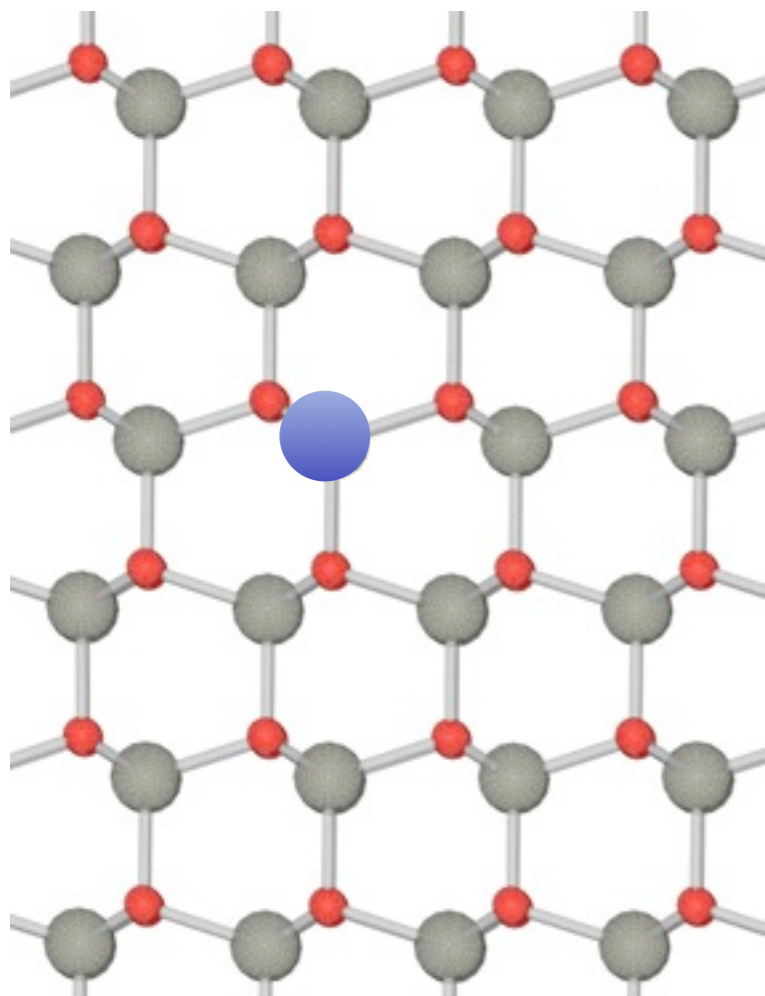


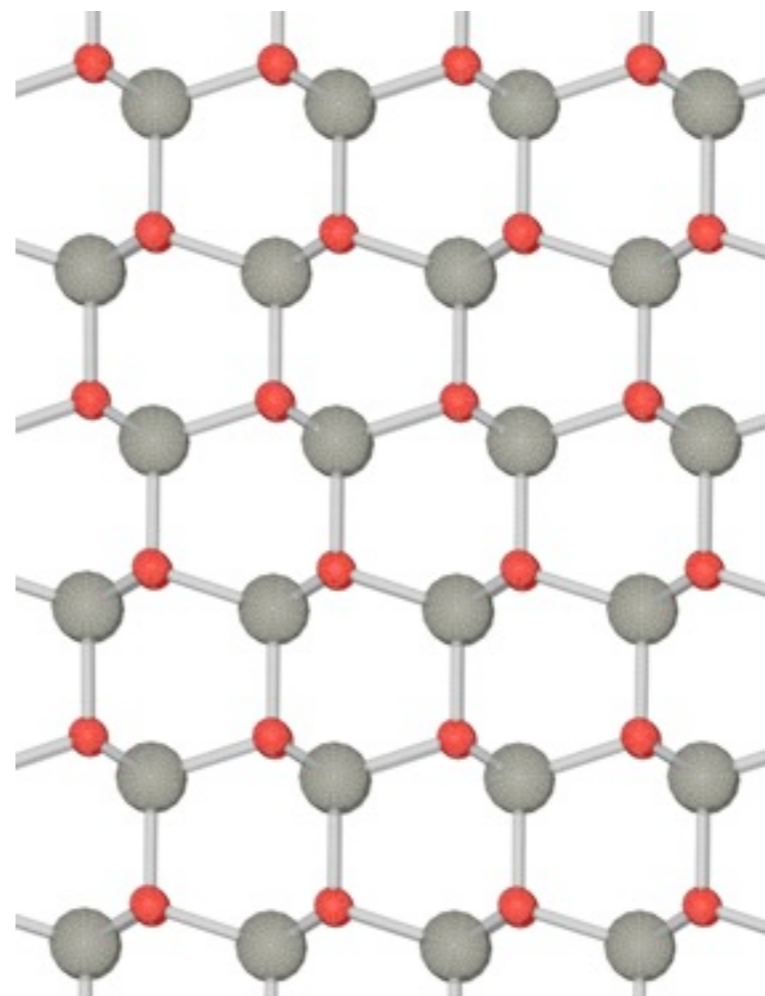
# **First-principles calculations of point defects in semiconductors: Formation energies, electronic properties, and diffusion**

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47048 Duisburg, Germany

# Methodological aspects



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# Doping of a semiconductor

- What does it mean ?
  - introduce an impurity that may occur in different charge states
  - the charge state should be accessible by putting a small amount of energy into the system ( $\sim$  thermal energy);  
e.g. for silicon, P may be  $P^0$  or  $P^+$ , Al may be  $Al^0$  or  $Al^-$
  - the ionized impurity may act as a donor or acceptor,  
e.g. for  $P^+$ , there is a rather extended conduction-band-like electron around P; for  $Al^-$ , there is an extended unoccupied, valence-band-like state around Al
- shallow and deep donors (or acceptors)
  - **shallow:** the wavefunction of the ionized impurity state has strong orbital character from the conduction band (the valence band)
  - **deep:** the wavefunction of the ionized impurity state has the orbital character of the impurity

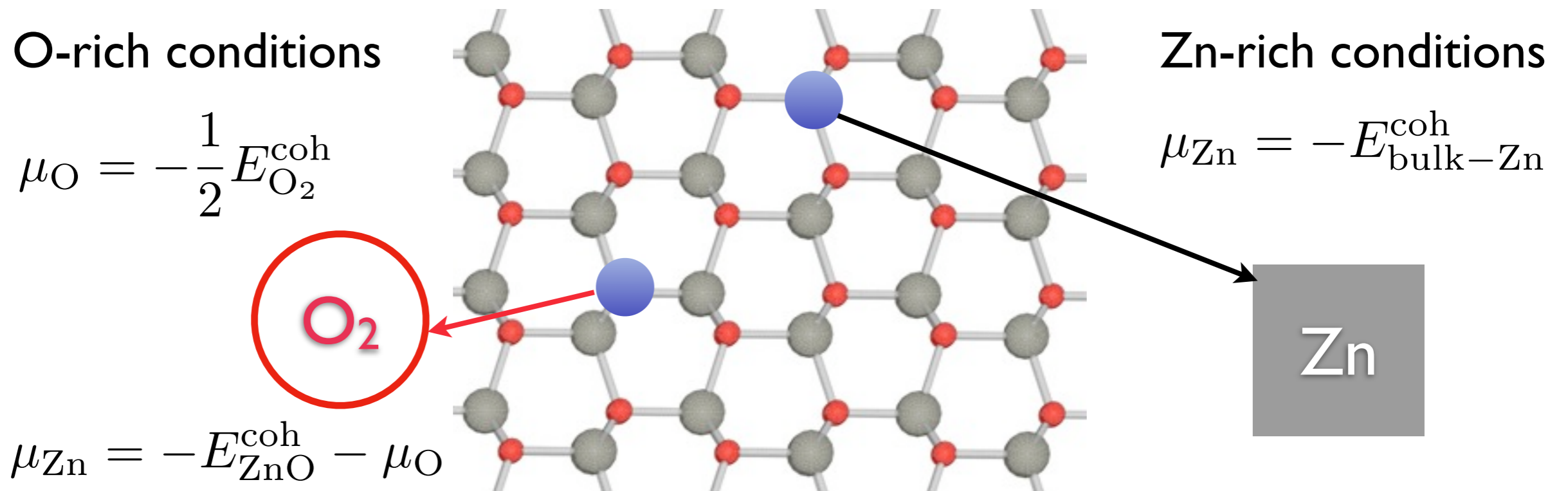
# Formation energy

- thermodynamics:** Formation energy must be calculated with respect to reservoirs  
 Example: carbon (C) in ZnO

$$E_{\text{form}}[\text{C}] = E_{\text{tot}}[\text{C/ZnO}] - E_{\text{tot}}[\text{ZnO}] - \sum_{\text{species } i} n_i \mu_i$$

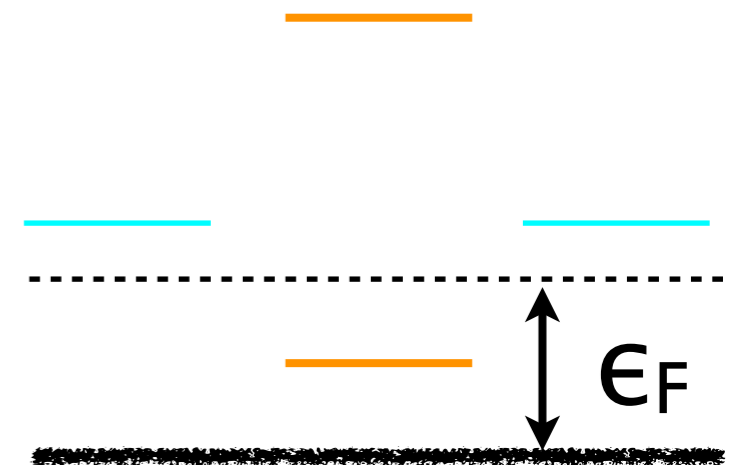
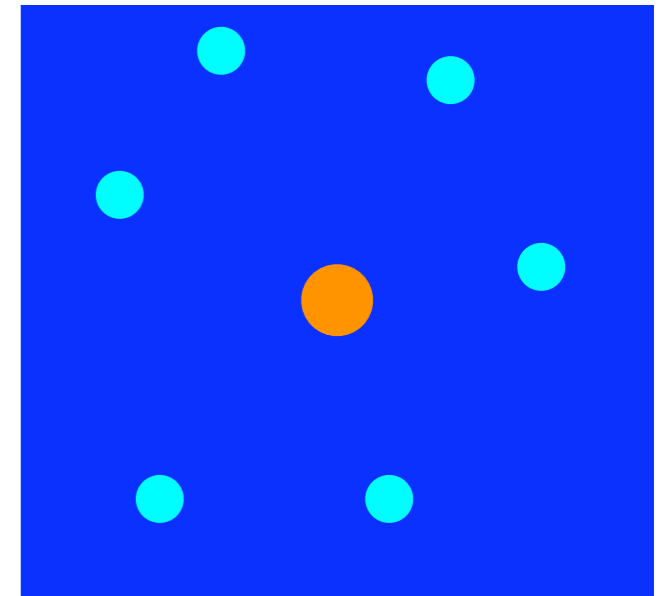
$$\mu_{\text{Zn}} + \mu_{\text{O}} = \mu_{\text{ZnO}}$$

- in crystal growth:** limiting values for the chemical potentials can be defined in terms of “growth conditions”



# Charge transfer level

- Apart from intentional doping, a semiconductor always contains (small concentrations of) intrinsic defects or other, unwanted impurities
- The chemical potential of the electrons (the “Fermi level”) is not an intrinsic property, but rather is determined by overall charge neutrality
- We want a descriptor for the behavior of a dopant in the environment of a “real” semiconductor: if the chemical potential hits the **charge transfer level**  $\epsilon(q'|q)$  of the dopant, it will be 50:50 ionized or neutral



$$E_{\text{form}}[C^{(q)}] = E_{\text{tot}}[C/\text{ZnO}^{(q)}] - E_{\text{tot}}[\text{ZnO}] - \sum_{\text{species } i} n_i \mu_i + q(\epsilon_F + E_v)$$

# Basics about DFT

- The total energy is a unique functional of the electronic density (Hohenberg-Kohn theorem)
- consider an **auxiliary system** of non-interacting electrons to estimate the kinetic energy (Kohn-Sham)

$$E_{\text{KS}} = T_0 + \int d\mathbf{r} v^{(0)}(\mathbf{r})n(\mathbf{r}) + E_{\text{Hartree}}[n] + E_{\text{XC}}[n] + V^{\text{ion-ion}}$$

$$\left( -\frac{\hbar^2 \nabla^2}{2m} + v[n](\mathbf{r}) \right) \psi_j(\mathbf{r}) = \varepsilon_j \psi_j(\mathbf{r})$$

$$v[n](\mathbf{r}) = v^{(0)}(\mathbf{r}) + V_{\text{Hartree}}[n](\mathbf{r}) + V_{\text{XC}}[n](\mathbf{r})$$

- for the “rest”  $E_{\text{XC}}$ , approximations may be used, e.g. LDA, GGA, ...

# Known problems with DFT

- The **Kohn-Sham levels** cannot (and should not, for any “reasonable” approximation to  $E_{XC}$ ) describe the **band gap** in a semiconductor
- The **self-interaction error** is not fully cancelled by an approximate  $E_{XC}$
- Both deficiencies pose the question if **charge transfer** between an impurity and a host can be described correctly in a KS scheme
- in particular, how to find the charge transfer level in the (too small) KS band gap ?

# How to obtain the charge transfer level

- **DON'T** use the Kohn-Sham level of the defect
- **BETTER:** compare the **total energies** of a neutral defect and a charged defect in a compensating background

$$E_{\text{form}}[C^{(q-1)}](\varepsilon_{\text{F}}) = E_{\text{form}}[C^{(q)}](\varepsilon_{\text{F}}) \implies \varepsilon(q-1|q) = \varepsilon_{\text{F}}$$

- alternative (for deep defects): “Slater’s transition state” rule applied to levels of the neutral and the charged system

$$\varepsilon(0|1) = \frac{\varepsilon_{H+1}(N) + \varepsilon_{H+1}(N+1)}{2}$$

- **BEST:** calculate the electronic structure in the GW method and use the result to apply a correction to DFT total energies

# Generalized Kohn-Sham schemes

- use explicit self-interaction corrections via “Hubbard  $U$ ” acting on some (e.g. 3d, 4f) **occupied** orbitals

- use the (orbital-dependent) Fock operator to calculate the exchange contribution

$$\hat{F}\psi_i(\mathbf{r}) = -e^2 \sum_{j,\text{occ}} \psi_j(\mathbf{r}') \int d\mathbf{r} \frac{\psi_j^*(\mathbf{r})\psi_i(\mathbf{r})}{|\mathbf{r} - \mathbf{r}'|}$$

- various realizations:

hybrid functionals (PBE0, B3LYP, ..):

mixing factor  $\alpha$  of Fock exchange and parameterized  $E_X$

$$v[\{\psi_i[n](\mathbf{r})\}] = v^{(0)}(\mathbf{r}) + V_{\text{Hartree}}[n](\mathbf{r}) + \alpha\hat{F}[\{\psi_i(\mathbf{r})\}] + (1 - \alpha)V_X[n](\mathbf{r}) + V_C[n](\mathbf{r})$$

range separation (screening of Fock kernel):

HSE functionals

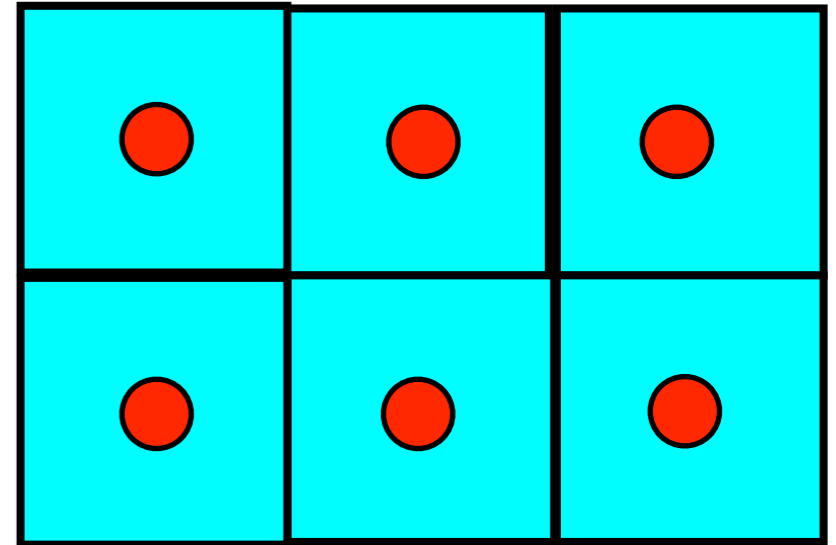
- compromise between good total energies (and lattice constants, ...) and energy levels accessible to a physical interpretation

- mixing parameter  $\alpha$  obtainable via Janak's theorem

[S. Lany & A. Zunger, PRB **81**, 205209 (2010)]

# Supercell calculations

- large supercells (several hundred atoms) required to minimize defect-defect interactions
- in plane-wave codes: homogeneous compensating background is easily realized



- Makov-Payne correction for **charged** defects [PRB **51**, 4014 (1995)]

$$E_{\text{form}}[C^{(q)}] \rightarrow E_{\text{form}}[C^{(q)}] + \frac{\alpha}{2\epsilon L} q^2 + \frac{2\pi Q}{3\epsilon L^3} q$$

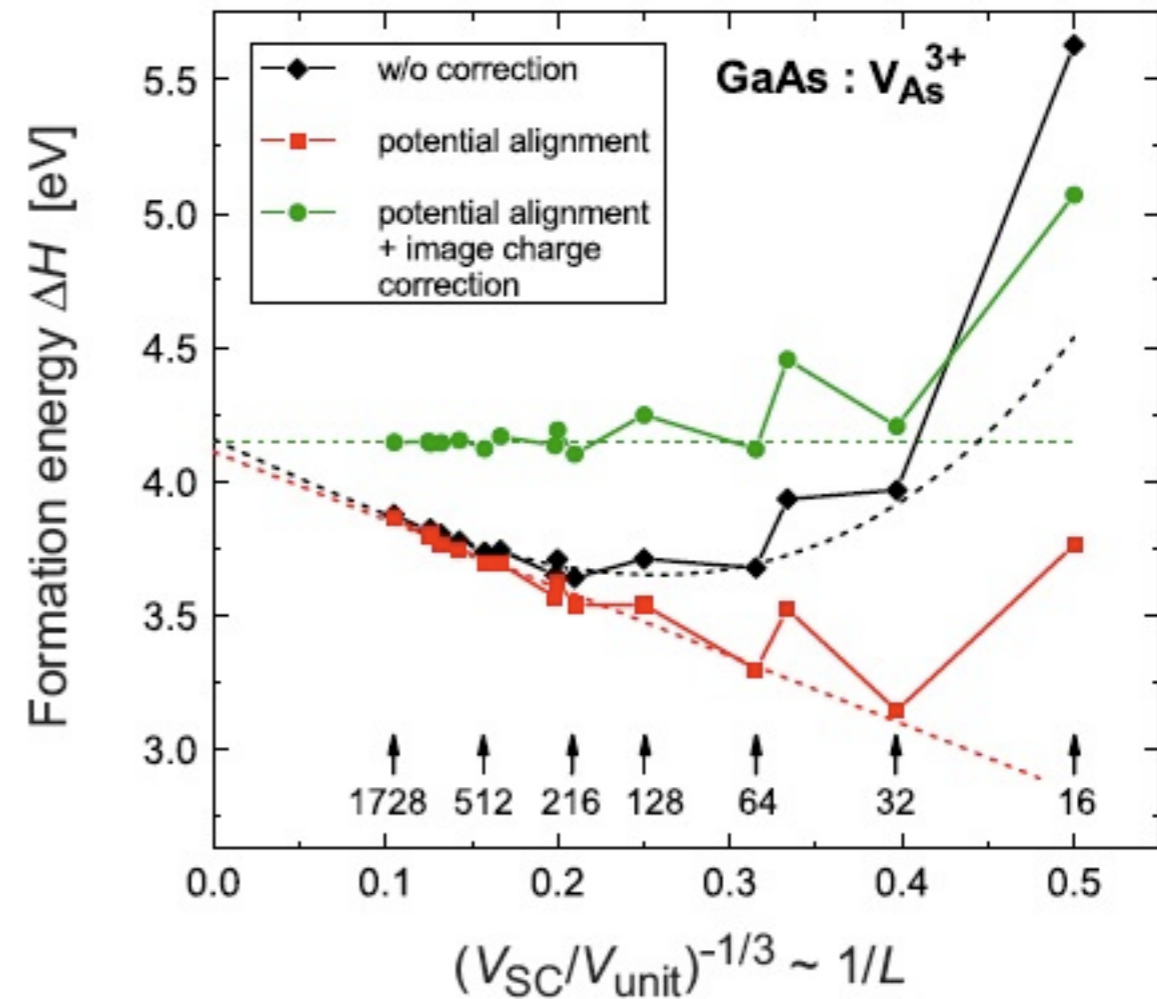
- in practice: linear (in  $q$ ) term is calculated as a potential-alignment correction  $\Delta V$

$$E_{\text{form}}[C^{(q)}] \rightarrow E_{\text{form}}[C^{(q)}] + \frac{\alpha}{2\epsilon L} q^2 + \Delta V q$$

# Periodic-image correction

- Example: oxygen vacancy in ZnO

S. Lany and A. Zunger,  
PRB **78**, 235104 (2008)

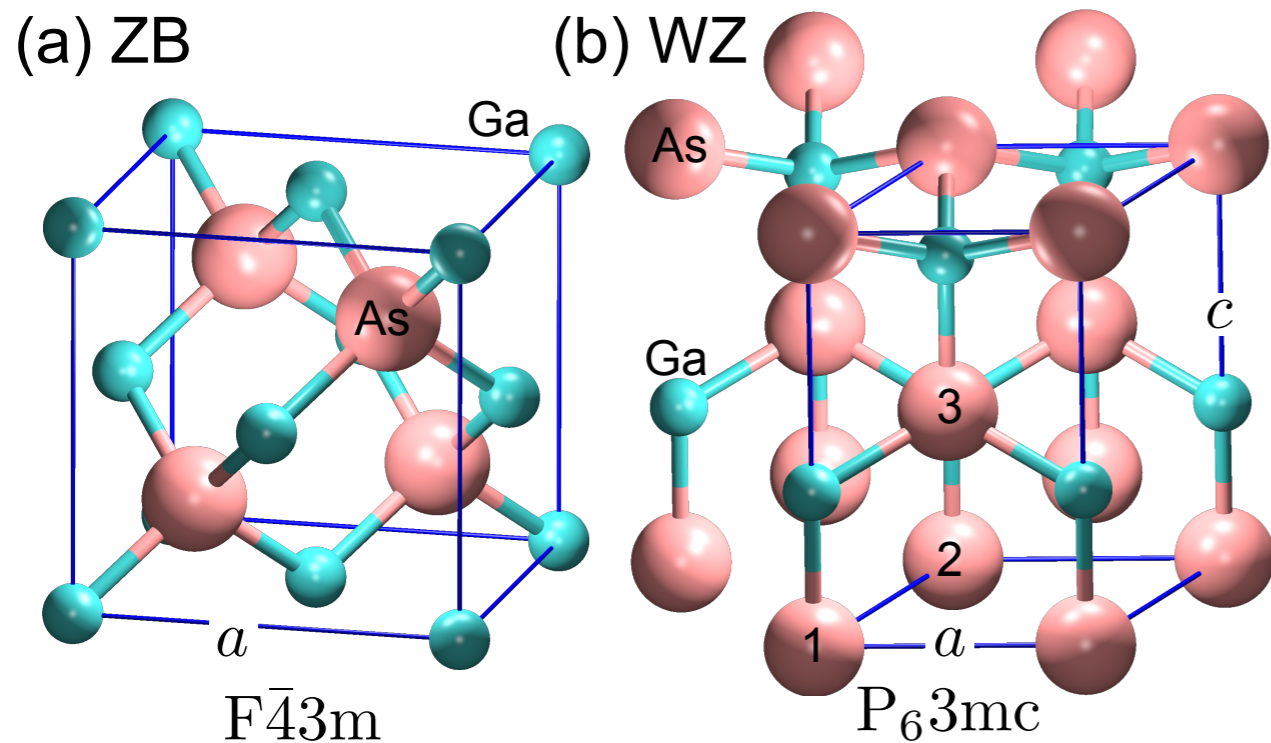


- N.B.: Corrections must be applied with great care, in particular if defect wavefunction is long-ranged, see eg. C. Freysoldt, J. Neugebauer & C. van de Walle, PRL **102**, 016402 (2009)

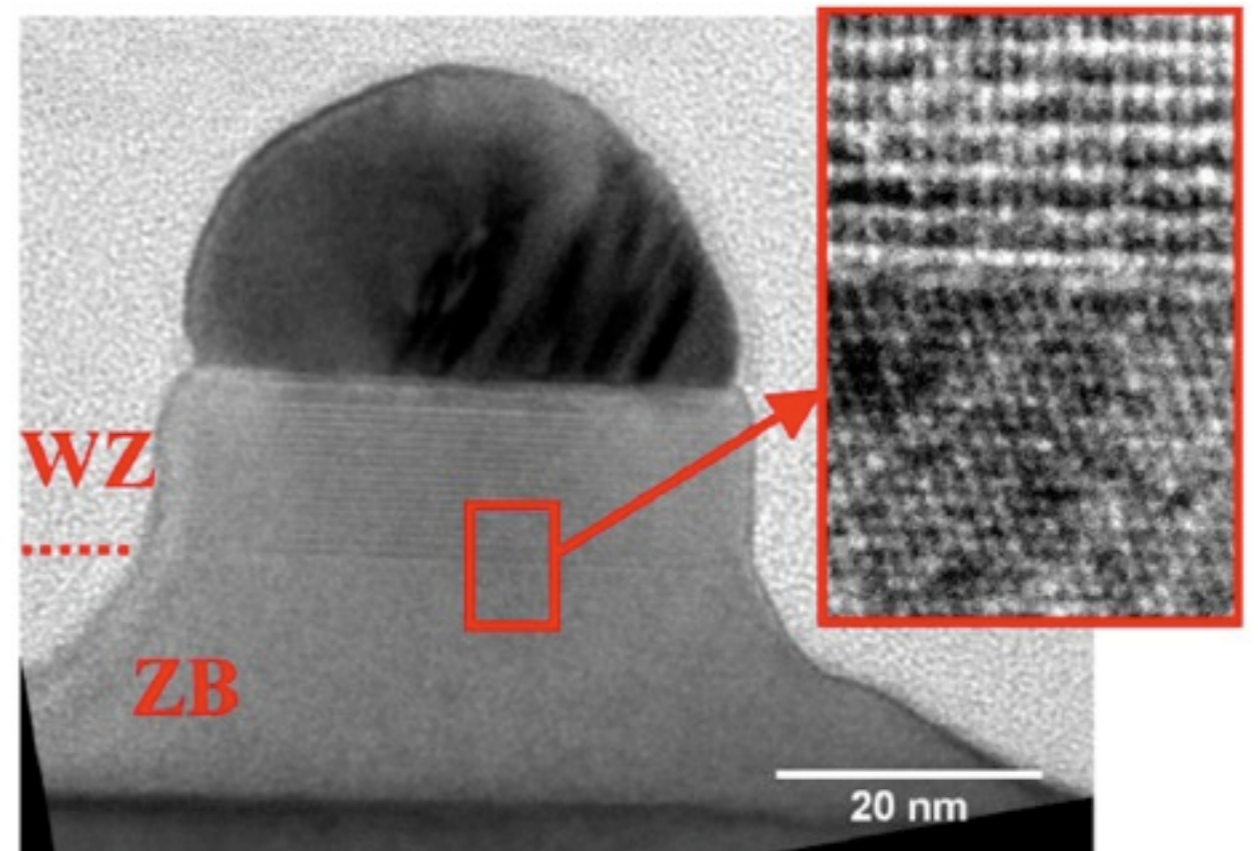
# Application I: Defects in wurtzite GaAs

# Polytypism in GaAs nanopillars

- self-assembled growth of GaAs nanopillars possible both with metal catalyst (gold) or “self-catalysed”
- growth zone below the metal particle
- solubility of Ga (As) is high (low) in the metal particle



F. Glas, J.-C. Harmand & G. Patriarche,  
PRL **99**, 146101 (2007)



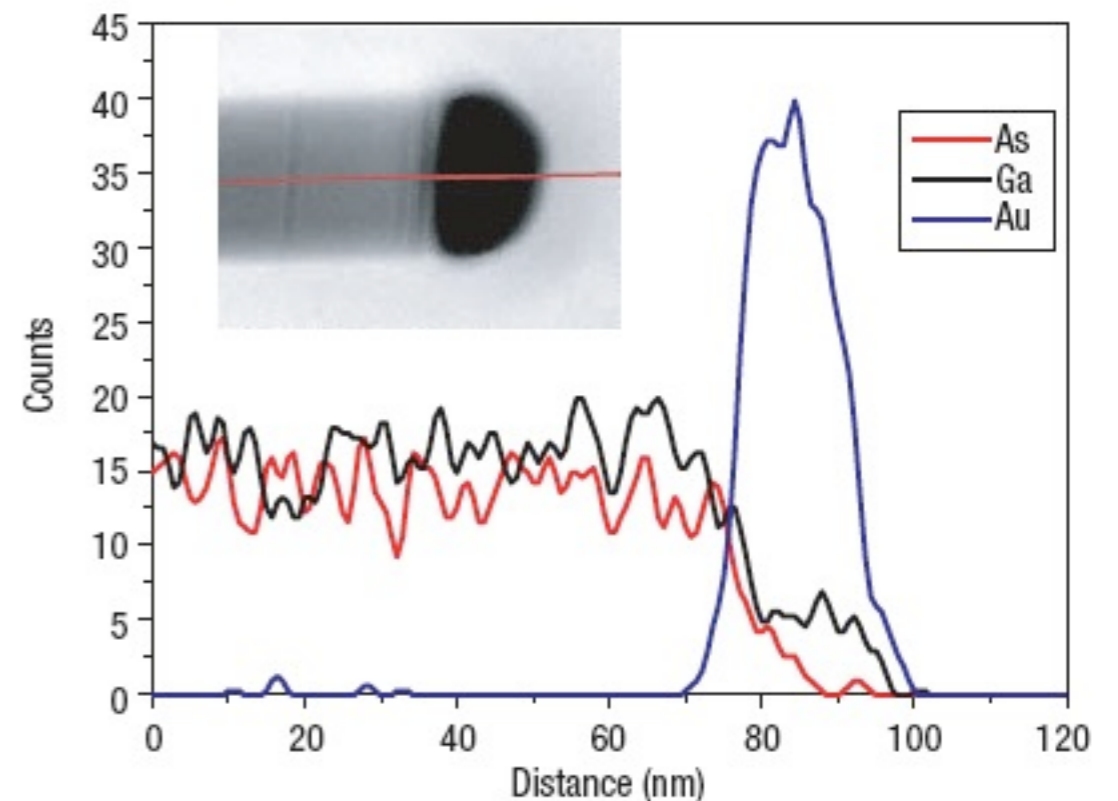
# Non-stoichiometric growth zone

- relative deficiency of As (compared to Ga) in as-grown nanopillars
- possible incorporation of Au in Au-catalysed growth
- disappointingly low carrier lifetimes in GaAs nanopillars due to recombination at point defects ?

## Topics to be studied:

- Formation energy of defects in **both** wurtzite and zincblende GaAs
- deep charge transfer levels due to Au impurities ?
- mobility of As vacancy ?  
(important for possibility of healing by anneal cycle)

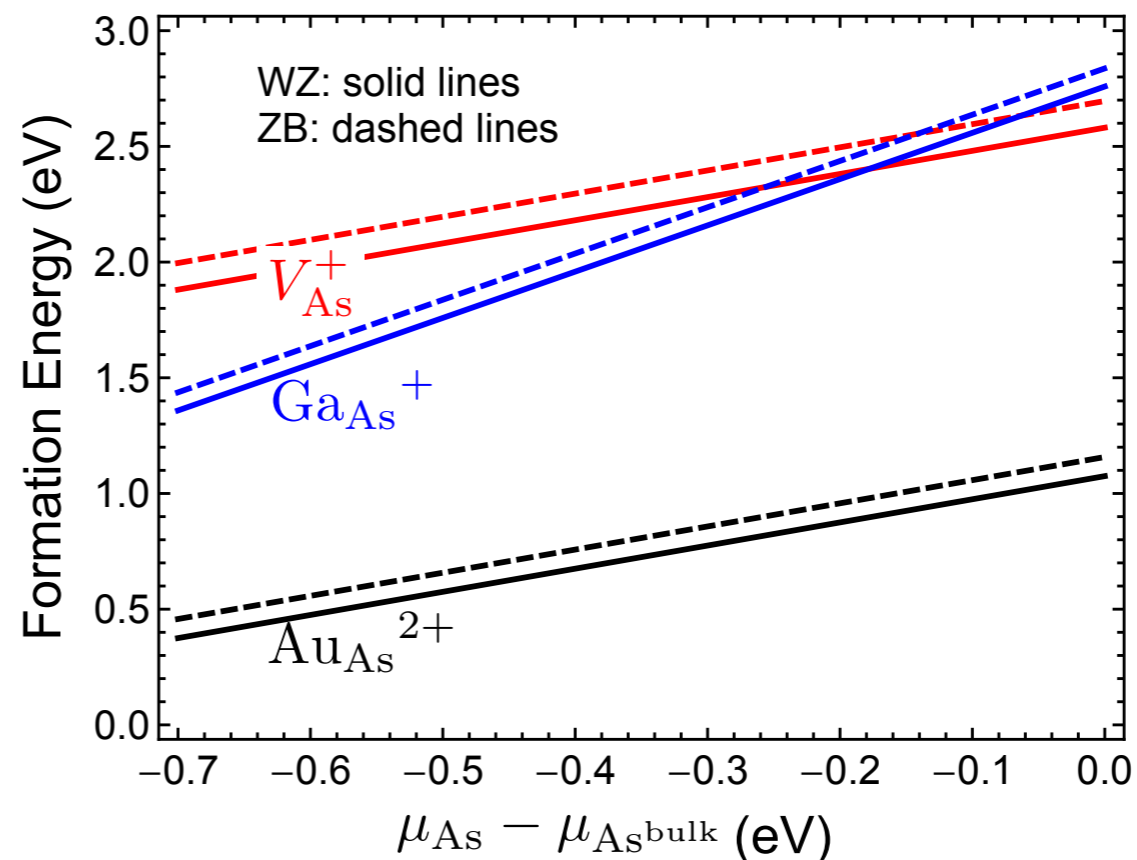
element-specific XEDS analysis  
A.I. Persson et al.,  
Nature Materials **3**, 667 (2004)



# Growth-related point defects

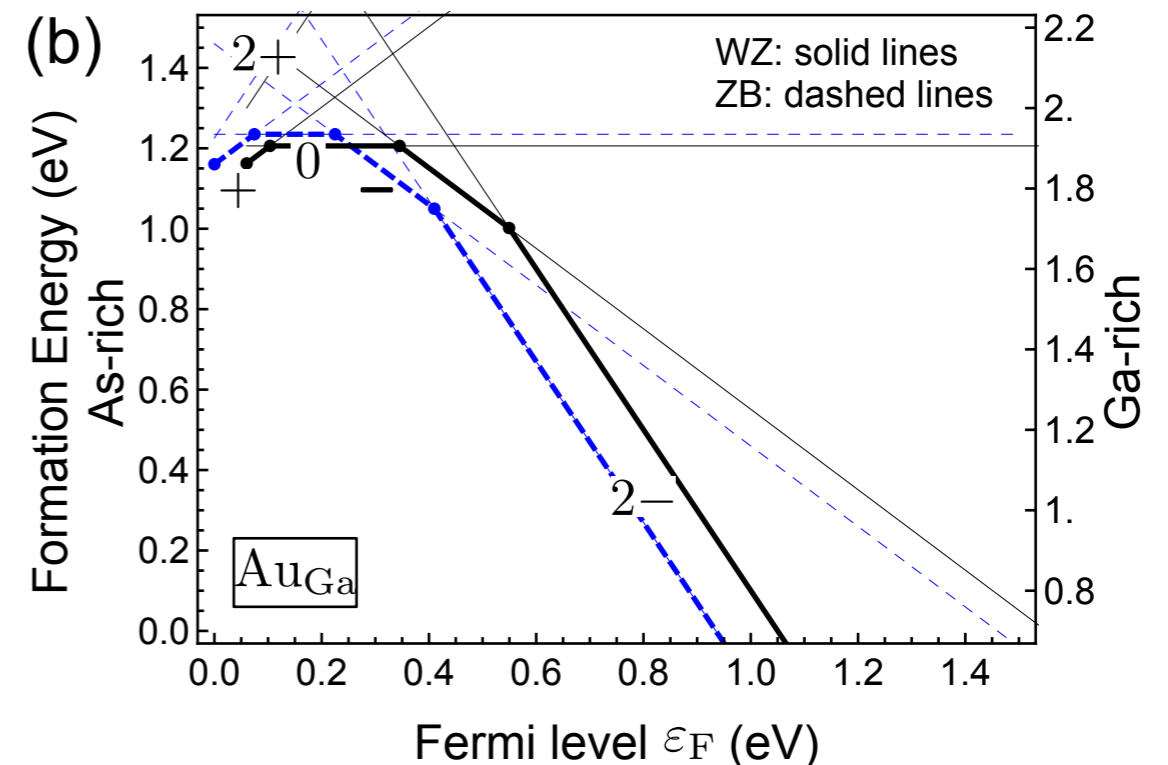
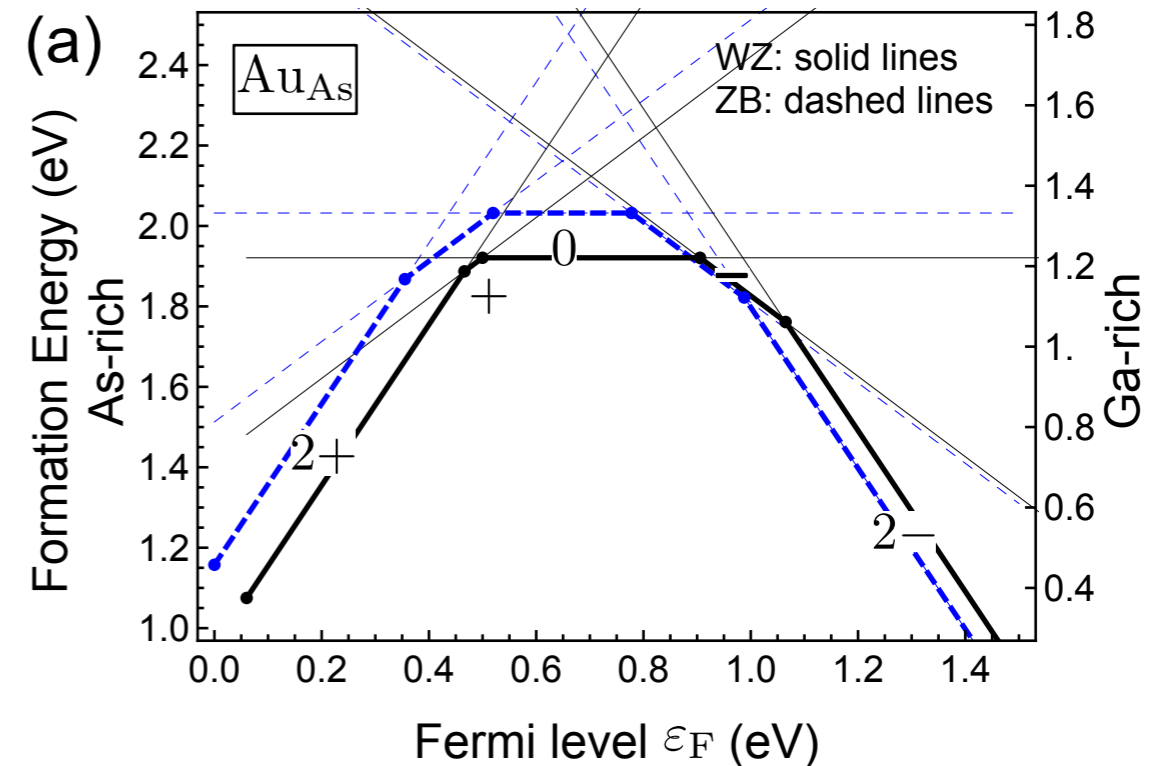
- Calculations performed with PBE functional in 216-atom (ZB) or 96-atom (WZ) supercell
- Growth with Au catalyst particle: substitutional Au defects are lower in energy than intrinsic  $V_{As}$  or  $Ga_{As}$  defects
- self-catalysed growth (no Au):  $V_{As}$  or  $Ga_{As}$  may form in low concentrations, depending on growth conditions

defects related to deficiency of As in *p*-type GaAs



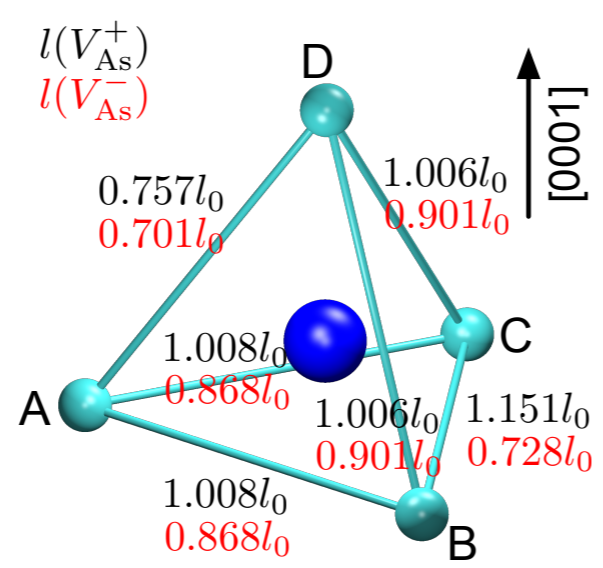
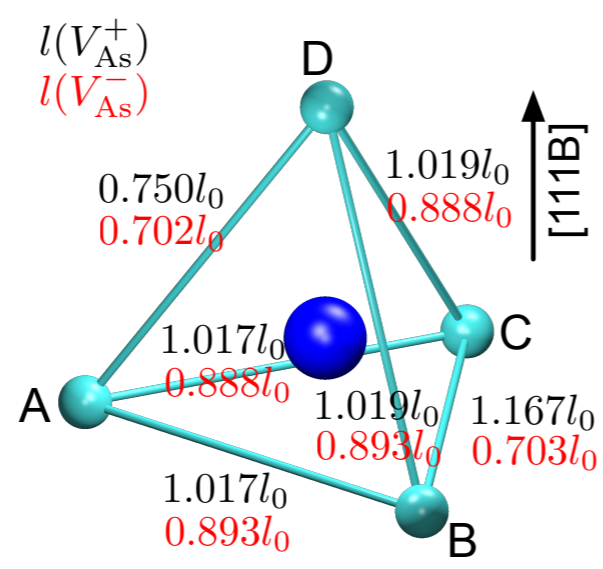
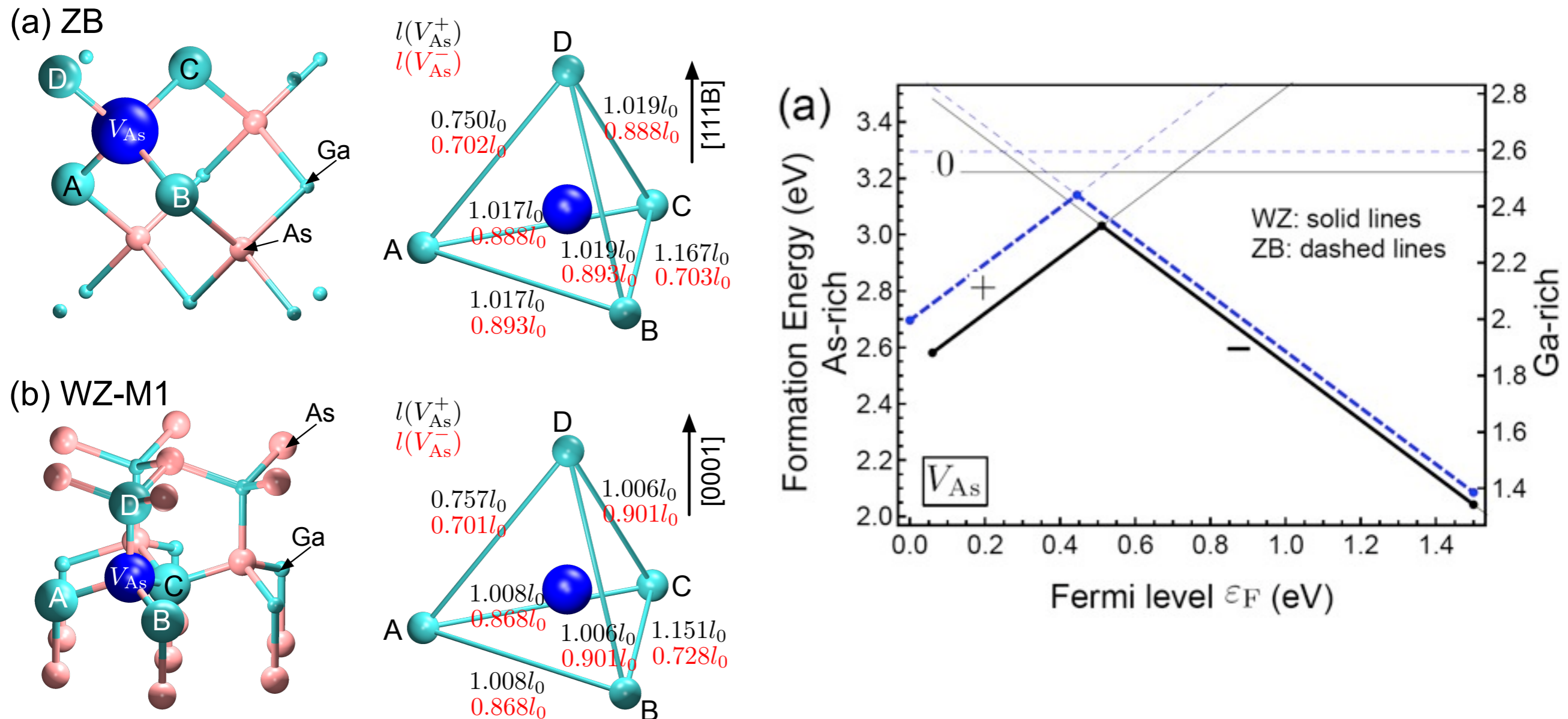
# Gold as deep center in GaAs

- formation energy for  $\text{Au}_{\text{Ga}}$  generally lower than for  $\text{Au}_{\text{As}}$
- various charge states from  $2+$  to  $2-$
- experiment: charge transfer level  $\varepsilon(0|-)$  at  $\sim 0.4$  eV seen in deep-level transient spectroscopy



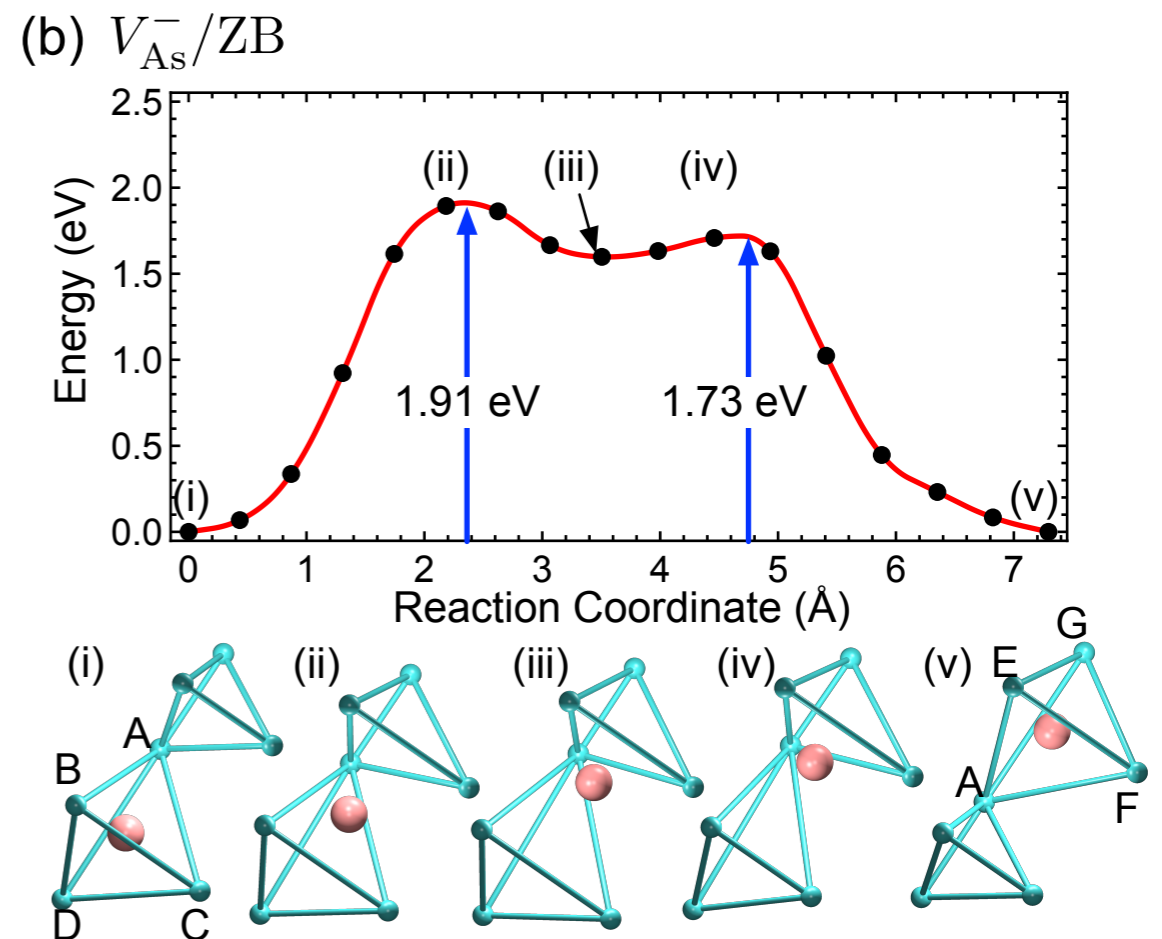
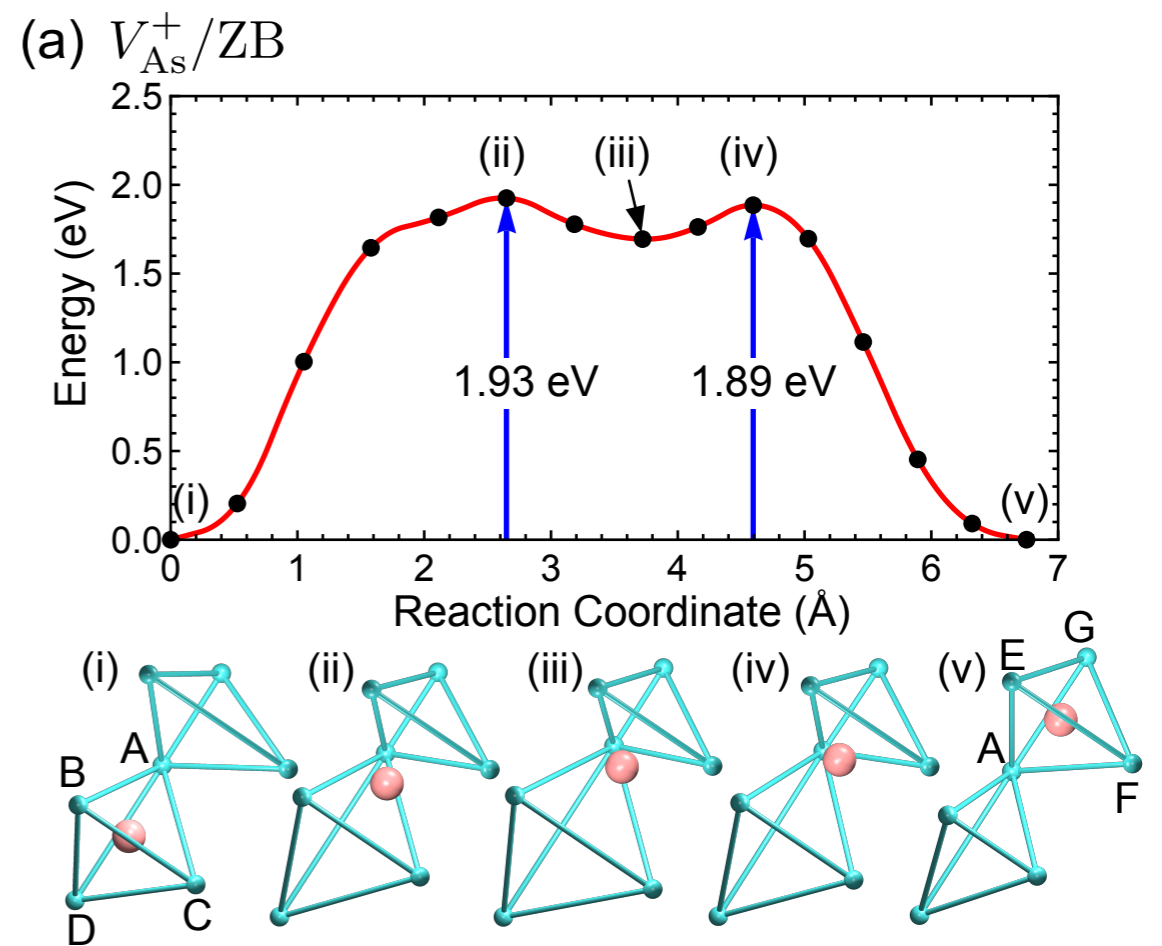
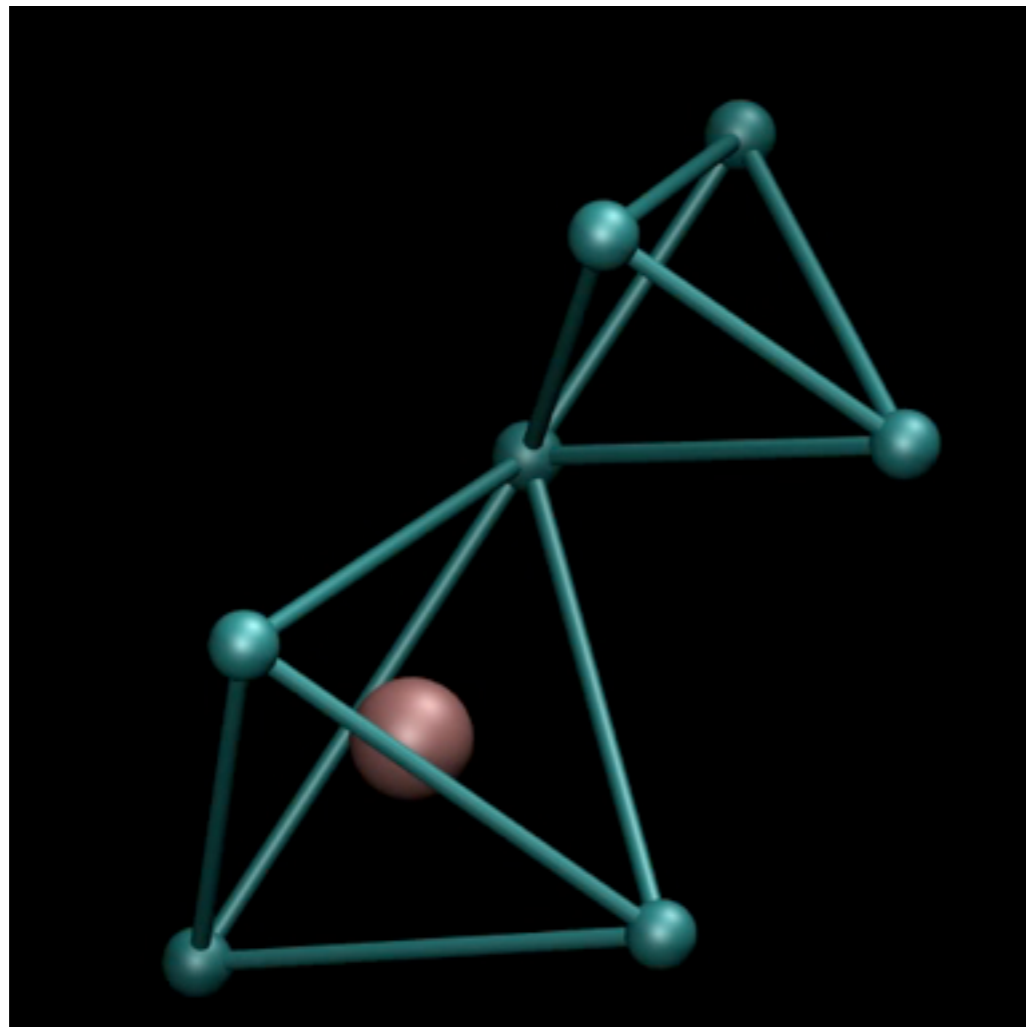
# Arsenic vacancies

- unstable as neutral vacancy (a so-called “negative-U” system)
- strong contraction of the vacancy tetrahedron for negatively charged vacancy, as bonding linear combination of Ga dangling bonds becomes occupied



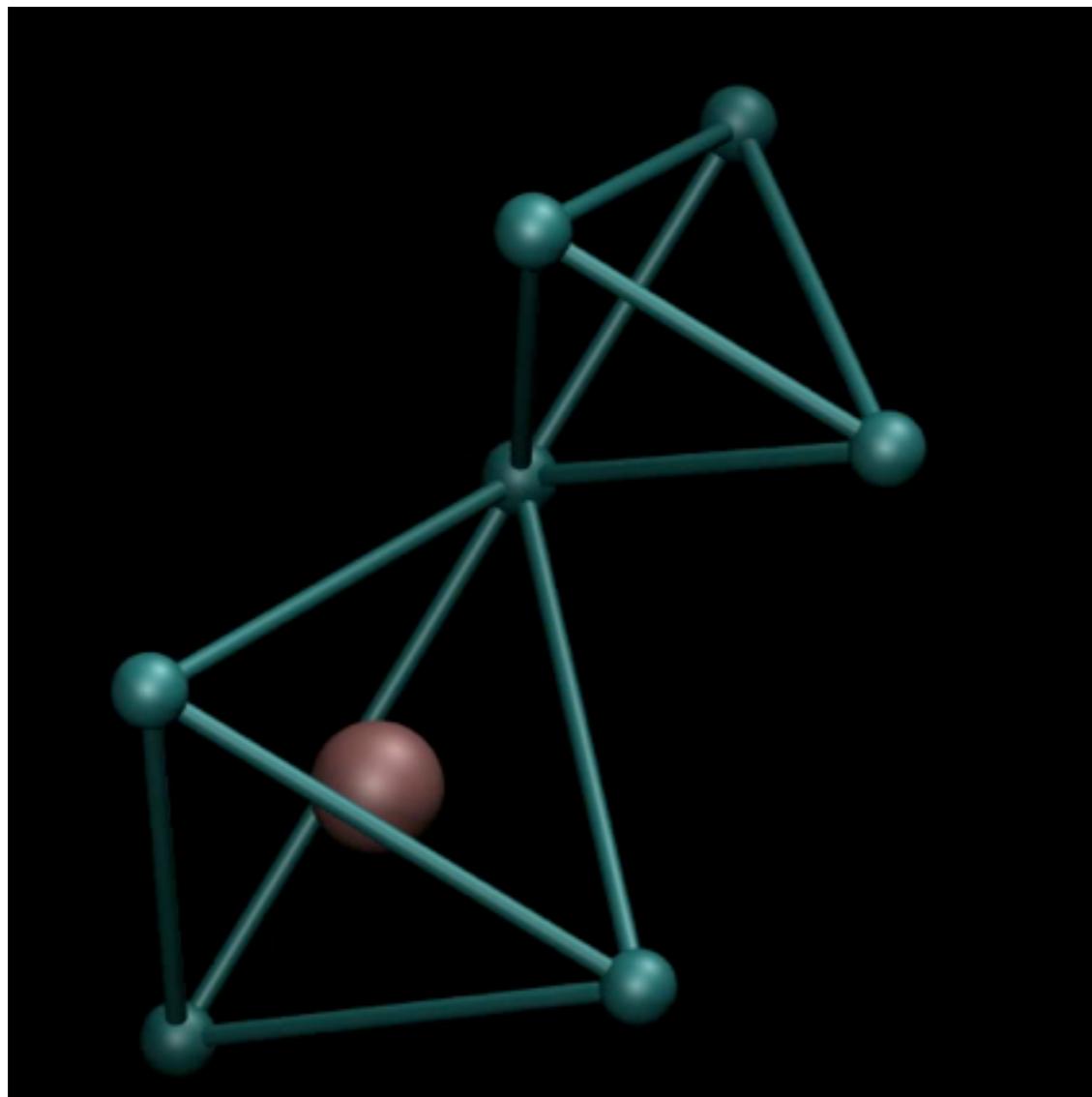
# $V_{As}$ diffusion in ZB

- octahedral interstitial as intermediate minimum
- Asynchronous diffusion of electronic charge and of As atom

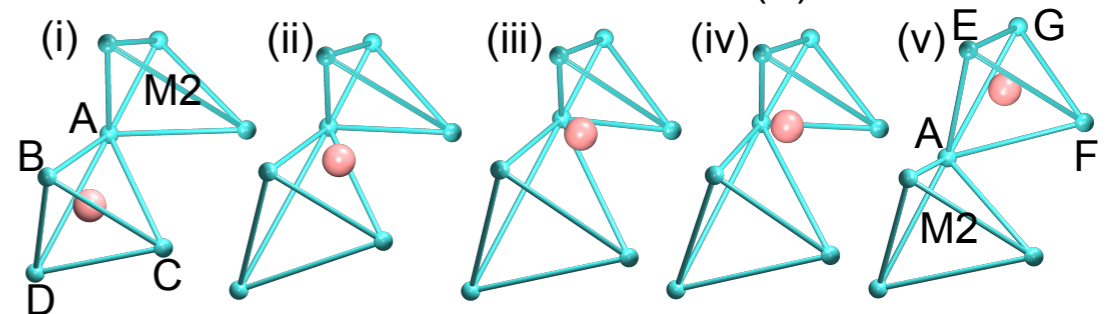
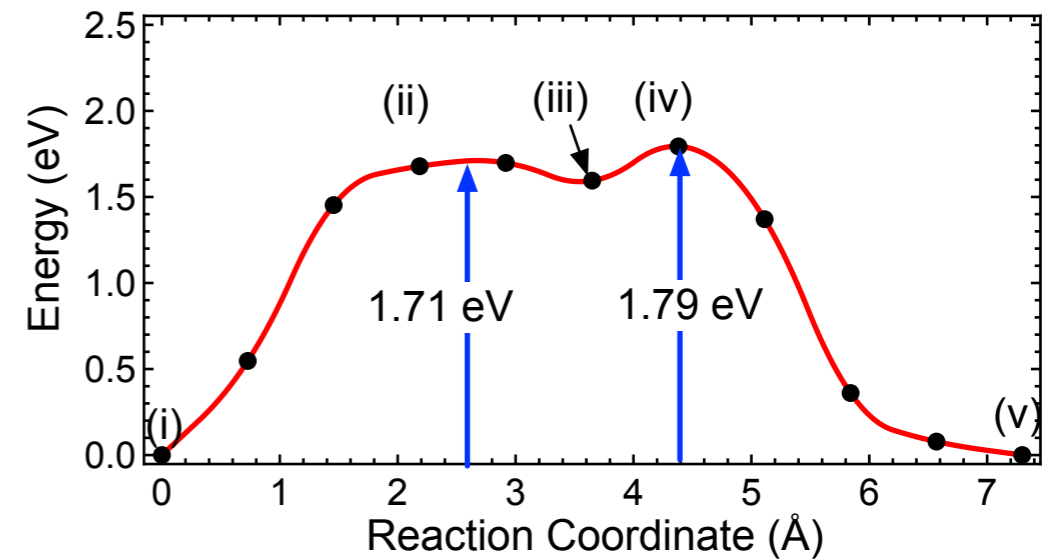


# $V_{As}$ diffusion in WZ

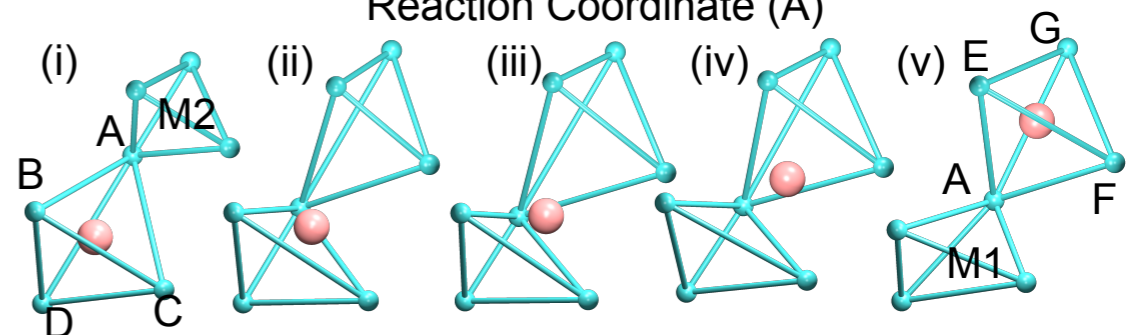
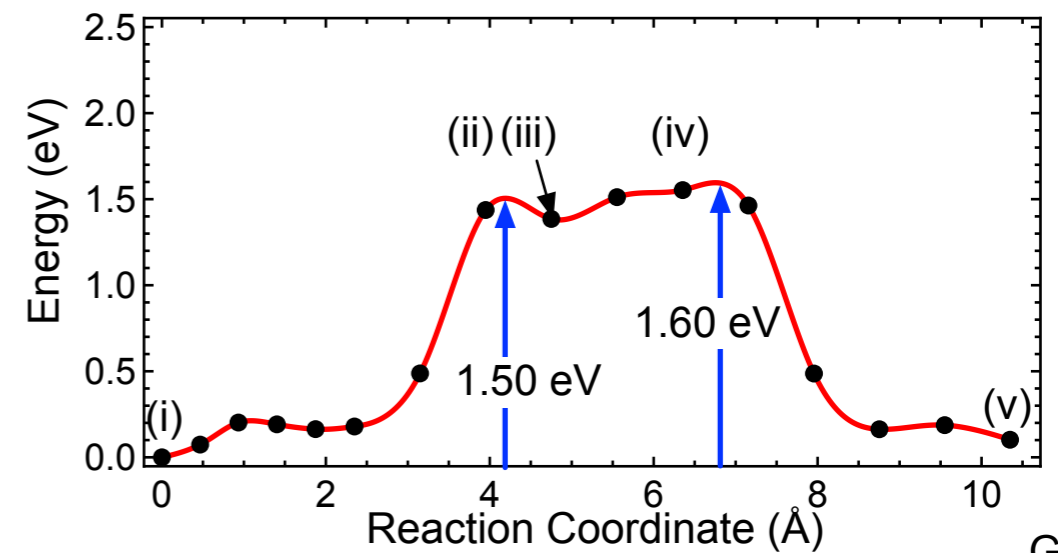
- in wurtzite  $ab$ -plane, barriers are lower as compared to ZB case



(a)  $V_{As}^+ / WZ - ab$

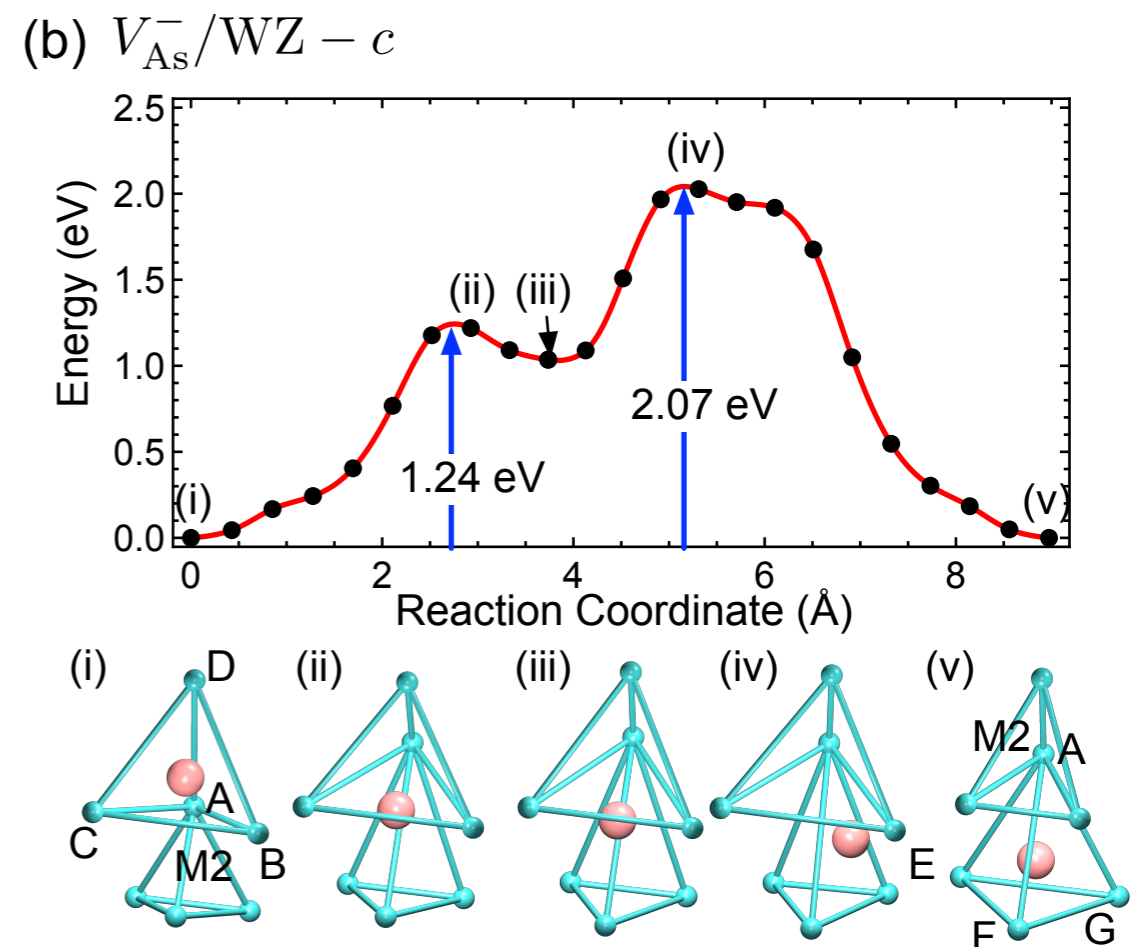
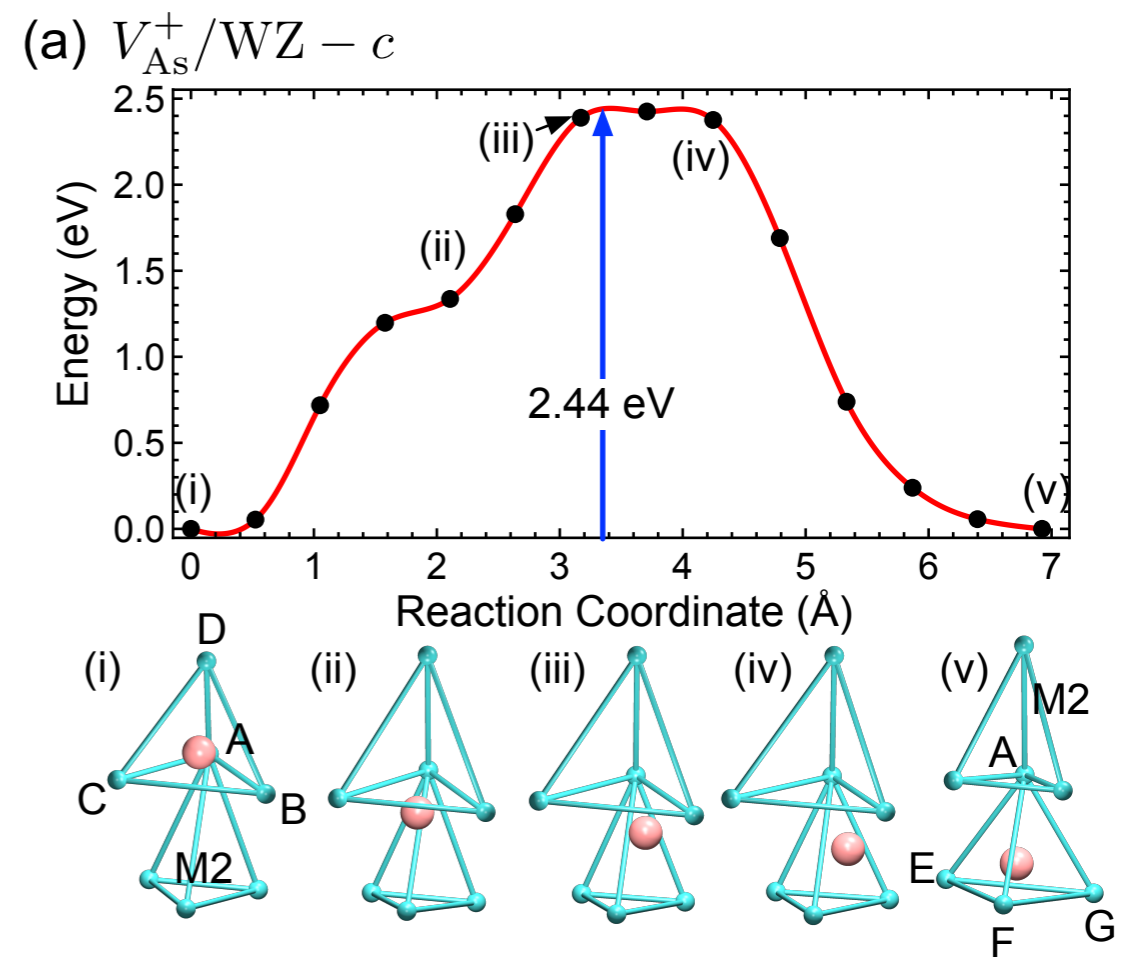
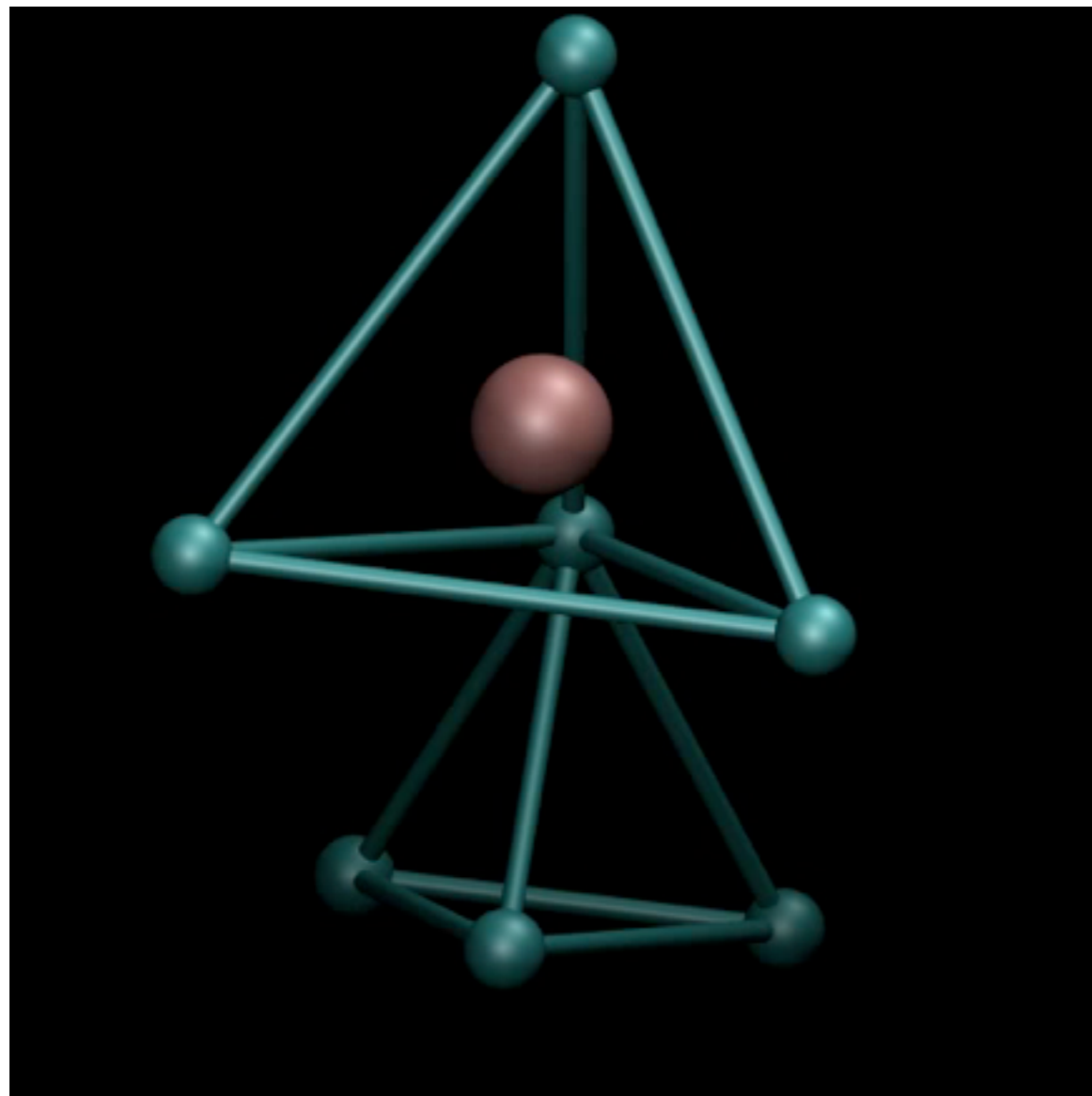


(b)  $V_{As}^- / WZ - ab$



# $V_{As}$ diffusion in WZ

- along the c-axis,  $V_{As}$  needs to go a detour to avoid crossing the Ga-Ga coordination line  $\rightarrow$  higher barrier than in ZB

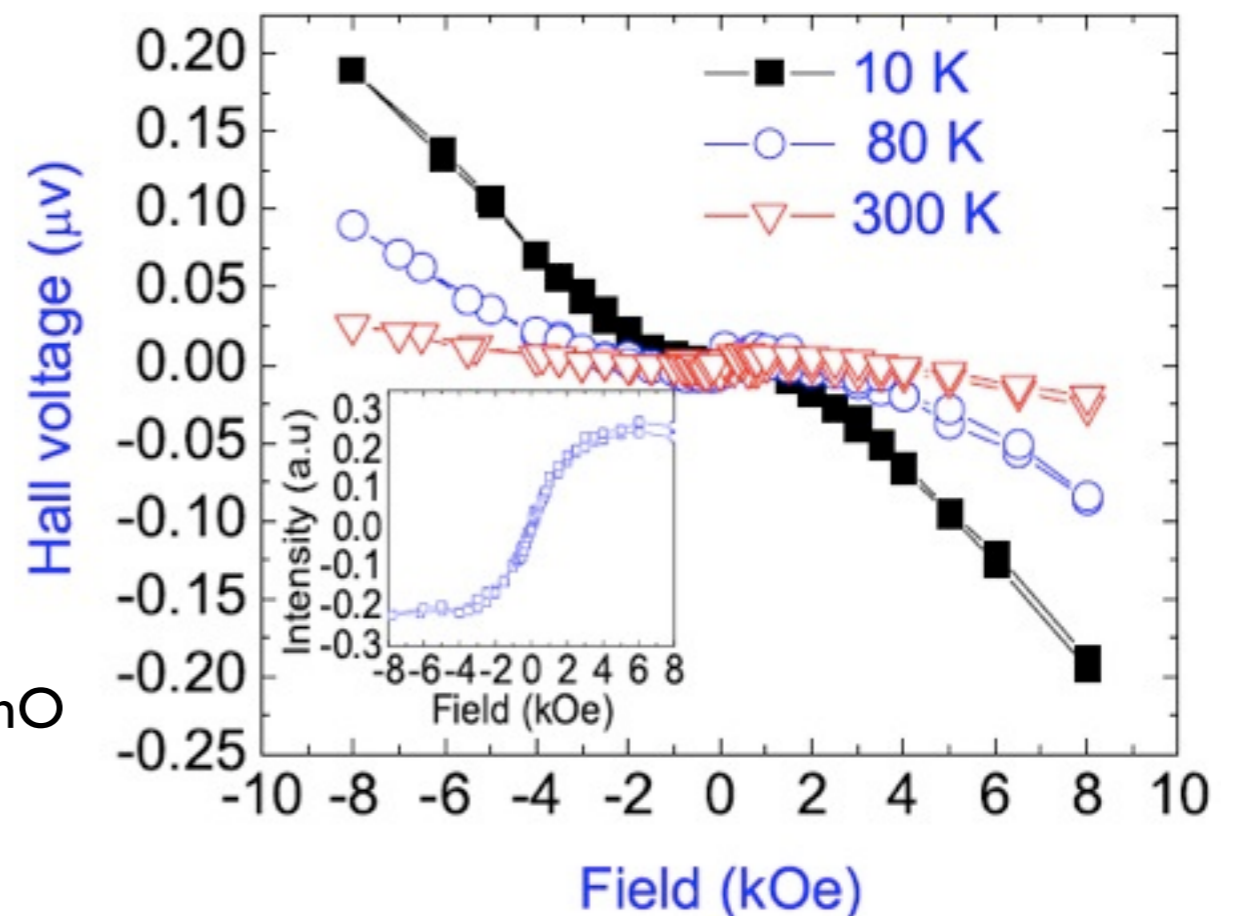


# Application II: Impurity pairing for carbon in ZnO

# Motivation

- ZnO is a low-cost wide-gap semiconductor that can be synthesised in bulk crystalline form
- **Drawback:** no reproducible strategy for  $p$ -type doping available. Could C act as acceptor in ZnO ?
- **Ferromagnetism** in ZnO induced by carbon ?  
Host-mediated magnetic interaction between C-impurities would require high C concentrations. Pairing of C energetically favorable ?

Anomalous Hall effect in n-type C-doped ZnO  
H. Pan et al., PRL **99** (2007), 127201

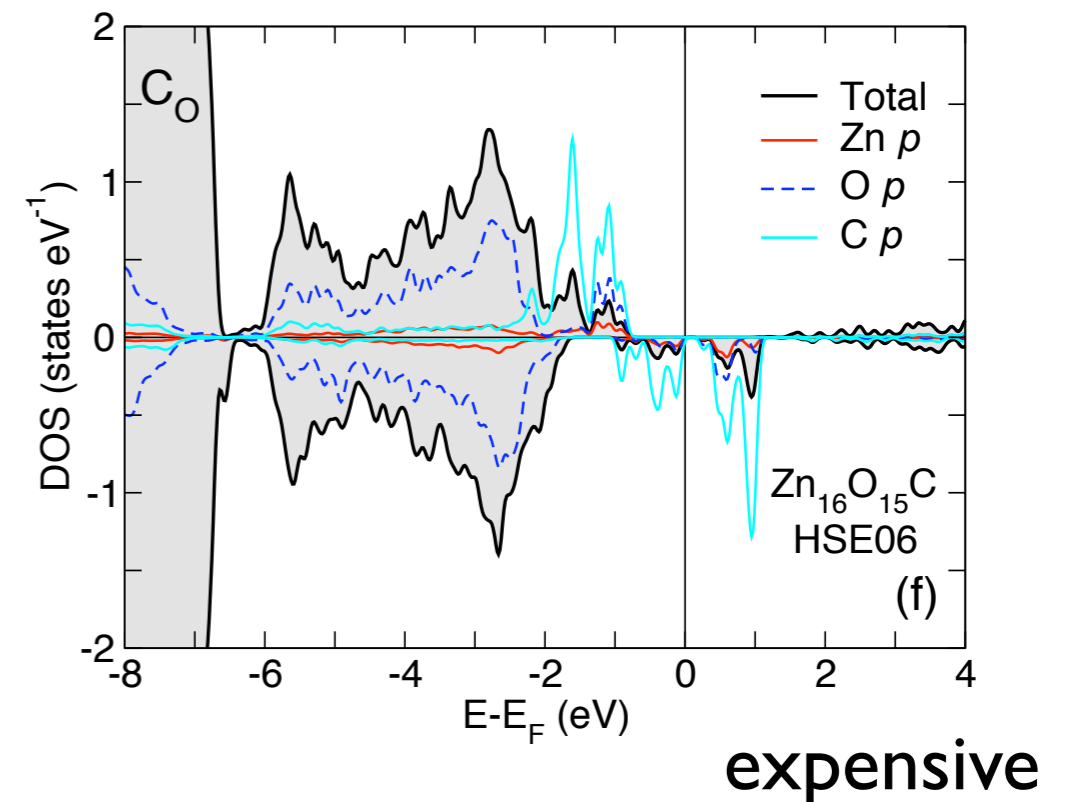
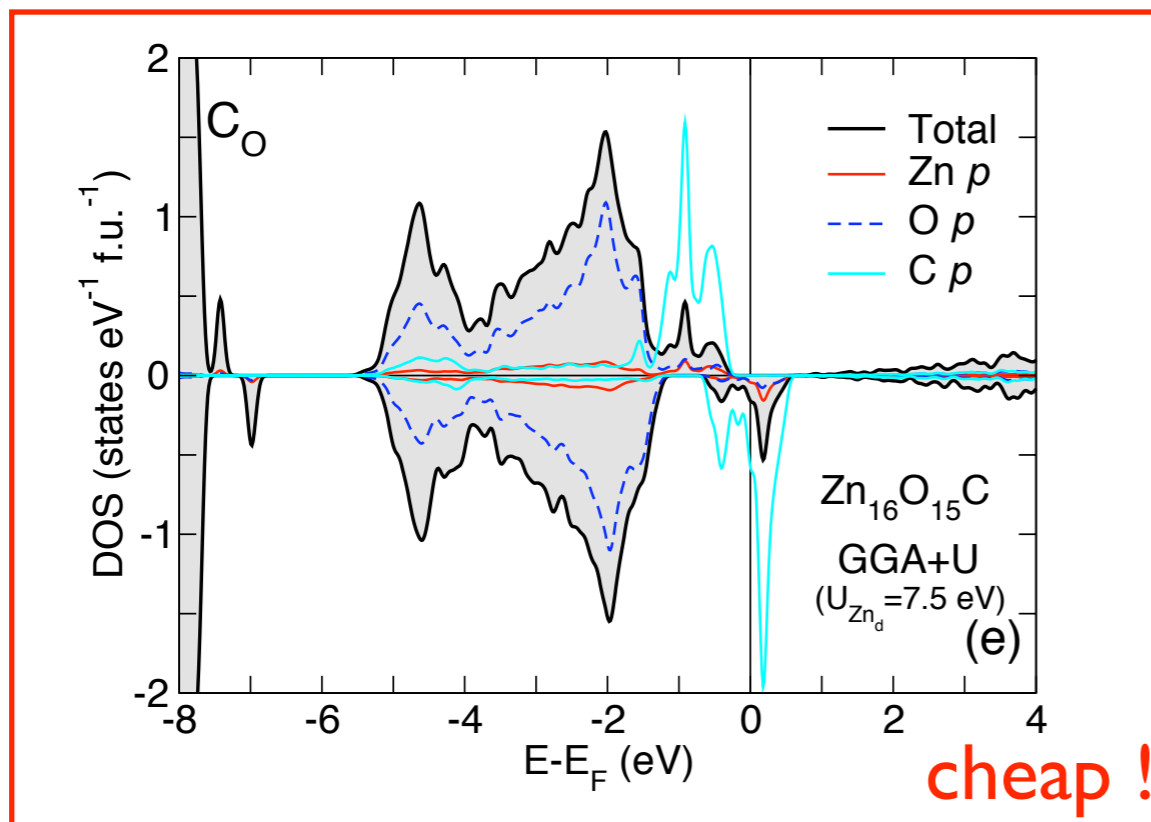
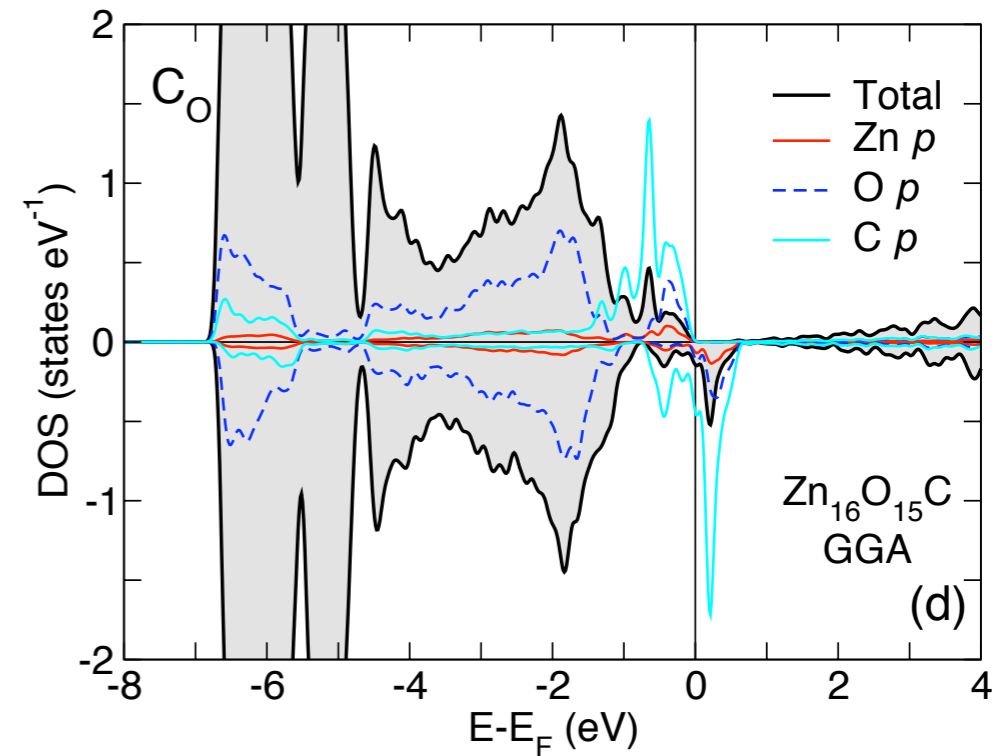


# Method comparison

- In LDA and GGA, the oxygen-derived valence band lies too high up in energy  
→ spurious hybridization between C and O orbitals

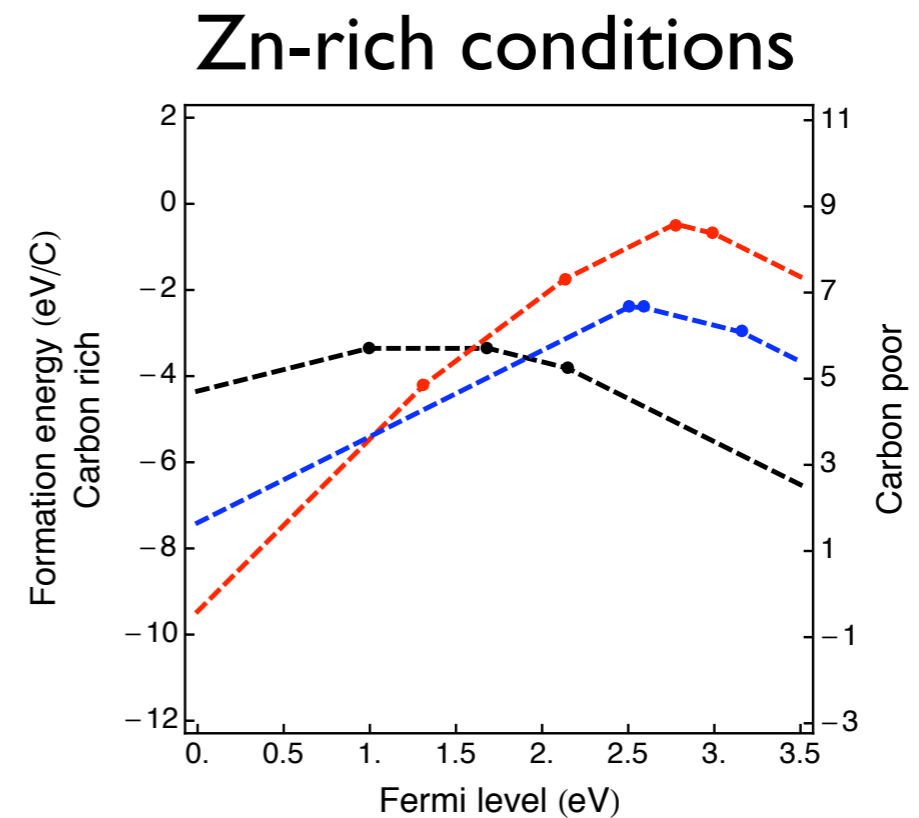
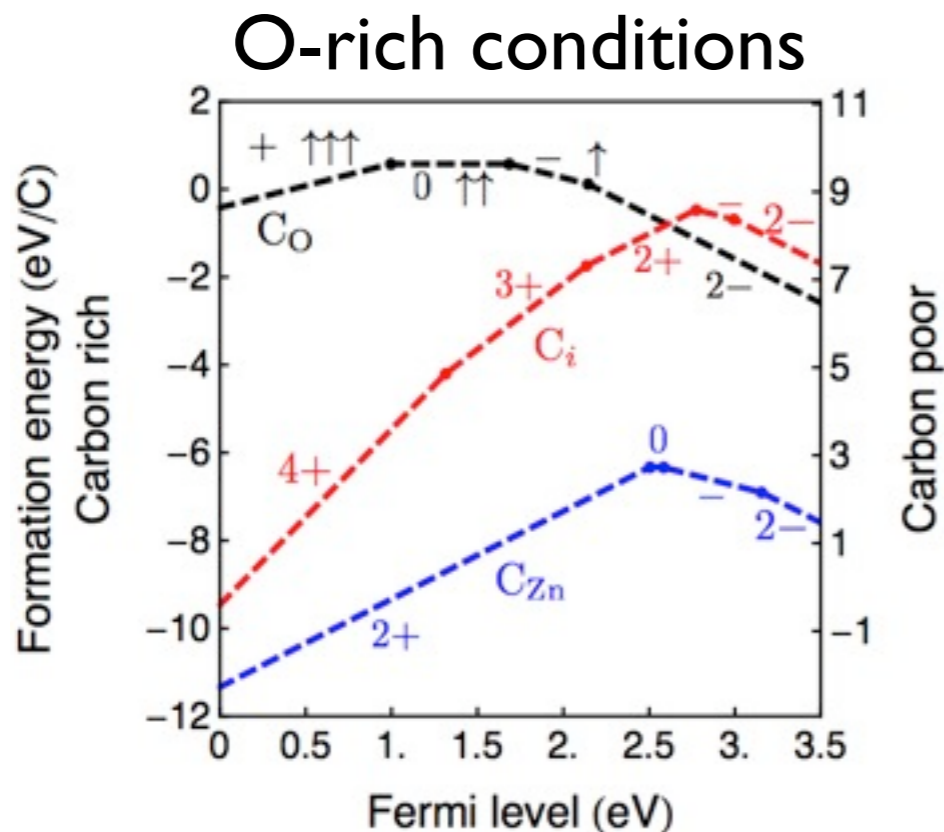
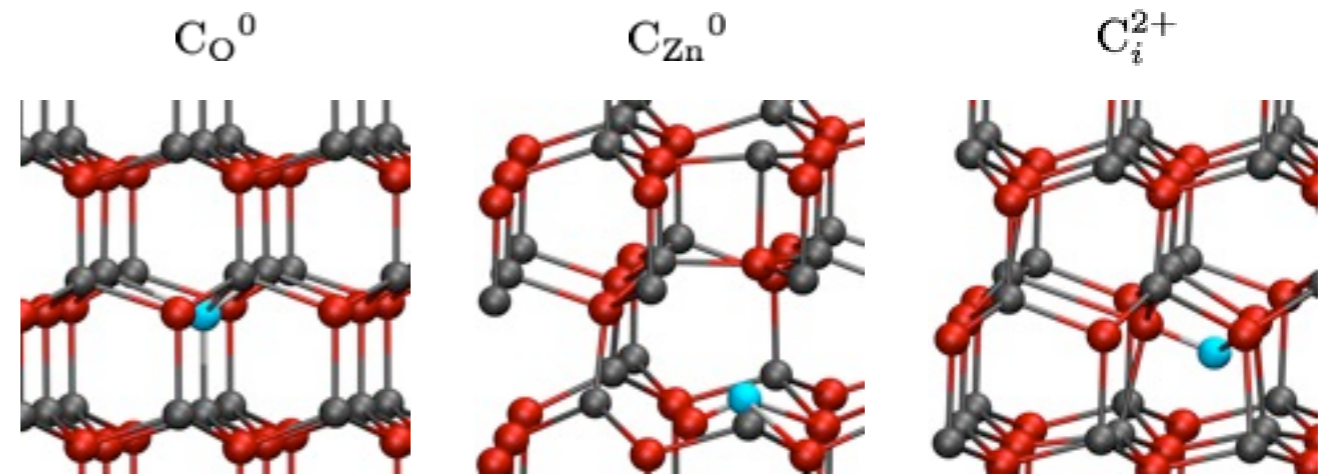
S. Nayak et al. PRB **86**, 054441 (2012)

- correction schemes



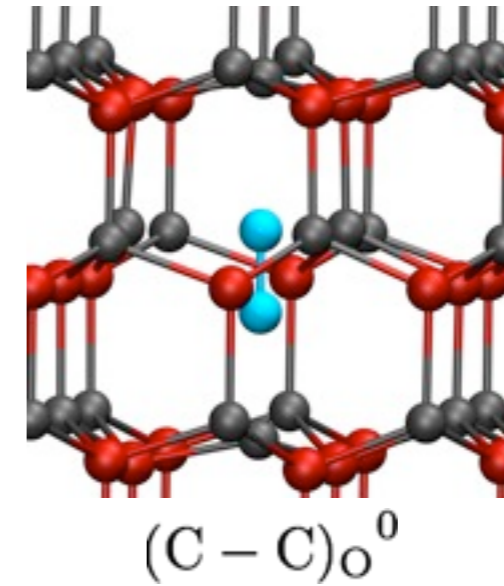
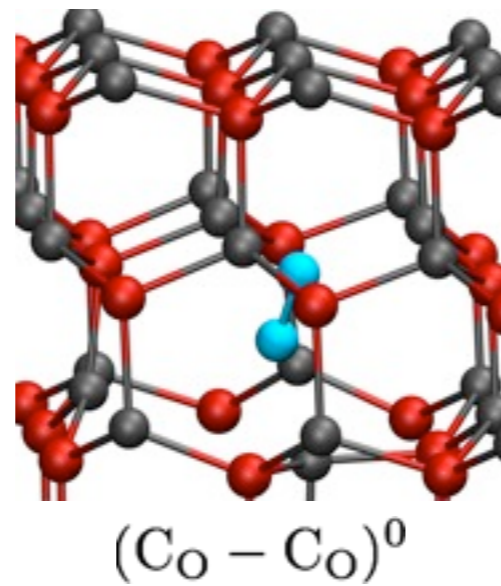
# Single impurities

- Reaction of C (stemming from di-ethyl-zinc from the precursor) with  $O_2$  to CO is thermodynamically more favorable than incorporation
- Under O-rich conditions,  $C_{Zn}$  is the thermodynamically most stable single impurity

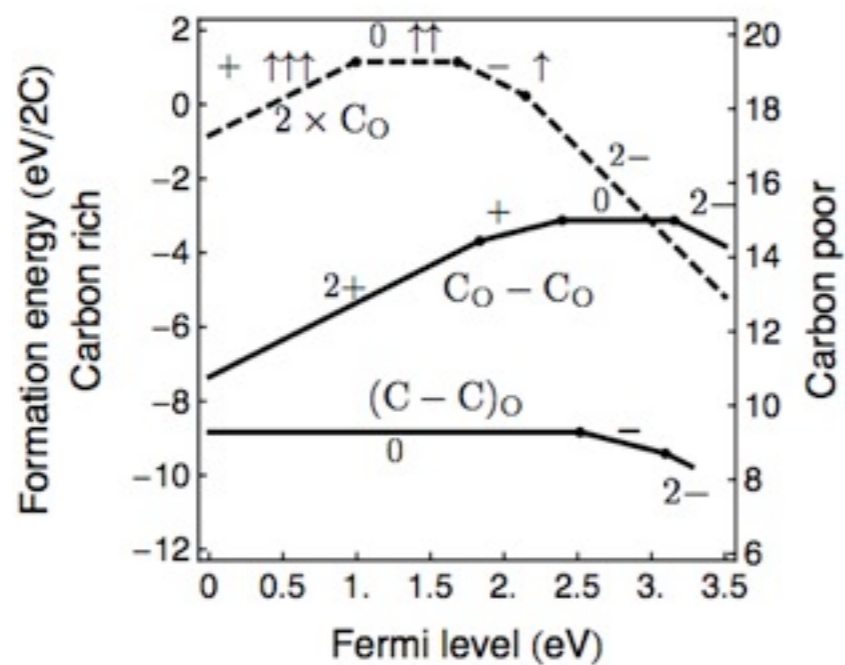


# Defect molecule at oxygen site

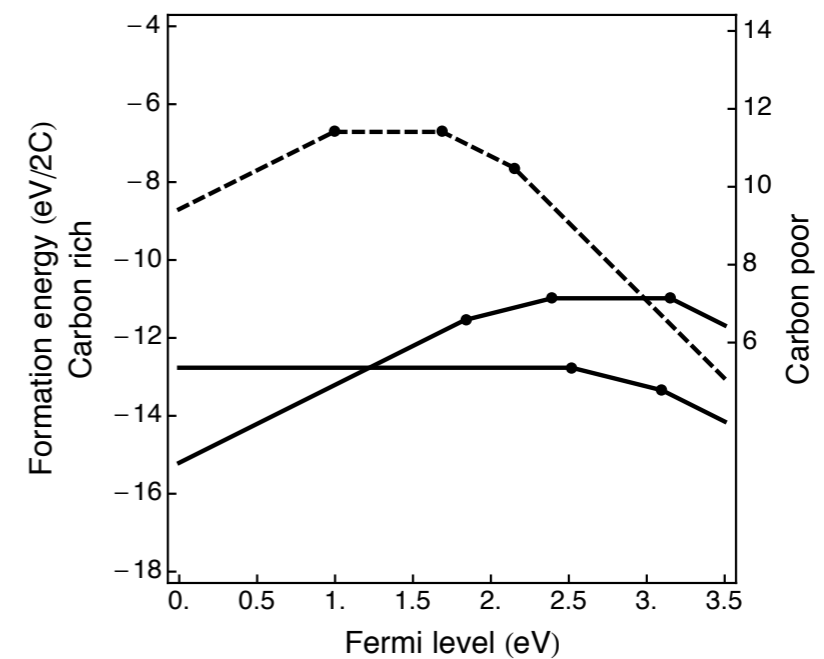
- Formation of  $C_2$  molecule lowers the formation energy compared to the isolated defects



## O-rich conditions

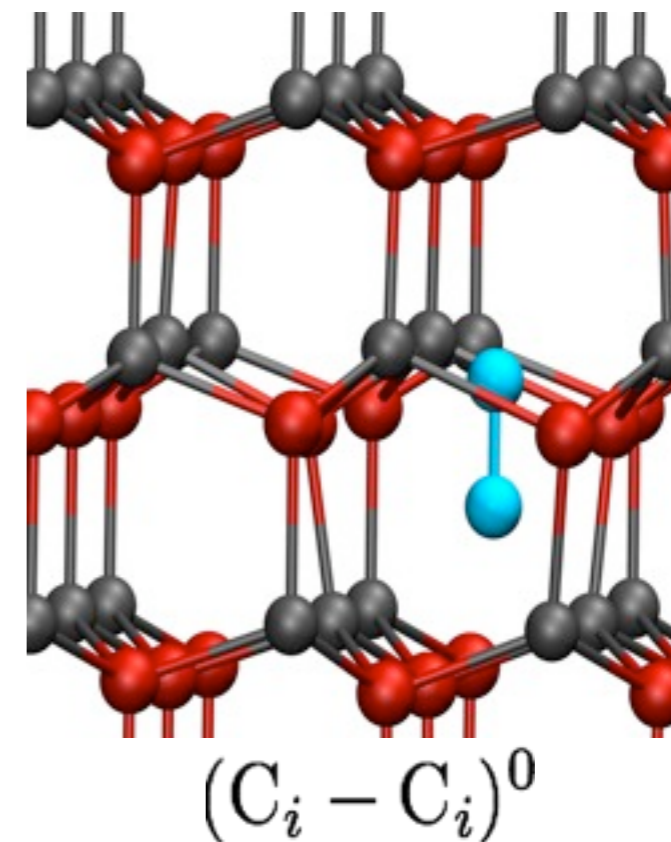
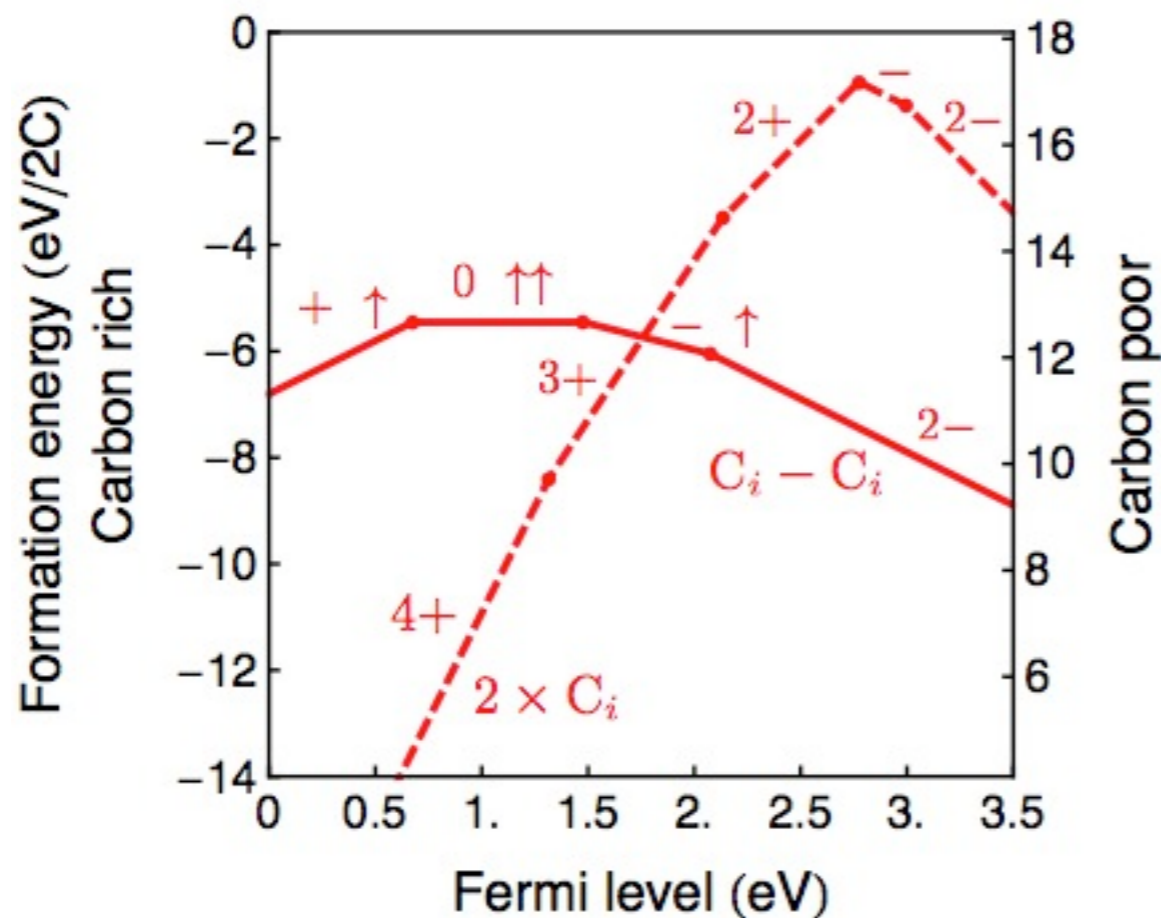


## Zn-rich conditions



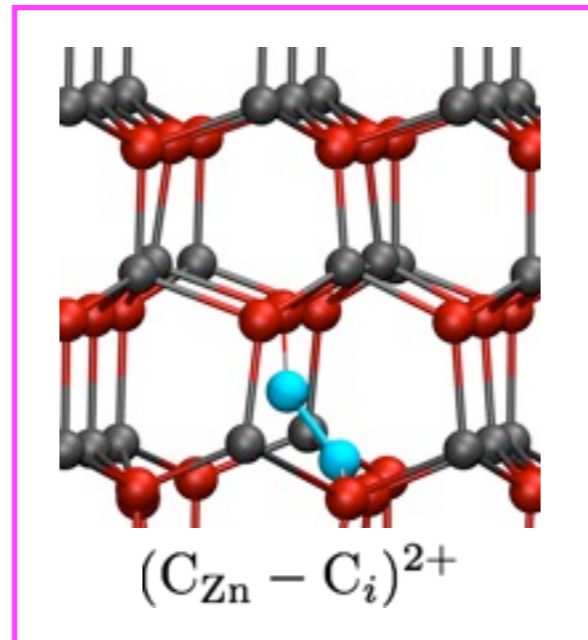
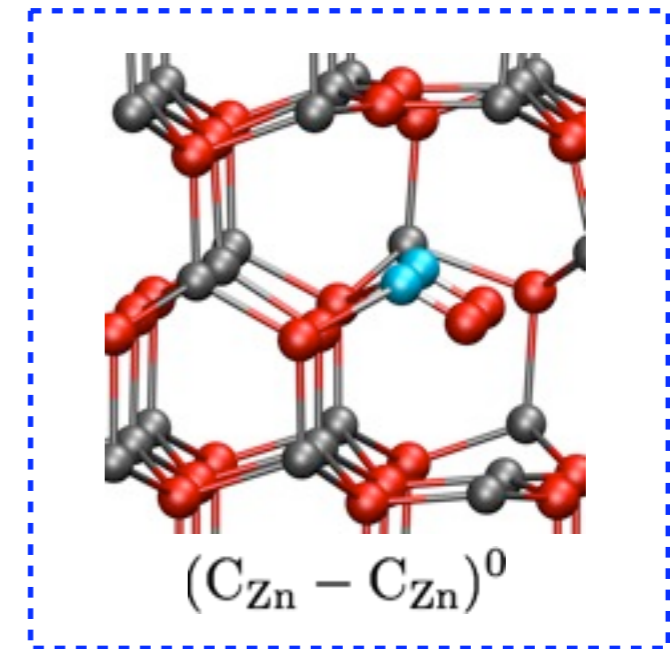
# Defect molecule at interstitial site

- In *p*-type ZnO, pairing is energetically more costly than single interstitials.
- Under *n*-type conditions, undissociated  $C_2^{2-}$  molecules or  $C_2^-$  radicals (magnetic moment !) may persist.

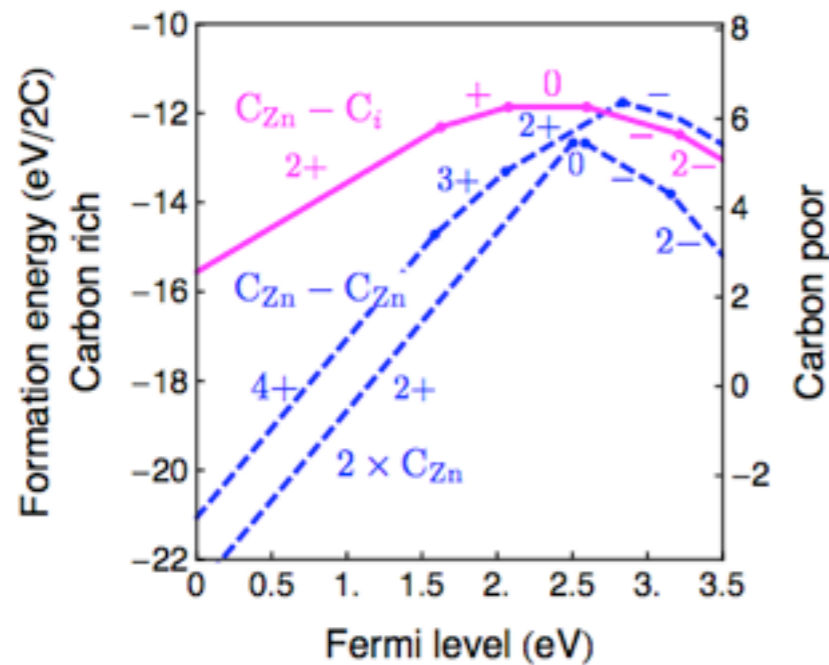


# Defect molecule at Zn site

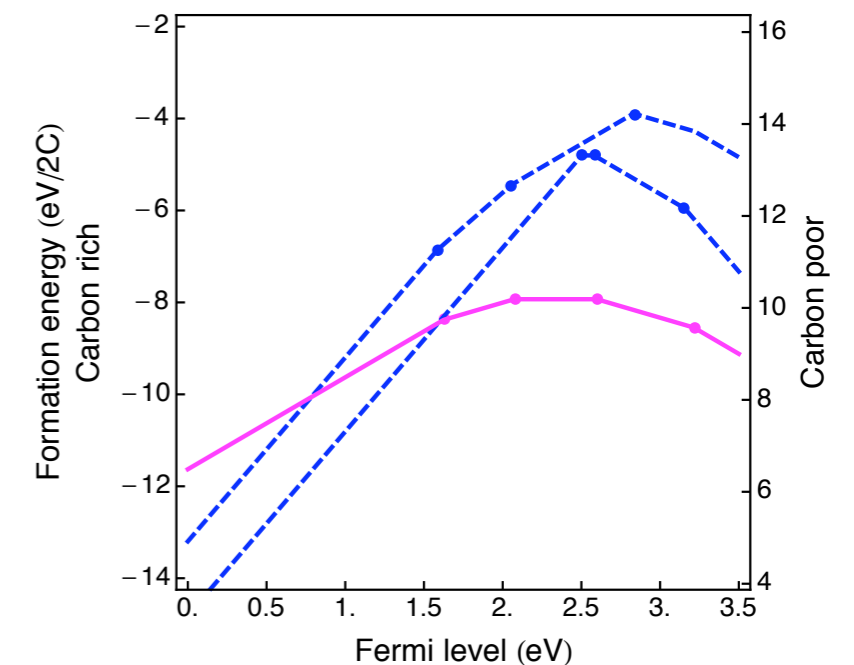
- Overall,  $C_{Zn}$  is the most stable species
- Pair of  $C_{Zn}$  at neighboring sites is less stable
- $C_{Zn} - C_i$  complex may play a role under Zn-rich conditions



## O-rich conditions

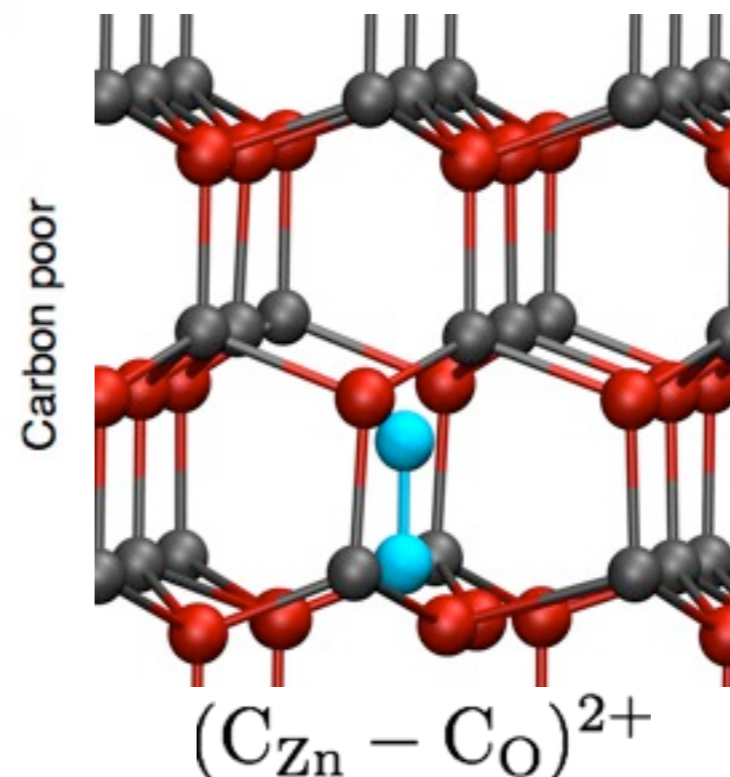
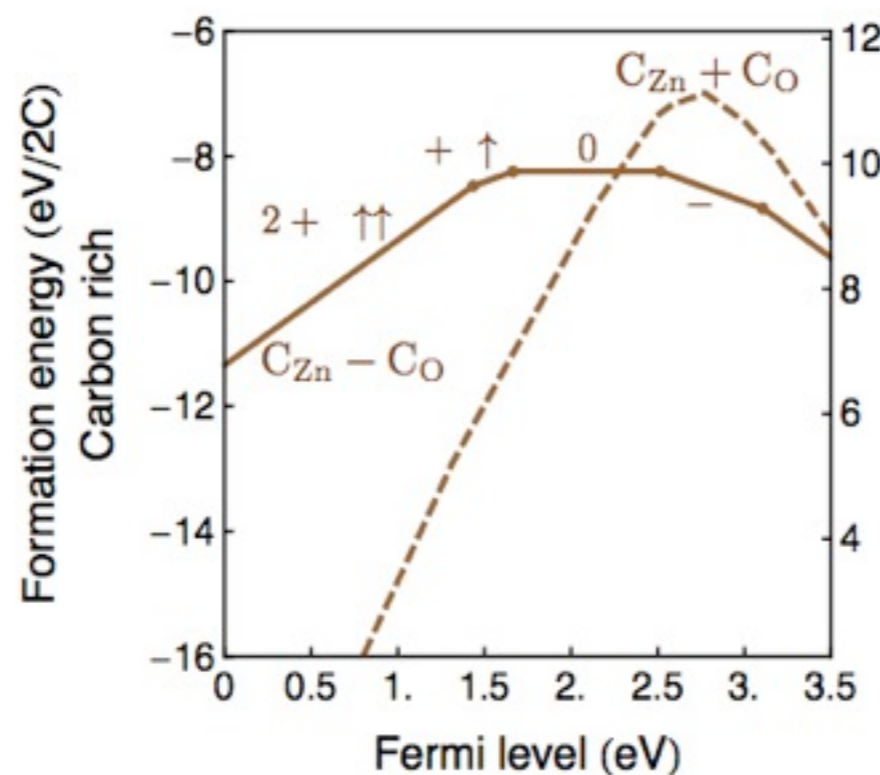


## Zn-rich conditions



# C<sub>2</sub> substituting a ZnO unit

- Pairs are less stable than 2C<sub>Zn</sub>, and less stable than C<sub>O</sub> + C<sub>Zn</sub> under *p*-type conditions.
- Molecular magnetic moment possible, but only in the (metastable) *p*-type conditions

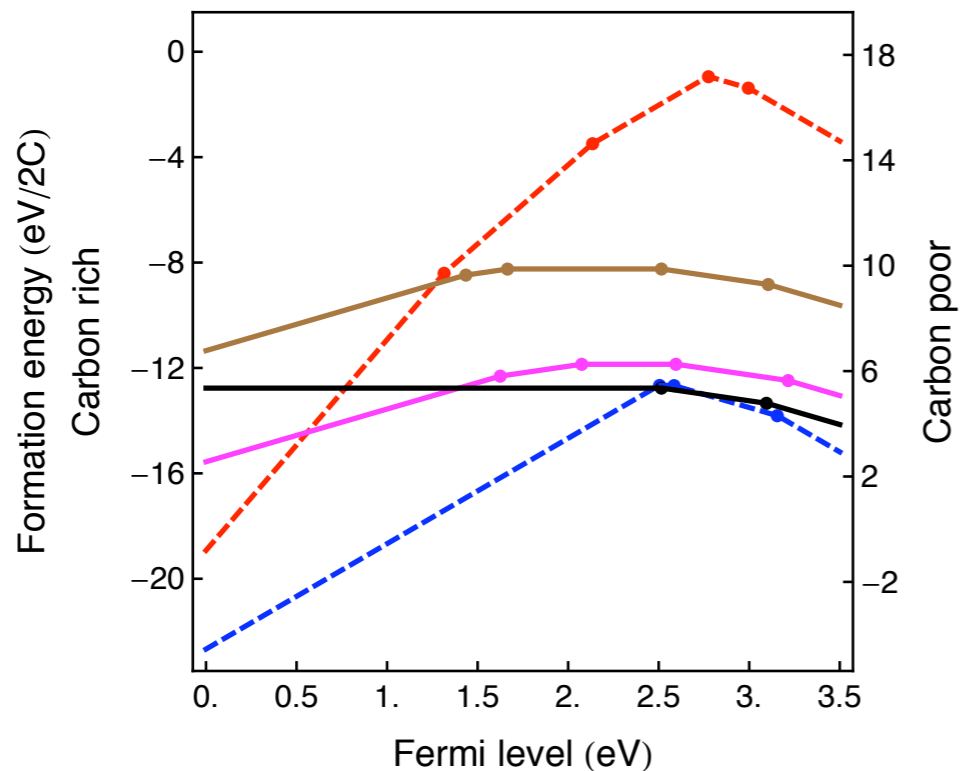


S. Sakong & P. Kratzer,  
Semicond. Sci. Technol. **26**, 014038 (2011)

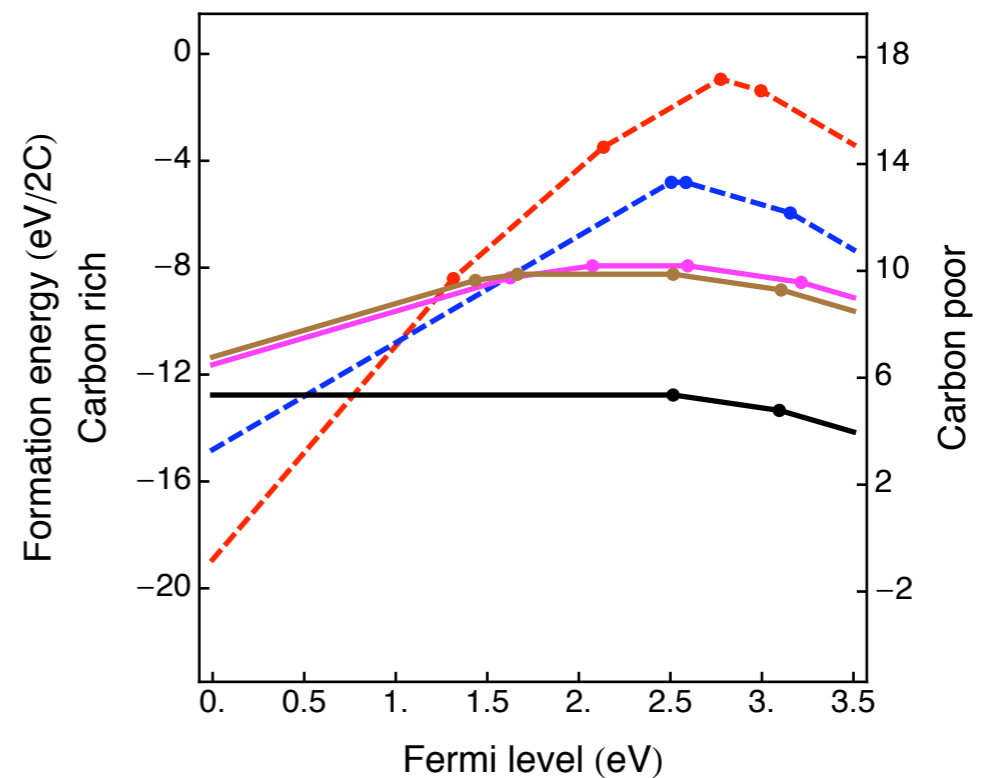
# Comparison of stable defect pairs

- **O-rich conditions:**  $C_{Zn}$ , no pairing
- **Zn-rich conditions:**  $C_i^{3+}$  in *p*-type regime,  $(C-C)_O$  in *n*-type regime

O-rich conditions

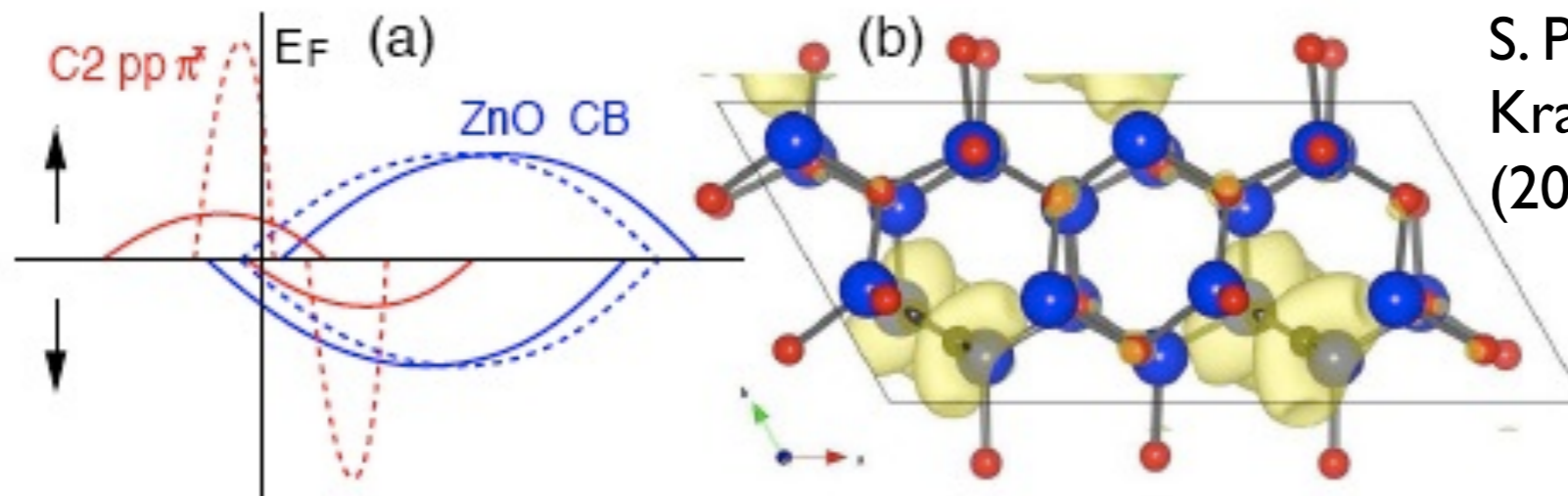


Zn-rich conditions



# Magnetism due to molecular triplet

- formation of a  $(C-C)_O$  out of two  $C_O$  in *n*-type ZnO
- studied by HSE calculation with  $\alpha=0.37$  (band gap 3.3 eV)
- antibonding  $C_2$  triplett becomes resonant with ZnO conduction band
- ferromagnetic interaction favored over antiferromagnetic by 36 meV



H. Wu, A. Stroppa, S. Sakong,  
S. Picozzi, M. Scheffler & P.  
Kratzer, PRL **105**, 267203  
(2010)

# Summary

## GaAs nanopillars:

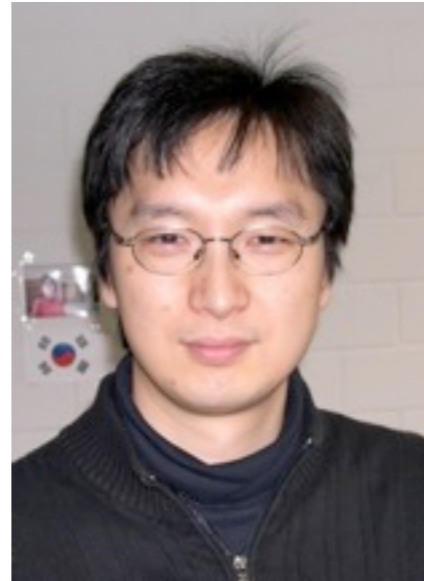
- Au impurities have lower formation energy than intrinsic defects
- In contrast to zincblende, arsenic vacancy diffusion is anisotropic in wurtzite GaAs (slow along  $c$ -axis)
- for a wurtzite nanopillar of 20 nm in diameter,  $V_{As}$  can be annealed in 30 min at  $T=700K$

## C-doped ZnO:

- Under O-rich conditions,  $C_{Zn}$  strongly favored
- $C_O$  stable under Zn-rich conditions, but not useful as acceptor for  $p$ -type doping
- C-induced magnetism in n-type ZnO could be explained by  $V_O+(C-C)_O$  triplets resonating with the conduction band

# Acknowledgements

- Sung Sakong



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- funding

