Dynamic Programming Applications in Defining Cortical Manifold Boundaries

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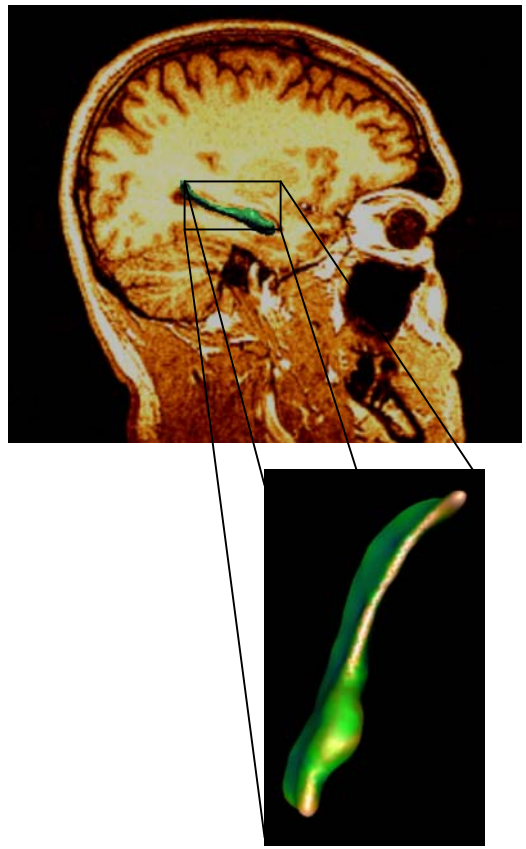
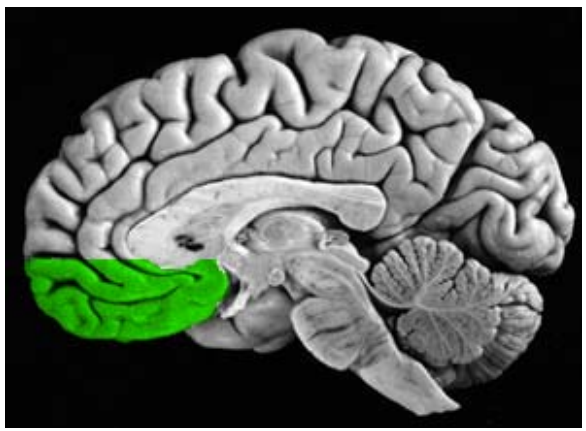
Geometries of the Brain

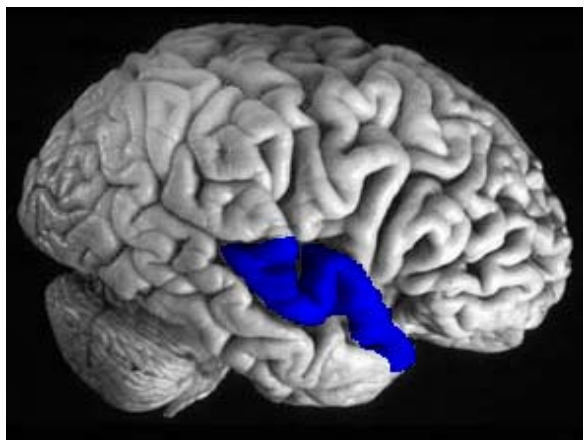
Cingulate gyrus
Conte Center (WUSTL)
Alzheimer's Center (WUSTL)

Occipital cortex
Barta & Yantis (JHU)





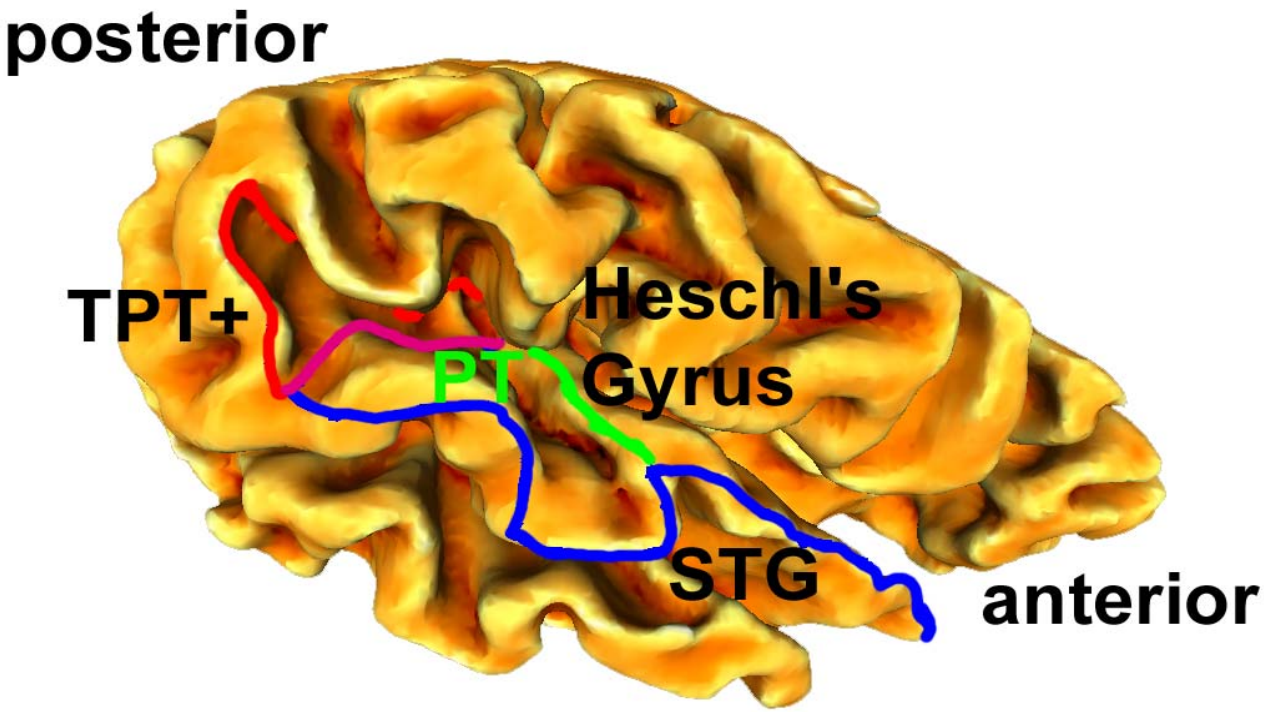
Medial prefrontal cortex
Depression
Botteron, Todd (WUSTL)



Temporal lobe
Barta, Boatman (JHU)
Pearlson (Yale)

Hippocampus
Conte Center (WUSTL)
Csernansky

A Reconstructed gray/white surface of a temporal lobe



- surface is a triangulated graph of edges and nodes
- curvature can be computed at each node
- define principal curves: sulcal and gyral

Ratnanather et al, 2003
Khaneja et al, 1998
Bakircioglu et al, 1999
Bartesaghi and Sapiro, 2001

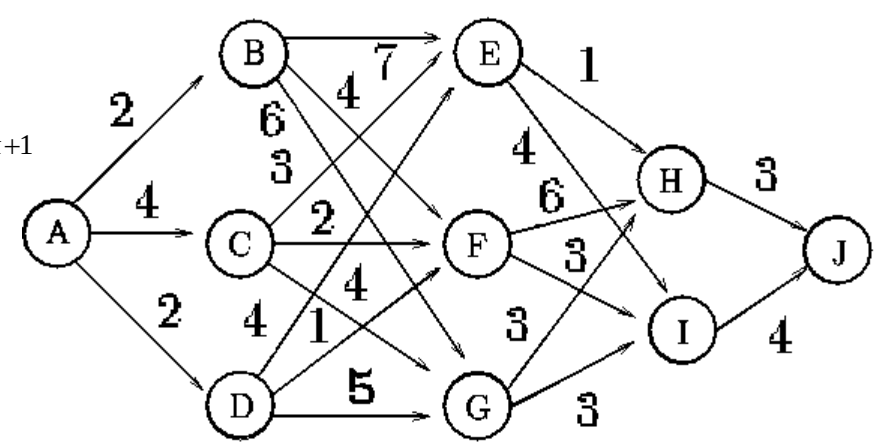
Dynamic programming: a control optimization problem used in industry e.g. optimizing costs in shipping or employee travelling

- developed to solve sequential, or multi-stage, decision problems
- optimize a cost function usually quadratic, sometimes subject to constraints
- recursive calculations from final stage to initial stage ensures optimal solution can be obtained

let $c^k(x_k, x_{k+1})$ be the cost from state x_k to x_{k+1}

aim to find path π that minimizes

$$J^{\pi}(x_o) = \sum_{k=0}^{k=N-1} c^k(x_k, x_{k+1})$$



Bertsekas (1987, 2000)

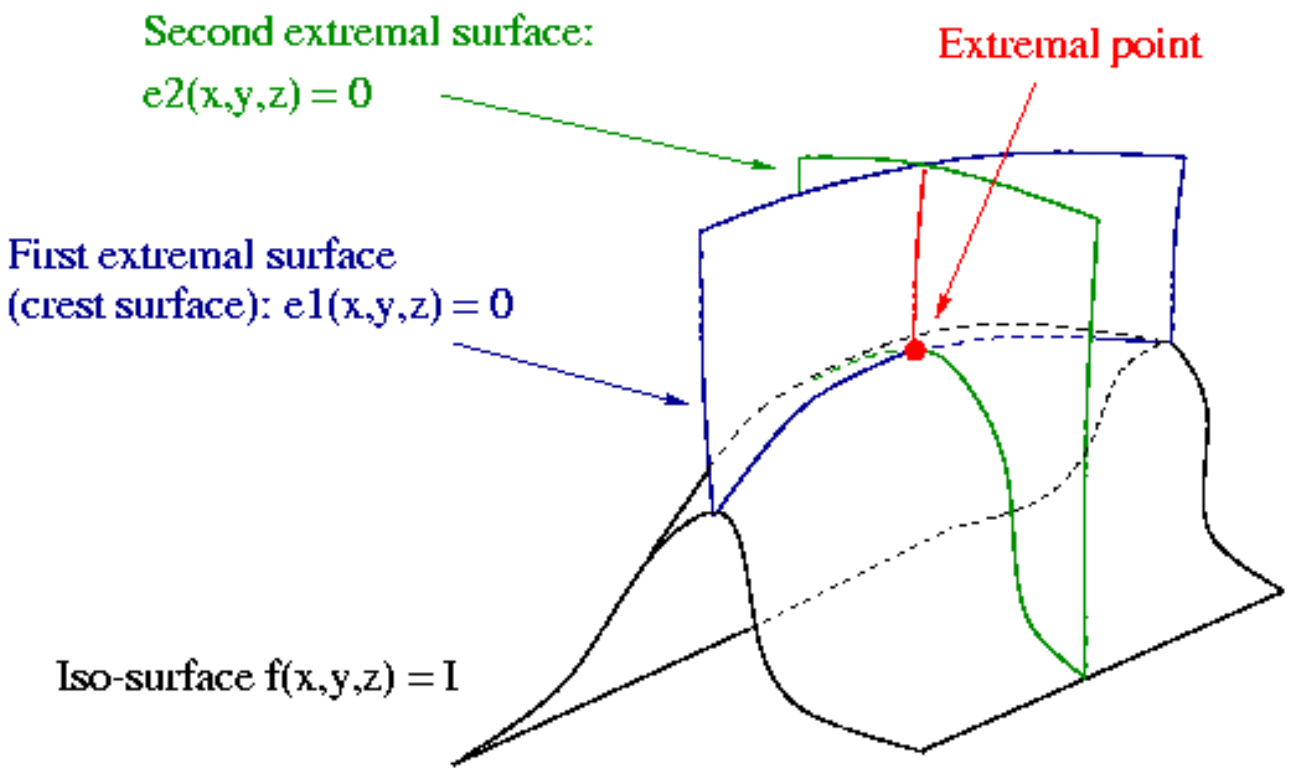
Crest lines: fundus beds are the deepest valleys of sulci

•Fundus beds are ridge curves or crest lines – locus of points of maximal positive curvature

• think of creating rivers from point A to B in a mountainuous terrain such as the Rockies

Crest Lines and Extremal Points

$f(x,y,z) = I$ $\quad e_1 = \nabla k_1 . t_1 = 0$ $\quad e_2 = \nabla k_2 . t_2 = 0$



Epidaure Lab – 1st group to classify gyri and sulci: see Subsol (1998)

Computing curvature of generated triangulated graph

Quadratic patch approximating surface S at p

$$x(u,v) = p + uE_{1p} + vE_{2p} + ([u,v]S_p[u,v]^t)E_{3p}$$

Shape operator $$S_p = \begin{pmatrix} c_{20} & c_{11} \\ c_{11} & c_{02} \end{pmatrix}$$

Coordinate patch $x(u,v) = (u, v, f(u,v))$

$$f(u,v) = \tfrac{1}{2}(c_{20}u^2 + 2c_{11}uv + c_{02}v^2)$$

Unit vector tangent to S $\quad u = (u_1, u_2)^t$

Normal curvature $\kappa(u) = u^t S_p u$ max and min κ_1, κ_2

Principal directions $\vec{t}_1, \vec{t}_2$ $\qquad S_p \vec{t}_i = \kappa_i \vec{t}_i$

Mean Curvature: $H = \dfrac{1}{2} Tr S_p = \dfrac{c_{20} + c_{02}}{2} = (\kappa_1 + \kappa_2)/2$

Gauss Curvature: $G = \kappa_1 \kappa_2$

Several algorithms: Hamman (1993), Joshi et al (1995), Meyer, Desbrun et al (2002),……..

Principal curves, ridge curves and crest lines

surface in $\mathbb{R}^3$: $z = f(x, y)$

tangent space of point $p = (x, y, f(x, y))$

vectors: $u_1 = (1, 0, f_x)^t, u_2 = (0, 1, f_y)^t$

Shape operator $S_p = A^{-1}B$

$$A = \begin{bmatrix} 1 + f_x^2 & f_x f_y \\ f_x f_y & 1 + f_y^2 \end{bmatrix}, \quad B = (1 + f_x^2 + f_y^2)^{-1/2} \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}.$$

- along a principal curve, the principal curvature can have extrema, i.e. ridge points: the locus of which are ridge curves
- two sets correspond to larger and smaller eigenvalue
- ridge points classified according to max or min curvature
- crest lines are loci of points whose maximal curvature is a local extremum along a principal curve: $\langle \nabla \kappa_m, t_m \rangle_{\mathbb{R}^3} = 0$

Dynamic programming generation of principal curves

- define boundaries of subgraphs as sulcal and gyral curves as well as geodesics
- automated algorithm based on dynamic programming (Khaneja et al., 1998)
- sequentially additive energy associated with candidate curves minimized
- s and t are points on smooth connected surface $S \subset \mathbb{R}^3$
- if $\mathcal{K}$ is largest maximal curvature, then define cost of a candidate curve $\alpha(s,t)$ as $\int_{\alpha(s,t)} (\kappa_{max}(x) - \mathcal{K})^2 d\alpha$
- select principal curve by minimizing over all such paths
- for S symmetrical about crest line: $(\kappa_{max}(x) - \mathcal{K}) \langle \nabla \kappa_{max}, t_{max} \rangle_{\mathbb{R}^3} = 0$
- thus $\langle \nabla \kappa_{max}, t_{max} \rangle_{\mathbb{R}^3} = 0$
- for sulcal regions $\kappa_{\max}$ is constant and minimizer gives shortest path
- for gyri: maximal negative curvature i.e. reverse signs

Dynamic Programming

- define platelet $\mathcal{P}_i$ of point i as the set of triangles with vertices (j_1, j_2, j_3) sharing x_i as a common vertex

$$\mathcal{P}_i = \cup\{(j_1, j_2, j_3) \mid x_i = x_{j_1} \text{or } x_{j_2} \text{or } x_{j_3}\}$$

- define a path on the surface routed and terminated respectively at nodes s and t on surface as

$$(s = j_1, j_2), \ldots, (j_{k-1}, j_k), \ldots, (j_{N-1}, t = j_N)$$

such that $j_k \in \mathcal{P}_{j_{k-1}} \; \forall k$ and the collection of all paths connecting (s,t) as $\alpha(s,t) \in \mathcal{P}_{s,t}(S)$

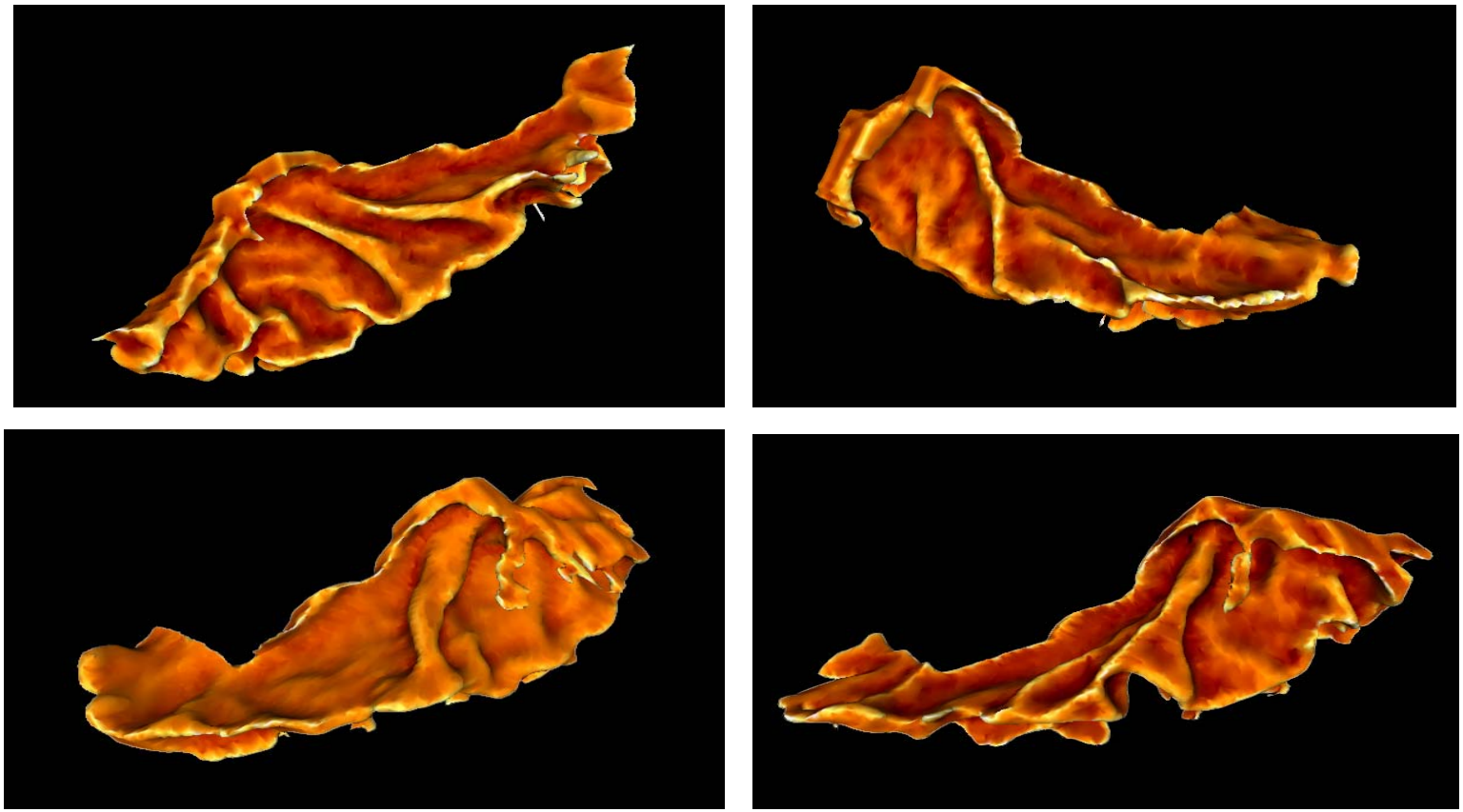
- discrete fundus beds $\mathcal{F}$ and N-length geodesics $\mathcal{G}$ are cost minimizing paths (using corrected trapezoid integration):

$$\mathcal{F}(s,t) = \underset{\alpha(s,t)\in\mathcal{P}_{s,t}(S)}{\text{argmin}} \sum_{k=1}^{N} d(j_k, j_{k+1}) \text{ where}$$

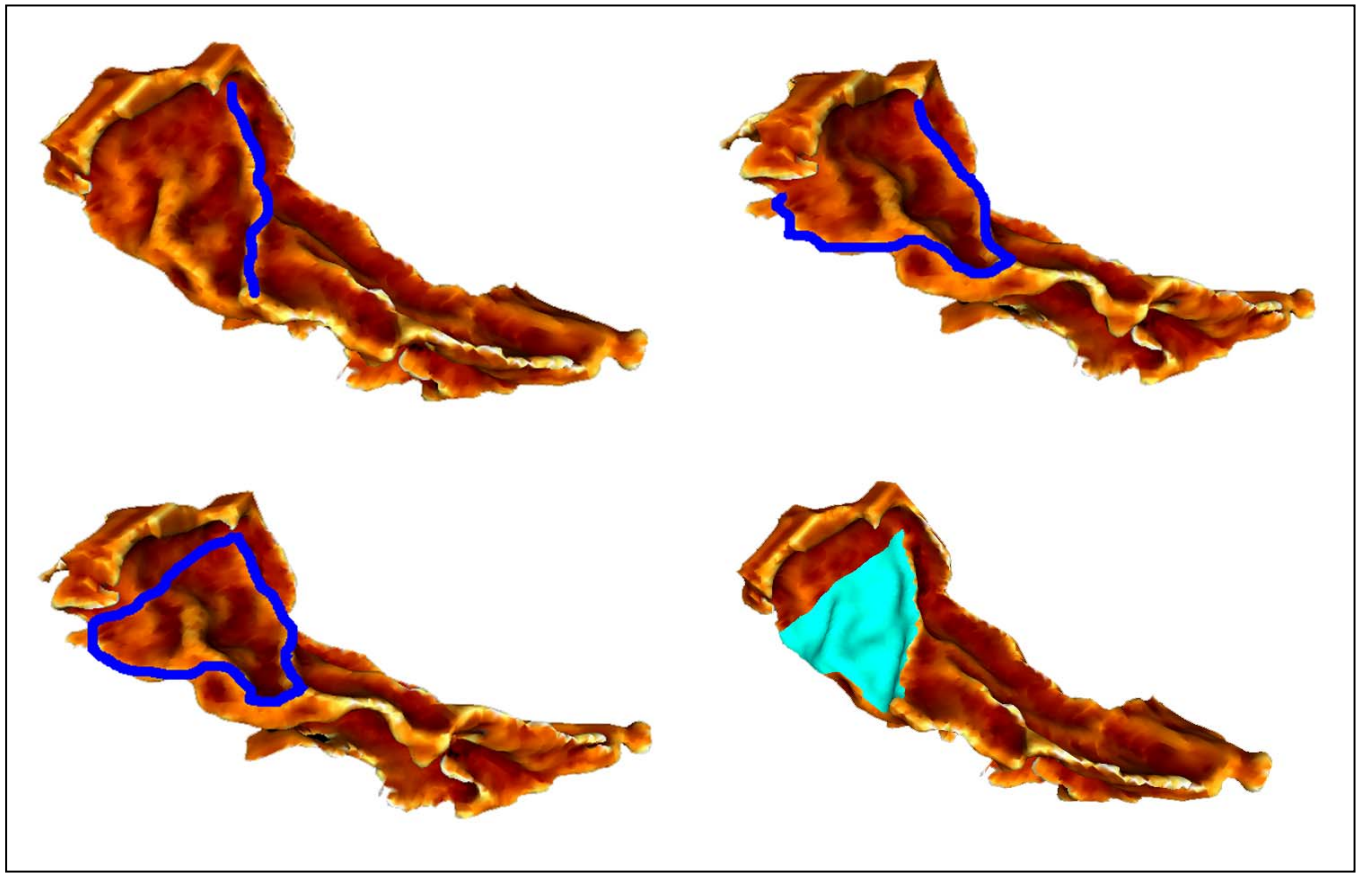
$$d(j_k, j_{k+1}) = \left(\frac{(\kappa_{max}(x_{j_k}) - \mathcal{K})^2}{2} + \frac{(\kappa_{max}(x_{j_{k+1}}) - \mathcal{K})^2}{2} + \frac{(\kappa_{max}(x_{j_k}) - \kappa_{max}(x_{j_{k+1}}))^2}{6} \right) \times \left\| x_{j_k} - x_{j_{k+1}} \right\|$$

$$\mathcal{G}(s,t) = \underset{\alpha(s,t)\in\mathcal{P}_{s,t}(S)}{\text{argmin}} \sum_{k=1}^{N} d(j_k, j_{k+1}) \text{ where } d(j_k, j_{k+1})^2 = (x_{1,j_k} - x_{1,j_{k+1}})^2 + (x_{2,j_k} - x_{2,j_{k+1}})^2 + (x_{3,j_k} - x_{3,j_{k+1}})^2$$

Reconstructed STG surfaces



Application of dynamic programming

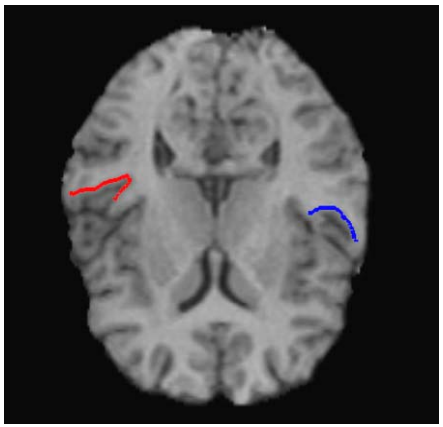


Comparison between automated and semi-automated generated Planum Temporales

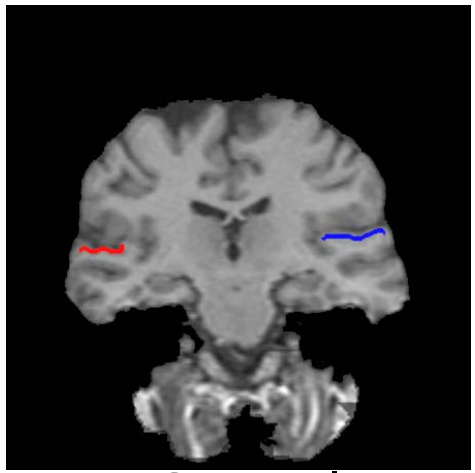
Auto

Semi-auto

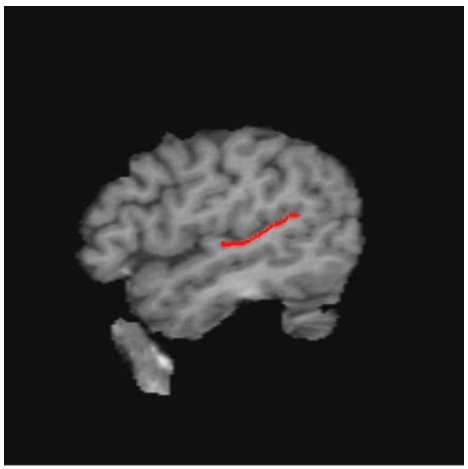
Orthogonal views of reconstructed surface of PT



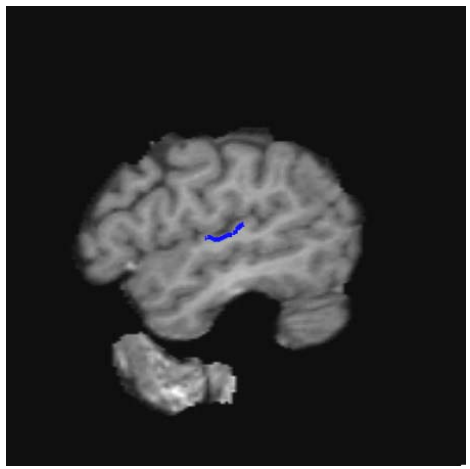
Axial



Coronal



Sagittal, L



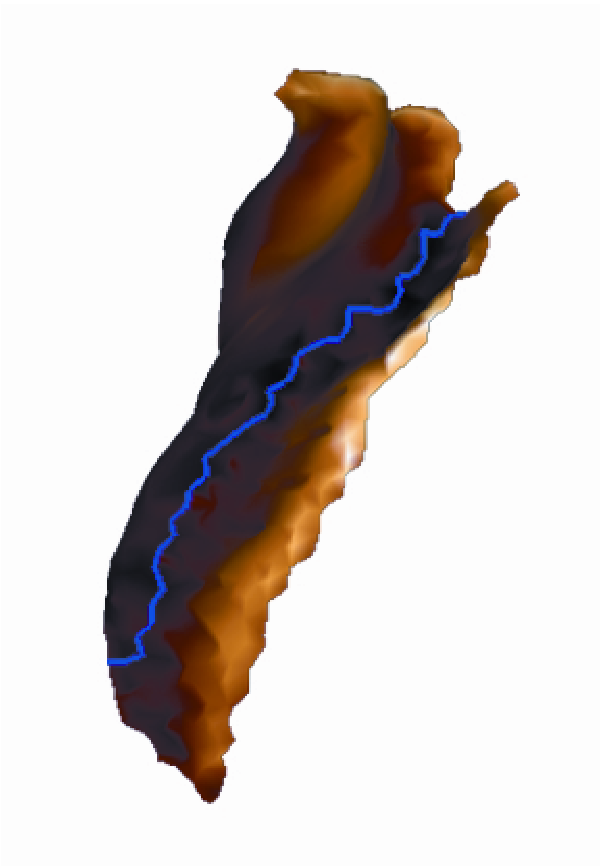
Sagittal, R

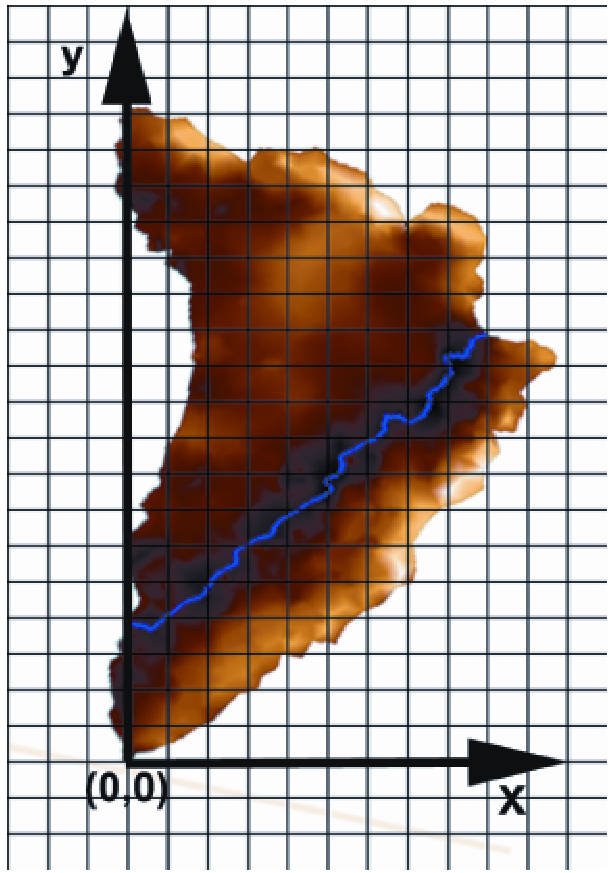
Surface Area (mm^2)

	L auto	L hand	R auto	R hand
1	553.66	612.83	289.92	299.73
2	932.70	939.67	463.67	423.30
3	527.81	532.59	655.19	665.19
4	652.83	621.97	411.80	361.69
5	746.92	695.45	513.05	500.14
6	607.88	617.05	529.68	553.97
7	314.18	284.79	701.92	692.77
8	582.30	664.47	516.59	539.62
9	628.30	570.98	754.13	709.64
10	565.47	618.02	266.19	289.40

reliability index between hand and 1st auto: 0.96 for left and 0.98 for right
reliability index for repeated auto reconstruction: 0.96 for left and 0.94 for right

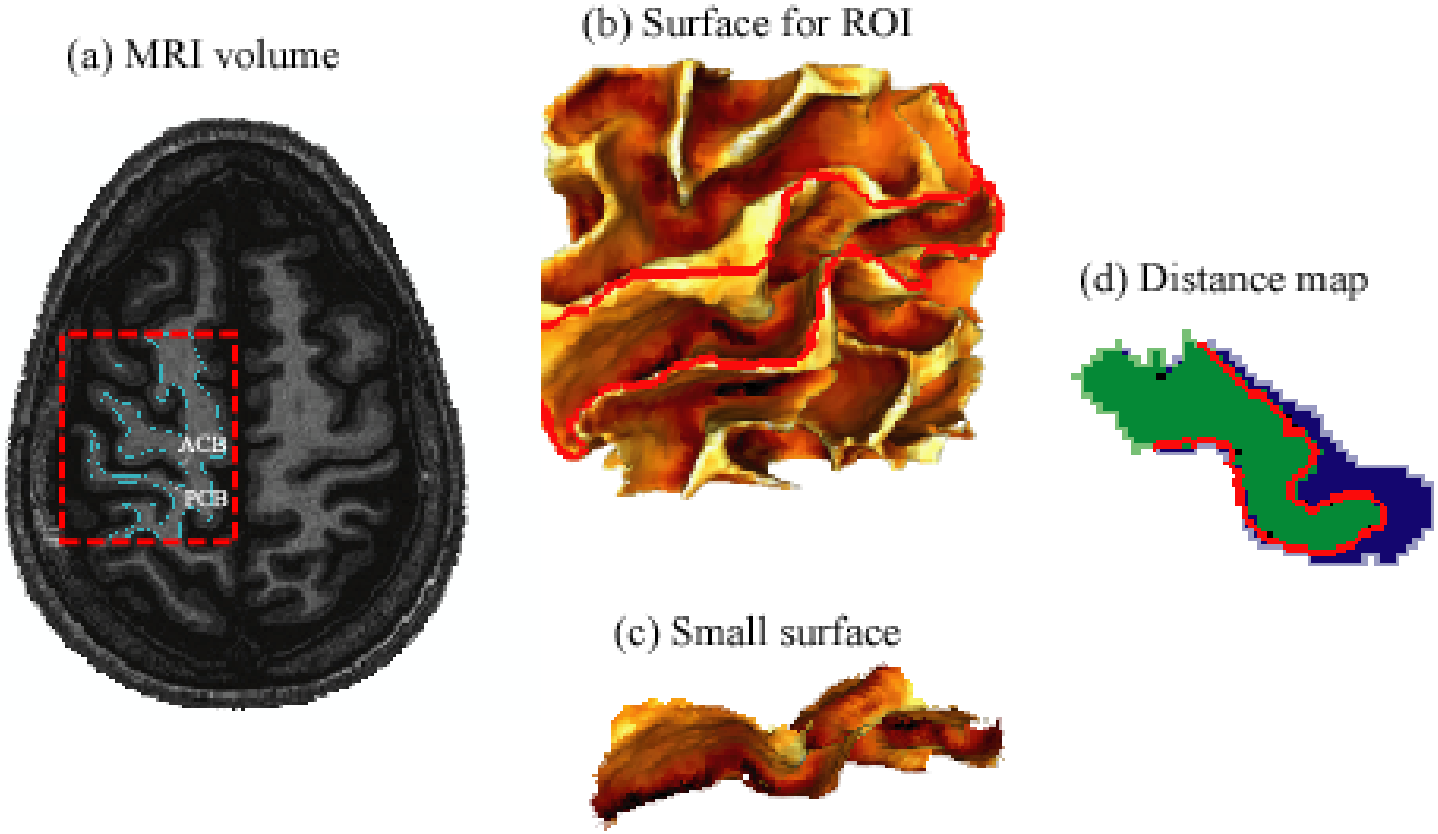
The coordinatized PT manifold





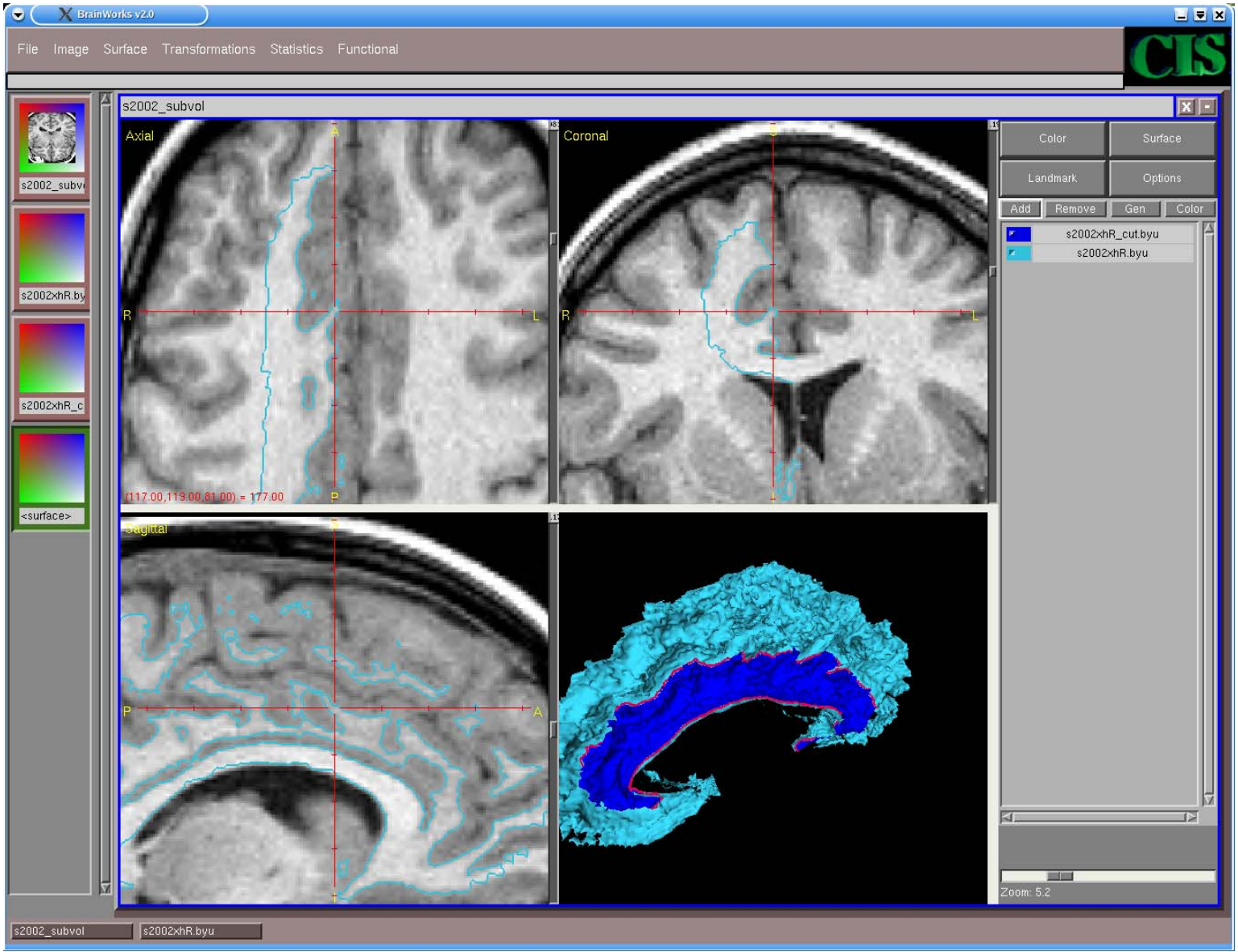
BrainWorks®

Other applications: sub-surface distance maps



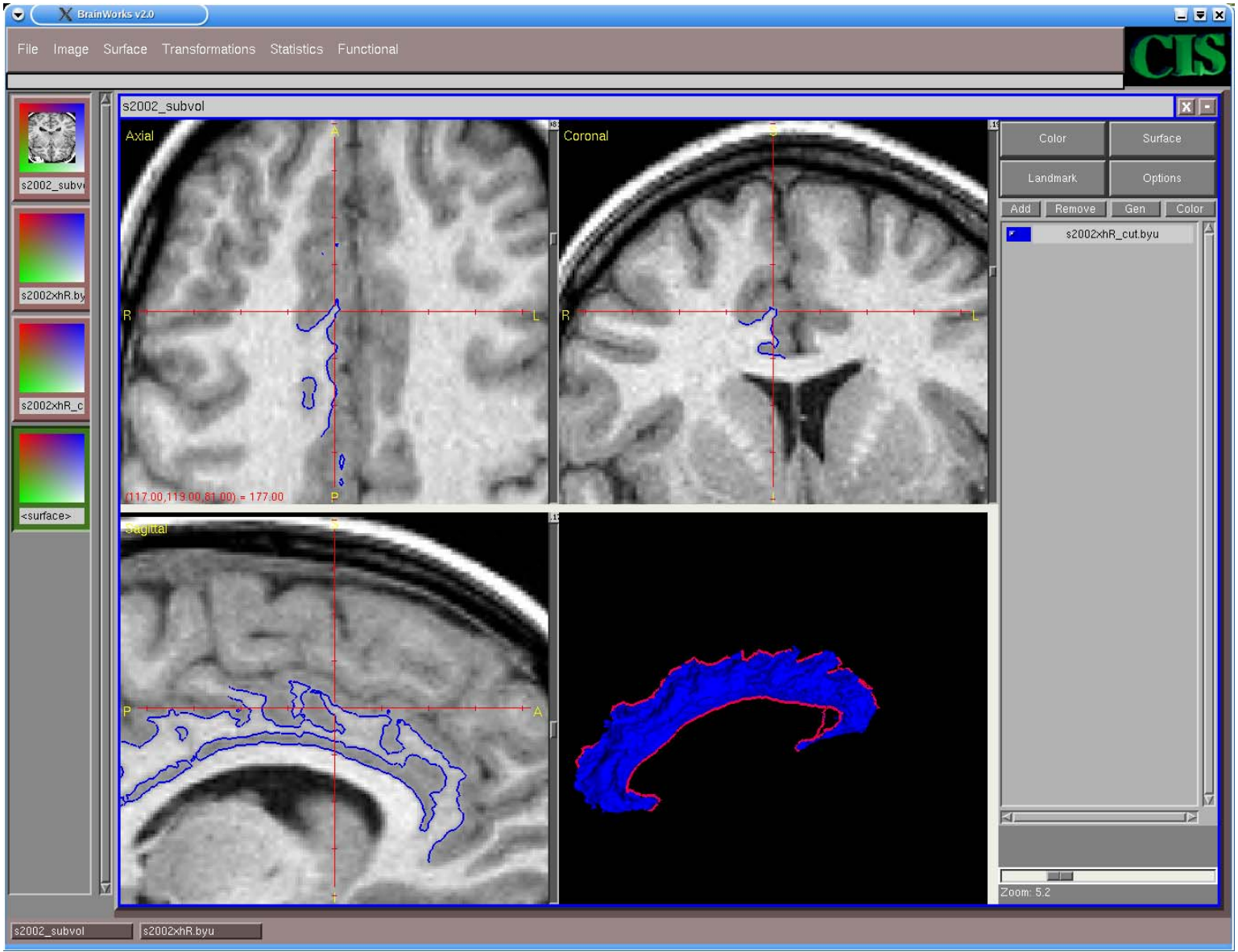
- Qui et al 2004 (Human Brain Mapping)
- Ratnanather et al, 2004 (Psychiatry Reseach NeuroImaging – in press)
- Botteron et al 2004 (Human Brain Mapping)
- Miller et al 2003 (PNAS)

Defining boundary of Cingulate Gyrus Surface: before



Ratnanather et al, 2004 (Psychiatry Reseach NeuroImaging – in press)
Data from Csernansky (WU, Conte Center)

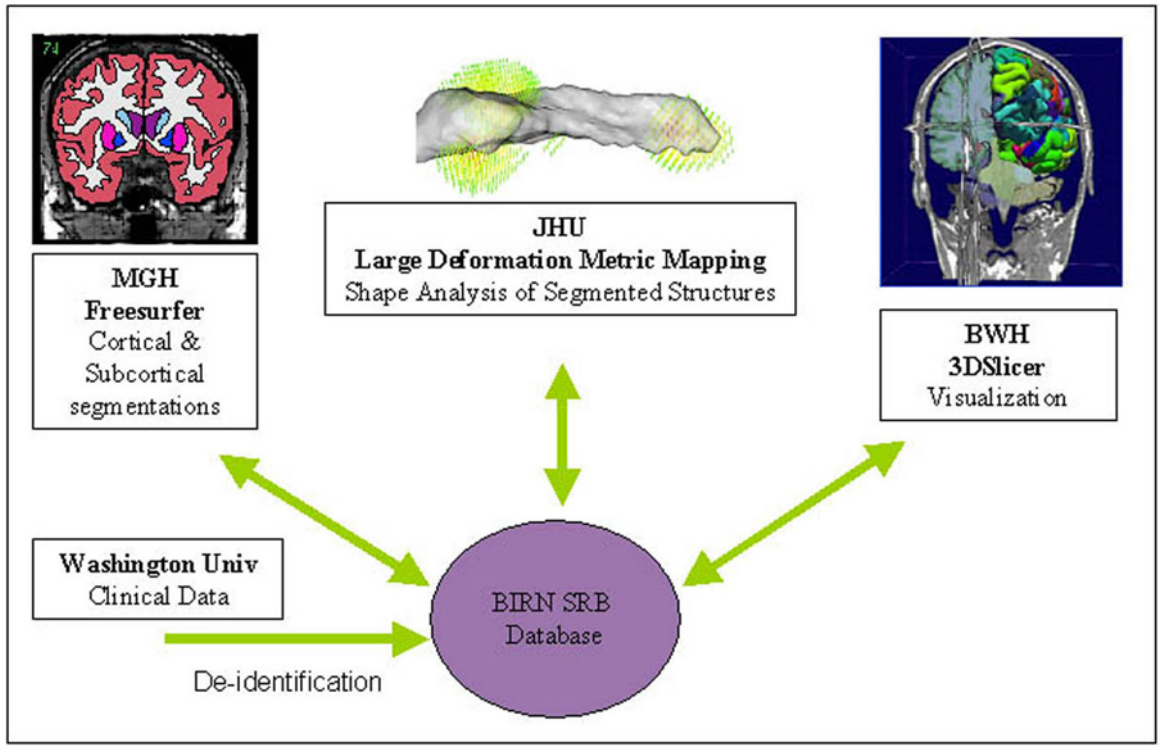
Defining boundary of Cingulate Gyrus Surface: after



Tracking not possible if surface is not topology correct – used Han algorithm (could use Shattuck & Leahy)

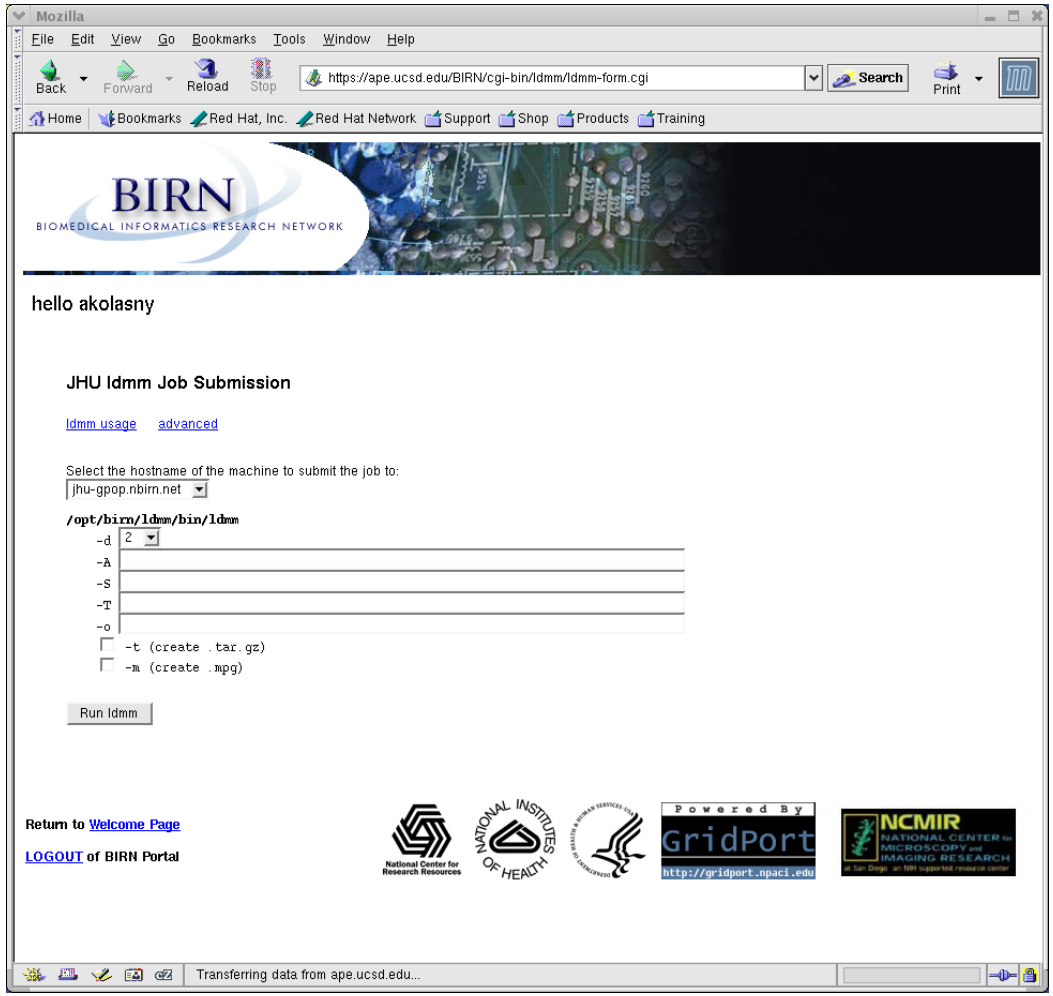
Ratnanather et al, 2004 (Psychiatry Reseach NeuroImaging – in press)
Data from Csernansky (WU, Conte Center)

Biomedical Informatics Research Network – a new paradigm



- use high speed networks to processing cortical data from multiple sites

Biomedical Informatics Research Network – a new paradigm



- could apply Large Deformation Diffeomorphic Metric Mapping to cortical surfaces via web portal

Online References

- see www.cis.jhu.edu/education/intrometricpatterntheory - for interactive tutorials on dynamic programming, groups, matrix groups, lie groups, orbits, deformable templates and metric distances

- can download executable for generating principal curves using the alternative method of implicit surfaces from http://mountains.ece.umn.edu/~abarte/3Distance/curves.html

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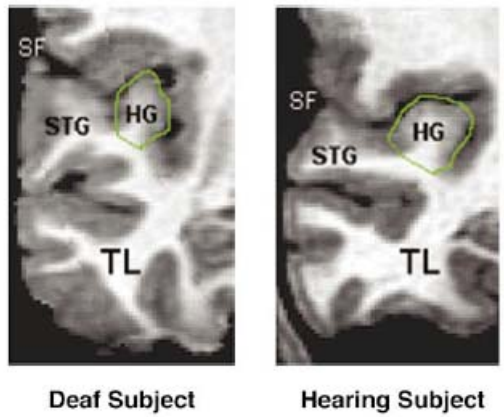
The deafened auditory cortex: another challenge for computational anatomy

- 1 in 10 deaf or hard of hearing
- 20,000 babies born with profound hearing impaired
- very few leave school or go to college with excellent speech or use hearing aids
- those who do had good auditory training or learnt to lipread
- personal exp suggest great variability in deaf brain
- HOW DO WE FIND OUT WHAT WORKS?
- no difference in Heschl's Gyrus volume (i.e. deafferentiation has no effect)
- larger grey/white matter ratios in Heschl's Gyrus in deaf subjects
- less white matter in both left and right Heschl's Gyrus in deaf subjects
- deaf subjects have higher grey/white matter ratios in the superior temporal gyrus (which has connections with the motor regions)



Emmorey et al. (2003)