Seismic compression

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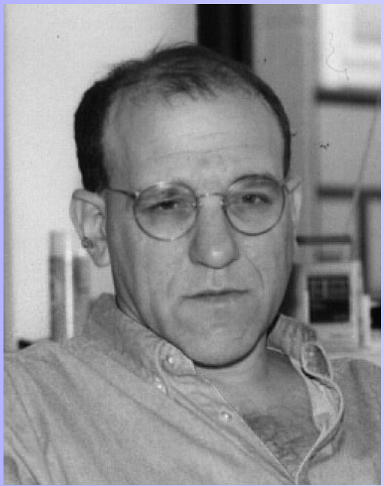
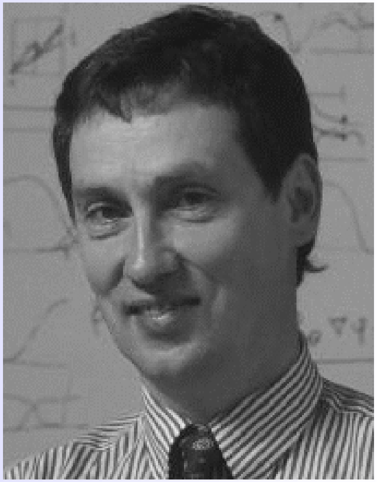
Collaborators


Amir Averbuch


Raphy Coifman

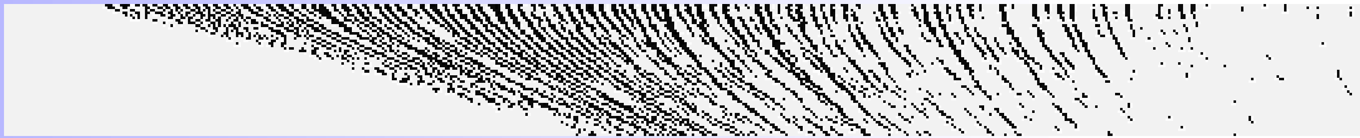

Jan-Olov Strömberg


Anthony Vassiliou

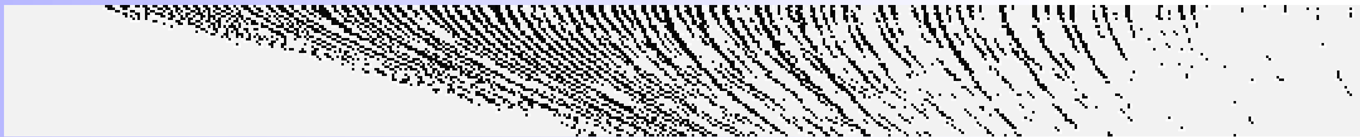
Introduction

- seismic data: HUGE ($\sim$ 100 Tbyte)
- datasets: large dynamic range, geometric features: important
- seismic data: highly oscillatory
- Wavelet compression of seismic data [Ergas et al., 1995, Reiter and Heller, 1994, Vassiliou and Wickerhauser, 1997]
- But: rapid variations of intensity $\rightarrow$ many fine scale coefficients
- very diffuse representations in a standard wavelet basis

Marine shot gathers



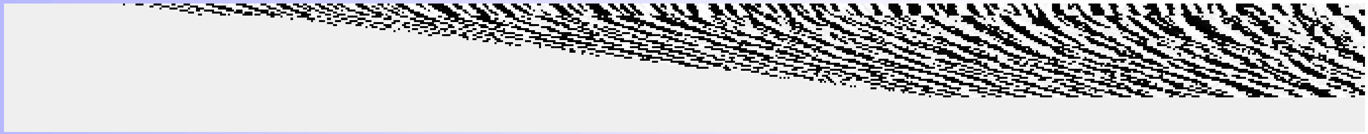
original, range$=[-280\ 10^6, 280\ 10^6]$, size $=2048 \times 96$



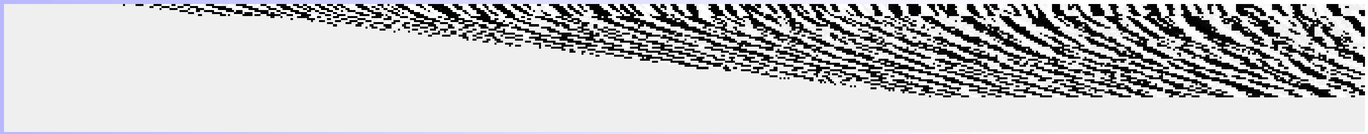
Compression by 20, SNR=35.

Error, range $=[-10^6, 10^6]$.

Common depth gathers



original, range=$[-4000, 4000]$, size=1472×224



Compression by 20, SNR=46.

Error, range $= [-10, 10]$.

Performance comparison

compression ratio	wavelet (SNR)	LCT (SNR)
10	94	63
20	48	48
30	33	37
40	26	29
50	22	25

Common depth gathers

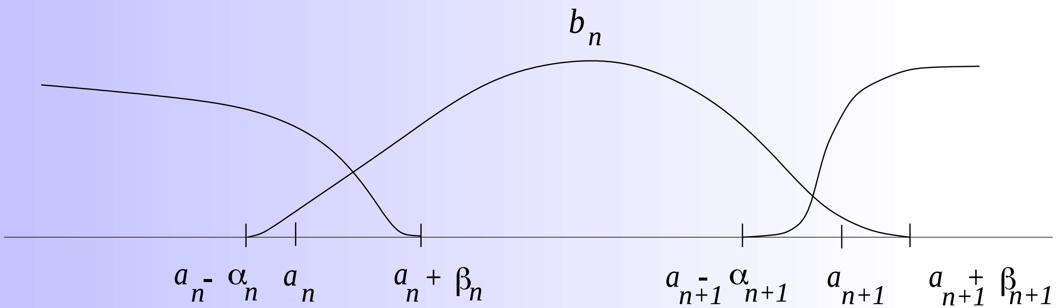
compression ratio	wavelet (SNR)	LCT (SNR)
10	48	55
20	36	43
30	31	37
40	27	33
50	25	30

Marine shot gathers

$$\mathrm{SNR} = \frac{l^2(f)}{l^2(f - \tilde{f})} \tag{1}$$

Local cosine outperform wavelets for seismic datasets !

Adaptive smooth local cosine transforms



- biorthogonal bases
- $\mathbb{R} = \bigcup_{n=-\infty}^{n=+\infty} [a_n, a_{n+1}[$
- neighborhood around each point a_n: $[a_n - \alpha_n, a_n + \beta_n]$

$$a_n + \beta_n < a_{n+1} - \alpha_{n+1}. \qquad (2)$$

- b_n : bell function lives over the interval $[a_n - \alpha_n, a_{n+1} + \beta_{n+1}]$.

Dual bells

$$\tilde{b}_n(x) = \begin{cases} \theta_n(x) b_{n-1}(2a_n - x) & \text{if} \quad x \in [a_n - \alpha_n, a_n + \beta_n] \\ \dfrac{1}{b_n(x)} & \text{if} \quad x \in [a_n + \beta_n, a_{n+1} - \alpha_{n+1}] \\ \theta_{n+1}(x) b_{n+1}(2a_{n+1} - x) & \text{if} \quad x \in [a_{n+1} - \alpha_{n+1}, a_{n+1} + \beta_{n+1}] \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$\theta_n(x) = \frac{1}{b_n(x)\, b_{n-1}(2a_n - x) + b_n(2a_n - x)\, b_{n-1}(x)} \tag{4}$$

Biorthogonal local cosine bases

DCT-IV:

$$c_{n,k}(x) = \sqrt{\frac{2}{a_{n+1} - a_n}} \cos\left[(k + 1/2)\frac{x - a_n}{a_{n+1} - a_n}\right] \tag{5}$$

We define the family

$$w_{n,k} = b_n(x)\, c_{n,k}(x) \tag{6}$$

and the dual family:

$$w_{n,k} = \tilde{b}_n(x)\, c_{n,k}(x) \tag{7}$$

Biorthogonal local cosine bases

Lemma 1 *$w_{n,j}$ and $\tilde{w}_{n,j}$ are Riesz biorthogonal bases:*

$$\int w_{n,j}(x)\, \tilde{w}_{k,m}(x)\, dx = \delta_{j,k}\delta_{n,m} \tag{8}$$

$\forall \mathbf{x} \in L^2(\mathbb{R})$,

$$\mathbf{x}(x) = \sum_{n,j} x_{n,j} w_{n,j}(x) \qquad \textit{with} \quad x_{n,j} = \int \mathbf{x}(x)\tilde{w}_{n,j}(x)dx \tag{9}$$

$$\mathbf{x}(x) = \sum_{n,j} \tilde{x}_{n,j} \tilde{w}_{n,j}(x) \qquad \textit{with} \quad \tilde{x}_{n,j} = \int \mathbf{x}(x) w_{n,j}(x)dx \tag{10}$$

Choice of the bell function

- biorthogonal bases: more freedom to select the bells b_n
- b_n can be optimized
- seismic data : oscillatory $\rightarrow$ [Matviyenko, 1996]
- other choices: reproduce constants [Jawerth and Sweldens, 1995], or polynomials [Bittner, 1999]

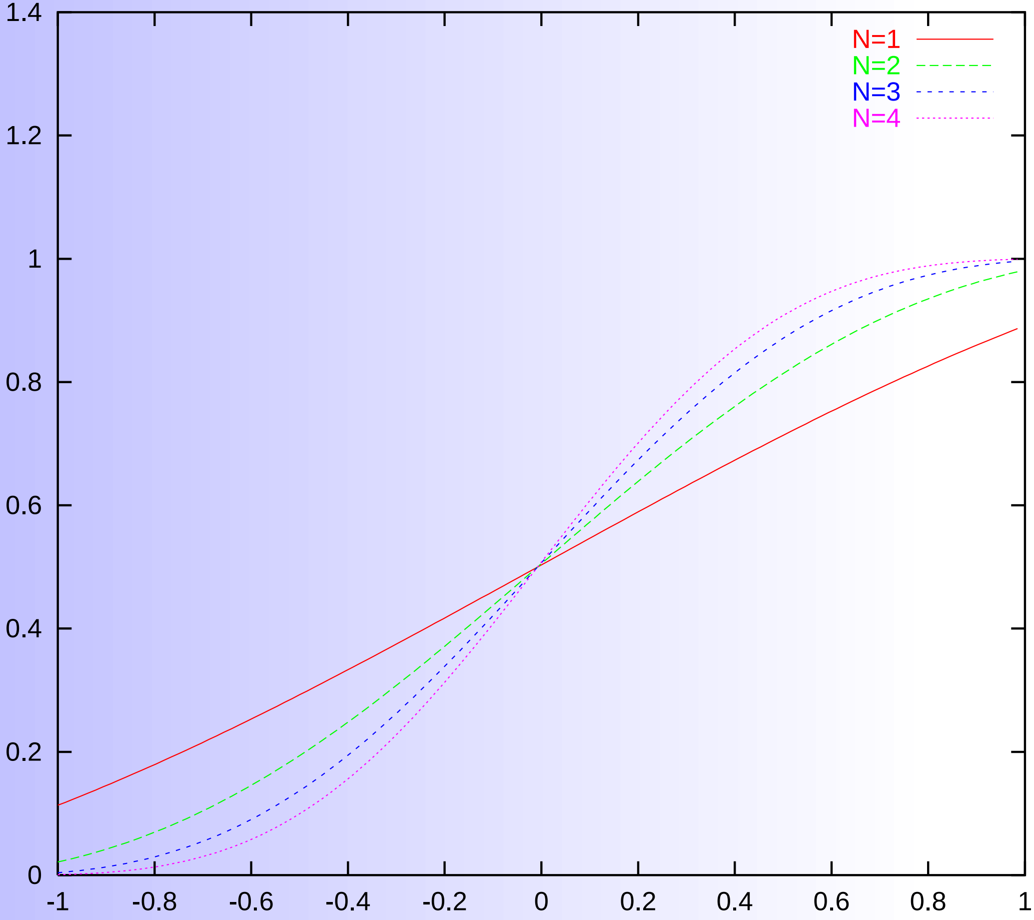
Bells of Matviyenko

- class of signals: $\mathbf{x} = \cos(\omega k + \varphi)$
- minimum number of coefficients $x_{n,j}$ to reconstruct $\mathbf{x}$ up to an error ε
- $$\begin{aligned} 0 &\le b(x) \le 1 \\ 0 &\le \tilde{b}(x) \le (\sqrt{(}2) + 1)/2 \end{aligned} \tag{11}$$

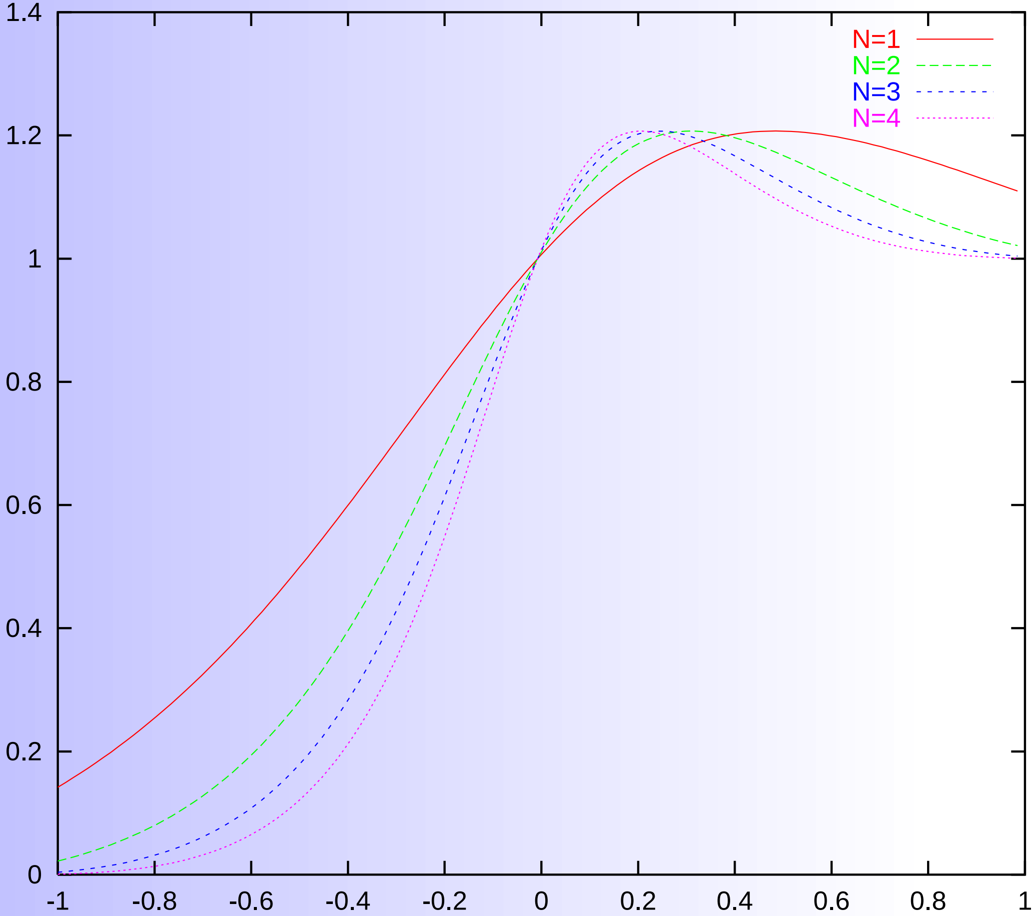
Bells of Matviyenko

$$b^N(x) = \begin{cases} \frac{1}{2}\left(1 + \sum_{n=0}^{N-1} g_n \sin(n+1/2)\pi x\right) & \text{if} \quad x \in [-1/2, 1/2] \\ \frac{1}{2}\left(1 + \sum_{n=0}^{N-1} (-1)^n g_n \cos(n+1/2)\pi x\right) & \text{if} \quad x \in [1/2 : 3/2] \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

coefficients g_n are calculated numerically



Optimized bells. $N = 1, 2, 3, 4$



Optimized dual bells. $N = 1, 2, 3, 4$

Adaptive segmentation

- quadtree segmentation
- preserve the original aspect ratio of the data
- anisotropic segmentations [Bennett, 2000]
- depth first approach: minimize memory
- extension on the borders

Choice of a cost function

- "entropy" [Coifman and Wickerhauser, 1992]

$$h(\mathbf{x}) = -\sum_k \frac{|x_k|^2}{\|\mathbf{x}\|^2} \log \frac{|x_k|^2}{\|\mathbf{x}\|^2} \qquad (13)$$

- Rate distortion [Ramchandran and Vetterli, 1993]

$$\max_{\lambda} \left\{ \min_{T \in \mathcal{T}} \left\{ \sum_{\text{node} \in T} \min_{q \in \mathcal{Q}} \{ D_{\text{node}}(q) + \lambda R_{\text{node}}(q) \} \right\} \right\} \qquad (14)$$

$\lambda \in \mathbb{R}$
$\mathcal{Q}$: set of all quantizers
$\mathcal{T}$: set of all bases T with quadtree structure
$\rightarrow$ very high computational complexity !
$\rightarrow$ first order entropy estimates only

Choice of a cost function

- cost function: estimate of the actual rate achieved by each node
- mimics the actual scalar quantization, and entropy coding
- much faster to compute
- composed of two complementary terms:
  - $c_1(\mathbf{x})$: cost of coding the sign and the magnitude of the non zero output levels of the scalar quantizer,
  - $c_2(\mathbf{x})$: cost of coding the locations of the non zero output levels (significance map),

Choice of a cost function

- $c_1(\mathbf{x}) = \sum_{k/Q(x_k)\neq 0} \max\left(\log_2 |Q(x_k)|, 0\right)$
- fast implementation: representation of floating numbers
- $c_2(\mathbf{x}) = -N\left(p\log_2(p) + (1-p)\log_2(1-p)\right)$
- first order entropy of a Bernoulli process: each coefficient x_k is significant with a probability p

Fast DCT-IV

- DCT-IV : FFT of half length
- DCT-IV coefficients, $\hat{x}(j), \quad j = 0, \ldots, N-1$ of the sequence $x(n), \quad n = 0, \ldots, N-1$

$$\begin{aligned}
\hat{x}(j) &= \mathrm{Re}\left(e^{-\frac{ij\pi}{2N}} \sum_{n=0}^{N/2-1} y(n) e^{-\frac{2i\pi jn}{N}} \right) \\
\hat{x}(N-j-1) &= -\mathrm{Im}\left(e^{-\frac{ij\pi}{2N}} \sum_{n=0}^{N/2-1} y(n) e^{-\frac{2i\pi jn}{N}} \right)
\end{aligned} \tag{15}$$

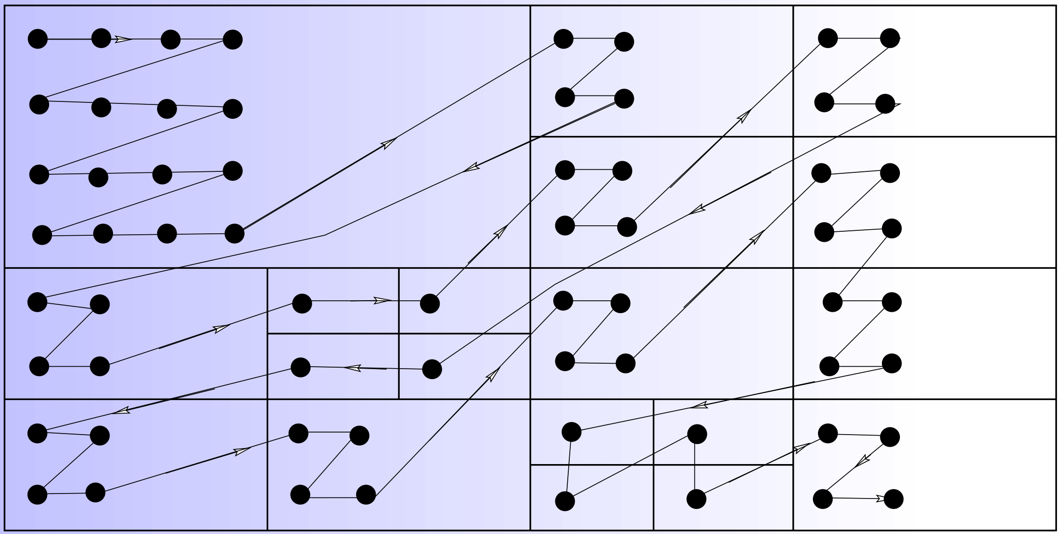
with

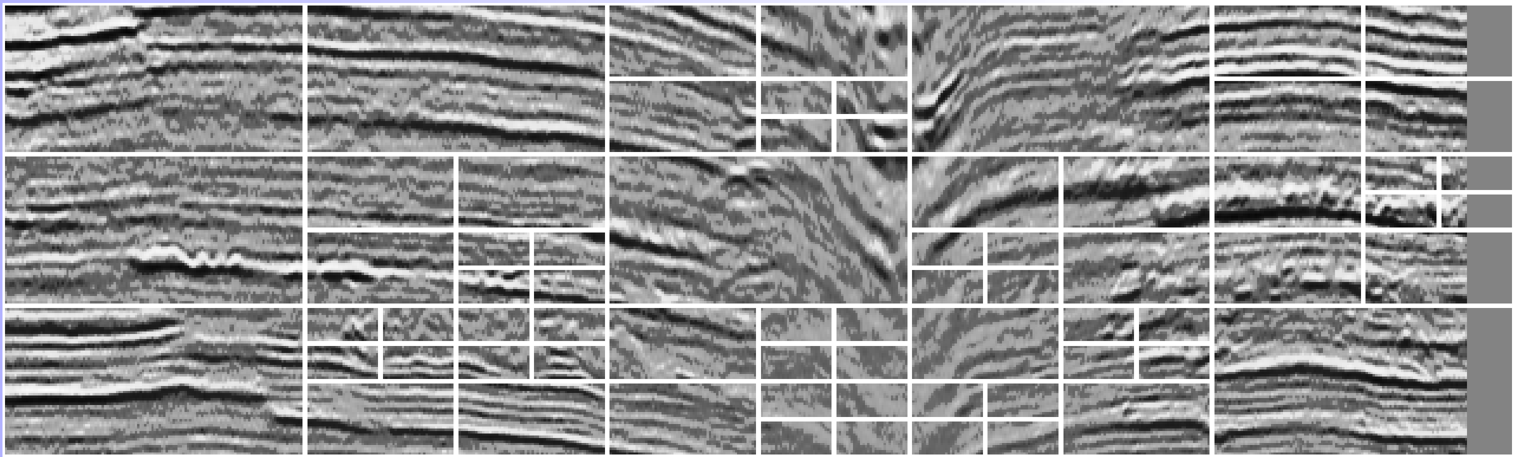
$$y(n) = (x(2n) + i\, x(N-2n-1))\, e^{-\frac{i(n+1/4)\pi}{N}} \tag{16}$$

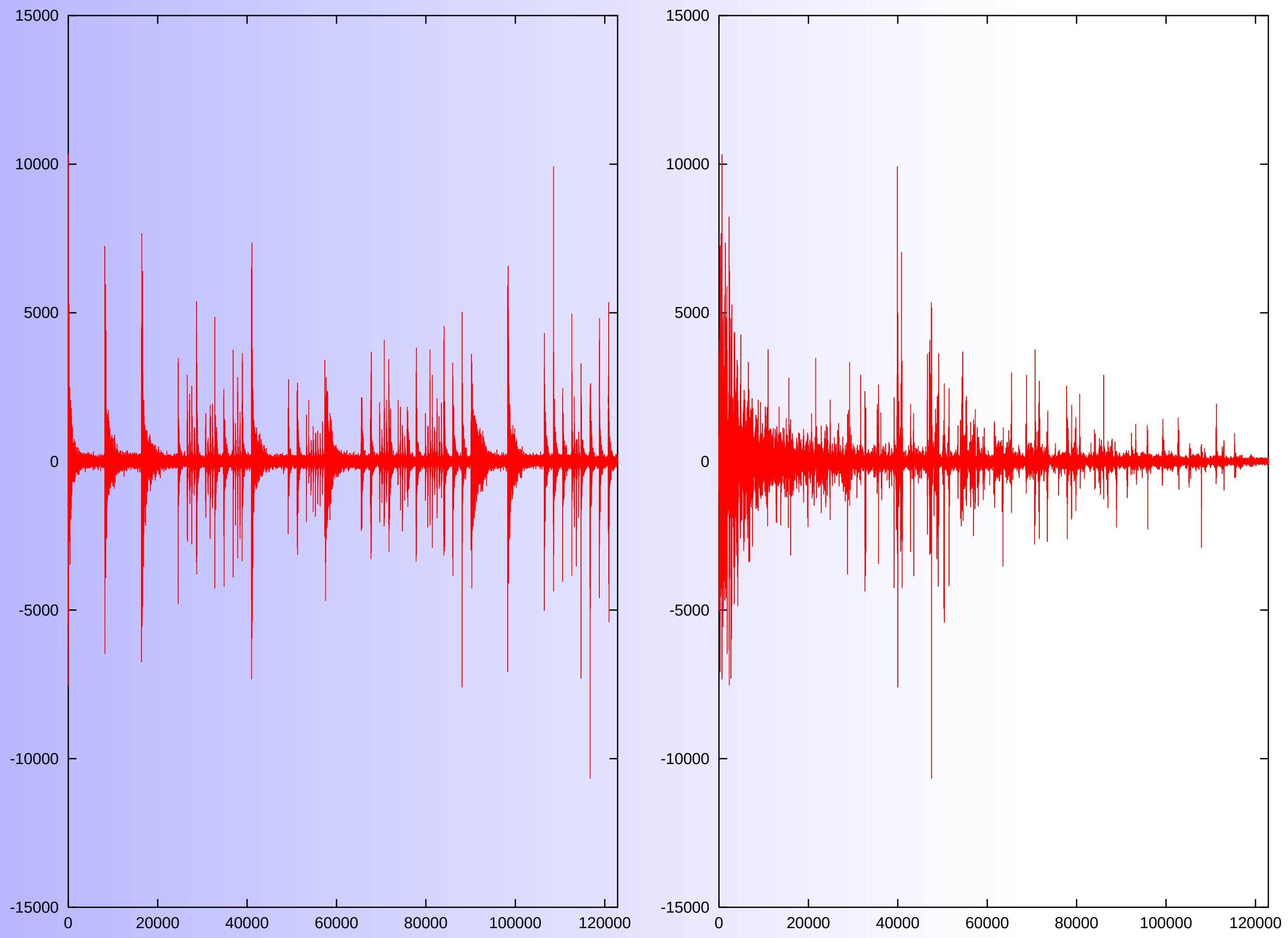
Scanning the coefficients

- scanning coefficients by increasing frequency
- each LCT block divided into a fixed number of frequency subsets
- gather from all the LCT blocks all the coefficients that are in the same subset

Scanning the coefficients



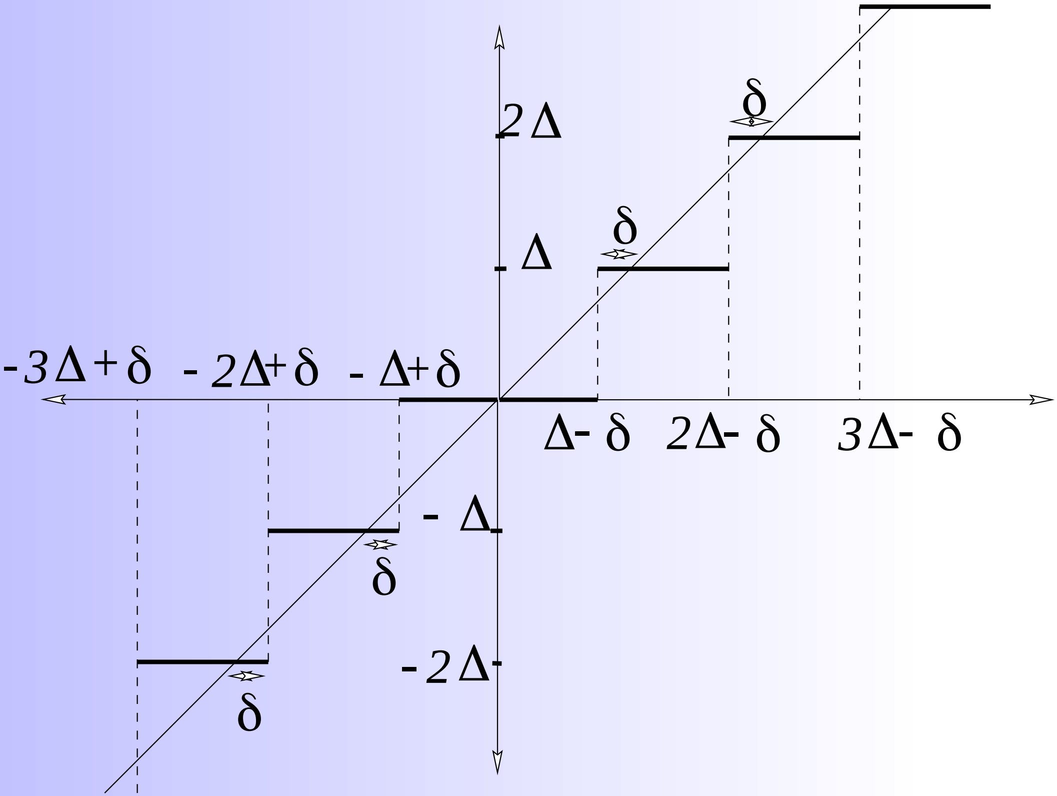



15000
10000
5000
0
-5000
-10000
-15000
0
20000
40000
60000
80000
100000
120000

Laplacian based scalar quantization

- distribution of the cosine coefficients: Laplacian [Birney and Fischer, 1995]
- near optimal scalar quantizer [Sullivan, 1996]
  - $[-\Delta + \delta, \Delta - \delta]$, the symmetric dead-zone ,
  - Δ, the quantizer step size,
  - δ, the reconstruction offset

Laplacian based scalar quantization



Scalar quantizer, with a dead zone.

Entropy coding

- significance map: n_C order arithmetic coder
- signs of the output levels: packed
- magnitude of the output levels: variable length encoded
- best basis geometry: adaptive arithmetic coder.

Experiments

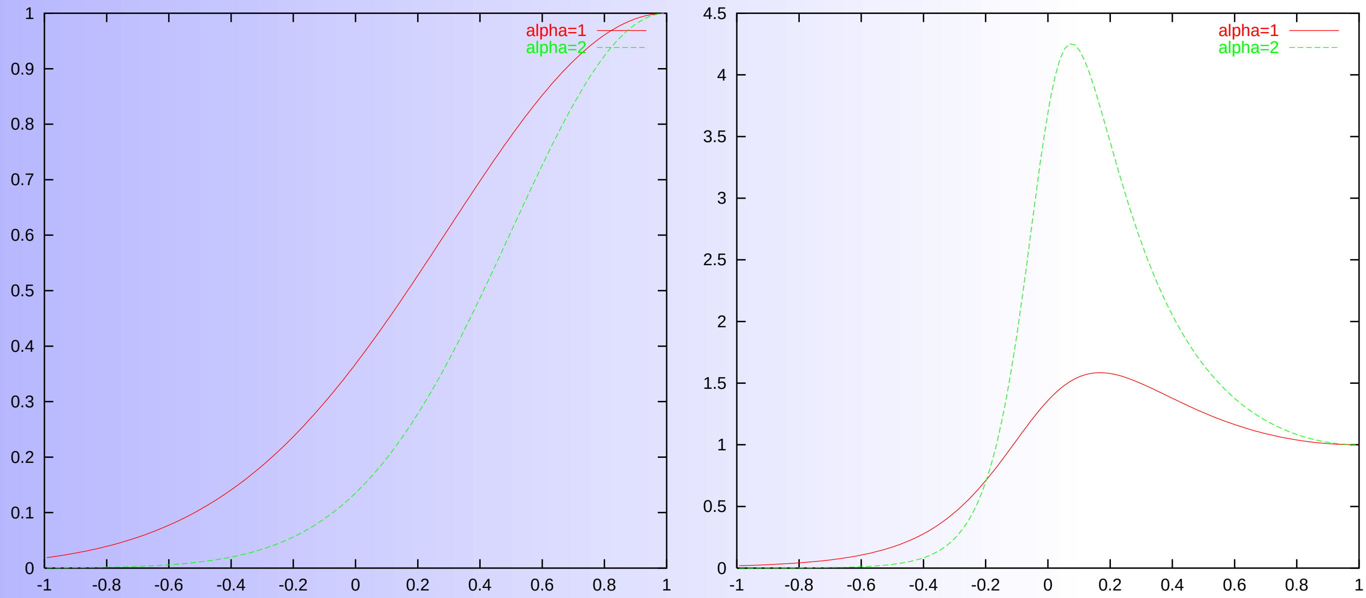
- Fast Local Cosine Transform (FLCT) coder and decoder
- actual bit stream: size equal to the targeted budget

Comparison of the bells

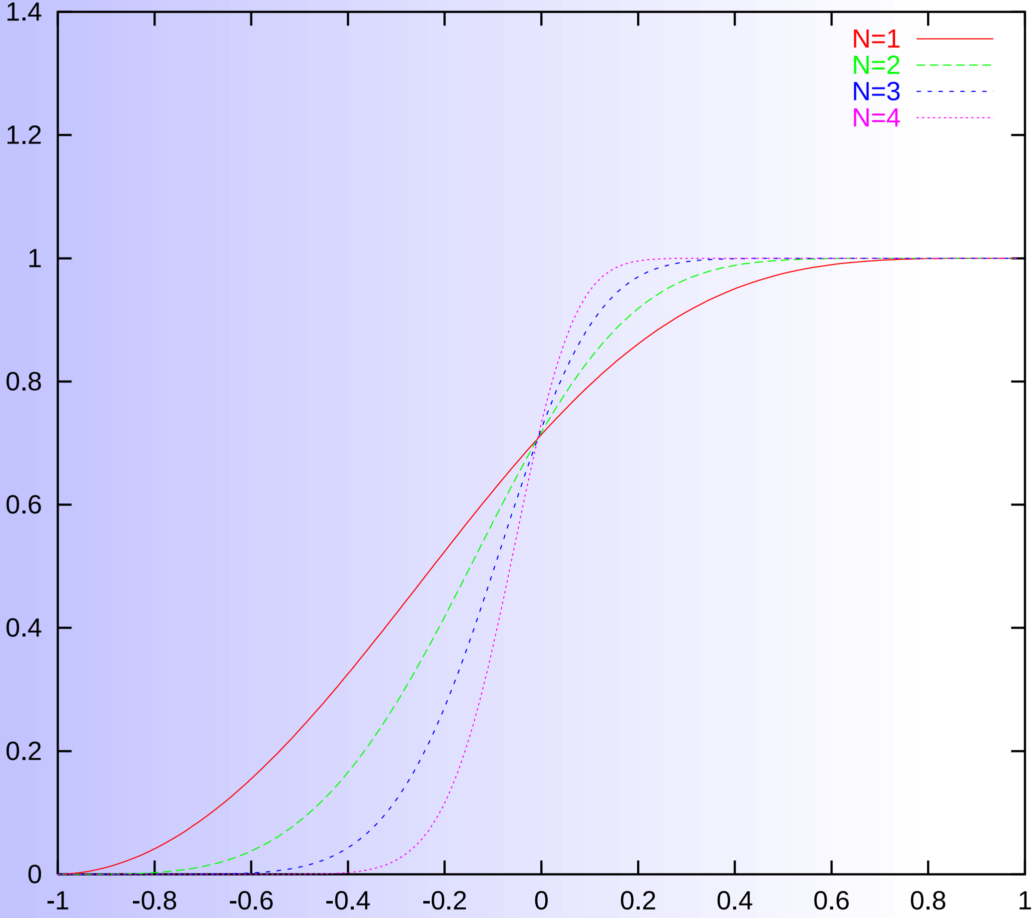
- Standard bell:

$$
\begin{aligned}
b^N(x) &= \sin\left[\frac{\pi}{4}(1 + x_N)\right] \\
x_j &= \sin(\frac{\pi}{2} x_{j-1}) \\
x_0 &= x
\end{aligned}
\tag{17}
$$

- Optimized bell: Matviyenko
- Gaussian $b(x) = e^{-\alpha(x - a_n)^2}$



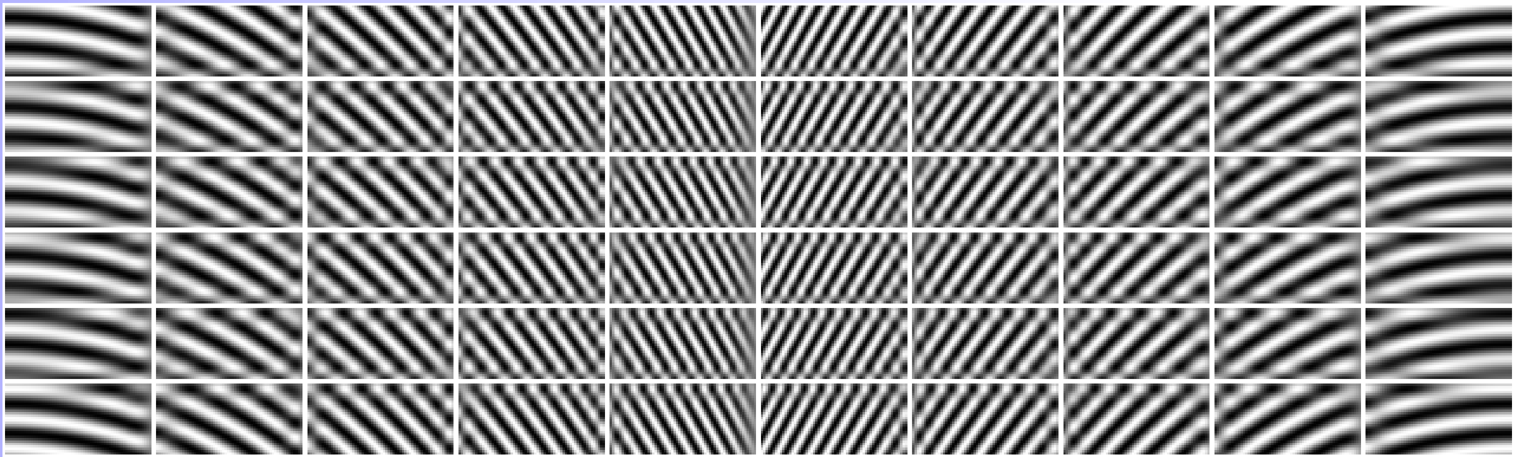
Gaussian bells. $\alpha = 1, 2$

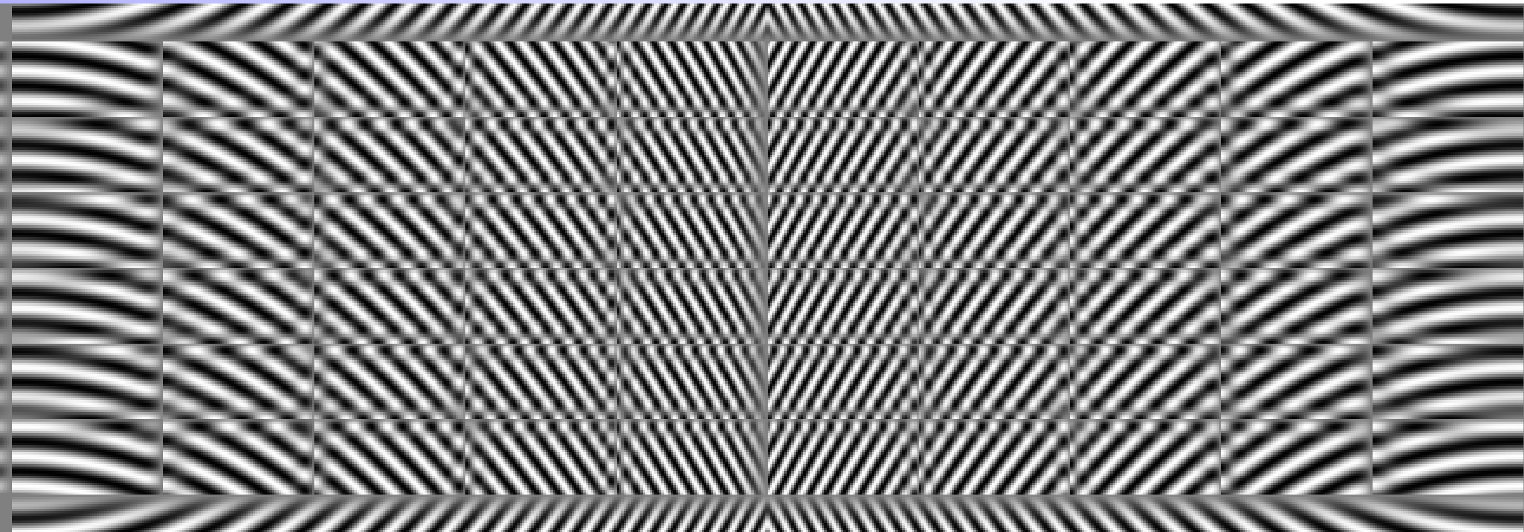


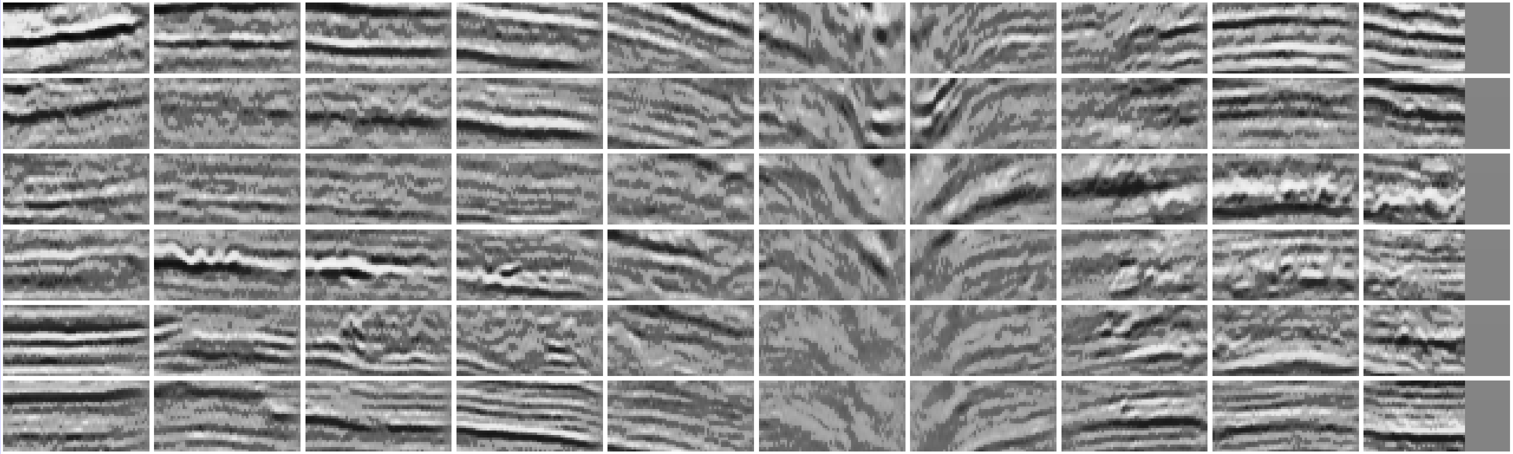
Standard bells. $N = 1, 2, 3, 4$

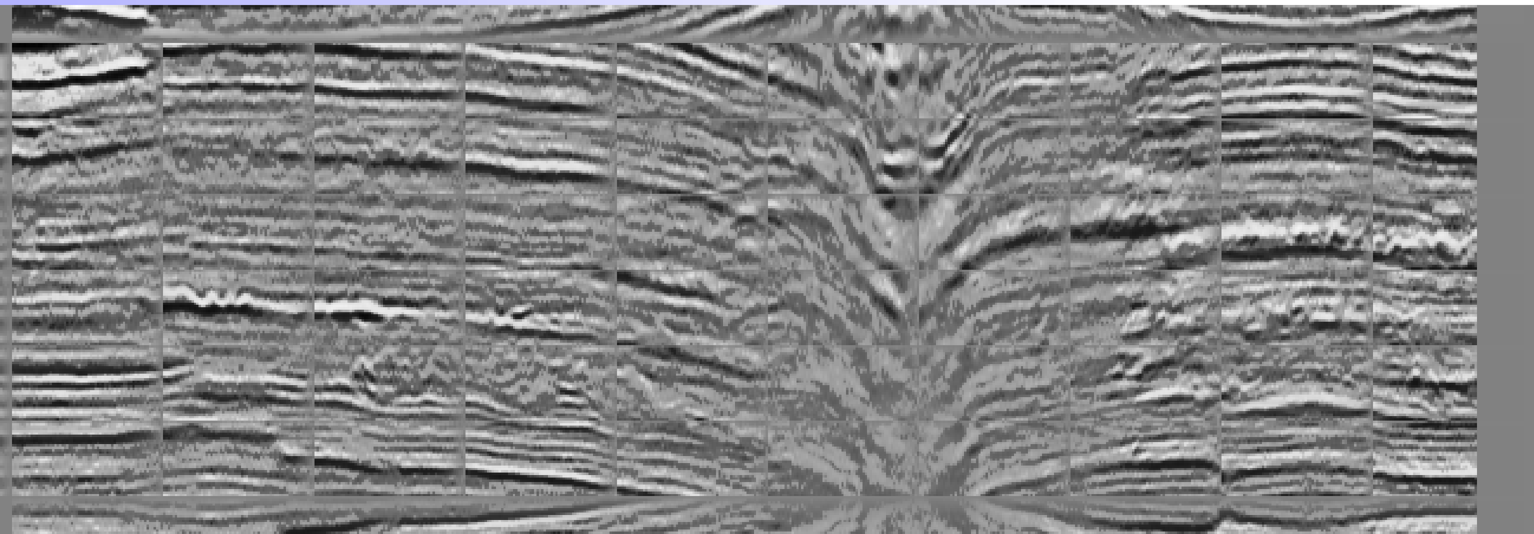
Test images

- Synthetic image: warped cosines, 192×640
- Seismic data: 2D slice 192×640
- uniform grid: blocks of size 32×64

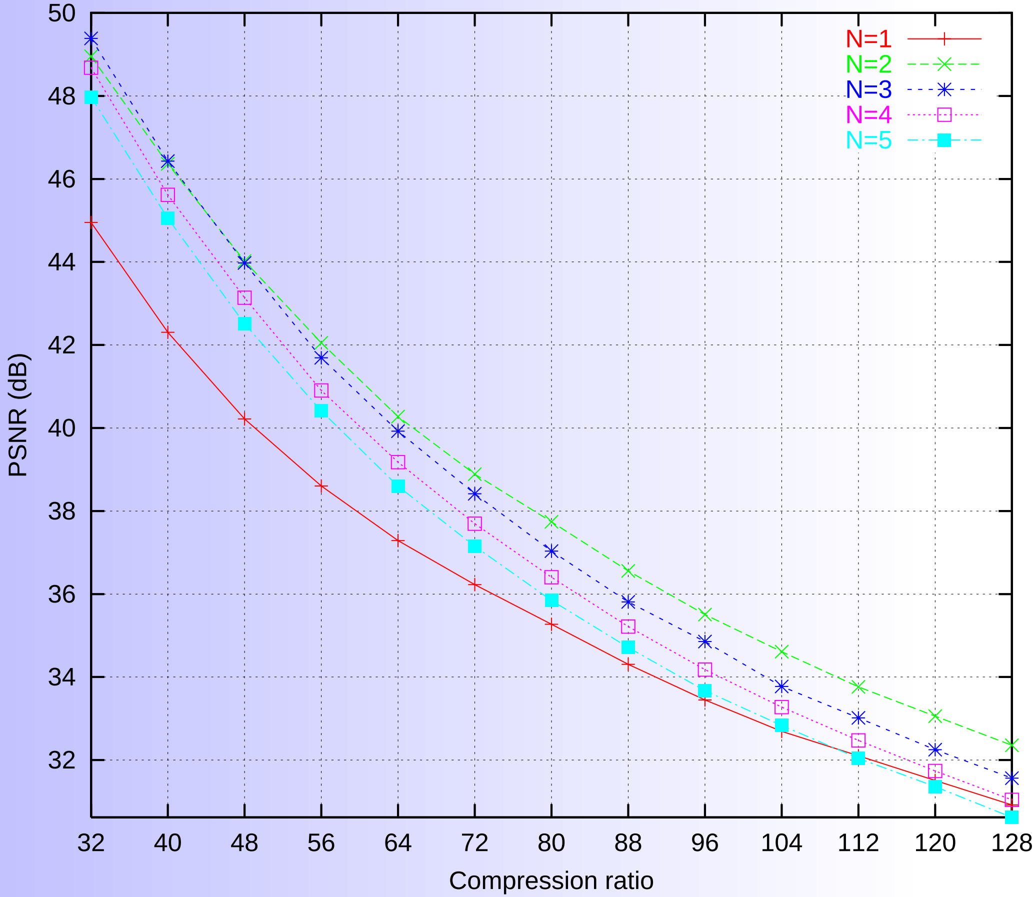




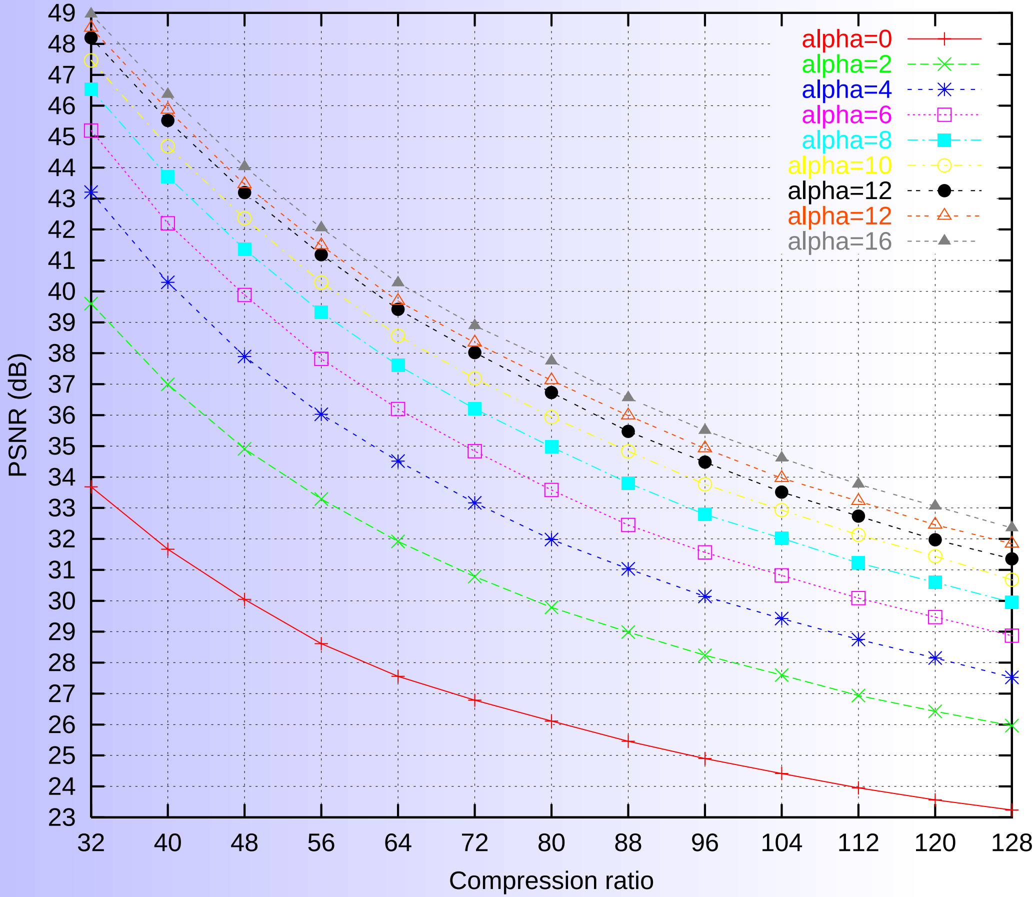




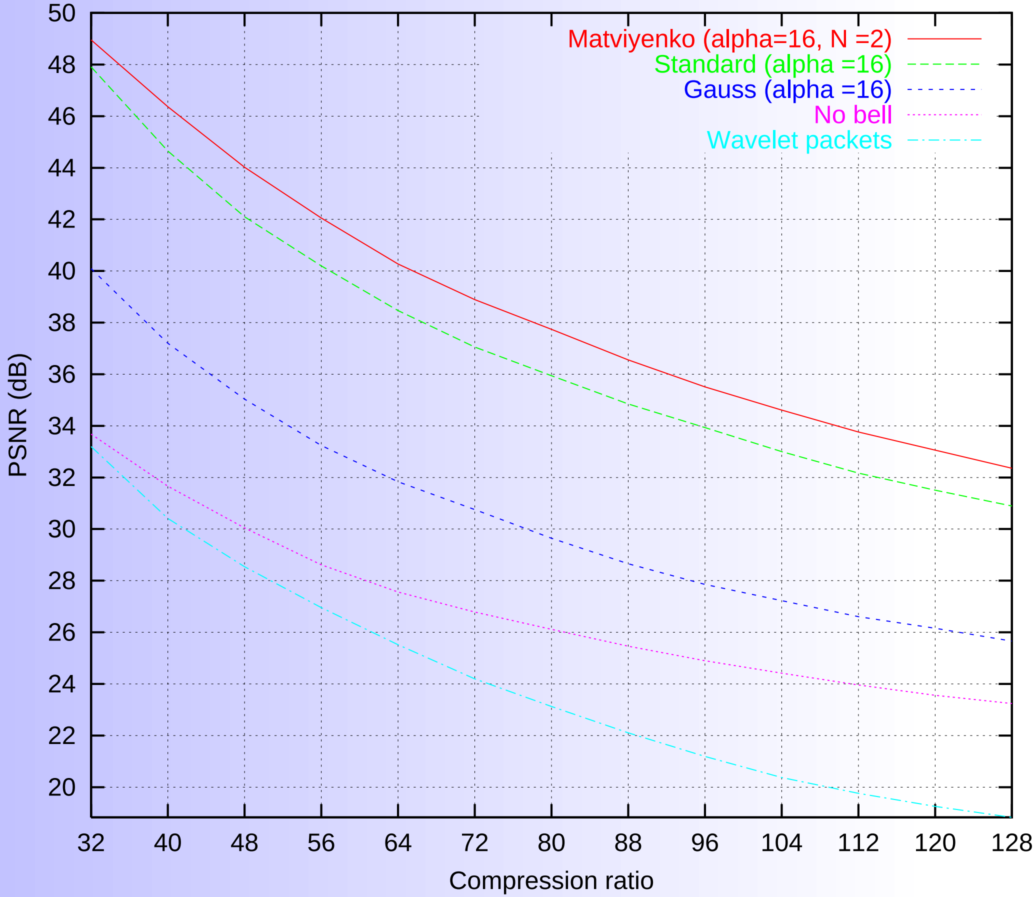
Synthetic data : optimal N for optimized bell



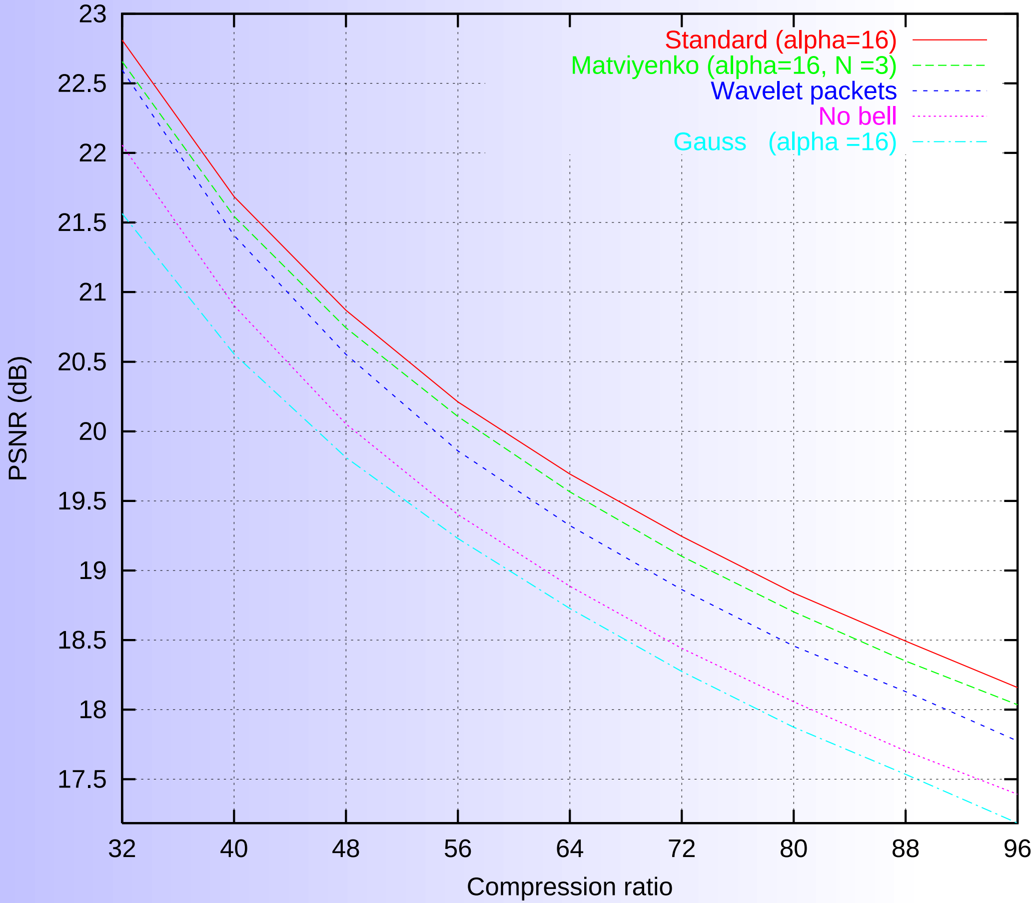
Synthetic data : optimal overlap for optimized bell



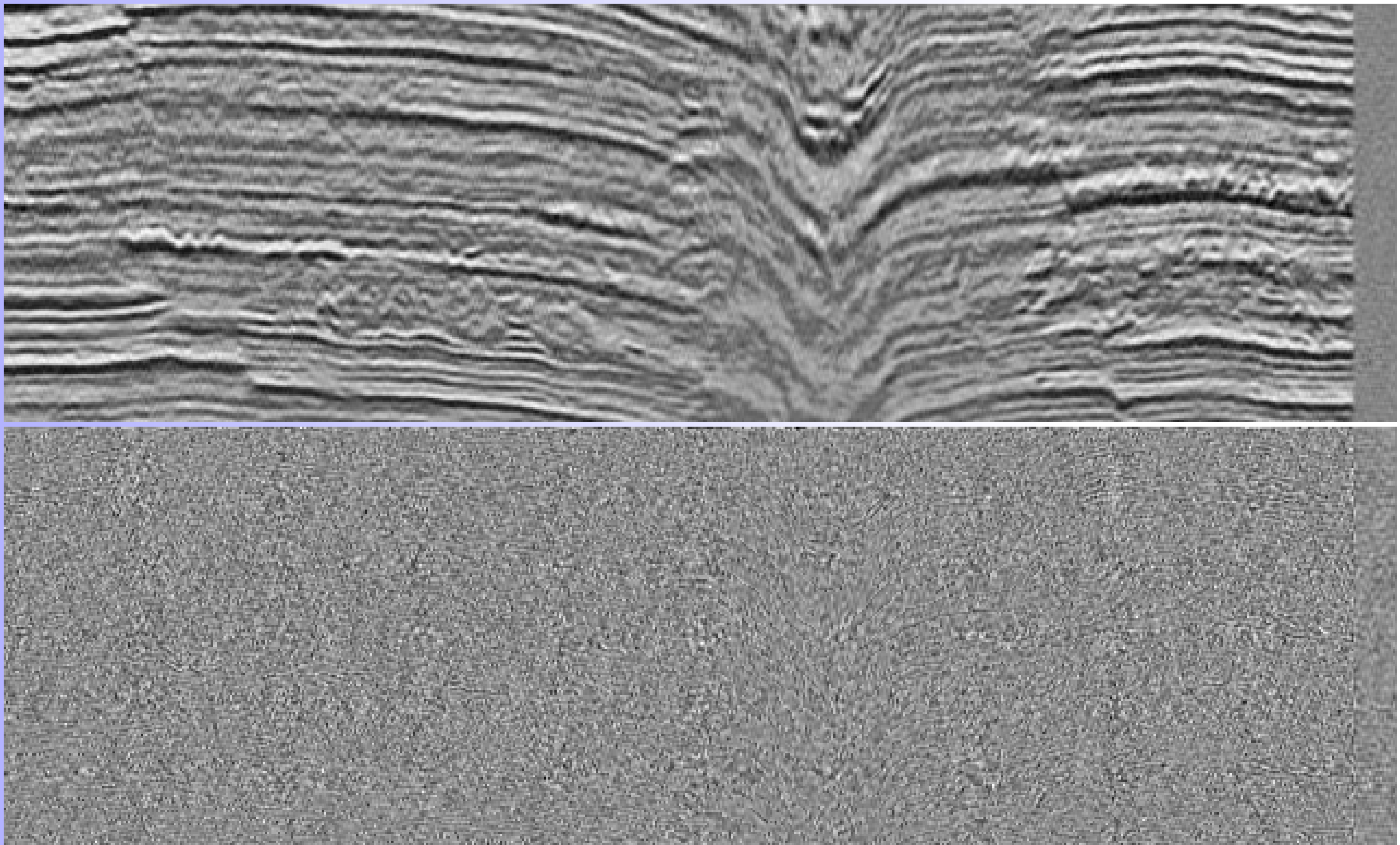
Synthetic data : comparison of the bells



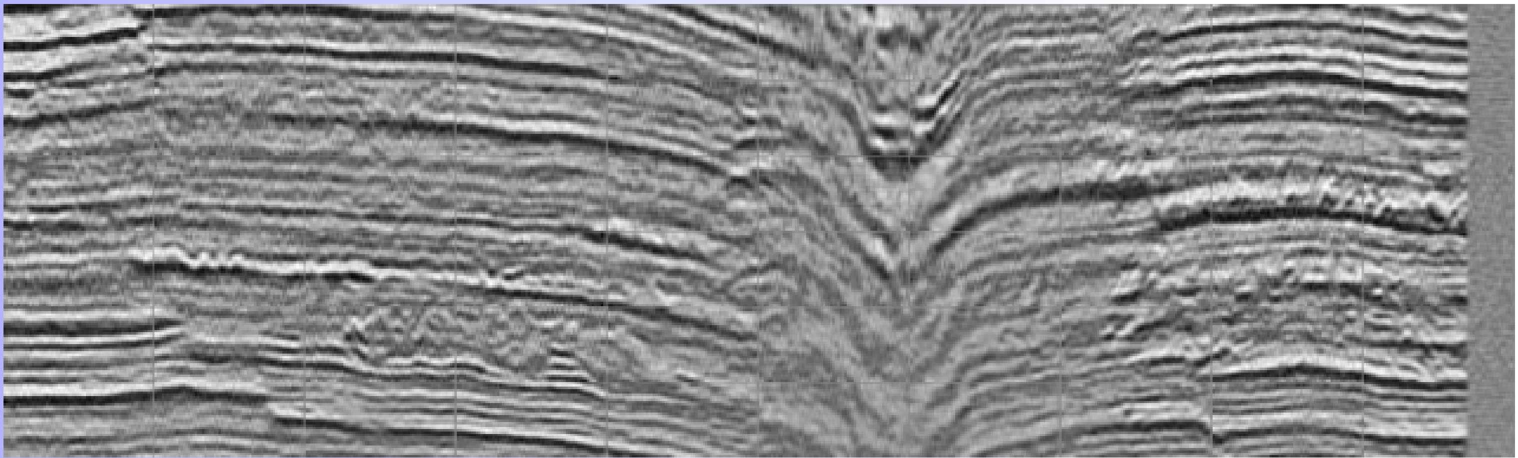
Seismic data : comparison of the bells

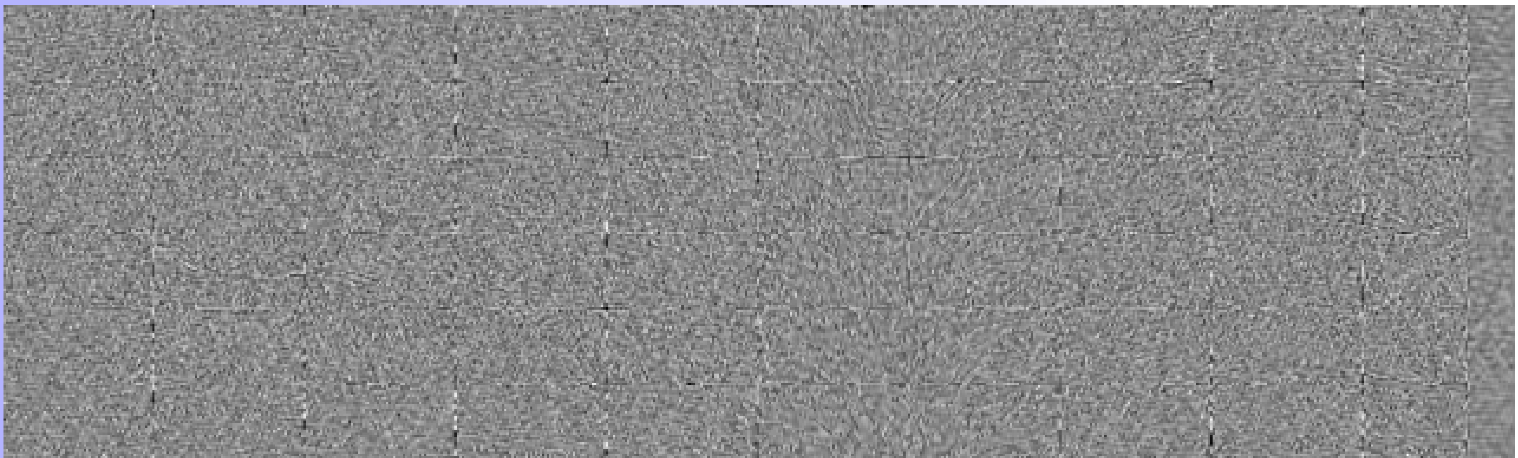


Compression 50, Matviyenko ($N = 3$), PSNR =20.56 dB



Compression 50, no bell, PSNR = 19.87 dB





Conclusion

- compression of highly oscillatory signals: local cosine transforms
- optimal spatial tiling
- design of the window
- full 3-D compression: fast 3-D FFT
- The future: curvelets ?

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