Tubular Surface Evolutions for Segmentation of Tubular Structures With Applications to the Cingulum Bundle From DW-MRI

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IPAM Brain Imaging Workshop

Joint Work With ...

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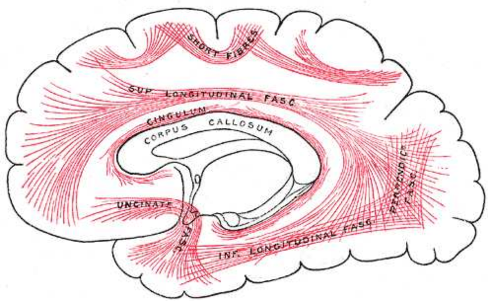
- Schools of ECE and BME, Georgia Tech/Emory

Outline

Outline

The Cingulum Bundle and Other Fiber Pathways



- 5-7mm in diameter fiber bundle: interconnects limbic system
  - fibers are mostly parallel, sometimes intersecting
- forms a “ring-like belt” around the corpus callosum
- Involved with executive control and emotional processing
- May be linked to *schizophrenia*

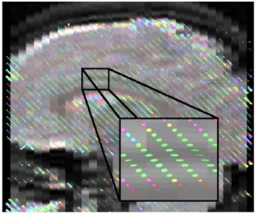
Imaging the Cingulum Bundle in the Brain: DW-MRI

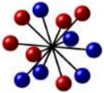
direction of
magnetic field

$$I : \mathbb{R}^3 \times \mathbb{S}^2 \longrightarrow \mathbb{R}^+$$

spatial
locations

image
intensity





$[0{,}0{,}0]^T$ $[1{,}1{,}0]^T$ $[0{,}1{,}1]^T$ $[1{,}0{,}1]^T$ $[0{,}1{,}-1]^T$ $[1{,}-1{,}0]^T$ $[1{,}0{,}1]^T$

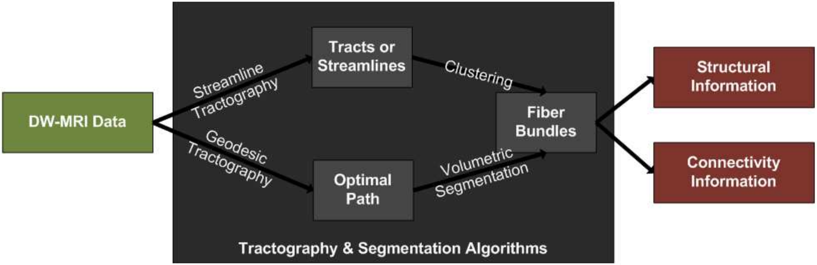
Imaging the Cingulum Bundle in the Brain: DW-MRI

We show visualization of DW-MRI and the cingulum bundle: Movie.

Outline

DW-MRI for Structural and Connectivity Information

Overview of Our Approach for Fiber Bundle Analysis

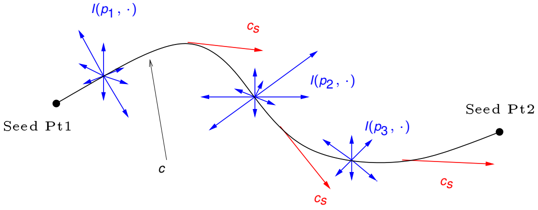


Our Approach: Geodesic Tractography

Detecting A Single Fiber (Melonakos et al., IEEE PAMI 2008)

Given two seed points, find *optimal path* between them.
Let $c : [0, 1] \to \mathbb{R}^3$

$$E(c) = \int_c \overset{\text{based on DW-MRI}}{\psi}(\underset{\text{position}}{c(s)}, \overset{\text{tangent}}{c_s(s)})\, \underset{\text{arclength}}{ds}$$



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based on DW-MRI

tangent

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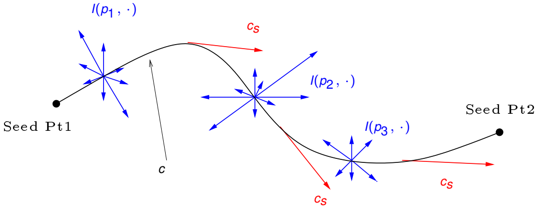
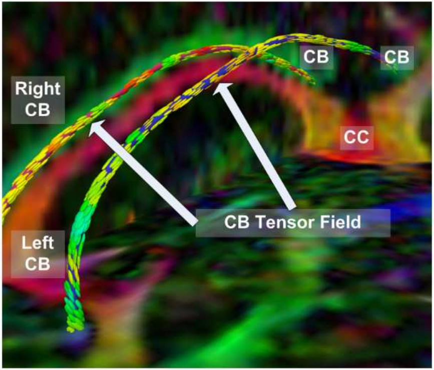
position

arclength



Globally minimize: fast sweeping (Kao et al. 2003) for some ψ

Our Approach: Geodesic Tractography



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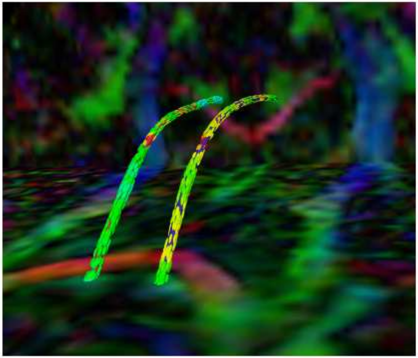

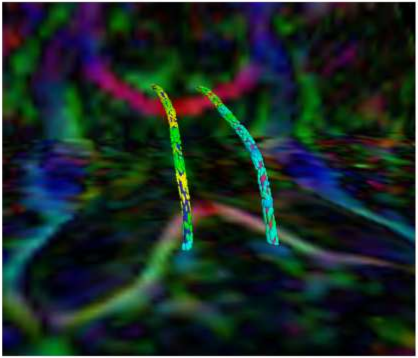

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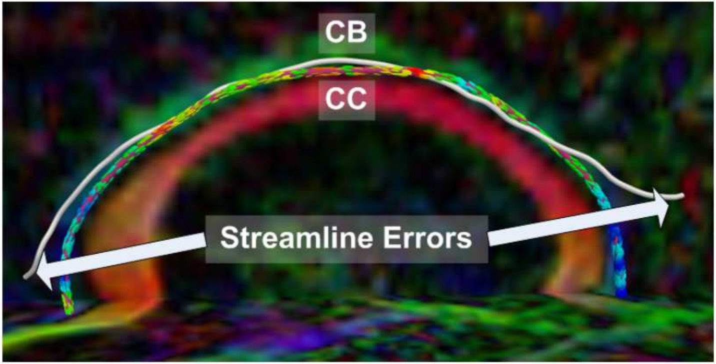


Our Approach: Volumetric Segmentation Method

Volumetric Surface Methods Applied to DW-MRI

Surface Obtained From DT-MRI

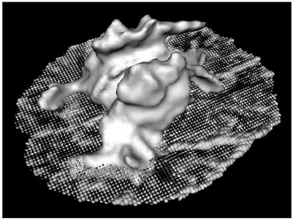
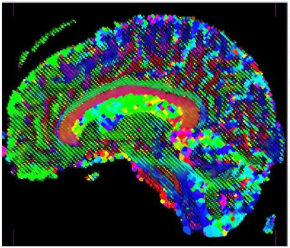


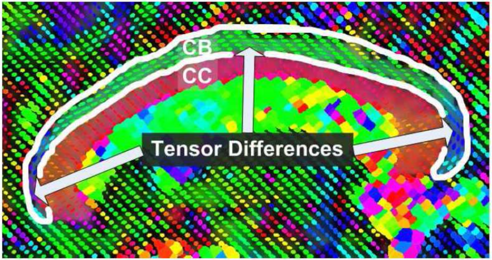
Image from Lenglet et al., Trans. Med. Imaging, 2006

- DTI Volumetric Segmentation:
  - Region-Based Methods (e.g. Lenglet et al., Wang and Vemuri)
  - Edge-Based Method (Melonakos et al.)
- We Tailor Above Methods to Fiber Bundles
  - *Shape Prior Needed*
  - Challenge: Non-homogeneity of statistics of the cingulum bundle

Non-Uniform Statistics of Cingulum Bundle

Sagittal Slice of DT-MRI of a Brain





CB = Cingulum bundle
CC = Corpus Callosum

Outline

Modeling the Cingulum Bundle as a Tubular Surface

Why Model the Cingulum Bundle as a Tubular Surface?

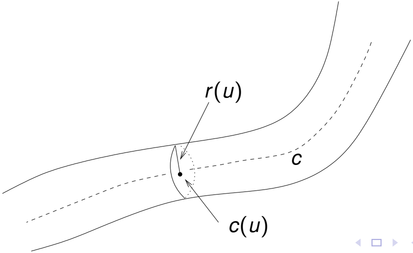
- **Natural Shape Prior**:
  - Cingulum Bundle is approximately tubular
  - DW-MRI is noisy and filled with irrelevant features; cingulum bundle hard to segment without prior
- **Significant Dimension Reduction**:
  - Segmentation reduced from detecting a *surface* to a *curve*
- **Statistical Shape Analysis of Tubular Surfaces is Easier**
  - Main point of segmenting the cingulum bundle: *population studies*, where statistical analysis must be performed to compare controls and disease cases

Modeling the Cingulum Bundle as a Tubular Surface

- Given *center-line*: $c : [0, 1] \to \mathbb{R}^3$, and *radius function*: $r : [0, 1] \to \mathbb{R}^+$
- Define the *tubular surface*, $S : \mathbb{S}^1 \times [0, 1] \to \mathbb{R}^3$, as

$$S(\theta, u) = c(u) + r(u)[n_1(u) \cos\theta + n_2(u) \sin\theta]$$

  where $n_1, n_2 : [0, 1] \to \mathbb{R}^3$ are normals to the curve c: orthonormal, smooth, and $c'(u) \cdot n_i(u) = 0$
- **Tubular Surface Identified With a 4-D Curve**: $S \Leftrightarrow \tilde{c} = (c, r) \in \mathbb{R}^4$



Outline

Segmentation Algorithm: Variational Approach

- Formulate energies on $4D$ curves, $\tilde{c}$, ($S \Leftrightarrow \tilde{c}$)
- *Weighted length* energies:

$$E(\tilde{c}) = \int_{\tilde{c}} \underbrace{\Psi}_{\text{Local Cost}}(\overbrace{\tilde{c}(\tilde{s})}^{\text{Position \& Radius}}, \overbrace{\frac{c'(\tilde{s})}{|c'(\tilde{s})|}}^{\textit{Tangent}\text{ of Centerline}})\, \underbrace{d\tilde{s}}_{\text{4D curve arclength}}, \quad \tilde{c} = (c, r)$$

Segmentation Algorithm: Variational Approach

- Formulate energies on $4D$ curves, $\tilde{c}$, ($S \Leftrightarrow \tilde{c}$)
- *Weighted length* energies:

Position & Radius

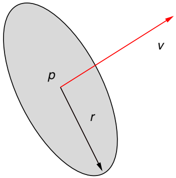
Tangent of Centerline

$$E(\tilde{c}) = \int_{\tilde{c}} \Psi\left(\tilde{c}(\tilde{s}), \frac{c'(\tilde{s})}{|c'(\tilde{s})|}\right) \mathrm{d}\tilde{s}, \quad \tilde{c} = (c, r)$$

Local Cost

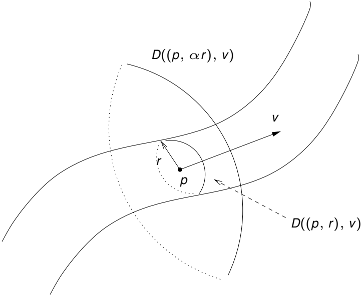
4D curve arclength

- $\Psi(\tilde{p} = (p, r), v)$ to incorporate statistics of DW-MRI *local* to $\tilde{p}, v$
- Rather than one set of global statistics



Segmentation Algorithm: Variational Approach

Example 1: Choice of Ψ



- Let I be DW-MRI
- Ψ_1 minimized when *mean* of I inside disc, $\mu_{D(p,r,v)}$, *maximally* different from *mean* of I outside, e.g.

$$\Psi_1(\tilde{p}, v) = \frac{1}{1 + \left\|\mu_{D(\tilde{p},v)} - \mu_{D((p,\alpha r),v)\setminus D(\tilde{p},v)}\right\|^2}$$

Segmentation Algorithm: Variational Approach

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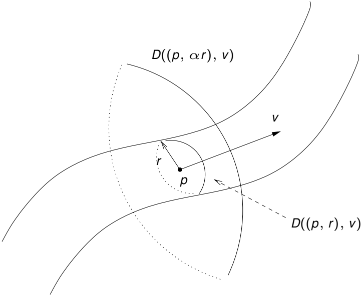

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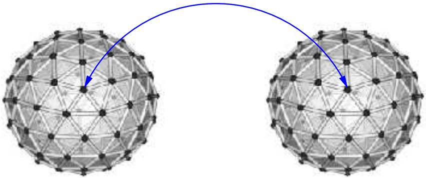
Need to define mean and norm for pixel-wise DW-MRI data

Segmentation Algorithm: Variational Approach

Defining Mean and Norm for DW-MRI (Easier than DT-MRI)

- DW-MRI: $I : \mathbb{R}^3 \times \mathbb{S}^2 \to \mathbb{R}^+$
- DW-MRI sampled uniformly directionally at each pixel, $p \in \mathbb{R}^3$
- Addition: add *corresponding* values at directions



$I(p_1, \cdot)$ $I(p_2, \cdot)$

Segmentation Algorithm: Variational Approach

Defining Mean and Norm for DW-MRI (Easier than DT-MRI)

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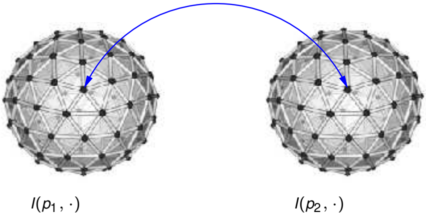
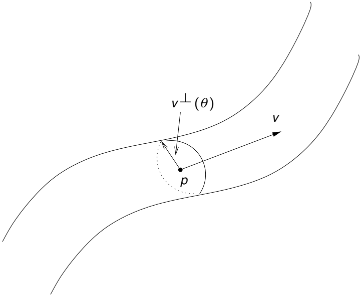


Given $f_1, \cdot, f_n : \mathbb{S}^2 \to \mathbb{R}^+$ (DW-MRI at n different spatial locations):

$$\text{mean of } f_1, \dots, f_n(v) := \frac{1}{n} \sum_{i=1}^{N} f_i(v), \qquad \|f_i\|^2 = \int_{\mathbb{S}^2} |f_i(v)|^2 \, dS(v)$$

Segmentation Algorithm: Variational Approach

Example 2: Choice of Ψ



$$\Psi_2(p,r,v) = r\int_0^{2\pi} \phi(p + rv^{\perp}(\theta))\,\mathrm{d}\theta$$

$$v^{\perp}(\theta) = n_1\cos\theta + n_2\sin\theta$$

$$\phi(x) = \frac{1}{|B(x,R)|}\int_{B(x,R)} \|I(y,\cdot) - \mu_{B(x,R)}(\cdot)\|^2\,\mathrm{d}y$$

- $B(x,R)$ is a ball
- ϕ is an “edge-detector”
- Corresponding energy to Ψ_2 is related to a weighted surface area.

Energy Optimization: Gradient Descent/Ascent

$$E(\tilde{c}) = \int_{\tilde{c}} \Psi(\tilde{c}(\tilde{s}), \tilde{c}_{\tilde{s}}(\tilde{s}))\, \mathrm{d}\tilde{s}$$

- Why Gradient Ascent/Descent?
  - Global techniques (e.g. minimal paths) do not apply to *direction-based* energies
  - Not interested in global optimum: Ψ_2
- Gradient flow: $\partial_t \tilde{c} = \pm \nabla E(\tilde{c})$

Gradient Flow of Tubular Energy

Given the energy

$$E(\tilde{c}) = \int_{\tilde{c}} \Psi(\tilde{c}(\tilde{s}), \tilde{c}_{\tilde{s}}(\tilde{s}))\, d\tilde{s}$$

we get the following gradient flow:

2nd covariant derivative

$$\tilde{c}_t = \pm(\tilde{c}_{\tilde{s}} \cdot \Psi_{v\tilde{p}} - \Psi_{\tilde{p}})^{\perp} \pm (\Psi + \Psi_{vv})\tilde{c}_{\tilde{s}\tilde{s}}$$

projection onto $c_{\tilde{s}}^{\perp}$

curvature vector

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curvature vector

Well-posedness: $\Psi + \Psi_{vv}$ must be positive definite (negative definite for ascent flow)

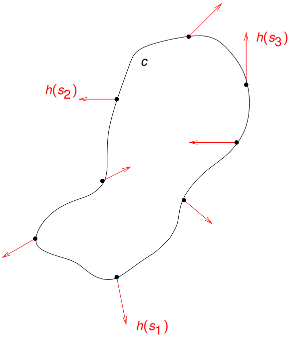
- For Ψ_1 and Ψ_2, this condition is NOT satisfied
- Flow is **ILL-POSED**

Calculating Gradient Flows of Geometric Energies

1. Compute

$$\underbrace{dE(c) \cdot h = \frac{d}{dt} E(c + th)\Big|_{t=0}}_{\text{change in } E \text{ in direction } h}$$

for generic c and h.



$c : S^1 \to \mathbb{R}^2$, and $h : S^1 \to \mathbb{R}^2$ is a vector field on c (*i.e.*, a perturbation or deformation of c)

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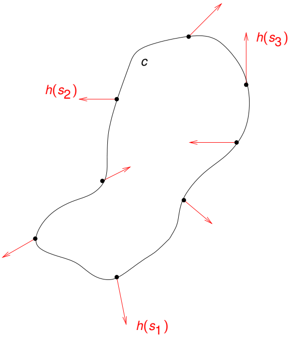

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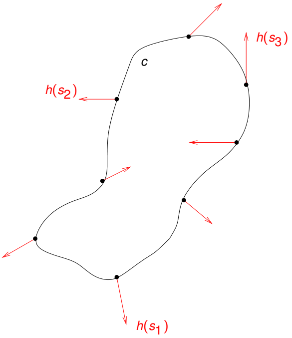
3. v is the *gradient*: direction which maximizes E *fastest*.



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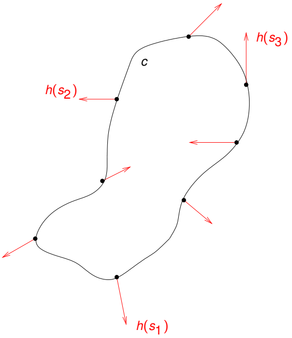
3. v is the *gradient*: direction which maximizes E *fastest*.
4. *Gradient descent flow*: $\partial_t C = -v(C)$.



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Traditional Norm That Led To Ill-posed Flows: $\mathbb{L}^2$

Norm on deformations assumed in deformable model literature:
Geometric $\mathbb{L}^2$-type norm

deformation of c

$$\underbrace{\|h\|^2_{c,\mathbb{L}^2}}_{\text{curve}} := \int_c |h(s)|^2 \underbrace{ds}_{\text{arclength}}$$

Gradient Depends on *Norm* on Deformations of Curve

Proposition

The gradient $\nabla E(c)$ is the vector in $T_c M$ that satisfies (if $dE(c) \neq 0$)

$$\frac{dE(c) \cdot (\nabla E(c))}{\|\nabla E(c)\|_c} = \sup_{h \in T_c M \setminus \{0\}} \frac{|\, dE(c) \cdot h|}{\|h\|_c}.$$

- Thus, gradient is the most *efficient* perturbation, *i.e.,* maximizes

$$\frac{\text{change in energy by moving in direction } h}{\text{cost of moving in direction } h}$$

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- By choosing different $\|\cdot\|_c$ than we obtain a different path to minimize E **without changing** E.

Proposed Norm: Sobolev-type Norm

Geometric Sobolev-type norm (open curves)

length of curve

deformation of c

$$\underbrace{\|h\|^2_{c,Sob}}_{\text{curve}} = \overbrace{L}^{\text{length of curve}} \int_c |\underbrace{D_s}_{\text{derivative wrt arclength}} \overbrace{h(s)}^{\text{deformation of } c}|^2 \, \underbrace{ds}_{\text{arclength}}$$

curve

derivative wrt arclength

arclength

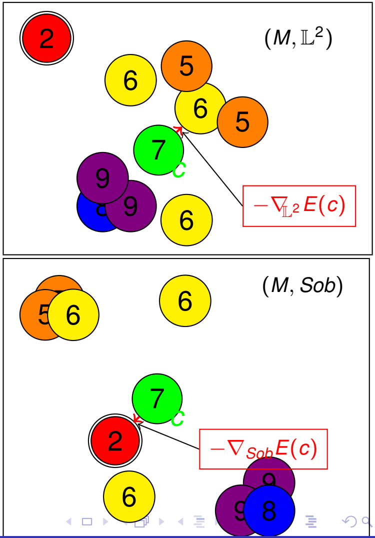
(Sundaramoorthi et al., Int. J. Computer Vision 2007, 2008, IEEE Trans. PAMI 2008 and separately Charpiat et al. Int. J. Computer Vision 2007)

Original Motivation for Sobolev Norms

Energy, E, **is not** changing; the scale that is used to measure cost (length) of a perturbation **is** changing.

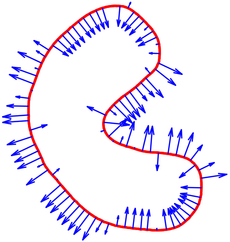
Diagram:

- Represents local neighborhood of curve $c \in M$.
- Spatial scale is relative to “distance” in M measured through $\|\cdot\|_{H^0}$ or $\|\cdot\|_{H^1}$.
- 6 means energy $E = 6$ at particular point (curve).

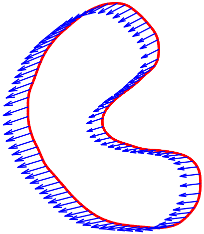


Original Motivation for Sobolev Norms

Sobolev Norms Favor Coarse Scale Motions



vs.



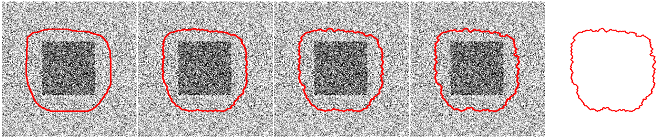
$$\|h\|^2_{c,H^0} := \int_c |h(s)|^2 \, ds$$

$$\|h\|^2_{c,Sob} = L \int_c |D_s h(s)|^2 \, ds$$

Original Motivation: Sobolev Norms Robust to Noise and Local Minima

Region-Based Segmentation: (E = Chan-Vese energy, TIP 2001)

$-\nabla_{\mathbb{L}^2} E + \alpha \kappa \mathcal{N}$



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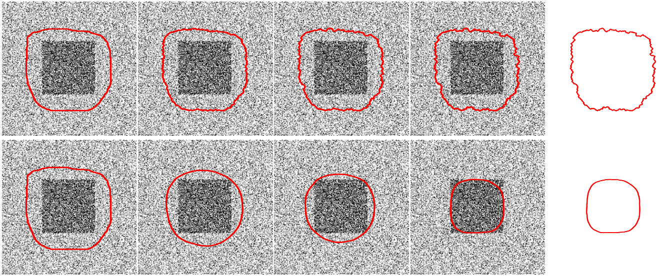
Region-Based Segmentation: ($E =$ Chan-Vese energy, TIP 2001)

$$-\nabla_{\mathbb{L}^2} E + \alpha \kappa \mathcal{N}$$

$$-\nabla_{\mathbb{L}^2} E + \beta \kappa \mathcal{N}$$

$$(\beta >> \alpha)$$



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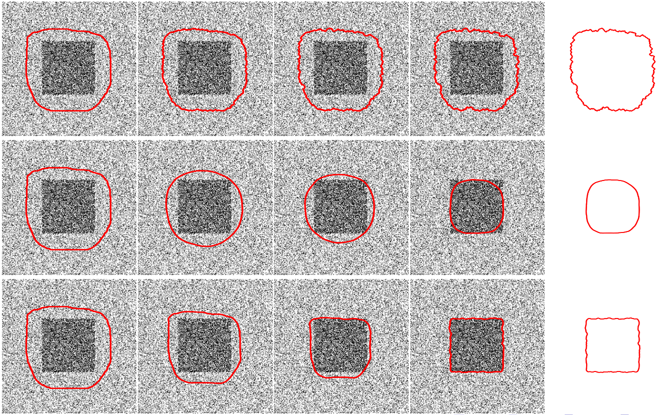
$(\beta >> \alpha)$

$-\nabla_{Sob} E$



Sobolev Stabilizes $\mathbb{L}^2$ Flows Involving Length

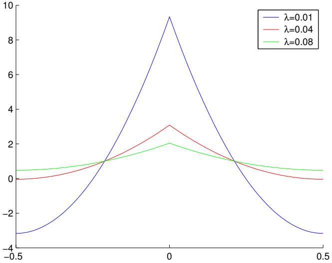
Let E be an energy, one can show that

$$\nabla_{Sob} E = K * \nabla_{\mathbb{L}^2} E$$

important property:

$$K''(s) = \frac{1}{L^2}\left(\frac{1}{L} - \delta(s)\right)$$

Plot of K:



If $E(c) =$ length of c, then $\nabla_{\mathbb{L}^2} E(c) = -c_{ss}$ and

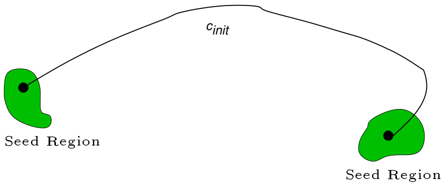
$$\nabla_{Sob} E(c) = K * (-c_{ss}) = \frac{c - \overline{c}}{L}$$

($\overline{c}$ is the centroid of c).

Fiber Bundle Extraction: Two Step Approach

Given: Two seed regions (beginning and end of fiber bundle)

1. Find an open curve, $c_{init} : [0, 1] \to \mathbb{R}^3$ in fiber bundle.
   - Rough estimation of *a curve* in bundle required (e.g. geodesic tractography)



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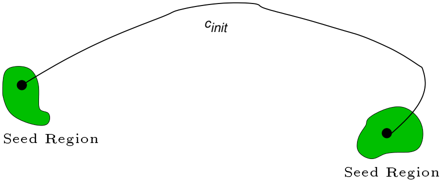

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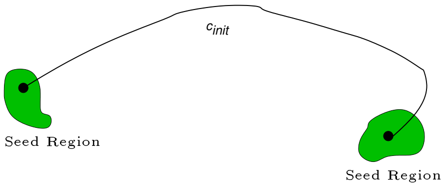

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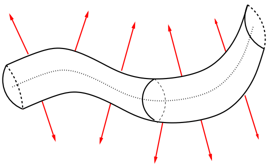
3. Optimize tubular surface energy

Tubular Energy Optimization Procedure

Iterate:

1. Evolve interior of 4D curve with fixed endpoints

$$\begin{aligned}\tilde{c}_t &= \pm\nabla_{\mathrm{Sob}} E(\tilde{c}) \\ &= \pm K(\Psi_{\tilde{p}}) \pm \partial_{\hat{s}} K\left(\widehat{\Psi_v \sqrt{1 + (r_{\tilde{s}}/|c_{\tilde{s}}|)^2}} + \Psi \tilde{c}_{\tilde{s}}\right),\end{aligned}$$



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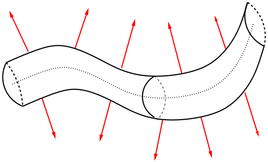
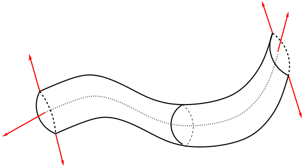


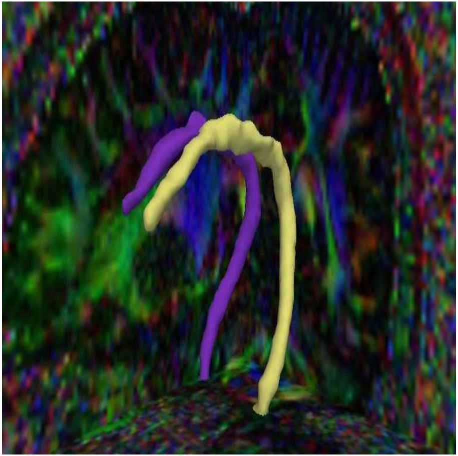
2. *Evolve the endpoints* of 4D curve (valid for e.g. Ψ_2)

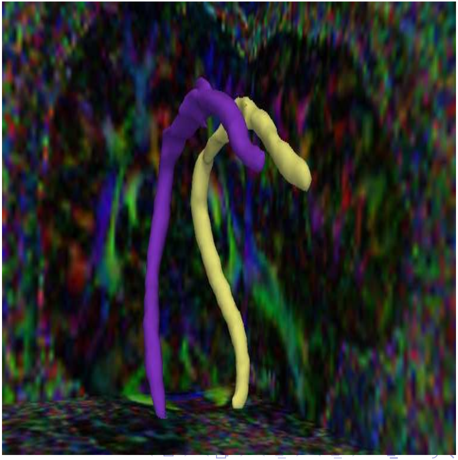
$$\tilde{c}_t(0) = \mp \widehat{\Psi_v} \sqrt{1 + \left(\frac{r_{\tilde{s}}}{|c_{\tilde{s}}|} \right)^2} \mp \Psi \tilde{c}_{\tilde{s}}$$
$$\tilde{c}_t(1) = \pm \widehat{\Psi_v} \sqrt{1 + \left(\frac{r_{\tilde{s}}}{|c_{\tilde{s}}|} \right)^2} \pm \Psi \tilde{c}_{\tilde{s}}$$



Result of the Segmentation Method

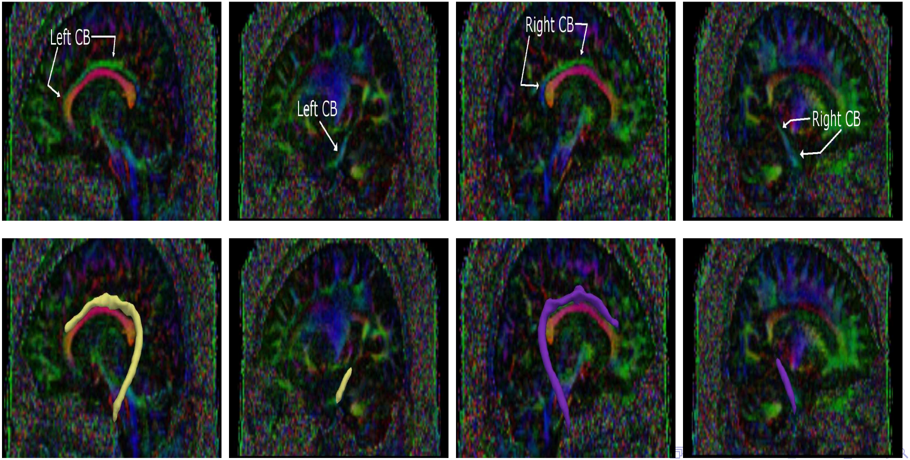
3D Views of Result





Some Results of the Segmentation Method

A Slice-Wise View of Result



Summary

- Method for fiber bundle extraction:
  - E.g. Geodesic tractography for *single* fiber
  - Initialization for volumetric segmentation
- Modeled certain fiber bundles as tubular region
- Future Work: Statistical analysis of cingulum bundles and functions defined on the cingulum bundle
  - Tubular model makes it easy

Thank you.