IPAM MGA Tutorial: Multiscale Problems in Geophysics

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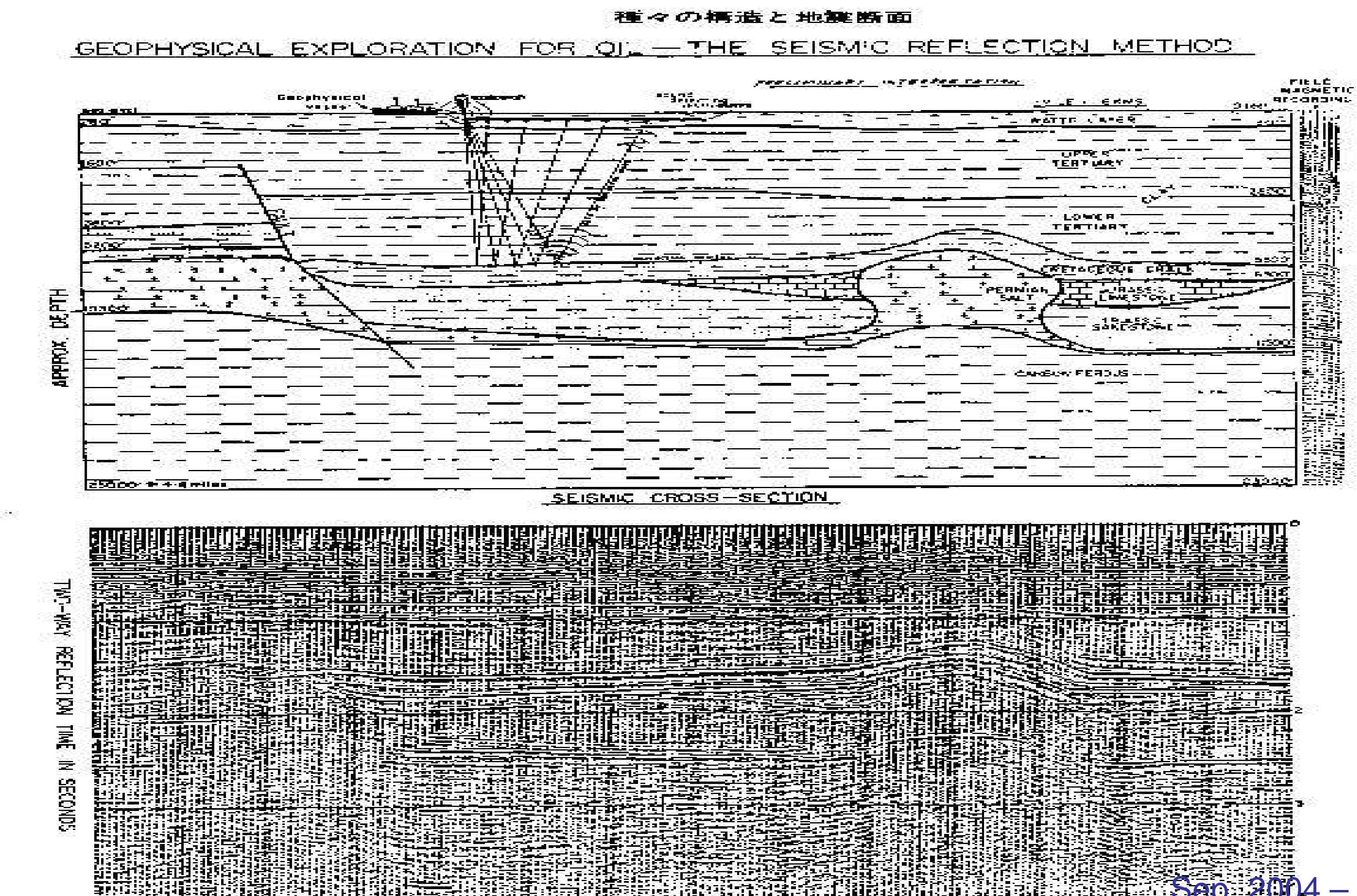
Outline

- Multiscale Nature of Geophysical Measurements
- Importance of Geometry, Textures, and Clustering: High Relevance to MGA2004
- Large Scale: Seismic Signal/Noise Separation
- Small Scale: Fracture and Vug Detection from Electrical Conductivity Images
- High-Dimensional Clustering Problem in Exploration Geophysics
- Summary

Multiscale Nature of Geophysical Measurements

- Large Scale: Surface Seismic Survey:
  - used for structural mapping of the entire reservoir volume
  - wide spatial coverage: $\approx$ 10km x 10km x 3km
  - spatial sampling: 25m x 25m
  - can resolve features > 30-40ft (9-12m)
  - vertical axis: time

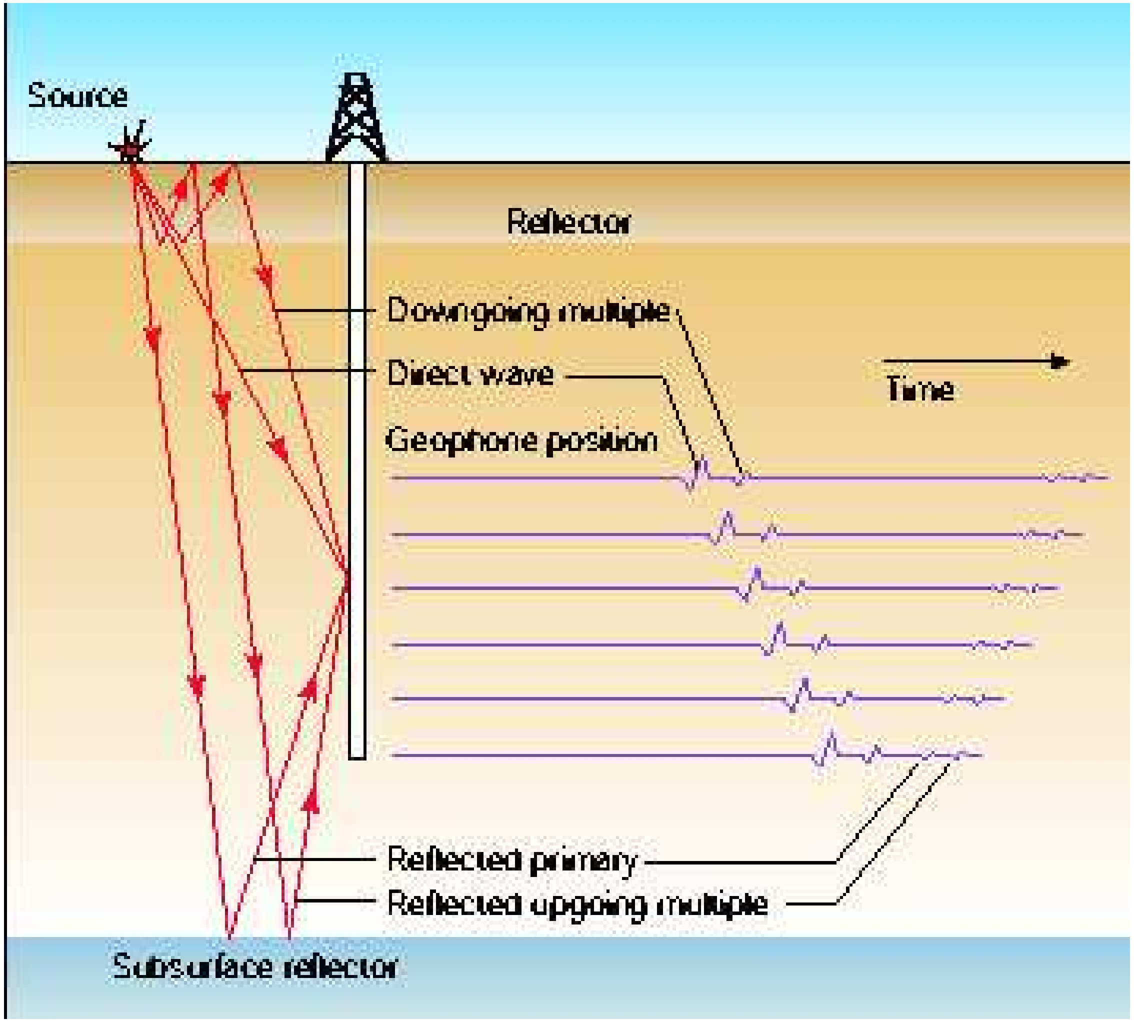
Surface Seismic Survey



Multiscale Nature of Geophysical Measurements . . .

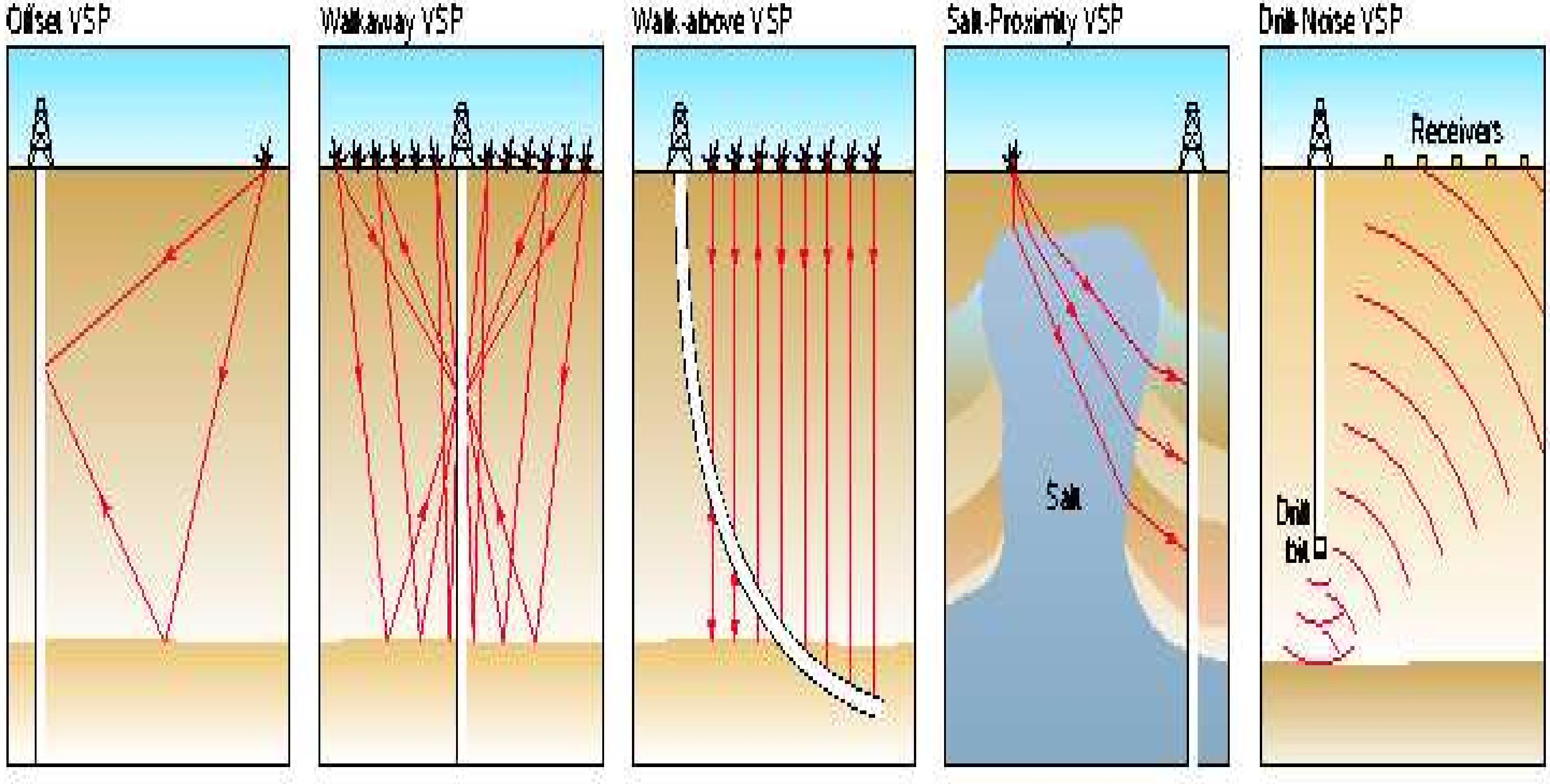
- Medium Scale: Borehole Seismics:
  - receivers are in a borehole/well
  - can record not only reflected waves but also direct waves
  - can record reflected waves with less attenuation
  - used for linking surface seismic information and well log information (e.g., sonic logs)
  - depth sampling: 20-50m (wish: < 10m)

Borehole Seismics



courtesy: Schlumberger

Borehole Seismics . . .



courtesy: Schlumberger

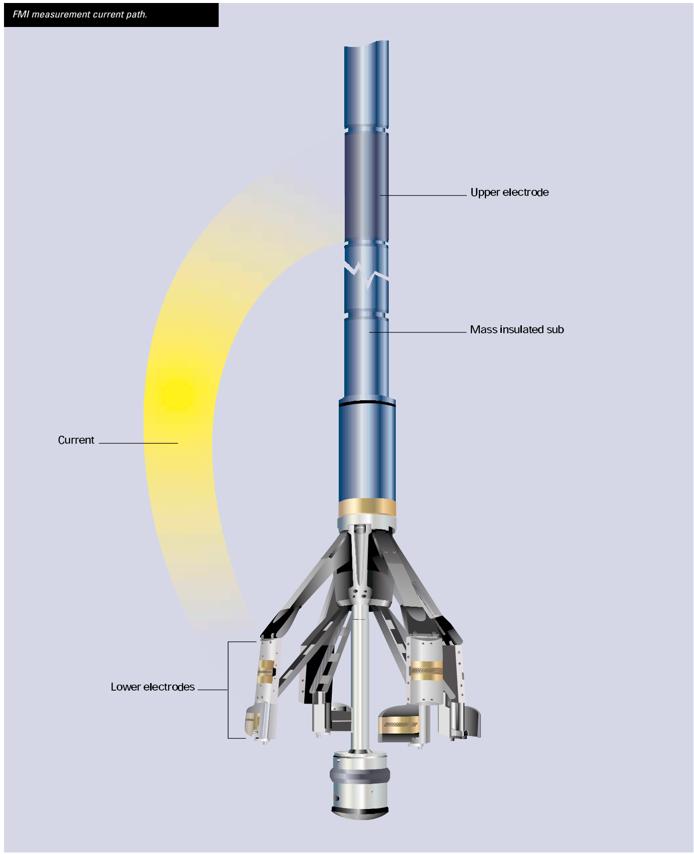
Multiscale Nature of Geophysical Measurements . . .

- Small Scale: Wireline measurements, cores:
  - used for detailed geological/geophysical information in the vicinity of wells
  - spatial coverage: $\approx$ 1m x 1m x 3km
  - can resolve features $\approx$ 0.5-3ft (0.15-1m) or less
  - vertical axis: depth
  - cores provide absolute truth, but expensive
  - can characterize not only elastic but also electrical and nuclear properties of subsurface formations

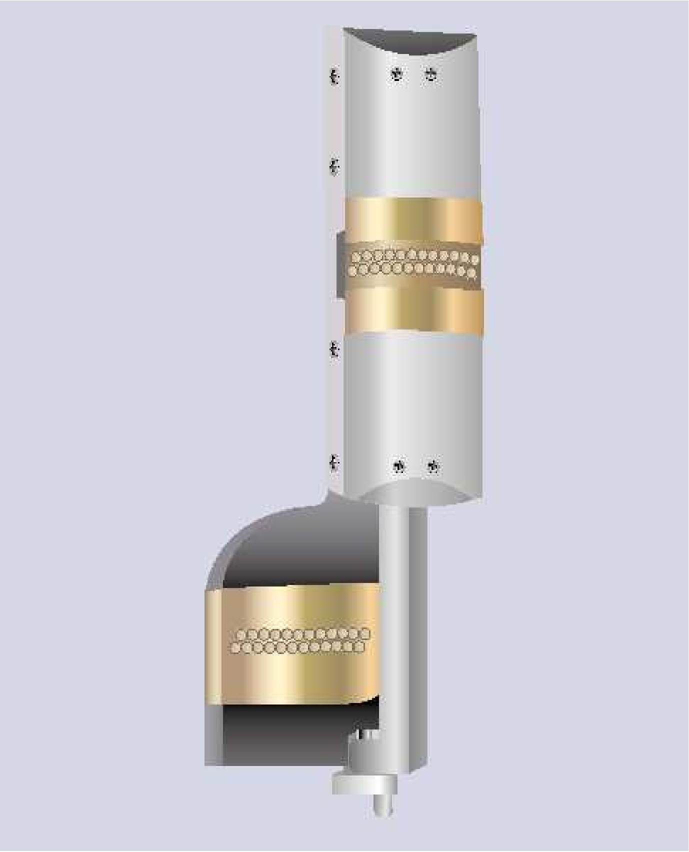
Electrical Conductivity Image ($\approx$ 2.5ft x 2.3ft)

courtesy: Schlumberger

Electrical Conductivity Imaging Tool



(a) Tool



(b) Pad/Flap

courtesy: Schlumberger

Multiscale Nature of Geophysical Measurements . . .

- Large Scale Geometry
- Medium/Small Scale Geometry
- Textures (Stratigraphy, Lithology, Facies)
- Reconciliation of multiscale info for better models
- Highly Relevant to MGA 2004
- This tutorial: introduction of problems + some attempts with older techniques
- My hope: provoke the audience so that some of you may be interested in applying new techniques discussed in MGA 2004 to these problems!

Extraction of Large Scale Geometric Objects

Harlan, Claerbout, & Rocca (1984)

- Motivation: Separate geologic events (layer boundaries) from other events and noise. Other events could be interference patterns, scattering from faults in the form of hyperbolas, etc.
- Measurement: Seismic signals (large scale)
- Beginning of “multilayered transforms,” “coherent structure extraction,” and ICA
- Method: iteration of “focusing” transforms + soft thresholding

Harlan, Claerbout, & Rocca (1984) . . .

- Define “focusing” as an invertible linear transformation making the data more statistically independent (or less statistically dependent)
- A transform focusing signal (pattern) must also defocus noise
- Focused signal becomes more non-Gaussian; defocused noise becomes more Gaussian
- In the transformed domain, apply filtering or soft-thresholding operations to extract signal or separate signal from noise

Harlan, Claerbout, & Rocca (1984) . . .

A stacked seismic section = $\sum$ of

- Geologic component $\approx$ linear events
- Diffraction component $\approx$ hyperbolic events
- Noise component $\approx$ white Gaussian noise $+ \alpha$.

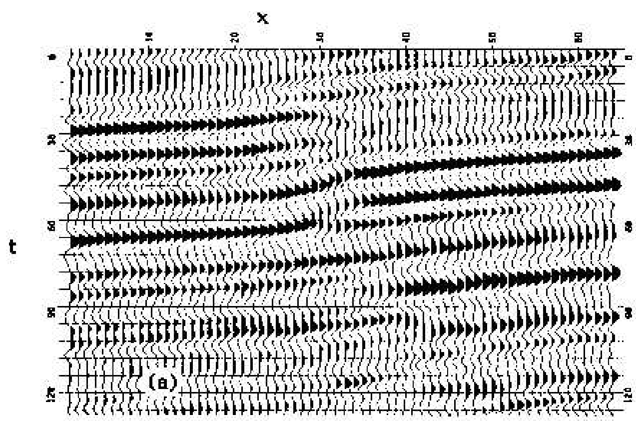
Radon Transform

The 2D-line version for $f(x, y)$ can be stated as follows:

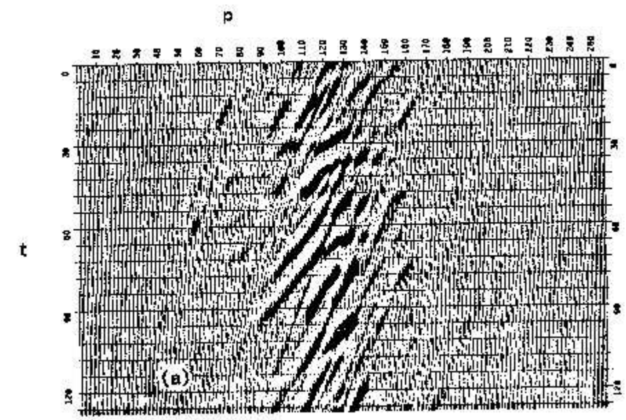
$$R_f(\tau, p) = \iint f(x, t)\, \delta(t - \tau - px)\, \mathrm{d}x\, \mathrm{d}t.$$

- Has a huge list of applications (X-ray tomography, geophysics, wave propagation, . . .)
- Can be generalized: integration of an n-dimensional function over m-dimensional geometric objects
- Inversion formulas exist as Beylkin showed yesterday
- Lines in an image are transformed to sharp peaks in (τ, p)-domain

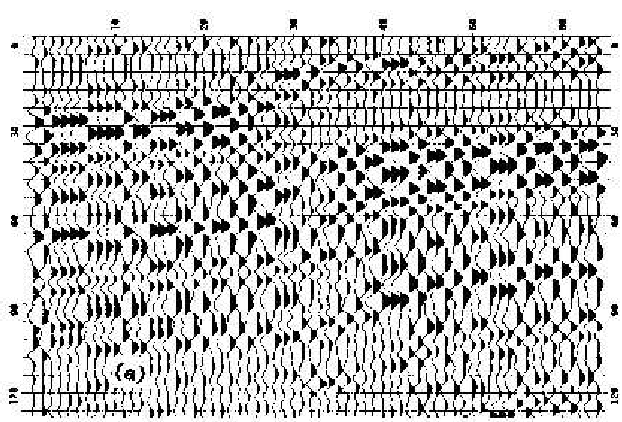
Example 1



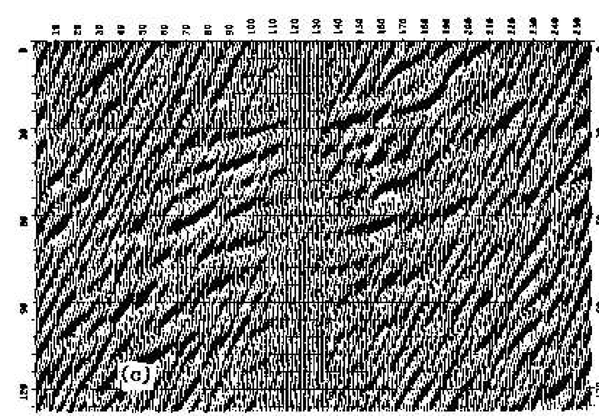
(a) Original



(b) $\tau - p$

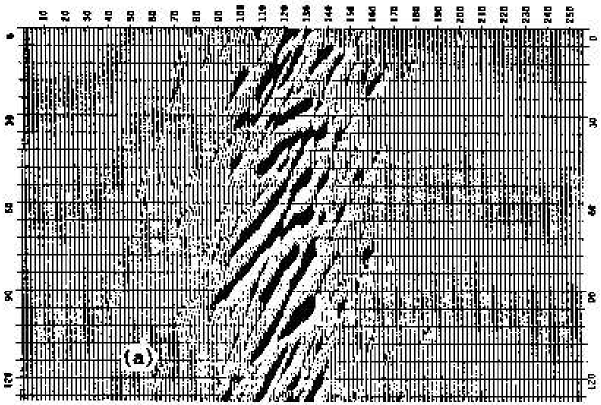


(c) Randomized

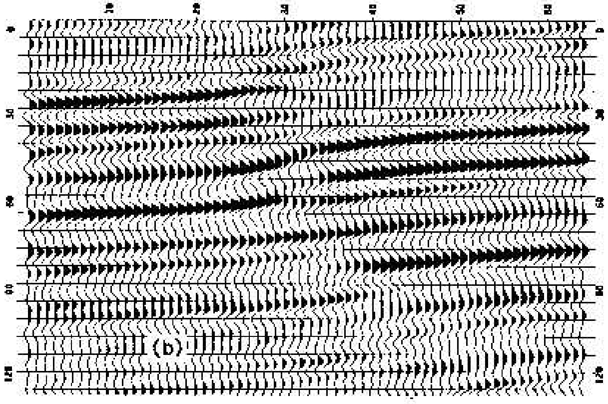


(d) $\tau - p$

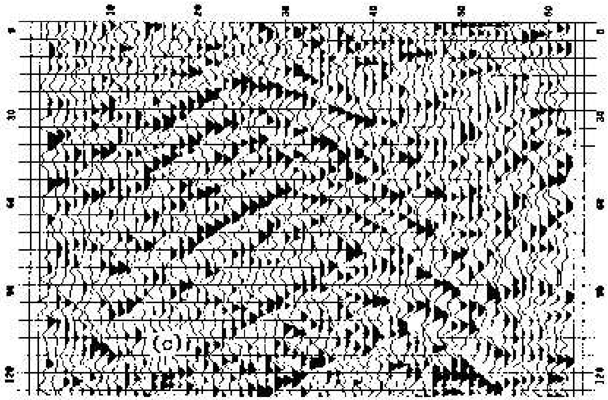
Example 1 . . .



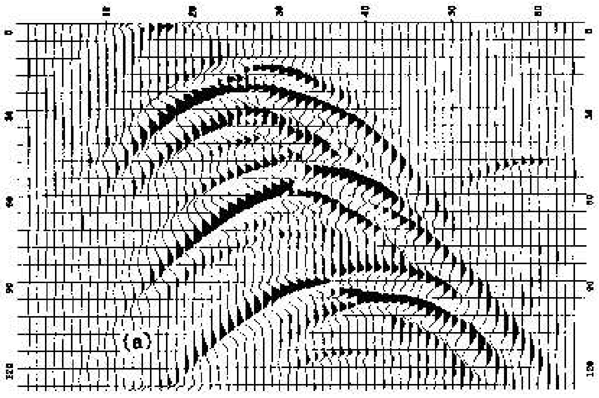
(e) Thresholded



(f) Recon

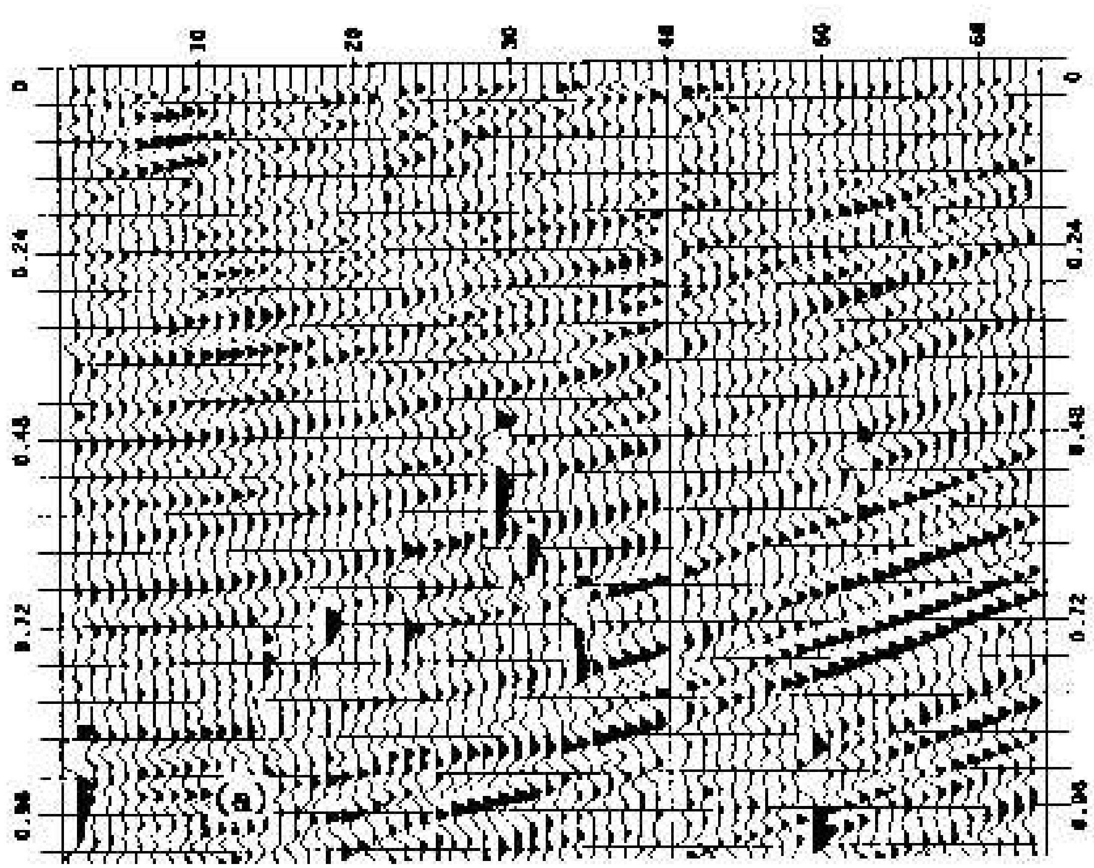


(g) Residual

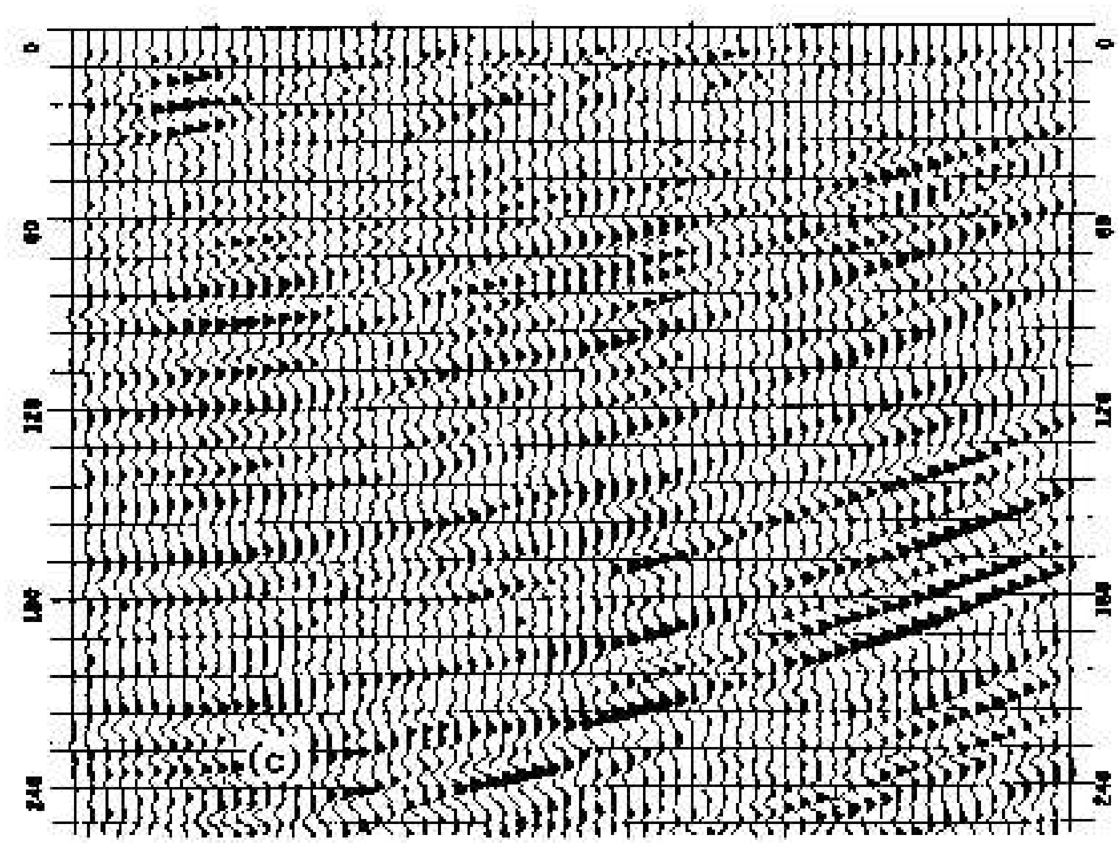


(h) Hyp. Ext.

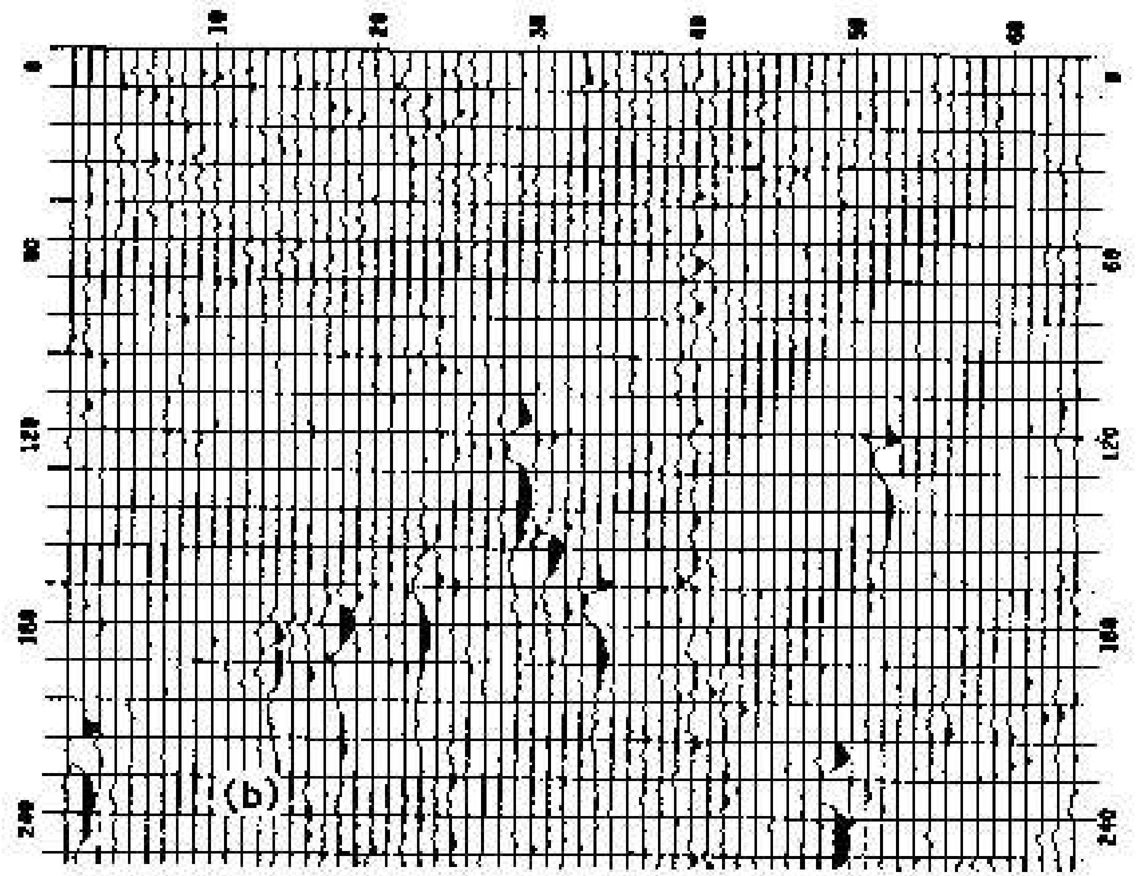
Example 2: Original



Example 2: Reconstruction



Example 2: Residual



Challenges

- Fast Radon and Generalized Radon Transforms $\Longrightarrow$ Beylkin's Fast Radon Transforms and USFFT
- Apply curvelets/ridgelets $\Longrightarrow$ F. Herrmann
- 3D is a big issue

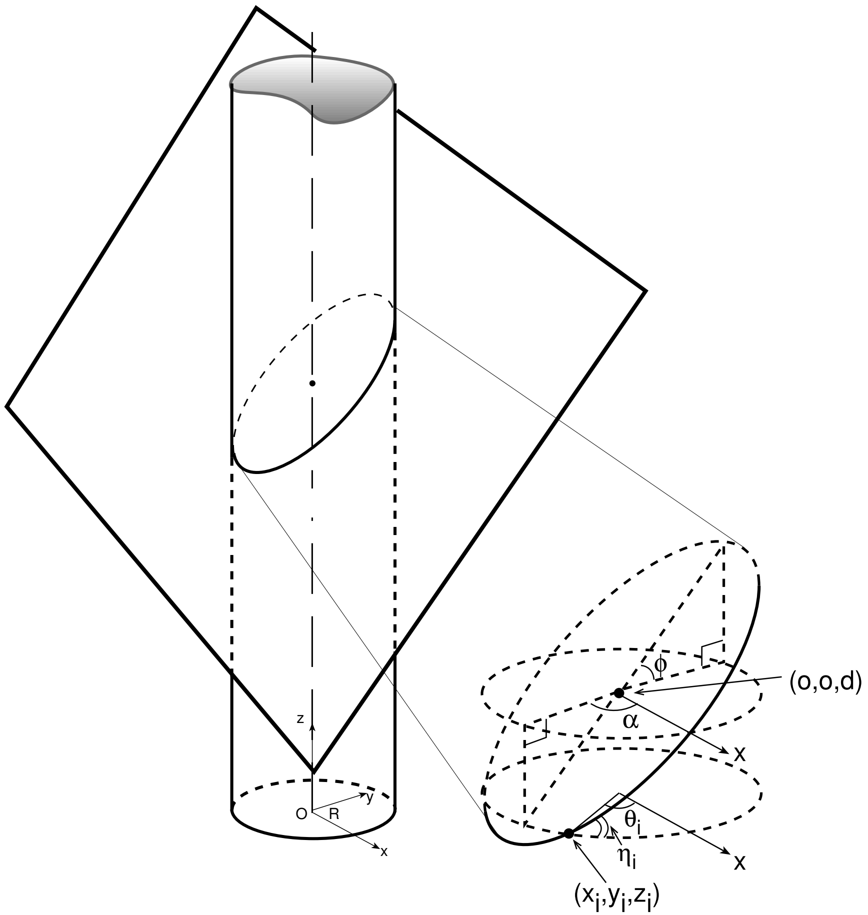
Fracture Plane Detection from Borehole Images

- Objective: detect and characterize fracture planes striking a borehole
- Measurement: borehole images (either electric or acoustic), medium$\sim$small scale
- Method: Hough transform
- Fracture plane can be parameterized by

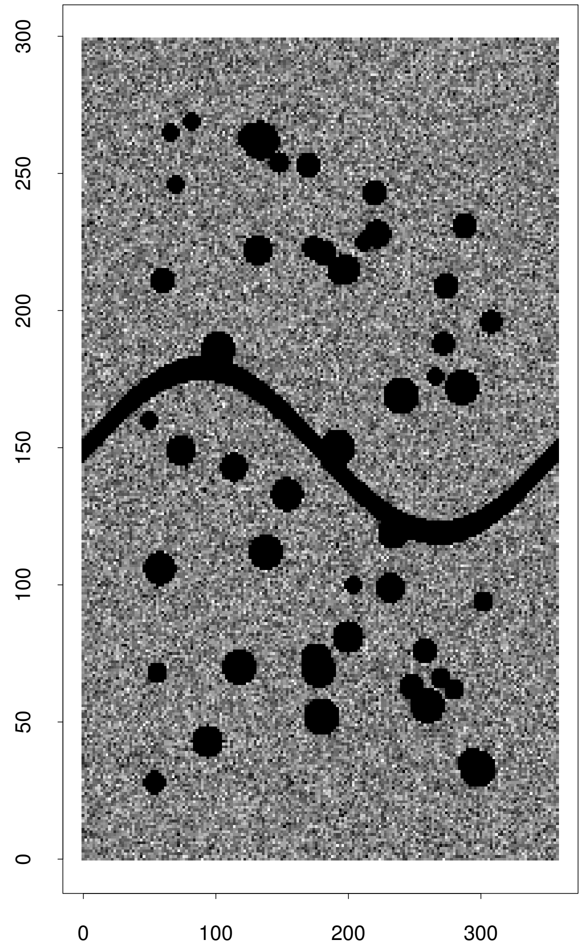
$$ax + by + z = d.$$

- The name of the game is to estimate the parameters (a, b, d) from available borehole images.

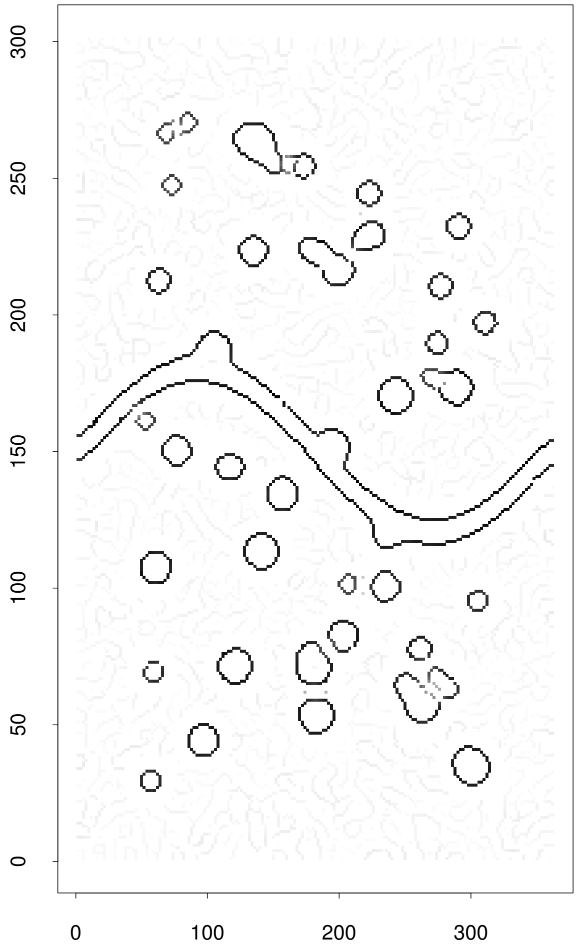
Fracture Plane Cutting a Borehole



A Synthetic Borehole Image (Unfolded)



Detected Edges



An Edge Element vs Fracture Geometry

- An edge location (x_0, y_0, z_0) and an edge orientation η_0 constrain the fracture plane:

$$
\begin{aligned}
ax_0 + by_0 + z_0 &= d \\
ay_0 - bx_0 &= R \tan \eta_0 .
\end{aligned}
$$

- These two equations specify a straight line in the (a, b, d)-space:

$$
\begin{aligned}
a &= a(d) = \frac{d - z_0}{R^2} x_0 + \frac{y_0 \tan \eta_0}{R} \\
b &= b(d) = \frac{d - z_0}{R^2} y_0 - \frac{x_0 \tan \eta_0}{R} .
\end{aligned}
$$

Hough Transform

is a popular technique to detect certain geometric patterns from the edges in images. Informally, it is related to Radon transform. For line detection in 2D:

$$H(a,b) = \iint \Theta(|\nabla_\epsilon f(x,y)|)\, \delta(b + ax - y)\, \mathrm{d}x\, \mathrm{d}y,$$

where Θ is a thresholding operation to make a binary image, ∇_ϵ is a regularized derivative with a characteristic scale ϵ. But the HT more heavily utilizes the duality between the feature space and the parameter space.

Hough Transform . . .

- Hough (1962) patented the original method for binary images
- Ballard (1981) generalized to arbitrary shape with gradient info
- Illingworth & Kittler (1988) surveyed and listed 136 papers
- Leavers (1993) surveyed and listed 173 papers

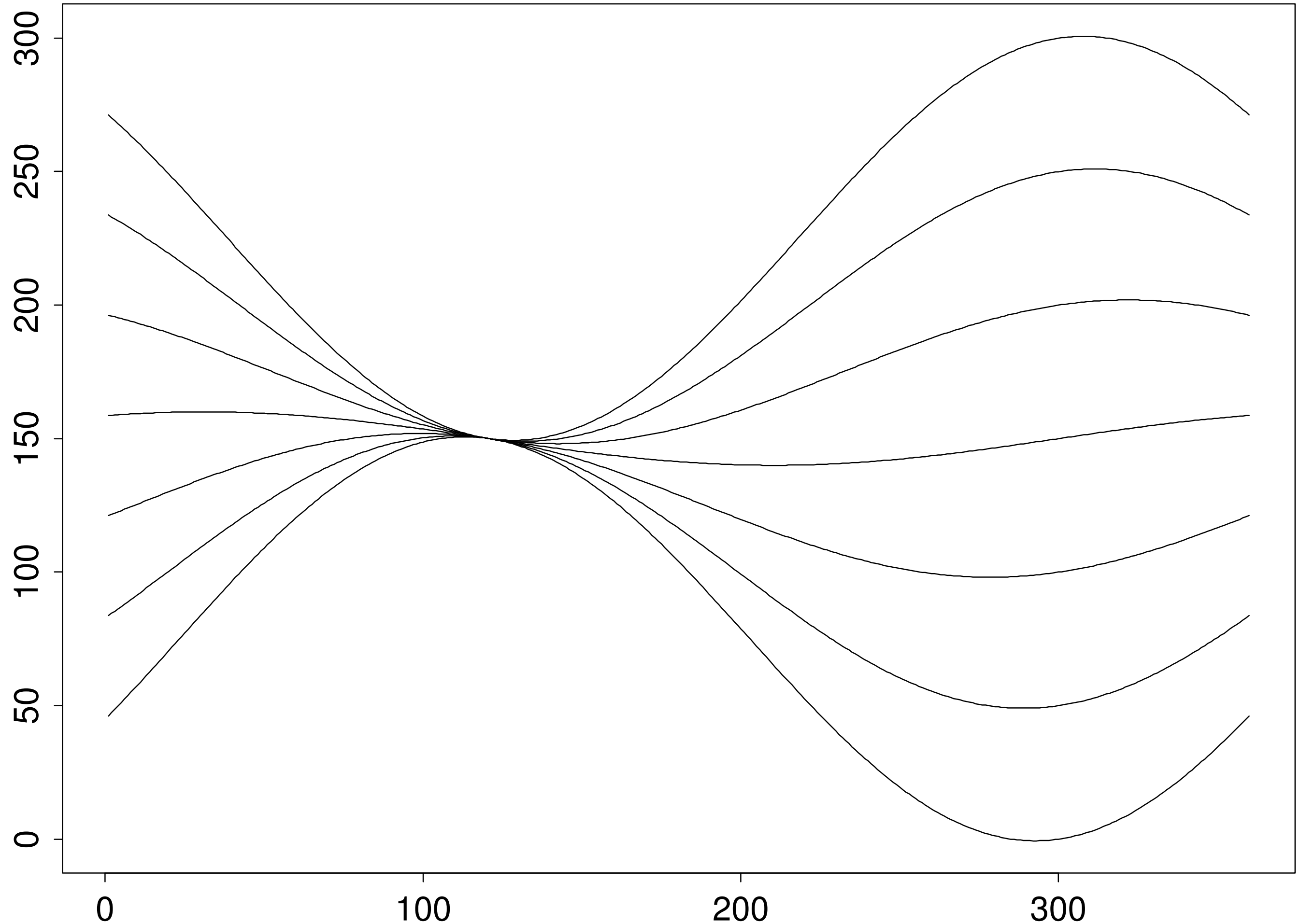
Hough Transform ...

- Compute edge positions and orientations (gradient)
- Prepare the parameter space (called accumulator array) whose axes are the parameters specifying shapes (e.g.,lines: gradient and y-intercept, circles: center and radius)
- For each edge, vote for all possible (specific) shapes in the accumulator array
- Accumulate the votes for all (significant) edges
- Local maxima in the accumulator array identify concrete shapes

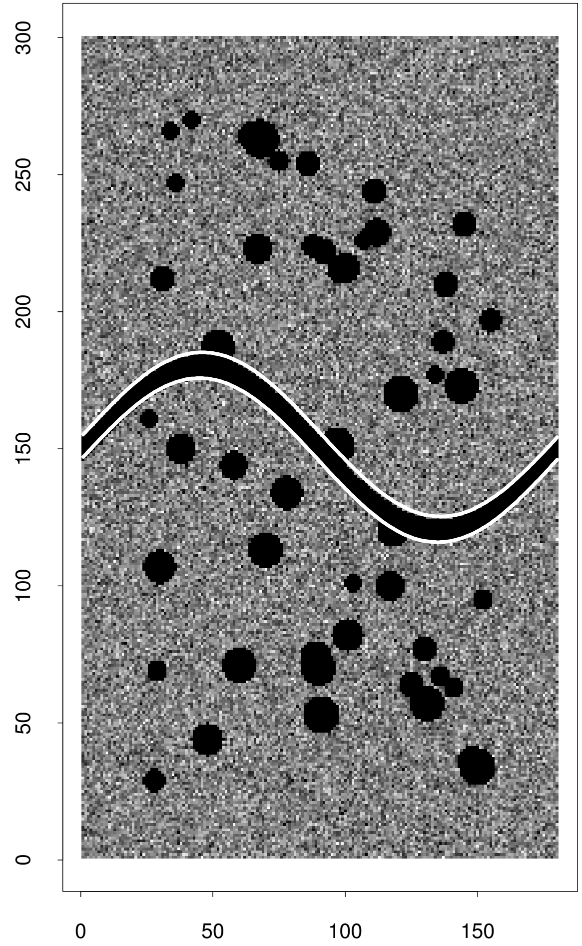
Hough Transform . . .

- + Very robust, insensitive to broken patterns and noises
- + Can detect multiple or intersecting objects
- + Can be more effective given edge orientation information
- - Computationally slow (voting process)
- - High storage (memory) requirement
- Discretization of parameter space $\leftrightarrow$ template matching in feature space

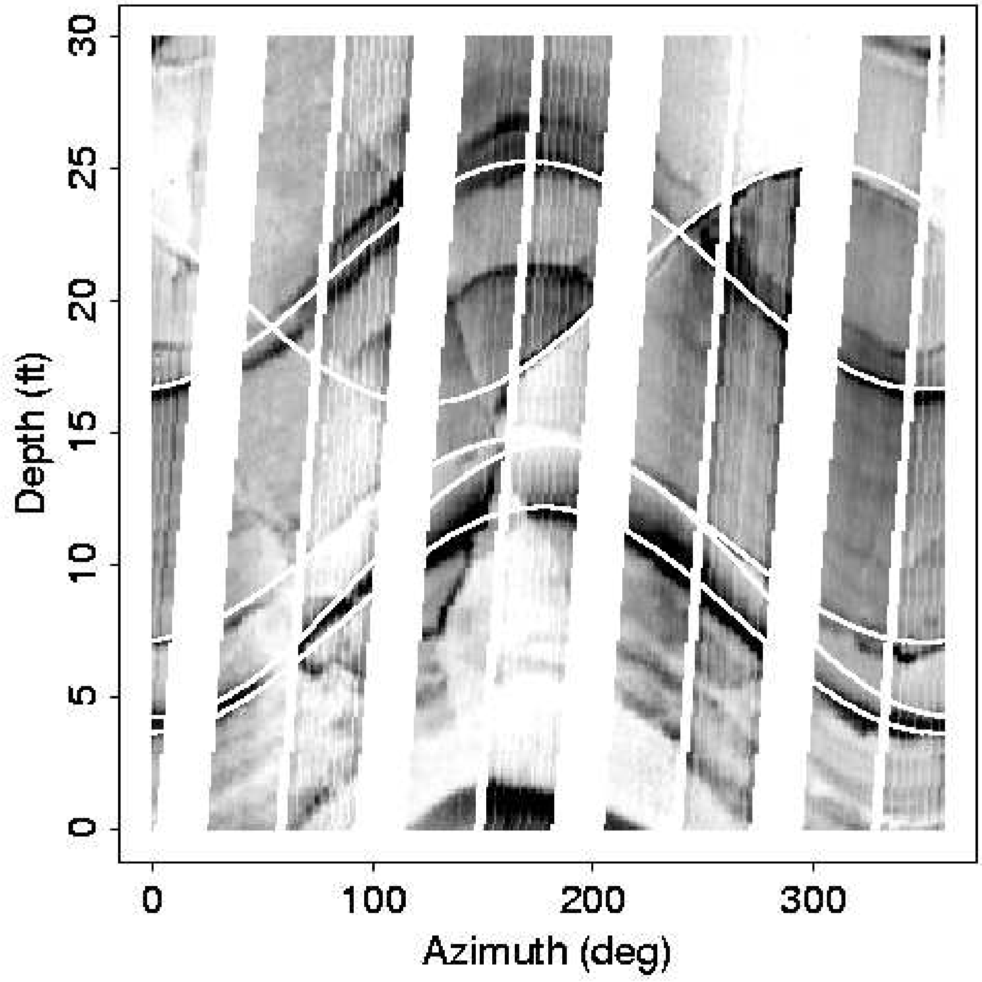
Edge Orientation Constrains Possible Planes



Detected Fracture Planes



Real Data Result



courtesy: Schlumberger

Detection of Vugs/Elliptic Shapes

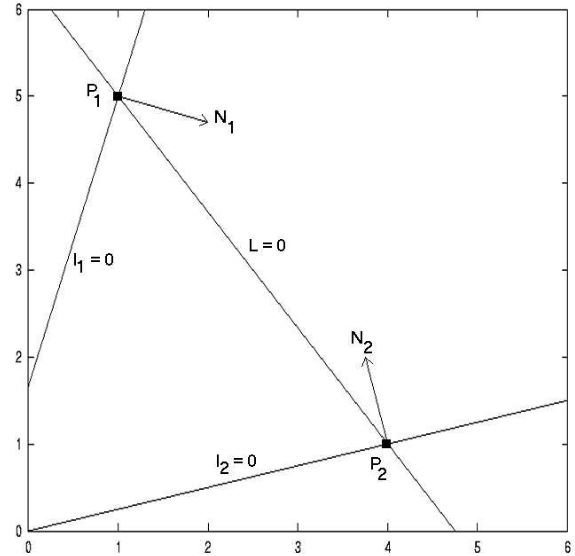
- Objective: detect and characterize vugs (elliptical cavity or void in a rock) $\Longrightarrow$ porosity, depositional information
- Measurement: borehole images (electric or acoustic), small scale
- Method: Hough transform for ellipses

Hough Transform for Ellipses

- 5 parameters (center coordinate (x, y), lengths of major/minor axes (a, b), and orientation of major axis β) are required to specify each ellipse
- Voting in the 5 dimensional accumulation array is costly
- Found lower dimensional strategy if one needs only certain combinations of parameters such as center positions and areas of ellipses (N. Bennett, R. Burridge, & NS)

Hough Transform for Ellipses . . .

- Each pair of edges in an image determines a one-parameter family of ellipses (an exercise of projective geometry).
- Let $P_i = (x_i, y_i)$ and $\boldsymbol{n}_i = (p_i, q_i)$, $i = 1, 2$ be the positions and the normal vectors of a pair of edges.



Hough Transform for Ellipses . . .

- Then, the following represents a conic section $C(x, y)$:

$$C(x, y; \lambda) \triangleq L^2(x, y) - \lambda \ell_1(x, y)\ell_2(x, y) = 0,$$

where

$$L(x, y) \triangleq \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

is the line $\overline{P_1 P_2}$,

$$\ell_i(x, y) \triangleq p_i(x - x_i) + q_i(y - y_i) = 0$$

is the tangent line at P_i.

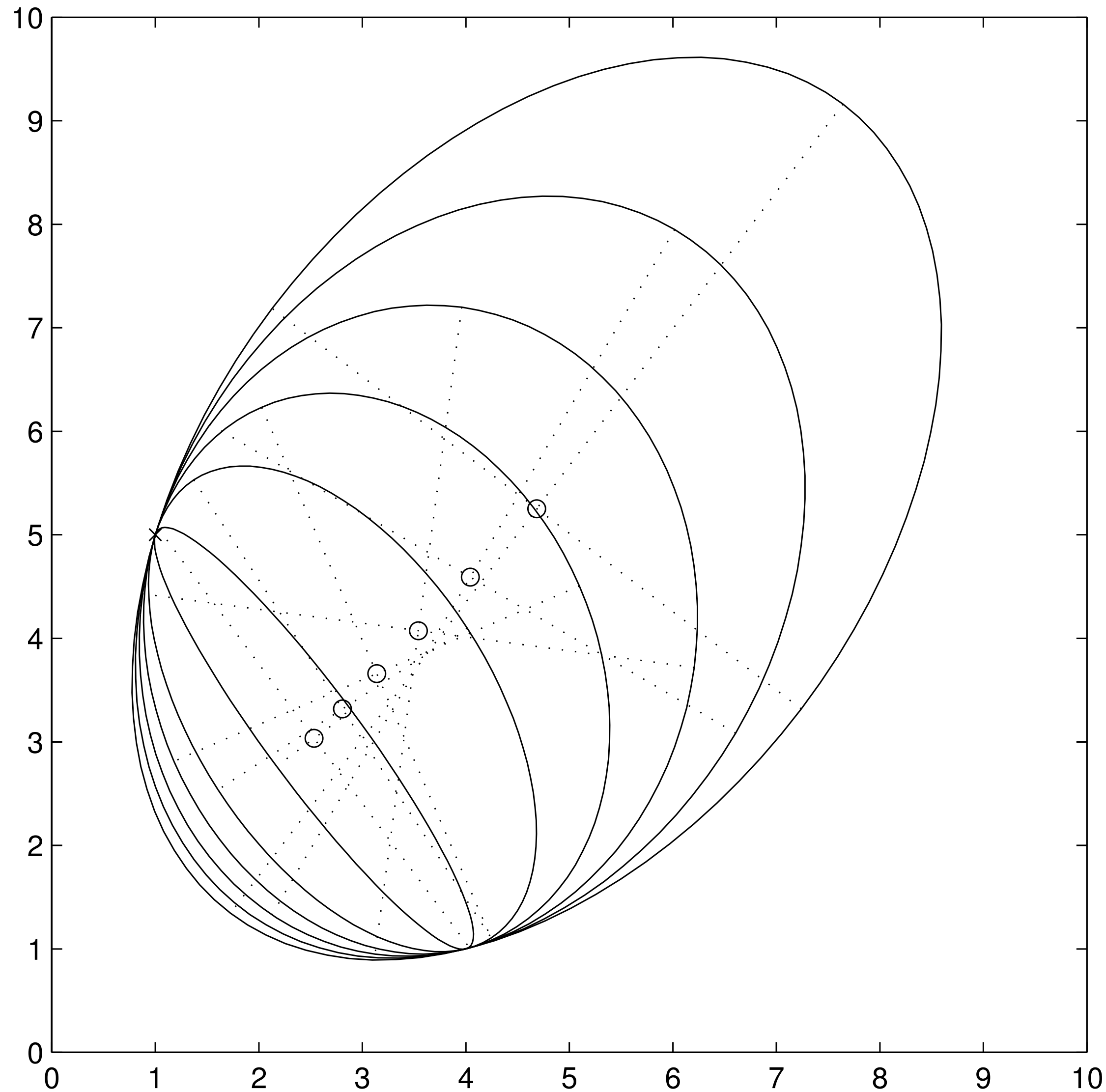
Hough Transform for Ellipses . . .

- If $\lambda \in (0, \lambda_0)$, then $C(x, y; \lambda)$ represents an ellipse where

$$\lambda_0 = 4(\boldsymbol{n}_1 \cdot \overrightarrow{P_1P_2})(\boldsymbol{n}_2 \cdot \overrightarrow{P_2P_1})/(p_1q_2 - p_2q_1)^2.$$

- Other cases: $\lambda < 0$ or $\lambda > \lambda_0$: hyperbola; $\lambda = 0$: line $\overline{P_1P_2}$; $\lambda = \lambda_0$: parabola.
- All the geometric quantities of interest (e.g., all those 5 parameters as well as its area: πab) can be written as a function of λ.

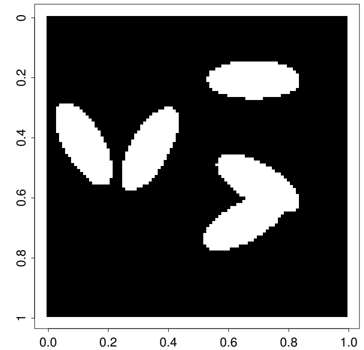
Hough Transform for Ellipses . . .



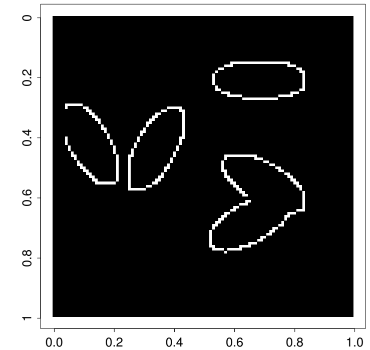
Hough Transform for Ellipses . . .

- Position and area of ellipses are of particular interest
- Need to consider only $3D$ curve $(x_0(\lambda), y_0(\lambda), \pi a(\lambda) b(\lambda))$ instead of resolving 5 parameters
- For each pair of all the strong edges (or its random subset), vote for all ellipses lying along this curve
- Pick the local maxima in this 3D voting space (accumulator array)

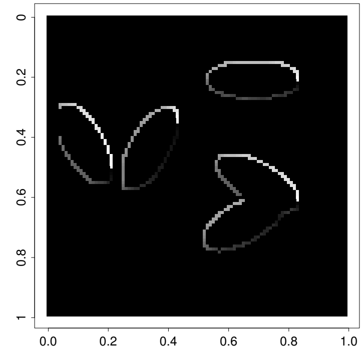
Five Ellipses



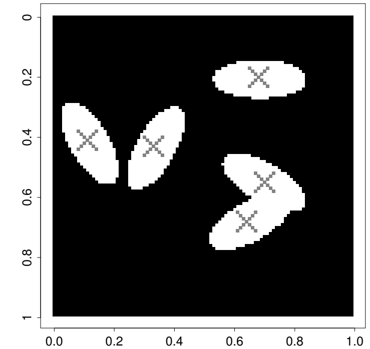
(a) Original



(b) Edge Mag.

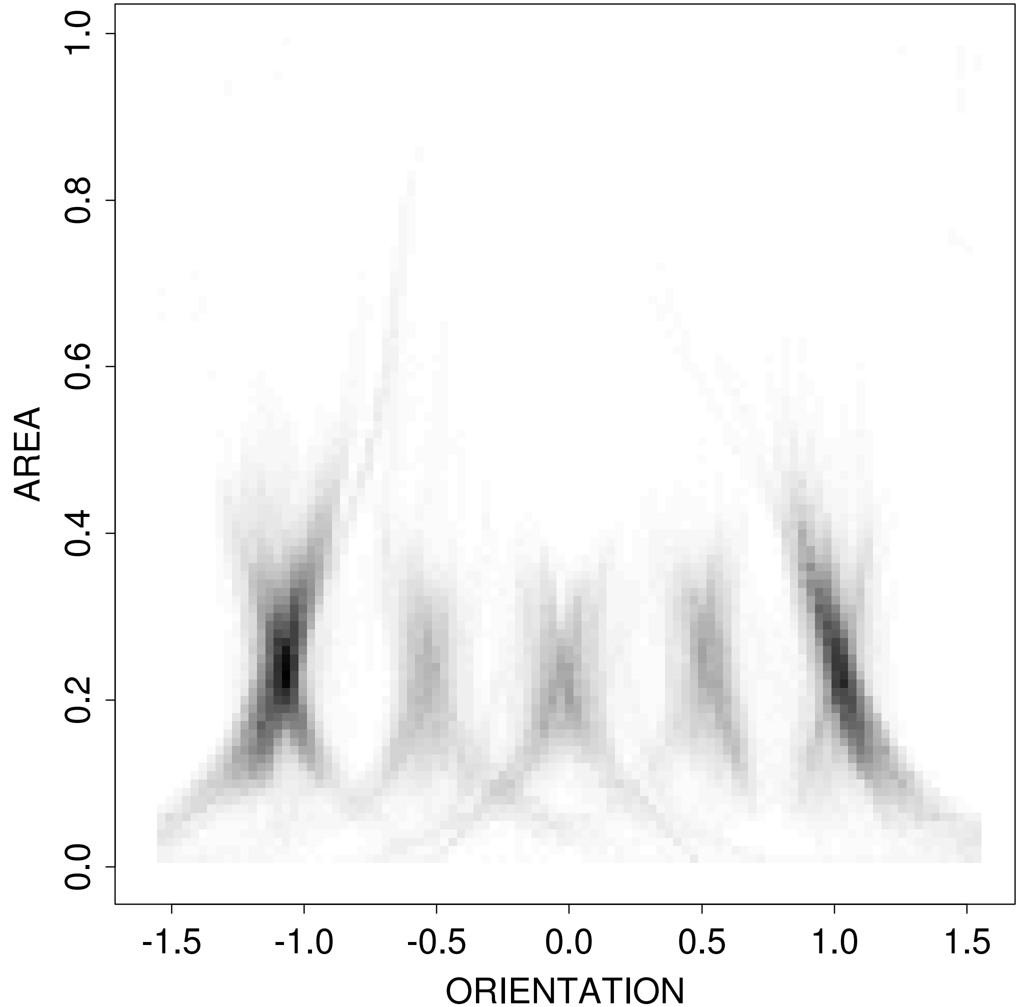


(c) Edge Ori.



(d) Detected Cnt.

Five Ellipses: Orientation vs Area



Challenges

- Fast Hough Transform via multiscale?
- Will beamlets or its generalized version help?
- Other geometric shapes?
- Remember that there are strong needs in many applications to specific geometric shapes from images!

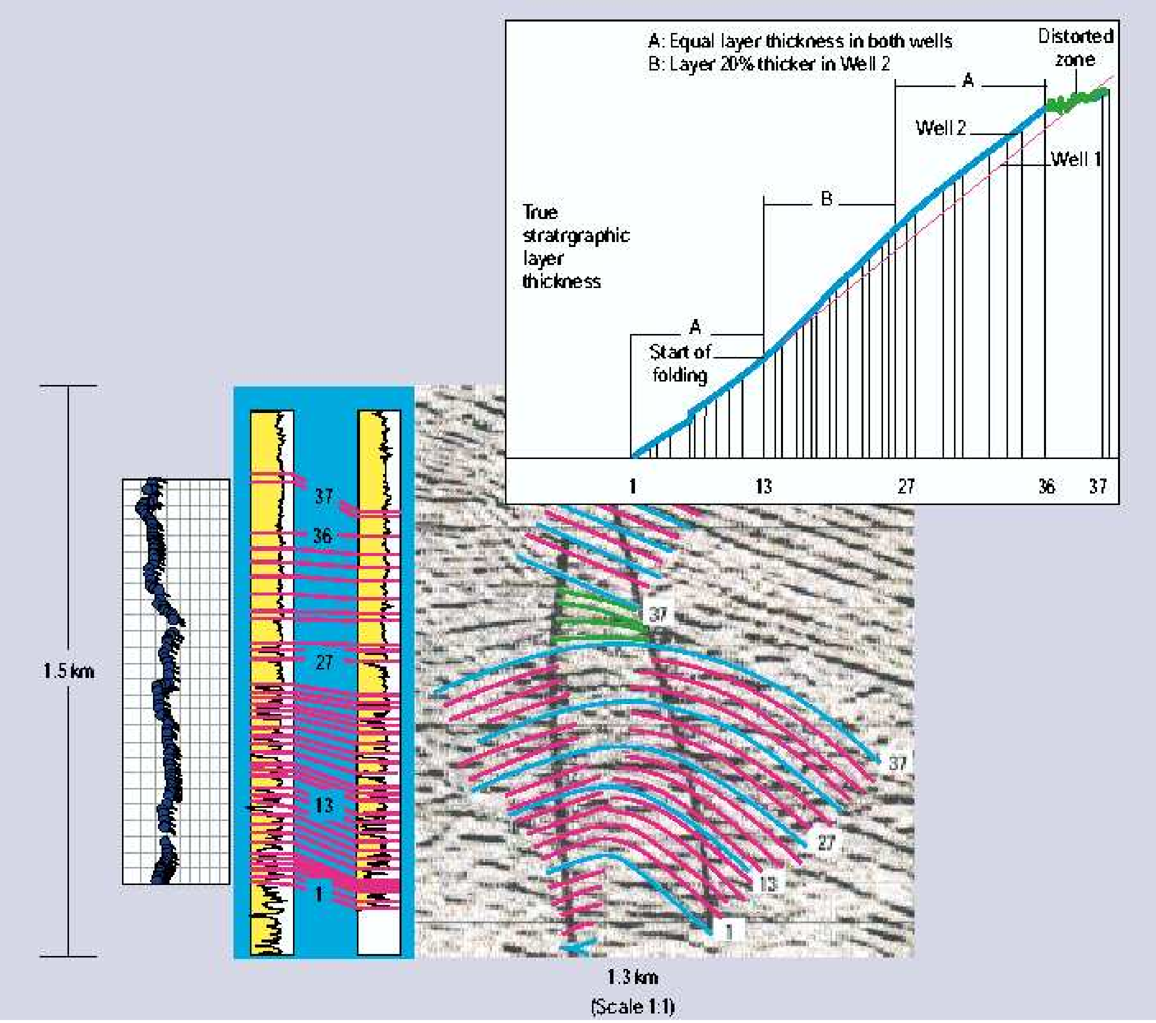
A Library of Focusing Transformations

- Each focusing transformation focuses a specific geometric object
- Can repeat extraction of different objects sequentially
- A Library of Focusing Transformations
- $Data = Objects_1 + Objects_2 + \cdots$,
  e.g., A borehole image = fractures + vugs + residuals.

Reconciliation of Different Scales

- Small$\Longrightarrow$Large: Geometric information from smaller scale measurements serves as constraints for models of seismic imaging and inversion
- Large$\Longrightarrow$Small: Geometry from surface seismic helps well-to-well correlation (for avoiding “local minima”)
- Easier said than done

Reconciliation of Different Scales . . .



courtesy: Schlumberger

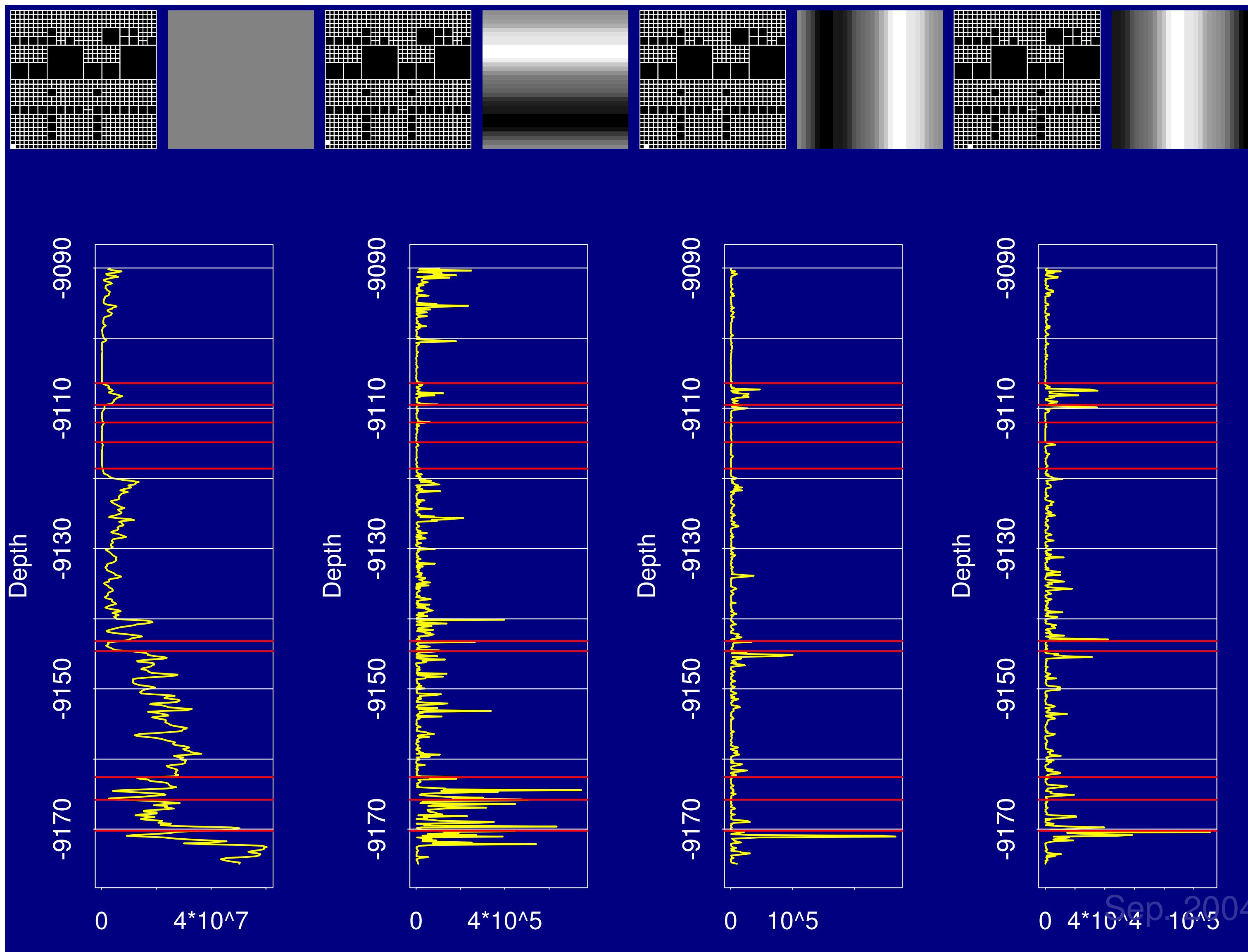
Clustering of Multiscale Geophysical Information

- Objective: identify clusters of various attributes of measurements; associate them with the facies (rock types); compare them with cores
- Measurements include:
  - The standard well logs (porosity, density, . . .)
  - Statistics (mean, variance, skewness, kurtosis) of borehole images (electric) with sliding windows;
  - Wavelet features of borehole images (electric) with sliding windows
- Method: Self-Organizing Map (SOM) algorithm

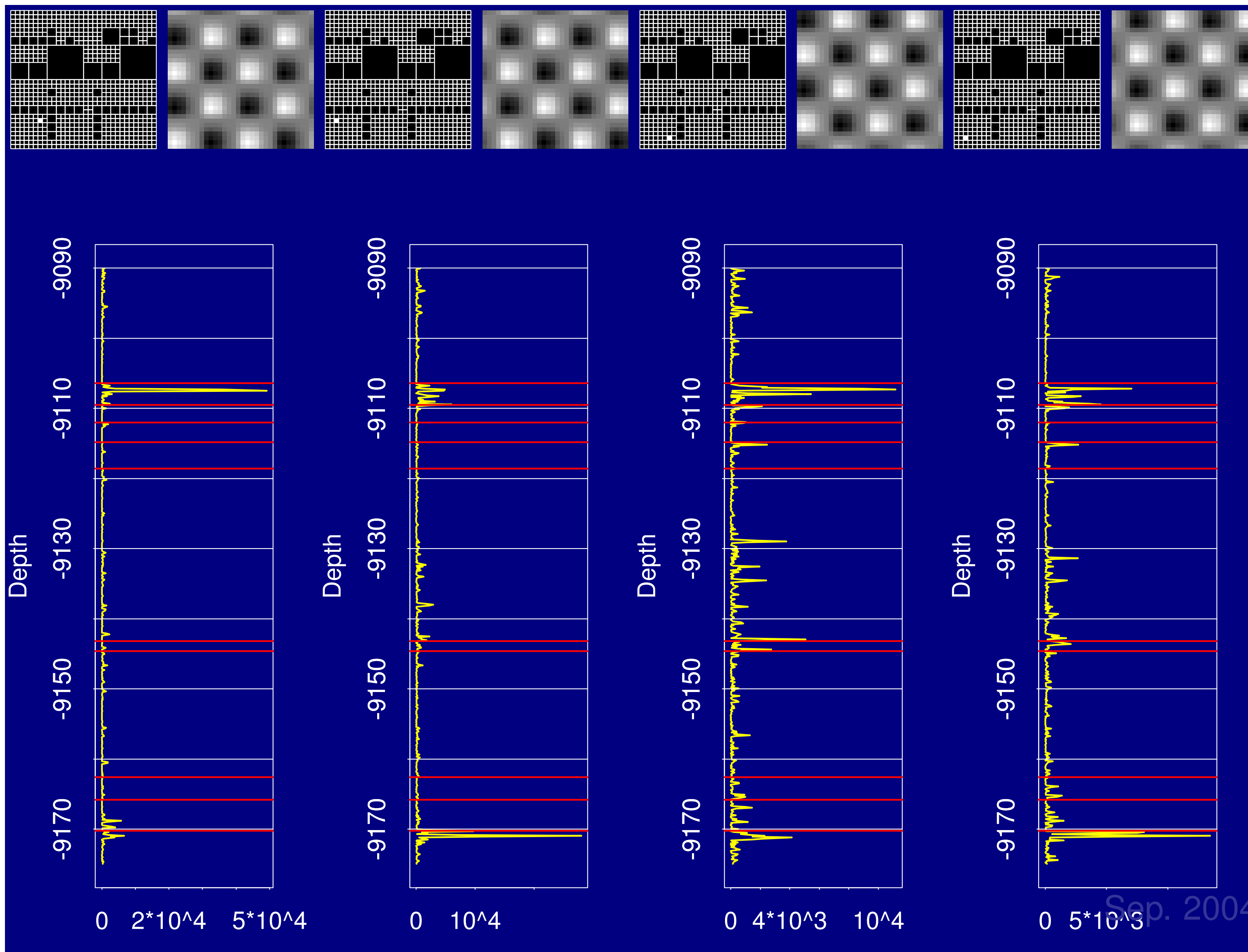
Wavelet-Based Texture Features

- Decompose available images into local frequency components via wavelet packets
- Select the best basis that captures majority of energy of images
- Represent images based on the selected basis functions
- Supply the square of the top k (say, 100) coefficients to SOM

Facies Classification from Wavelet Features



Facies Classification from Wavelet Features . . .



Self-Organizing Maps (SOM)

- A biologically motivated clustering method proposed by T. Kohonen ($\sim$ 1981)
- Viewed as a nonlinear projection of the probability density function $p(\boldsymbol{x})$ of high dimensional input vector $\boldsymbol{x} \in \mathbb{R}^d$ onto the 2D plane
- Neighboring points in $\mathbb{R}^d$ are mapped to neighboring points in $\mathbb{R}^2$
- Also viewed as a vector quantization: approximate any input vector by the closest vector in a set of reference vectors
- The Euclidean distance is often used.

Self-Organizing Maps (SOM) ...

- Consider a 2D array of nodes organized as a hexagonal lattice.
- At a node located at $\boldsymbol{r}_i$, an initial reference (random) vector $\boldsymbol{m}_i(0) \in \mathbb{R}^d$ is associated, $i = 1, \ldots, N$.
- A sequence of input vectors $\boldsymbol{x}(t) \in \mathbb{R}^d$, $t = 0, 1, \ldots$ are compared with the reference vectors $\{\boldsymbol{m}_i(t)\}$.
- For each t, the "best-matching" node $(\boldsymbol{r}_c, \boldsymbol{m}_c(t))$ with $\boldsymbol{x}(t)$ is found:

$$c = \arg \min_{1 \le i \le N} \|\boldsymbol{x}(t) - \boldsymbol{m}_i(t)\|,$$
$$\|\boldsymbol{x}(t) - \boldsymbol{m}_c\| = \min_{1 \le i \le N} \|\boldsymbol{x}(t) - \boldsymbol{m}_i(t)\|.$$

Self-Organizing Maps (SOM) ...

- Then, the neighboring nodes are updated by the following rule (learning process):

$$\boldsymbol{m}_i(t+1) = \boldsymbol{m}_i(t) + h_{ci}(t)[\boldsymbol{x}(t) - \boldsymbol{m}_i(t)],$$

  where $h_{ci}(t)$ is called the neighborhood function, a non-negative smoothing kernel over the lattice points.

- Normally, $h_{ci}(t) = h(\|\boldsymbol{r}_c - \boldsymbol{r}_i\|, t)$ with $h(\cdot, t) \downarrow 0$ as $t \uparrow \infty$, $h(r, \cdot) \downarrow 0$ as $r \uparrow \infty$.

Self-Organizing Maps (SOM) ...

- Typical examples of $h_{ci}(t)$:

$$h_{ci}(t) = \alpha(t) \exp\left(-\|\boldsymbol{r}_c - \boldsymbol{r}_i\|^2 / 2\sigma^2(t)\right),$$

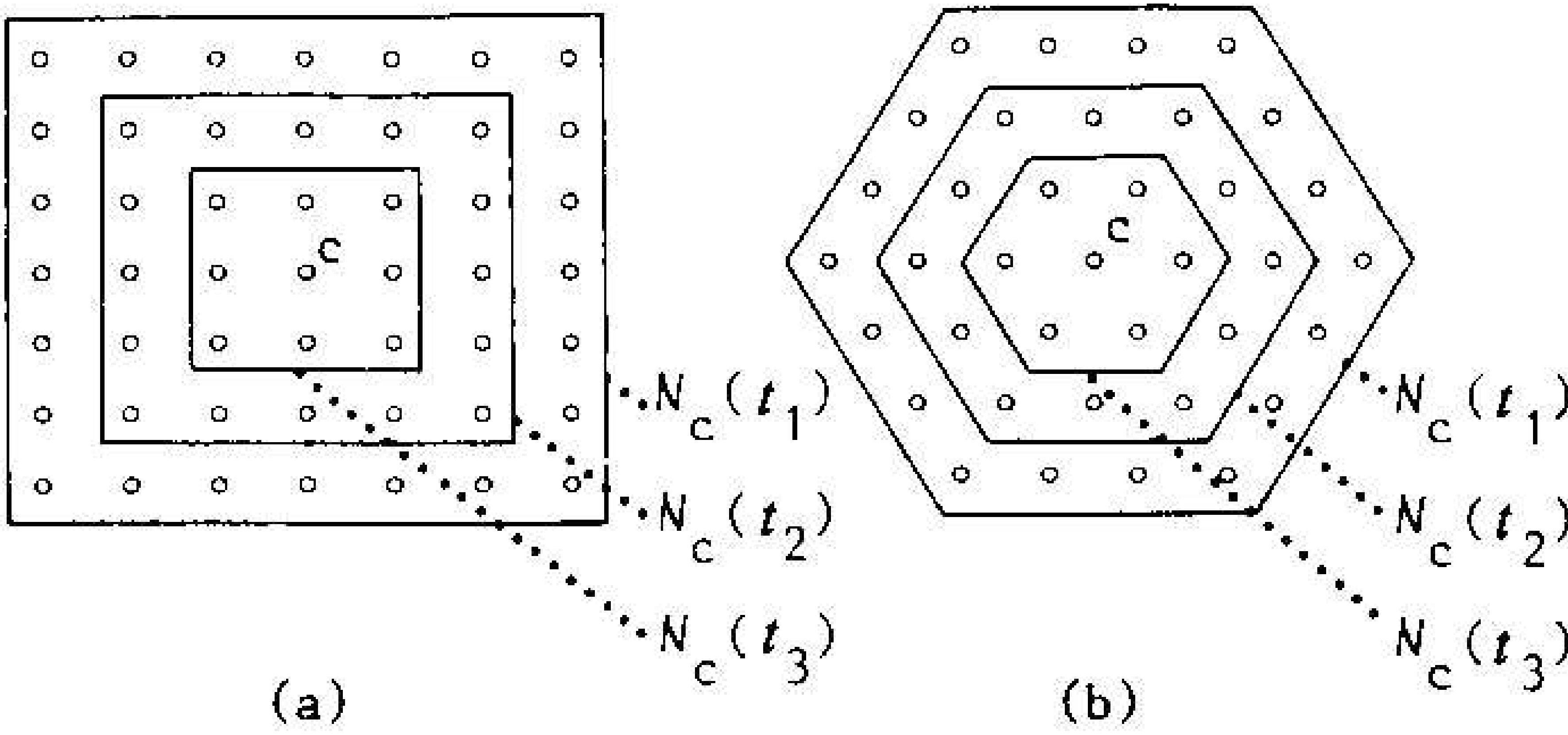
or

$$h_{ci}(t) = \alpha(t)\chi_{N_c(t)}(i),$$

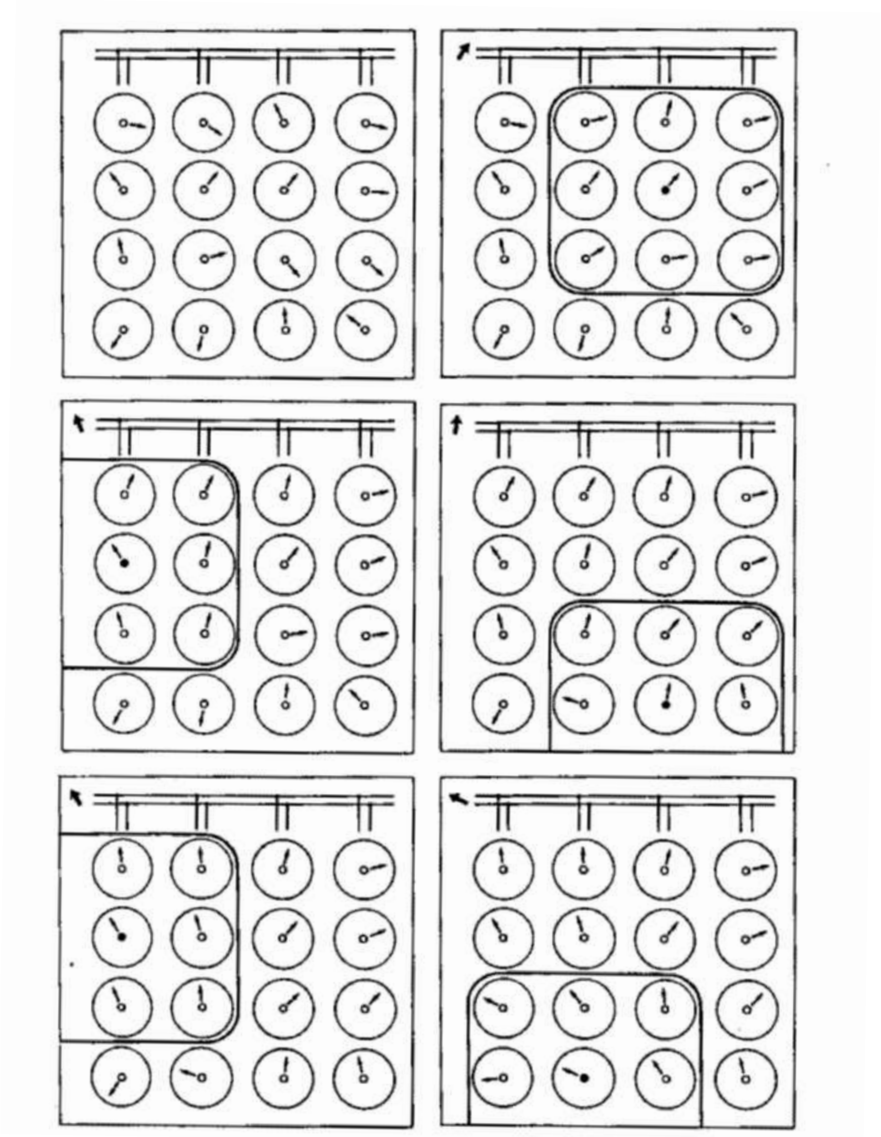
where $\alpha(t)$ is a learning-rate factor $0 < \alpha(t) < 1$, $\sigma(t)$ is an effective width of the kernel, and $N_c(t)$ is a neighborhood set of the best node c, all of which are non-increasing functions of t.

- Plasticity ...

Self-Organizing Maps (SOM) ...

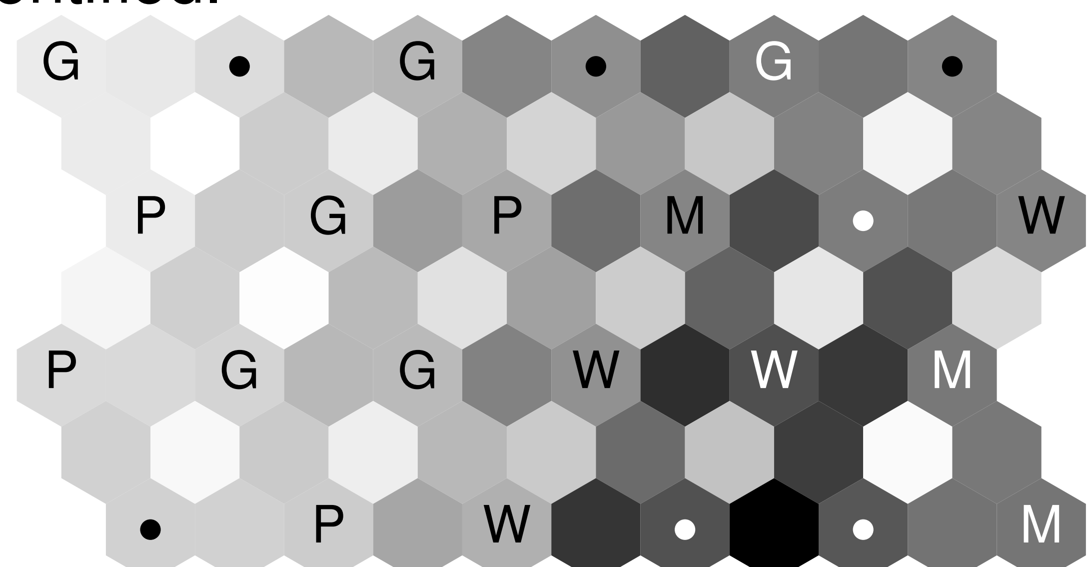


Self-Organizing Maps (SOM): An Example

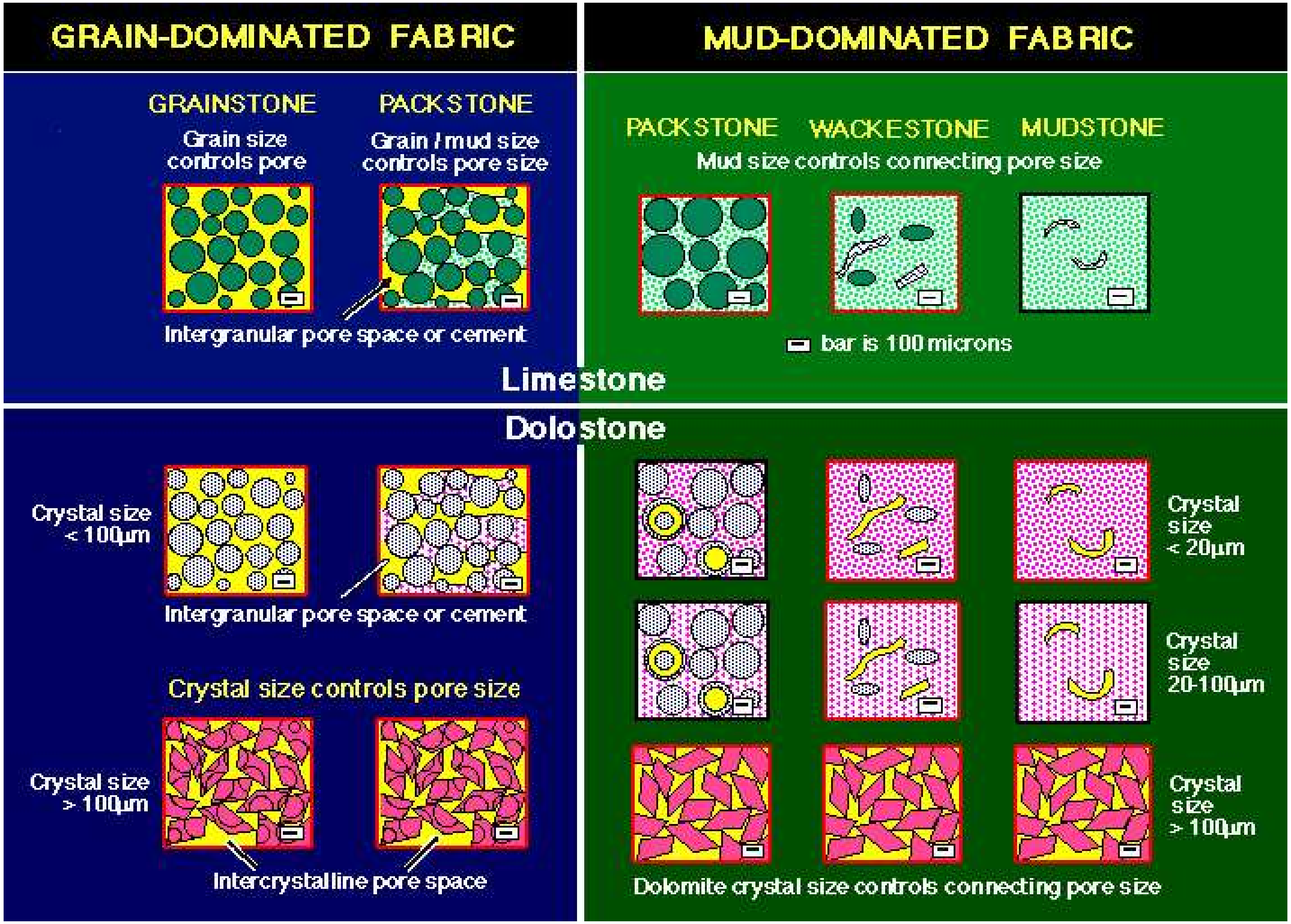


Facies Classification from Wavelet Features

- Feature vectors where the ground truth of facies were obtained from thin sections were fed to the trained SOM, which resulted in the labels (G, P, M, W).
- Clusters corresponding Grainstone-Packstone (grain supported) and Mudstone-Wackstone (mud supported) were identified.



Facies Class



courtesy: Bureau of Economic Geology: UT Austin

Remarks on SOM

- \- Many parameters to be specified, and the performance critically depends on some of them.
- \- Precise mathematical analysis of convergence is tough (proof done only for 1D array of nodes).
- \+ Each iteration is computationally fast.

Challenges

- Apply “Diffusion Maps” to the data!
- Issue of normalization of different measurements
- Separation of environmental effects and true geophysical properties of rocks and formations from measurements (the uncertainty principle!)

Summary

- Reviewed three geophysical problems
- Detection and description of geometry followed by characterization of texture
- Scale of Measurements: vast range
- Dimensions of Measurements: high
- Classical techniques have been used
- Can significantly improve via new techniques discussed in this IPAM MGA program
- May be able to provide data if interested

References

- W. S. Harlan, J. F. Claerbout, and F. Rocca: “Signal/noise separation and velocity estimation,” *Geophysics*, vol.49, pp.1869–1880, 1984.
- N. N. Bennett, R. Burridge, and N. Saito, “A method to detect and characterize ellipses using the Hough transform,” IEEE Trans. Pattern Anal. Machine Intell., vol. 21, no. 7, pp.652–657, 1999.
- N. Saito, N. N. Bennett, and R. Burridge, “Methods of Determining Dips and Azimuths of Fractures from Borehole Images,” US Patent Number 5,960,371, Grant Date: 9/28/99.
- N. Saito, A. Rabaute, and T. S. Ramakrishnan, “Method for Interpreting Petrophysical Data,” UK Patent Number GB2345776, Grant Date: 1/16/01.

References on Radon Transforms

- G.Beylkin: “Discrete Radon Transform,” IEEE Trans. Acoust., Speech, Signal Processing, vol.ASSP-35, no.2, pp.162–172, 1987.
- S.R.Deans: The Radon Transform and Some of Its Applications, Wiley Interscience, 1983.
- I.M.Gel’fand, M.I.Graev, and N.Ya.Vilenkin: Generalized Functions, vol.5, Integral Geometry and Representation Theory, Academic Press, 1966.
- L.A.Santaló: Integral Geometry and Geometric Probability, Addison-Wesley, 1976.
- L.A.Shepp and J.B.Kruskal: “Computerized tomography: The new medical x-ray technology,” Amer. Math. Monthly, vol.85, pp.402–439, 1978.

References on Hough Transform

- J.Illingworth and J.Kittler: "A survey of the Hough transform," Comput. Vision, Graphics, Image Processing, vol.44, pp.87–116, 1988,
- V.F.Leavers: "Which Hough transform?" CVGIP: Image Understanding, vol.58, pp.250–264, 1993, *This article cites 173 articles!*
- J.Princen, J.Illingworth, J.Kittler: "A formal definition of the Hough transform: Properties and relationships," J. Math. Imaging and Vision, vol.1, pp.153–168, 1992.

References on SOM

- T. Kohonen: *Self-Organizing Maps*, Springer-Verlag, 1995.