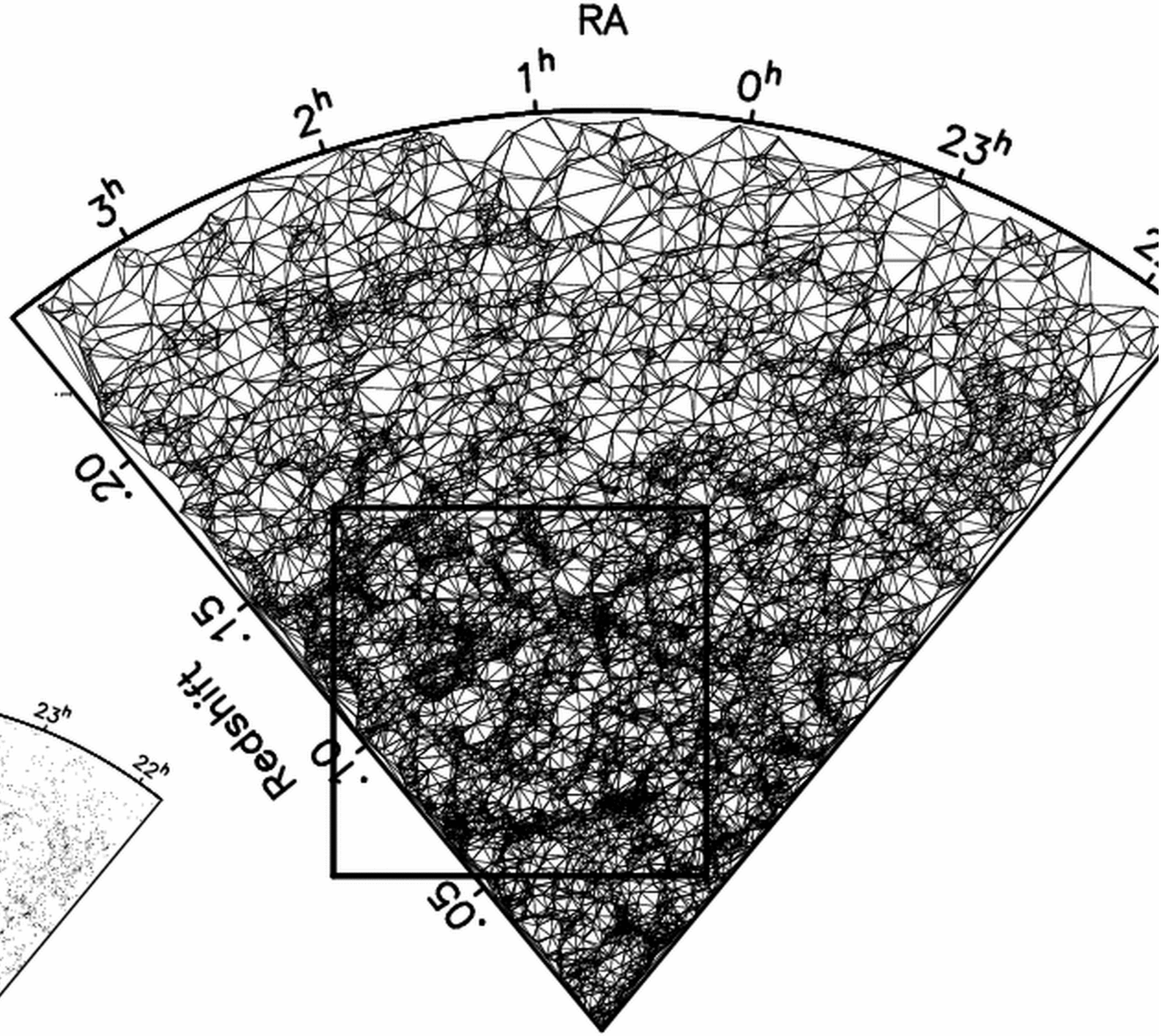
Delaunay Tessellation Field Estimator

Tessellations:

Sensitivity to Point Process

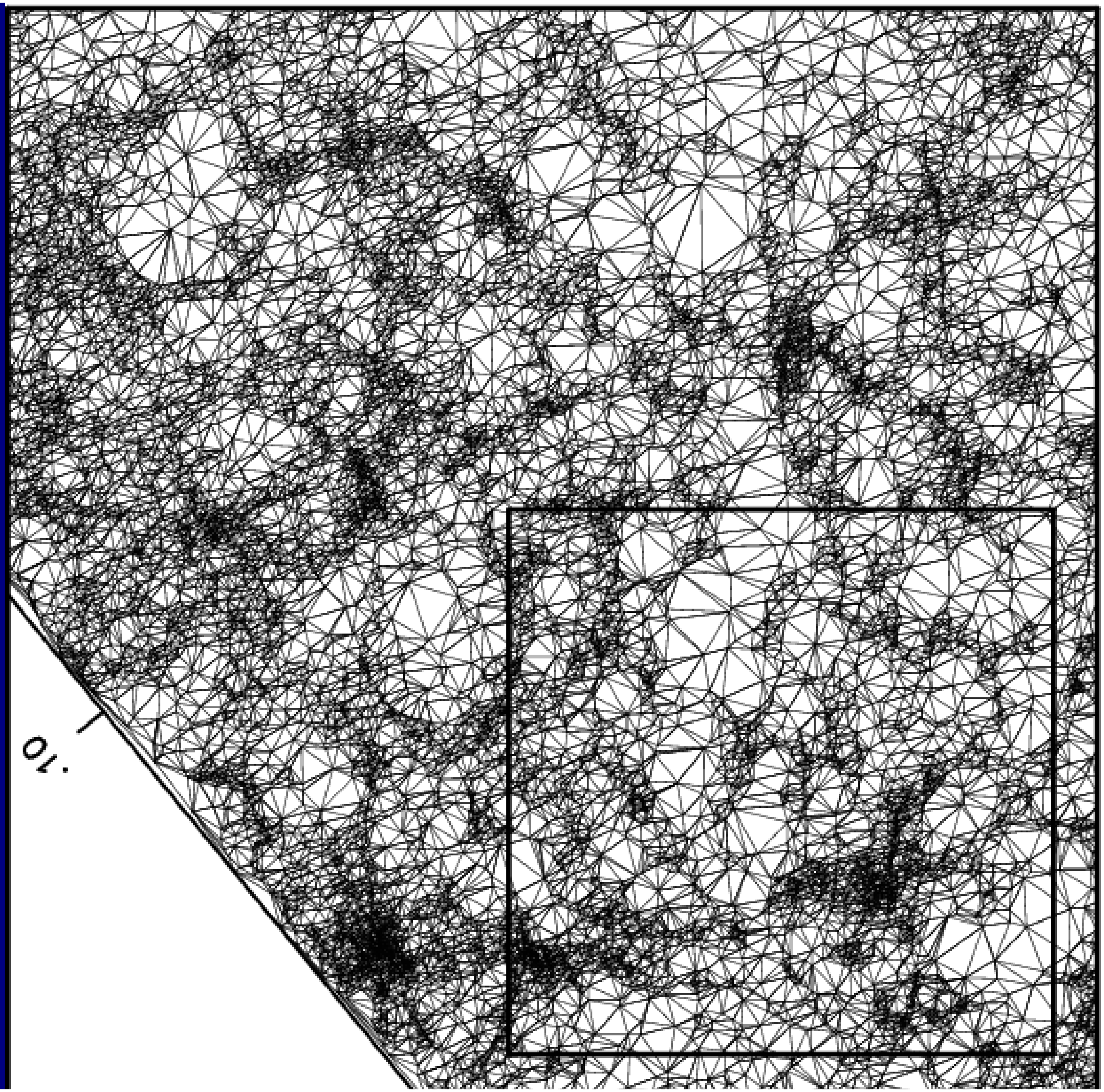
Delaunay 2dF Galaxy Redshift Survey





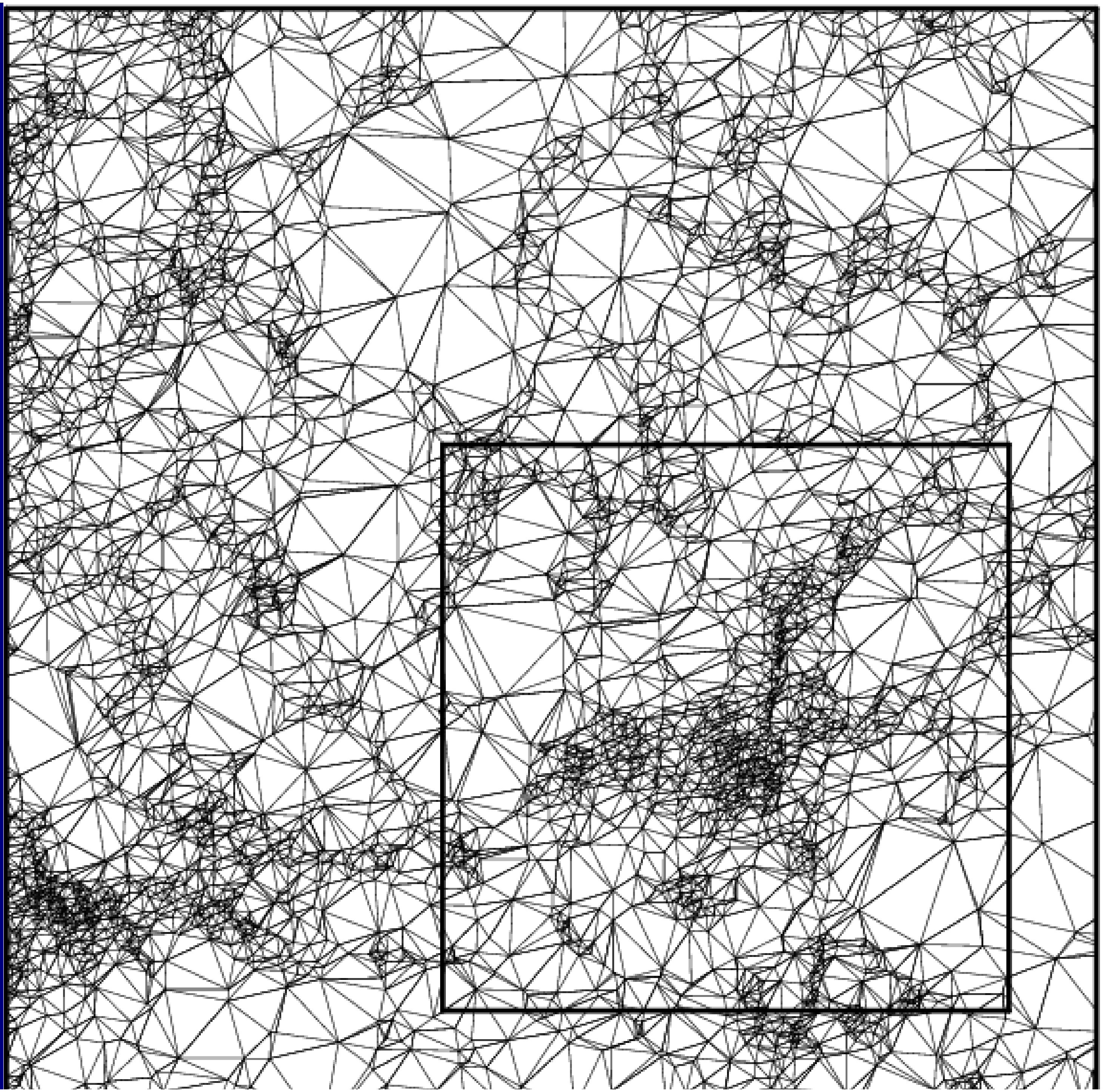
Delaunay

2dF Galaxy Redshift Survey



Delaunay

2dF Galaxy Redshift Survey



Delaunay

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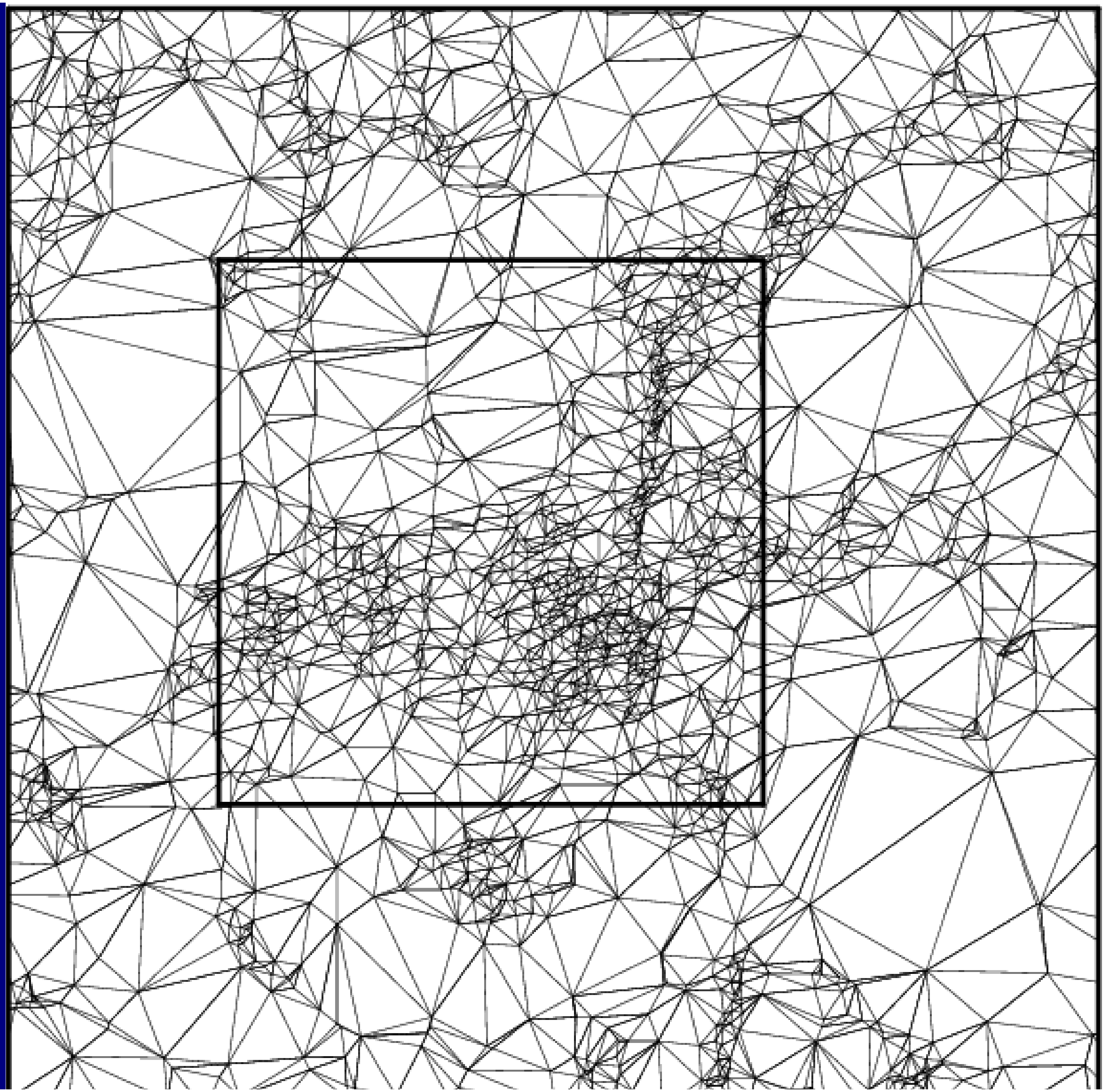
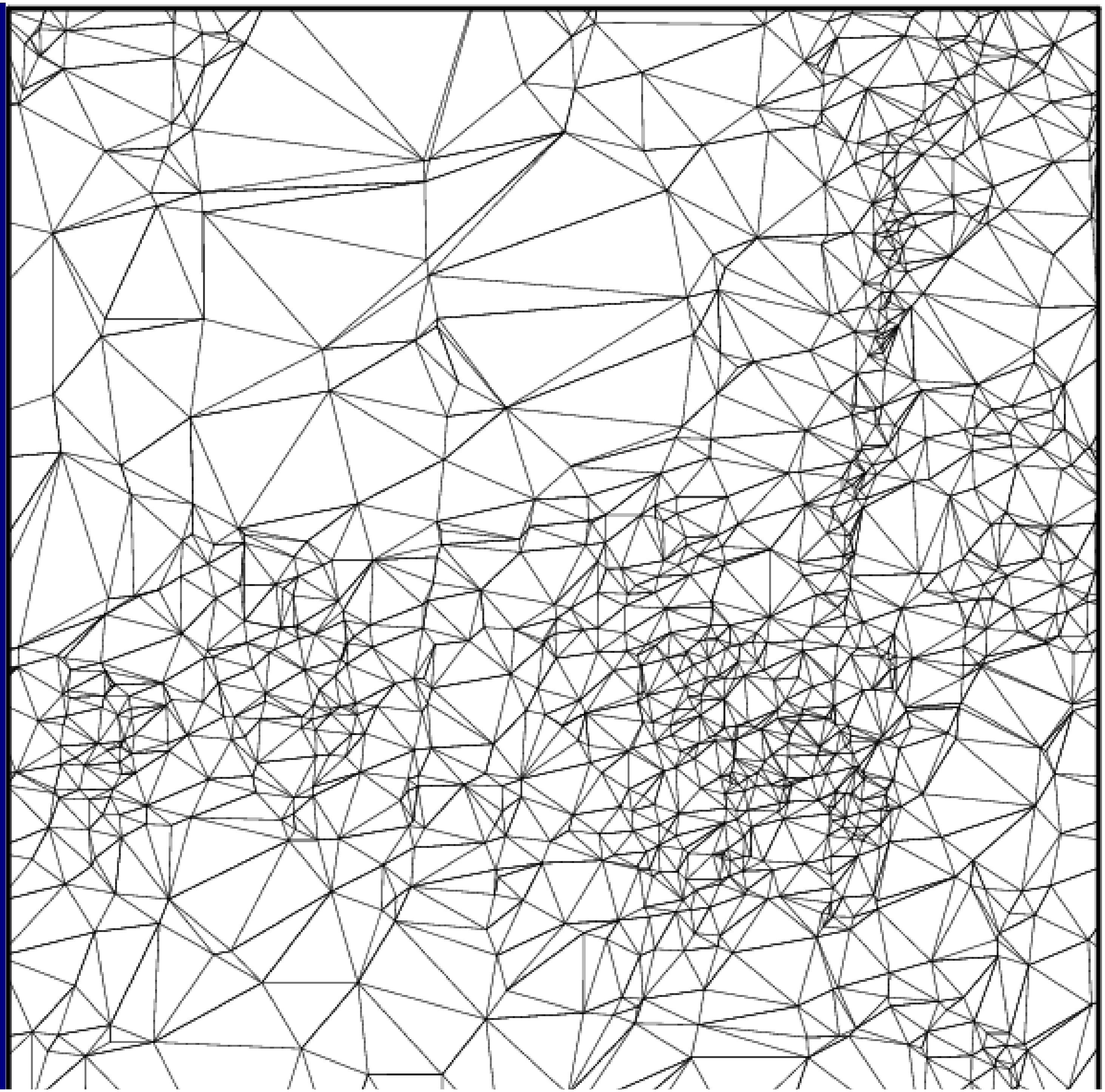


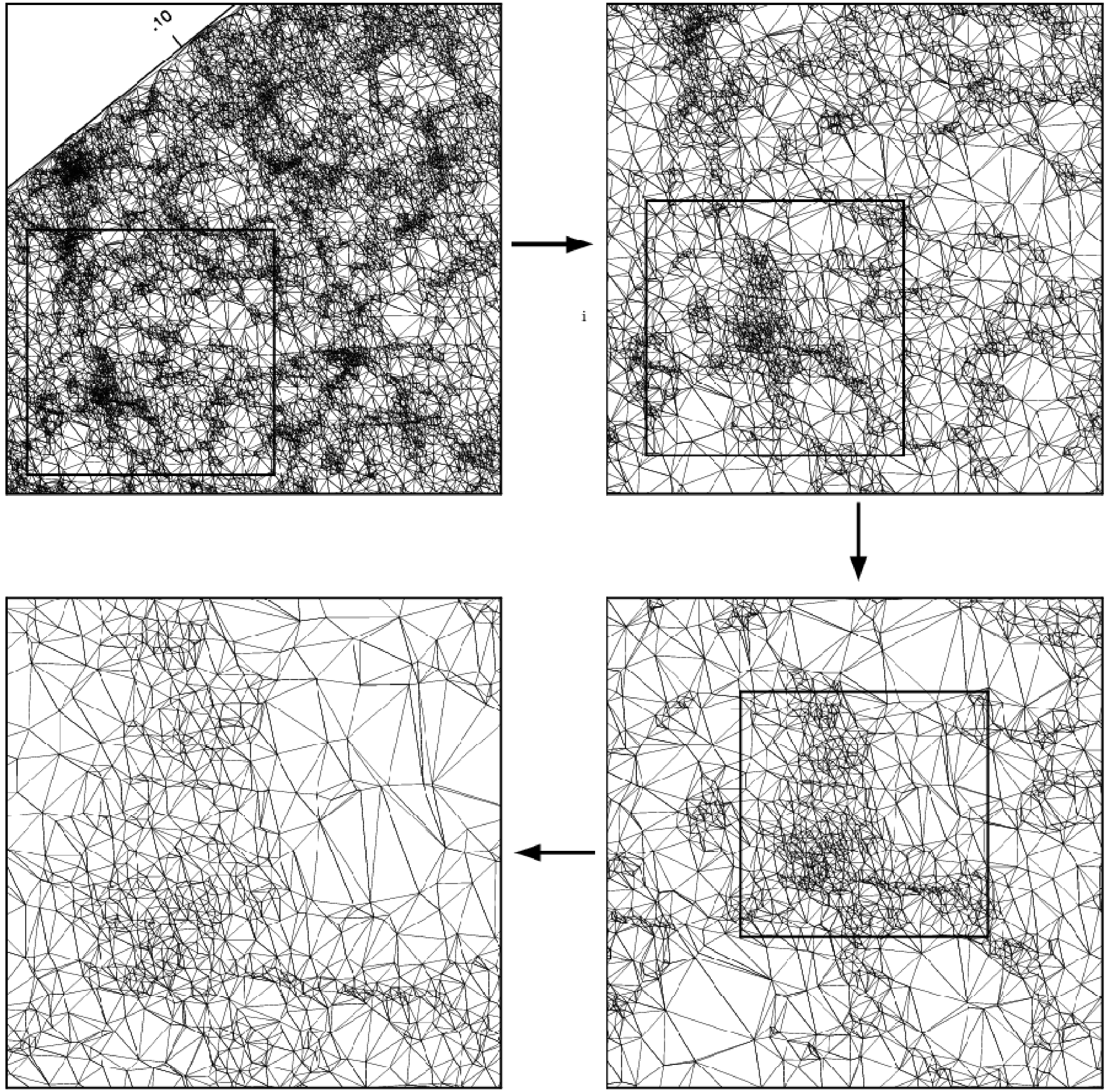
Delaunay

2dF Galaxy Redshift Survey



Delaunay

2dF Galaxy Redshift Survey



Delaunay Tessellation Field Estimator

Mass- vs. Volume-Weighted

Filtering

Volume- vs. Mass- Weighting

$$f_{volume}(\mathbf{x}) \equiv \frac{\int d\mathbf{y}\, f(\mathbf{y})\, W(\mathbf{x}-\mathbf{y})}{\int d\mathbf{y}\, W(\mathbf{x}-\mathbf{y})}$$

Volume-Weighted

Two different ways to filter a field f(x) with a filter W(x-y):

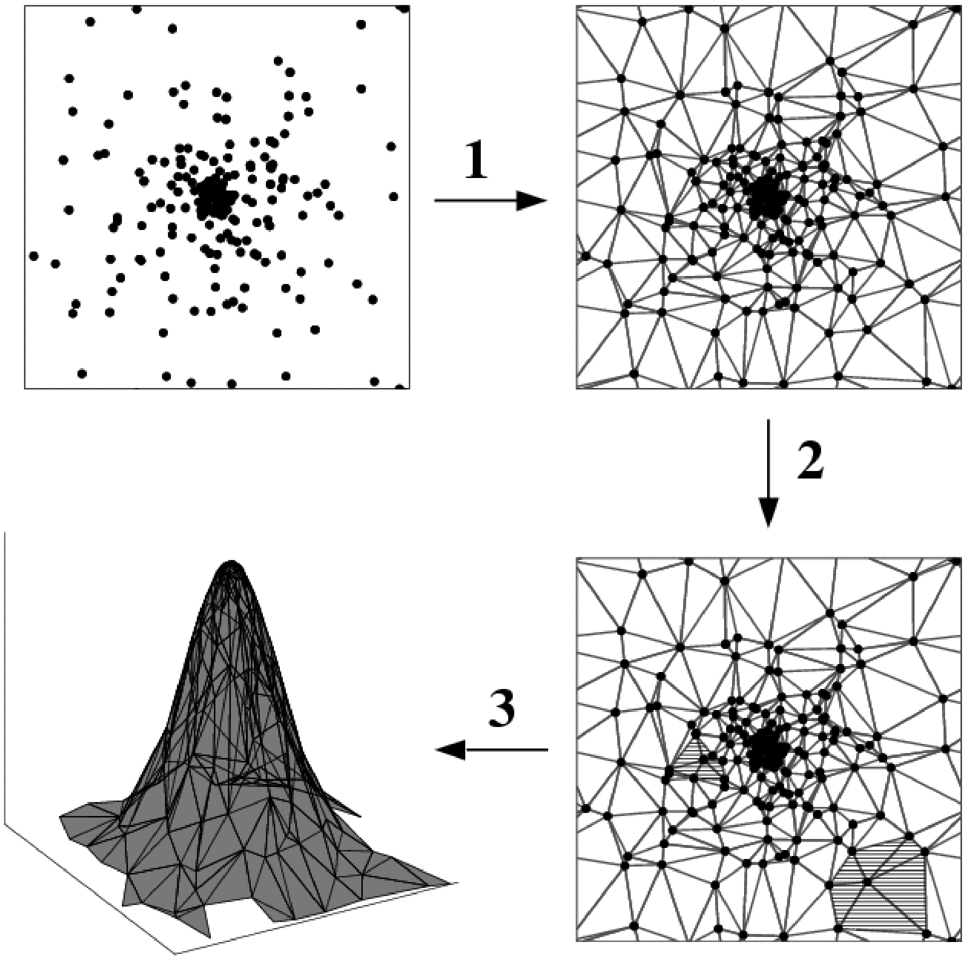
Mass-Weighted

$$f_{mass}(\mathbf{x}) \equiv \frac{\int d\mathbf{y}\, f(\mathbf{y})\, \rho(\mathbf{y})\, W(\mathbf{x}-\mathbf{y})}{\int d\mathbf{y}\, \rho(\mathbf{y})\, W(\mathbf{x}-\mathbf{y})}$$

DTFE Procedure

DTFE Procedure

DTFE reconstruction procedure:



Summary

I. Point Sampling
II. Construction Delaunay Tessellation
III. Determination Field Values
IV. Calculation Field Gradient in Delaunay cell
V. - Interpolation to locations x
   - Image construction through interpolation to ordered locations
VI. Processing field

DTFE Procedure

1. Point Sampling:

- Density Field Reconstruction:

  strict requirement of discrete sample to be a fair sample

  underlying continuous density field:

  Inhomogeneous Poisson Process

- Field Interpolation:

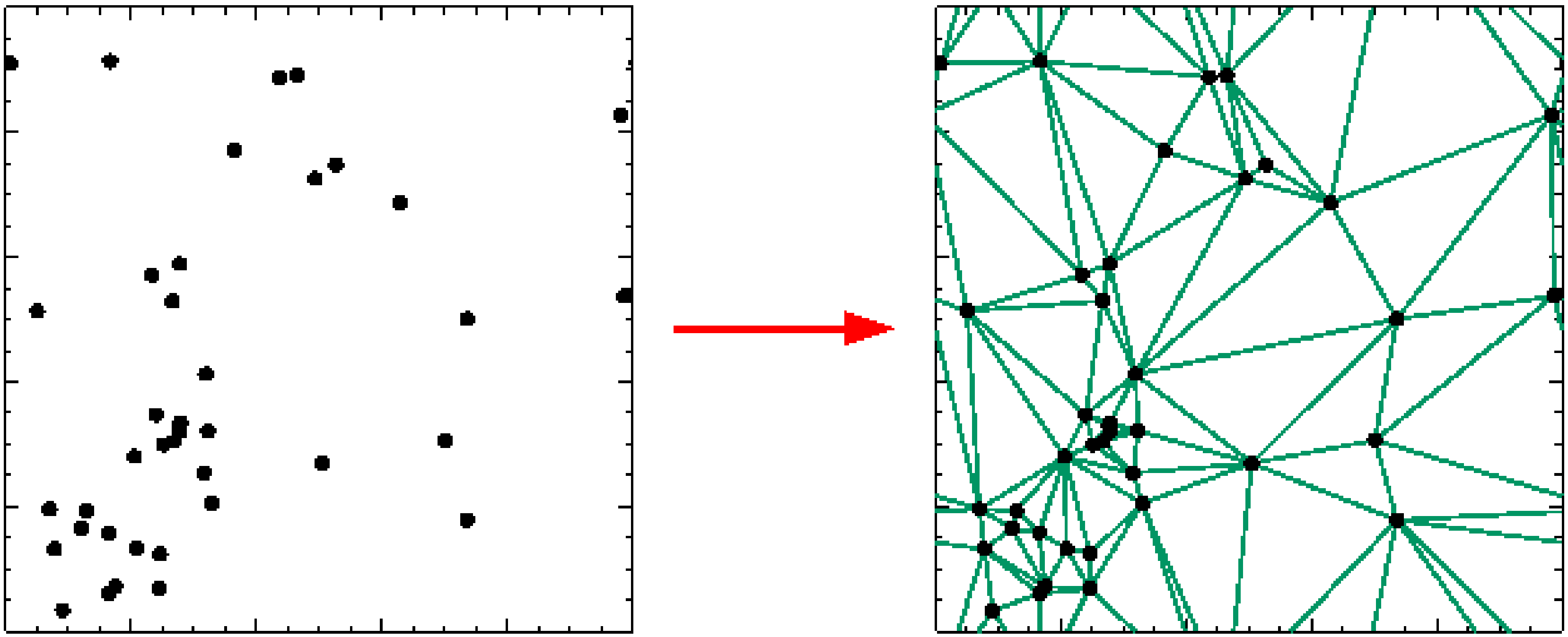
  Interpolation well-defined and sensible

  not on scales over which "orbit-crossing" occurred

DTFE Procedure

II. Delaunay Tessellation:

- construct Delaunay tessellation point distribution



Delaunay Tetrahedron: Set of 4 nuclei, circumscribing sphere not containing any of the other nuclei

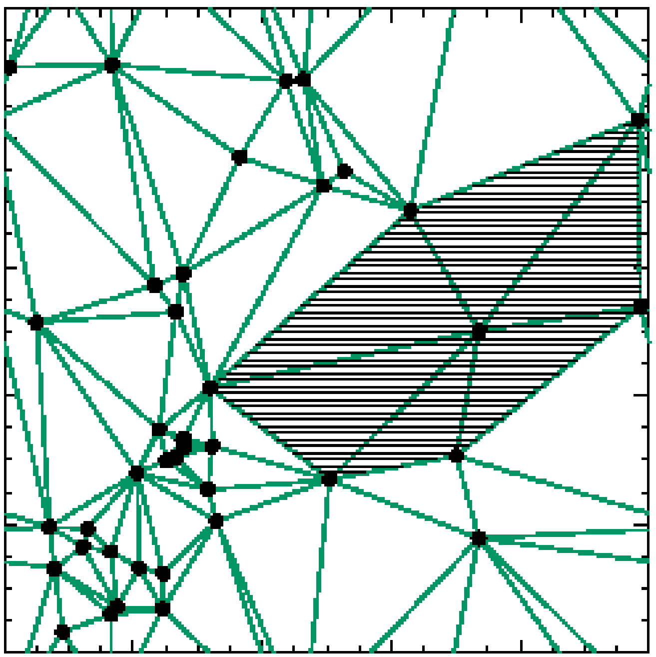
DTFE Procedure

II. Field values at point locations:

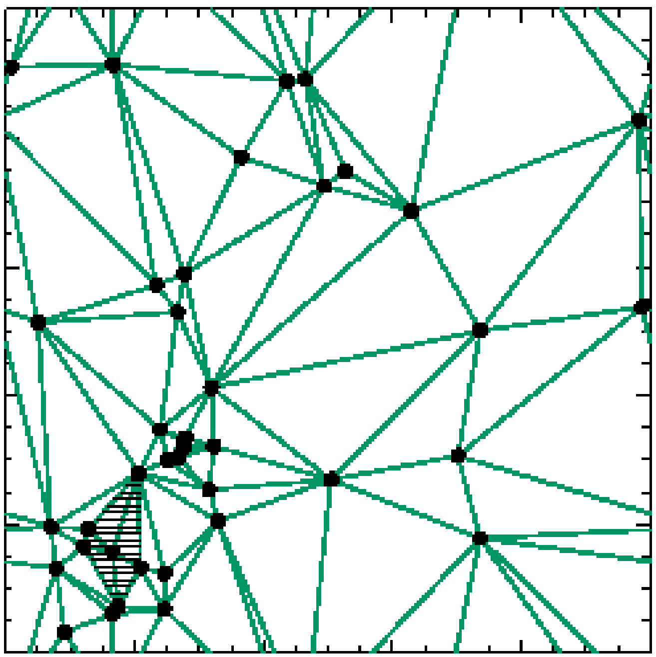
- "measured" field: field values f_i at point locations i
- density field: inverse volume

contiguous Voronoi cell

$$\hat{\rho}_i = \frac{C_D}{V(\mathcal{W}_i)}$$

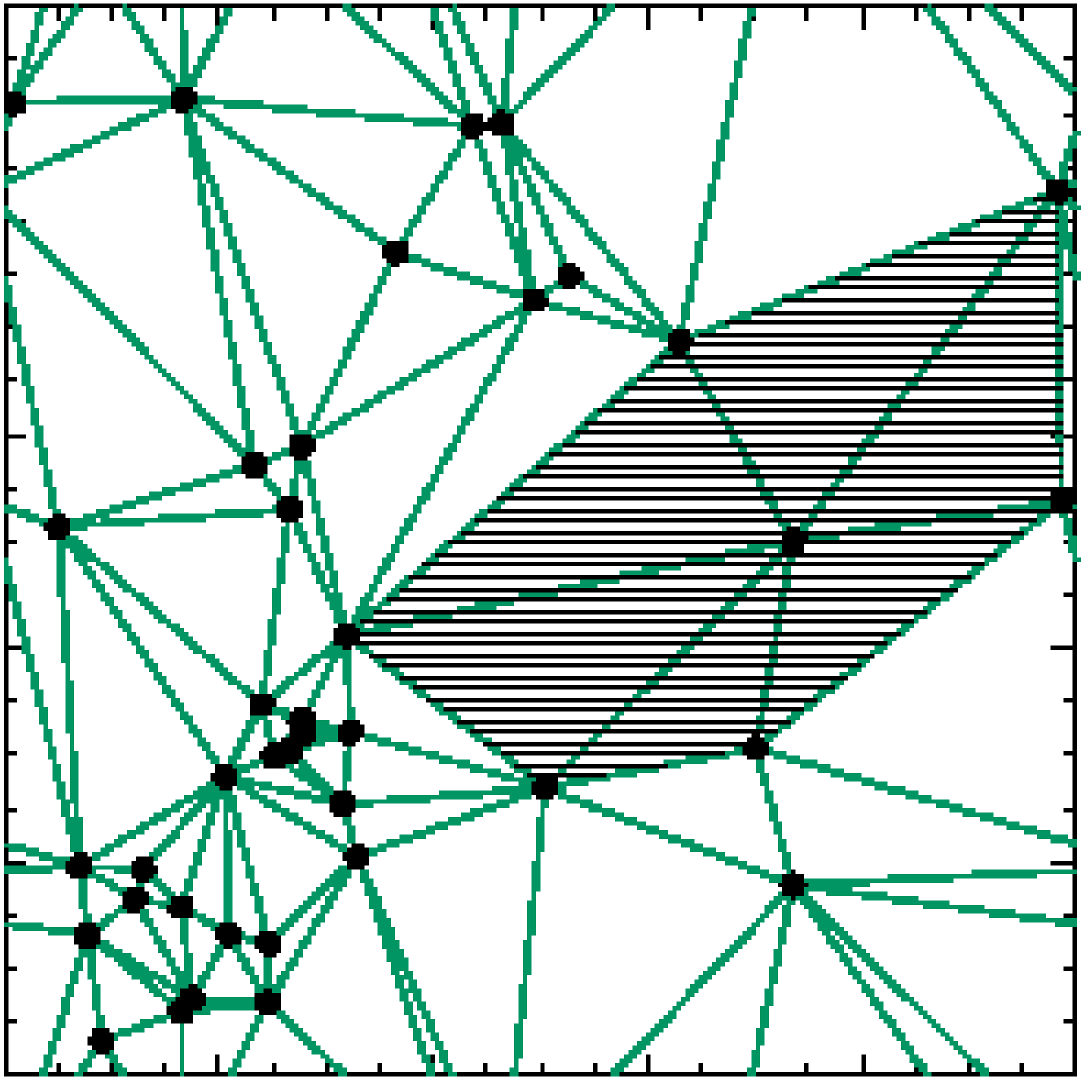


low density: large volume



high density: small volume

DTFE Procedure



Contiguous Voronoi Cell

For nucleus i:

the union of all Delaunay tetrahedra $T_{i,j}$ of which the nucleus i is one of the 4 vertices.

$$\mathcal{W}_i \equiv \cup_j \mathcal{T}_{i,j}$$

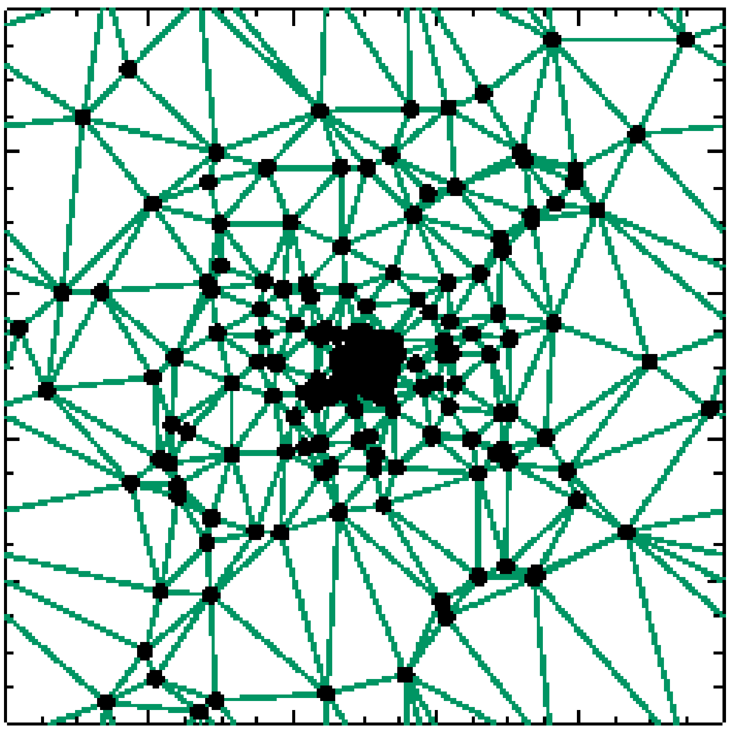
$$\Downarrow$$

$$V(\mathcal{W}_i) = \sum_{j=1}^{N_{\mathcal{T},i}} V(\mathcal{T}_{i,j})$$

DTFE Procedure

IV: Computation Field Gradients

- in each Delaunay cell compute field gradient from (1+D) field values at its (1+D) vertices.



$$\hat{f}(\mathbf{x_1}) = f(\mathbf{x}_0) + \widehat{\nabla f}|_j \cdot (\mathbf{x_1} - \mathbf{x}_0)$$
$$\vdots$$
$$\hat{f}(\mathbf{x_D}) = f(\mathbf{x}_0) + \widehat{\nabla f}|_j \cdot (\mathbf{x_D} - \mathbf{x}_0)$$

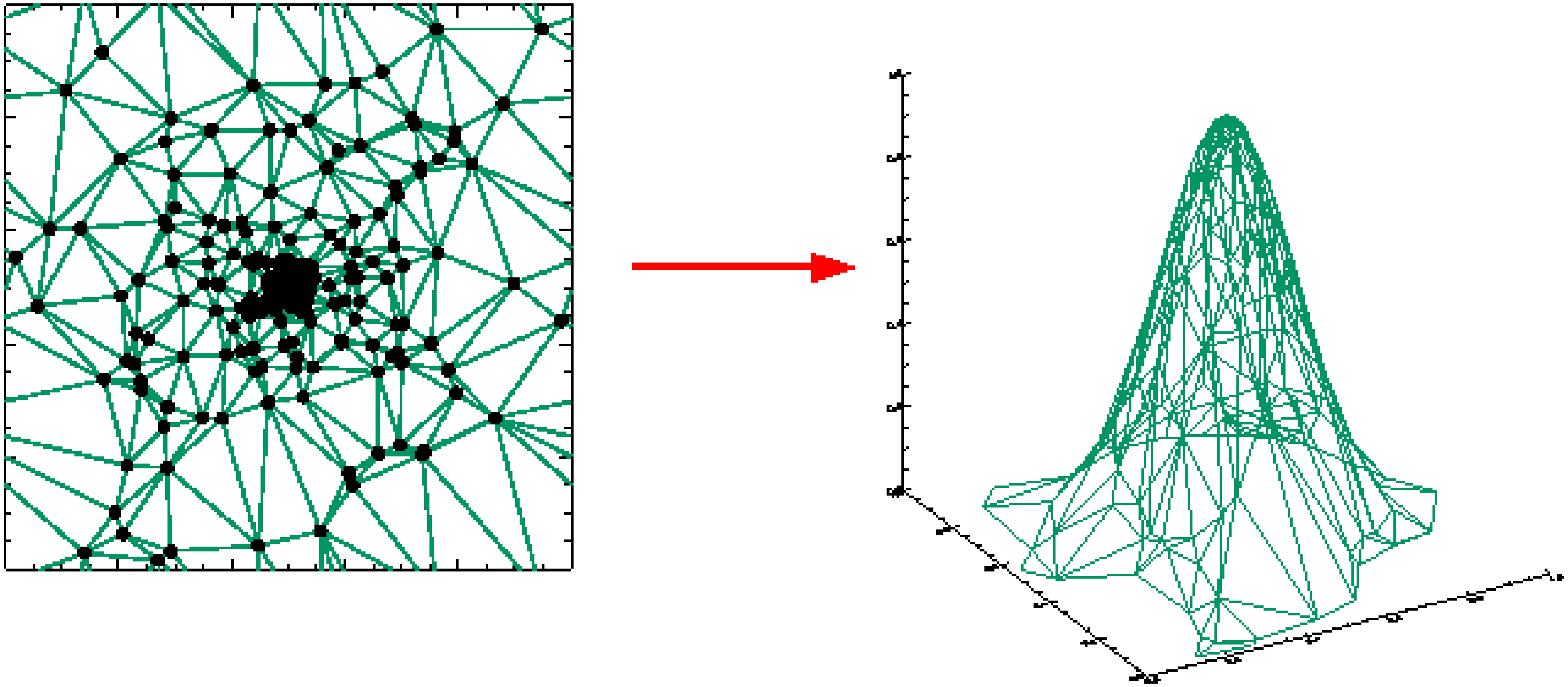
$$\widehat{\nabla f}|_j$$

DTFE Procedure

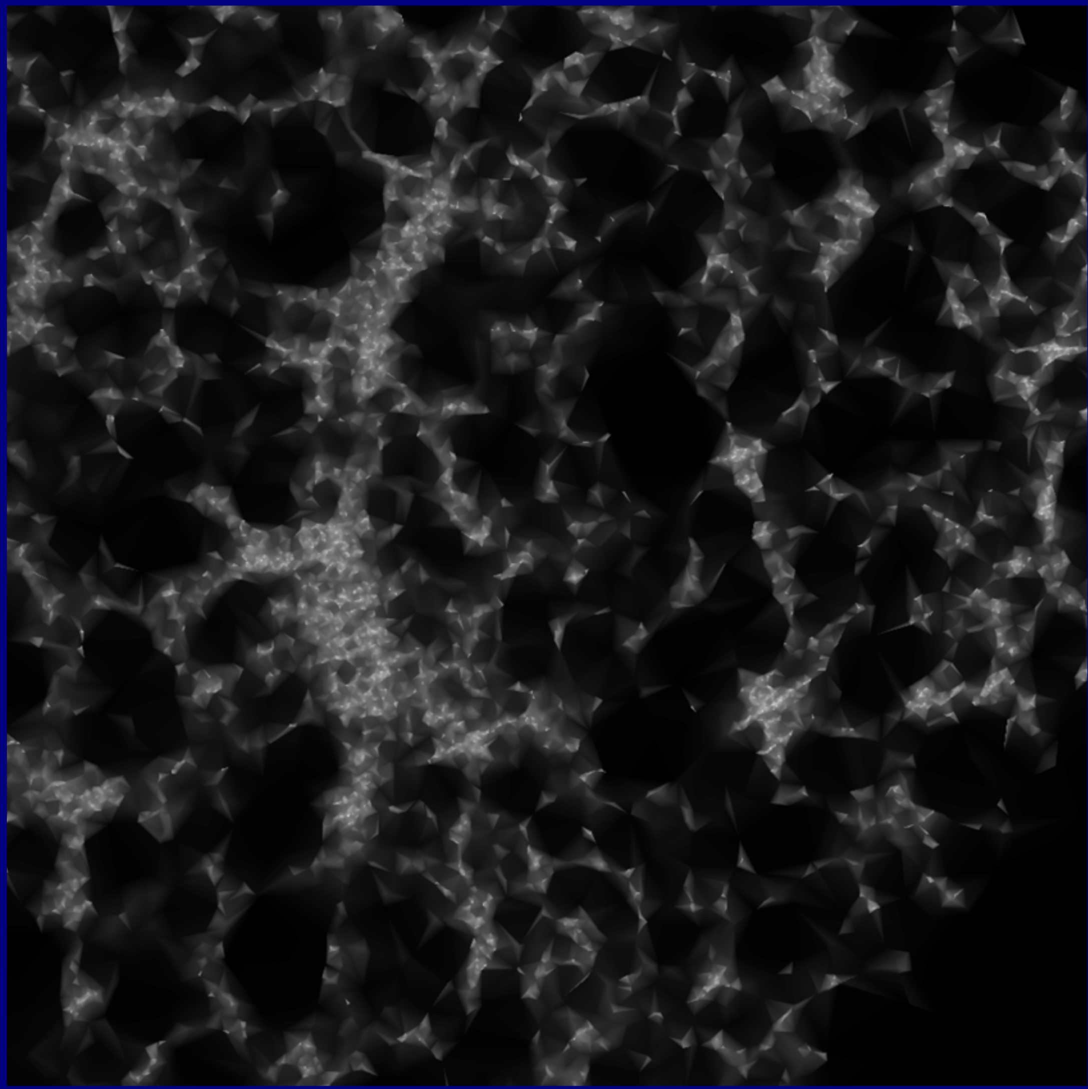
V: Linear Field Interpolation:

- determination field values at each location by linear interpolation

$$\hat{f}(\mathbf{x}) = f(\mathbf{x}_0) + \widehat{\nabla f}|_j \cdot (\mathbf{x} - \mathbf{x}_0)$$



DTFE Procedure

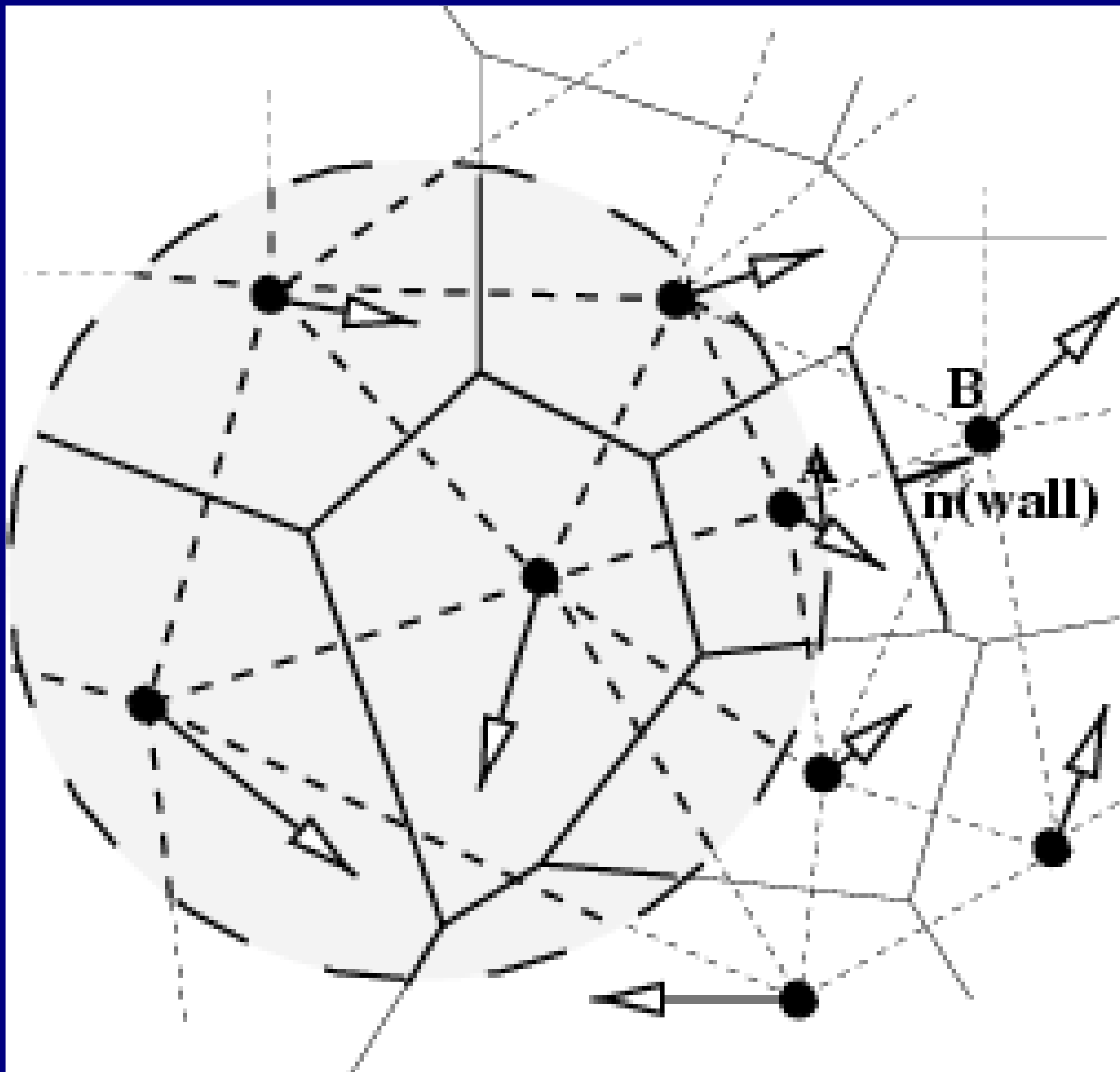


VIa. Processing

- Images, Maps ...

...

DTFE Procedure

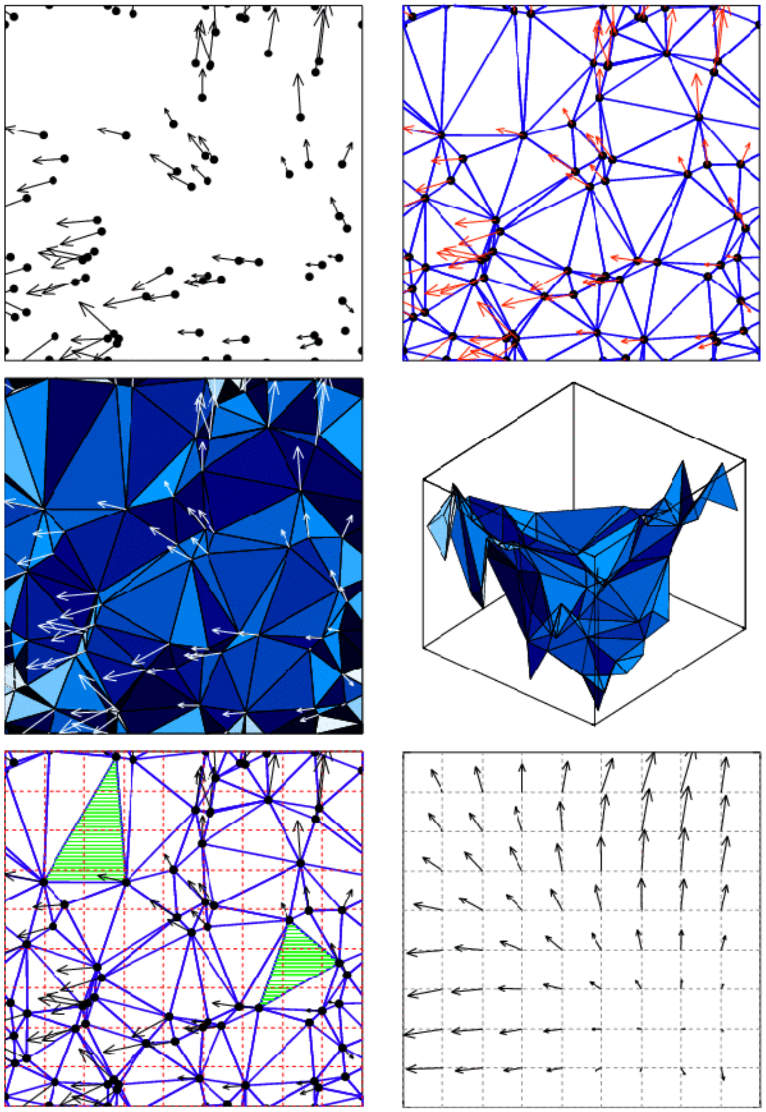


VIb. Processing

- filtering,
- (auto)correlations
- geometric &
  topological measures
  (e.g. SURFGEN)

...

DTFE Procedure



Process Example

velocity field

From discrete velocity measurement to continuous velocity field.

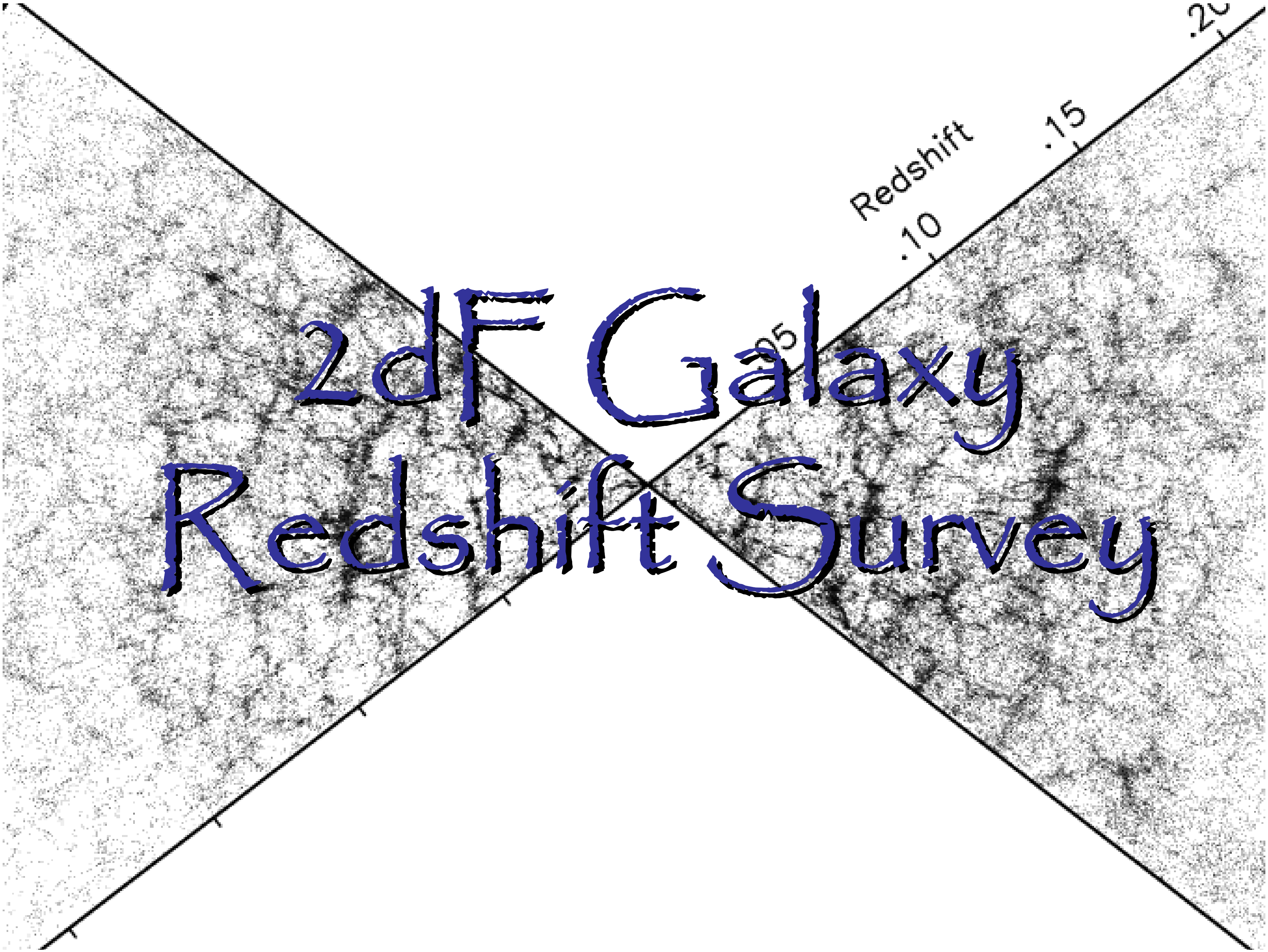
Delaunay Tessellation Field Estimator

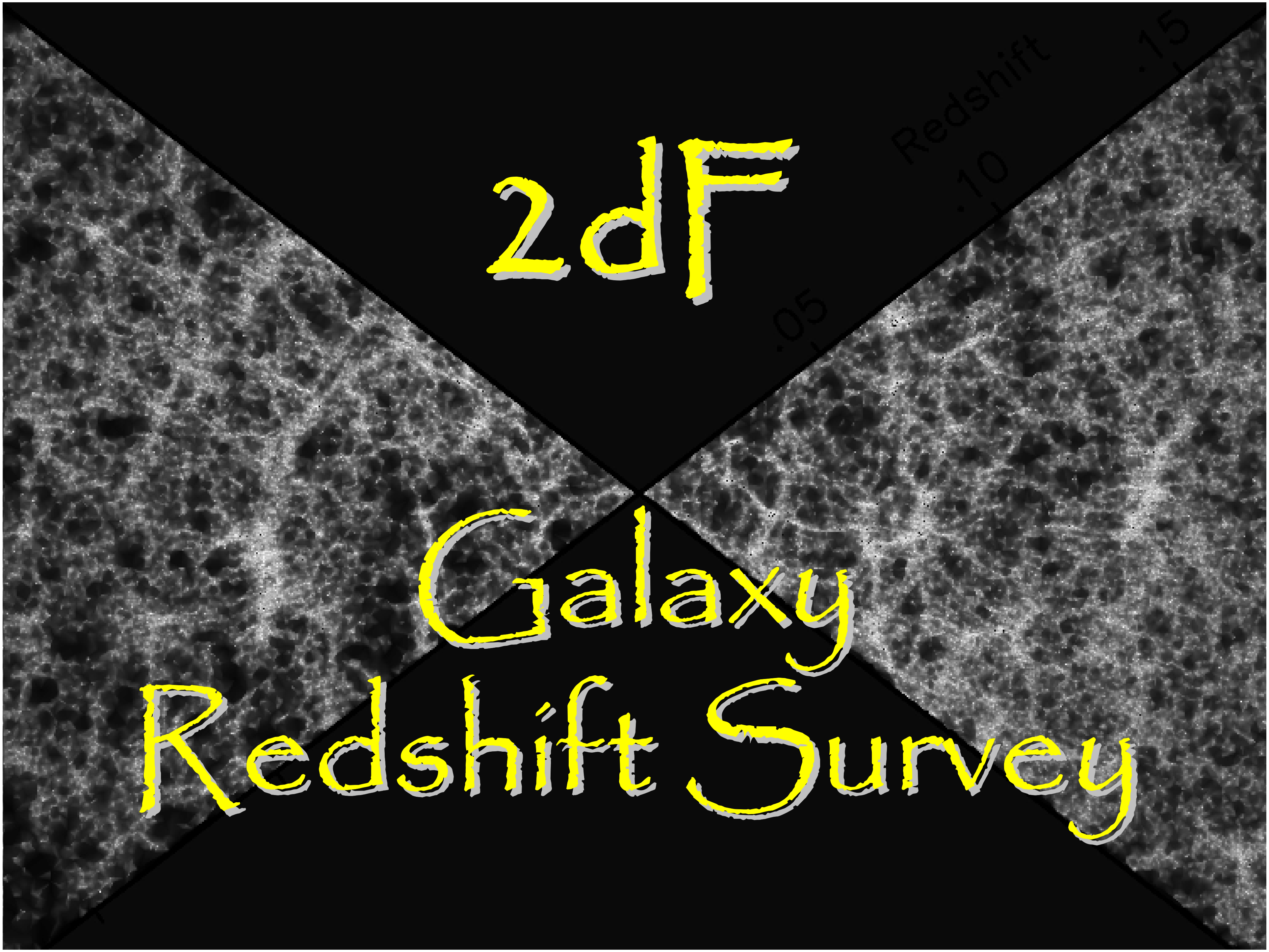
Example: The Cosmic Foam

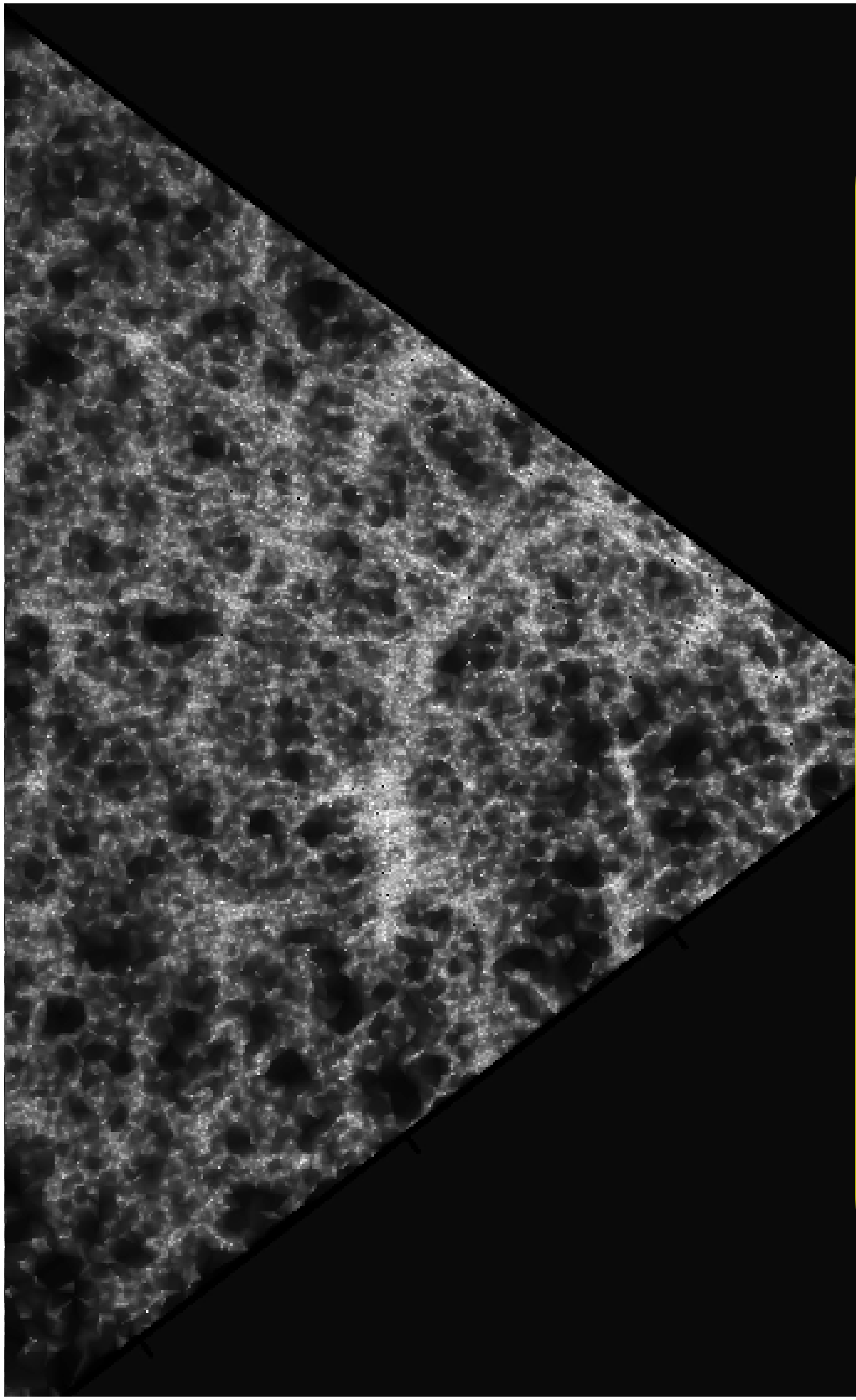
the Cosmic Foam
&
Spatial Patterns

Traced by
Galaxy Redshifts

2dF Galaxy Redshift Survey




2dF
Galaxy
Redshift Survey
Redshift
.05
.10
.15



Spatial Galaxy Distribution

galaxies assembled in a Foamlike Network

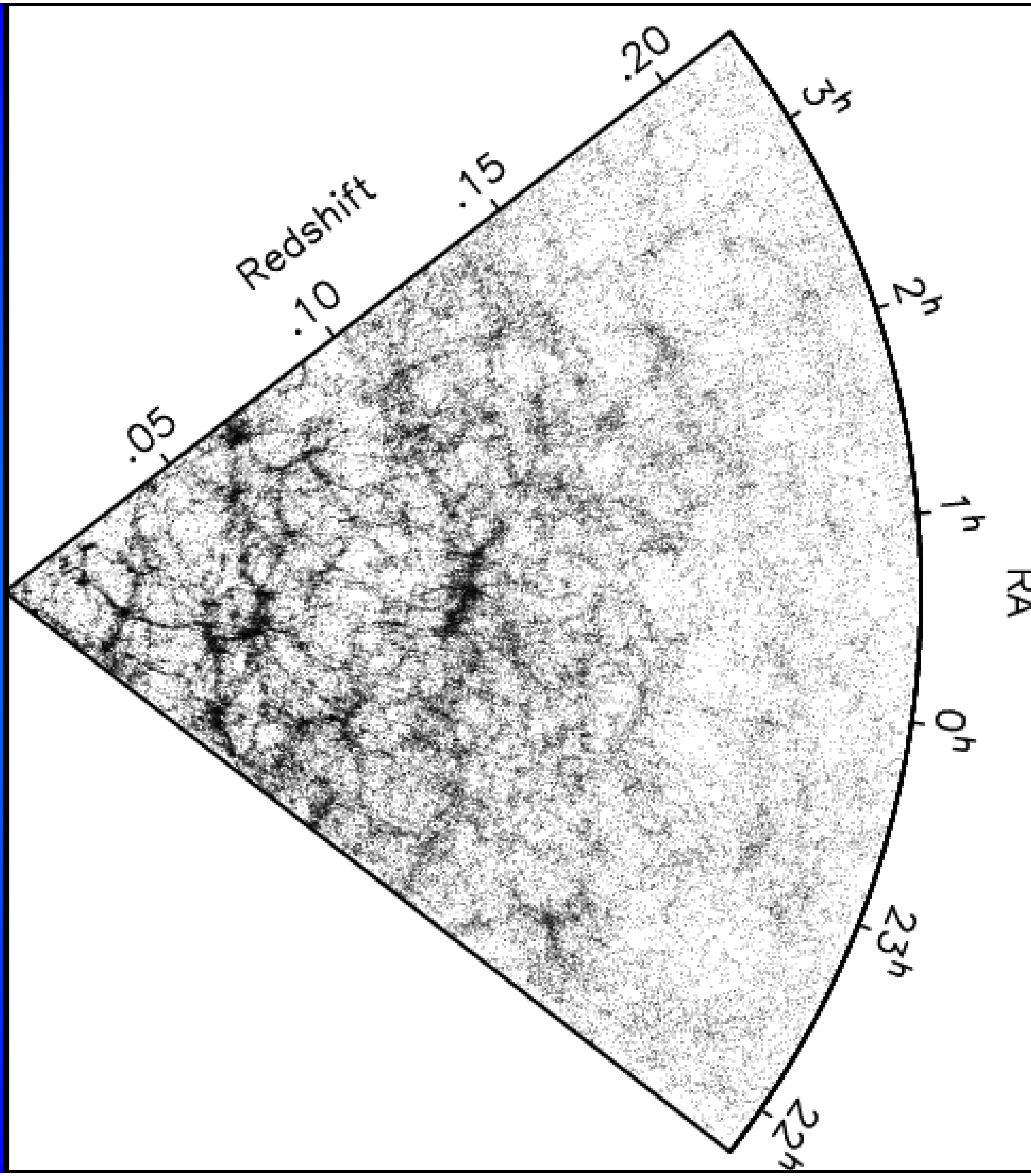
Foam Elements:

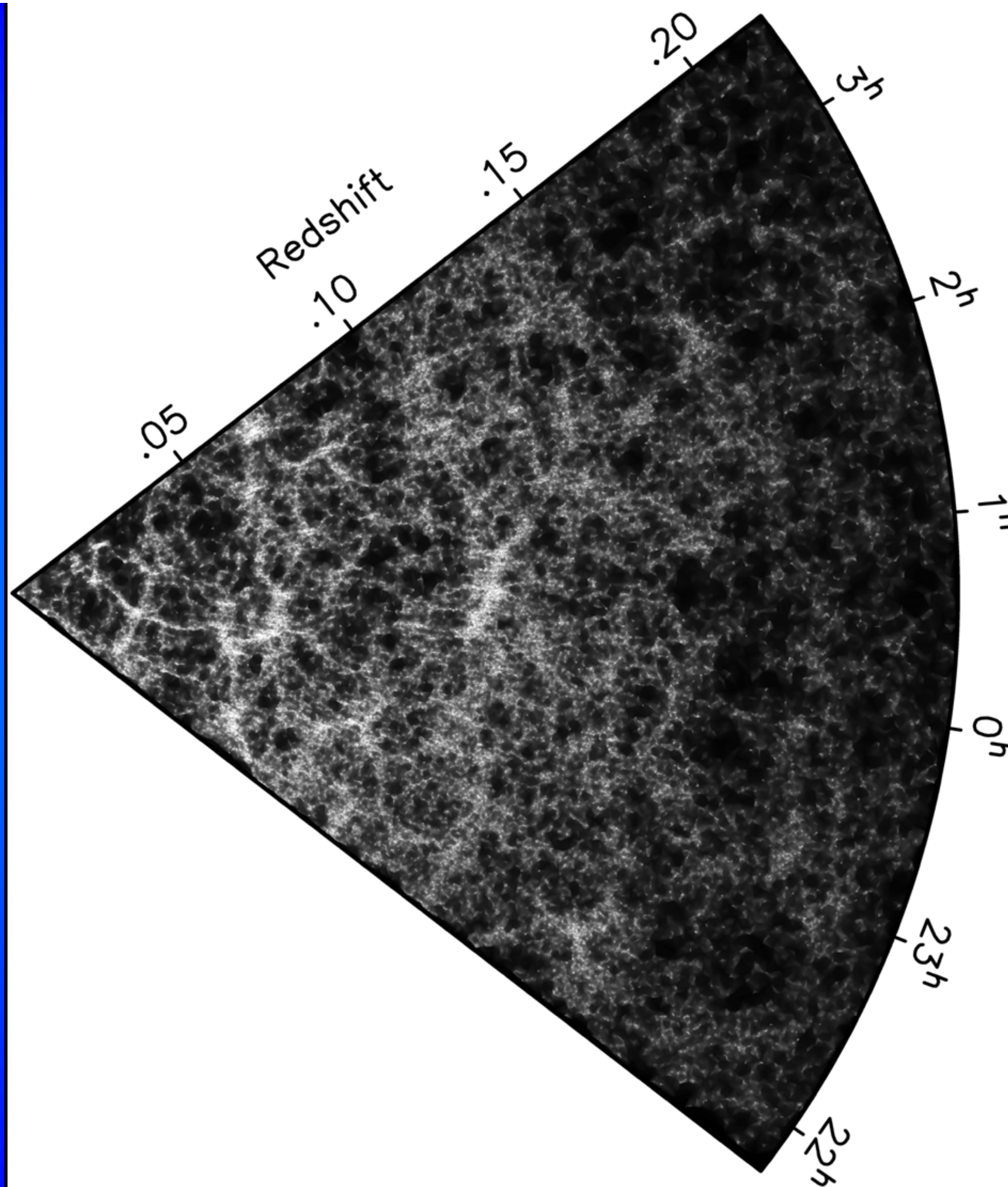
+ Walls
+ Filaments
+ Clusters
+ Voids

Spatial Organization:

Non-trivial stochastic pattern of filaments and walls - filaments marking the dense ridges of the walls – which join up at high-density junctions, sites of rich clusters.

Walls, filaments and clusters framed in spatial network in which their spatial distribution is dictated by their location on the surface of roundish and near-empty voids.


Redshift
.05
.10
.15
.20
RA
3h
2h
1h
0h
23h
22h


Redshift
.05
.10
.15
.20
3h
2h
1h
0h
23h
22h

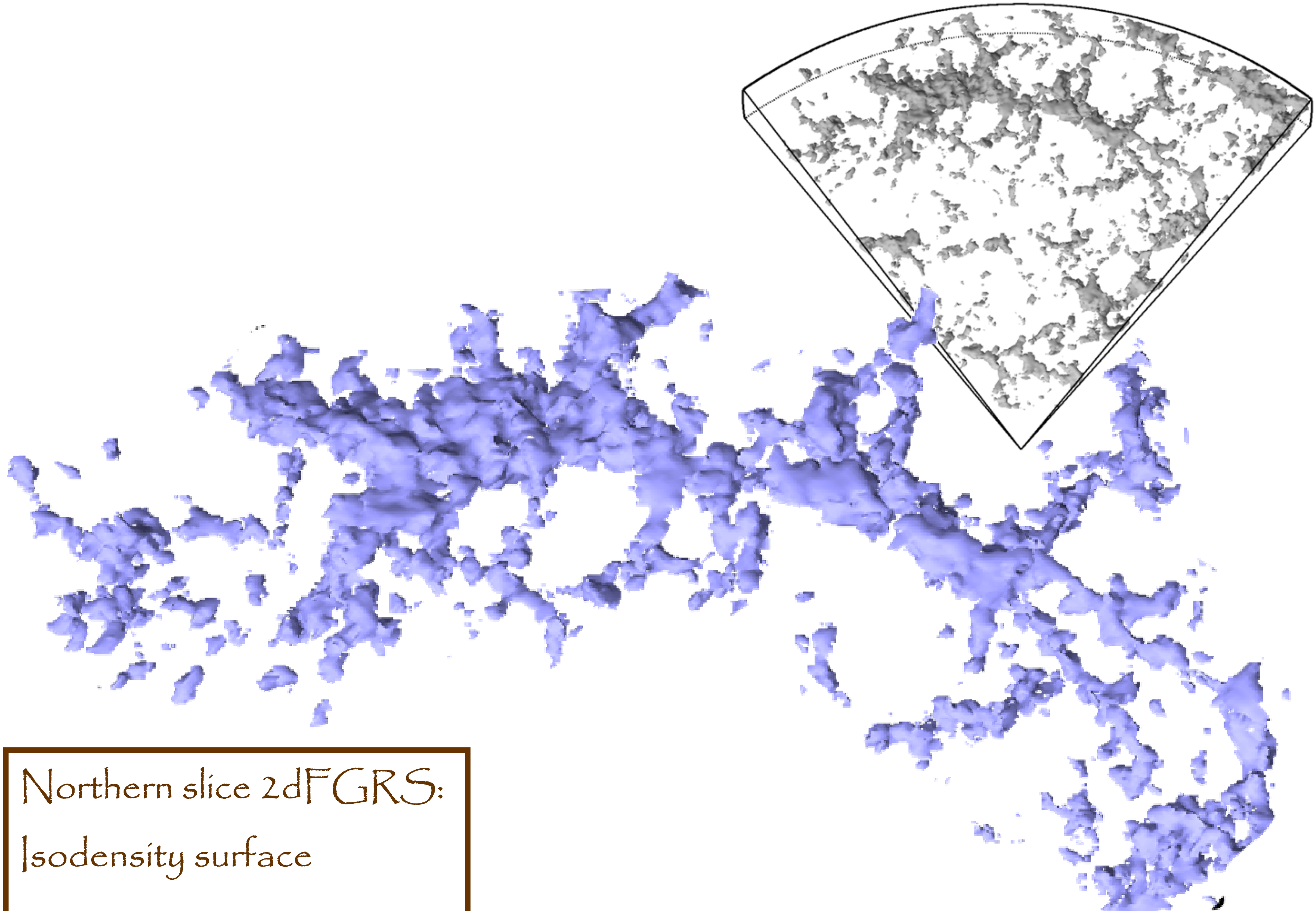
Southern slice 2dFGRS:

Isodensity surfaces reveal

Intricate 3-D topology

Northern slice 2dFGRS:

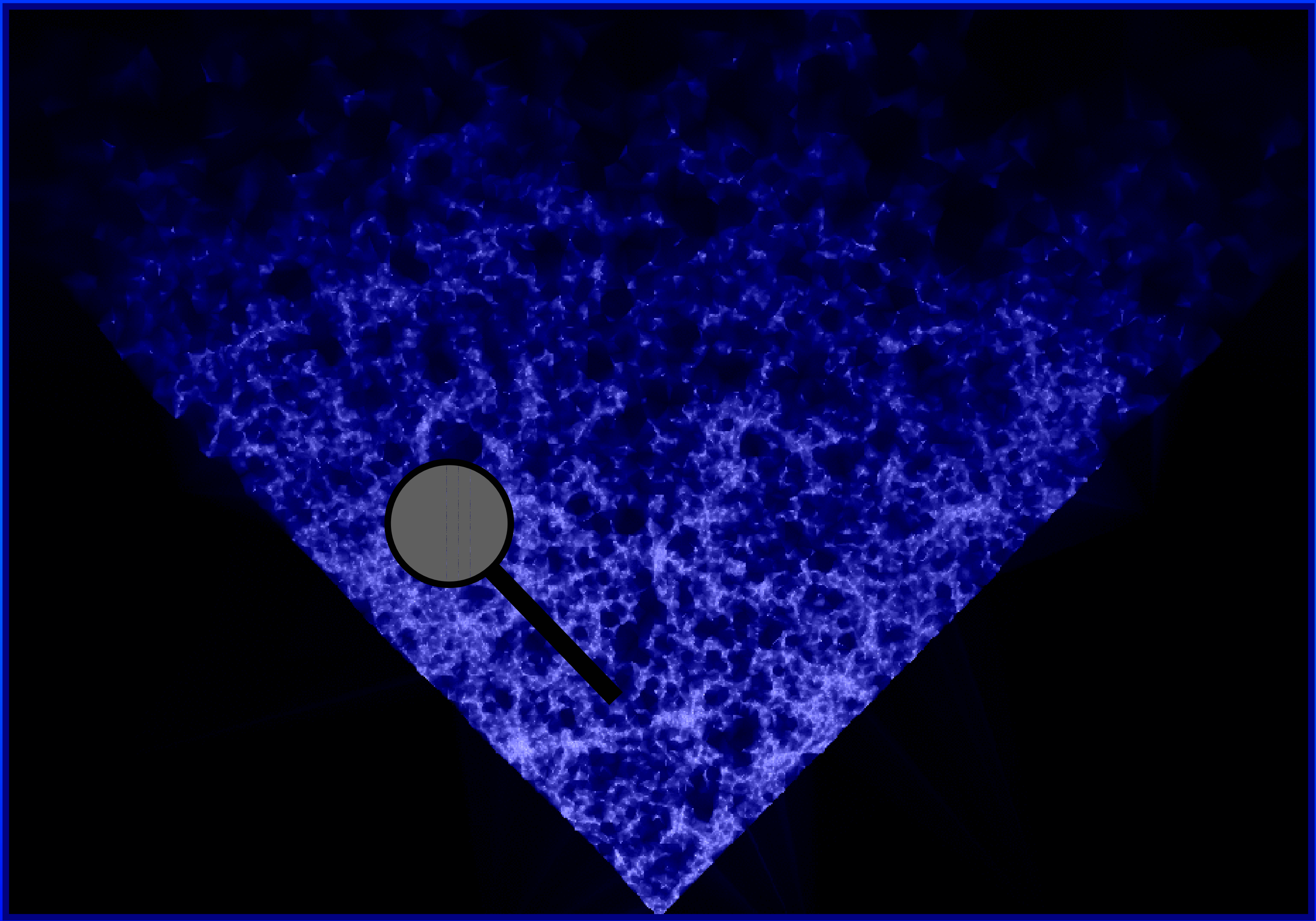
Isodensity surface



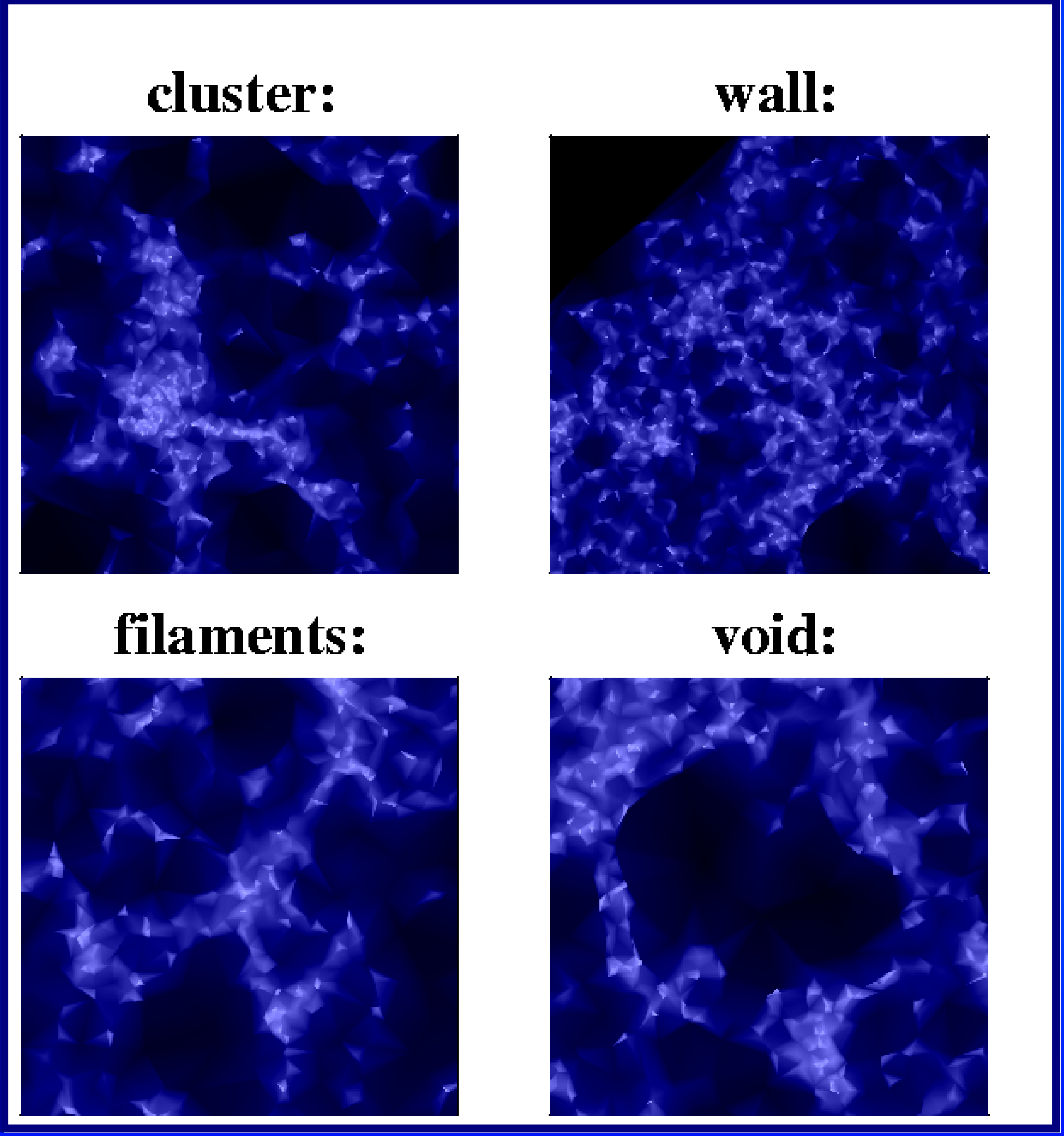
Northern slice 2dFGRS:

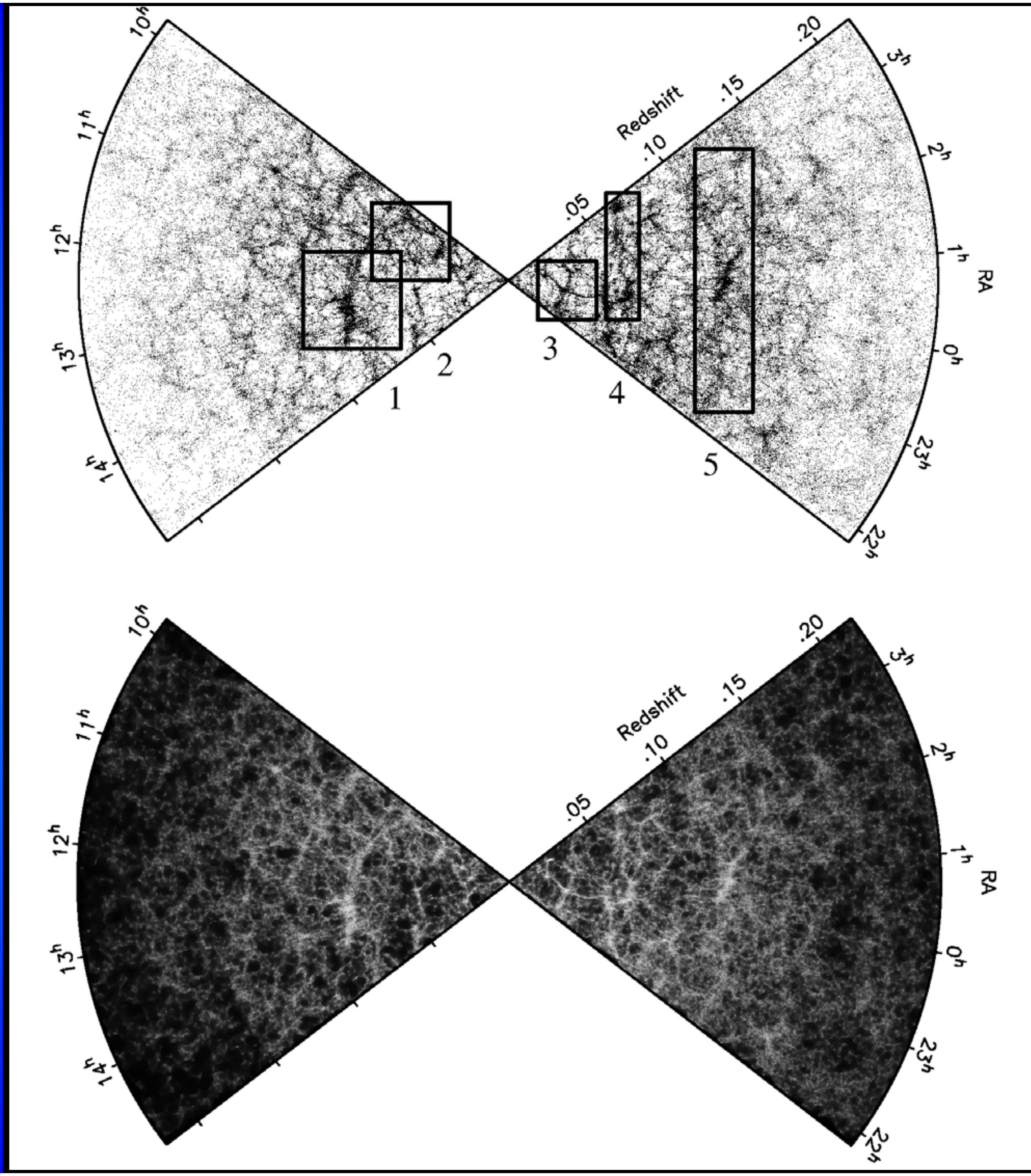
Isodensity surface

Structure Identification

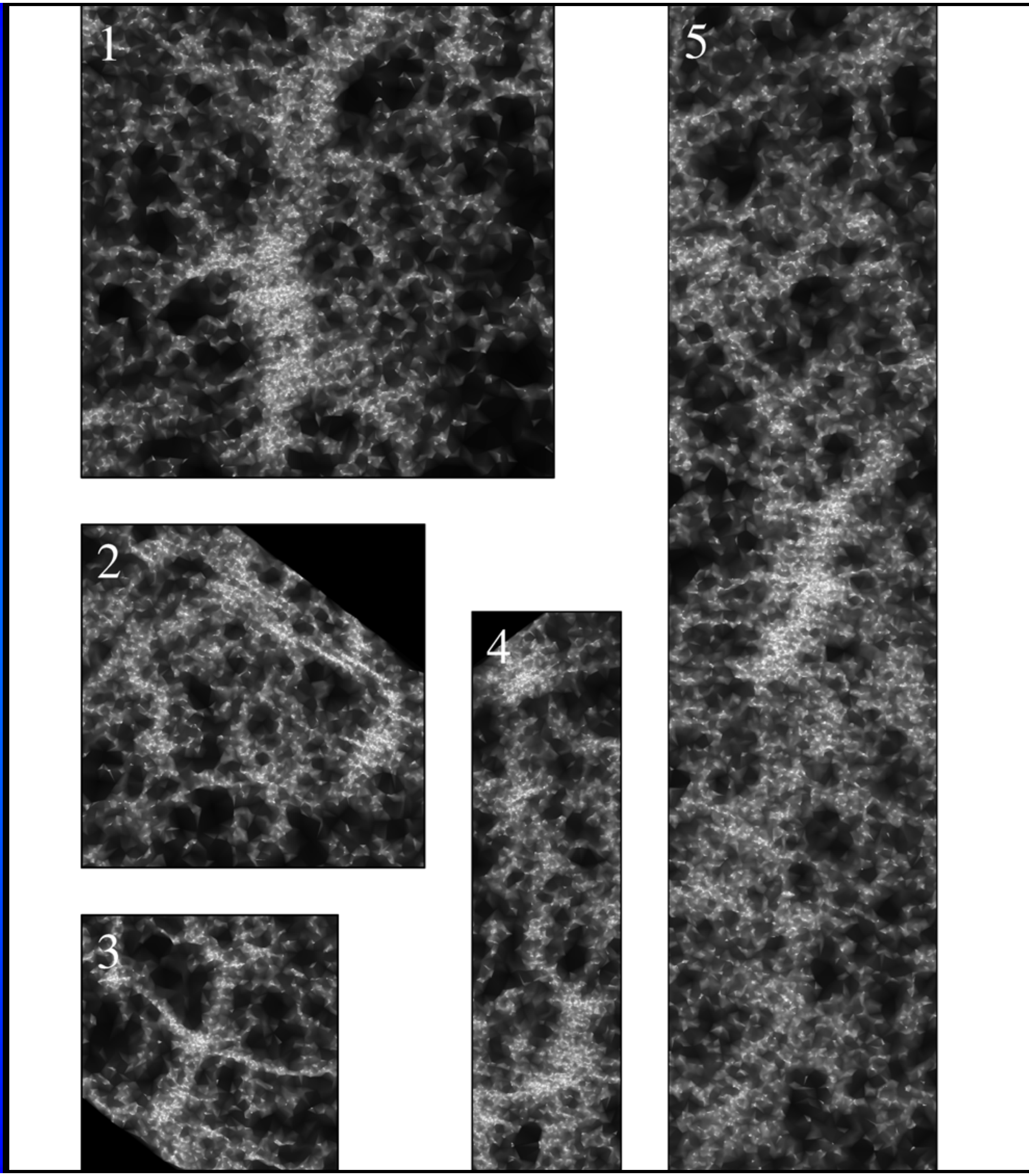


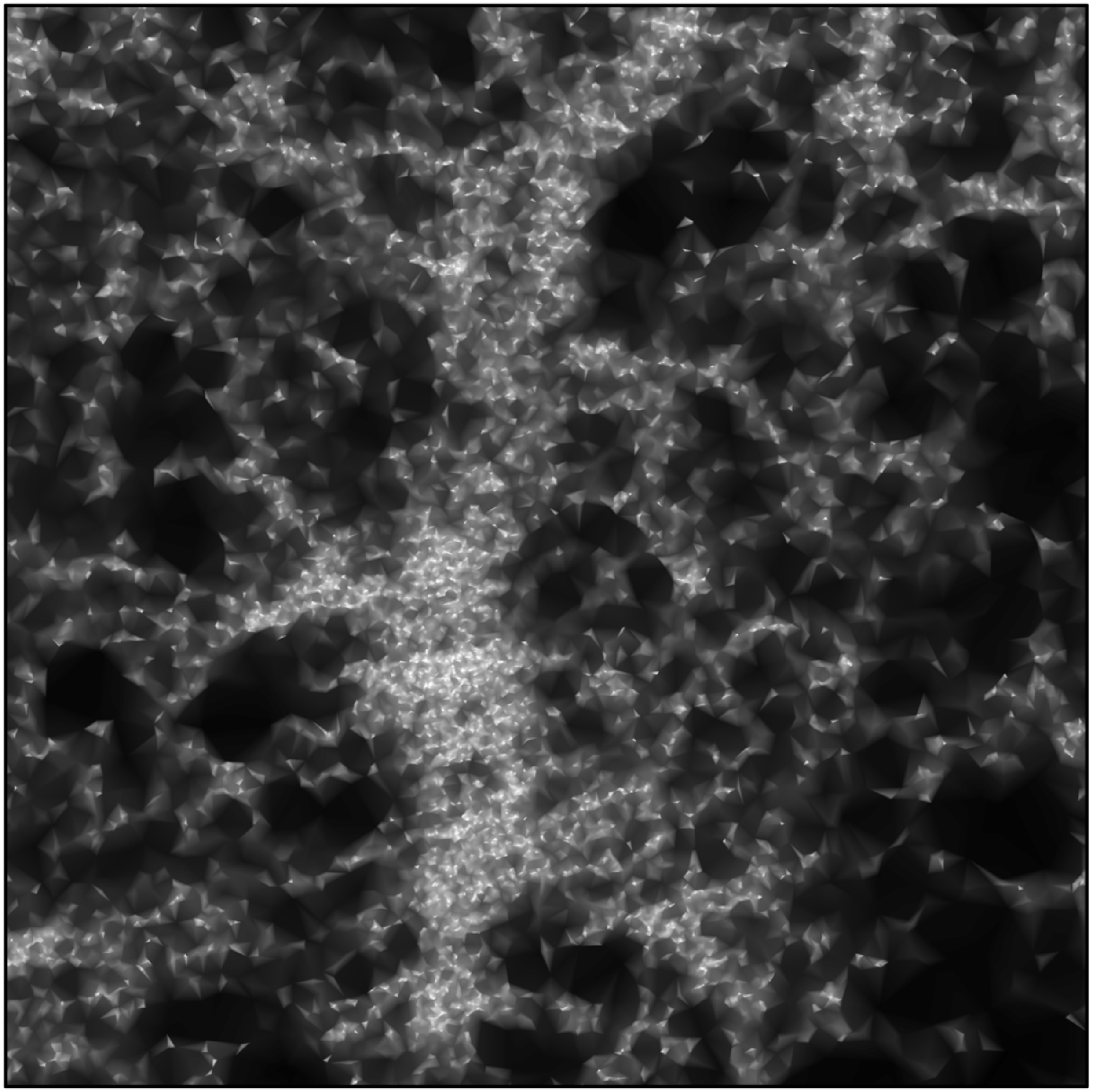
Structure Identification

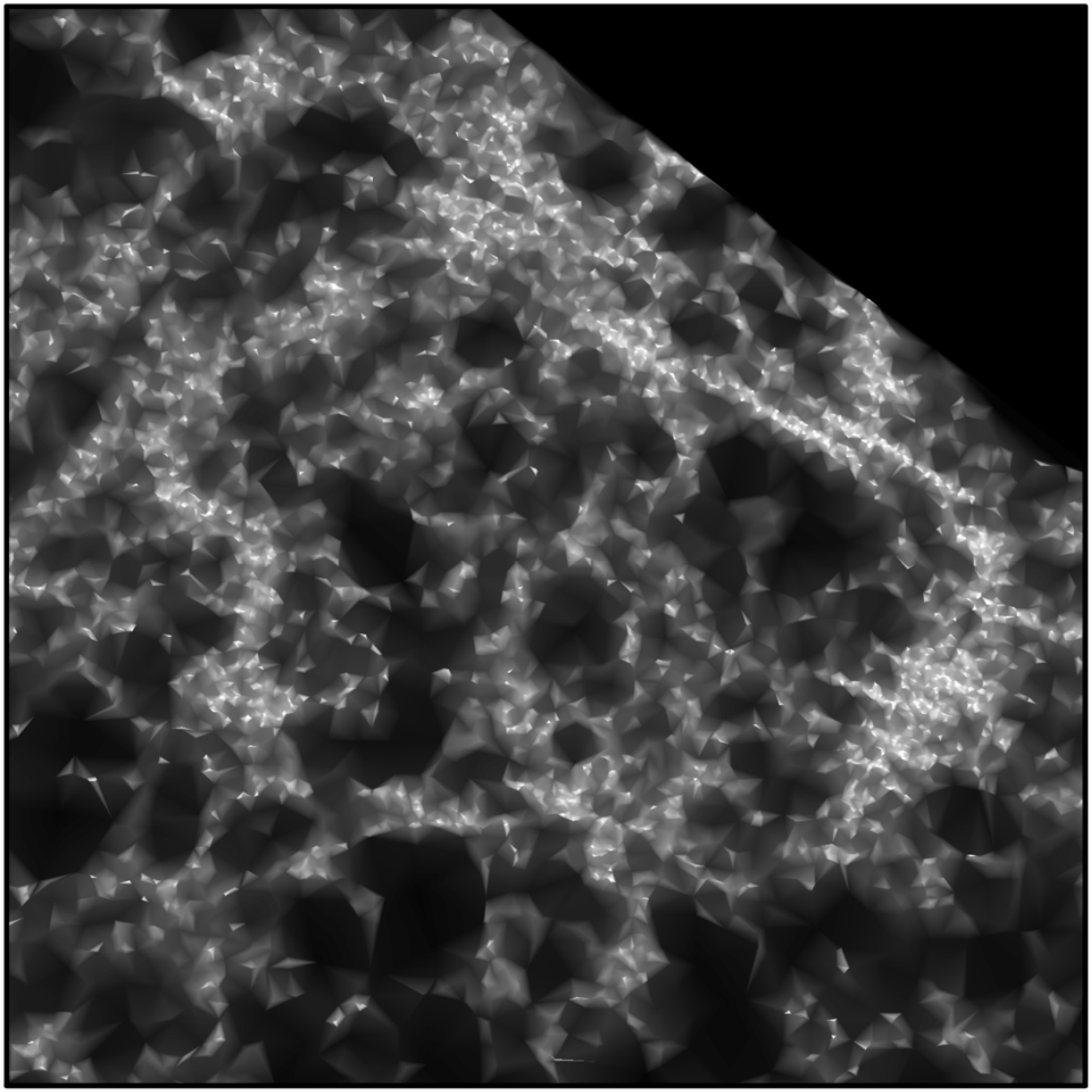


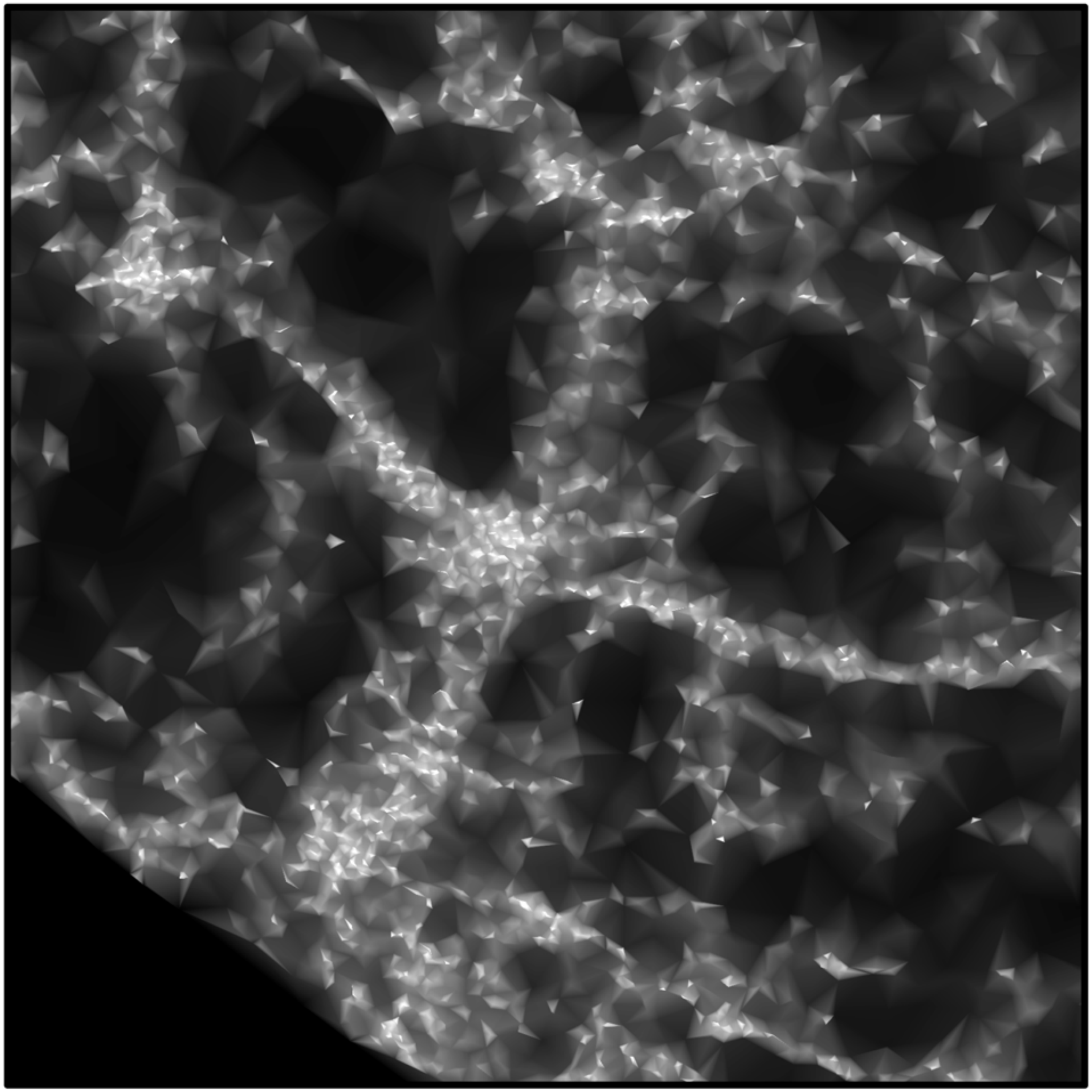


10h
11h
12h
13h
14h
Redshift
.05
.10
.15
.20
3h
2h
1h
0h
23h
22h
RA
1
2
3
4
5


1
2
3
4
5







Delaunay Tessellation Field Estimator

DTFE Characteristics

DTFE Characteristics

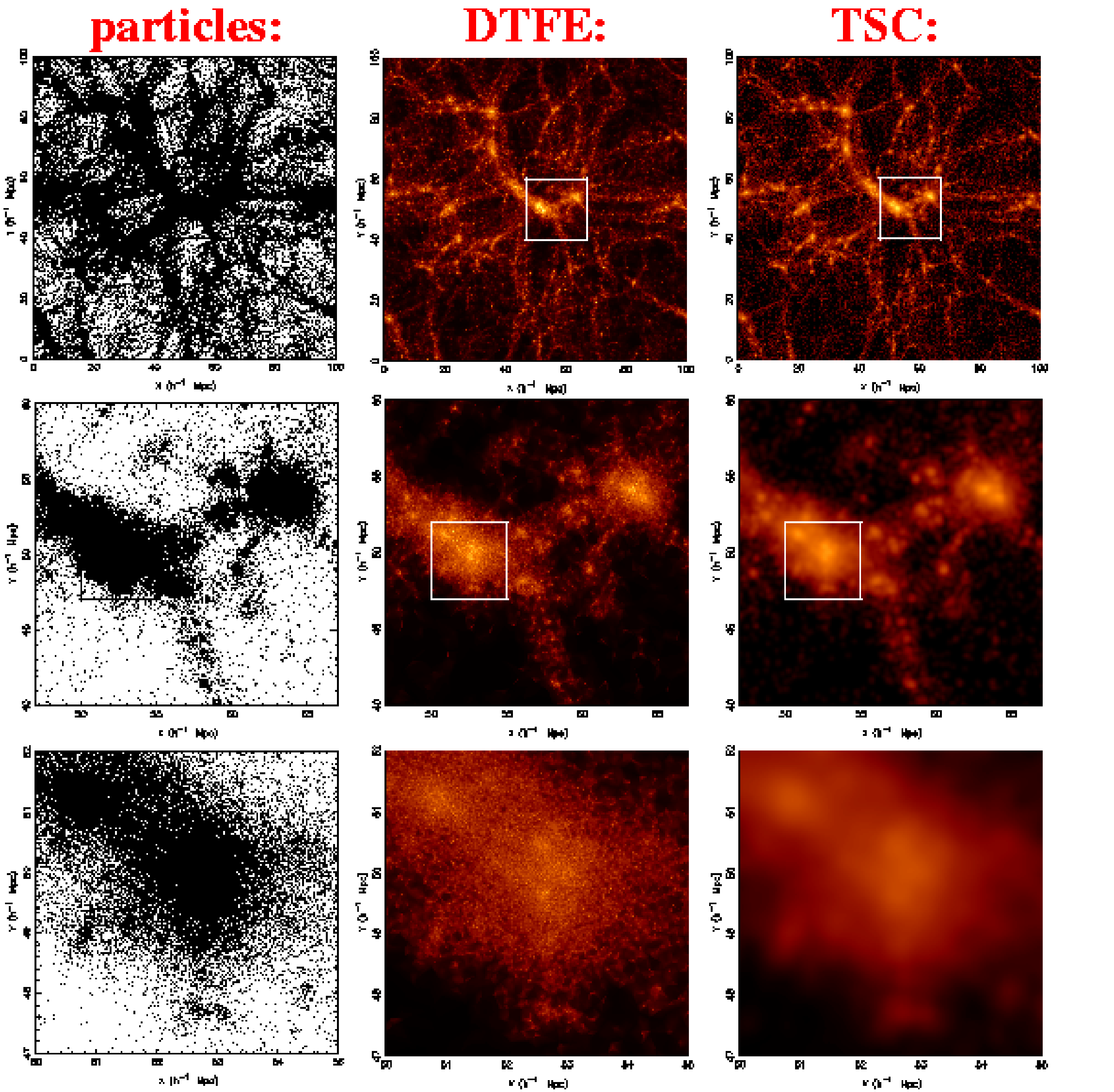
1. Spatial Resolution,
   Dynamic Range & Substructure
2. Suppression Shot-noise
3. Anisotropic Patterns
4. Structural Hierarchy & Scaling
5. Field Universality

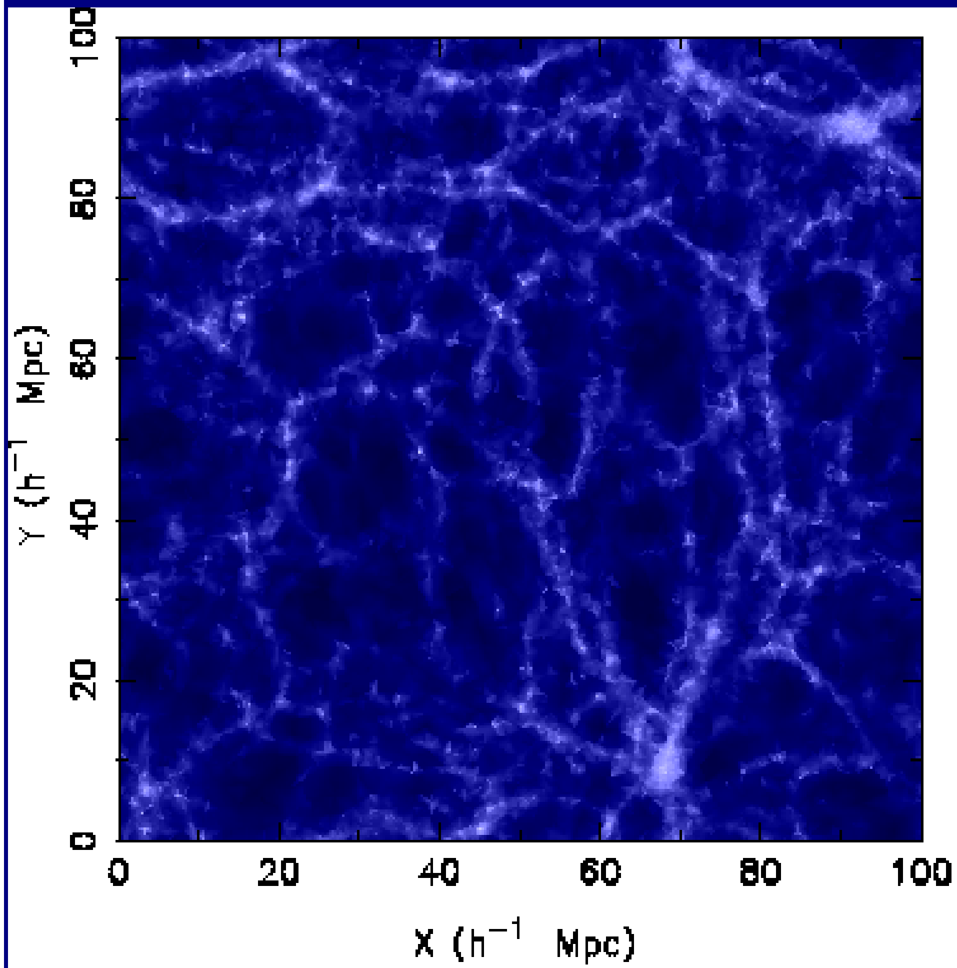
DTFE Characteristics

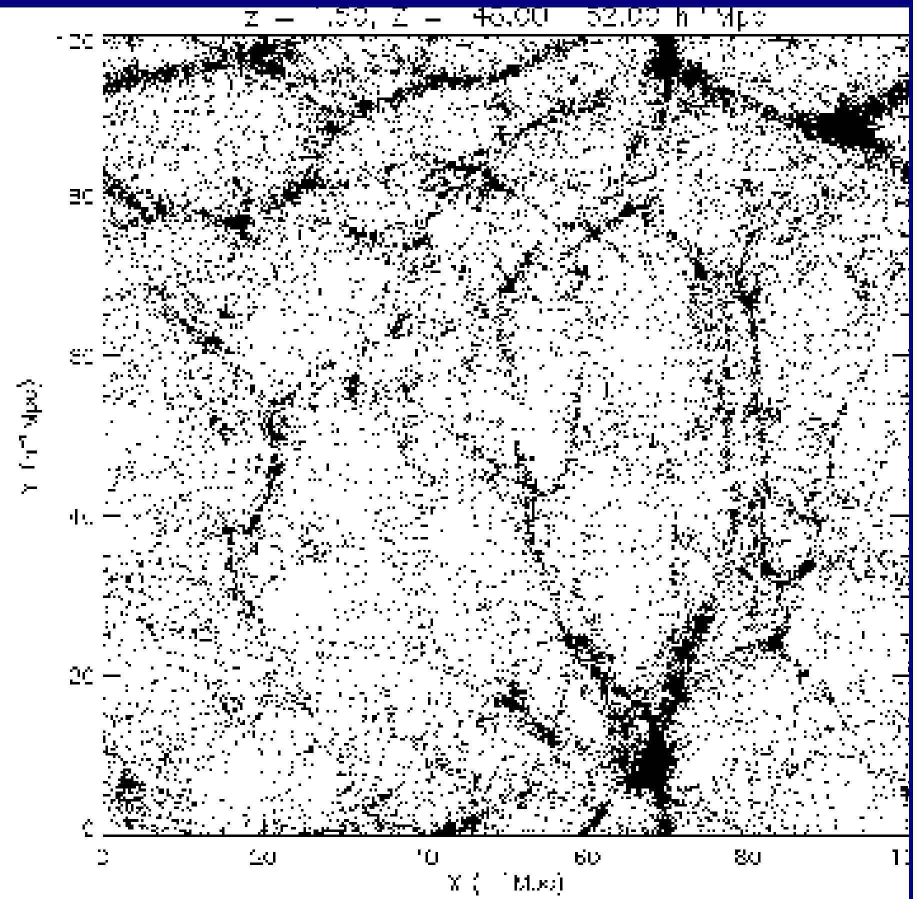
1.

Spatial Resolution

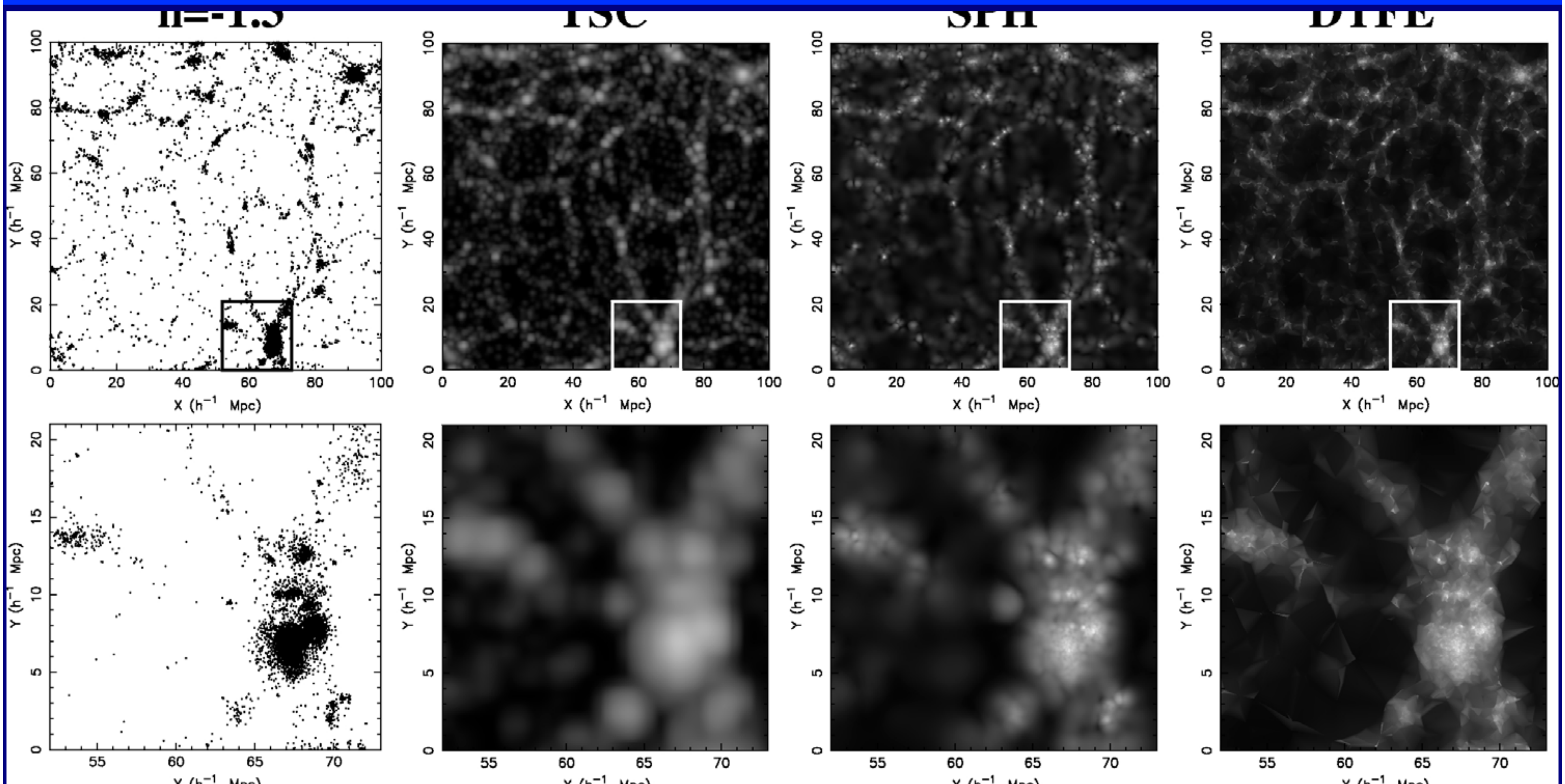
& Dynamic Range

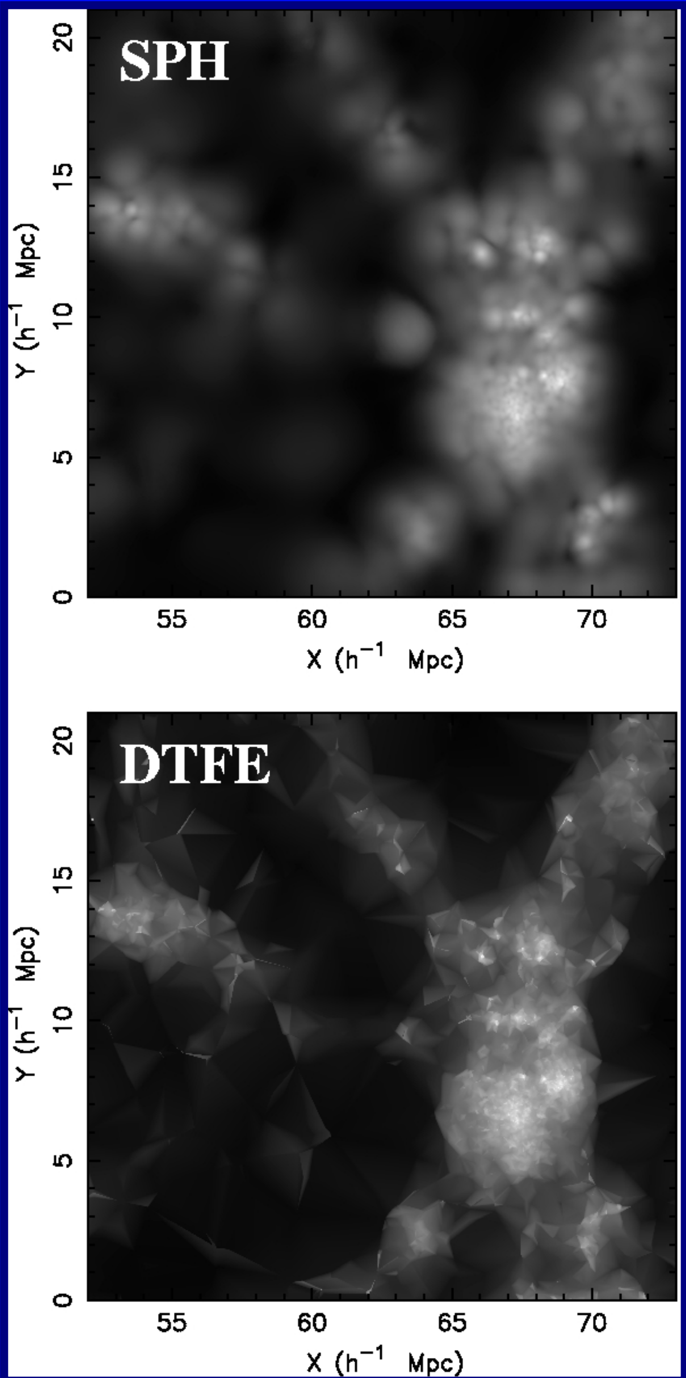

particles:
DTFE:
TSC:





n=-2


TSC
SPH
DTFE
X (h-1 Mpc)
Y (h-1 Mpc)



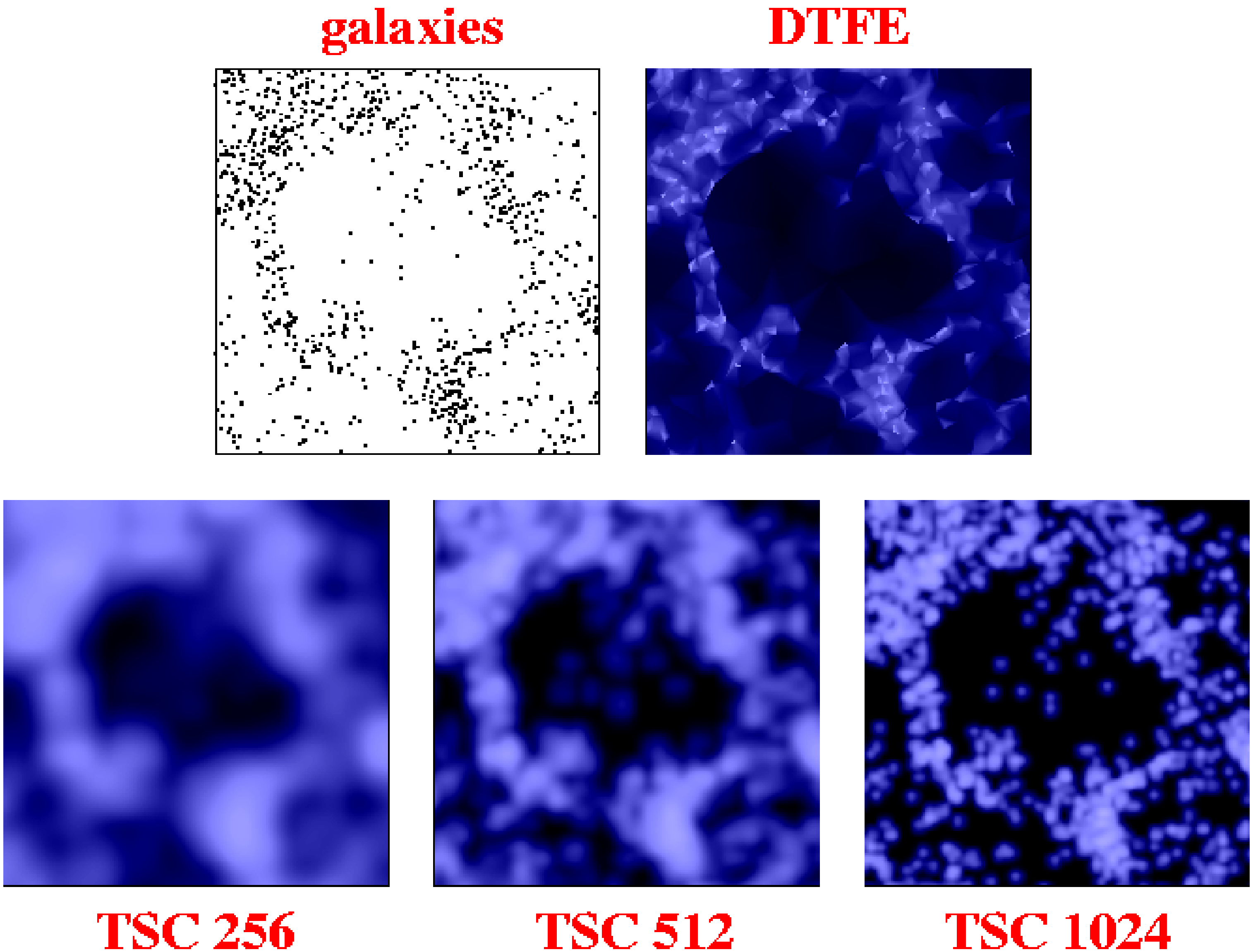
Dynamic Range and Resolution of Tessellation techniques has been recognized in various studies. This involves mainly the Voronoi tessellations generate by galaxy or particle distribution.

Cluster finding:	Ebeling & Wiedemann 1992
	Kim et al. 2001
Halo finding/structure:	Neyrinck et al. 2004
Halo phase-space:	Arad, Klypin & Dekel 2004

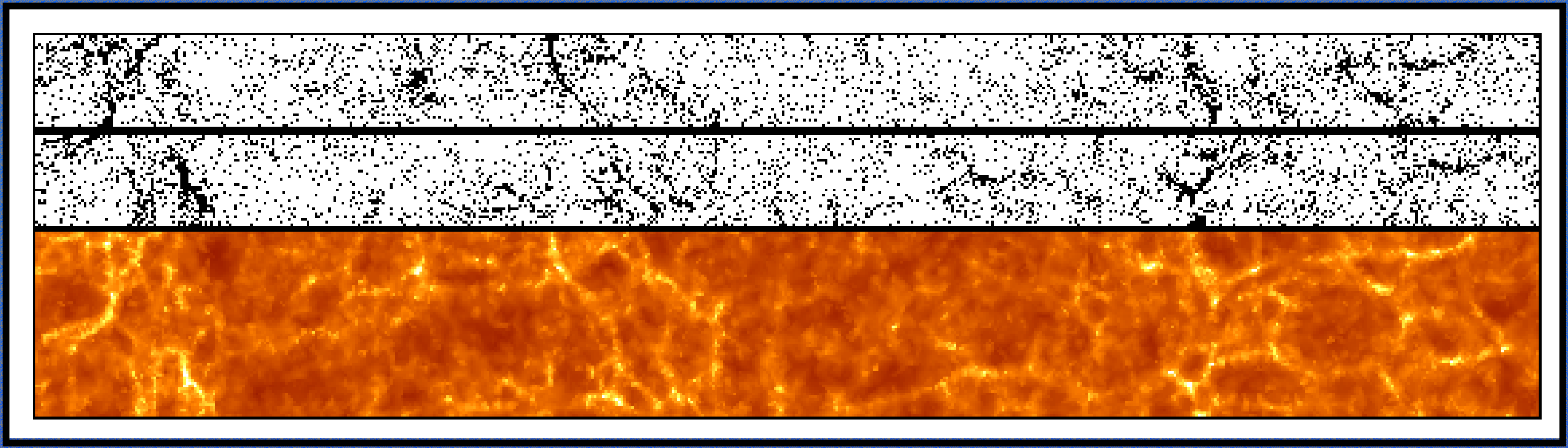
Testing DTFE

2.

Shot-Noise Suppression


galaxies
DTFE
TSC 256
TSC 512
TSC 1024

Superhubble Void Flow

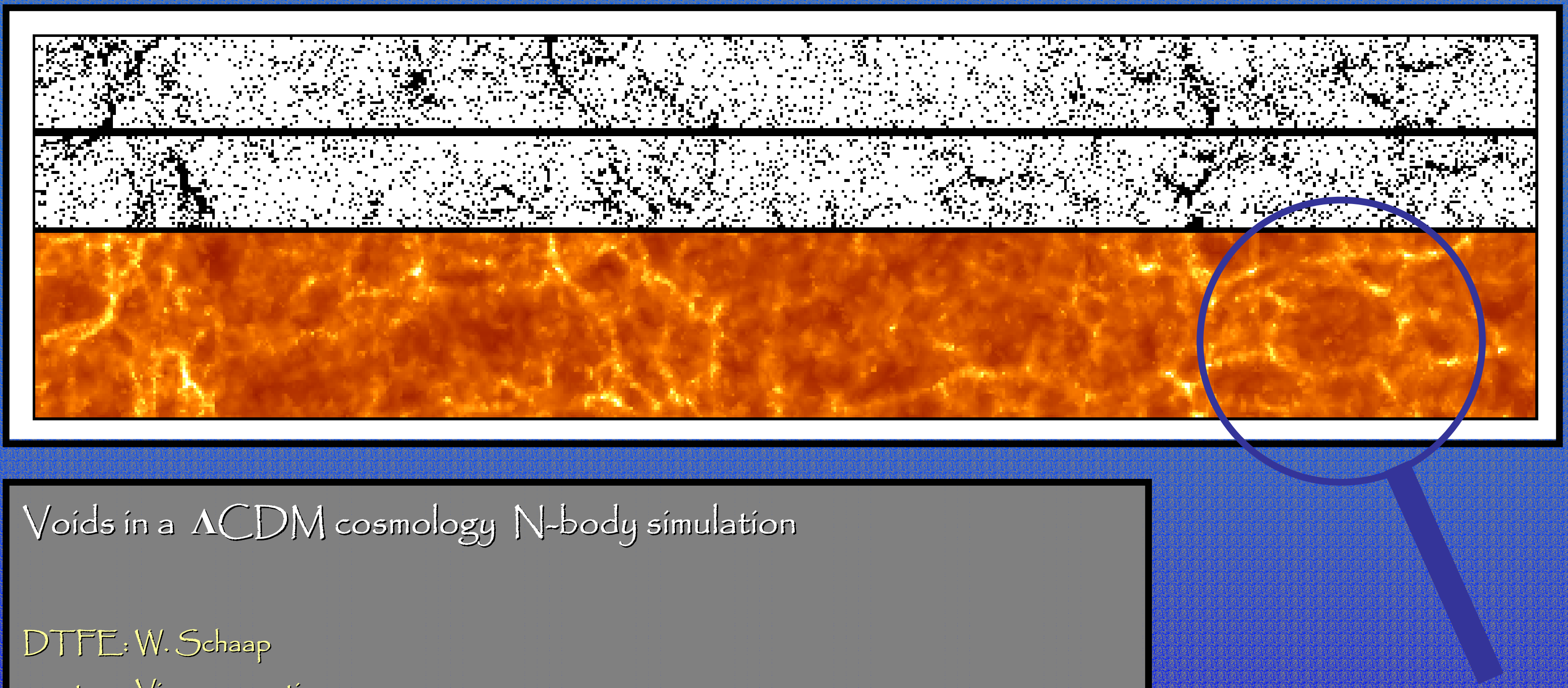


Voids in a ΛCDM cosmology N-body simulation

DTFE: W. Schaap

courtesy: Virgo consortium

Superhubble Void Flow

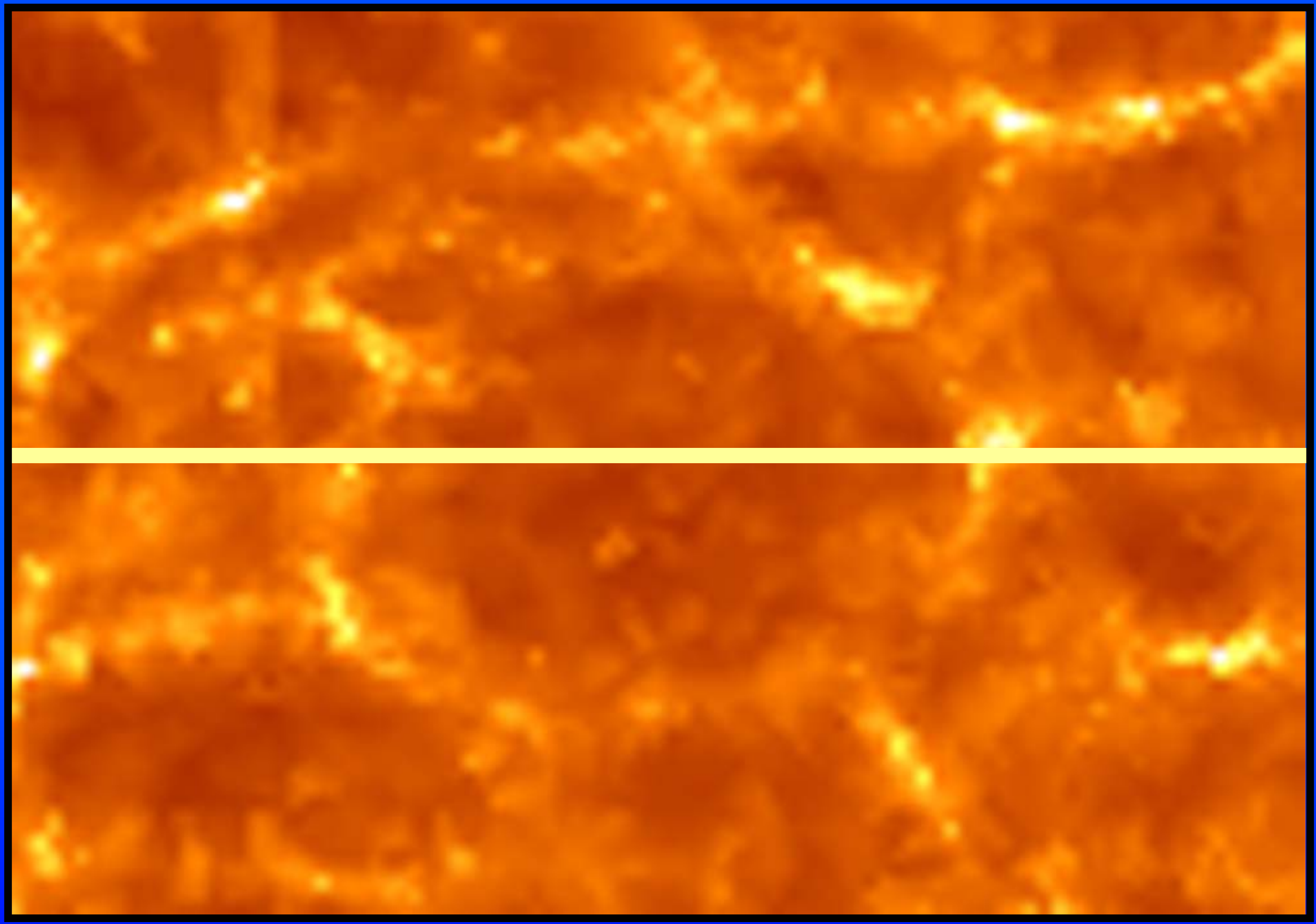


Voids in a ΛCDM cosmology N-body simulation

DTFE: W. Schaap

courtesy: Virgo consortium

Superhubble Void Flow



Voids in a
ΛCDM cosmology
N-body simulation

DTFE: W. Schaap
courtesy: Virgo consortium

Superhubble Void Flow

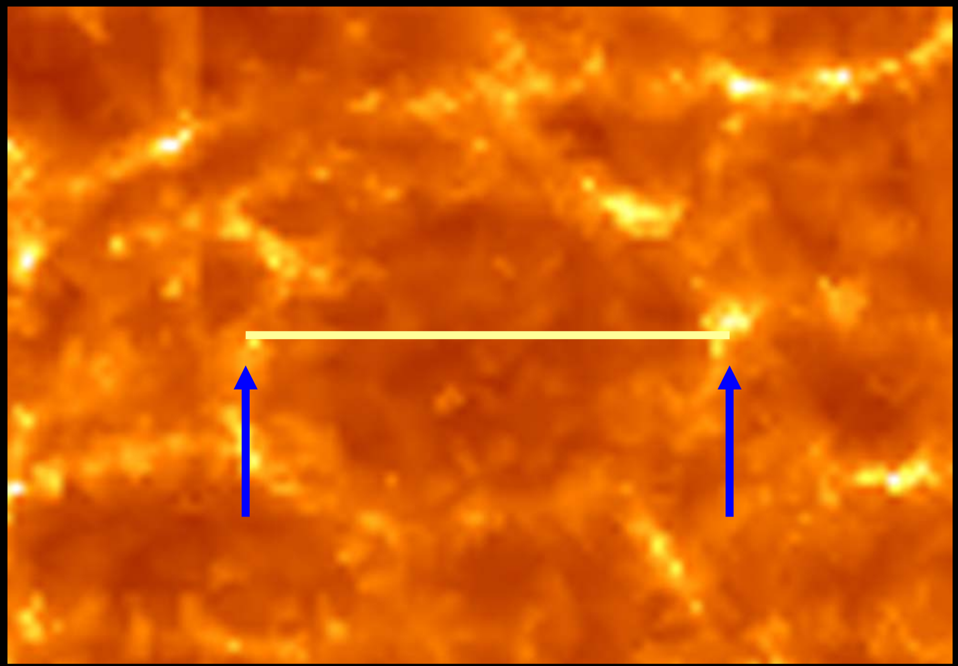
``Superhubble Bubble"

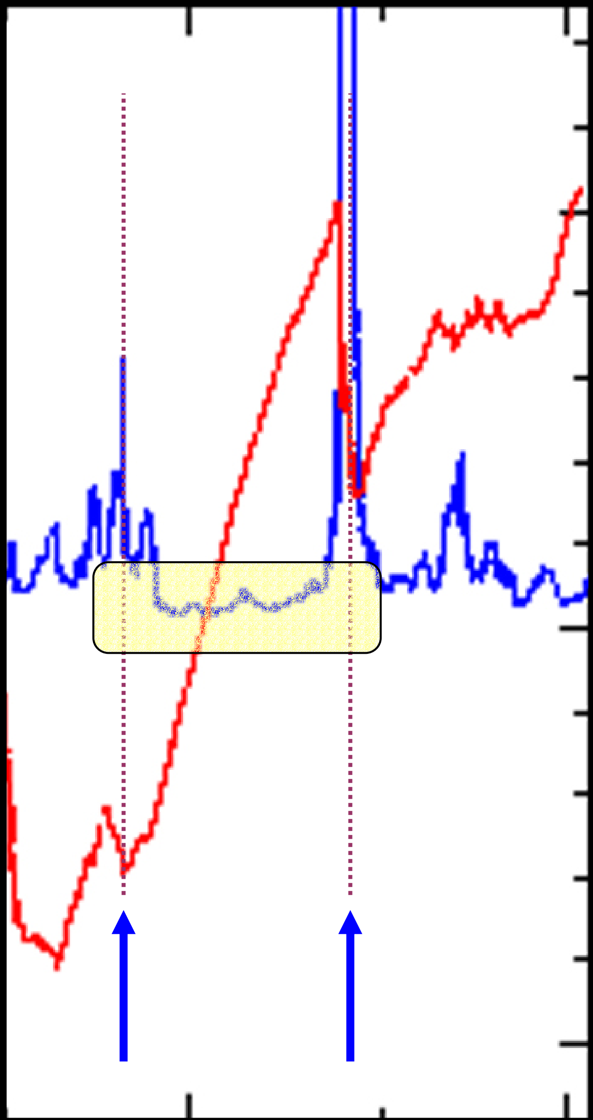
- void evacuation results into tophat interior
- flow field: velocity divergence constant

``Super-Hubble flow"

Void in a
ΛCDM
cosmology
N-body
simulation

DTFE:
W. Schaap
courtesy:
Virgo consortium



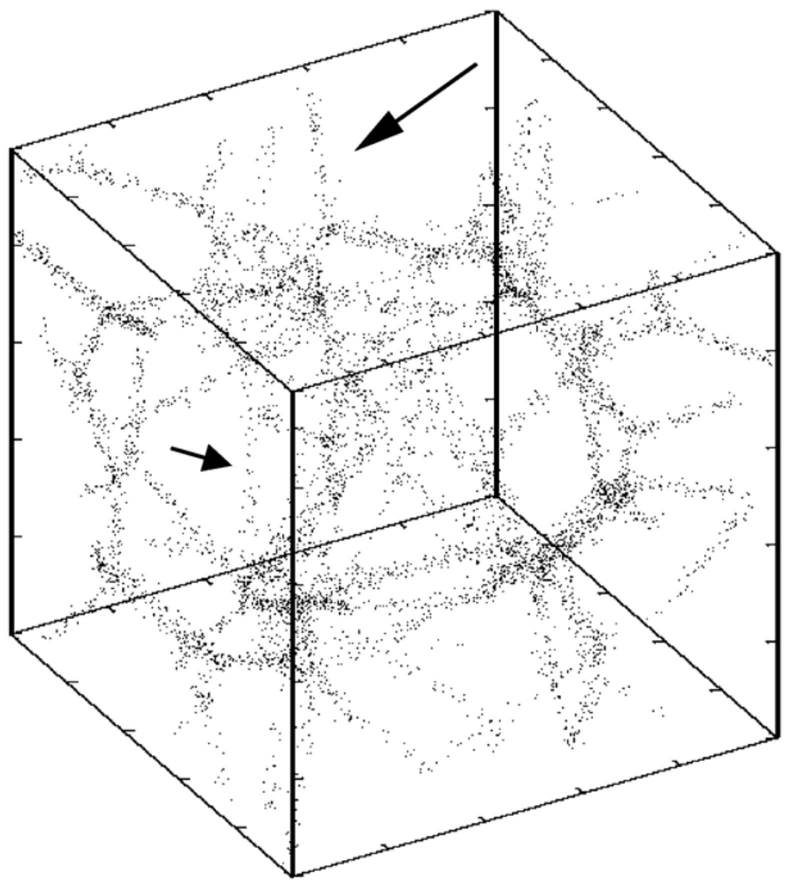


DTFE Characteristics

3.

Anisotropic

Patterns



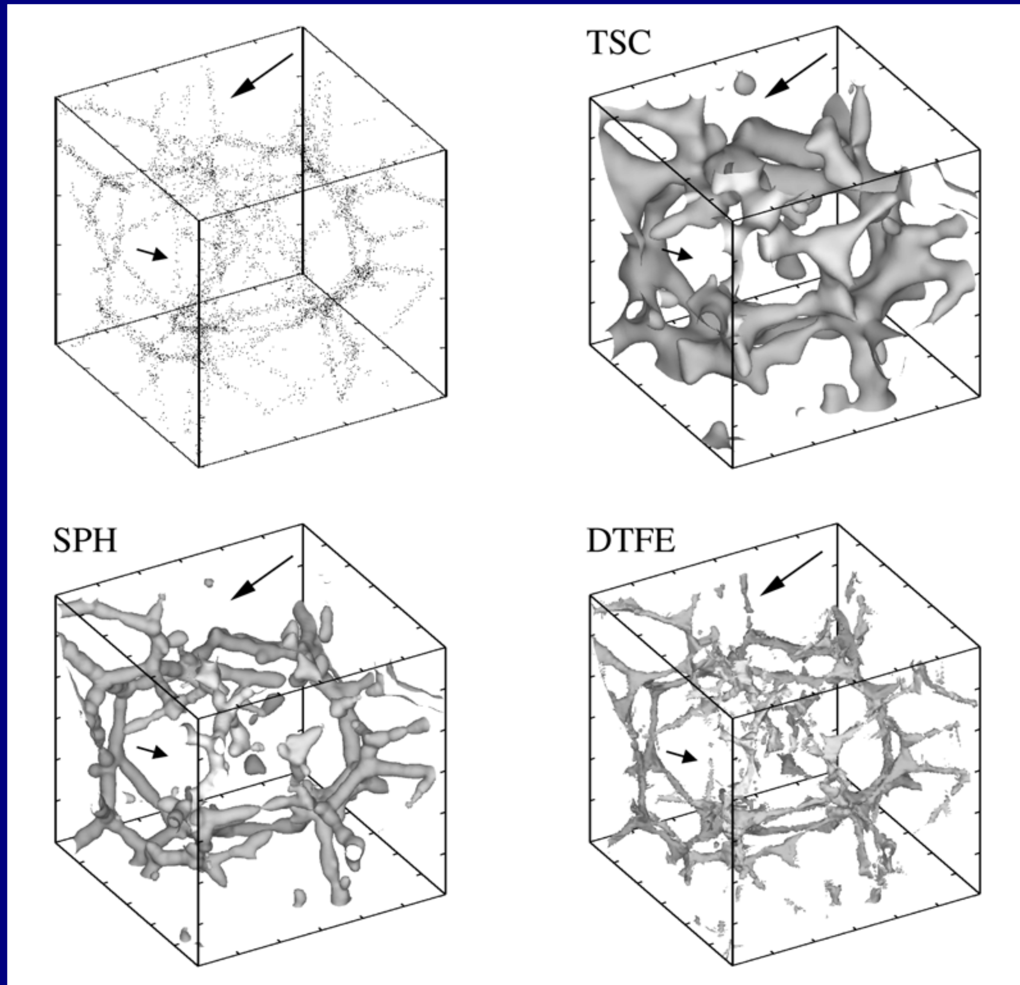
DTFE & Anisotropy

Anisotropic Patterns

Heuristic Model:

Voronoi Template Model

- Filamentary



DTFE & Anisotropy

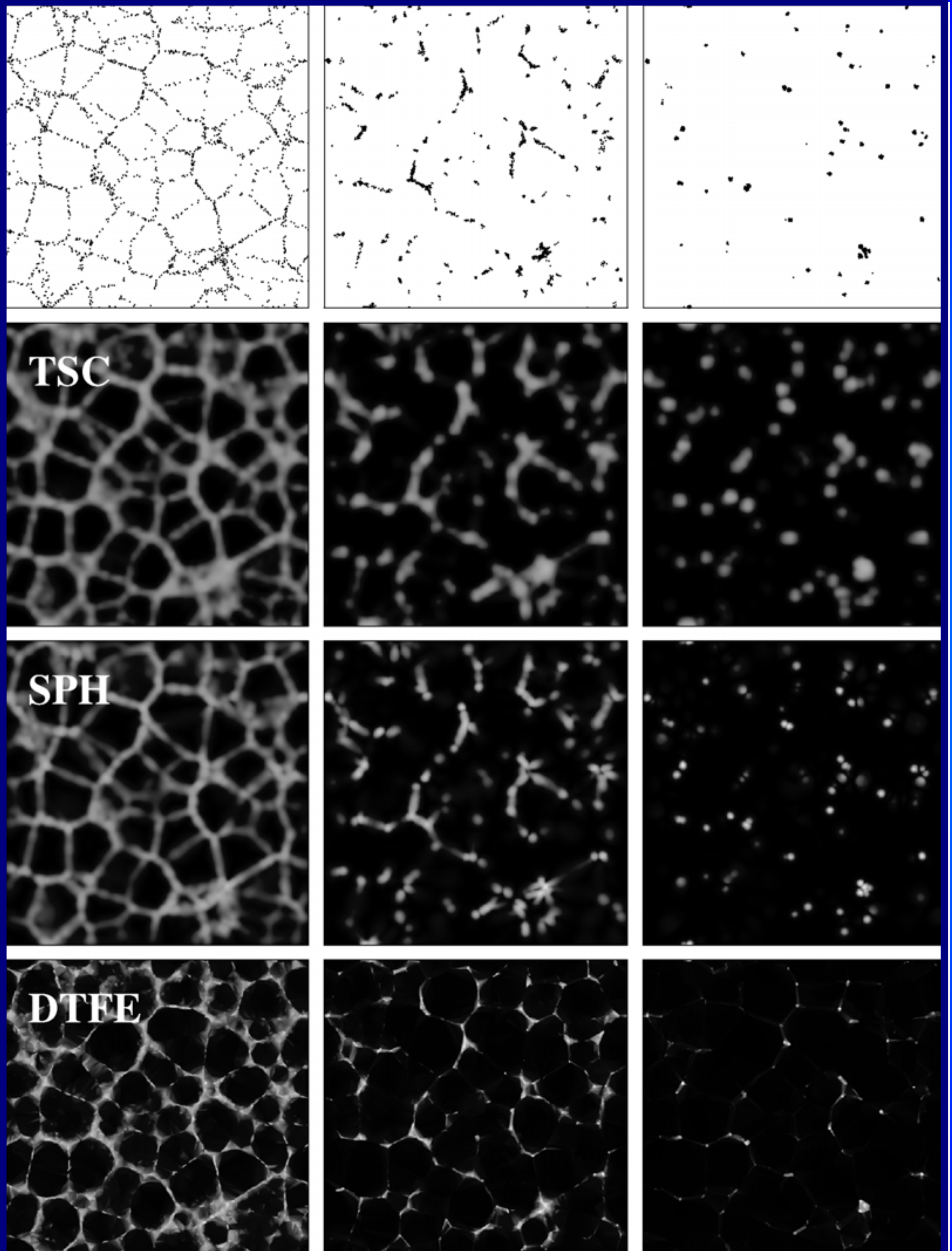
Anisotropic Patterns

Heuristic Model:

Voronoi Template Model

- Filamentary

DTFE & Anisotropy



Anisotropic Patterns

Heuristic Models:

Voronoi Template Model

- Wall-like
- Filamentary
- Clumpy

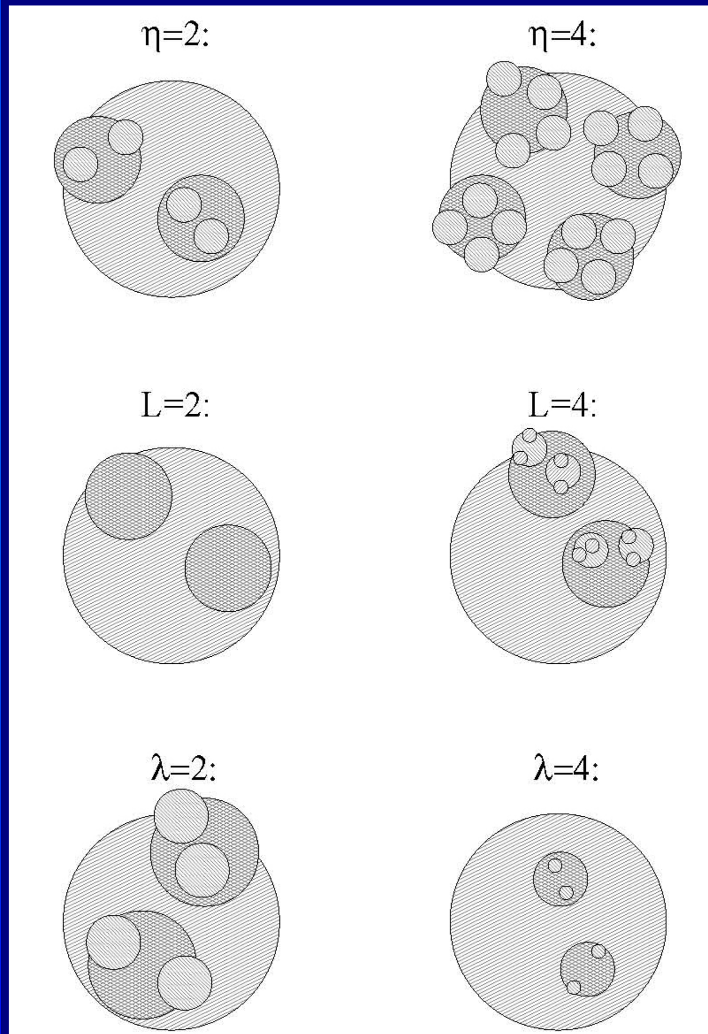
DTFE Characteristics

4.

Structural

Hierarchy & Scaling

DTFE Hierarchy & Scaling

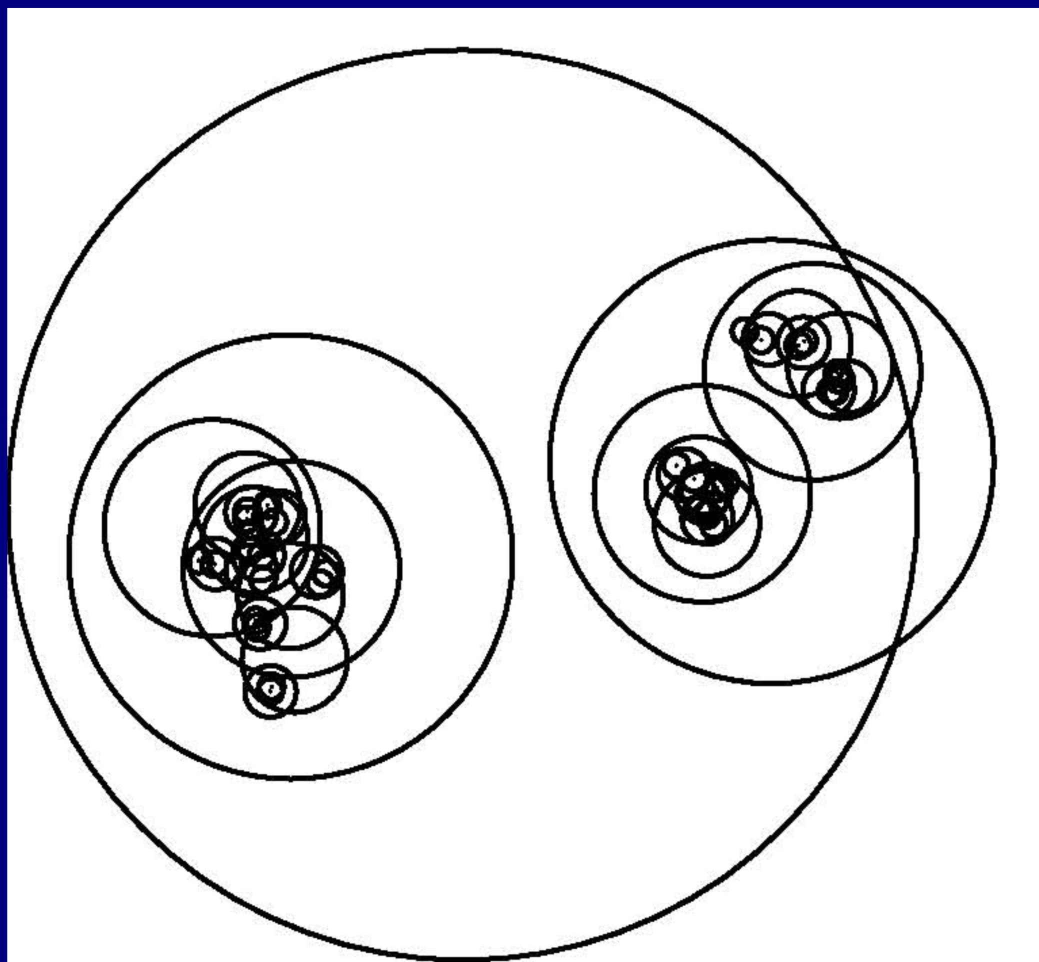


Hierarchically Embedded Substructures

Heuristic Model:

Soneira-Peebles

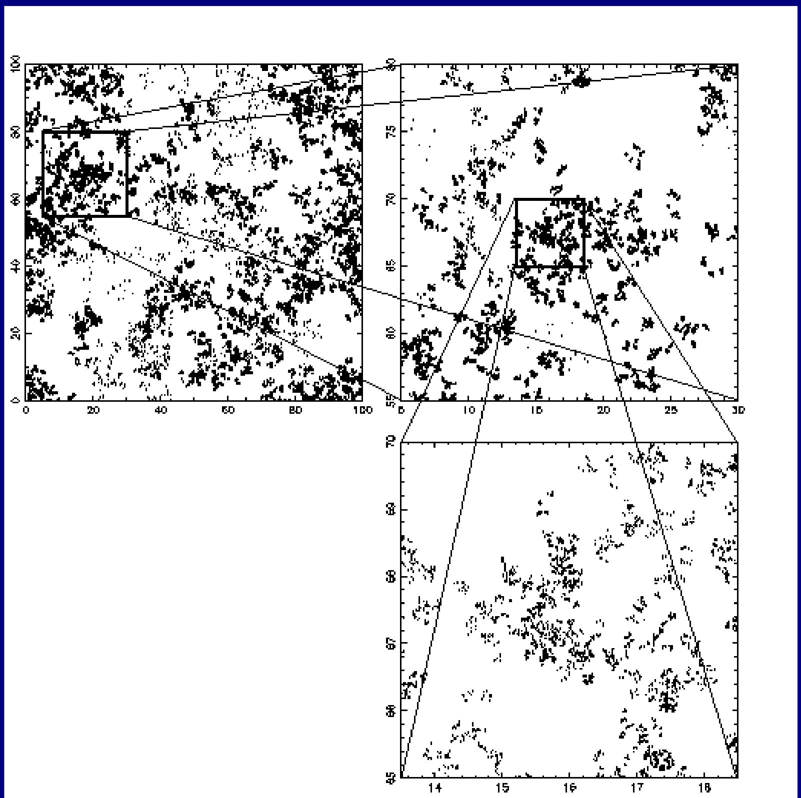
DTFE Hierarchy & Scaling

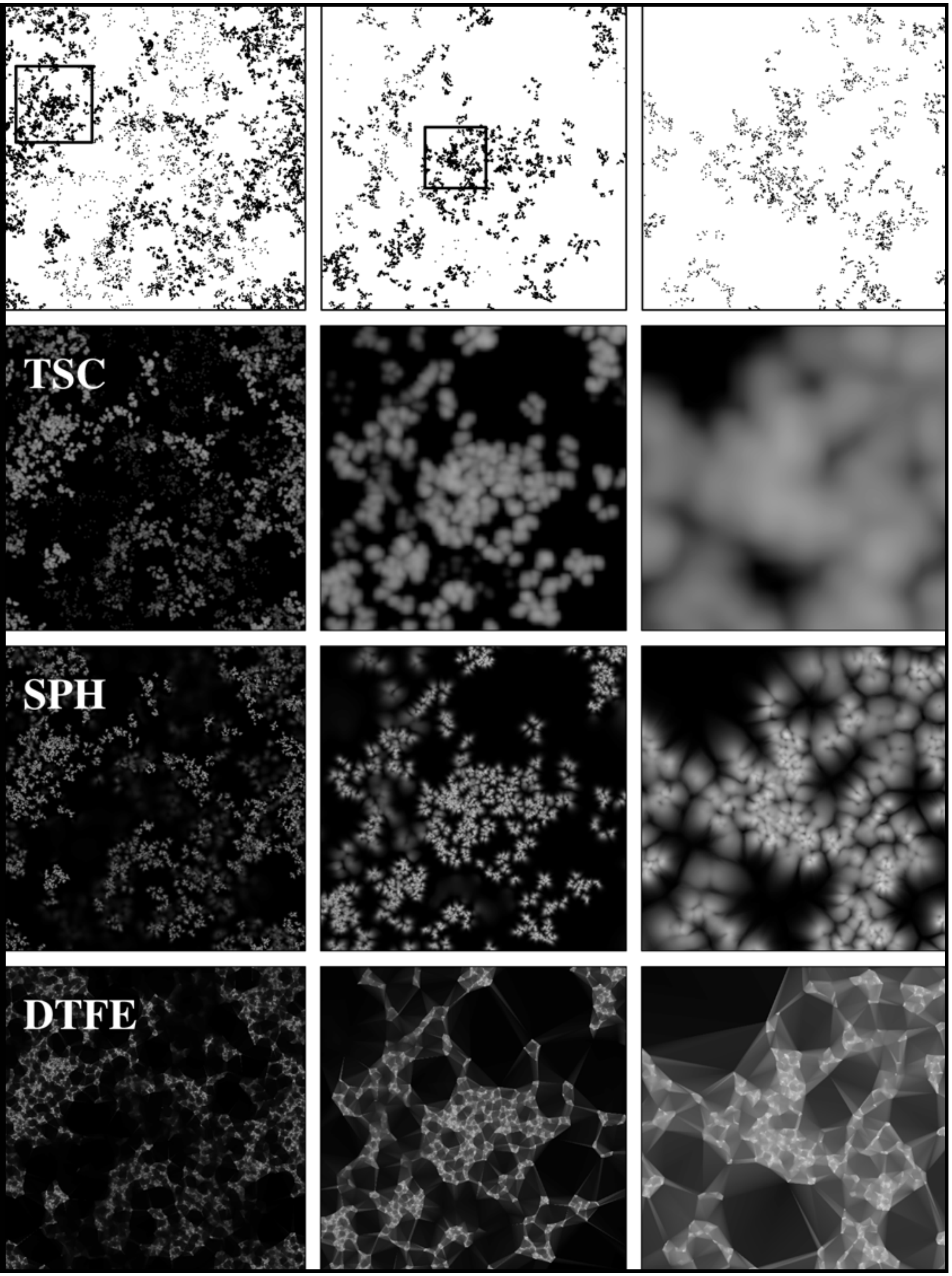


Hierarchically Embedded Substructures

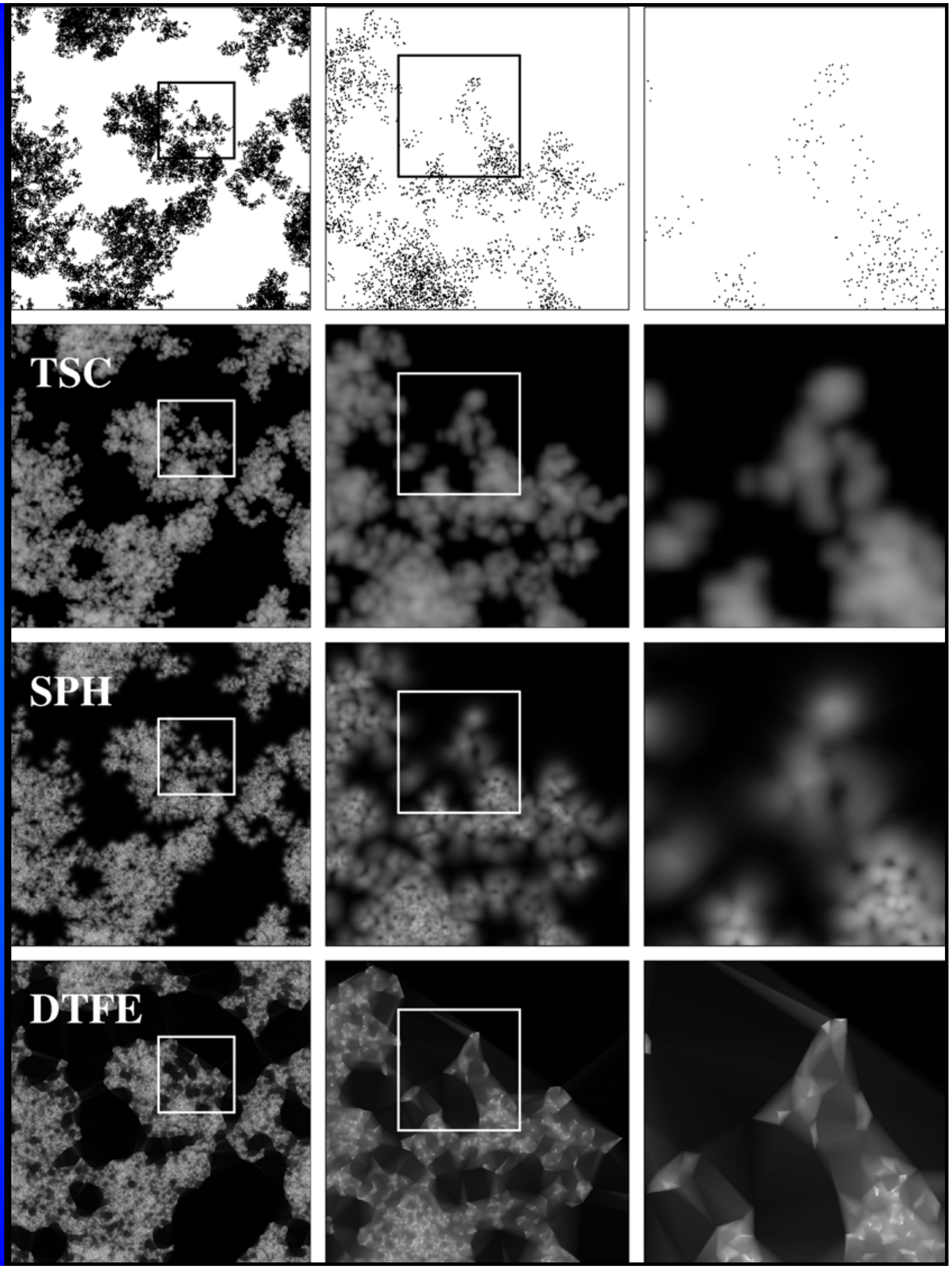
Heuristic Model: Soneira-Peebles

DTFE Scaling

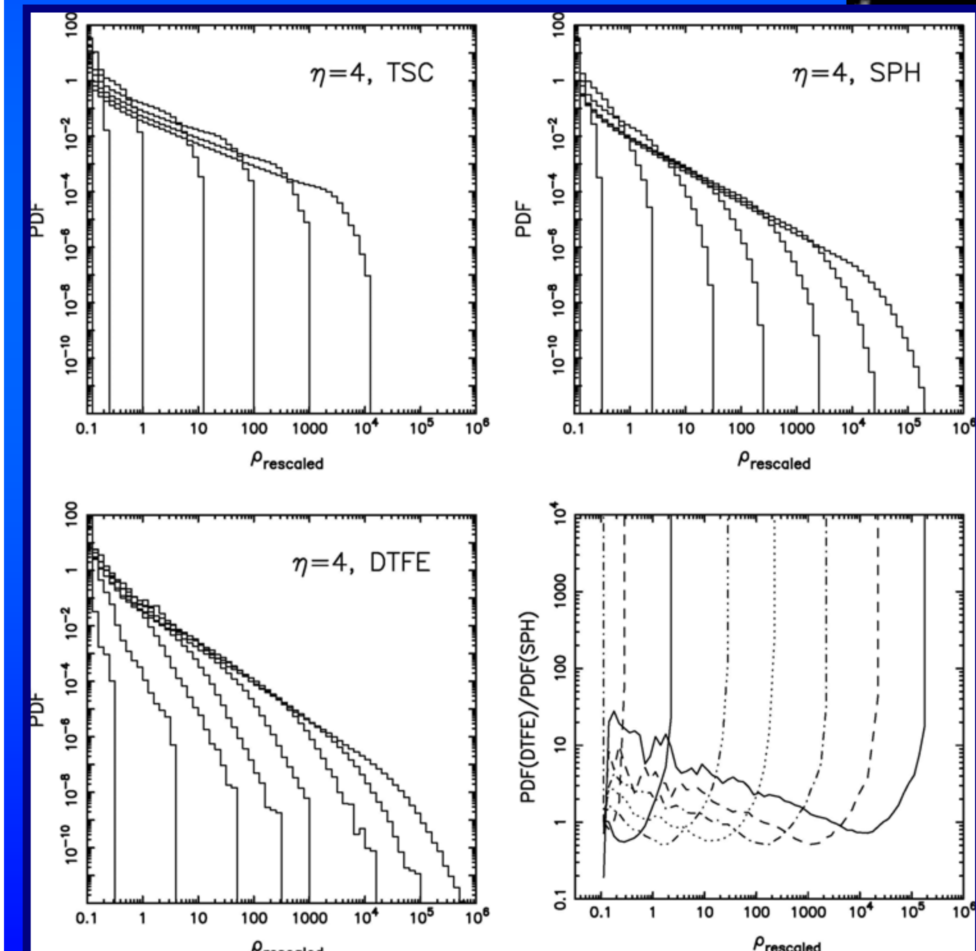


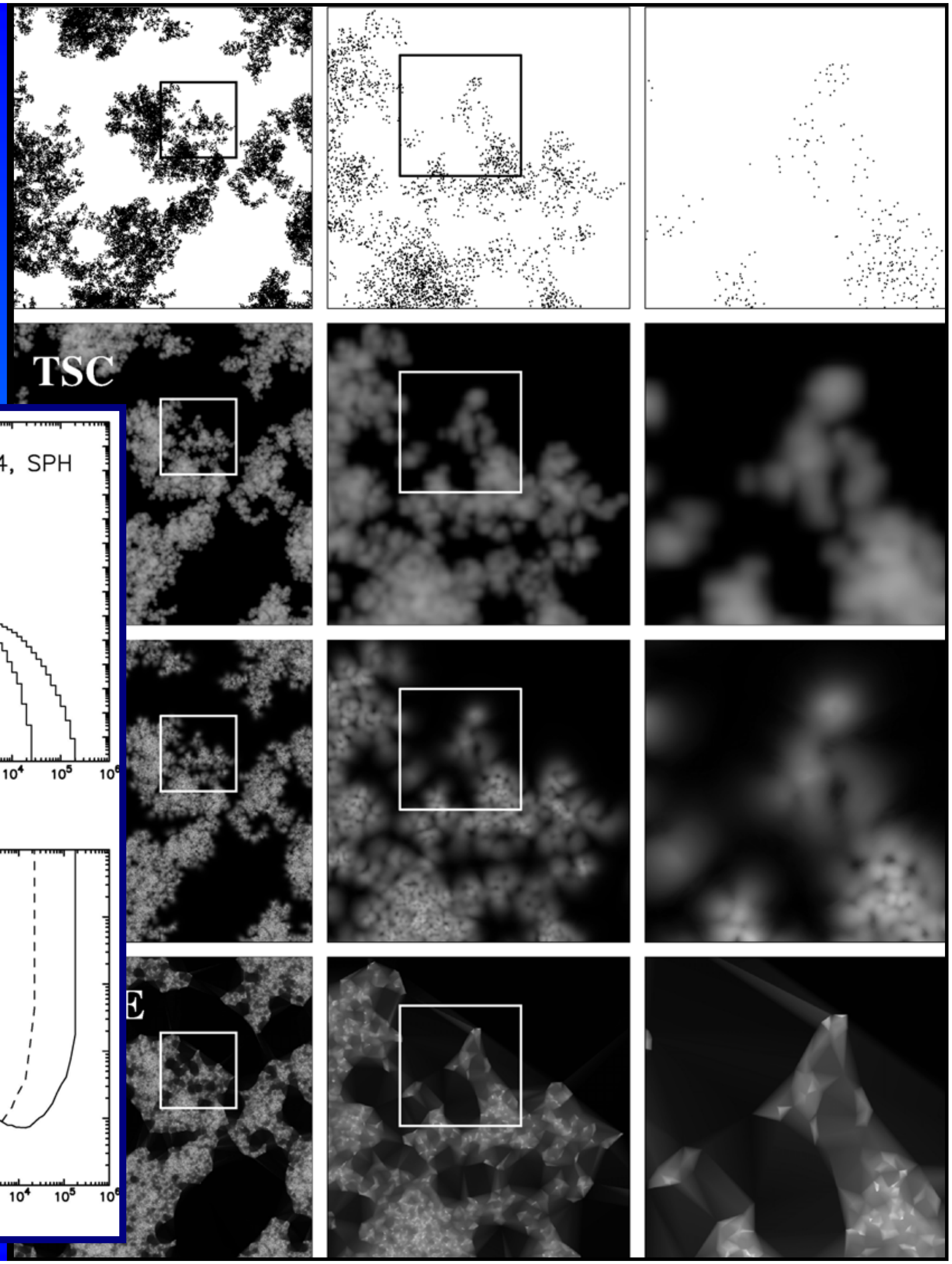


DTFE Scaling

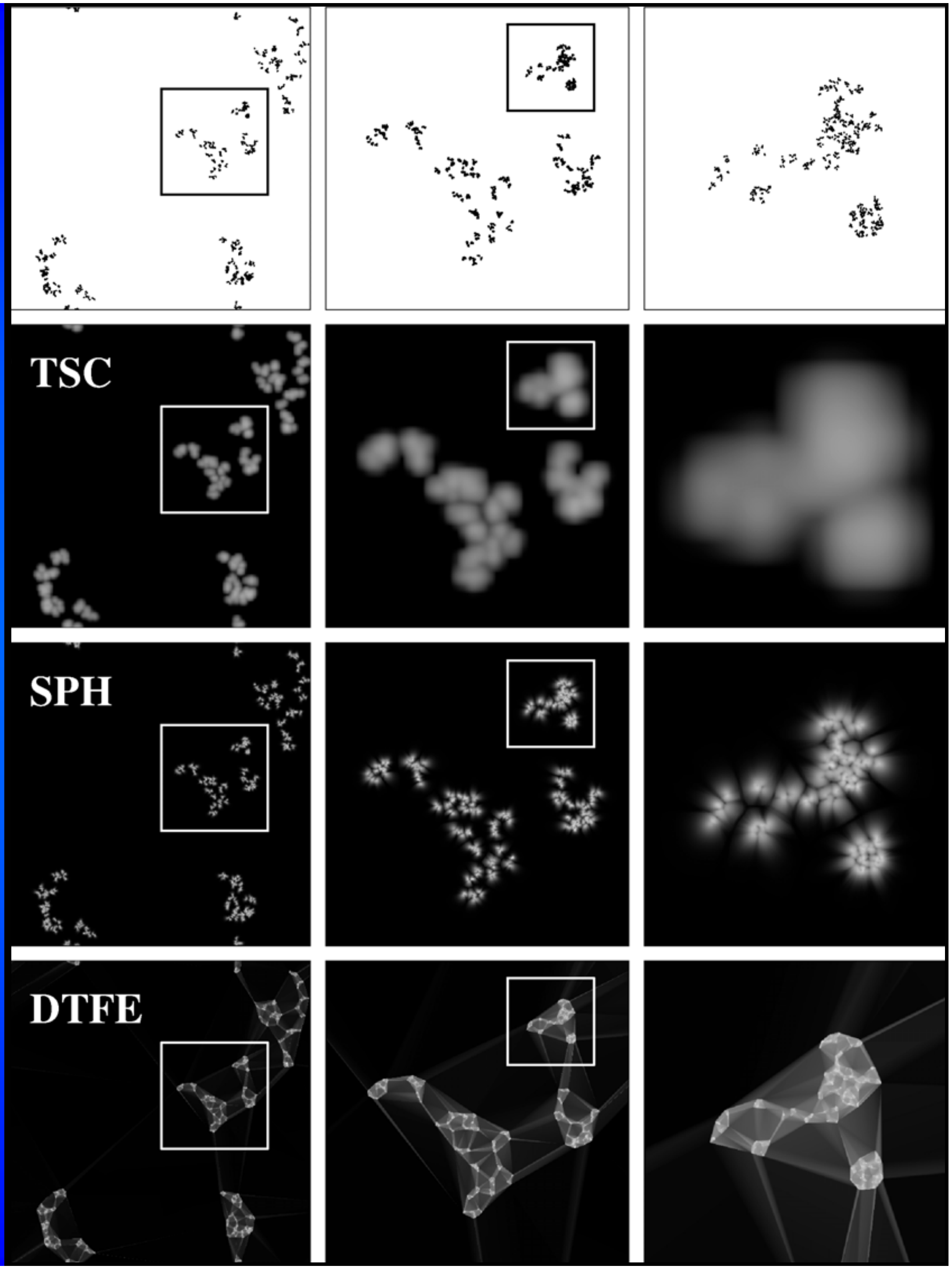


DTFE Scaling

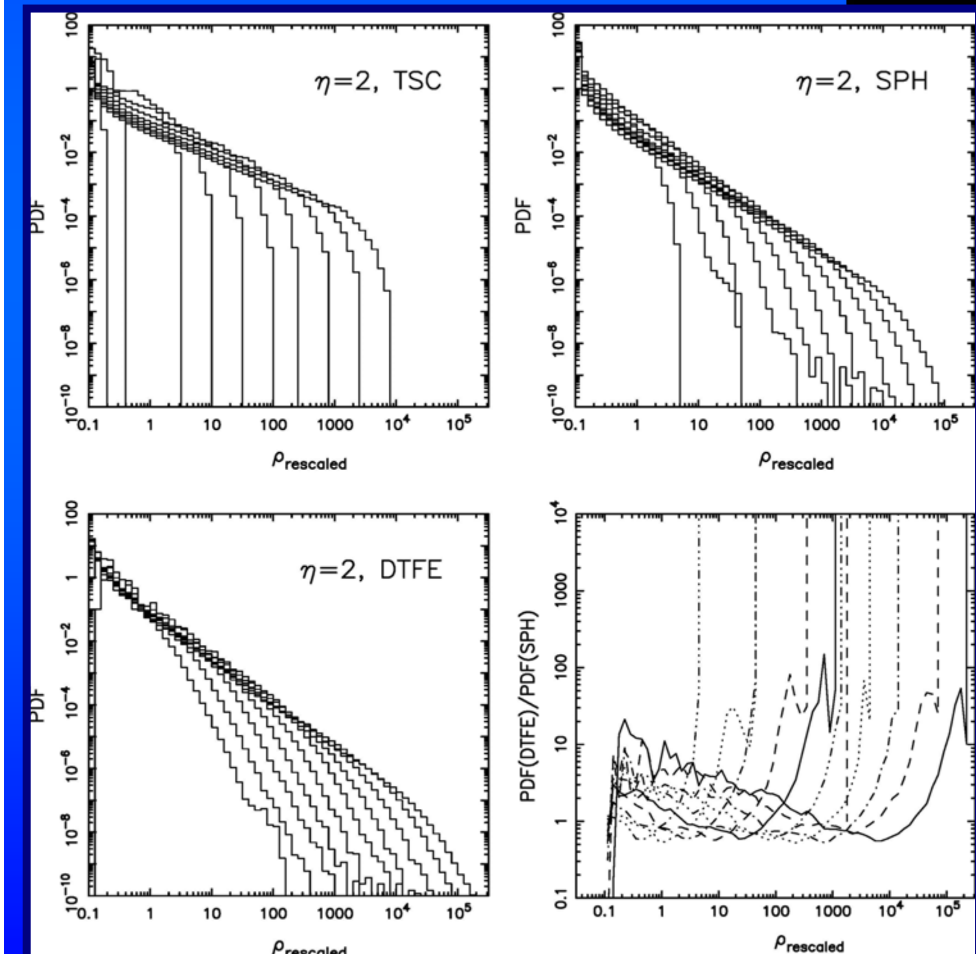


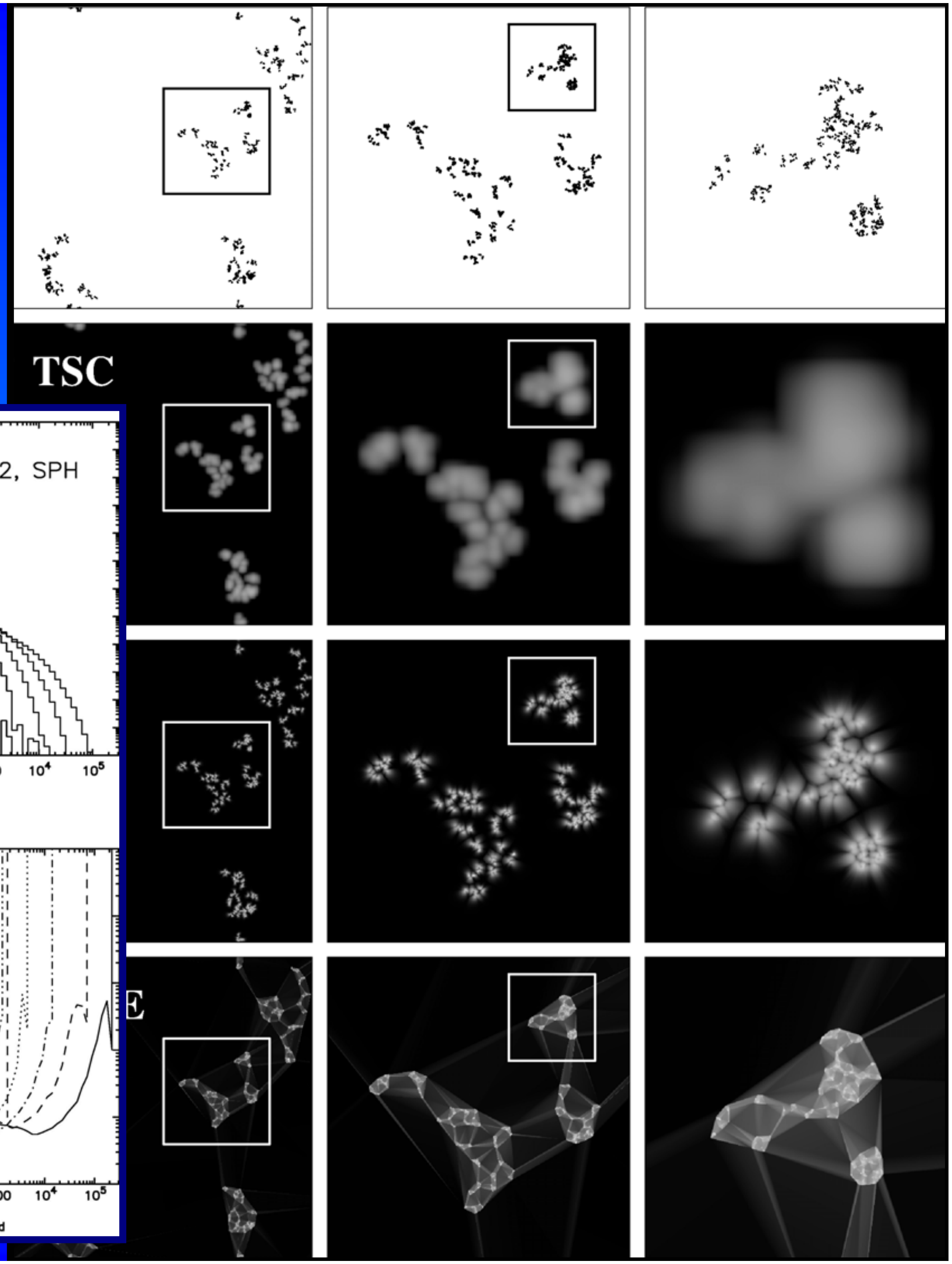


DTFE Scaling

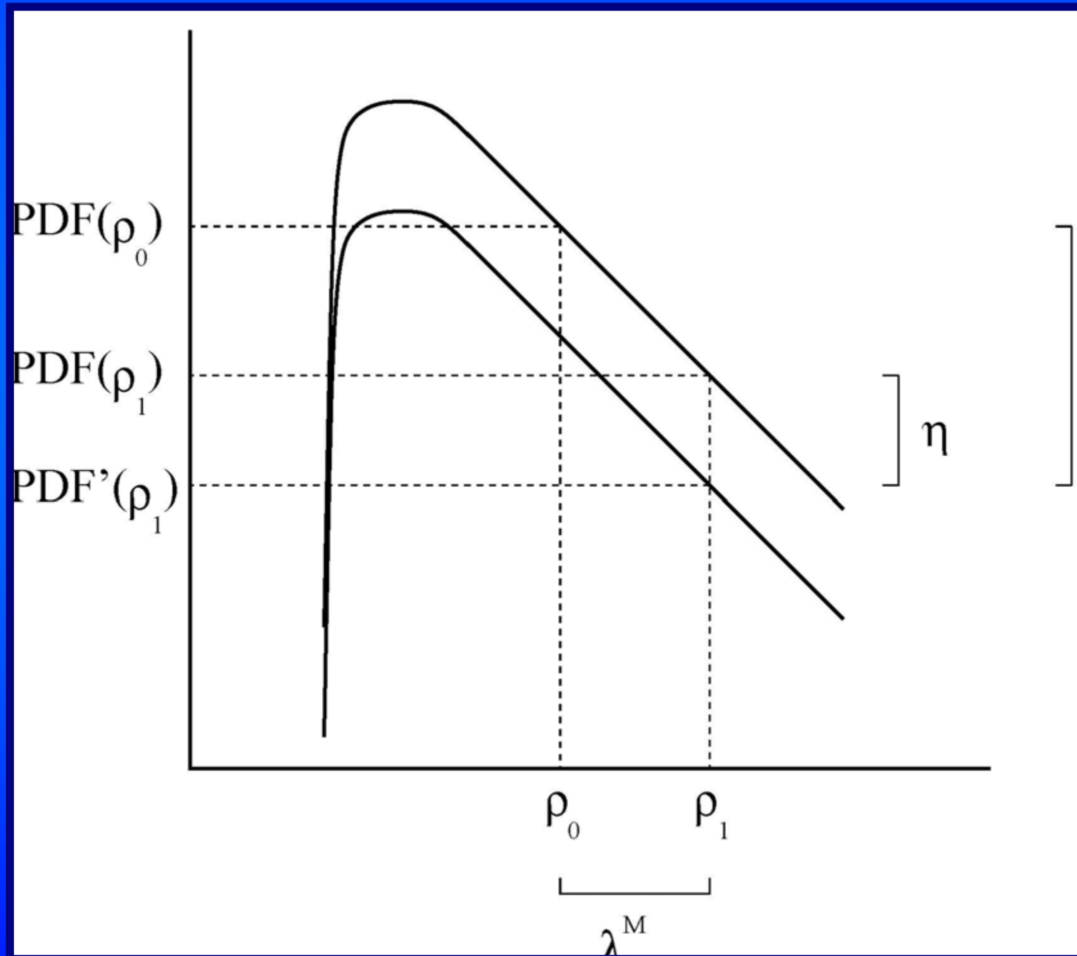


DTFE Scaling





DTFE Hierarchy & Scaling



DTFE Hierarchy & Scaling

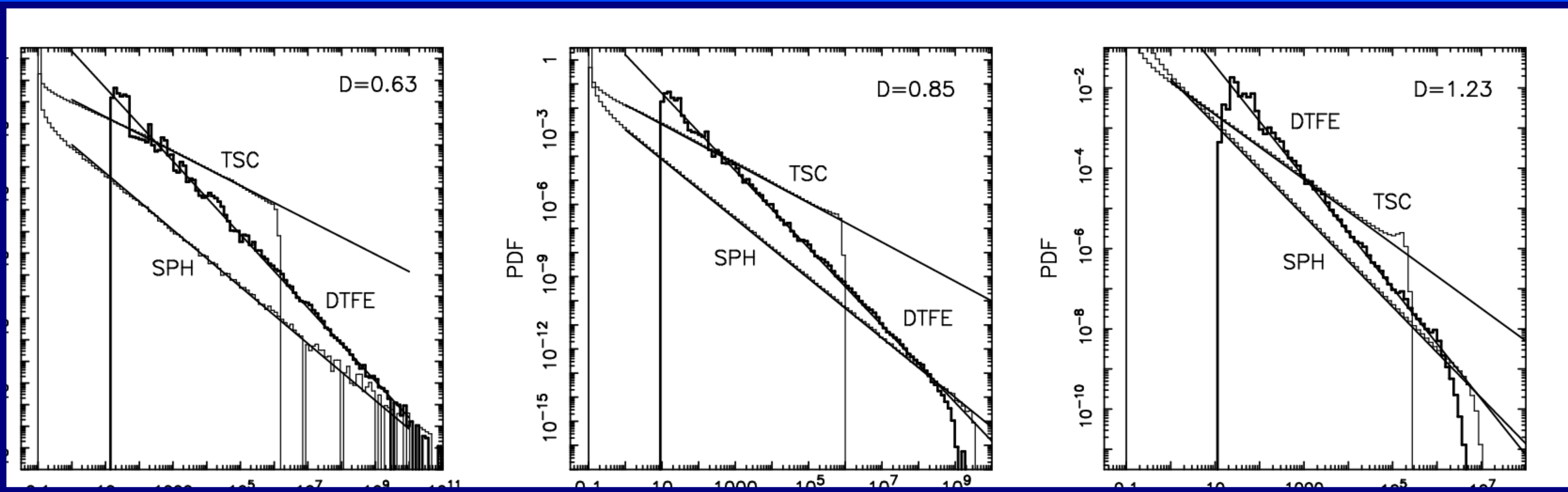


Table 4.2 — Slopes of the power-law region of the PDF of a Soneira-Peebles density field as reconstructed by the TSC, SPH and DTFE procedures. The theoretical value (Eqn. 4.13) is also listed. Values are listed for three different Soneira-Peebles realizations, each with a different fractal dimension D.

D	α(theory)	α(TSC)	α(SPH)	α(DTFE)
0.63	−1.69	−0.81	−1.32	−1.70
0.86	−1.57	−0.82	−1.24	−1.60
1.23	−1.39	−0.79	−1.13	−1.38

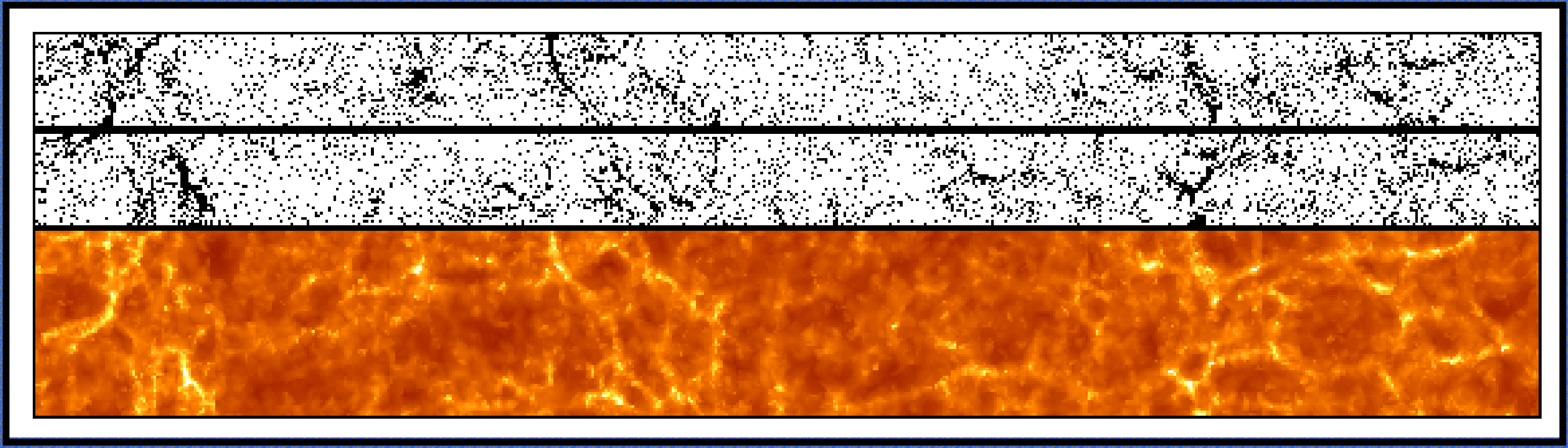
Testing DTFE

5.

Field

Universality

Density Structure & Cosmic Flow

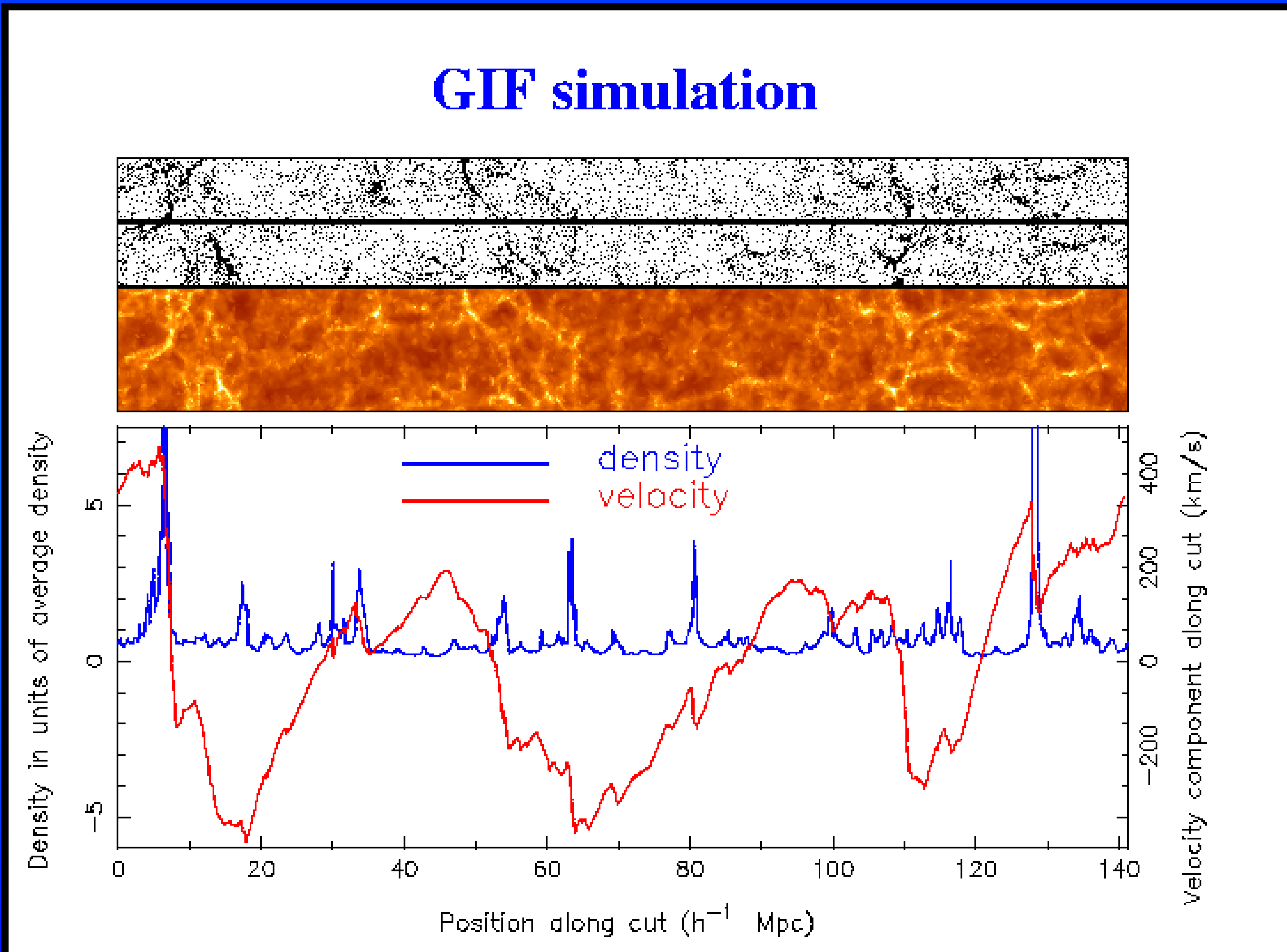


Voids in a ΛCDM cosmology N-body simulation

DTFE: W. Schaap

courtesy: Virgo consortium

Density Structure & Velocity Flows



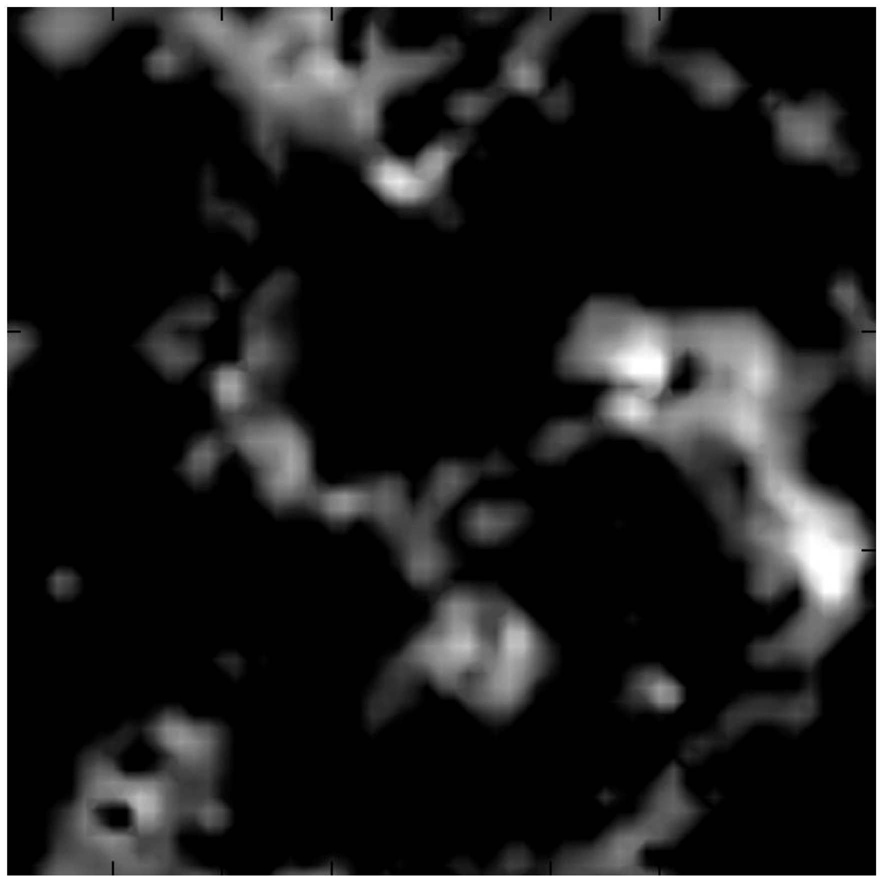
GIF Simulation

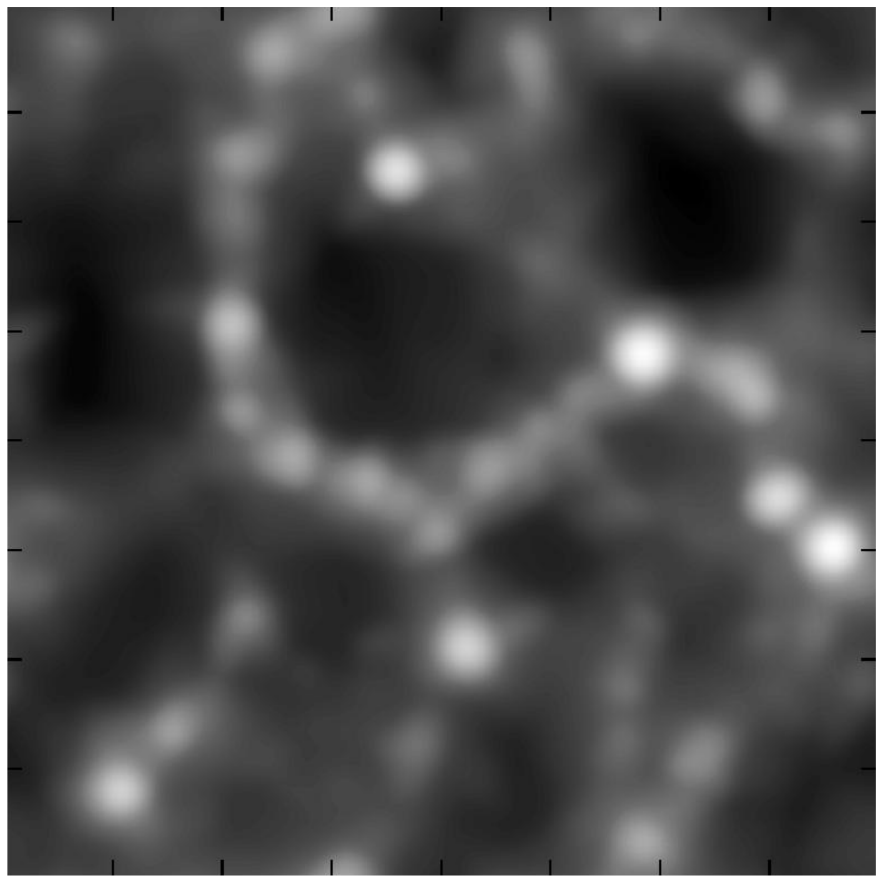
W.Schaap,
S. White,
R. van de Weygaert

Density Structure & Velocity Flows

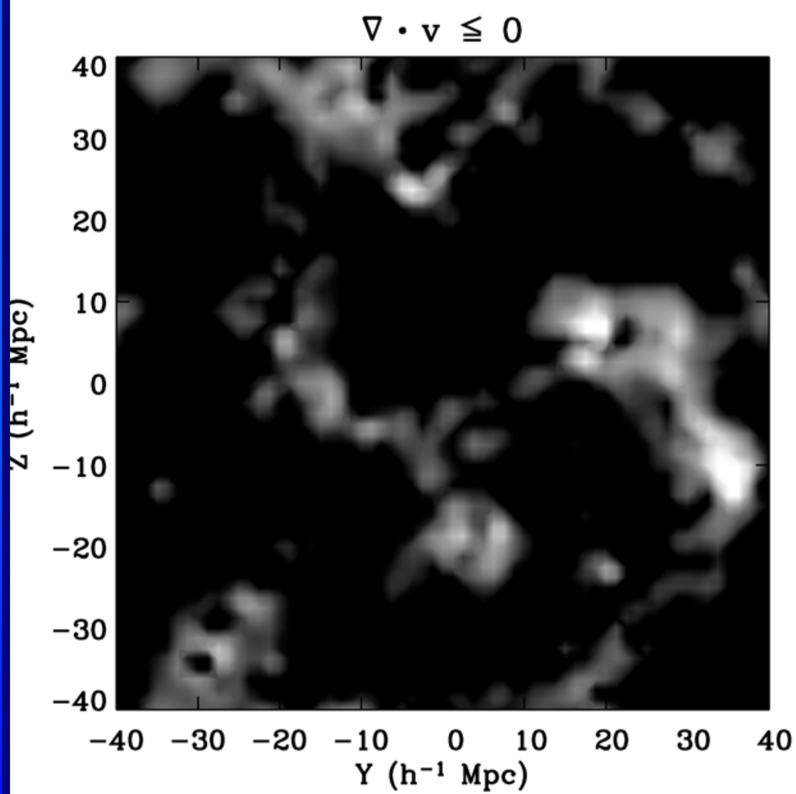
$\nabla \cdot \mathbf{v} < 0$: inflow

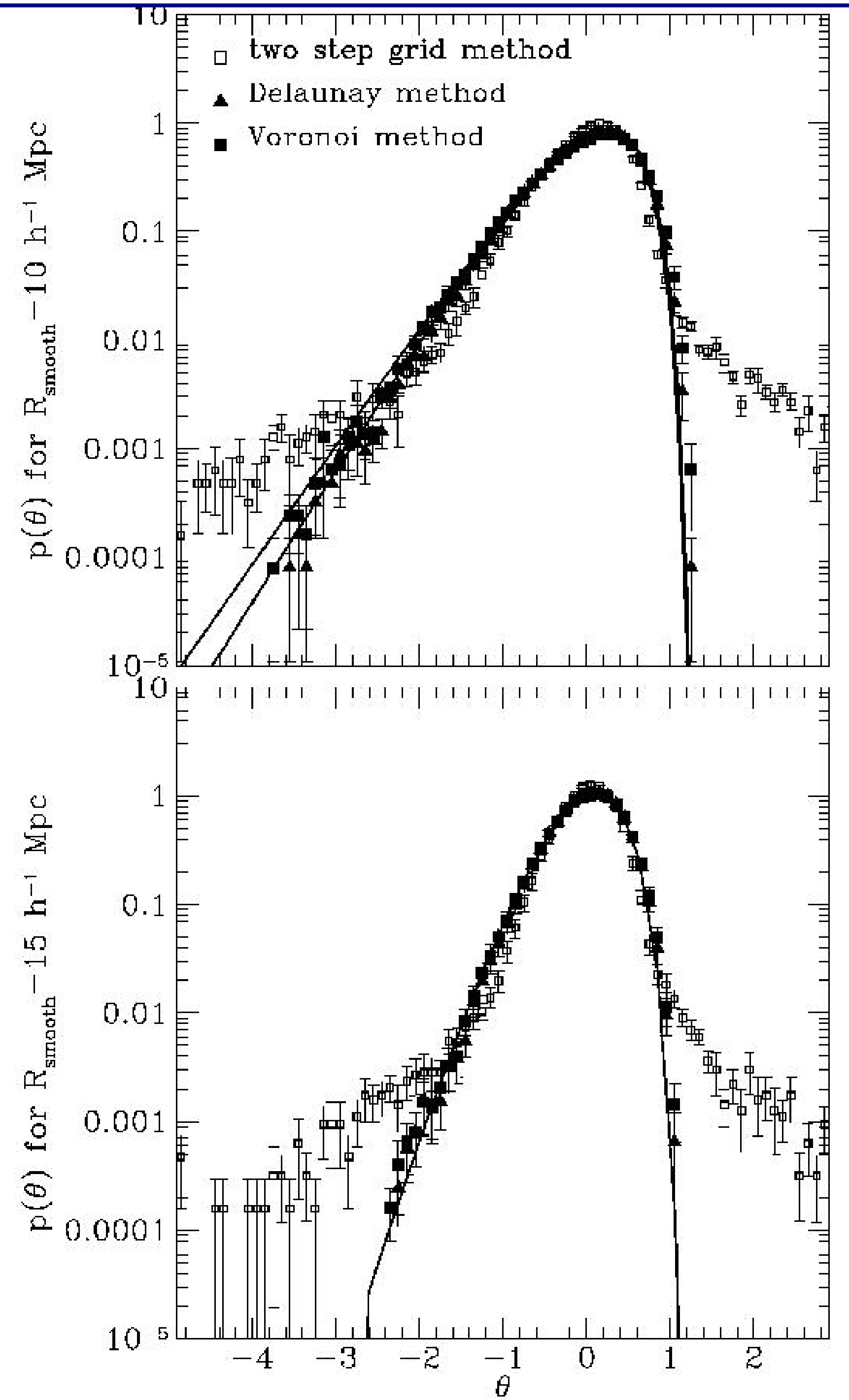
$$\delta(\mathbf{x}) \equiv \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}}$$



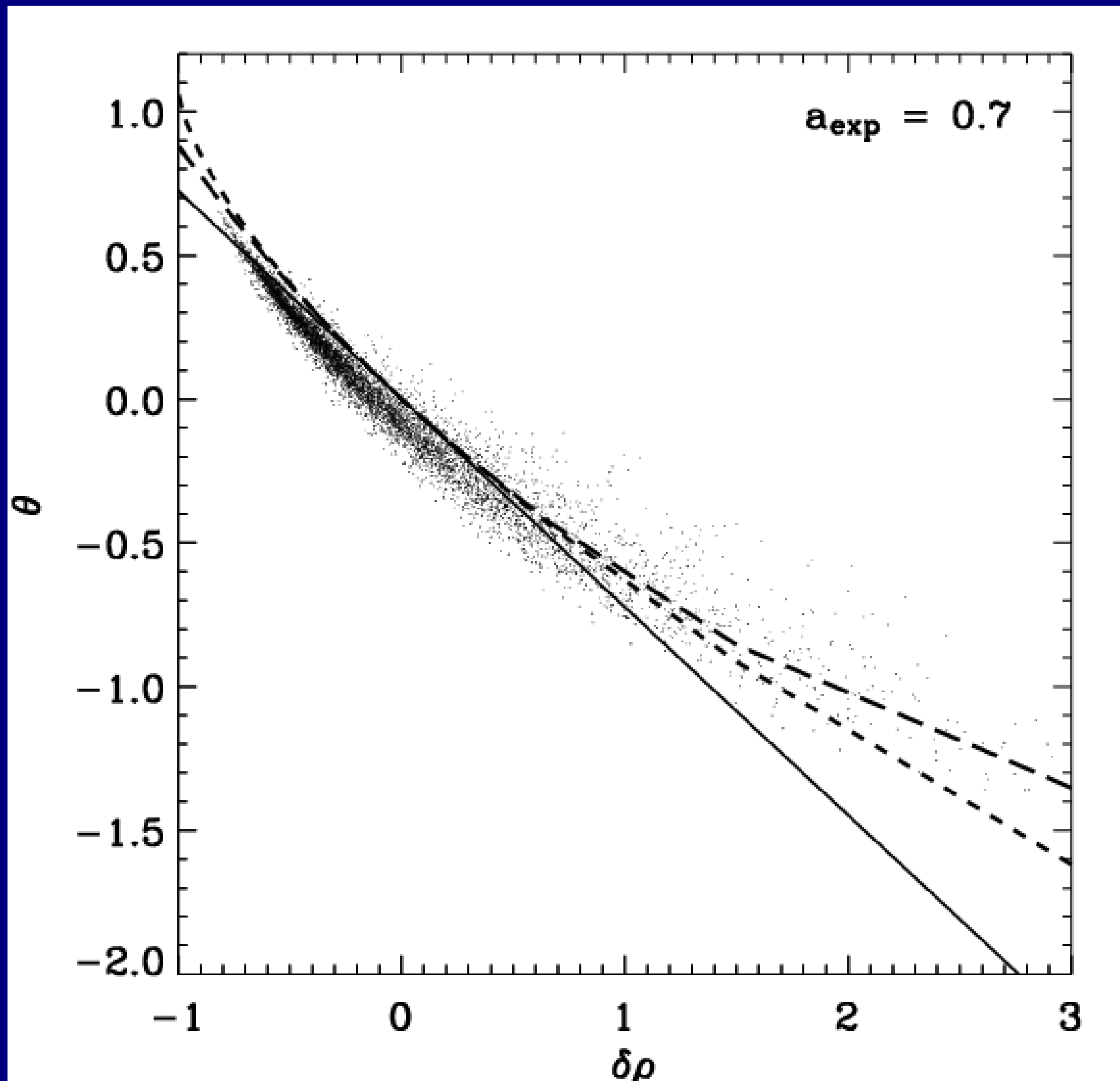


DTFE Velocity Field Divergence





Density Structure & Velocity Flows



The resulting DTFE

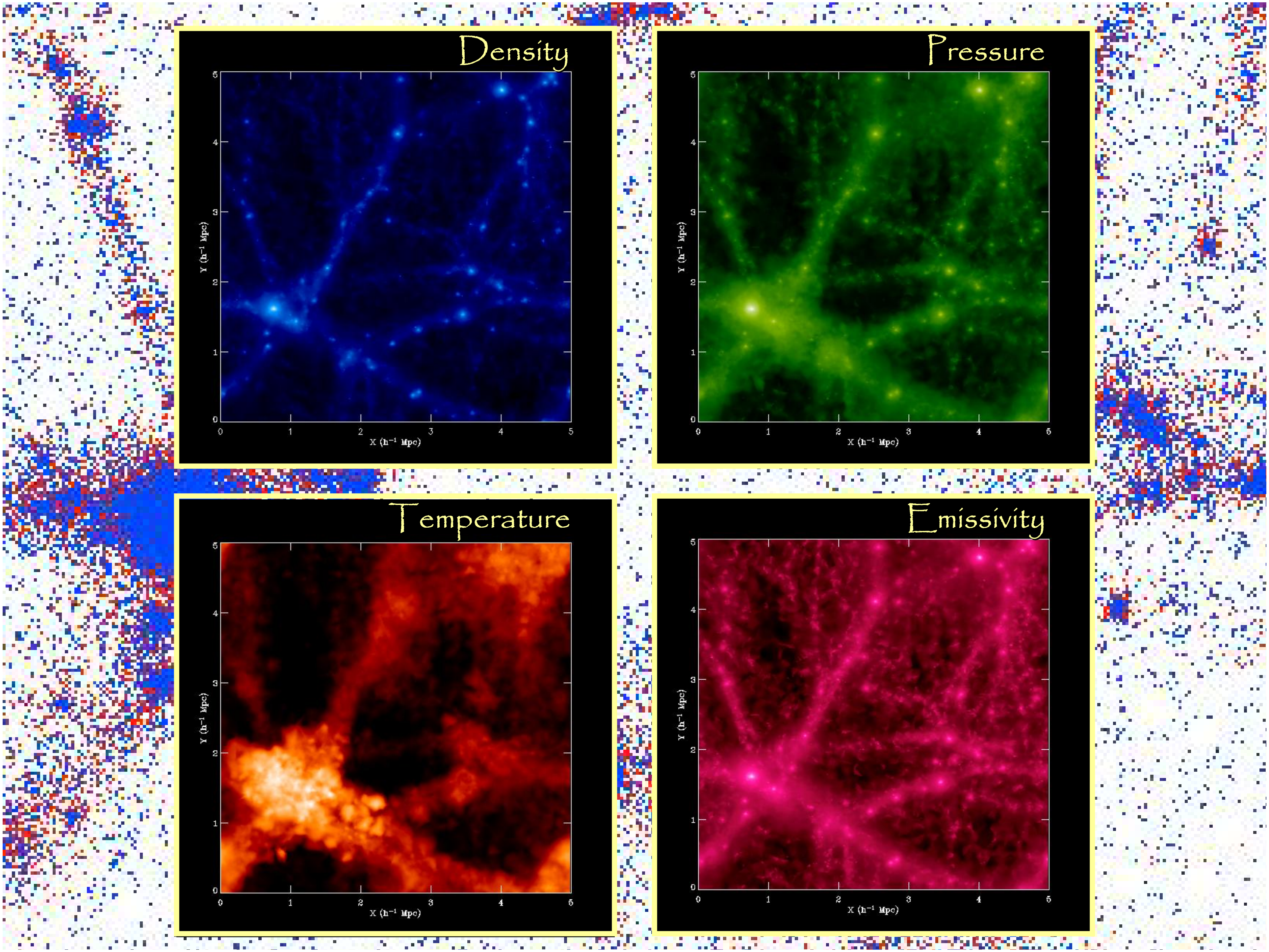
density-velocity divergence field relation

adheres closely to the

predictions of analytical perturbation theory:

$$\frac{1}{3}\frac{\nabla\cdot\mathbf{v}}{H_0} = \frac{f(\Omega_0)}{2}\left\{1 - (1+\delta)^{2/3}\right\}$$

(Bernardeau 1992)


Density
Pressure
Temperature
Emissivity
X (h⁻¹ Mpc)
Y (h⁻¹ Mpc)