THE TRANSMISSION-LINE PARADIGM FOR METAMATERIALS: FUNDAMENTALS & SELECTED APPLICATIONS

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CANADA
METAMATERIALS

META=“BEYOND” IN GREEK

Artificial materials with unusual electromagnetic properties that are difficult to encounter in nature.

ARTIFICIAL DIELECTRICS:


TRANSMISSION-LINE METAMATERIALS:

Artificial dielectrics synthesized by periodically loading a host transmission-line medium with R,L,C lumped elements: Periodicity $\ll \lambda$ (although non-periodic MTMs could also be defined).
J.B. Pendry

Artificial Molecules

1948

2001

2008
LEfT-HANDED $\varepsilon<0$ AND $\mu<0$

METHAMATERIALS

Veselago, 1960s

<table>
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<tr>
<th>$\varepsilon &gt; 0, \mu &gt; 0$</th>
<th>$\varepsilon &lt; 0, \mu &lt; 0$</th>
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<td>Regular Materials (right-handed)</td>
<td>Left-handed Materials</td>
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<tr>
<td>$n = -\sqrt{\varepsilon\mu}$</td>
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Backward Waves
NEGATIVE REFRACTION

Negative-Refractive-Index (NRI) Media
Reconceptualizing \( \varepsilon < 0 \) and \( \mu < 0 \) metamaterials

Start from the transmission-line representation of normal dielectrics:

\[
\begin{align*}
  j\omega \mu &= \frac{jX}{\Delta S} = \frac{j\omega L}{\Delta S} \Rightarrow \mu = \frac{L}{\Delta S} \\
  j\omega \varepsilon &= \frac{jB}{\Delta S} = \frac{j\omega C}{\Delta S} \Rightarrow \varepsilon = \frac{C}{\Delta S}
\end{align*}
\]

How to synthesize \( \varepsilon < 0, \mu < 0 \)?
Simply: Make the series reactance $X$ and shunt susceptance $B$ both negative!

\[ j\omega\varepsilon = \frac{jB}{\Delta S} = \frac{j(-1/\omega L)}{\Delta S} \Rightarrow \varepsilon = -\frac{1}{\omega^2 L\Delta S} \]

\[ j\omega\mu = \frac{jX}{\Delta S} = \frac{j(-1/\omega C)}{\Delta S} \Rightarrow \mu = -\frac{1}{\omega^2 C\Delta S} \]


Planar Negative Refractive Index Media Using Periodically $L-C$ Loaded Transmission Lines

George V. Eleftheriades, Senior Member, IEEE, Ashwin K. Iyer, Student Member, IEEE, and Peter C. Kremer

![Diagram of 1-D unit cell with host transmission line medium embedded as an equivalent series inductor and shunt capacitor.]

Fig. 6: 1-D unit cell with host transmission line medium embedded as an equivalent series inductor and shunt capacitor.

It is necessary to develop a procedure to modify the values of the discrete loading elements such that the value of $\beta$ is restored, particularly at the frequency of operation, to that obtained in the dimensionless case. That is, any design idea is to explicitly account for the contribution of the transmission-line parameters in the total inductance and capacitance per unit cell. Specifically, the per-unit-length transmission line parameters of the line $L$ and $C$ can be incorporated into the unit cell by modifying the cell as depicted for the 1-D case in Fig. 6.

Here, $C_0$ and $L_0$ are the new loading elements, and the total impedance in the series and shunt branches give the desired equivalent series capacitance $C_{1,LIM}$ and shunt inductance $L_{1,LIM}$, respectively, at the design frequency.

The required new series loading capacitance $C_0$ can be determined using the following equation:

$$\frac{1}{j\omega L_0} + \frac{1}{j\omega C_0} = \frac{1}{j\omega C_{1,LIM}}$$

which simplifies, through some manipulation, to the desired result

$$C_0 = C_{1,LIM} \frac{1 + \frac{\omega L_0}{\omega^2 C_{1,LIM} L_0 d}}{1}$$

Before specifying $L_0$, it is important to note that, in the 2-D version of the medium, it is necessary to further compensate for the effect of the host medium transverse to the direction of propagation. This is most clearly illustrated for the case of normal incidence, for which each of the two transverse transmission line segments becomes open-circuited at their centres, and, therefore, contribute capacitively to the unit cell. This is shown in Fig. 7 from the cross-section view.

![Diagram of top view of transmission line grid, showing open-circuits formed in the medium transverse to the direction of propagation, for the case of normal incidence.]

Fig. 7: Top view of transmission line grid, showing open-circuits formed in the medium transverse to the direction of propagation, for the case of normal incidence.

The equivalent material parameters of (12) and (13) may now be given by

$$\mu_S = \mu_{r,LIM} \mu_0 = \frac{1}{\sigma^2 C_{1,LIM} L_0} = \frac{L_0}{\sigma^2 C_{1,LIM} L_0}$$

$$\epsilon_S = \epsilon_{r,LIM} \epsilon_0 = \frac{1}{\sigma^2 L_{1,LIM} d} = \frac{2C_0}{\sigma^2 L_{1,LIM} d}$$

from which it can be seen that each of these parameters consists of a positive contribution due to the unloaded medium, and a negative contribution due to the loading (corresponding to an effective negative susceptibility).
Employing a higher level of precision in the approximation of (21) than (22) permits the analytic determination of the stopband limits

$$\cos \beta d \equiv 1 - \frac{\theta^2}{2} + \frac{Z Y}{2} + j \frac{Z}{Z_0} + \frac{Y}{Y_0} \theta.$$

(27)

Making the substitutions of (23), with $\theta = k d - \alpha d v$ (where $v$ is the phase velocity in the host medium), and setting $\beta = 0$ in (27), the solution of the resulting quadratic equation in $\omega$ yields the desired cut-off frequencies (see Fig. 5) as follows:

$$f_{c,1} = \frac{1}{2 \pi} \sqrt{\frac{v}{C_0 Z_0 d}}, \quad f_{c,2} = \frac{1}{2 \pi} \sqrt{\frac{v}{L_0 Y_0 d}}$$

(28)

Note that both cutoff frequencies in (28) tend to infinity as the cell dimensionality $d$ approaches zero, thus arbitrarily increasing the bandwidth of the first LH passband (see Fig. 5). Furthermore, closing this stopband by equating these two cutoff frequencies yields the matching condition

$$Z_0 = \sqrt{\frac{L_0}{C_0}}$$

(29)

which suggests that the width of the stopband may be controlled by adjusting the mismatch between the characteristic impedances of the host medium and the loading. It is clear from the lowest
2D Microstrip Implementation of NRI-TL Metamaterials

Distributed TL Network With Chip or Printed (gaps and vias) Loading Lumped Elements

IEEE Microwave and Wireless Components Letters
Effective Medium Approach for a Left-Handed Loaded Parallel-Plate Waveguide (PPW)

Ampere’s Law: \( \nabla \times \vec{H} = j \omega \varepsilon_0 \vec{E} + j \omega \vec{P}_e \)
\[ \vec{P}_e = -\frac{\vec{E}}{\omega^2 L_0 d} \] (x-z loop)

Faraday’s Law: \( \nabla \times \vec{E} = -j \omega \mu_0 \vec{H} - j \omega \mu_0 \vec{P}_m \)
\[ \vec{P}_m = -\frac{\vec{H}}{\mu_0 \omega^2 C_0 d} \] (y-z loop)

Equivalent PPW filled with effective media parameters

\[
\varepsilon_{\text{eff}} = \varepsilon_0 - \frac{1}{\omega^2 L_0 d}
\]
\[
\mu_{\text{eff}} = \mu_0 - \frac{1}{\omega^2 C_0 d}
\]
Spatial Harmonics in NRI-TL


\[ V(z) = \sum_{n=-\infty}^{+\infty} u_n e^{-j\beta_n z} \]

\[ \beta_n = \beta + \frac{2\pi n}{a} \]

Infinite number of forward \( n > 0 \) and backward harmonics \( n \leq 0 \)

Just how much power is contained in the fundamental \( n=0 \) BW harmonic?

Homogeneity coefficient for \( a/\lambda = \frac{1}{6.3} \)

In the homogeneous limit \( \beta a \ll 1, ka \ll 1 \) most of the power is carried by the fundamental (\( n=0 \)) spatial harmonic.
PHASE EVOLUTION: How does a Backward Wave Form?

On the interconnecting TL there is a phase delay; the phase advances from one unit cell to the next due to the phase jumps on the shunt inductors (for the current).

**On the AVERAGE the phase linearly advances with distance**

The departure from the average becomes smaller and smaller as the unit cell becomes smaller.
Backward wave on an NRI TL
1/p = normalized coupling coefficient between adjacent loops

NEGATIVE REFRACTION OF A GAUSSIAN BEAM IN NRI-TL METAMATERIALS

Power refracts negatively

Fig. 6. Schematic of the Gaussian beam simulation. Grey crosses represent unit cells, on the left hand side a PRI grid is excited by a Gaussian beam, the PRI interfaces a NRI. The Gaussian beam excitation is chosen such that the beam waist forms at the PRI-NRI interface. Towards the side edges the unit cells are terminated by a suitable termination, denoted by ‘T’.

Fig. 7. Simulation results for a Gaussian beam propagating in a grid of 257 × 340 network unit cells. The left half space is PRI, the right half space is NRI. (a) Linear magnitude plot of a beam at normal incidence. (b) Linear magnitude plot of a beam incident at 20°. (c) Phase plot of the normally incident beam. (d) Phase plot for the beam incident at 20°.
1D APPLICATIONS
A LEAKY BACKWARD-WAVE ANTENNA (fan beam)

BACKWARD Radiation from the FUNDAMENTAL Spatial harmonic

F = 15GHz

2D NRI-TL Leaky-Wave Antenna

Closed Stop Band

\[
\frac{Z_o}{\sqrt{2}} = \sqrt{\frac{L_o}{C_o}}
\]

Directivity

REDUCED BEAM-SQUINTING LEAKY-WAVE ANTENNA
(in planar CPS)

4.5 GHz  
5.0 GHz  
5.5 GHz

Zero-Degree Phase-Shifting Lines

Phase Compensation with RHM/LHM Lines

Conventional $1\lambda$ line

Measured vs. Simulated

COMPACT AND BROADBAND SERIES POWER DIVIDERS

Metamaterial 1:4 Divider

Transmission-Line 1:4 Divider

Non-Radiating Lines

DRAMATIC AREA REDUCTION
BROADBAND:

The phase $\phi$ of a unit-cell can be electronically tuned from -35° to +59° at 2.6GHz, while maintaining $S_{11}<-19$dB. Across the entire phase tuning range $S_{21}$ varies from -2.8dB to -3.8dB at 2.6GHz.

Steerable Series-fed Patch Array

- The array consists of 4 patch antennas and uses 3 inter-stage phase shifters.
- The entire antenna array (which includes: patches, feed-lines, and the phase shifters) is designed on a single 2-layer board.
- All the inter-stage phase shifter receive the bias and control signals from a ribbon cable on the bottom side of the board.

Experimental Results

The measured gain patterns versus the azimuthal angle

- The measured scan angle ranges from +18° to -27° at 2.4GHz by changing the TAI bias voltages and the varactor control voltage from 3V to 15V.
- The antenna gain changes from 8.4 to 7.1dBi across the entire 45° scan angle range
- Relative side-lobe level < -10dB
- Very little beam-squinting vs frequency

Electrically Small NRI-TL Zero-Index Antennas

Main idea: Wrap around a 0° MTM phase-shifting line to make a small resonant antenna

Measured Antenna Patterns

Measured radiation efficiency up to 70-80%
Bandwidth: 1-3% (-10dB point)

\[ W \times L \times H = \frac{\lambda}{11} \times \frac{\lambda}{14} \times \frac{\lambda}{31} \]

A Compact Zero-Index NRI-TL Metamaterial Antenna with Extended Bandwidth (double-tuned matching)

1. Doubly resonant MTM Structure

2. More than doubled the bandwidth compared with the singly resonant MTM antenna

‘DUAL-MODE’ BROADBAND NRI-TL MONOPOLE

Unloaded monopole antenna

Single NRI-TL cell loaded monopole

A single resonance at 6.4 GHz

A dual resonance at 3.5 GHz and 5.5 GHz.

BW_{-10dB} = 3.78GHz

High-Directivity Coupled-Line Coupler

Coupled Microstrip/NRI TLines:

Co-directional phase flow but contra-directional power flow!

Conventional Microstrip vs. MS/NRI Coupled-Line Coupler

- Equal length
- Equal line spacing
- Equal propagation constant

Metamaterial MS/NRI 3dB Coupler

Operates in coupled mode stop band
Arbitrary coupling levels by increasing coupler length
Metamaterial MS/NRI 3dB Coupler

Operates in coupled mode stop band
Arbitrary coupling levels by increasing coupler length
Metamaterial MS/NRI 3dB Coupler

Operates in coupled mode stop band
Arbitrary coupling levels by increasing coupler length

Phase Progression With Exponential Field Variation

Operation in Coupled-Mode Stop Band

\[ |V_1| \]

Input

Coupled

\[ |V_2| \]

Line 1 (MS)

Line 2 (NRI)

Isolated
3dB Coupler: Experimental Results

- Operating frequency – 3GHz
- Cell size – 4mm
- Line width – 2.34mm
- C - 1.3pF, L - 3.3nH
- #of unit cells – 6

FULLY PRINTED HIGH-DIRECTIVITY REFLECTOMETER

Solid Line: Measured

Coupling: $-27$ dB  
Isolation: $-72$ dB  
Directivity: $45$ dB  
@ $f=2.04$ GHz

2D AND VOLUMETRIC APPLICATIONS: SUPERLENSES
Focusing from Planar $n<0$ Slabs

Veselago’s Lens

- Flat but homogeneous lens
- Point-to-point focusing
- No optical axis

Negative-Refractive-Index (NRI) Lens
VOLUMETRIC “STACKED” NRI-TL MEDIUM
(layer-by-layer fabrication)

A source embedded in free-space

• A.K. Iyer and G.V. Eleftheriades, “A volumetric layered transmission-line metamaterial exhibiting a negative refractive index,”

• A.K. Iyer and G.V. Eleftheriades, “Characterization of a multilayered negative-refractive-index transmission-line (NRI-TL) metamaterial,”
NEGATIVE-REFRACTIVE-INDEX
TRANSMISSION-LINE (NRI-TL) SUPERLENS

Resolving two sources $\lambda/3$ apart @ 2.4GHz
Distance between source and image: 0.57 $\lambda$


“Isotropic” $n = -1$ evident from iso-frequency contour at $\omega_0$

Clear Bloch-wavefronts forming (macroscopic)
Transmission-Line MTM Cloaks:
Point Source Adjacent to a Metallic Cylinder

Without TL Cloak

With TL Cloak

M. Zedler and G.V. Eleftheriades, 2009
Unit Cell of K.G. Balmain’s Anisotropic TL Metamaterial

ANISOTROPIC RESONANCE-CONE METAMATERIALS USING CONTINUOUS METALLIC GRIDS OVER GROUND

\[ \beta_x(f_r) d_x + \beta_y(f_r) d_y = 2\pi \]

NOTE: NO LUMPED ELEMENTS OR VIAS ➔ SCALABLE
FOCUSING WITH CONTINUOUS HYPERBOLIC GRIDS OVER GROUND

F=10 GHz

Input

50 Ω Resistive terminations

Vias to Ground

v_p

v_g

v_p'

v_g'

k_x

k_y

k_x'

k_y'

12.12 mm

0.3 mm

10.1 mm
Simulation

Source = 1
Focus = 0.84

At Resonance (10 GHz)

Experiment

Source = 1
Focus = 0.763

At Resonance (10.3 GHz)

Source = 1
Focus = 0.764

Below Resonance (9.81 GHz)

Below Resonance (10.15 GHz)

A DIPLEXER BASED ON NEGATIVE REFRACTION AND SPATIAL FILTERING

Photonic-Crystal Metamaterial
No lumped elements (chip or printed)

Experiment

\[
\begin{align*}
S_{21} \quad \text{solid} \\
S_{31} \quad \text{dashed}
\end{align*}
\]

Transmission Coefficient

Frequency (GHz)

6.2 GHz
5.8 GHz

A Shifted-Bean Approach to Sub-wavelength Focusing

Slots are closely spaced (lambda/10) and close to resonance
The spot size is NOT sensitive to losses
Simple to construct/frequency scalable structure
Resonance enhances field transmission

Experimental Apparatus

5-slit Screen at $f = 10\text{GHz}$, $\lambda = 30\text{mm}$
measured results taken 7.5 mm above the screen (0.25\(\lambda\)) at 10 GHz
30 mm by 30 mm surface at 0.25 mm increments
shown at left in blue contours

satellite slots were covered by copper tape and
the single-slot pattern measured
FWHM contour shown in red

beam width along the x-axis 0.271\(\lambda\) and along
the z-axis is 0.385\(\lambda\)

G.V. Eleftheriades and A.M.H. Wong, “Holography inspired screens for sub-wavelength focusing in the near field,”
Detecting Thru Reflection with Sub-wavelength Resolution

For the two washer case, the array probe clearly resolves them at a separation of $0.4\lambda$ @ $0.25\lambda$ away (a single dipole probe cannot).

EMERGING TRENDS AND APPLICATIONS

UNIQUE PROPERTIES CAN LEAD TO UNIQUE APPLICATIONS (negative refraction, super-resolution, wavelength ~ freq, cloaking)

DIFFERENT PERSPECTIVE OF LOOKING AT THE WORLD!

RF/MICROWAVE PASSIVE COMPONENTS
SMALL ANTENNAS
ANTENNA BEAMFORMING
RCS MANAGEMENT/CLOAKING
MEDICAL IMAGING
TUNABLE AND ACTIVE METAMATERIAL STRUCTURES
EMI REDUCTION USING METAMATERIAL GROUNDS
THz COMPONENTS
BEYOND NRI METAMATERIALS