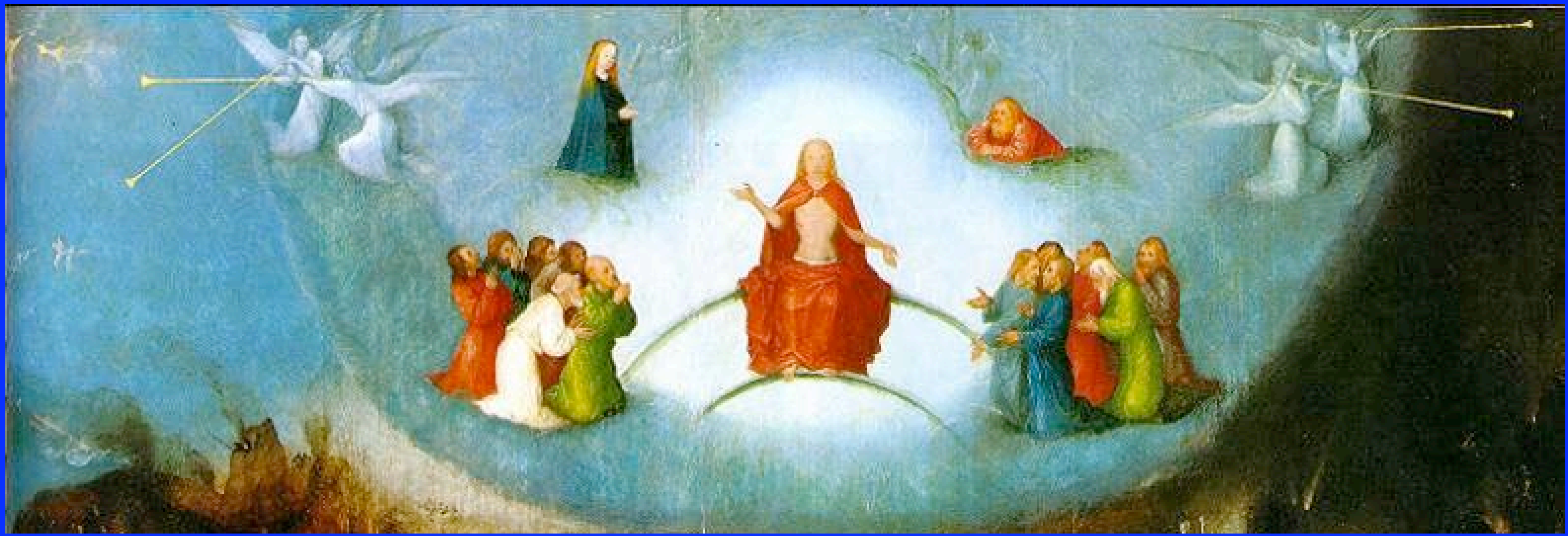
Phase Correlations in Harmonic Analysis of Cosmological Fluctuations

Peter Coles
(University of Nottingham)

OR….

The Quest for Cosmic Weirdness

“CONCORDANCE”



How Weird is the Universe?

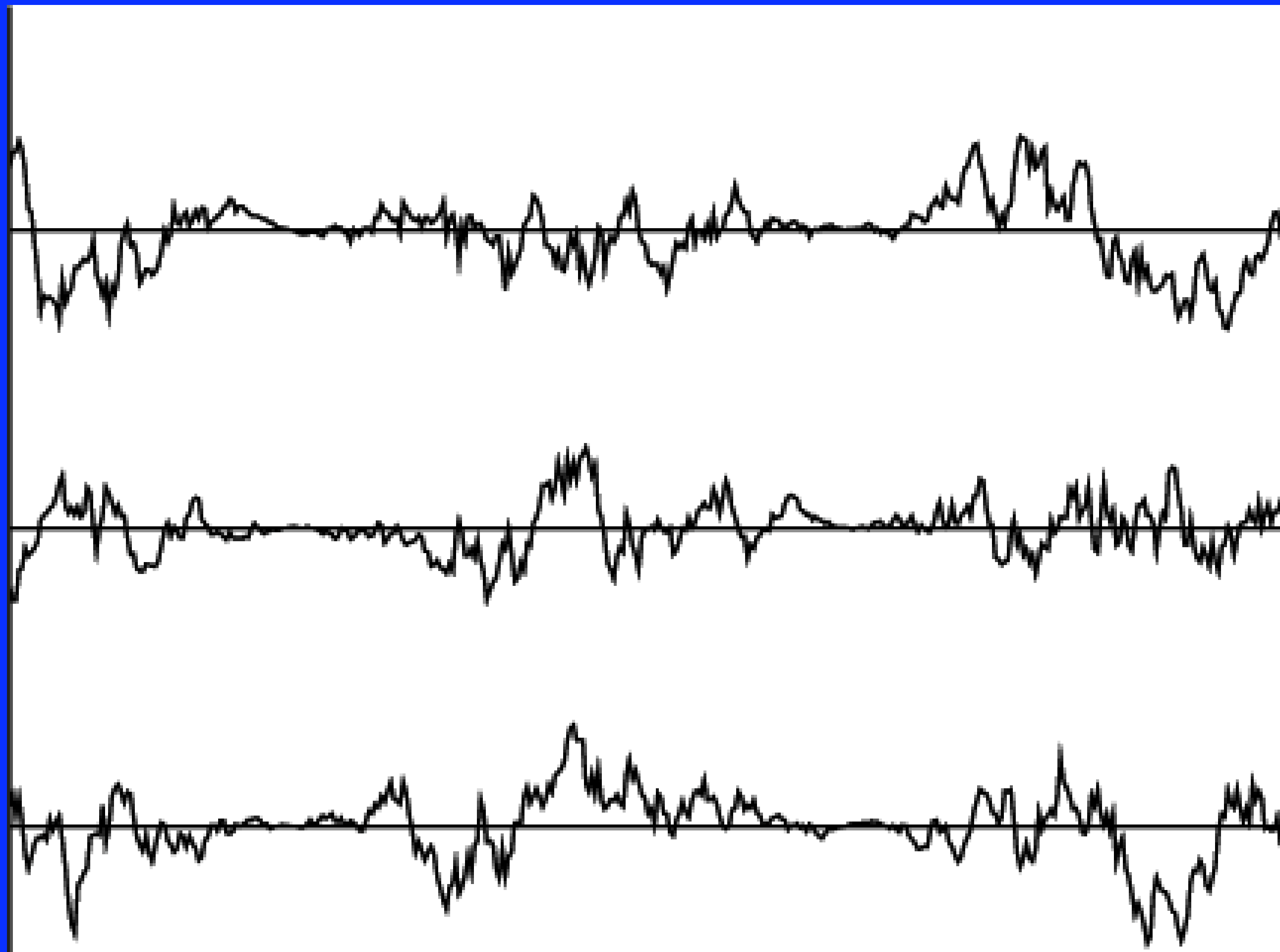
- The concordance cosmology is a “first-order” model
- In it (and other “first-order” models), the initial fluctuations were a statistically homogeneous and isotropic Gaussian Random Field (GRF)
- These are the “maximum entropy” initial conditions having “random phases” motivated by inflation.
- Weirdness = Non-Random Phases!
- Could be non-Gaussian, topologically non-trivial, non-linear (even with inflation), etc.
- Or masked by foreground contamination.
- Diagnosis needs appropriate statistical tools

OUTLINE

- The importance of phase information in cosmology
- Fourier phases in gravitational clustering
- Spherical Harmonic phases
- Illustration using preliminary WMAP data
- Some other funny properties of WMAP

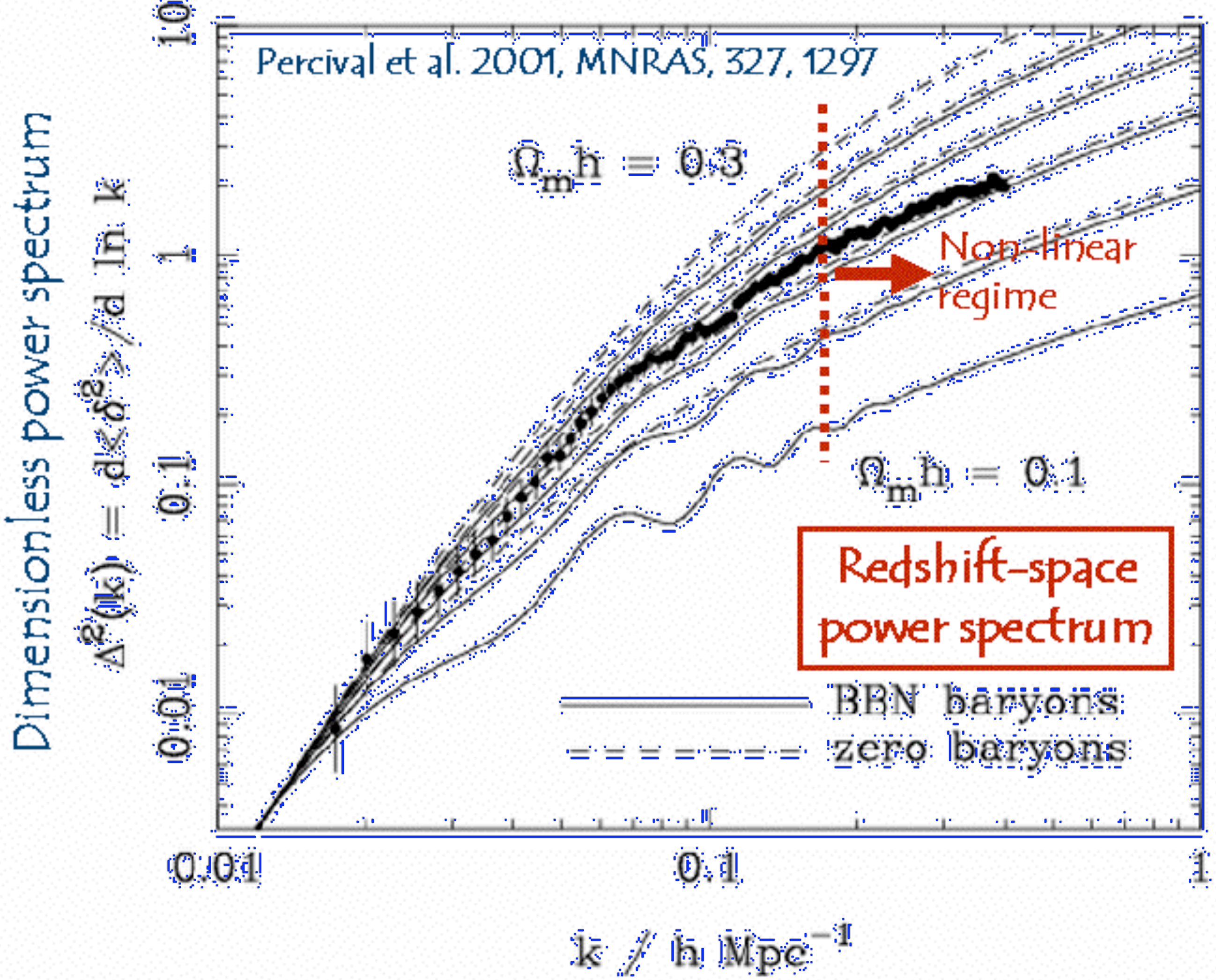
Fourier Phases

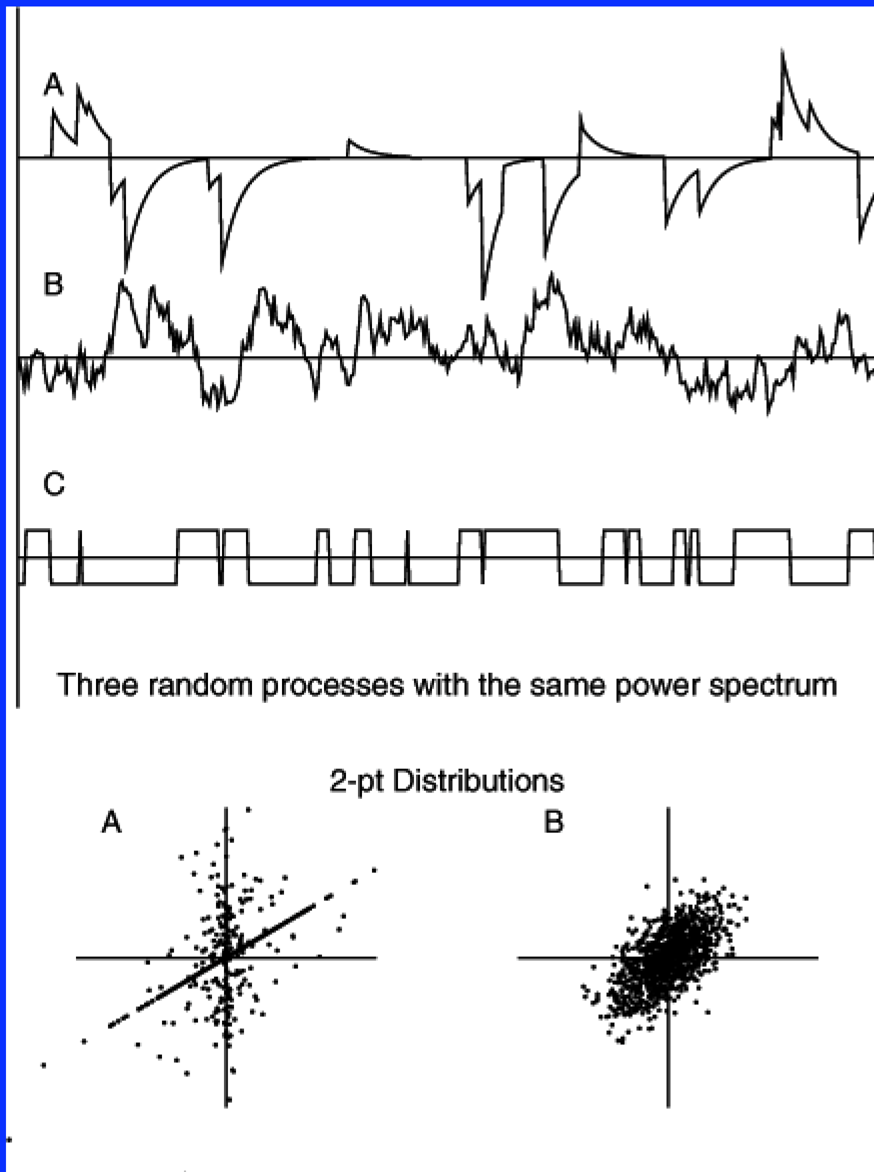
- The usual thing $\delta(x) = \sum_k \delta(k) \exp(ik \cdot x)$
- where $\delta(k) = |\delta(k)| \exp[i\varphi_k]$
- In a homogeneous and isotropic GRF then the phases φ are random...
- ..apart from $\delta(k) = \delta(-k)^*$
- ..as are differences, e.g. $\varphi_{k_1} - \varphi_{k_2}$

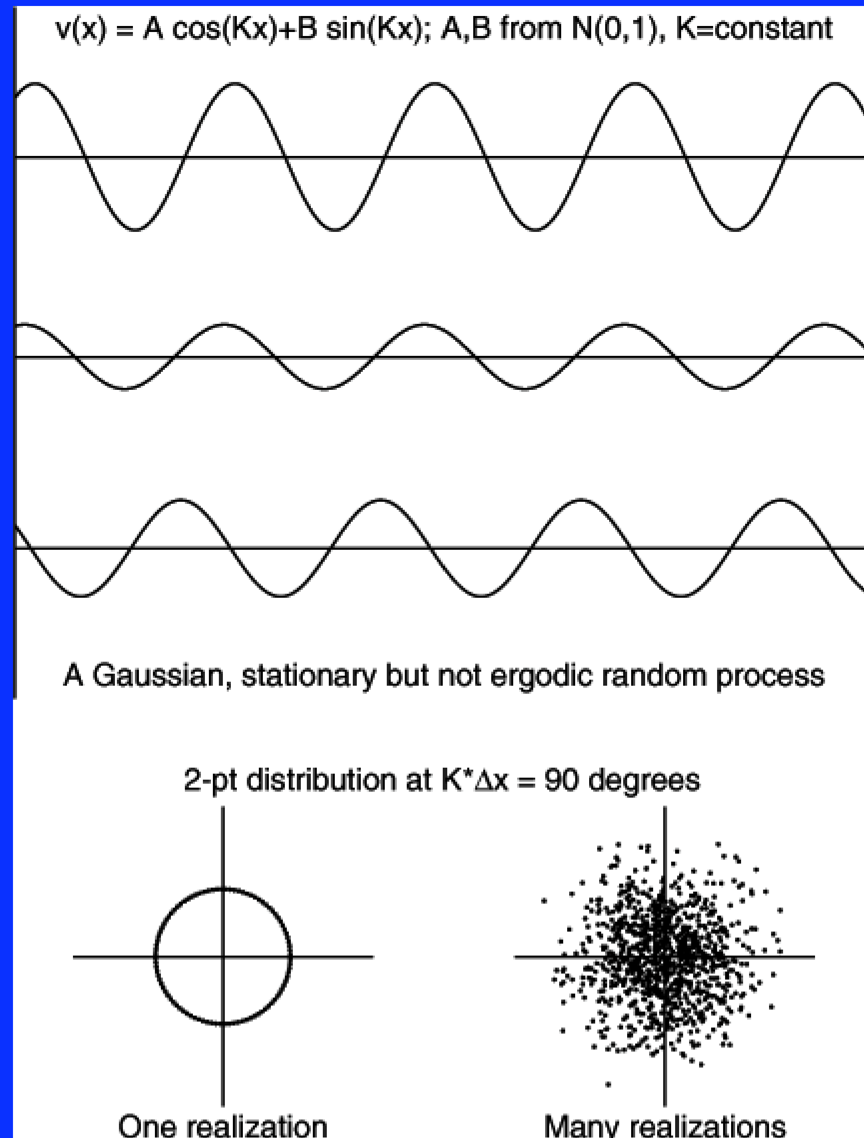


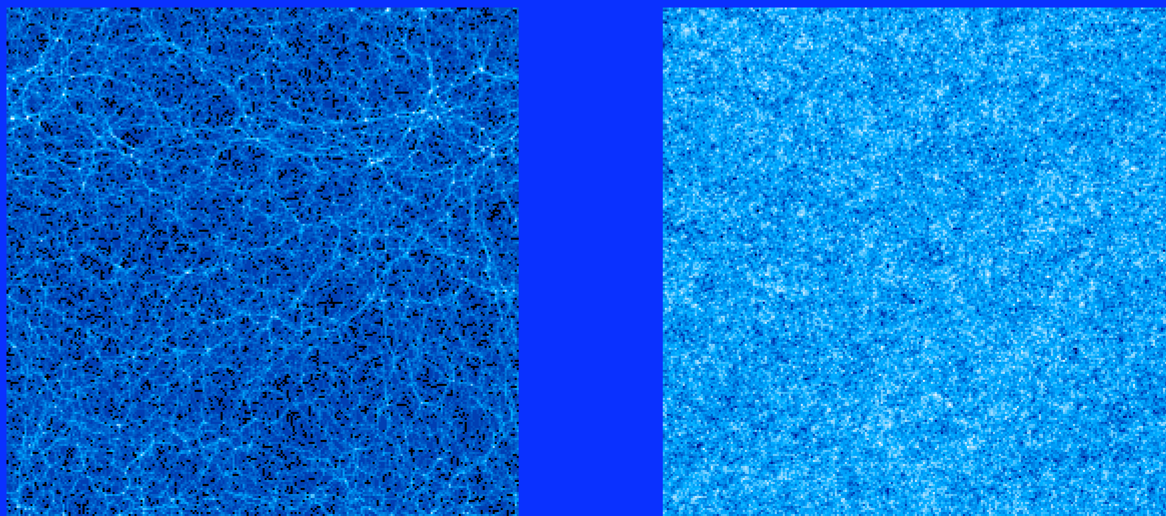
Three realizations of a Gaussian non-stationary process

The Galaxy Power Spectrum




A
B
C
Three random processes with the same power spectrum
2-pt Distributions
A
B


v(x) = A cos(Kx)+B sin(Kx); A,B from N(0,1), K=constant
A Gaussian, stationary but not ergodic random process
2-pt distribution at K*Δx = 90 degrees
One realization
Many realizations



Polyspectra and phases

- Power spectrum $P(k) = \langle \delta(k)\delta(-k) \rangle$
- Contains no phase information
- Bispectrum $B(k_1, k_2) = \langle \delta(k_1)\delta(k_2)\delta(-k_1 - k_2) \rangle$
- Straightforward for higher-order polyspectra
- These are all zero for random phases
- Non-linearities produce non-zero polyspectra, e.g. bispectrum measures quadratic non-linearity

Quadratic Phase Coupling

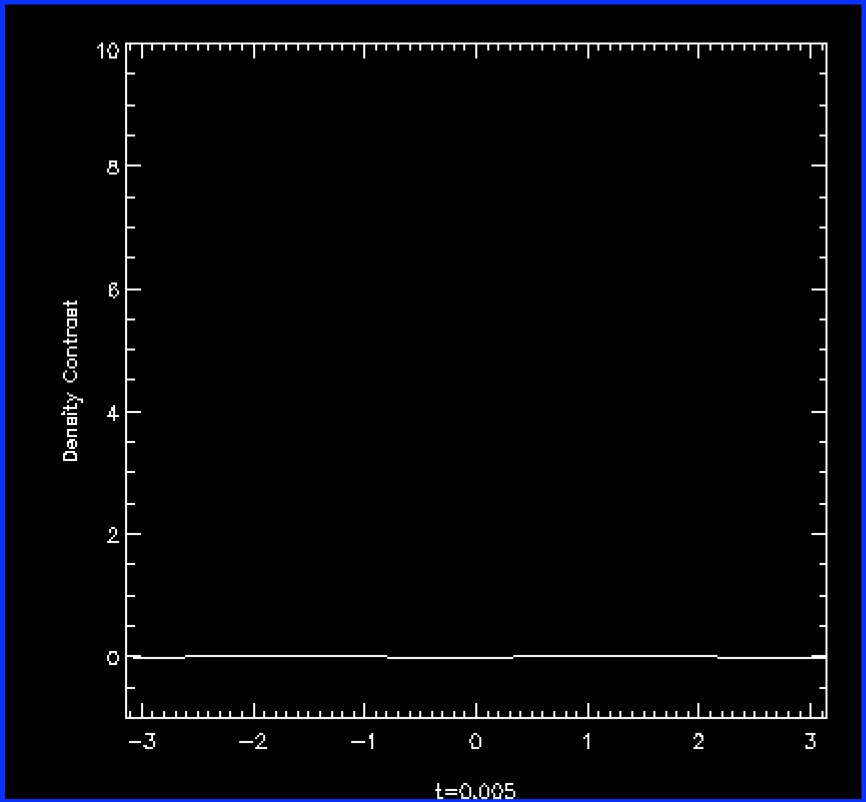
- E.g. Bispectrum $B(k_1,k_2) = \left\langle \delta(k_1)\delta(k_2)\delta(-k_1-k_2) \right\rangle$
- Consider $\delta = \delta_1 + \delta_2$
- Where $\delta_i = a_i \exp(ik_i x + \phi_i)$
- Squaring gives, e.g. $(2k_i, 2\phi_i)$
- And $(k_1 + k_2, \phi_1 + \phi_2)$
- So the phase of $b(k_1,k_2) = \delta(k_1)\delta(k_2)\delta(-k_1-k_2)$
- Is not random…

Gradients, Wrapping and Correlations

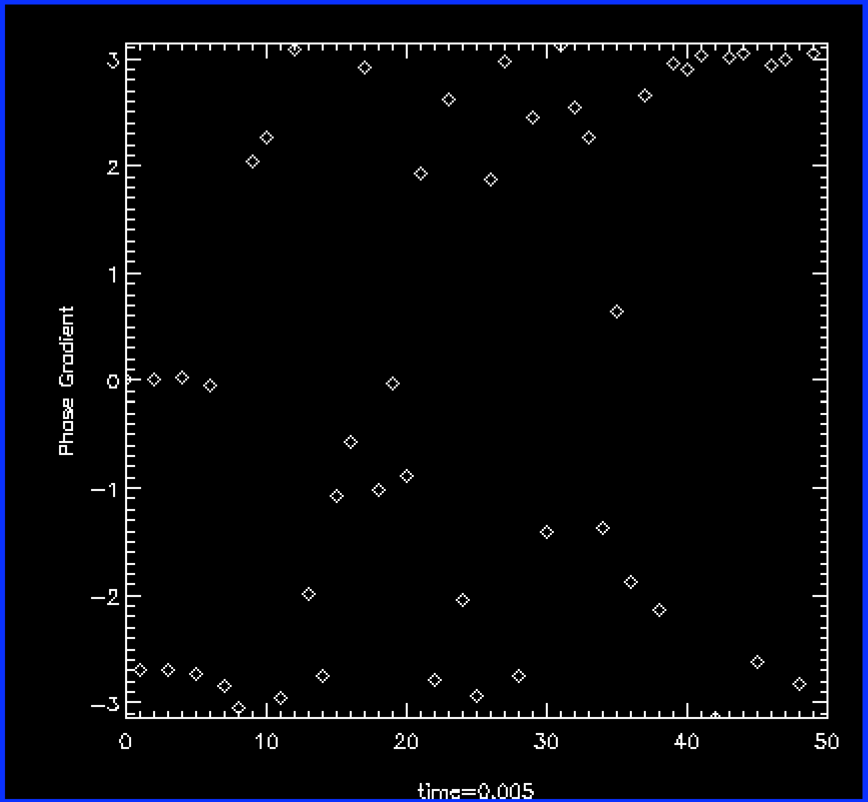
- Change origin by *x*, and φ_k changes by *kx*, but phase gradients change by a constant.
- During evolution ..

$$\varphi(t) = \varphi(0) + m.2\pi$$

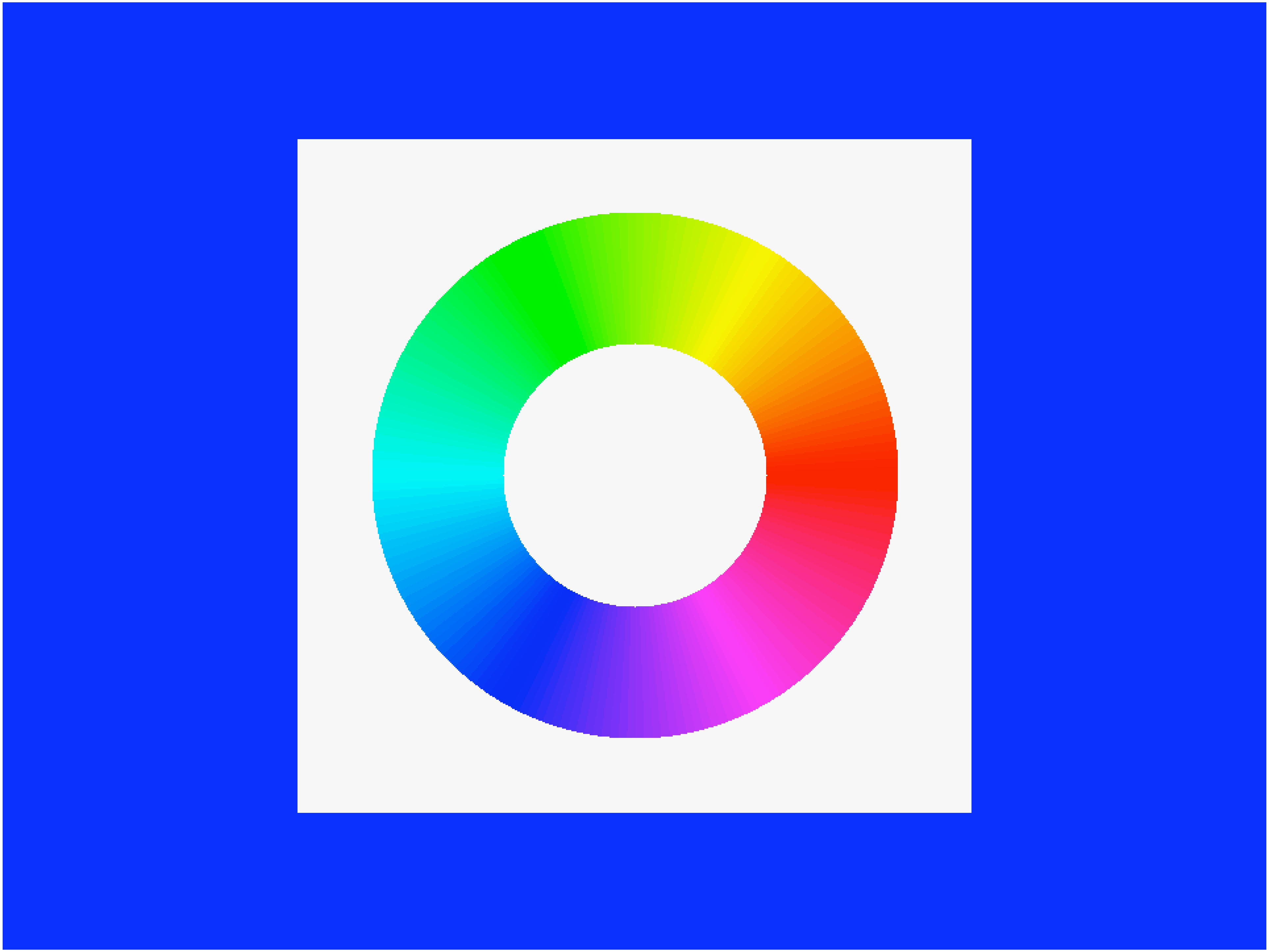
- where *m* can be very large. One-point distribution appears random.
- But phases of different modes are not independent…and phase differences between different modes are not uniform.

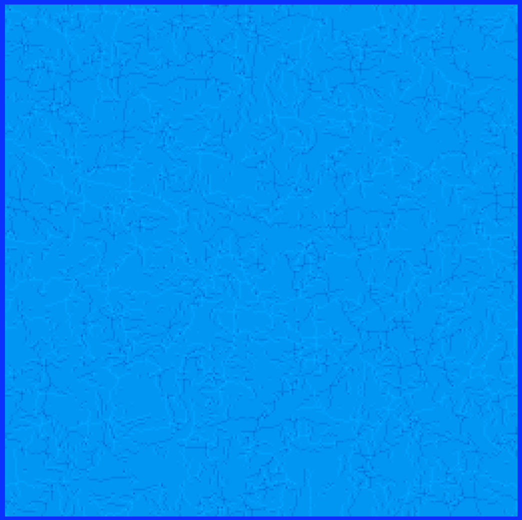


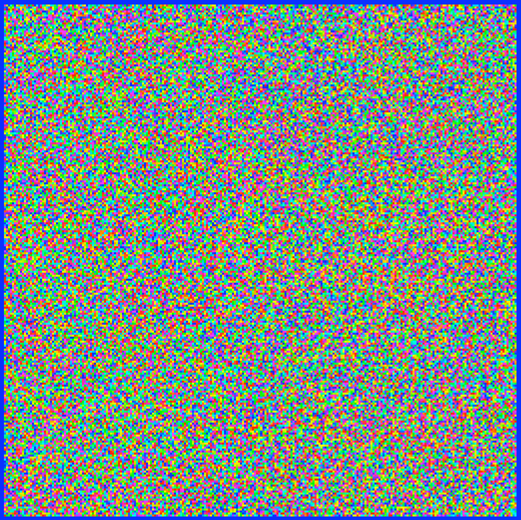
Density Contrast
10
8
6
4
2
0
−3
−2
−1
0
1
2
3
t=0.005

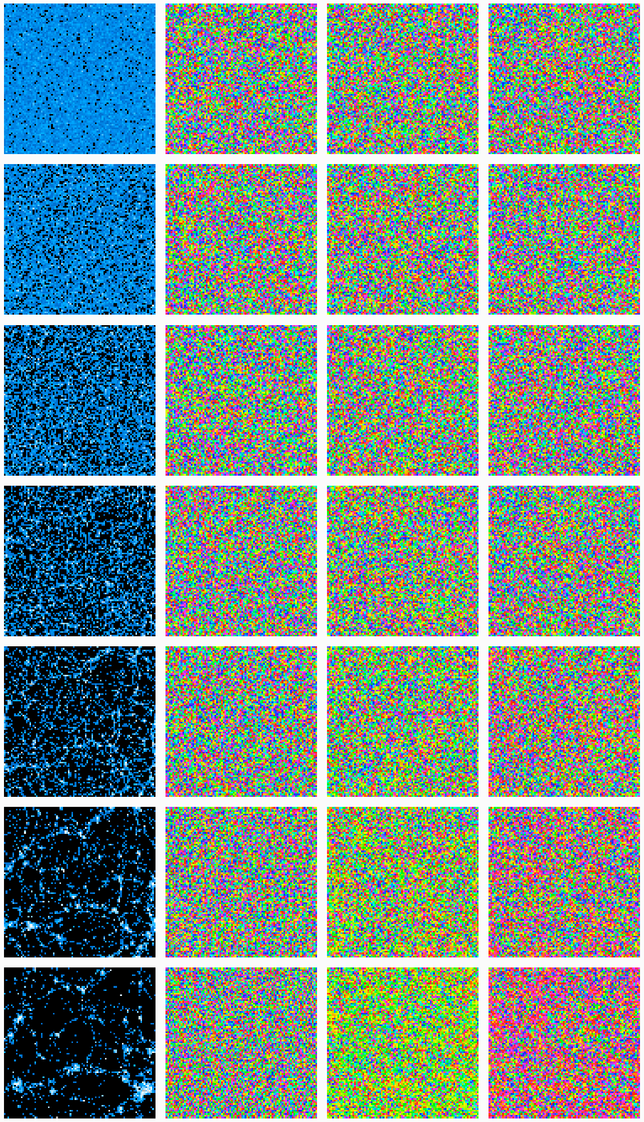


Phase Gradient
3
2
1
0
−1
−2
−3
0
10
20
30
40
50
time=0.005







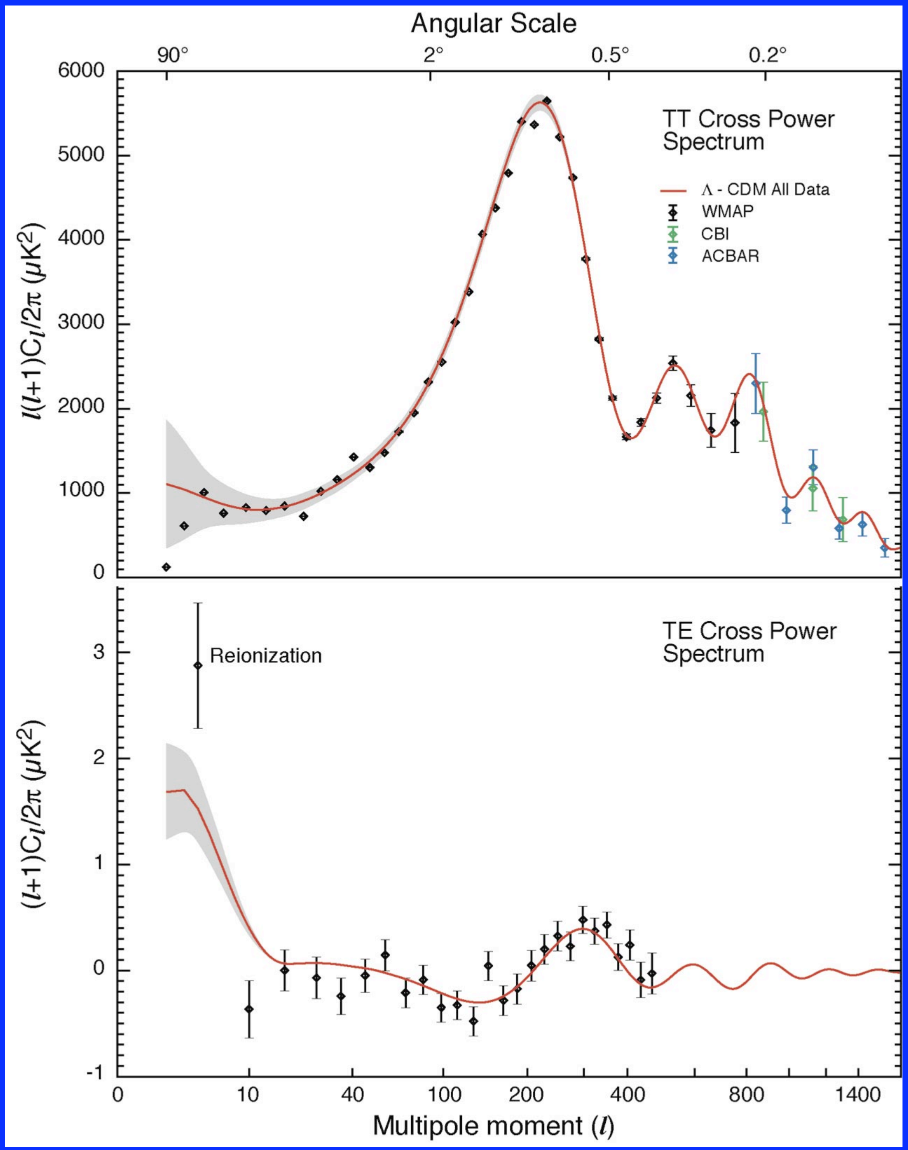


Some papers on Fourier Phases

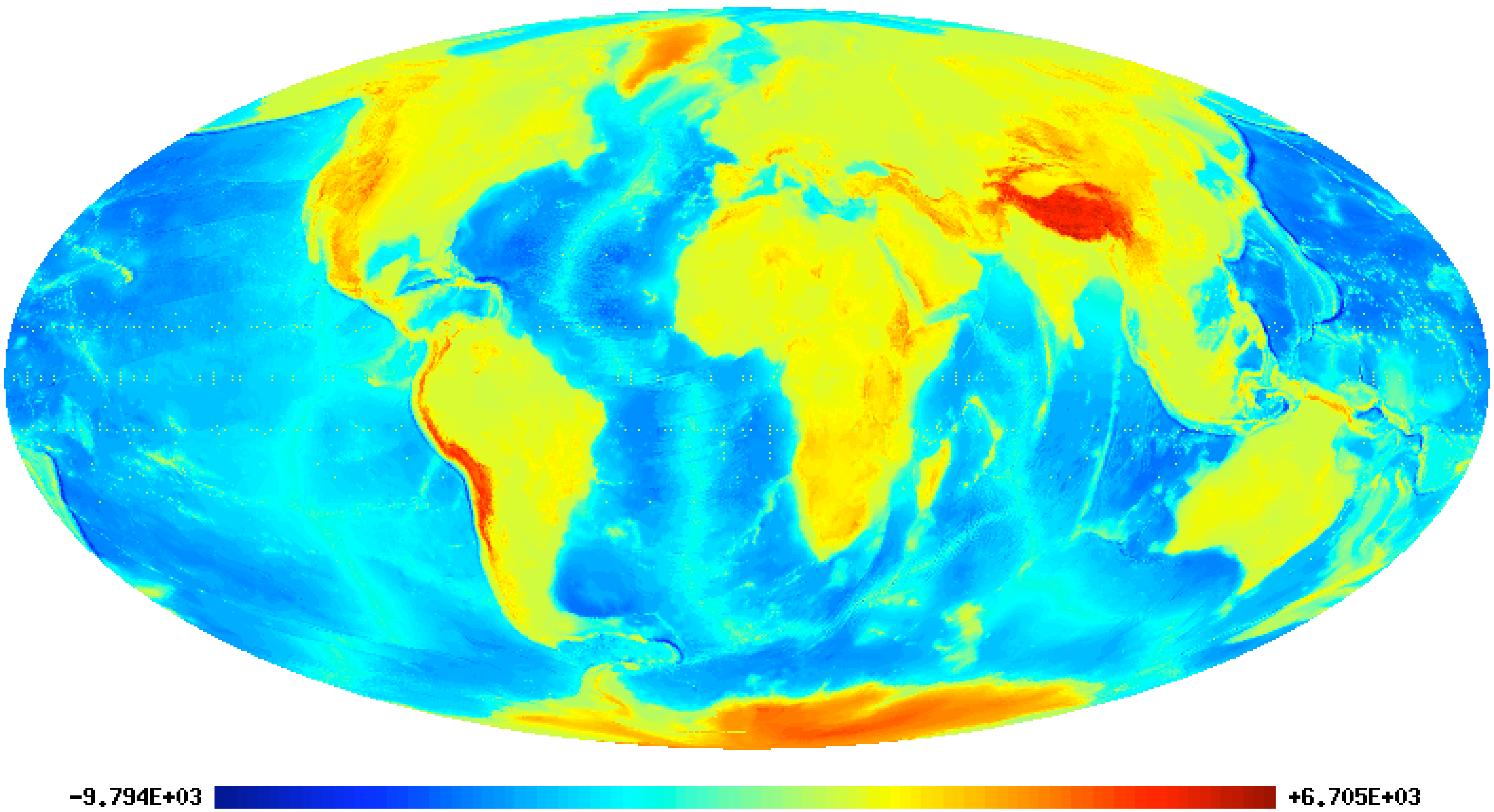
- Chiang & Coles, 2000, MNRAS, 311, 809-824
- Coles & Chiang, 2000, Nature, 406, 376-378
- Chiang, Coles & Naselsky, 2002, MNRAS, 337, 488-494
- Watts & Coles 2002, MNRAS, 338, 806
- Watts, Coles & Melott, 2003, ApJL, 589, L61
- For animations, etc, see also: http://www.nottingham.ac.uk/~ppzpc/phases/index.htm

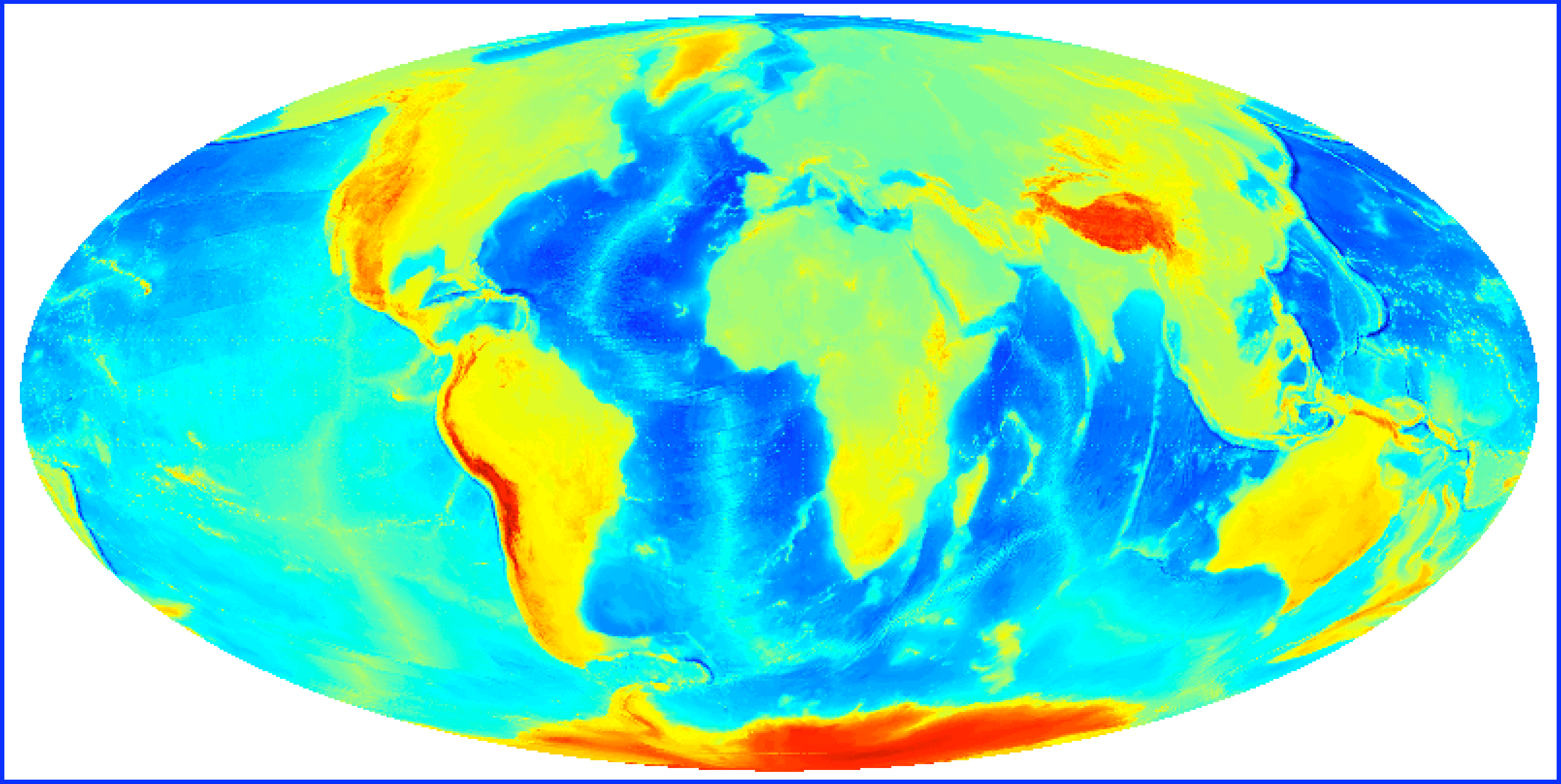
Spherical Harmonic Phases

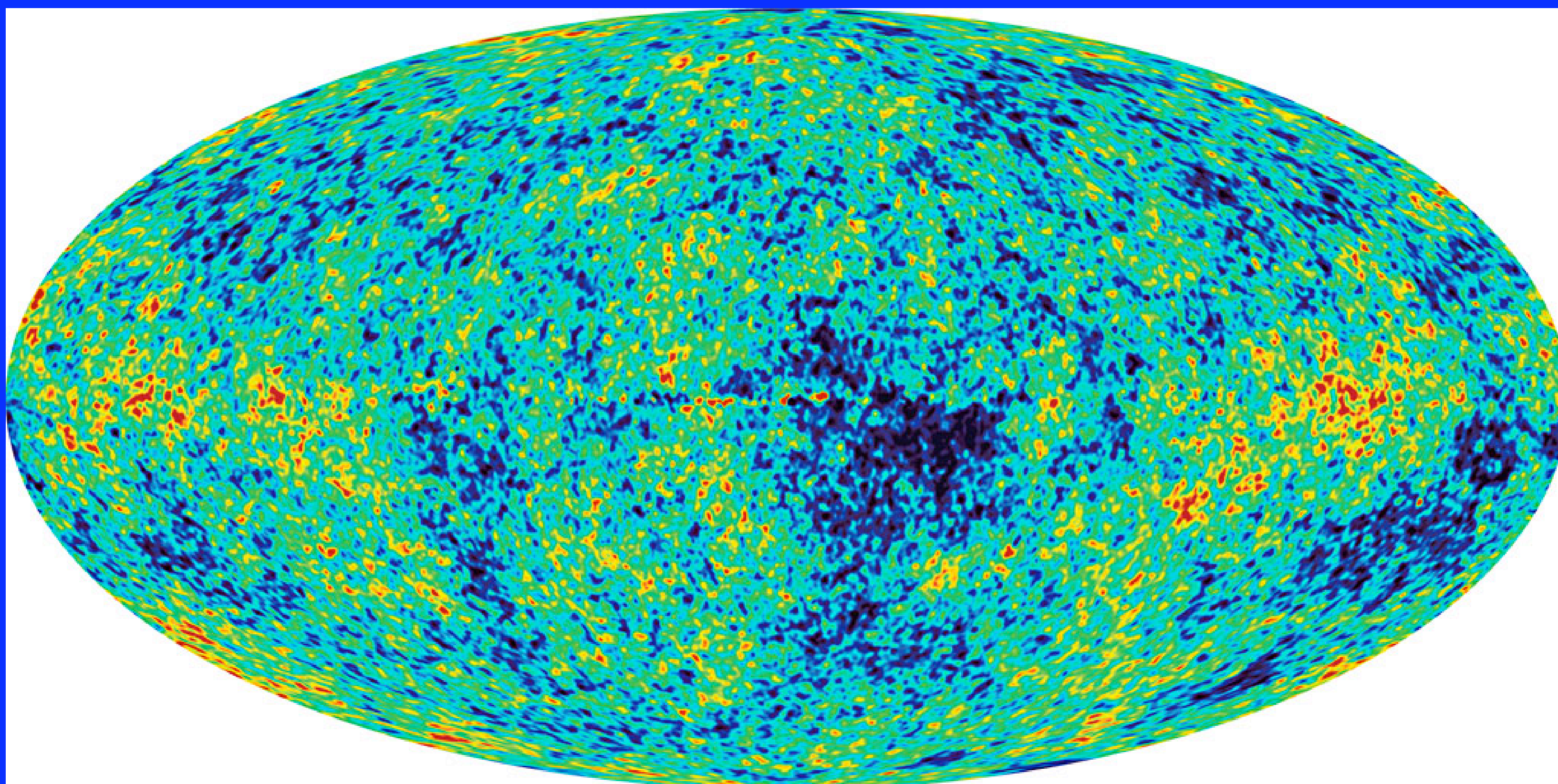
- The usual thing $\frac{\Delta T(\theta,\phi)}{T} = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=+l} a_{l,m} Y_{lm}(\theta,\phi)$
- where $a_{l,m} = |a_{l,m}| \exp\left[i\varphi_{l,m}\right]$
- If the fluctuations are a homogeneous and isotropic GRF then the phases $\varphi_{l,m}$ are random...
- ..apart from $a^{*}_{l,m} = a_{l,-m}$
- ..as are differences, e.g. $\varphi_{l,m} - \varphi_{l,m-1}$

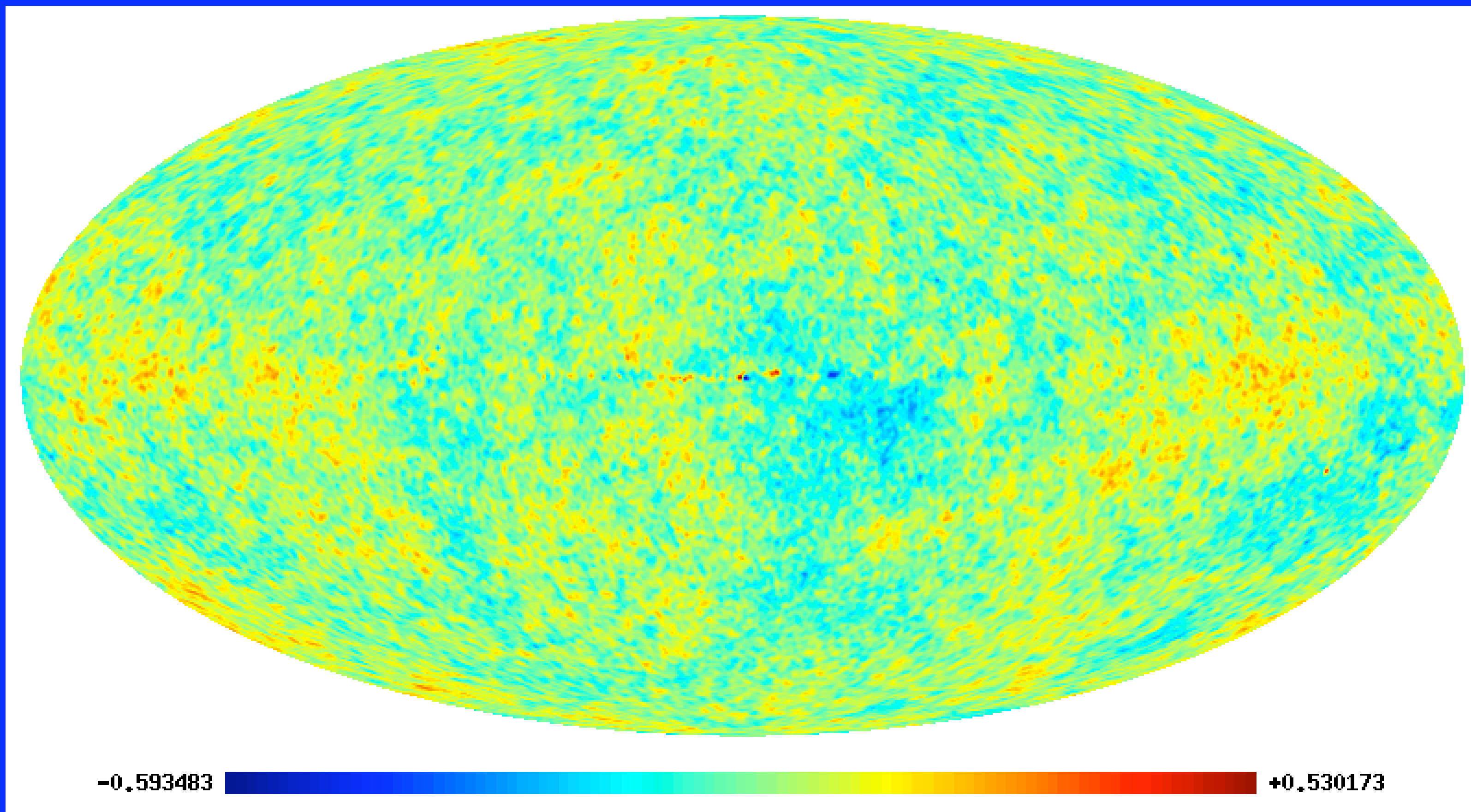


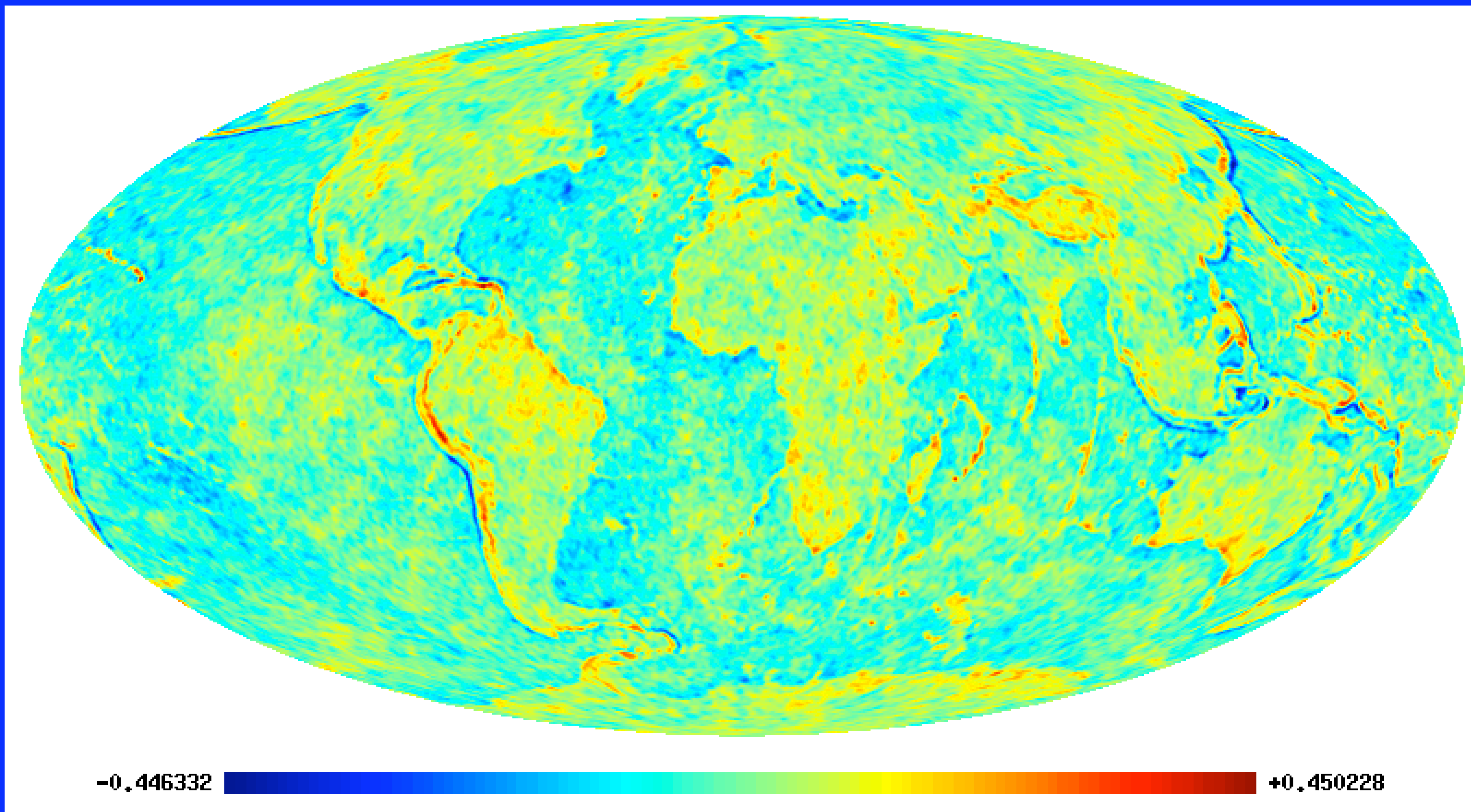
Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
l(l+1)C_l/2π (μK²)
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
3
2
1
0
-1
(l+1)C_l/2π (μK²)
Reionization
TE Cross Power Spectrum
0
10
40
100
200
400
800
1400
Multipole moment (l)

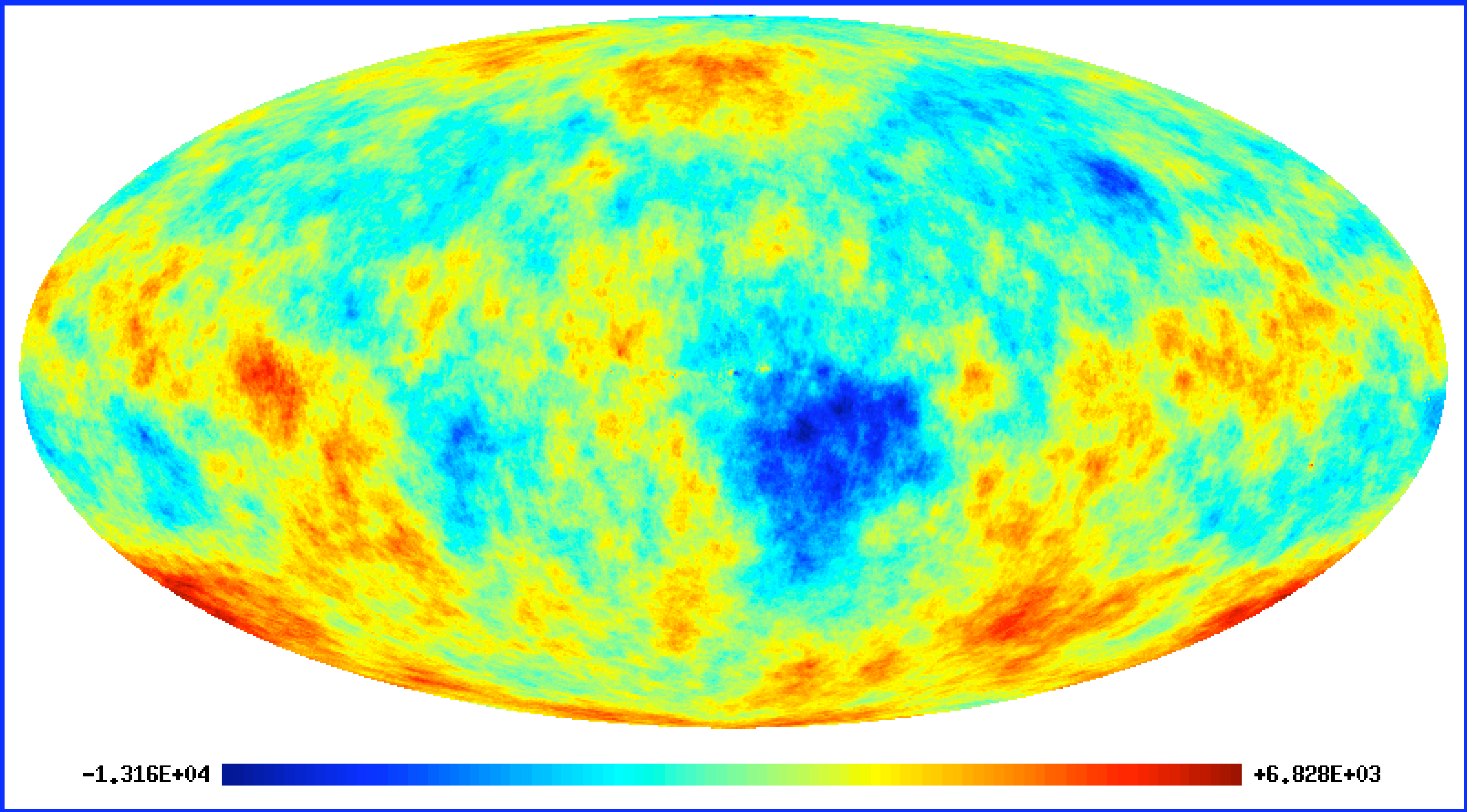

-9.794E+03
+6.705E+03



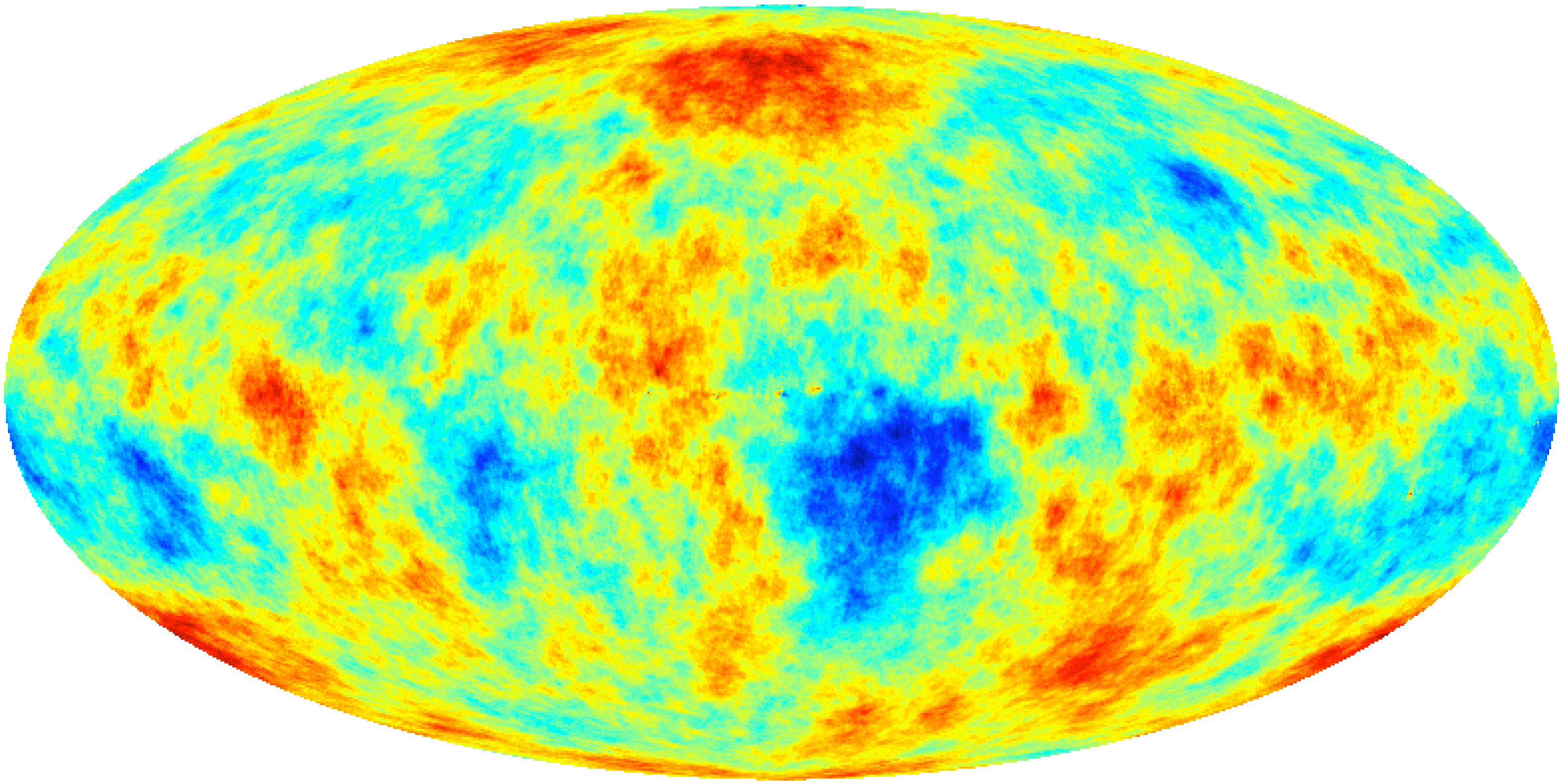


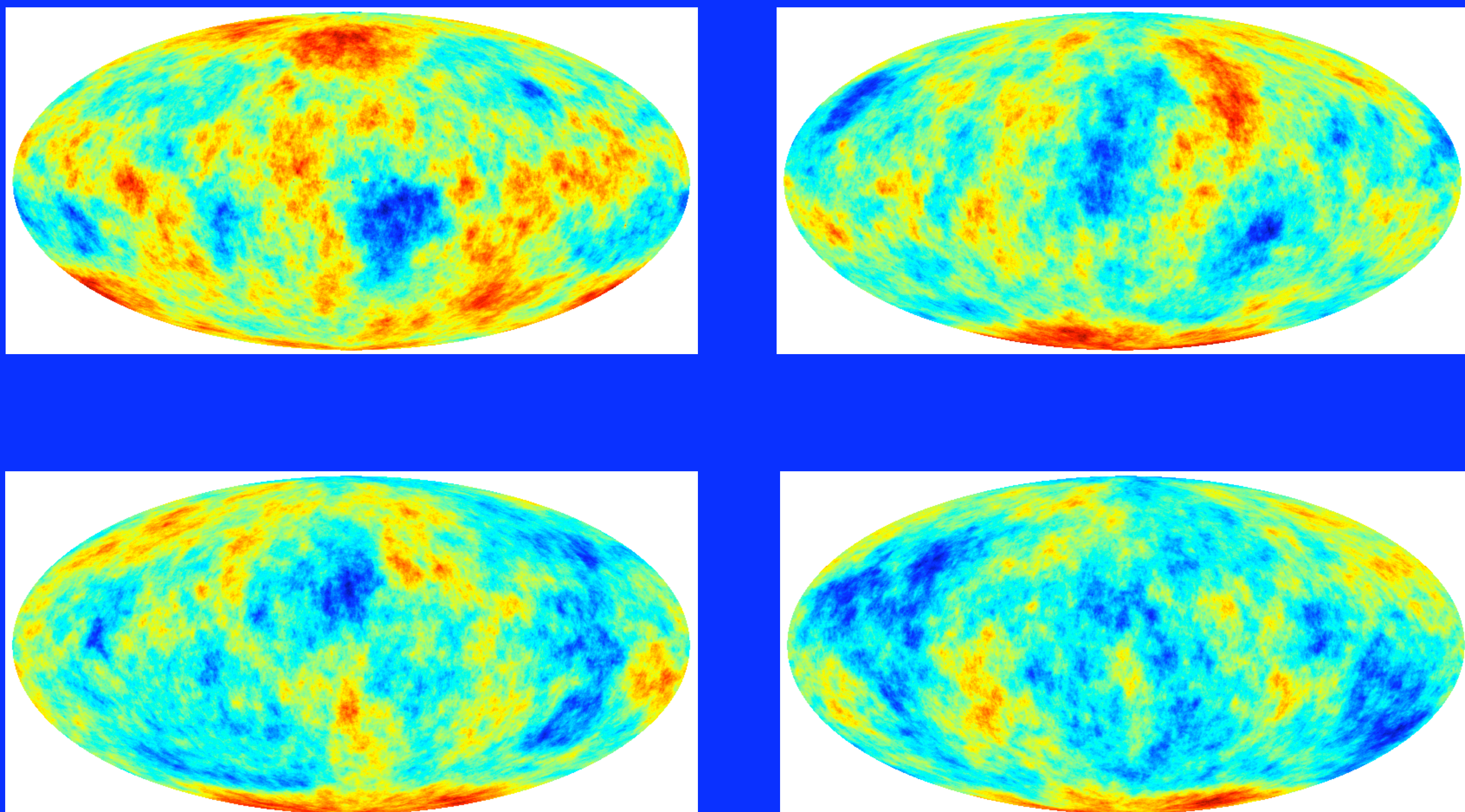

-0.593483
+0.530173


-0.446332
+0.450228



-1.316E+04
+6.828E+03





Comments

- The simplest statistics are distributions of phases and phase-differences: these are highly sensitive to departures from statistical homogeneity.
- Different signatures are more sensitive to non-Gaussianity in statistically homogeneous fields.
- BUT the phases vary in a complicated way under rotations
- AND they will not be random if there is a mask
- This is OK for a non-parametric approach, since it can all be included in fast Monte Carlo simulations

Apologies to Bayesians

- This is a frequentist approach…
- If you have a sufficiently well-developed alternative model, be Bayesian and infer parameters of a model (evidence, etc)
- If you don't, you simply have to try rejecting the null hypothesis using non-parametric methods
- It's a question of whether you test in model space or data space!

Kuiper's Statistic

- Non-parametric test for uniformity on the unit circle..c.f Kolmogorov-Smirnov
- Define

$$X_i = \vartheta_i \big/ 2\pi$$

- Then

$$S_n^+ = \max\left\{\frac{1}{n} - X_1, \frac{2}{n} - X_2, \ldots, 1 - X_n\right\}$$

- and

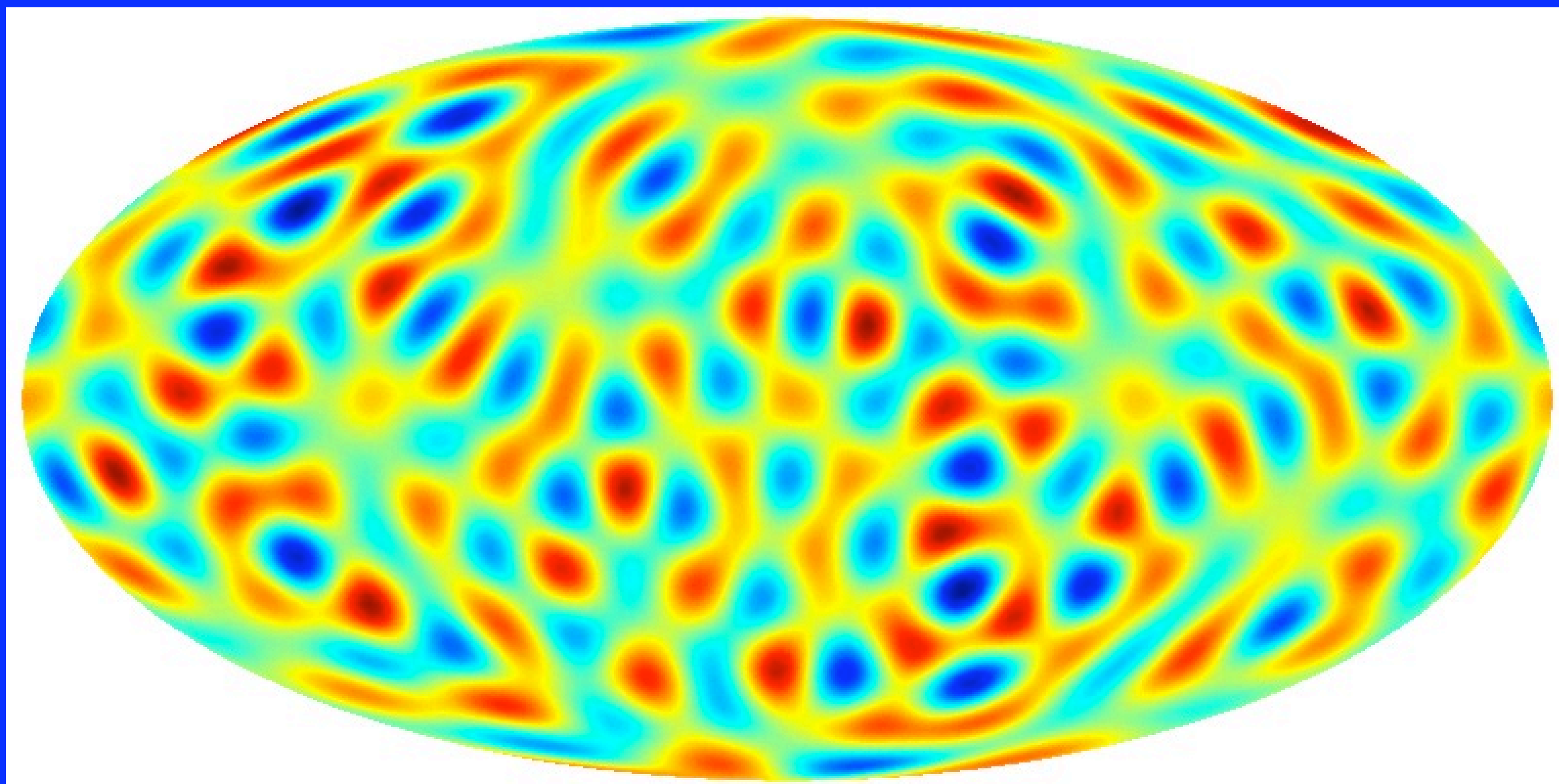
$$S_n^- = \max\left\{X_1, X_2 - \frac{1}{n}, \ldots, X_n - \frac{n-1}{n}\right\}$$

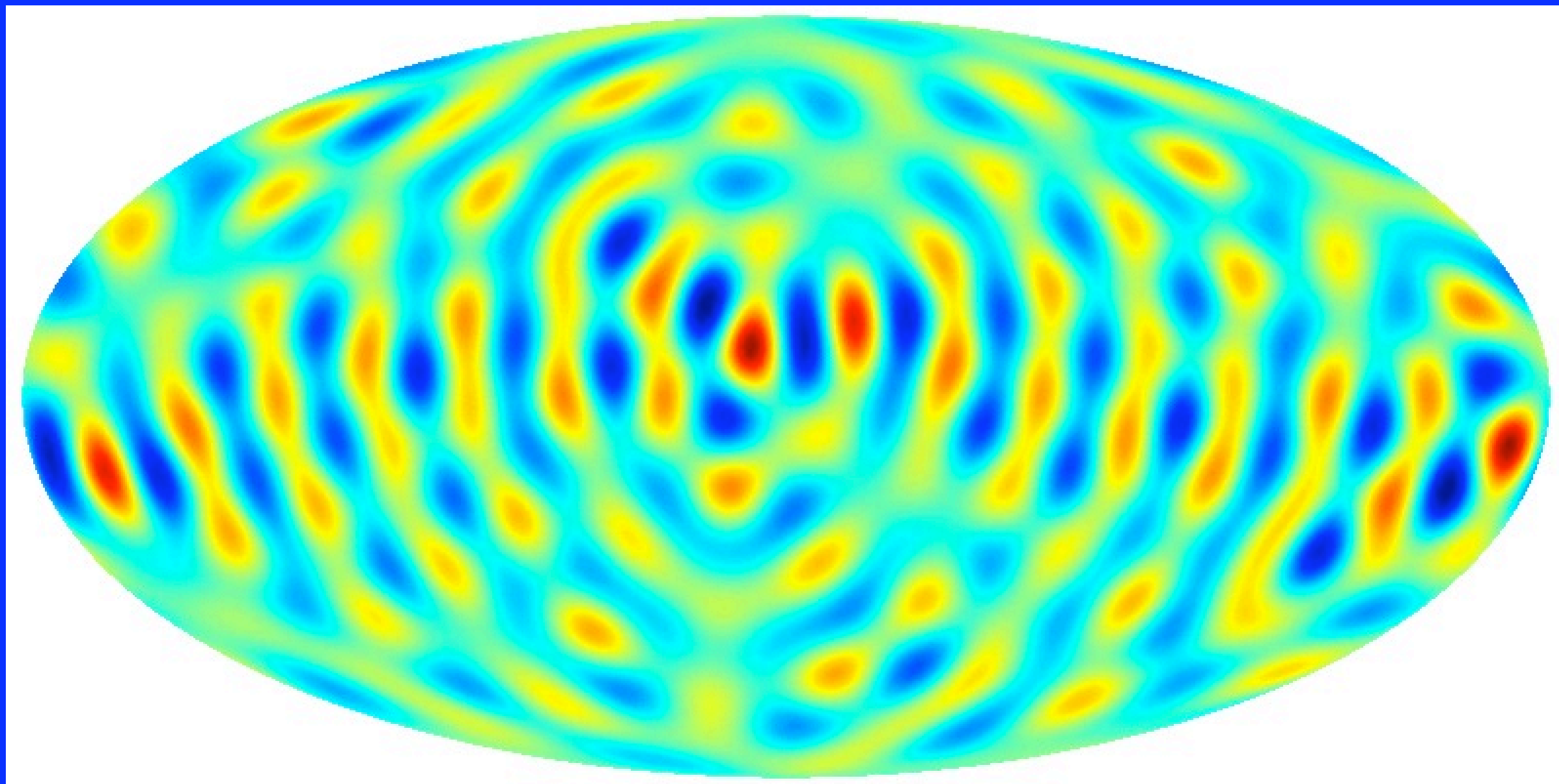
- the statistic is

$$V = \left\lfloor S_n^+ + S_n^- \right\rfloor A(n)$$

Results

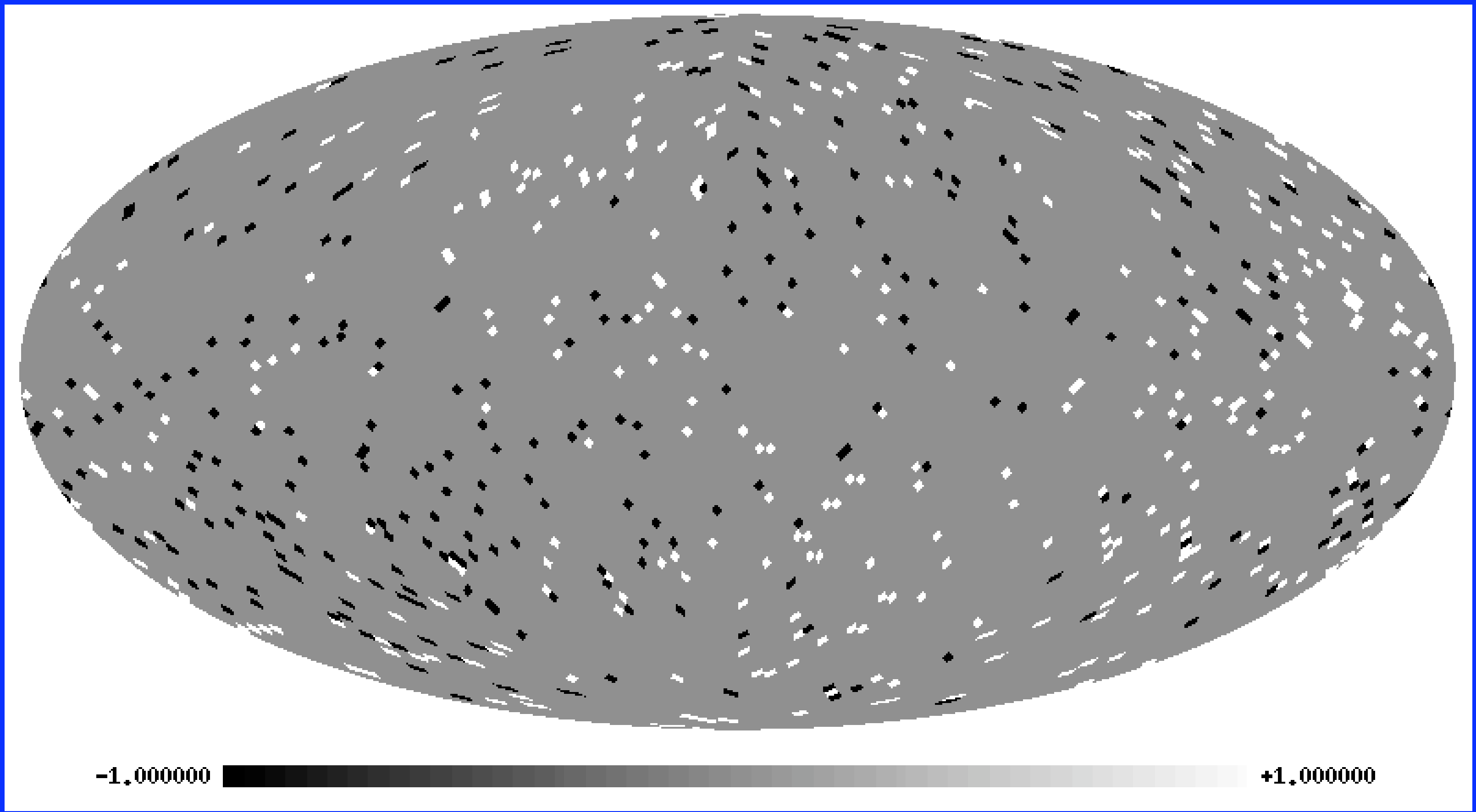
- Simplest thing is to do this with phase differences
- This is not a very good test of quadratic non-Gaussianity, but is good for statistical isotropy
- The ILC, TOH etc are all weird at $l=16$
- biggest departure in phase differences at fixed l, rather than fixed m
- More details in Coles et al., 2004, MNRAS, 350, 989





Weirdness in WMAP...

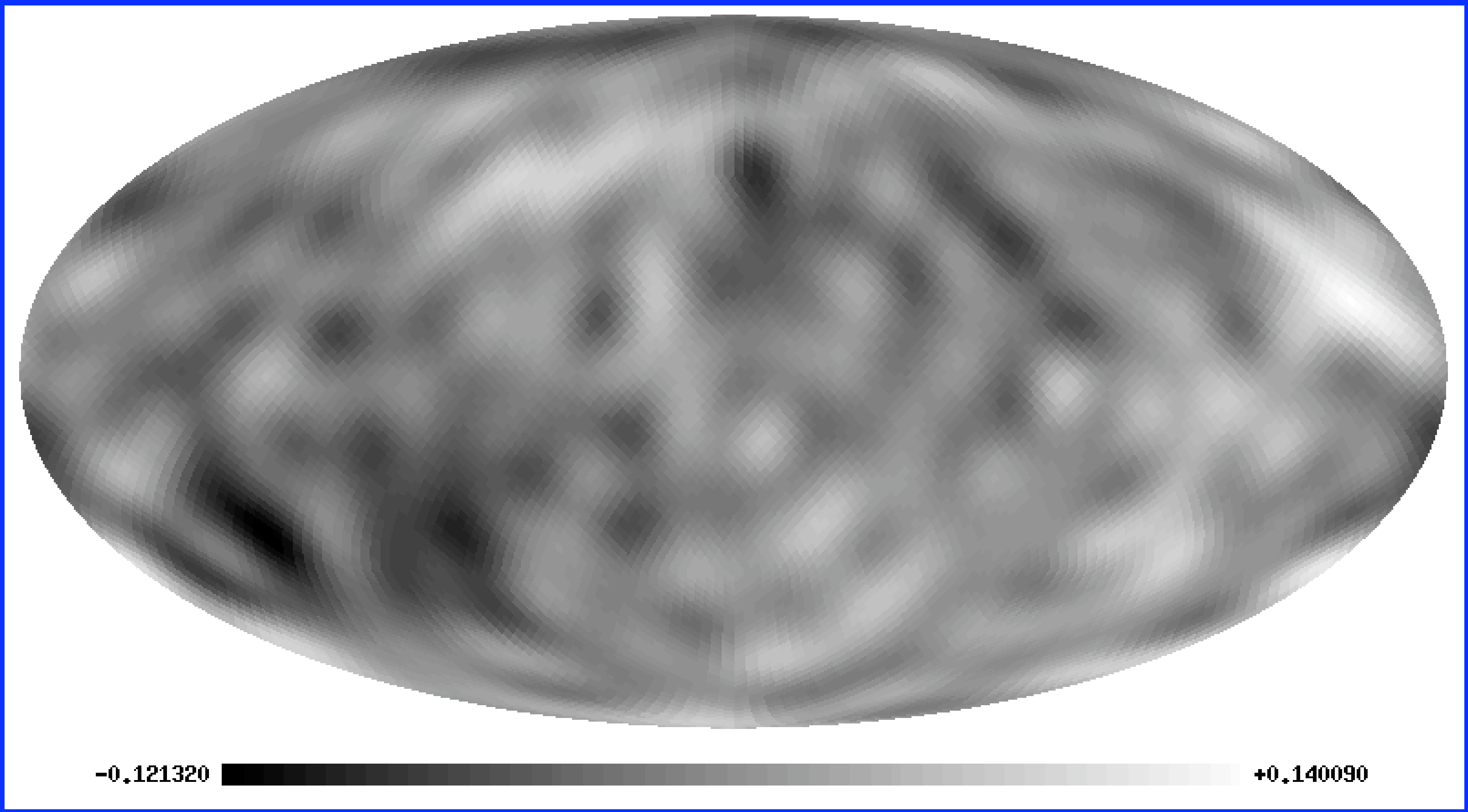
- Dineen & Coles, 2004, MNRAS, 347, 52, WMAP maps correlate with Faraday Rotation measures
- Eriksen et al.,2004, ApJ, 604, 14, WMAP North-South Divide
- Chiang et al. 2003, ApJ, 590, L65…mode correlations at high l
- The WMAP data are preliminary, the noise is known to be non-stationary, and the foreground subtraction is not perfect…maybe that's all there is to it!

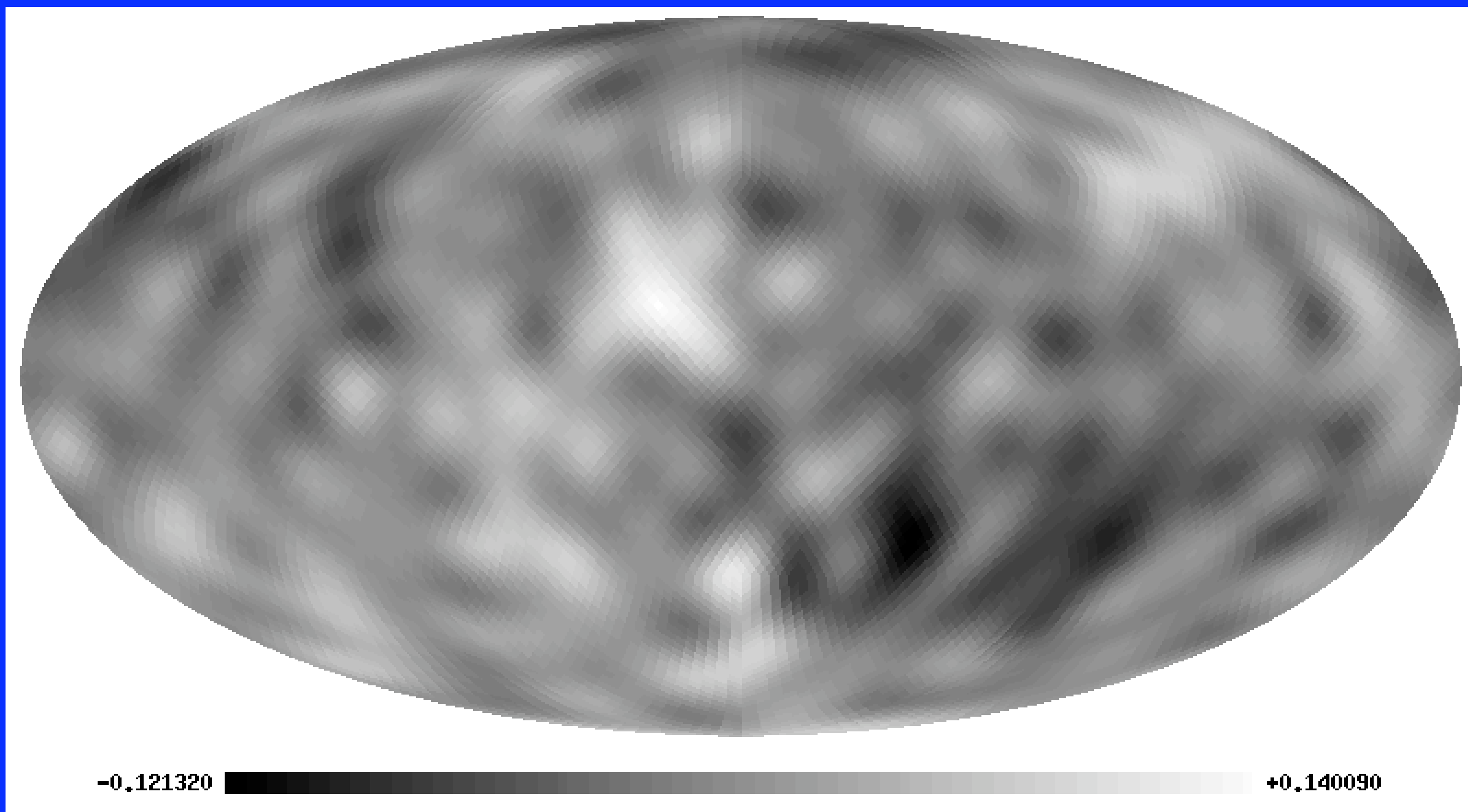


-1.000000
+1.000000

Faraday Rotation of the CMB

- RN measures the integral of the Galactic Magnetic field along the line of sight to the source
- Mapping the galactic magnetic field using RM values is *hard…*
- However, the rotation of the CMB polarisation is ~ 0.5 degrees at 30GHz
- Use CMB to map the Galaxy's B-field
- Similar scale of challenge to B-mode measurement
- Magnetic fields change E into B too…so this can confuse the primordial polarisation signal.


-0.121320
+0.140090

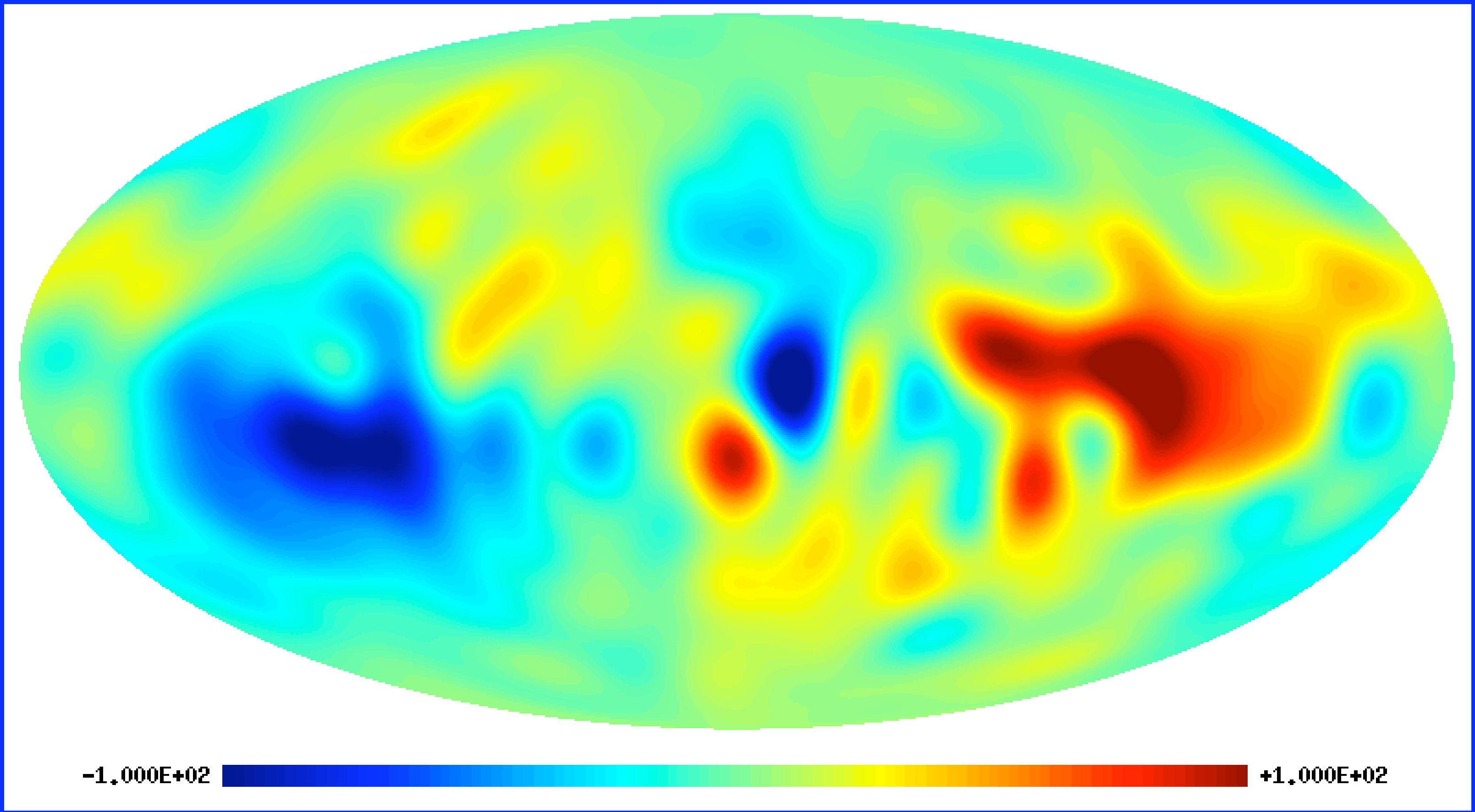


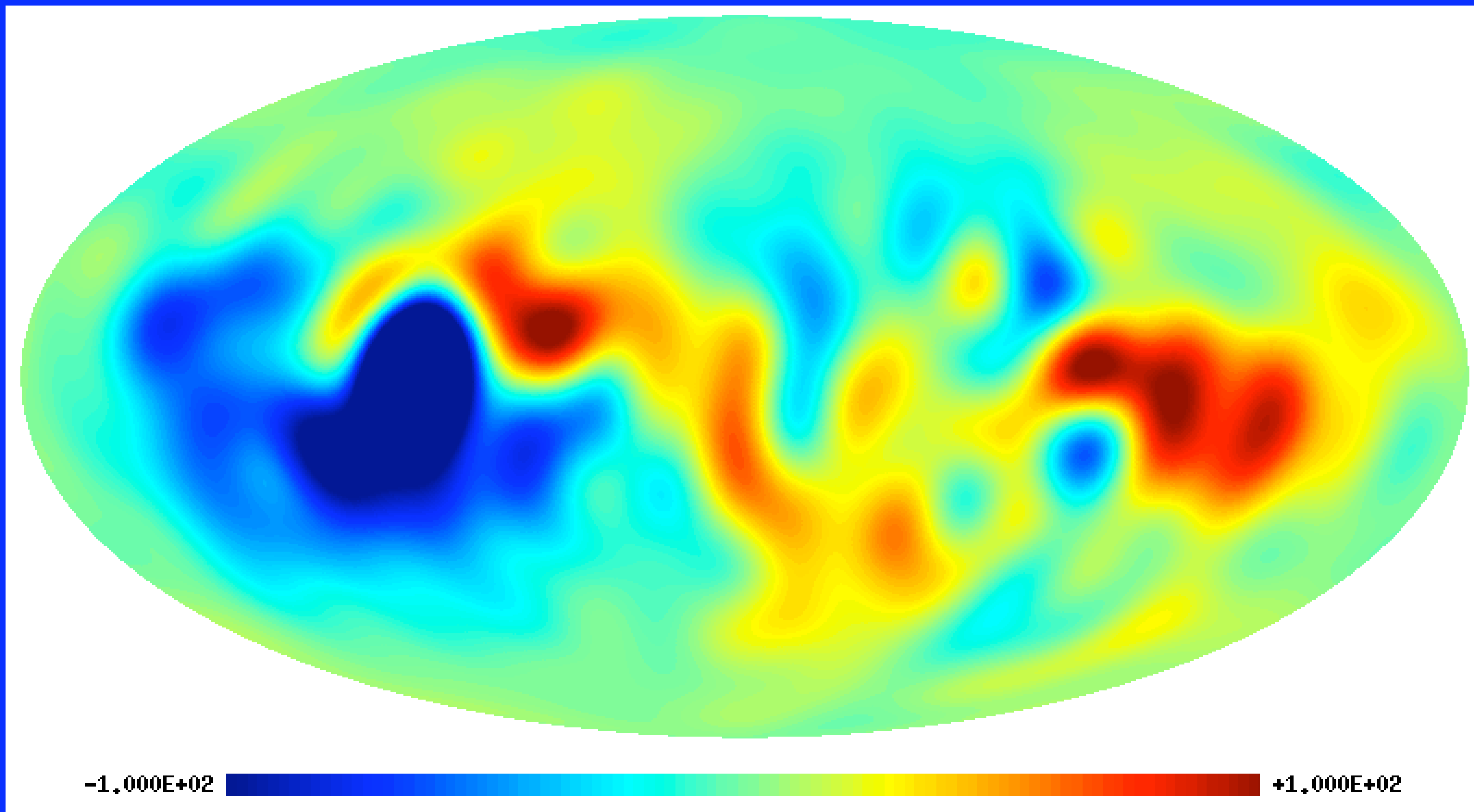
-0.121320
+0.140090

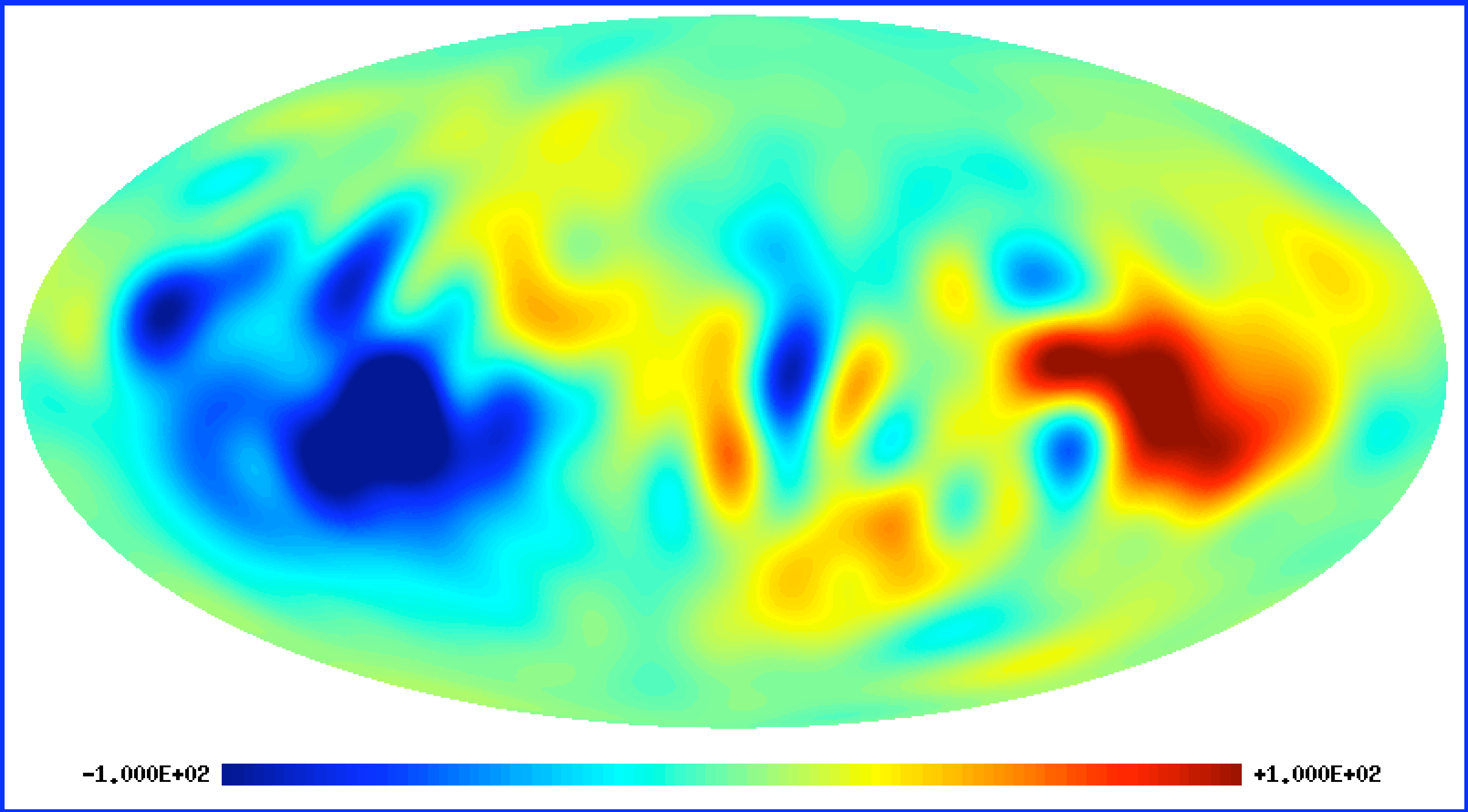
Mapping the Galactic Sky in RM

- We are presented with a non-uniform collection of sight lines
- A complex “mask”, through which the RM sky is seen
- Construct orthogonal modes on the “masked” sky
- The same procedure as is used for Galactic cuts and survey boundaries

Dineen & Coles, astro-ph/0410636

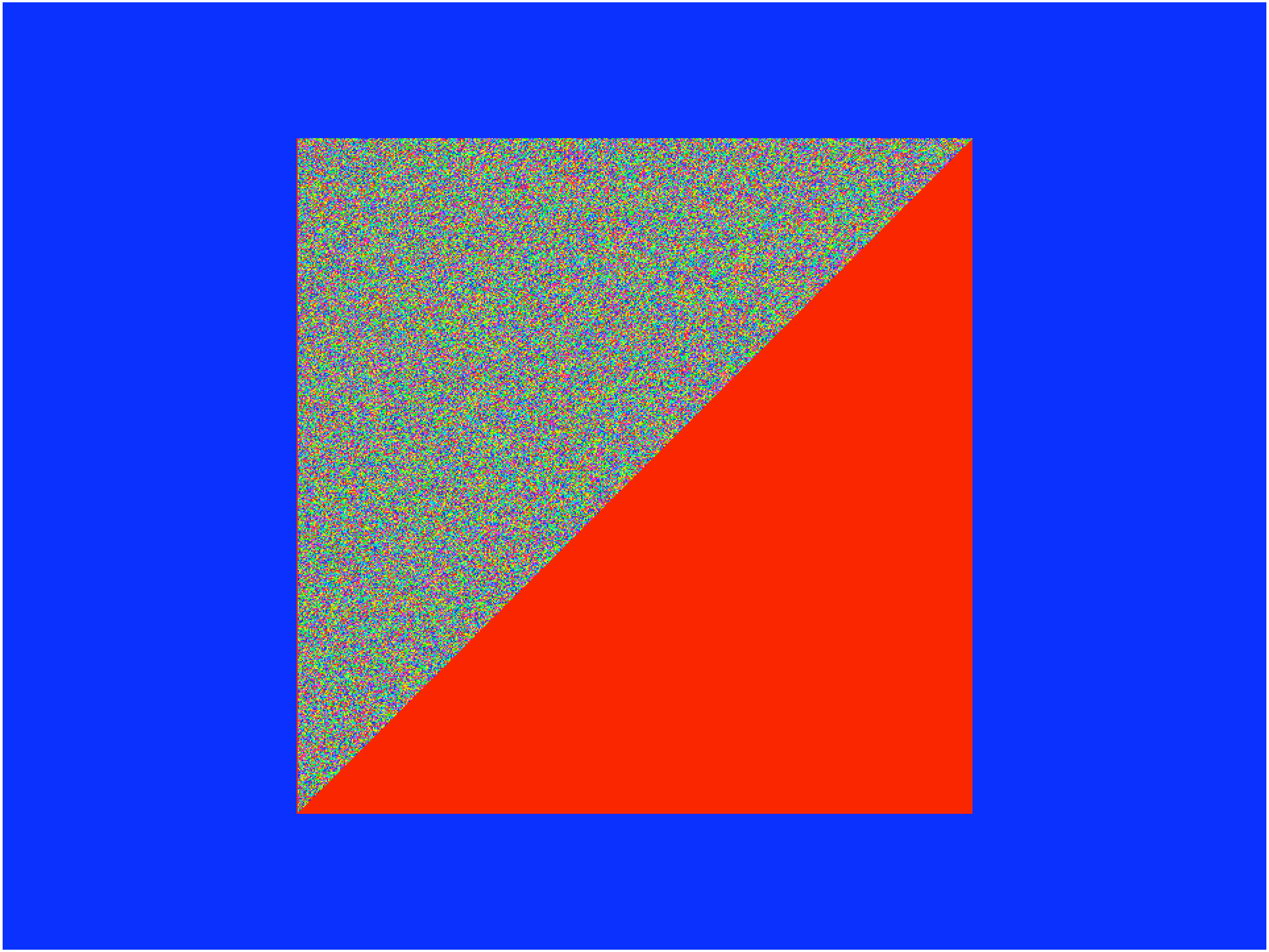


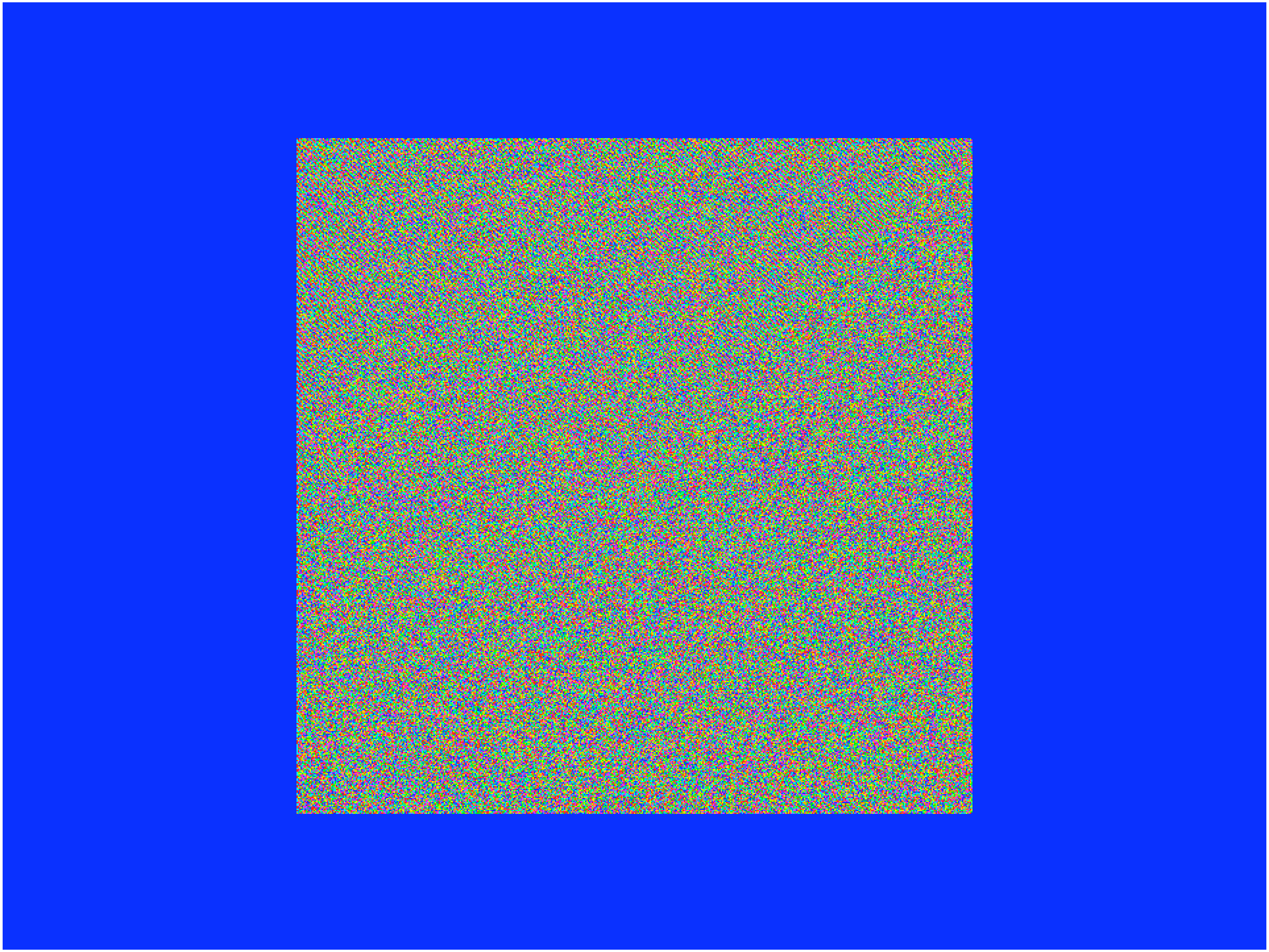

-1.000E+02
+1.000E+02


-1.000E+02
+1.000E+02

Weirdness in WMAP...

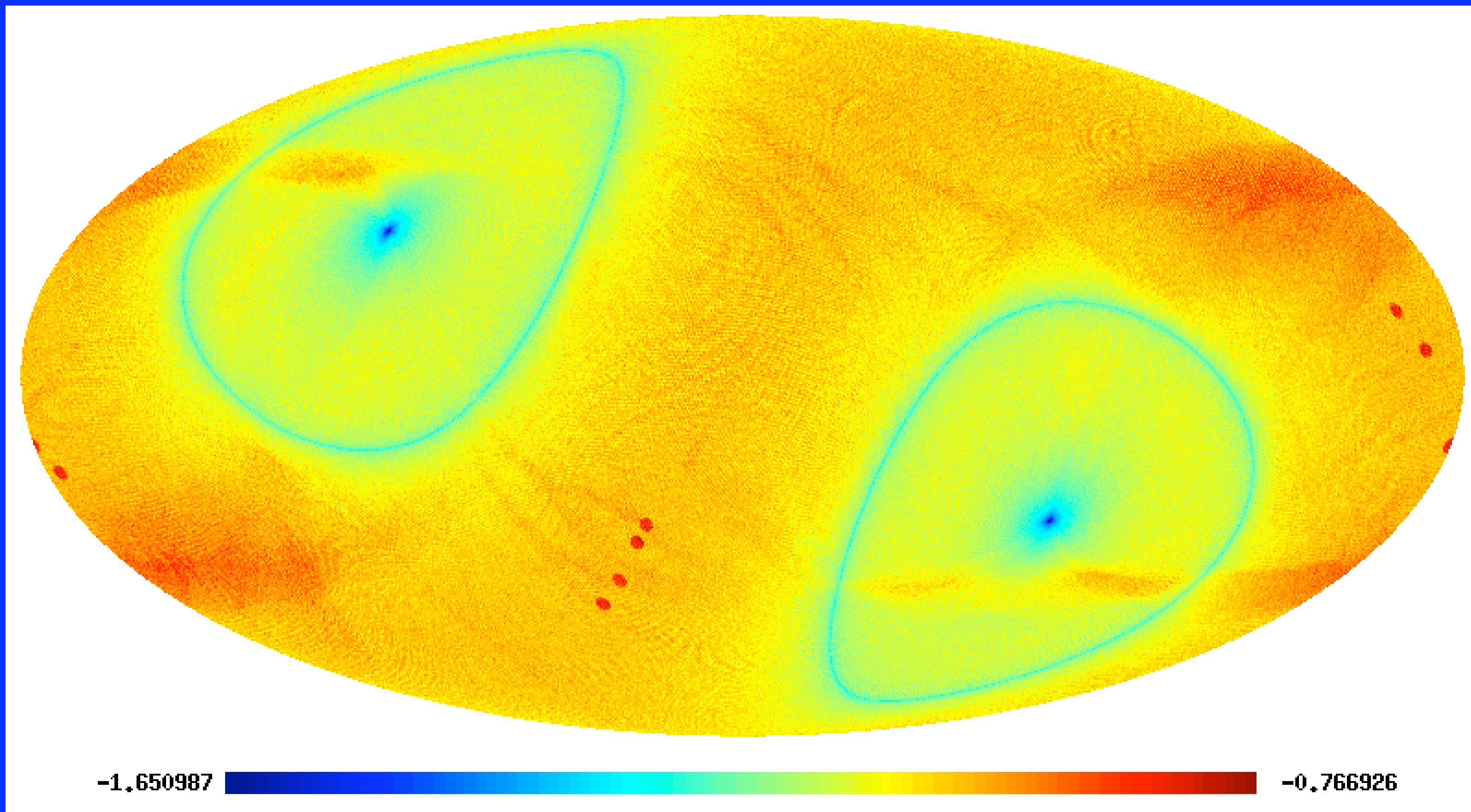
- Dineen & Coles, 2004, MNRAS, 347, 52, WMAP maps correlate with Faraday Rotation
- Eriksen et al.,ApJ, 605, 14, WMAP North-South Divide
- Chiang et al. 2003, ApJ, 590, L65…mode correlations at high l
- The WMAP data are preliminary, the noise is known to be non-stationary, and the foreground subtraction is not perfect…maybe that's all there is to it!





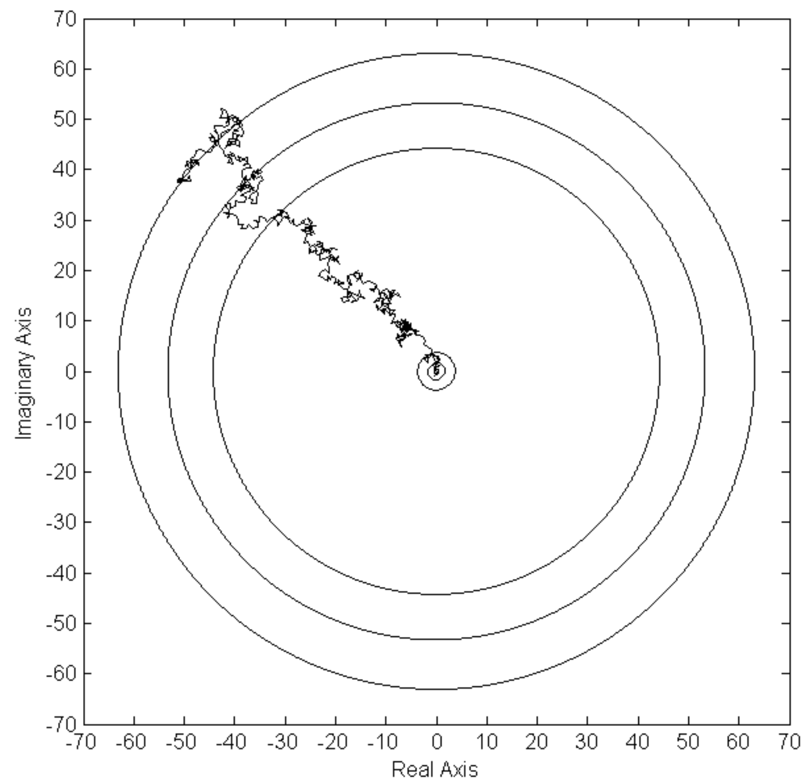
Weirdness in WMAP...

- Dineen & Coles, 2004, 347, 52, WMAP Temperature maps correlate with Faraday Rotation
- Eriksen et al.,ApJ, 605, 14, WMAP North-South Divide
- Chiang et al. 2003, ApJ, 590, L65…mode correlations at high l
- The WMAP data are preliminary, the noise is known to be non-stationary, and the foreground subtraction is not perfect…maybe that's all there is to it!


-1.650987
-0.766926

Random walks in Spherical Harmonic Space

(Stannard & Coles, astro-ph/0410633)



ps...also works on non-trivial topologies, but at fixed *m* rather than fixed *l* (Dineen, Rocha & Coles, astro-ph/0404356)