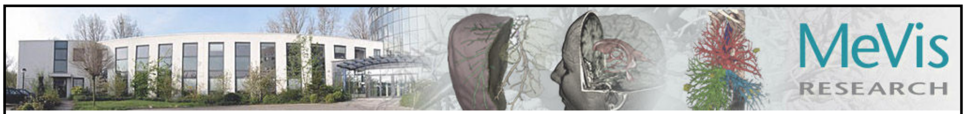

Adaptive Multivariate Modeling

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1 MeVis RESEARCH

Contents

- Linear models for fMRI analysis
- Constrained modeling
- Spatial basis functions
- Multivariate modeling
- Canonical correlation analysis (CCA)
- Results

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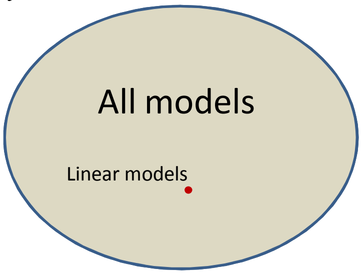
Linear models

$$y = kx + m$$



- Simple
- Easy to understand
- Predictable – no surprises
- Computationally efficient
- Theoretical results
- Bounds on non-linear performance
- "All models are wrong, some models are useful."
  George Box

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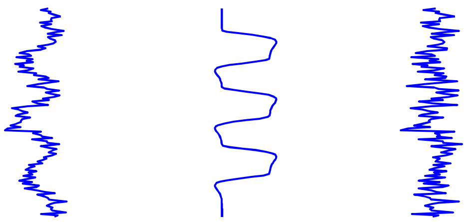
Linear models in fMRI

$$y(t) = \beta x(t) + \varepsilon(t)$$



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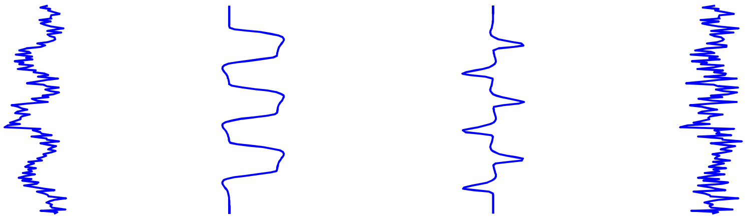
Linear models in fMRI

$$y(t) = \beta_1 x_1(t) + \beta_2 x_2(t) + \varepsilon(t)$$



Taylor expansion: $x(t+\Delta t) \approx x(t) + \Delta t\,\dot{x}(t)$

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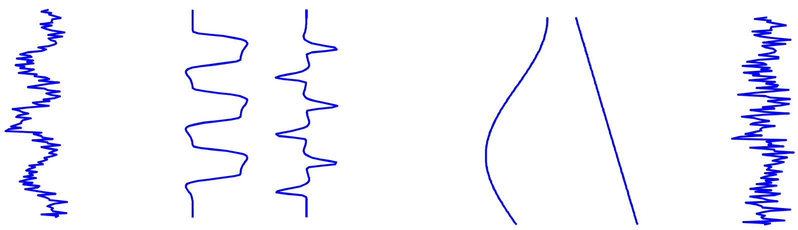
Linear models in fMRI

$$y(t) = \sum \beta_i x_i(t) + \sum \theta_j s_j(t) + \varepsilon(t)$$



$$\mathbf{y} = \mathbf{X\beta} + \mathbf{S\theta} + \mathbf{\varepsilon}$$

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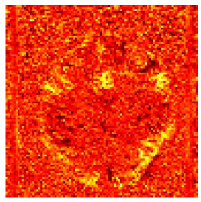
BOLD detection

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{S}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$

$$\lambda = \frac{\text{BOLD response energy}}{\text{Noise energy}}$$



t-value, *F*-value, likelihood ratio, correlation coefficient,…

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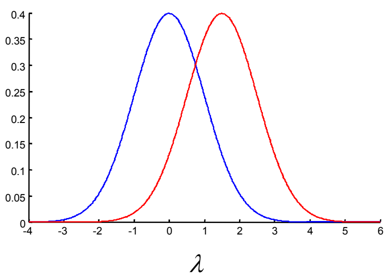
Detection performance

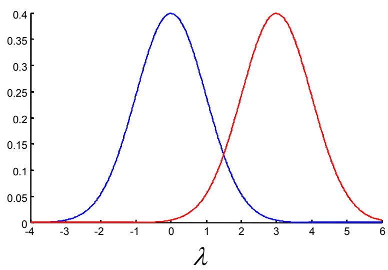
$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{S}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$

$$\lambda = \frac{\text{BOLD response energy}}{\text{Noise energy}}$$

$p(\lambda \mid \text{Inactive voxel})$

$p(\lambda \mid \text{Active voxel})$





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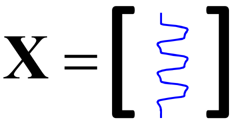
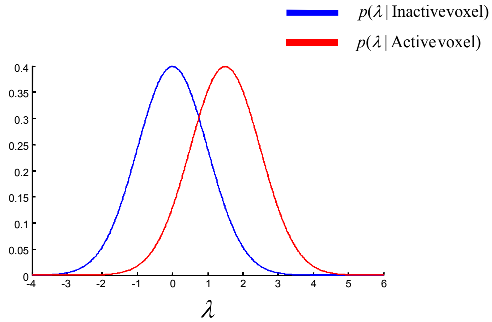
Model optimization

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{S}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$





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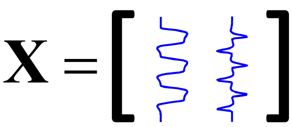
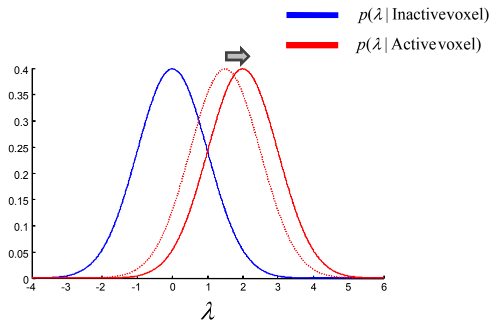
Model optimization

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{S}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$





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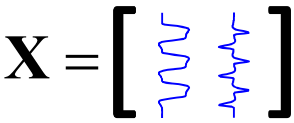
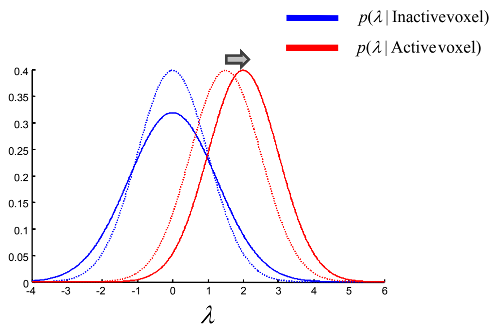
Model optimization

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{S}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$





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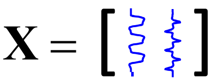
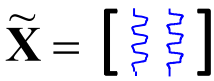
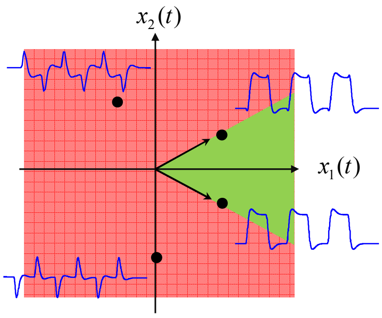
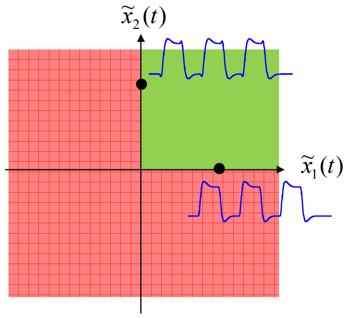
Constrained modeling



$$\tilde{x}_1(t) = x_1(t) - \alpha x_2(t)$$
$$\tilde{x}_2(t) = x_1(t) + \alpha x_2(t)$$







$$x(t + \Delta t) \approx x(t) + \Delta t\, \dot{x}(t)$$

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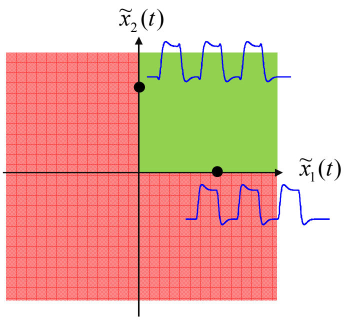
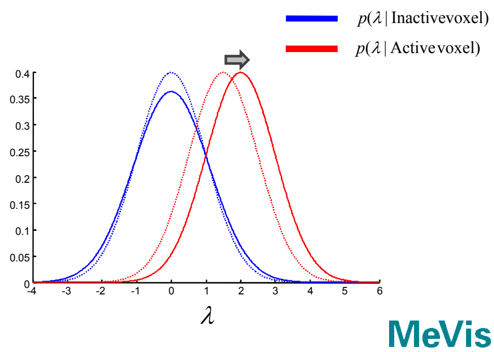
Constrained fitting

$$\mathbf{y} = \tilde{\mathbf{X}}\boldsymbol{\beta} + \mathbf{S}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$

$$\text{subject to } \boldsymbol{\beta} \geq \mathbf{0}$$

Lawson & Hanson (1974).
Solving Least Squares Problems





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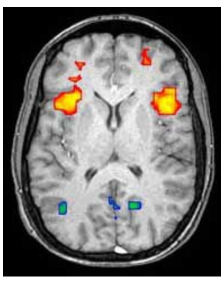
Linear spatial modeling?

Linear temporal modeling

$$x(t + \Delta t) \approx x(t) + \Delta t\, \dot{x}(t)$$

Spatial modeling



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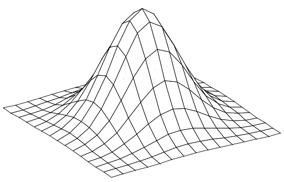

Linear models

- Simple
- Easy to understand
- Predictable – no surprises
- Computationally efficient
- Theoretical results (optimality)
- Bounds on non-linear performance

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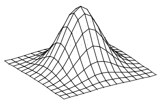
Spatial basis functions

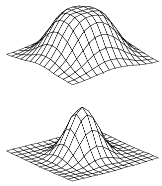
Gauss kernel

$$g(\mathbf{x},\sigma^2) \sim \exp\left(-\frac{\|\mathbf{x}\|^2}{2\sigma^2}\right)$$



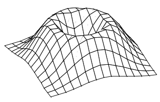
Taylor expansion

$$g(\mathbf{x},\sigma^2+\Delta\sigma^2) \approx g(\mathbf{x},\sigma^2) + \Delta\sigma^2 \frac{\partial g(\mathbf{x},\sigma^2)}{\partial\sigma^2}$$



Partial derivative

$$\frac{\partial g(\mathbf{x},\sigma^2)}{\partial\sigma^2} \sim \|\mathbf{x}\|^2 g(\mathbf{x},\sigma^2)$$



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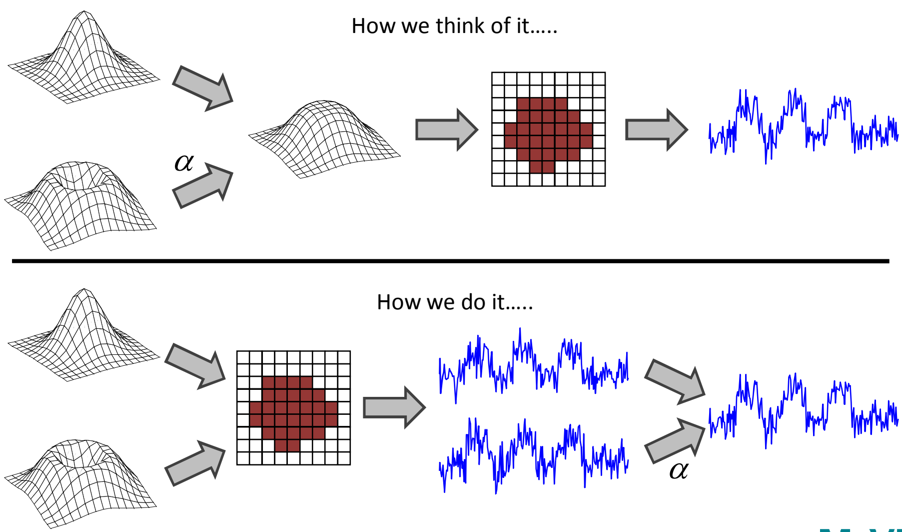
Spatial basis functions



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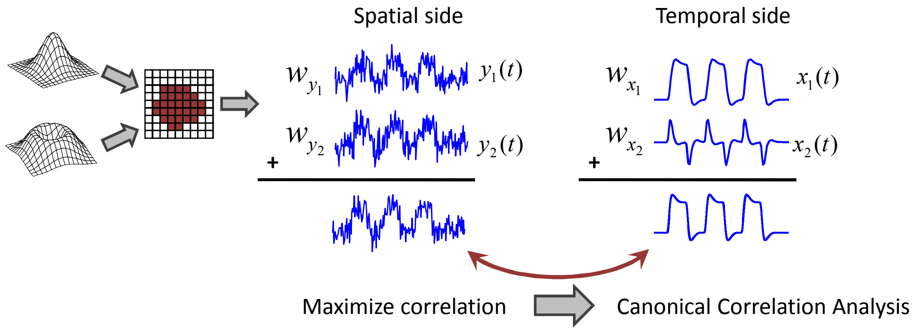
Multivariate modeling

Univariate model: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$

Multivariate model:



Repeat for every voxel!

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Canonical Correlation Analysis (CCA)

Harold Hotelling, 1936

Problem: Find vectors $\mathbf{w}_x$ and $\mathbf{w}_y$ so that

$\mathbf{w}_x^T\mathbf{x}(t) = w_{x_1}x_1(t) + \ldots + w_{x_m}x_m(t)$ and $\mathbf{w}_y^T\mathbf{y}(t) = w_{y_1}y_1(t) + \ldots + w_{y_n}y_n(t)$

are maximally correlated.

Solution:

$$\mathbf{C}_{xx}^{-1}\mathbf{C}_{xy}\mathbf{C}_{yy}^{-1}\mathbf{C}_{yx}\mathbf{w}_x = \rho^2\mathbf{w}_x$$
$$\mathbf{C}_{yy}^{-1}\mathbf{C}_{yx}\mathbf{C}_{xx}^{-1}\mathbf{C}_{xy}\mathbf{w}_y = \rho^2\mathbf{w}_y$$

$$\mathbf{C}_{xx} \sim \mathbf{x}\mathbf{x}^T$$
$$\mathbf{C}_{yy} \sim \mathbf{y}\mathbf{y}^T$$
$$\mathbf{C}_{xy} = \mathbf{C}_{yx}^T \sim \mathbf{x}\mathbf{y}^T$$

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Constrained/Restricted CCA

Das and Sen, 1994

Problem: Find vectors $\mathbf{w}_\mathbf{x} \geq 0$ and $\mathbf{w}_\mathbf{y} \geq 0$ so that

$$\mathbf{w}_\mathbf{x}^T \mathbf{x}(t) = w_{x_1} x_1(t) + \ldots + w_{x_m} x_m(t) \quad \text{and} \quad \mathbf{w}_\mathbf{y}^T \mathbf{y}(t) = w_{y_1} y_1(t) + \ldots + w_{y_n} y_n(t)$$

are maximally correlated.

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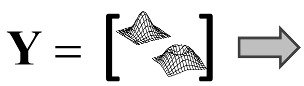
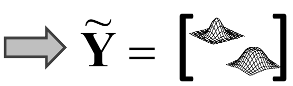
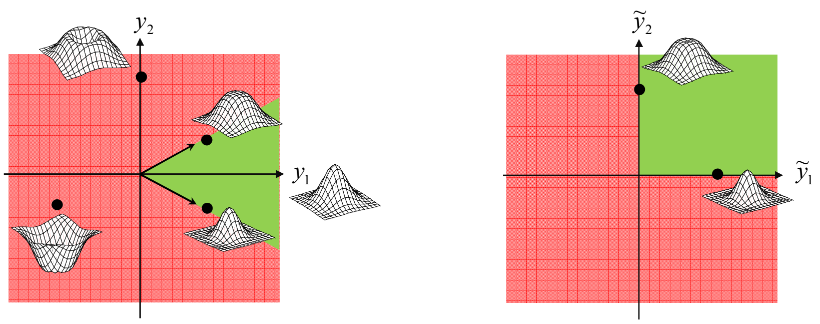
Constrained spatial modeling



$$\tilde{y}_1 = y_1 - \alpha y_2$$
$$\tilde{y}_2 = y_1 + \alpha y_2$$





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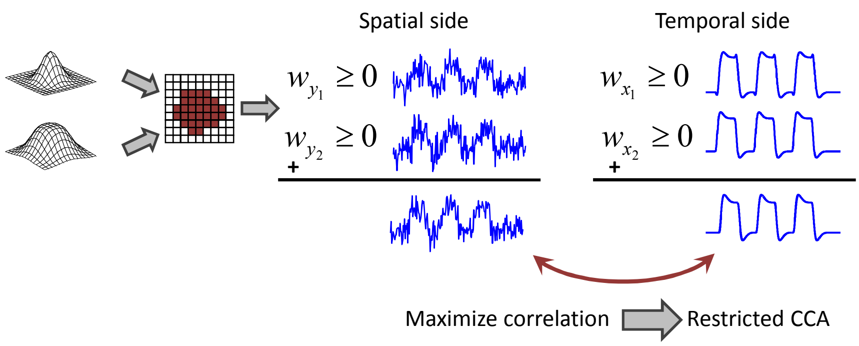
Constrained multivariate modeling



Repeat for every voxel!

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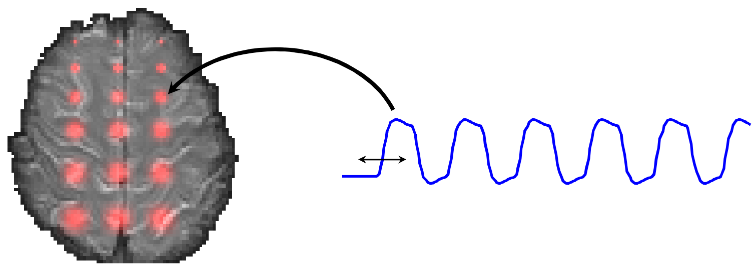
Results – Synthetic data



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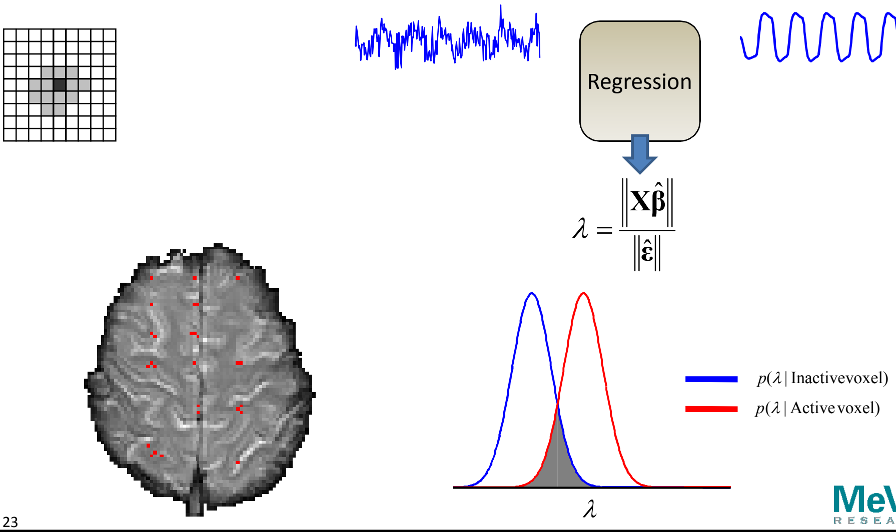
Univariate model



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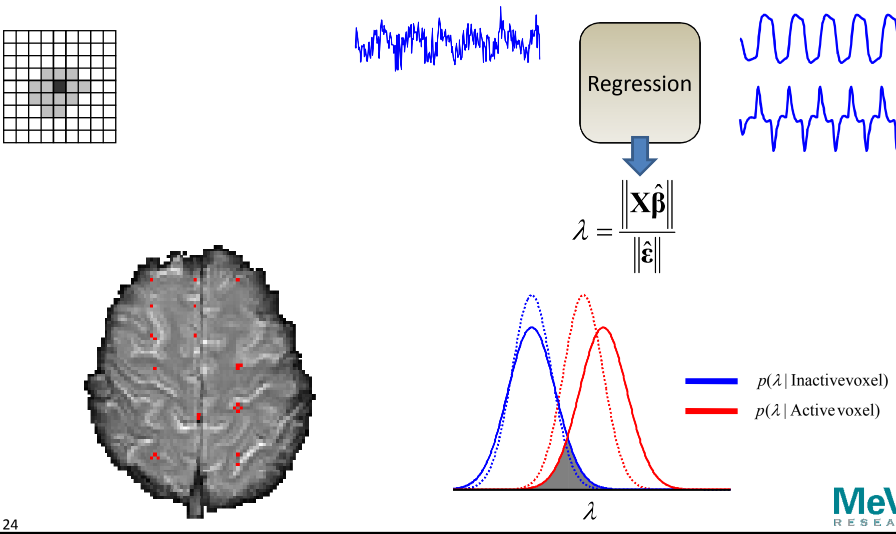
Univariate model (GLM)



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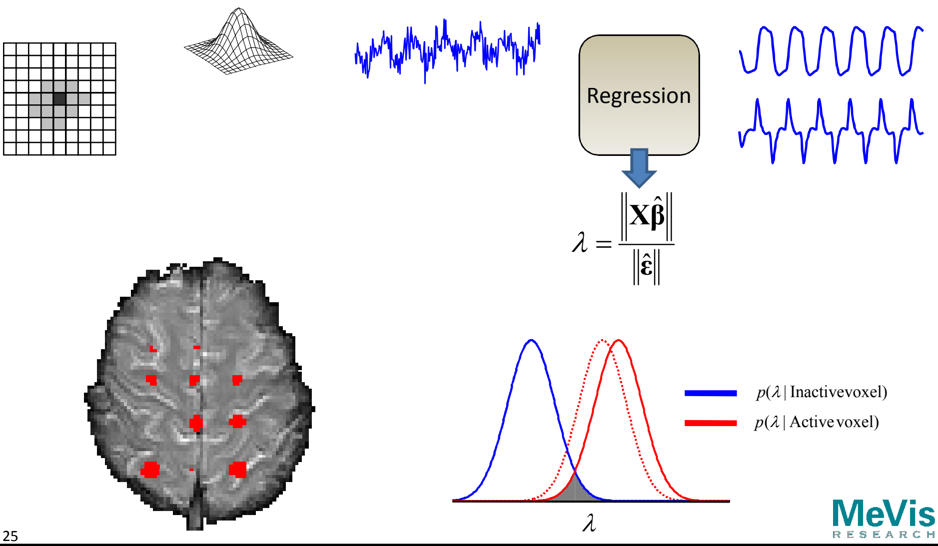
Univariate model (GLM) + Smoothing



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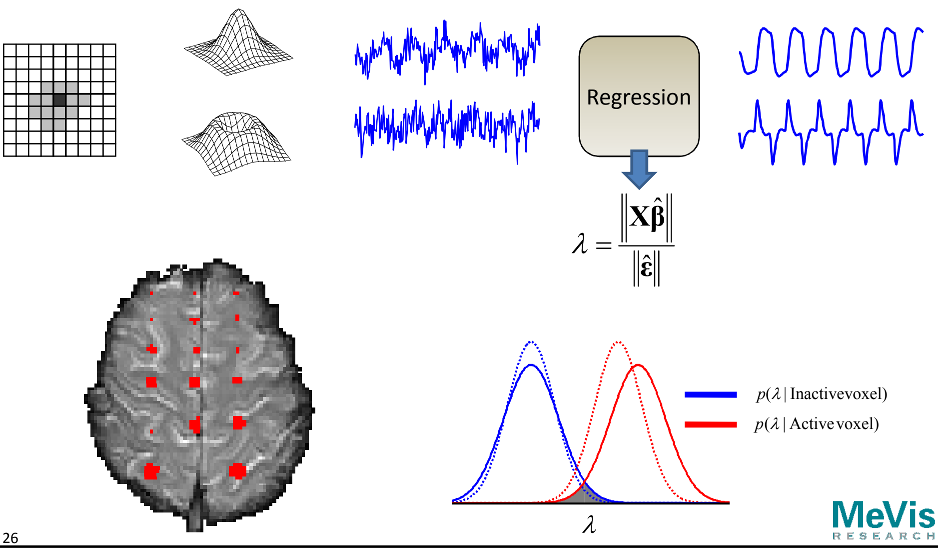
Multivariate modeling



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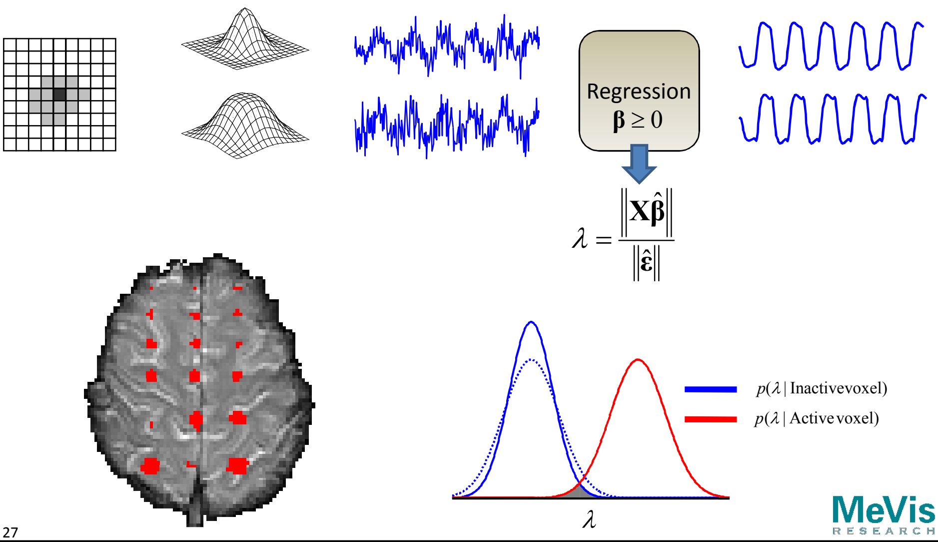
Constrained multivariate modeling



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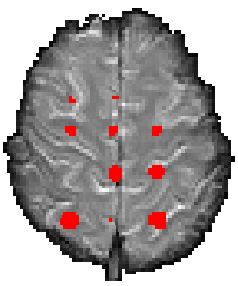
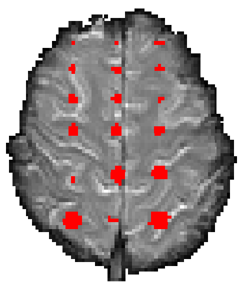
Comparison

GLM + smoothing



Constrained multivariate modeling



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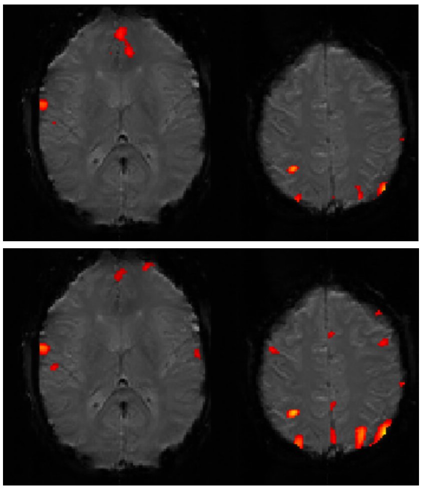
Results – fMRI data

GLM + smoothing

Constrained multivariate modeling



$p = 10^{-5}$

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Summary

- Constrained linear modeling
- Spatial linear modeling
- Constrained Canonical Correlation Analysis

Mass-univariate analysis

↓

Constrained mass-multivariate analysis

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Literature

Constrained & multivariate modeling, Canonical Correlation Analysis
O. Friman (2003)
Adaptive Analysis of Functional MRI Data
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Bayesian approach to constrained modeling
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NeuroImage, 21(4):1748-1761

Inference
K. Worsley & J. Taylor (2008).
Non-negative least-squares random field theory
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