IPAM MGA Tutorial: Multiscale Problems in Geophysics

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Outline

- Multiscale Nature of Geophysical Measurements
- Importance of Geometry, Textures, and Clustering: High Relevance to MGA2004
- Large Scale: Seismic Signal/Noise Separation
- Small Scale: Fracture and Vug Detection from Electrical Conductivity Images
- High-Dimensional Clustering Problem in Exploration Geophysics
- Summary
Multiscale Nature of Geophysical Measurements

- Large Scale: Surface Seismic Survey:
  - used for structural mapping of the entire reservoir volume
  - wide spatial coverage: $\approx 10\text{km} \times 10\text{km} \times 3\text{km}$
  - spatial sampling: $25\text{m} \times 25\text{m}$
  - can resolve features $> 30\text{-}40\text{ft} (9\text{-}12\text{m})$
  - vertical axis: time
Surface Seismic Survey

GEOPHYSICAL EXPLORATION FOR OIL — THE SEISMIC REFLECTION METHOD

[Diagram of seismic cross-section]

[Diagram of seismic cross-section]
Multiscale Nature of Geophysical Measurements …

- Medium Scale: Borehole Seismics:
  - receivers are in a borehole/well
  - can record not only reflected waves but also direct waves
  - can record reflected waves with less attenuation
  - used for linking surface seismic information and well log information (e.g., sonic logs)
  - depth sampling: 20-50m (wish: < 10m)
Borehole Seismics

courtesy: Schlumberger
Borehole Seismics ...
Multiscale Nature of Geophysical Measurements . . .

- Small Scale: Wireline measurements, cores:
  - used for detailed geological/geophysical information in the vicinity of wells
  - spatial coverage: $\approx 1\text{m} \times 1\text{m} \times 3\text{km}$
  - can resolve features $\approx 0.5$-3ft (0.15-1m) or less
  - vertical axis: depth
  - cores provide absolute truth, but expensive
  - can characterize not only elastic but also electrical and nuclear properties of subsurface formations
Electrical Conductivity Image ($\approx 2.5\text{ft} \times 2.3\text{ft}$)

courtesy: Schlumberger
Electrical Conductivity Imaging Tool

(a) Tool

(b) Pad/Flap

courtesy: Schlumberger
Multiscale Nature of Geophysical Measurements …

- Large Scale Geometry
- Medium/Small Scale Geometry
- Textures (Stratigraphy, Lithology, Facies)
- Reconciliation of multiscale info for better models
- Highly Relevant to MGA 2004

This tutorial: introduction of problems + some attempts with older techniques

My hope: provoke the audience so that some of you may be interested in applying new techniques discussed in MGA 2004 to these problems!
Extraction of Large Scale Geometric Objects

Harlan, Claerbout, & Rocca (1984)

- **Motivation:** Separate geologic events (layer boundaries) from other events and noise. Other events could be interference patterns, scattering from faults in the form of hyperbolas, etc.

- **Measurement:** Seismic signals (large scale)

- **Beginning of** “multilayered transforms,” “coherent structure extraction,” and ICA

- **Method:** iteration of “focusing” transforms + soft thresholding

- Define “focusing” as an invertible linear transformation making the data more statistically independent (or less statistically dependent)

- A transform focusing signal (pattern) must also defocus noise

- Focused signal becomes more non-Gaussian; defocused noise becomes more Gaussian

- In the transformed domain, apply filtering or soft-thresholding operations to extract signal or separate signal from noise
A stacked seismic section = \( \sum \) of

- **Geologic** component \( \approx \) linear events
- **Diffraction** component \( \approx \) hyperbolic events
- **Noise** component \( \approx \) white Gaussian noise + \( \alpha \).
Radon Transform

The 2D-line version for $f(x,y)$ can be stated as follows:

$$R_f(\tau, p) = \int \int f(x, t) \delta(t - \tau - px) \, dx \, dt.$$  

- Has a huge list of applications (X-ray tomography, geophysics, wave propagation, . . . )
- Can be generalized: integration of an $n$-dimensional function over $m$-dimensional geometric objects
- Inversion formulas exist as Beylkin showed yesterday
- Lines in an image are transformed to sharp peaks in $(\tau, p)$-domain
Example 1

(a) Original

(b) $\tau - p$

(c) Randomized

(d) $\tau - p$
Example 1 . . .

(e) Thresholded

(f) Recon

(g) Residual

(h) Hyp. Ext.
Example 2: Original
Example 2: Reconstruction
Example 2: Residual
Challenges

- Fast Radon and Generalized Radon Transforms
- Beylkin’s Fast Radon Transforms and USFFT
- Apply curvelets/ridgelets
- 3D is a big issue
Fracture Plane Detection from Borehole Images

- Objective: detect and characterize fracture planes striking a borehole
- Measurement: borehole images (either electric or acoustic), medium~small scale
- Method: Hough transform
- Fracture plane can be parameterized by

\[ ax + by + z = d. \]

- The name of the game is to estimate the parameters \((a, b, d)\) from available borehole images.
Fracture Plane Cutting a Borehole
A Synthetic Borehole Image (Unfolded)
Detected Edges
An Edge Element vs Fracture Geometry

- An edge location \((x_0, y_0, z_0)\) and an edge orientation \(\eta_0\) constrain the fracture plane:

\[
ax_0 + by_0 + z_0 = d \\
ay_0 - bx_0 = R \tan \eta_0.
\]

- These two equations specify a straight line in the \((a, b, d)\)-space:

\[
a = a(d) = \frac{d - z_0}{R^2} x_0 + \frac{y_0 \tan \eta_0}{R} \\
b = b(d) = \frac{d - z_0}{R^2} y_0 - \frac{x_0 \tan \eta_0}{R}.
\]
Hough Transform

is a popular technique to detect certain geometric patterns from the edges in images. Informally, it is related to Radon transform. For line detection in 2D:

\[
H(a, b) = \int\int \Theta(|\nabla_\epsilon f(x, y)|) \delta(b + ax - y) \, dx \, dy,
\]

where \( \Theta \) is a thresholding operation to make a binary image, \( \nabla_\epsilon \) is a regularized derivative with a characteristic scale \( \epsilon \). But the HT more heavily utilizes the duality between the feature space and the parameter space.
Hough Transform . . .

- Hough (1962) patented the original method for binary images
- Ballard (1981) generalized to arbitrary shape with gradient info
- Illingworth & Kittler (1988) surveyed and listed 136 papers
- Leavers (1993) surveyed and listed 173 papers
Hough Transform . . .

- Compute edge positions and orientations (gradient)
- Prepare the parameter space (called accumulator array) whose axes are the parameters specifying shapes (e.g., lines: gradient and $y$-intercept, circles: center and radius)
- For each edge, vote for all possible (specific) shapes in the accumulator array
- **Accumulate** the votes for all (significant) edges
- Local maxima in the accumulator array identify concrete shapes
Hough Transform …

+ Very robust, insensitive to broken patterns and noises
+ Can detect multiple or intersecting objects
+ Can be more effective given edge orientation information
- Computationally slow (voting process)
- High storage (memory) requirement

Discretization of parameter space ↔ template matching in feature space
Edge Orientation Constrains
Possible Planes
Detected Fracture Planes
Real Data Result

courtesy: Schlumberger
Detection of Vugs/Elliptic Shapes

- Objective: detect and characterize vugs (elliptical cavity or void in a rock) → porosity, depositional information
- Measurement: borehole images (electric or acoustic), small scale
- Method: Hough transform for ellipses
Hough Transform for Ellipses

- 5 parameters (center coordinate \((x, y)\), lengths of major/minor axes \((a, b)\), and orientation of major axis \(\beta\)) are required to specify each ellipse.

- Voting in the 5 dimensional accumulation array is costly.

- Found lower dimensional strategy if one needs only certain combinations of parameters such as center positions and areas of ellipses (N. Bennett, R. Burridge, & NS)
Each pair of edges in an image determines a one-parameter family of ellipses (an exercise of projective geometry).

Let $P_i = (x_i, y_i)$ and $n_i = (p_i, q_i)$, $i = 1, 2$ be the positions and the normal vectors of a pair of edges.
Hough Transform for Ellipses …

Then, the following represents a conic section \( C(x, y) \):

\[
C(x, y; \lambda) \triangleq L^2(x, y) - \lambda \ell_1(x, y) \ell_2(x, y) = 0,
\]

where

\[
L(x, y) \triangleq \begin{vmatrix}
    x & y & 1 \\
    x_1 & y_1 & 1 \\
    x_2 & y_2 & 1
\end{vmatrix} = 0
\]

is the line \( \overline{P_1P_2} \),

\[
\ell_i(x, y) \triangleq p_i(x - x_i) + q_i(y - y_i) = 0
\]

is the tangent line at \( P_i \).
If $\lambda \in (0, \lambda_0)$, then $C(x, y; \lambda)$ represents an ellipse where

$$\lambda_0 = 4(\mathbf{n}_1 \cdot \overrightarrow{P_1P_2})(\mathbf{n}_2 \cdot \overrightarrow{P_2P_1})/(p_1q_2 - p_2q_1)^2.$$ 

Other cases: $\lambda < 0$ or $\lambda > \lambda_0$: hyperbola; $\lambda = 0$: line $\overrightarrow{P_1P_2}$; $\lambda = \lambda_0$: parabola.

All the geometric quantities of interest (e.g., all those 5 parameters as well as its area: $\pi ab$) can be written as a function of $\lambda$. 

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Hough Transform for Ellipses
Position and area of ellipses are of particular interest.

Need to consider only 3D curve $(x_0(\lambda), y_0(\lambda), \pi a(\lambda)b(\lambda))$ instead of resolving 5 parameters.

For each pair of all the strong edges (or its random subset), vote for all ellipses lying along this curve.

Pick the local maxima in this 3D voting space (accumulator array).
Five Ellipses

(a) Original

(b) Edge Mag.

(c) Edge Ori.

(d) Detected Cnt.
Five Ellipses: Orientation vs Area
Challenges

- Fast Hough Transform via multiscale?
- Will beamlets or its generalized version help?
- Other geometric shapes?
- Remember that there are strong needs in many applications to **specific** geometric shapes from images!
A Library of Focusing Transformations

- Each focusing transformation focuses a specific geometric object
- Can repeat extraction of different objects sequentially
- A Library of Focusing Transformations

\[
Data = Objects_1 + Objects_2 + \cdots
\]
e.g., A borehole image = fractures + vugs + residuals.
Reconciliation of Different Scales

- Small $\rightarrow$ Large: Geometric information from smaller scale measurements serves as constraints for models of seismic imaging and inversion

- Large $\rightarrow$ Small: Geometry from surface seismic helps well-to-well correlation (for avoiding “local minima”)

- Easier said than done
Reconciliation of Different Scales...
Clustering of Multiscale Geophysical Information

- Objective: identify clusters of various attributes of measurements; associate them with the facies (rock types); compare them with cores

- Measurements include:
  - The standard well logs (porosity, density, ...)
  - Statistics (mean, variance, skewness, kurtosis) of borehole images (electric) with sliding windows;
  - Wavelet features of borehole images (electric) with sliding windows

- Method: Self-Organizing Map (SOM) algorithm
Wavelet-Based Texture Features

- Decompose available images into local frequency components via wavelet packets
- Select the best basis that captures majority of energy of images
- Represent images based on the selected basis functions
- Supply the square of the top $k$ (say, 100) coefficients to SOM
Facies Classification from Wavelet Features
Facies Classification from Wavelet Features...
Self-Organizing Maps (SOM)

- A biologically motivated clustering method proposed by T. Kohonen (~1981)

- Viewed as a **nonlinear** projection of the probability density function $p(x)$ of high dimensional input vector $x \in \mathbb{R}^d$ onto the 2D plane

- Neighboring points in $\mathbb{R}^d$ are mapped to neighboring points in $\mathbb{R}^2$

- Also viewed as a **vector quantization**: approximate any input vector by the closest vector in a set of **reference vectors**

- The Euclidean distance is often used.
Consider a 2D array of nodes organized as a hexagonal lattice.

At a node located at $r_i$, an initial reference (random) vector $m_i(0) \in \mathbb{R}^d$ is associated, $i = 1, \ldots, N$.

A sequence of input vectors $x(t) \in \mathbb{R}^d$, $t = 0, 1, \ldots$ are compared with the reference vectors $\{m_i(t)\}$.

For each $t$, the “best-matching” node $(r_c, m_c(t))$ with $x(t)$ is found:

$$c = \arg \min_{1 \leq i \leq N} \|x(t) - m_i(t)\|,$$

$$\|x(t) - m_c\| = \min_{1 \leq i \leq N} \|x(t) - m_i(t)\|.$$
Self-Organizing Maps (SOM) . . .

Then, the neighboring nodes are updated by the following rule (learning process):

\[ m_i(t + 1) = m_i(t) + h_{ci}(t)[x(t) - m_i(t)], \]

where \( h_{ci}(t) \) is called the neighborhood function, a non-negative smoothing kernel over the lattice points.

Normally, \( h_{ci}(t) = h(||r_c - r_i||, t) \) with \( h(\cdot, t) \downarrow 0 \) as \( t \uparrow \infty \), \( h(r, \cdot) \downarrow 0 \) as \( r \uparrow \infty \).
Typical examples of $h_{ci}(t)$:

$$h_{ci}(t) = \alpha(t) \exp \left( -\| r_c - r_i \|^2 / 2\sigma^2(t) \right),$$

or

$$h_{ci}(t) = \alpha(t) \chi_{N_c(t)}(i),$$

where $\alpha(t)$ is a learning-rate factor $0 < \alpha(t) < 1$, $\sigma(t)$ is an effective width of the kernel, and $N_c(t)$ is a neighborhood set of the best node $c$, all of which are non-increasing functions of $t$.

Plasticity . . .
Self-Organizing Maps (SOM) ...
Self-Organizing Maps (SOM): An Example
Facies Classification from Wavelet Features

Feature vectors where the ground truth of facies were obtained from thin sections were fed to the trained SOM, which resulted in the labels (G, P, M, W).

Clusters corresponding Grainstone-Packstone (grain supported) and Mudstone-Wackstone (mud supported) were identified.
Facies Class

**GRAIN-DOMINATED FABRIC**
- **GRAINSTONE**: Grain size controls pore
  - Intergranular pore space or cement
- **PACKSTONE**: Grain size controls pore size

**MUD-DOMINATED FABRIC**
- **PACKSTONE**: Mud size controls connecting pore size
- **WACKESTONE**: Mud size controls connecting pore size
- **MUDSTONE**: Mud size controls connecting pore size
  - Bar is 100 microns

**Limestone**
- Crystal size $< 100 \mu m$
  - Intergranular pore space or cement
- Crystal size $> 100 \mu m$
  - Intercrystalline pore space

**Dolostone**
- Crystal size $< 20 \mu m$
- Crystal size $20-100 \mu m$
- Crystal size $> 100 \mu m$
  - Dolomite crystal size controls connecting pore size

courtesy: Bureau of Economic Geology: UT Austin

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Remarks on SOM

- Many parameters to be specified, and the performance critically depends on some of them.

- Precise mathematical analysis of convergence is tough (proof done only for 1D array of nodes).

+ Each iteration is computationally fast.
Challenges

- Apply “Diffusion Maps” to the data!
- Issue of normalization of different measurements
- Separation of environmental effects and true geophysical properties of rocks and formations from measurements (the uncertainty principle!)
Summary

- Reviewed three geophysical problems
- Detection and description of geometry followed by characterization of texture
- Scale of Measurements: vast range
- Dimensions of Measurements: high
- Classical techniques have been used
- Can significantly improve via new techniques discussed in this IPAM MGA program
- May be able to provide data if interested
References


References on Radon Transforms

References on Hough Transform


References on SOM