Learning sparse representations of static and time-varying natural images

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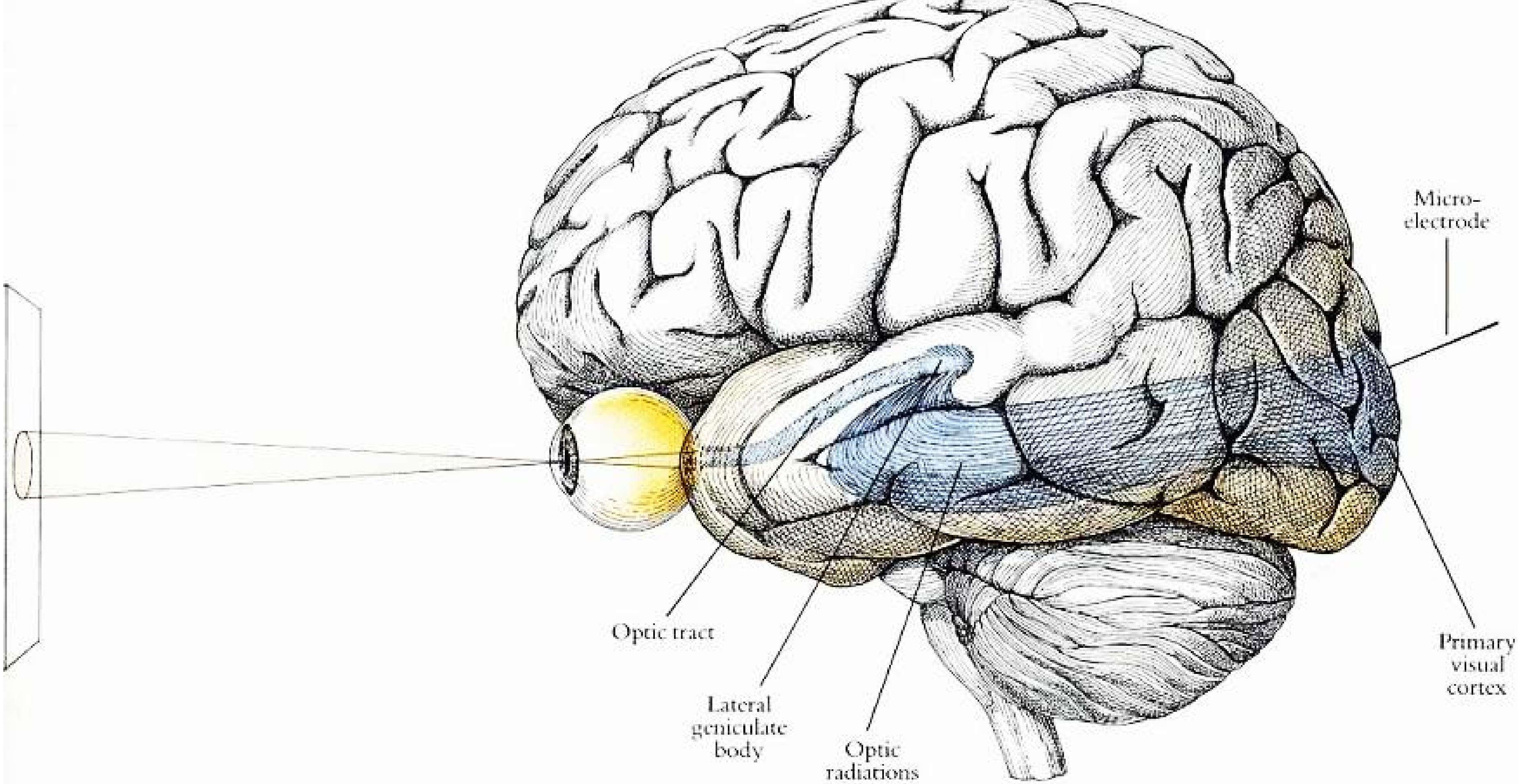
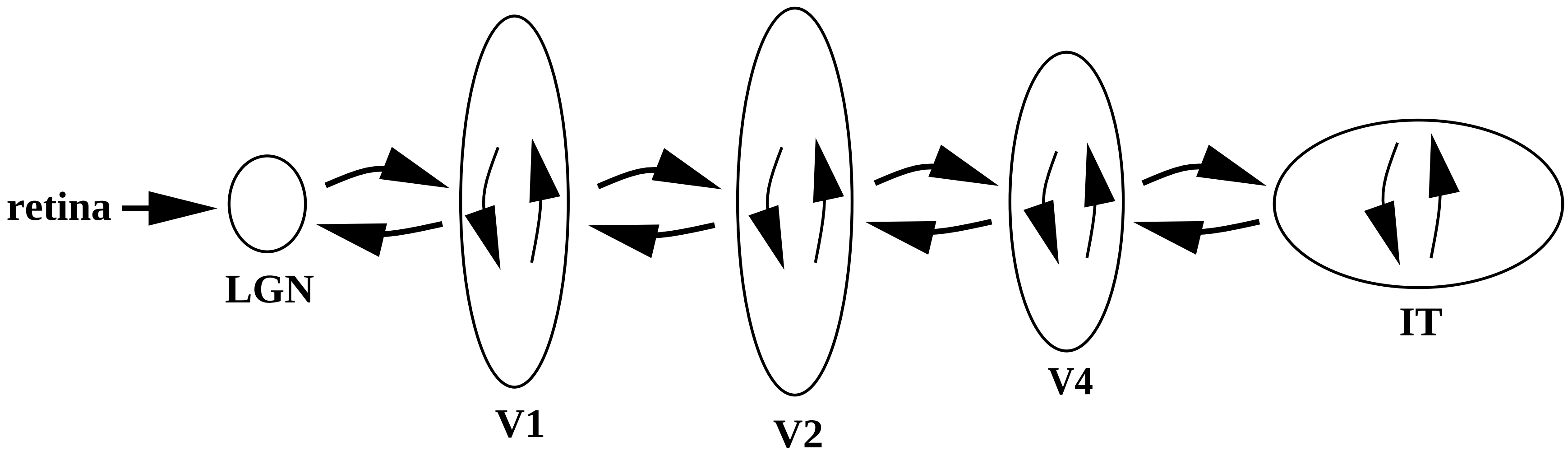
Main Points

- Vision as inference
- Learning sparse, overcomplete image representations
- Sparse coding in V1
- Learning shiftable basis functions

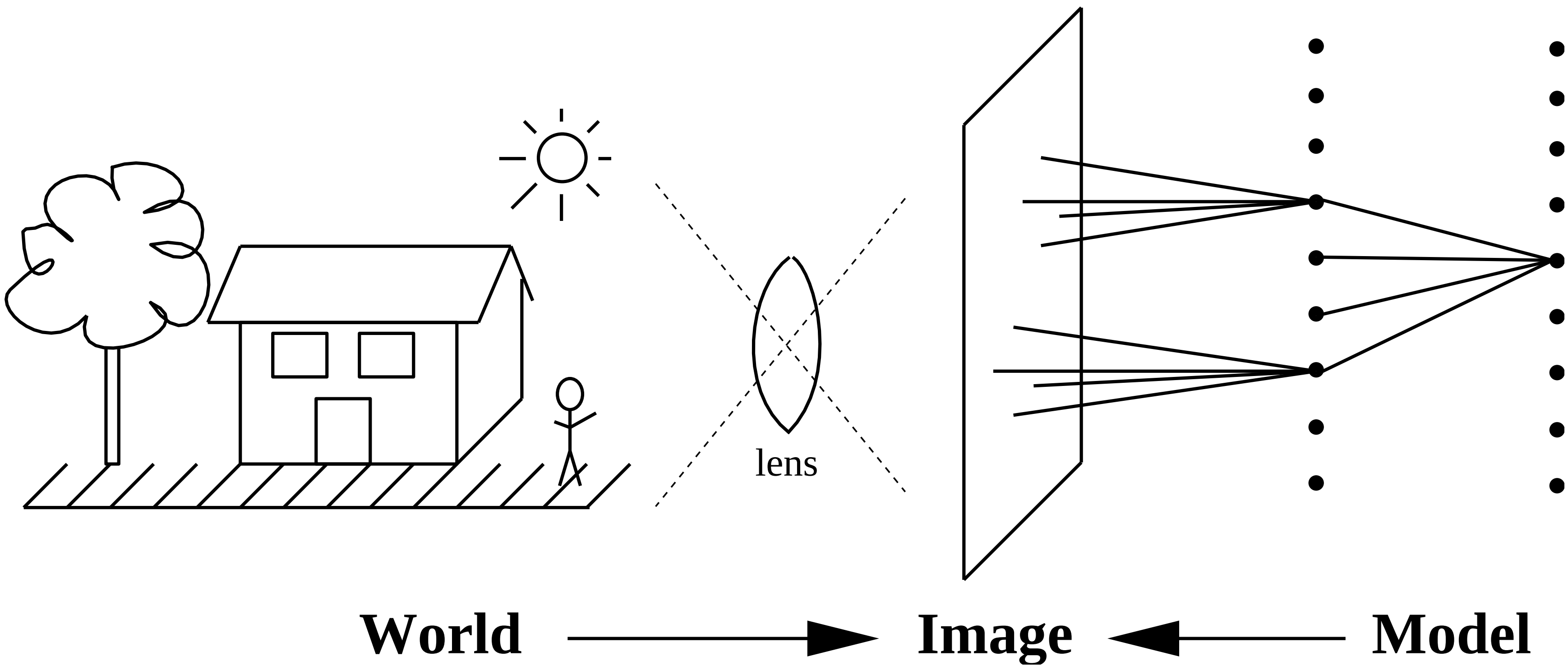


Micro-
electrode
Optic tract
Lateral
geniculate
body
Optic
radiations
Primary
visual
cortex

Recurrent computation is pervasive throughout cortex



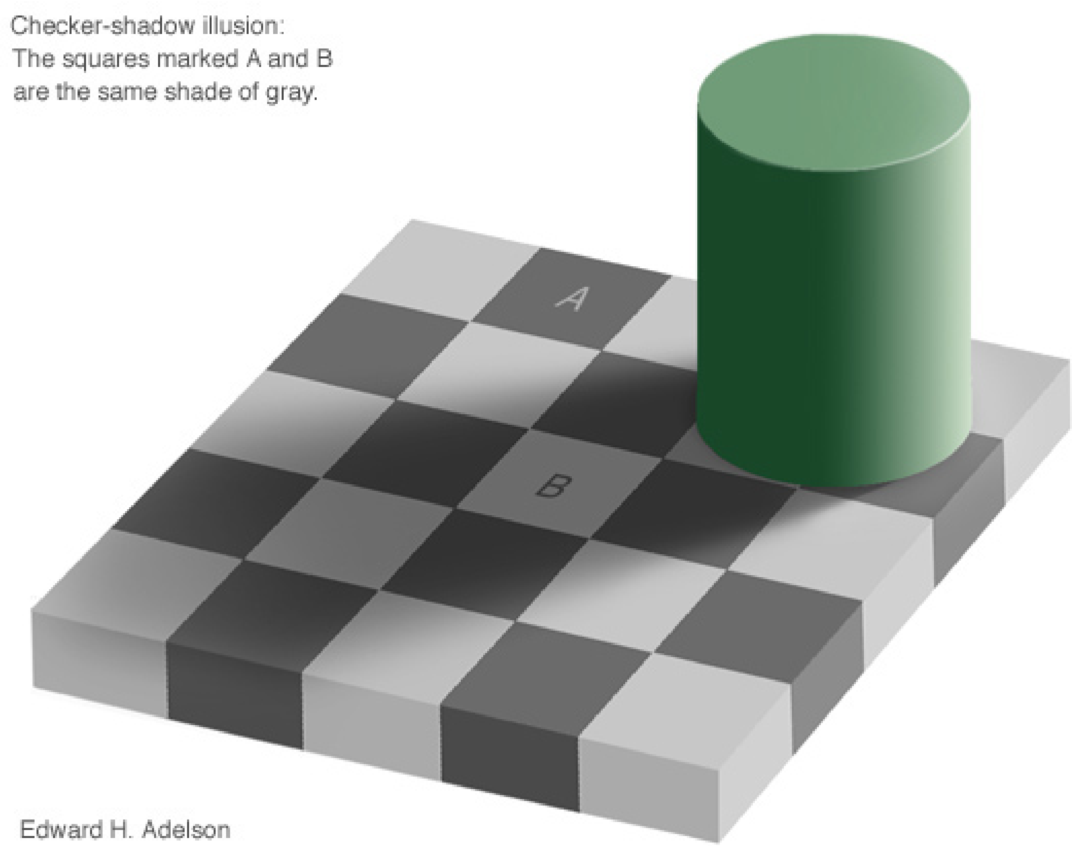
Vision as inference



How do you interpret an edge?



Lightness perception depends on 3D scene layout

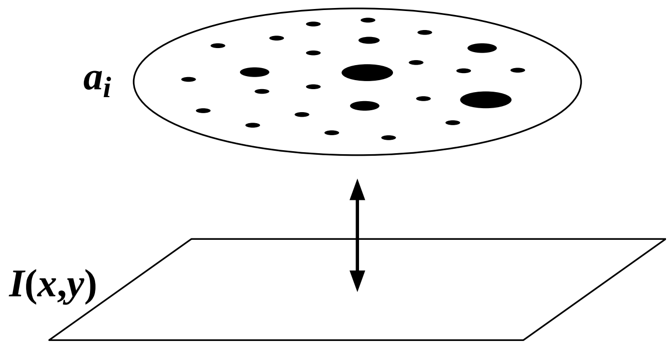


Bayes' rule

$$P(E|D) \propto \underbrace{P(D|E)}_{\substack{\text{how data is}\\ \text{generated by}\\ \text{the environment}}} \times \underbrace{P(E)}_{\substack{\text{prior beliefs}\\ \text{about the}\\ \text{environment}}}$$

$$\begin{aligned} E &= \text{the actual state of the environment} \\ D &= \text{data about the environment} \end{aligned}$$

Sparse coding



- Provides a simple description of images
- Makes image structure explicit $\longrightarrow$ Grouping
- Makes it easier to learn associations
- Field's (1987) analysis of simple-cell receptive fields suggests they have been optimized for sparseness.

Image model

$$I(x,y) = \sum_i a_i\,\phi_i(x,y) + \nu(x,y)\ .$$

$$P(\mathbf{a}|\mathbf{I},\theta) \propto P(\mathbf{I}|\mathbf{a},\theta)\,P(\mathbf{a}|\theta)$$

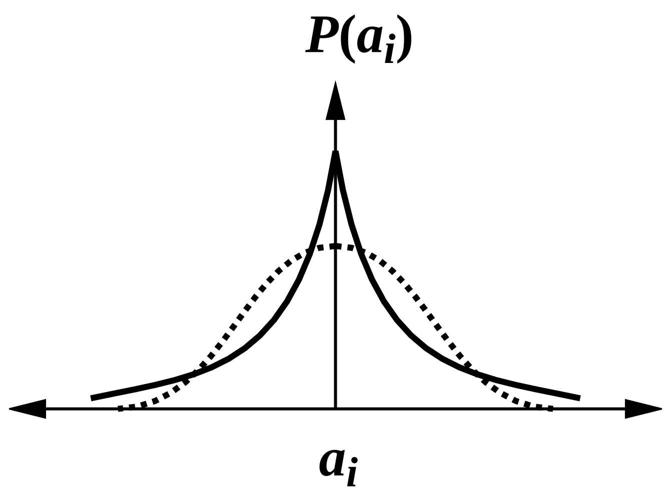
$$P(\mathbf{I}|\theta) = \int P(\mathbf{I}|\mathbf{a},\theta)\,P(\mathbf{a}|\theta)\,d\mathbf{a}$$

Goal: Find a set of basis functions $\{\phi_i\}$ for representing natural images such that the coefficients a_i are as sparse and statistically independent as possible.

Prior

- Factorial: $P(\mathbf{a}|\theta) = \prod_i P(a_i|\theta)$

- Sparse: $P(a_i|\theta) = \frac{1}{Z_S} e^{-S(a_i)}$



Objective functions for inference and learning

Inference (perception):

$$P(\mathbf{a}|\mathbf{I},\theta) \propto P(\mathbf{I}|\mathbf{a},\theta)\, P(\mathbf{a}|\theta)$$

Learning:

$$\langle \log P(\mathbf{I}|\theta) \rangle = \left\langle \log \int P(\mathbf{I}|\mathbf{a},\theta)\, P(\mathbf{a}|\theta)\, d\mathbf{a} \right\rangle$$

Energy function

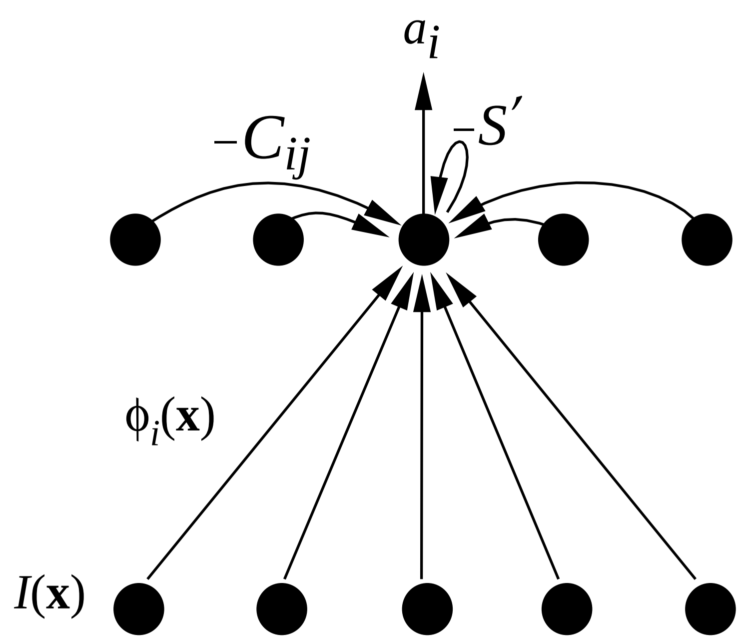
$$
\begin{aligned}
E &= \log P(\mathbf{a}|\mathbf{I},\theta) \\
&= \frac{\lambda_N}{2}\sum_{x,y}\left[I(x,y)-\sum_i a_i\phi_i(x,y)\right]^2+\sum_i S(a_i)+\text{const.}
\end{aligned}
$$

Dynamics

$\dot{a}_i \propto -\frac{\partial E}{\partial a_i}$:

$$
\begin{aligned}
\tau \dot{a}_i &= b_i - \sum_j C_{ij} a_j - S'(a_i) \\
b_i &= \lambda_N \sum_{x,y} \phi_i(x,y)\, I(x,y) \\
C_{ij} &= \lambda_N \sum_{x,y} \phi_i(x,y) \phi_j(x,y)
\end{aligned}
$$

Network implementation

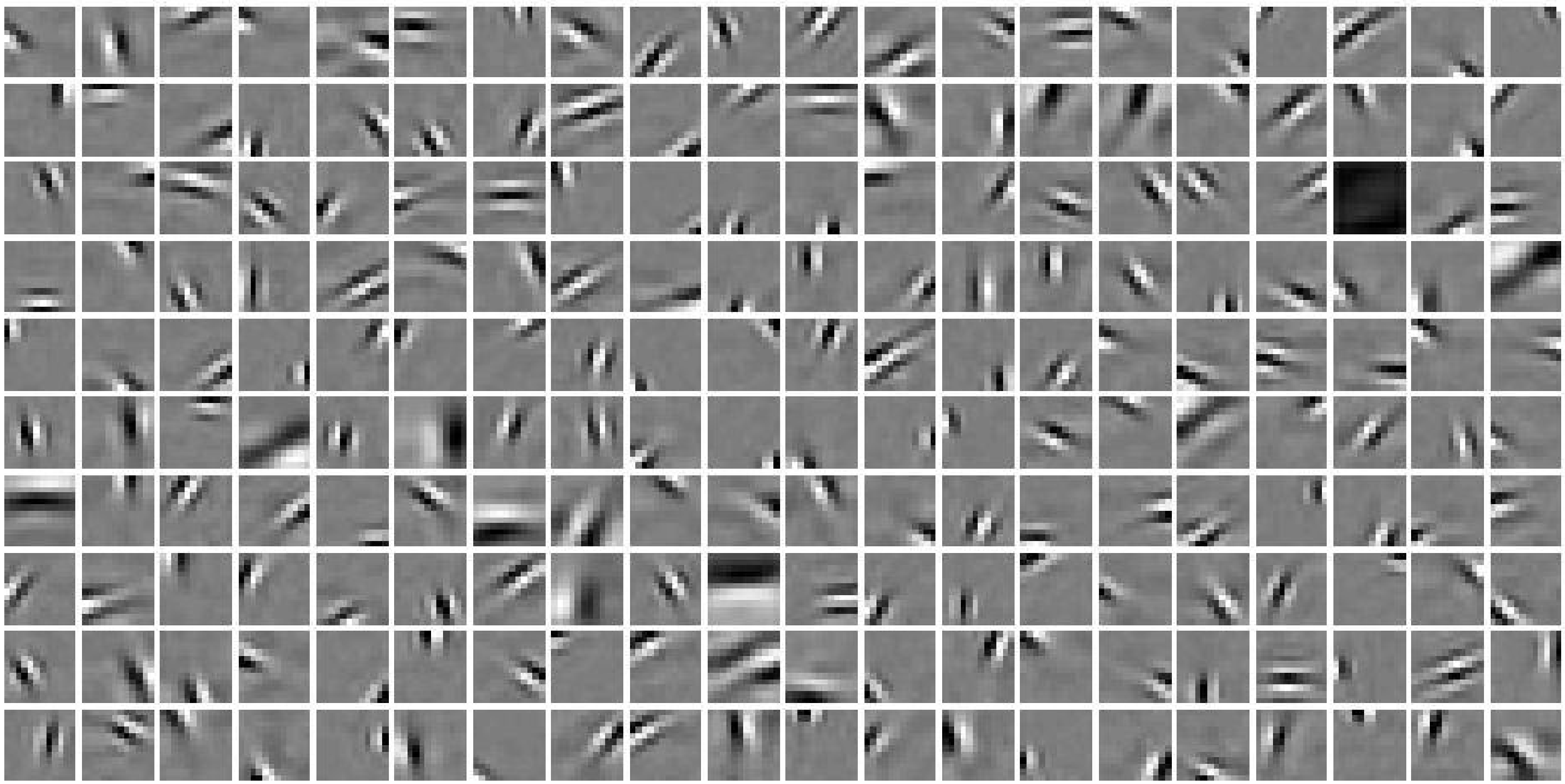


Learning

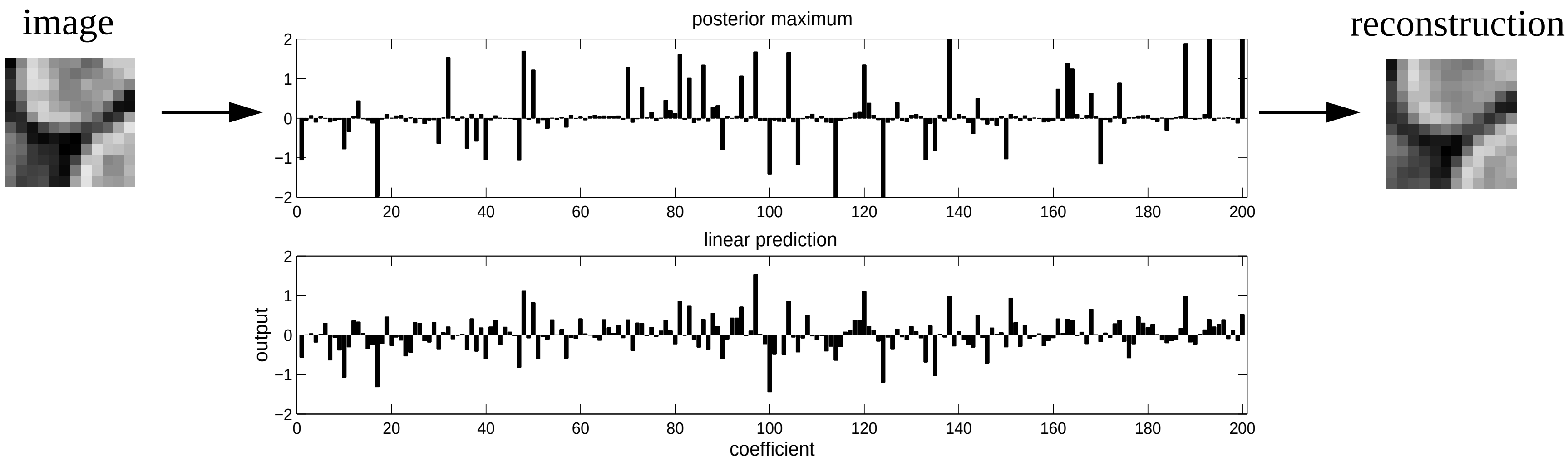
$\Delta\phi_i \propto -\left\langle \frac{\partial E}{\partial \phi_i} \right\rangle$:

$$
\begin{aligned}
\Delta\phi_i(x,y) &= \eta \, \langle a_i \, r(x,y) \rangle \\
r(x,y) &= I(x,y) - \sum_i a_i \phi_i(x,y)
\end{aligned}
$$

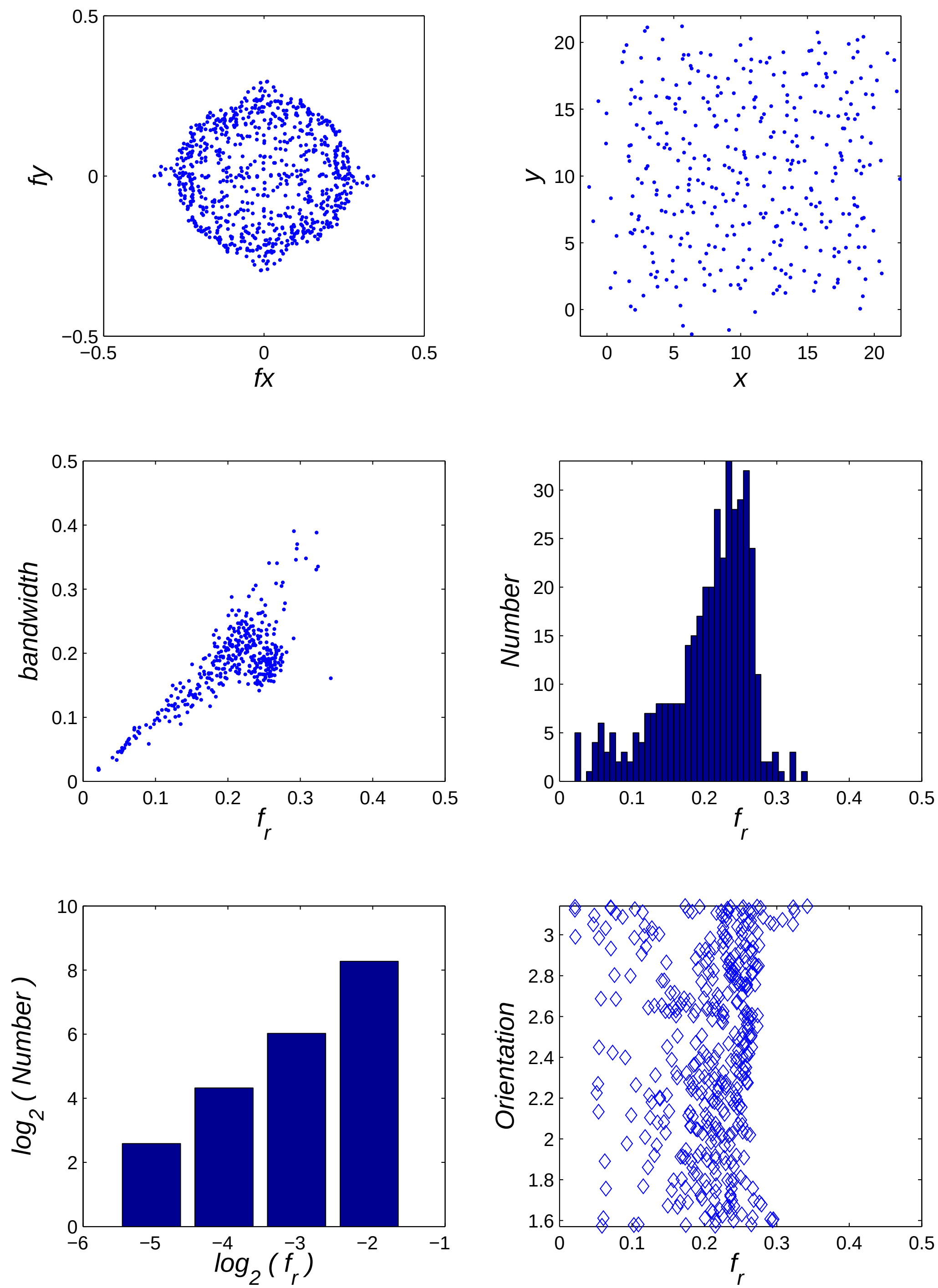
Learned basis functions (200, 12x12)

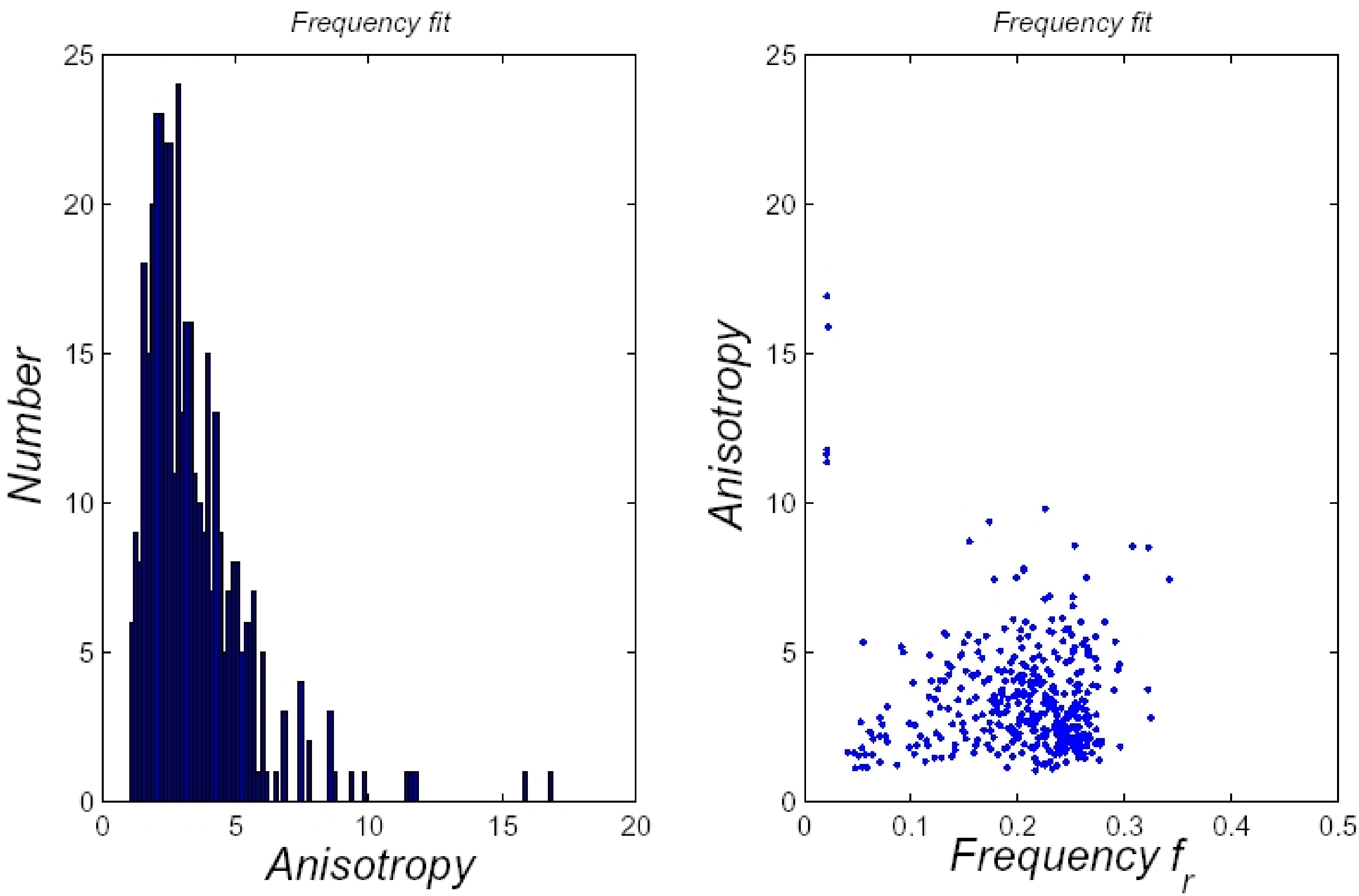


Sparsification



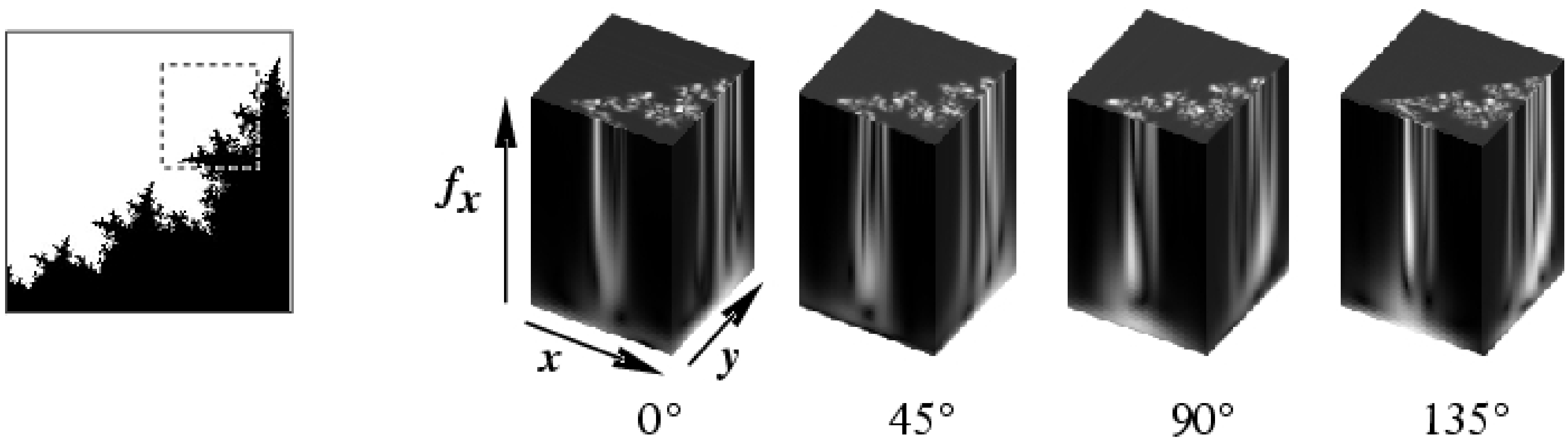
Tiling properties





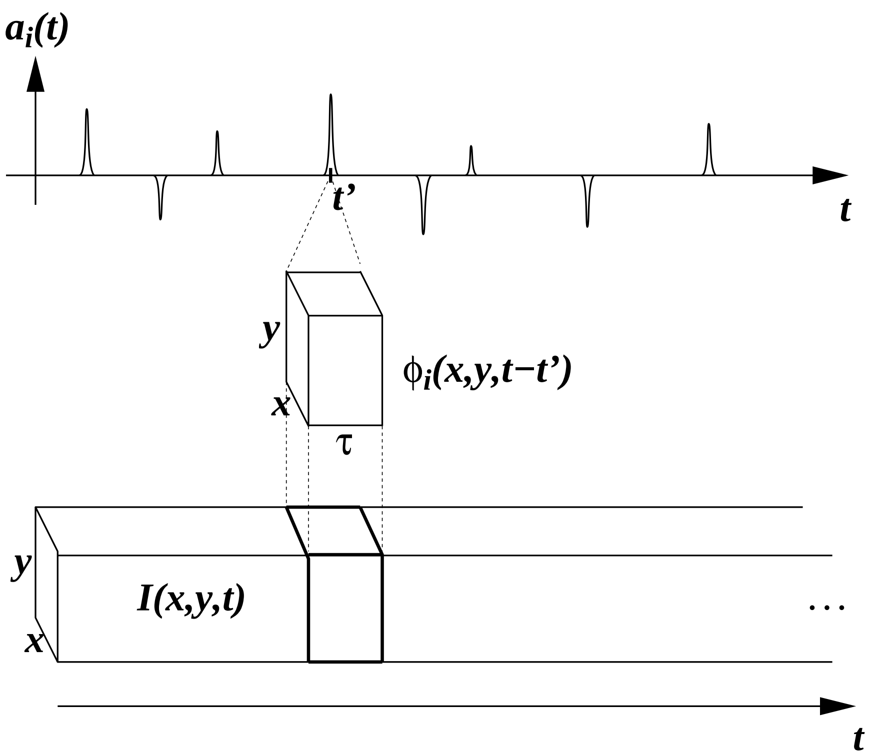
Frequency fit
Number
Anisotropy
0
5
10
15
20
25
Frequency fit
Anisotropy
Frequency f_r
0
0.1
0.2
0.3
0.4
0.5

Scale space cross-section of a fractal contour



Space-time image model

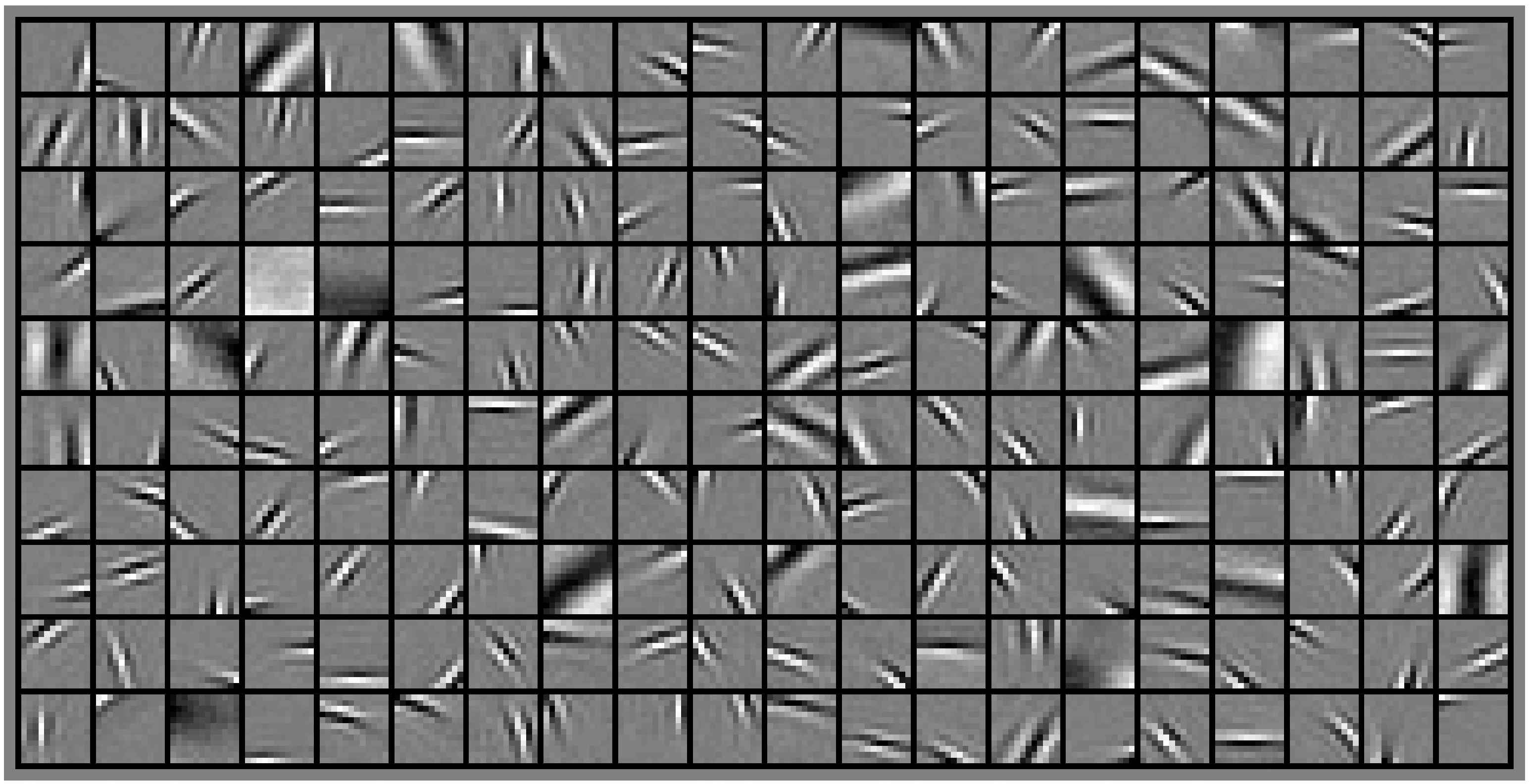
$$I(x, y, t) = \sum_i a_i(t) * \phi_i(x, y, t) + \nu(x, y, t)$$



Goal: Find a set of space-time basis functions $\{\phi_i\}$ for representing natural images such that the *time-varying* coefficients $a_i(t)$ are as sparse and statistically independent as possible *over both space and time.*

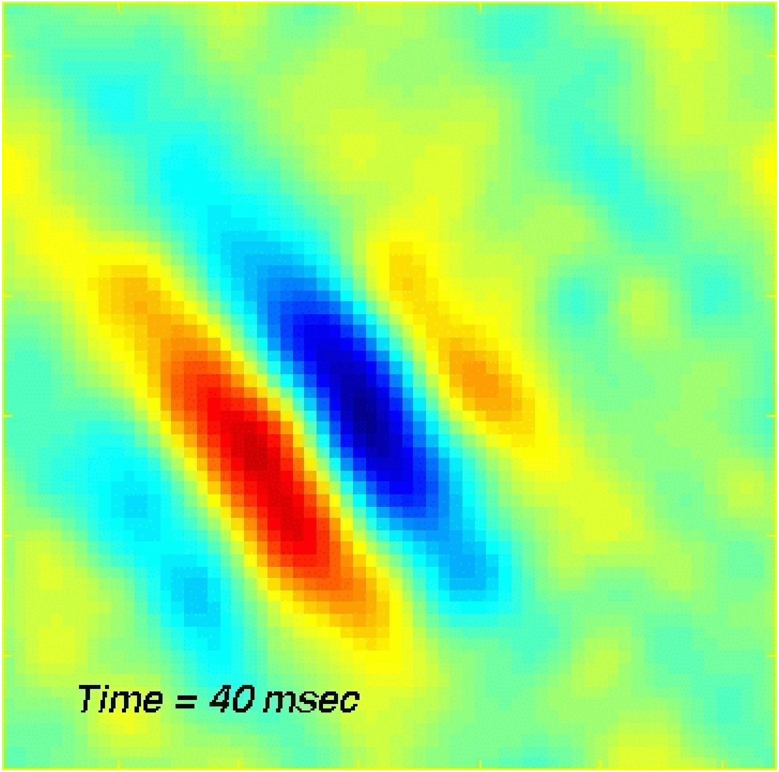
Learned space-time basis functions (200, $12 \times 12 \times 7$)

Training set: nature documentary

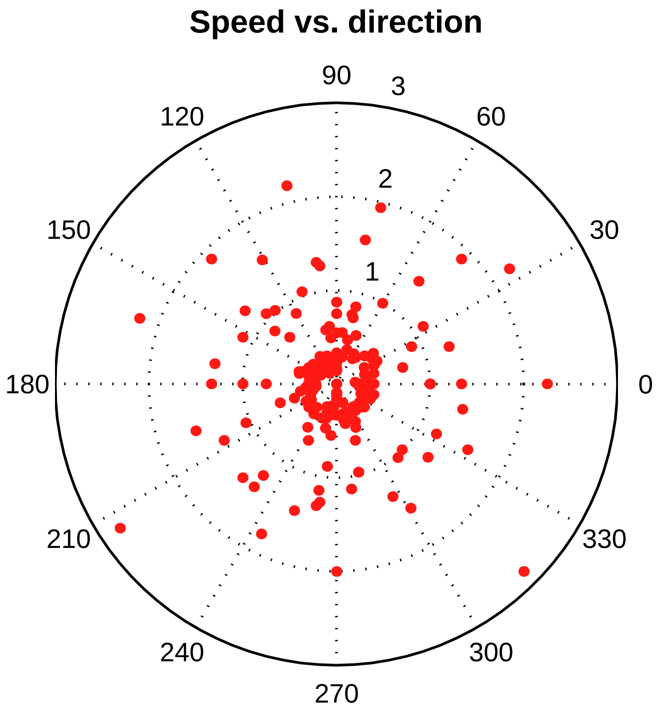


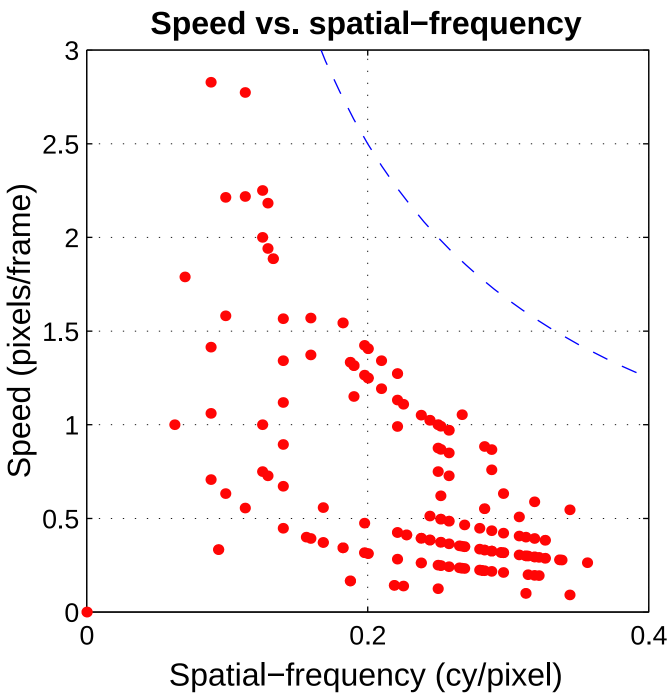
V1 space-time receptive field

(Courtesy of Dario Ringach)

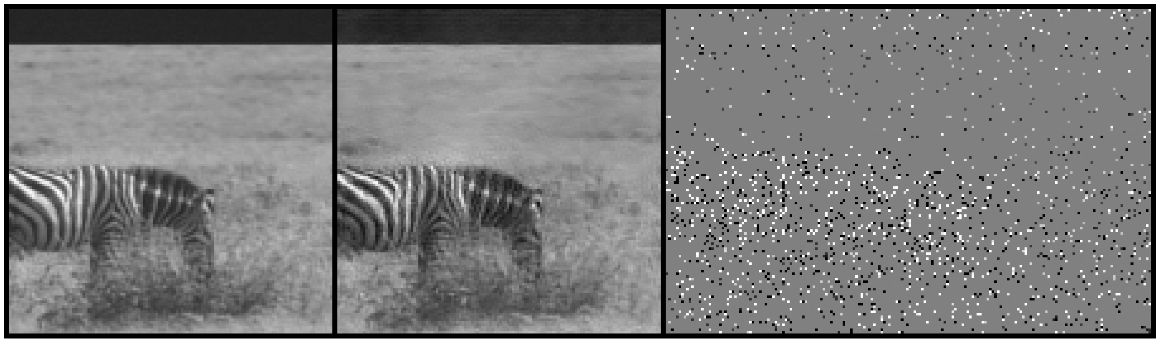


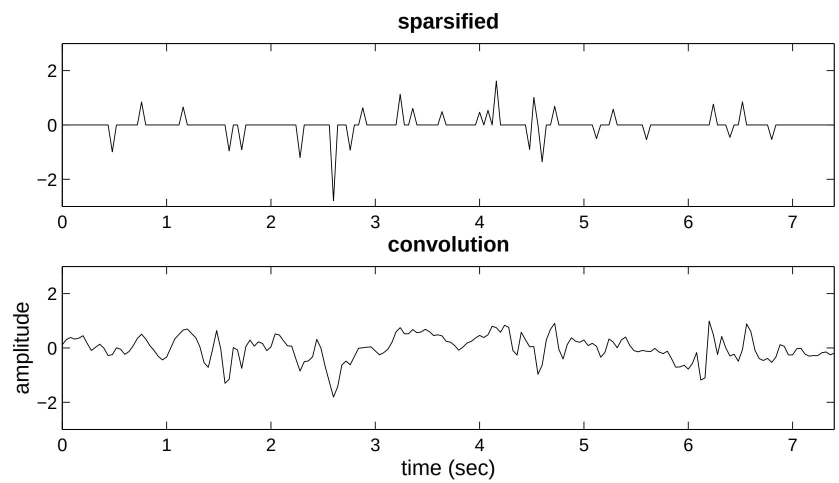
Basis function properties



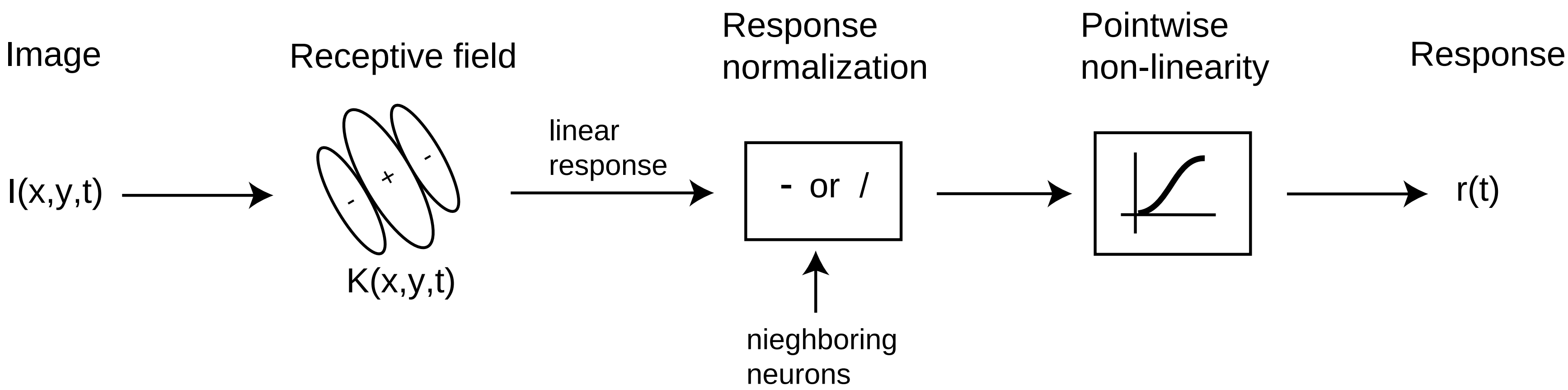


Spike encoding and reconstruction



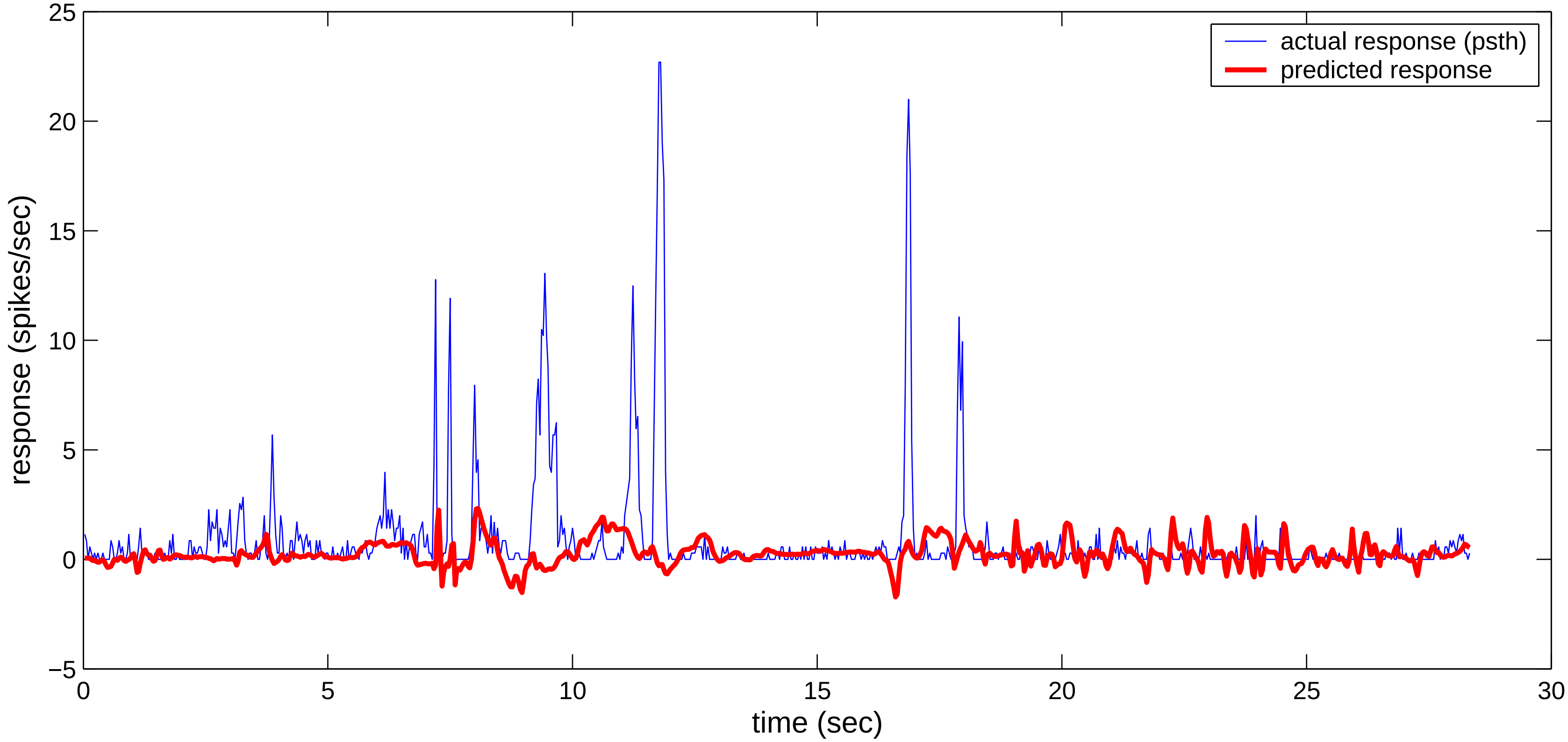


“Standard model” of V1 simple-cells

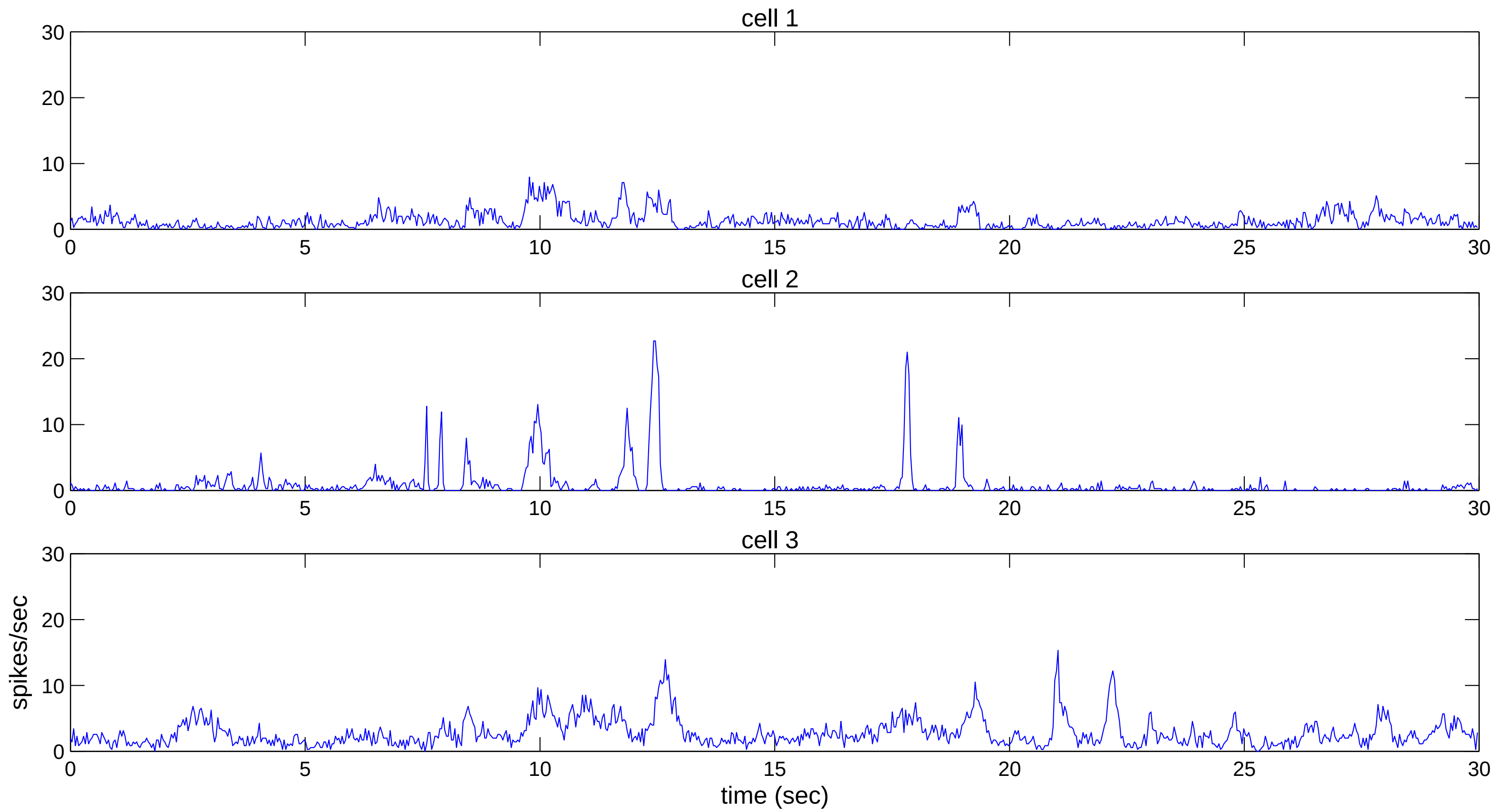


Evidence for sparse coding

Data from Gray lab (J. Baker and S. Yen)

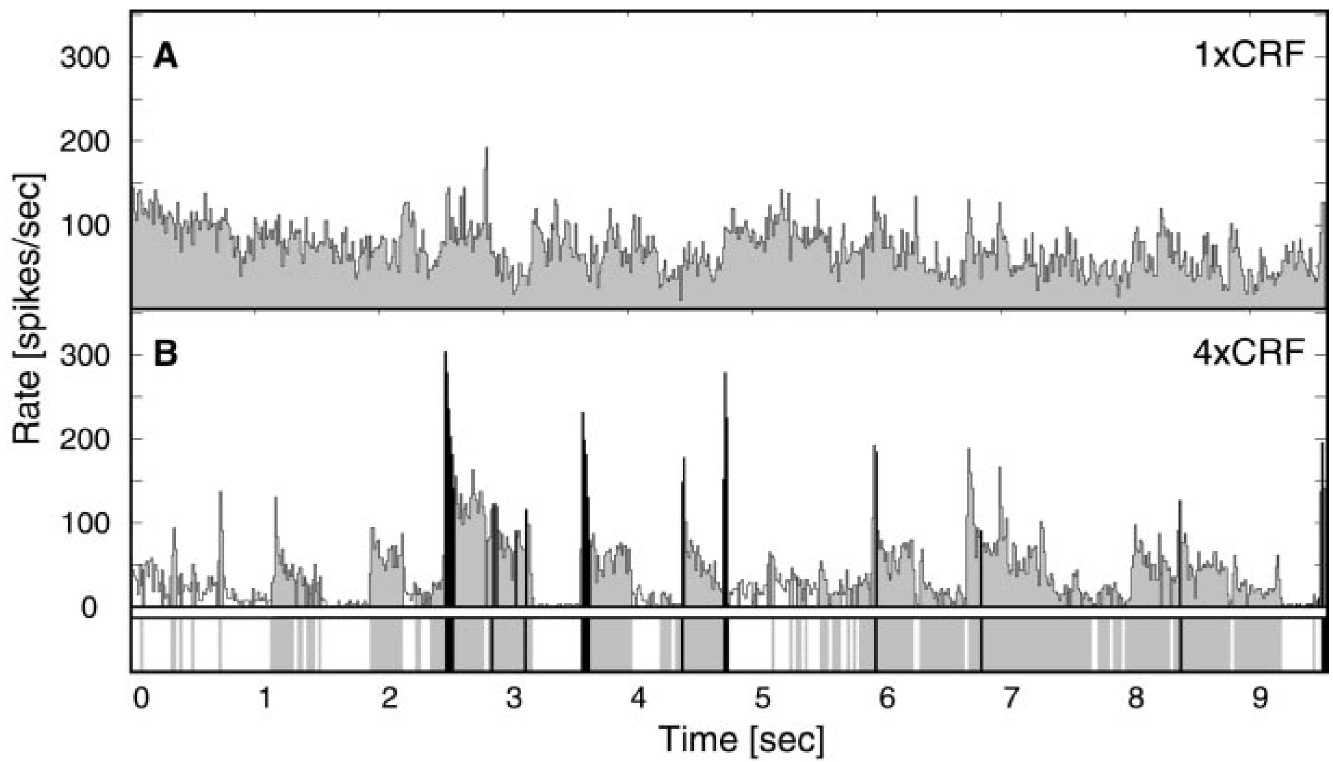


Responses of nearby units are heterogeneous



Context in natural scenes sparsifies responses

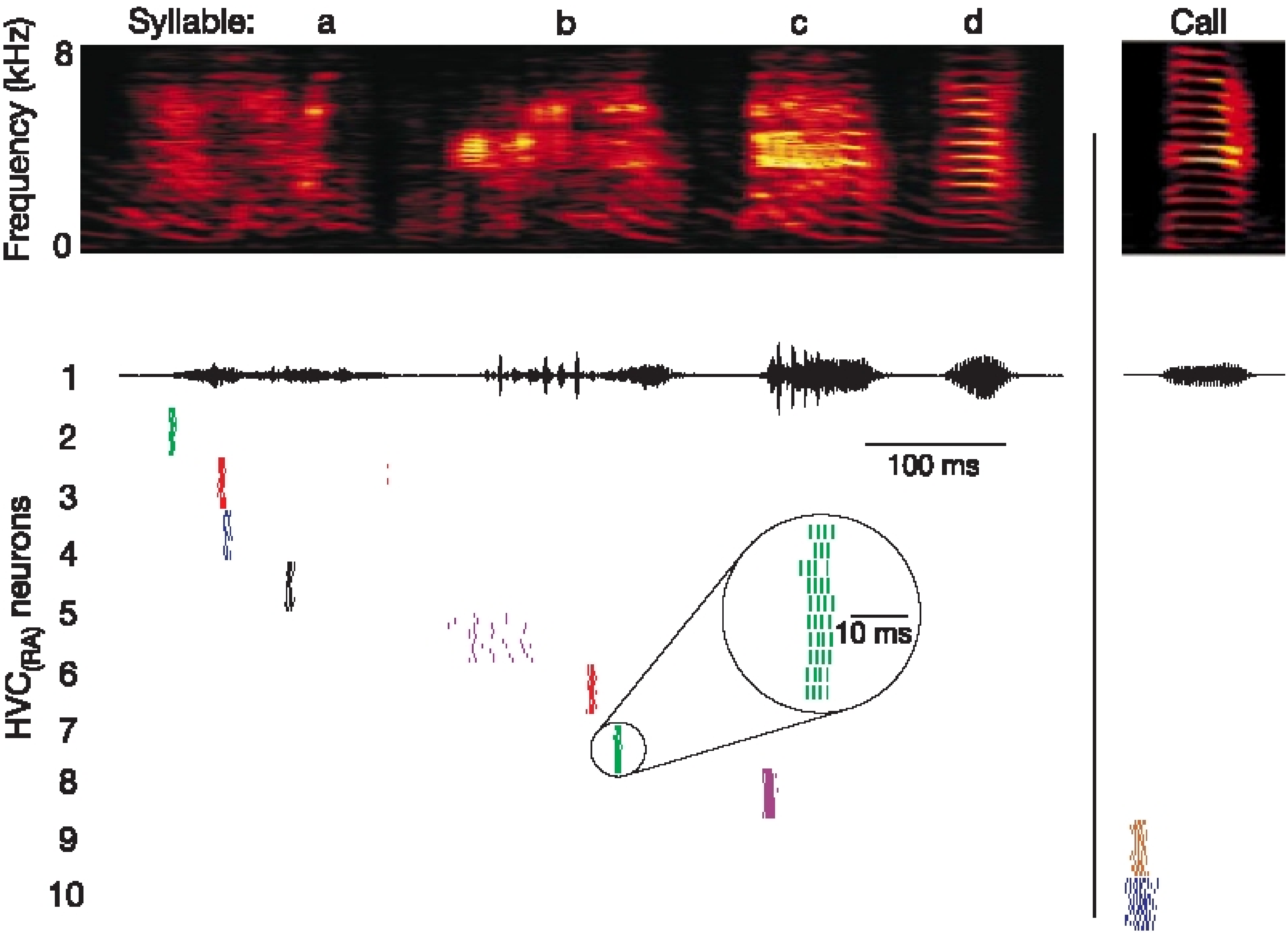
Vinje & Gallant (2000, 2002)



Extreme sparse coding

- Gilles Laurent - mushroom body, insect
- Michael Fee - HVC, zebra finch
- Tony Zador - auditory cortex, mouse
- Bill Skaggs - hippocampus, primate
- Harvey Swadow - motor cortex, rabbit
- Michael Brecht - barrel cortex, rat
- Christof Koch - inferotemportal cortex, human

Hahnloser RHR, Kozhevnikov AA, Fee MS (2002) An ultra-sparse code underlies the generation of neural sequences in a songbird. *Nature, 419*, 65-70.




California
SPARSE

Review article

Olshausen BA, Field DJ (2004) Sparse coding of sensory inputs. *Current Opinion in Neurobiology, 14*, 481-487.

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Problems with the current model

- Sparsification: small changes in the image could lead to drastic changes in the output representation.
- Factorial prior: coefficients exhibit strong dependencies, so the factorial prior is wrong.
- Linear model: how to extend to a hierarchical model?

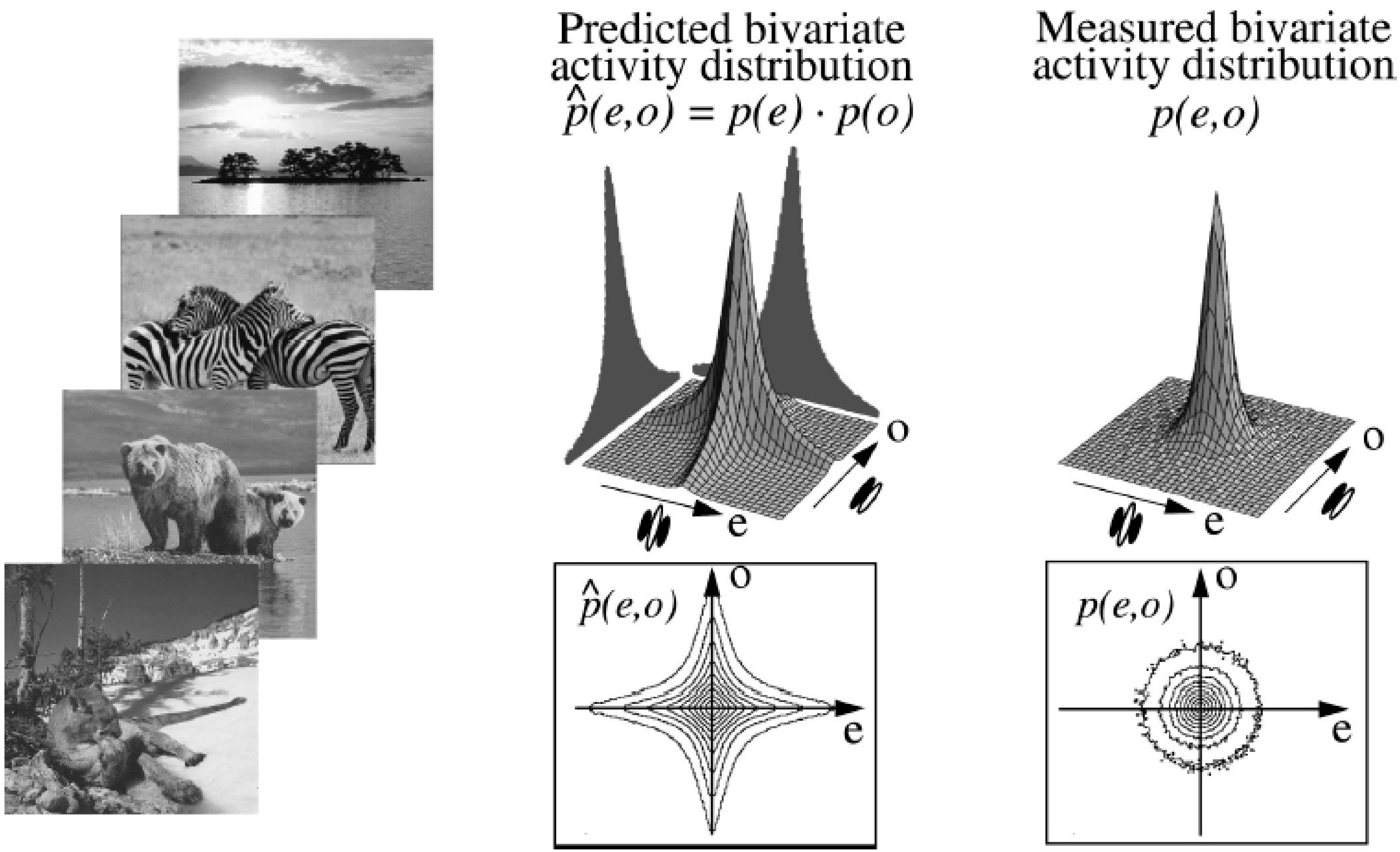
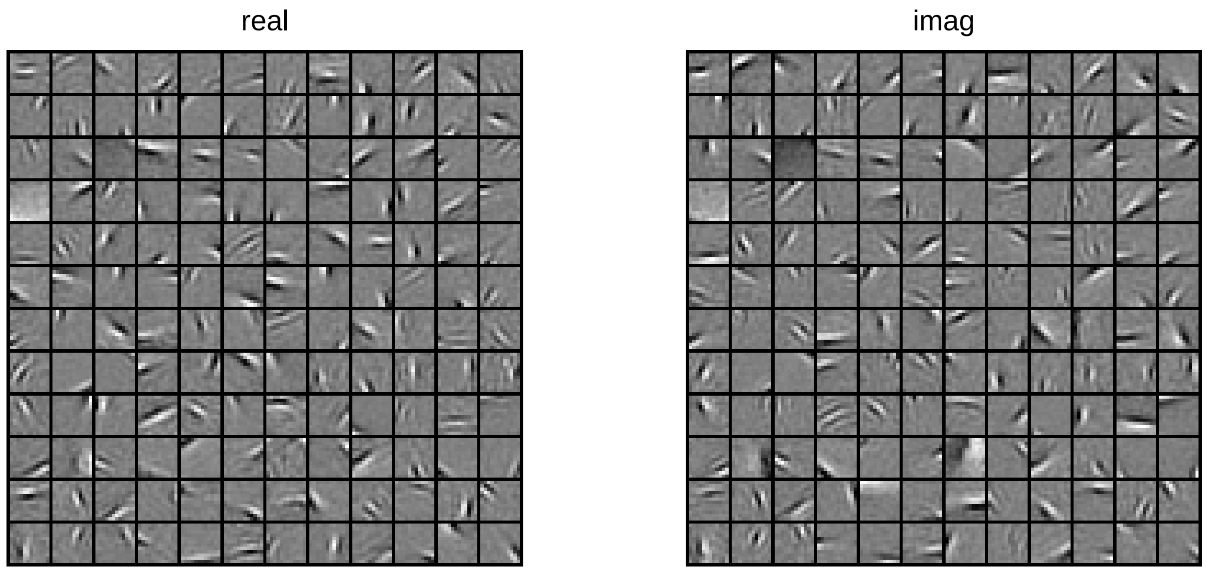

Predicted bivariate
activity distribution
$\hat{p}(e,o) = p(e) \cdot p(o)$
Measured bivariate
activity distribution
$p(e,o)$
o
e
o
e
$\hat{p}(e,o)$
o
e
$p(e,o)$
o
e

Image model with 'shiftable' basis functions

$$\begin{aligned} I(x) &= \sum_i \Re\{z_i\, \phi_i(x)\} \\ z_i &= a_i\, e^{j\,\alpha_i} \\ \phi_i(x) &= \phi_i^R(x) + j\,\phi_i^I(x) \end{aligned}$$

$$I(x) = \sum_i a_i \left[\cos\,\alpha_i\;\phi_i^R(x) + \sin\,\alpha_i\;\phi_i^I(x)\right]$$

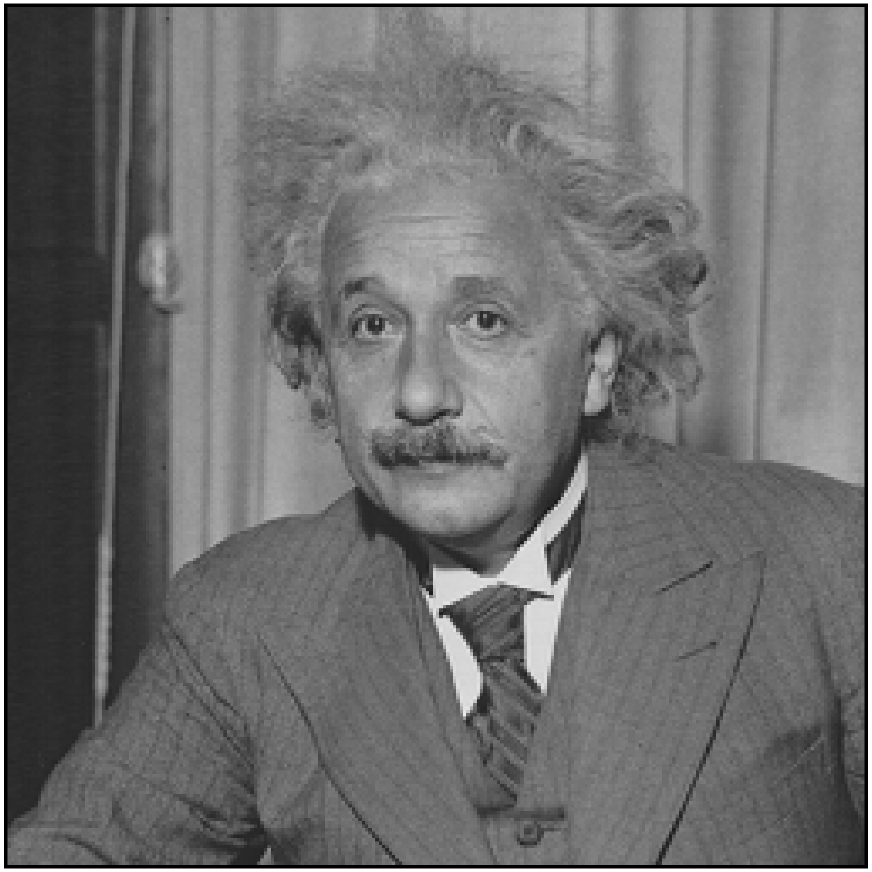
Learned complex basis functions (144, 12 $\times$ 12 patches)



animate!

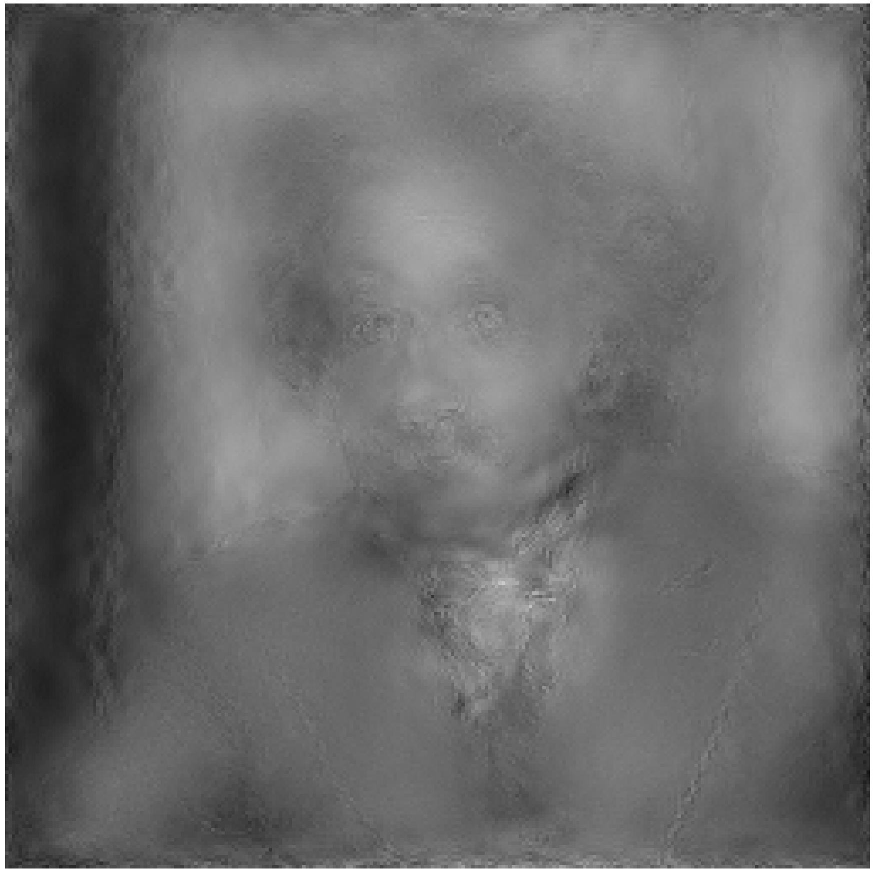
Local phase is important

Original image



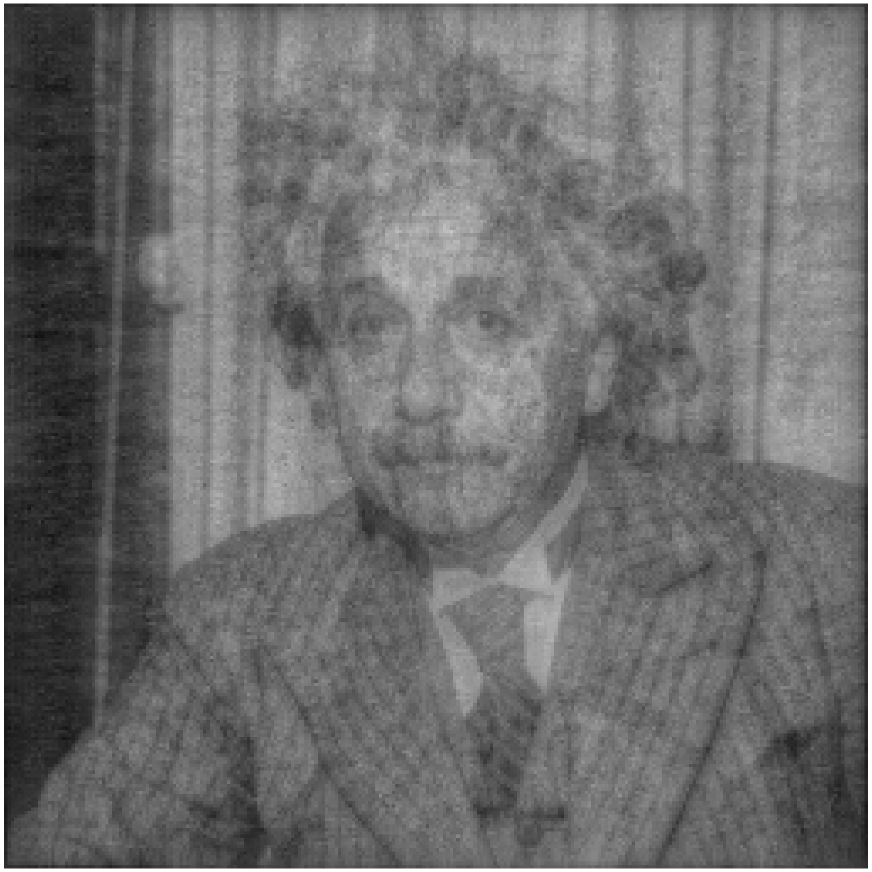
Local phase is important

Magnitudes only



Local phase is important

Phases only



Conclusions

- V1 neurons represent time-varying natural images in terms of sparse events.

- Joint dependencies among coefficients may be modeled with shiftable basis functions $\rightarrow$ neurons carry both amplitude and phase?

Further information and details

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