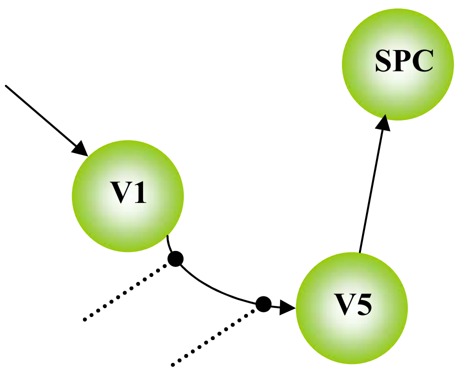
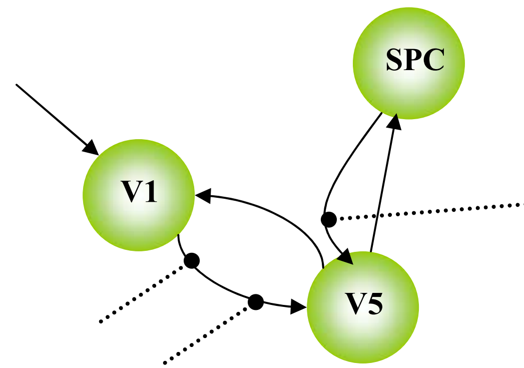
Dynamic Causal Models



Will Penny



Olivier David, Karl Friston, Lee Harrison,
Andrea Mechelli, Klaas Stephan

Wellcome Department of Imaging Neuroscience, ION, UCL, UK.

Mathematics in Brain Imaging, IPAM, UCLA, USA, July 22 2004.

Contents

- Neurodynamic model
- Hemodynamic model
- Model estimation and comparison
- Attention to visual motion
- Single word processing

Friston et al.(2003) NeuroImage, 19 (4), pp. 1273-1302.

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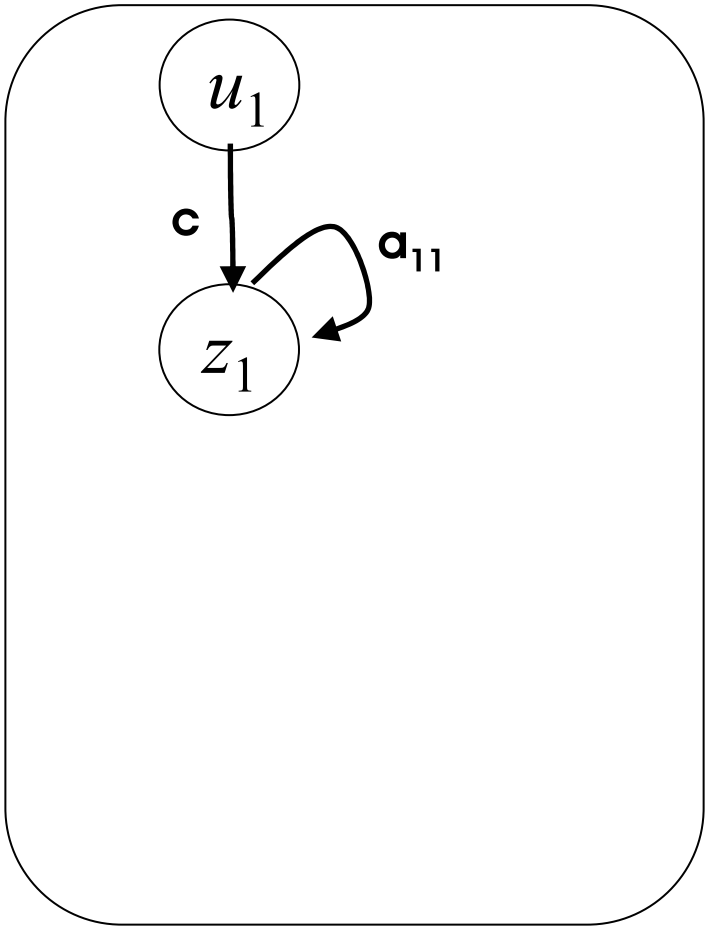
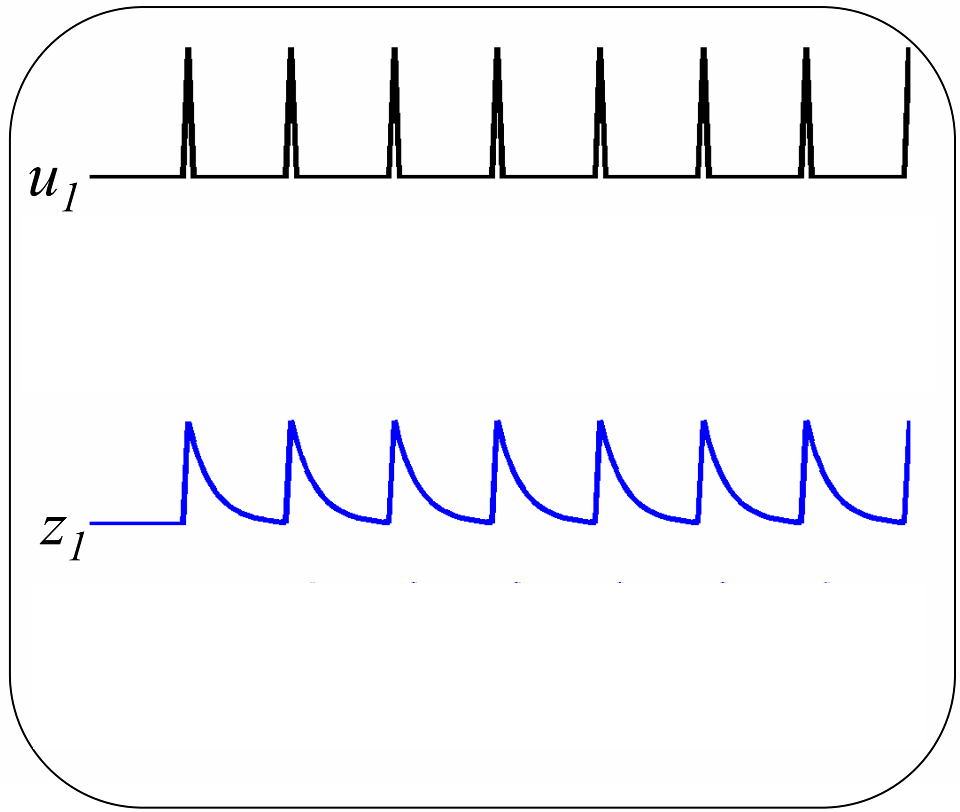
Single region

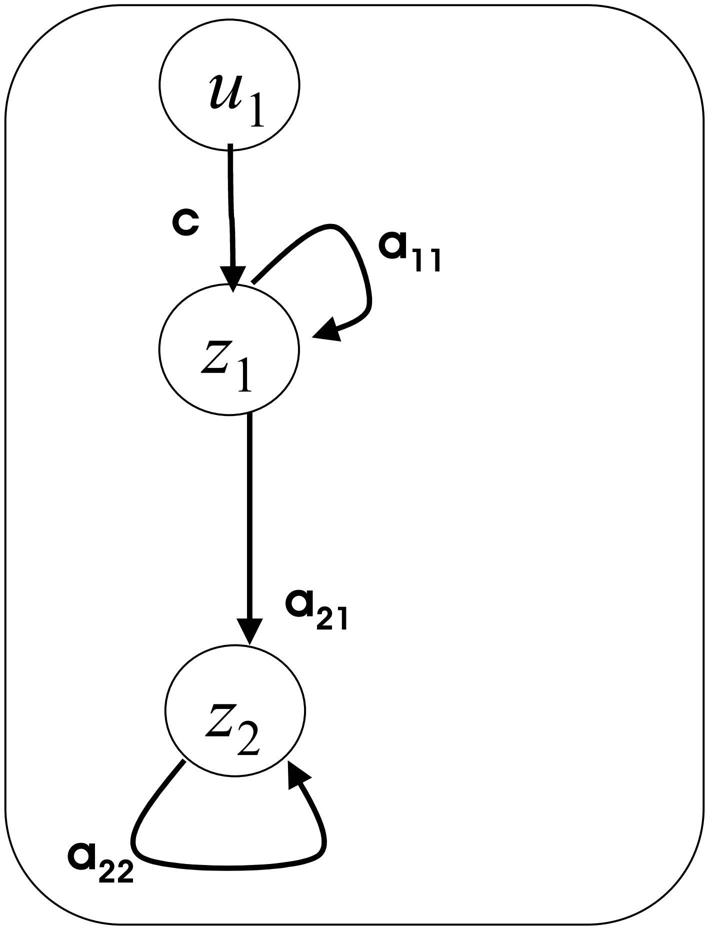
$$\dot{z}_1 = a_{11} z_1 + c u_1$$

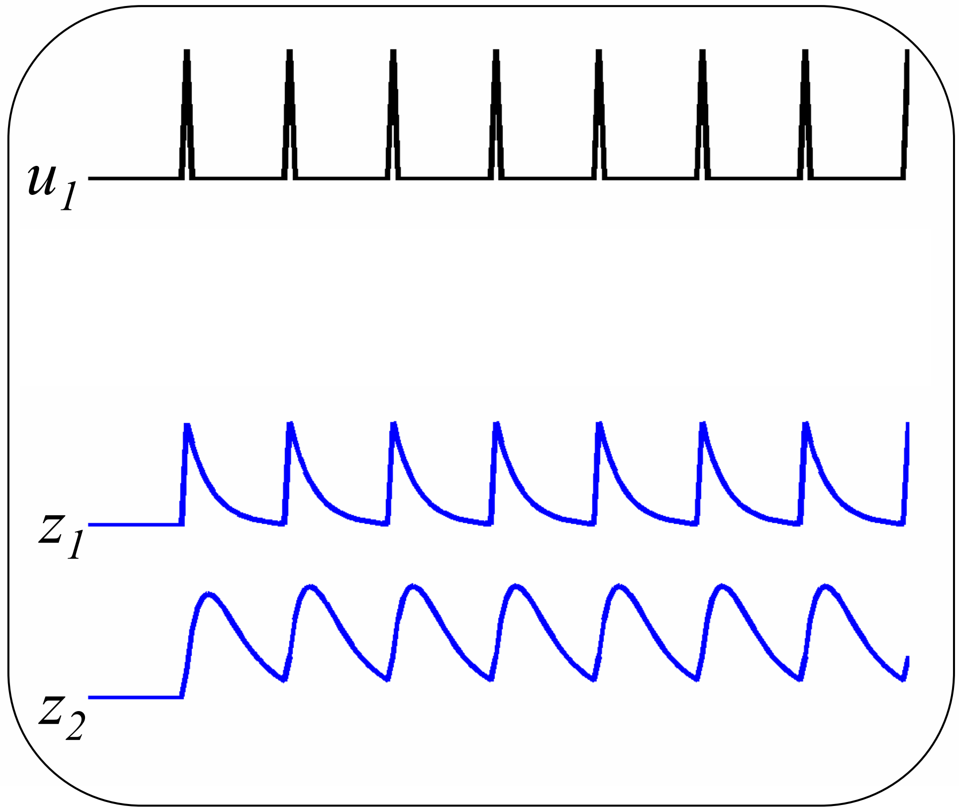




Multiple regions

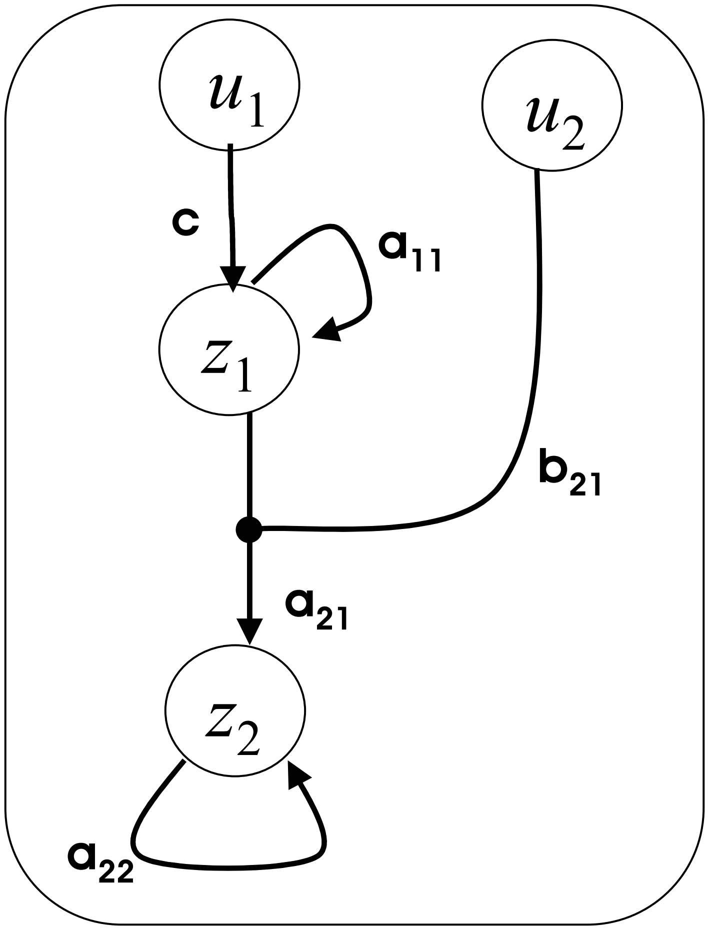
$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} c \\ 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

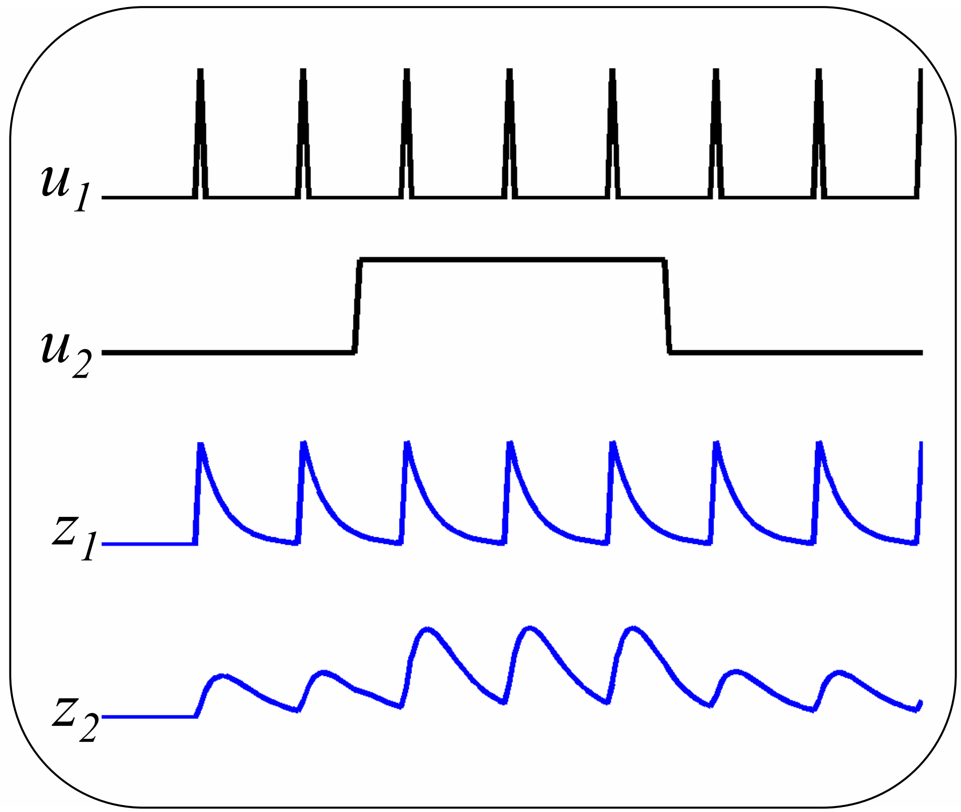




Modulatory inputs

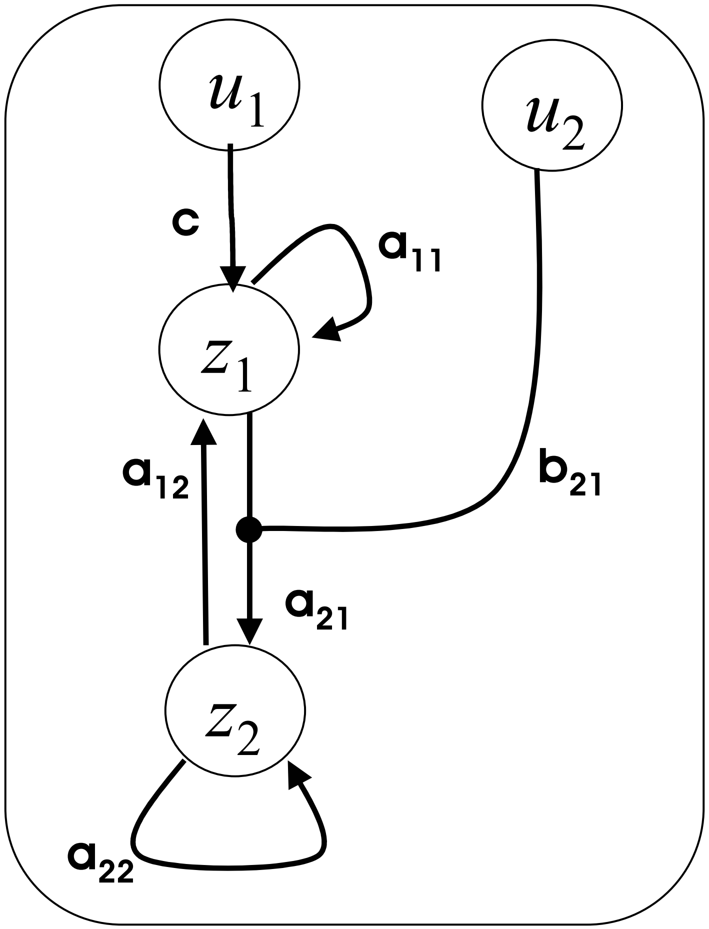
$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix}=\begin{bmatrix}a_{11} & 0\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}+u_2\begin{bmatrix}0 & 0\\ b_{21} & 0\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}+\begin{bmatrix}c\\ 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

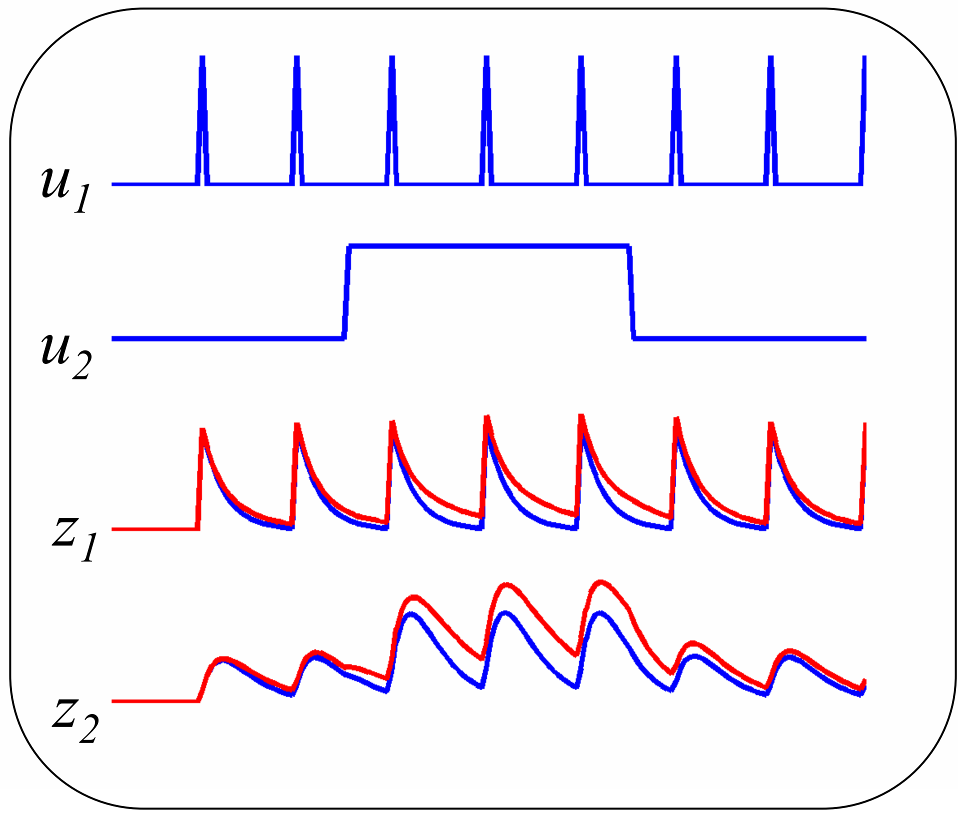




Reciprocal connections

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}+u_2\begin{bmatrix}0 & 0\\ b_{21} & 0\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}+\begin{bmatrix}c\\ 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$





Neurodynamics

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \sum_i \mathbf{u}_i \mathbf{B}_i \mathbf{z} + \mathbf{C}\mathbf{u}$$

Change in Neuronal Activity

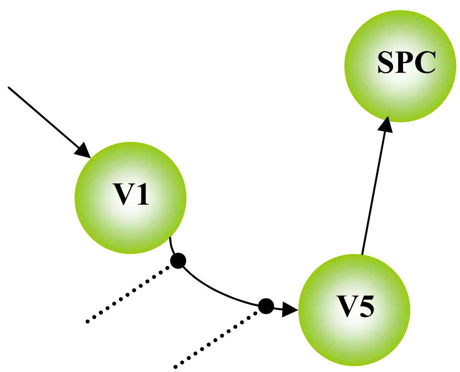
Neuronal Activity

Inputs

Intrinsic Connectivity Matrix

Modulatory Connectivity Matrices

Input Connectivity Matrix



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Hemodynamics

For each region:

Hemodynamic variables

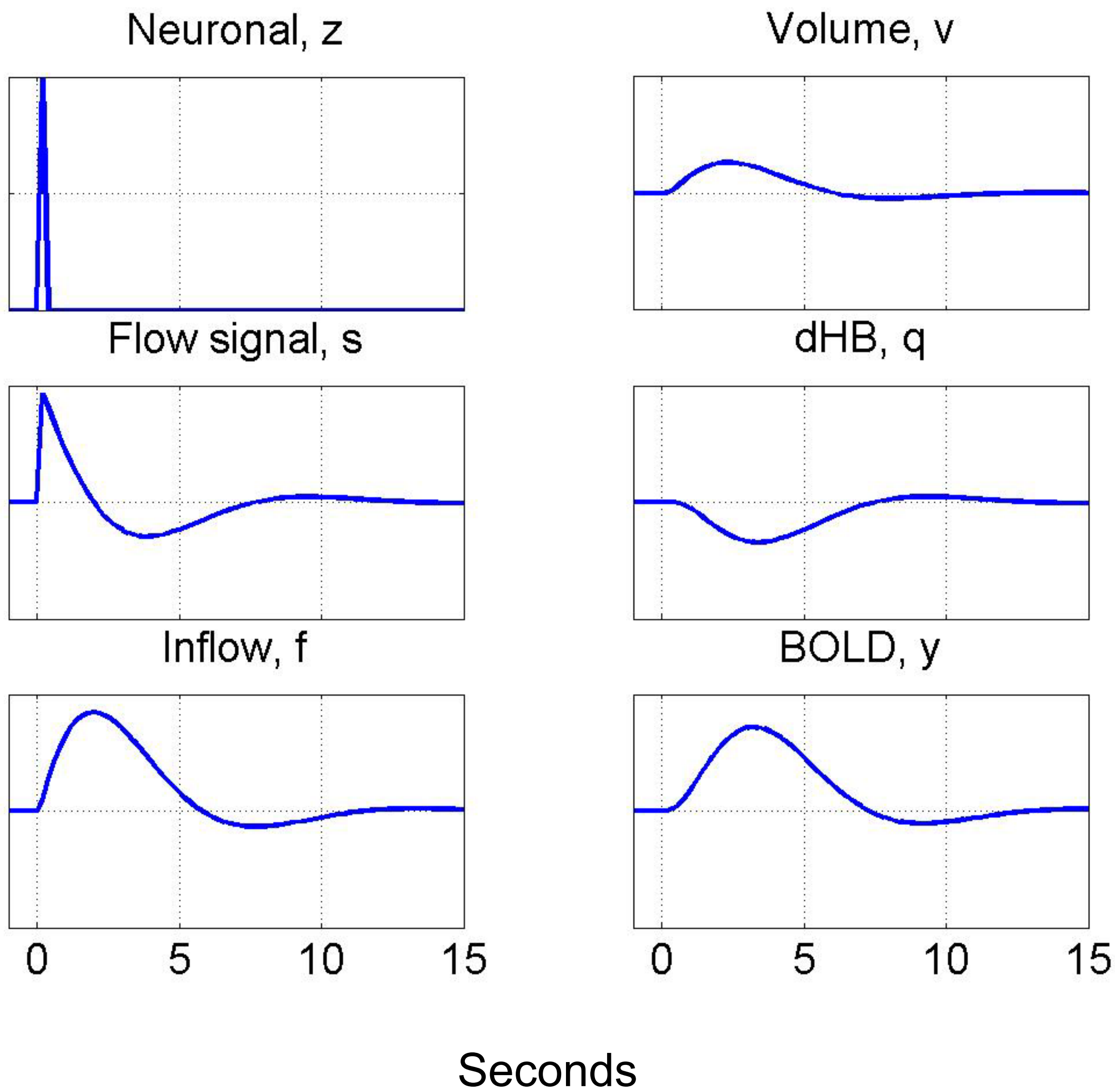
$$\mathbf{x} = [s, f, v, q]$$

Dynamics

$$\dot{\mathbf{x}} = g(\mathbf{x}, z, \mathbf{h})$$

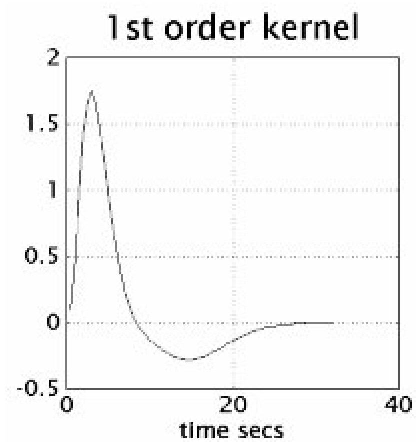
$$y = b(\mathbf{x})$$

Hemodynamic parameters



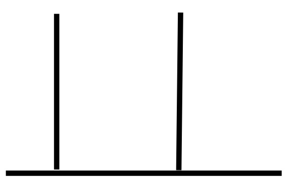
Hemodynamic saturation

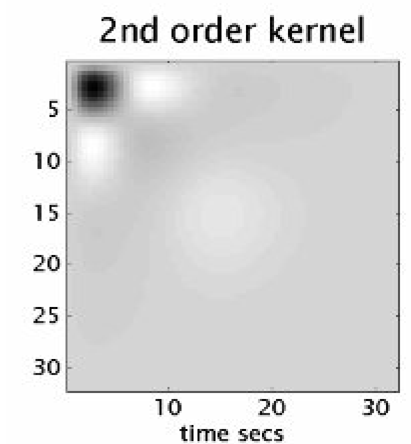
Neuronal impulse



Equivalent
input-output
functions

Neuronal impulses





Sub-linear and super-linear
responses to pairs of stimuli

Why have explicit models for neurodynamics and hemodynamics ?

For 4 event types u_1, u_2, u_3, u_4 :

In a GLM for a single region, **y=Xβ+e**, with 3 basis functions per event type (canonical,shifter, stretcher) there are 12 parameters to estimate. These relate hemodynamics *directly* to each stimulus.

In a (single region) DCM there are 4 neuronal efficacy parameters relating neuronal activity to each stimulus

$$\dot{z} = az + c_1 u_1 + c_2 u_2 + c_3 u_3 + c_4 u_4$$

And 5 hemodynamic parameters relating neuronal activity to the BOLD signal.

$$y = b(z, h)$$

A total of 9 parameters.

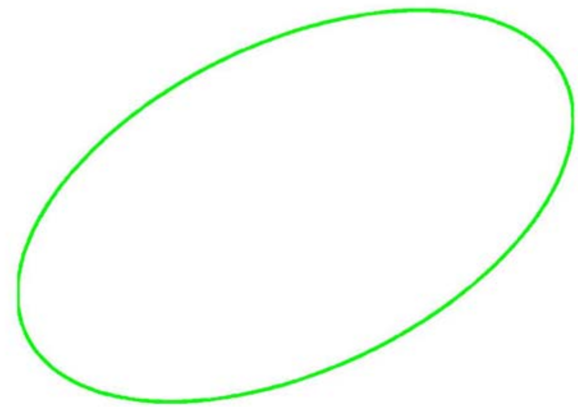
DCM Priors

Hemodynamics

E[h]

Rate of signal decay: 0.65
Elimination rate: 0.41
Transit time: 0.98
Grubbs exponent: 0.32
Oxygenation fraction: 0.34

Cov[h]



Neurodynamics

Stability priors ensure principal Lyapunov exponent is less than zero with high probability.

Contents

Bayesian Estimation

Normal densities

$$p(\theta) = N(\theta; \mu_p, \sigma_p^{\ 2})$$

$$p(y \mid \theta) = N(y; \theta, \sigma_e^{\ 2})$$

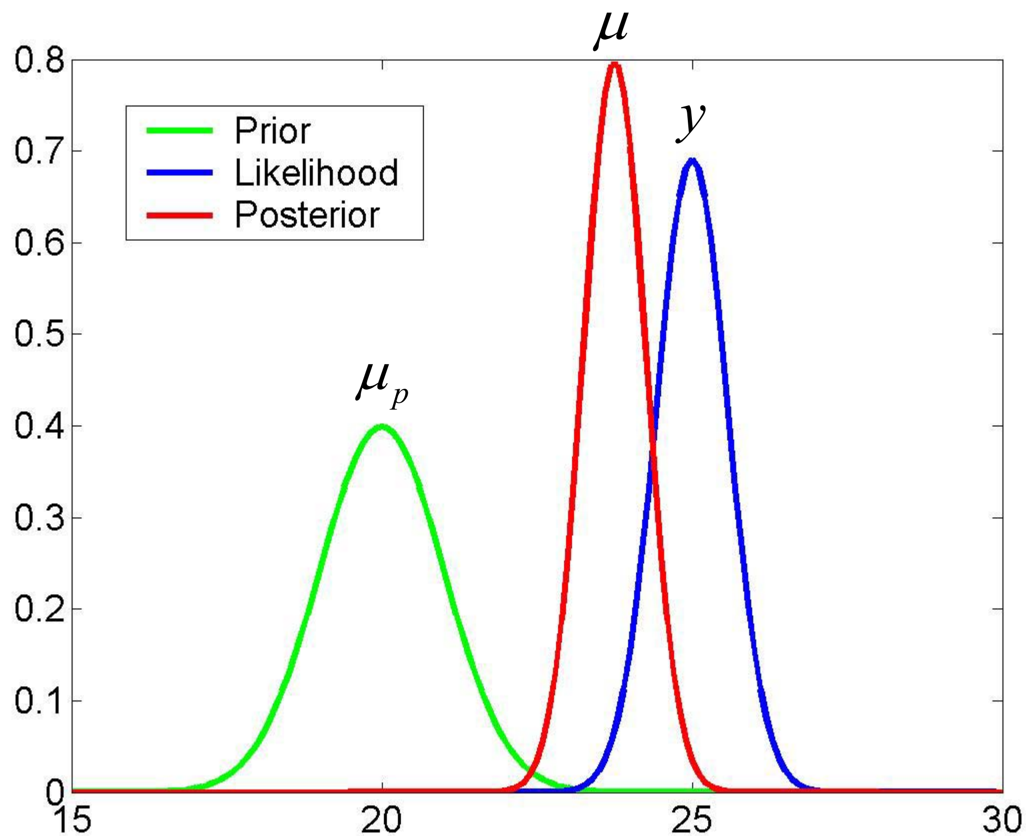
$$p(\theta \mid y) = N(\theta; \mu, \sigma^2)$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_e^{\ 2}} + \frac{1}{\sigma_p^{\ 2}}$$

$$\mu = \sigma^2 \left(\frac{1}{\sigma_e^{\ 2}} y + \frac{1}{\sigma_p^{\ 2}} \mu_p \right)$$

Relative Precision Weighting

$$y = \theta + e$$



Multiple parameters

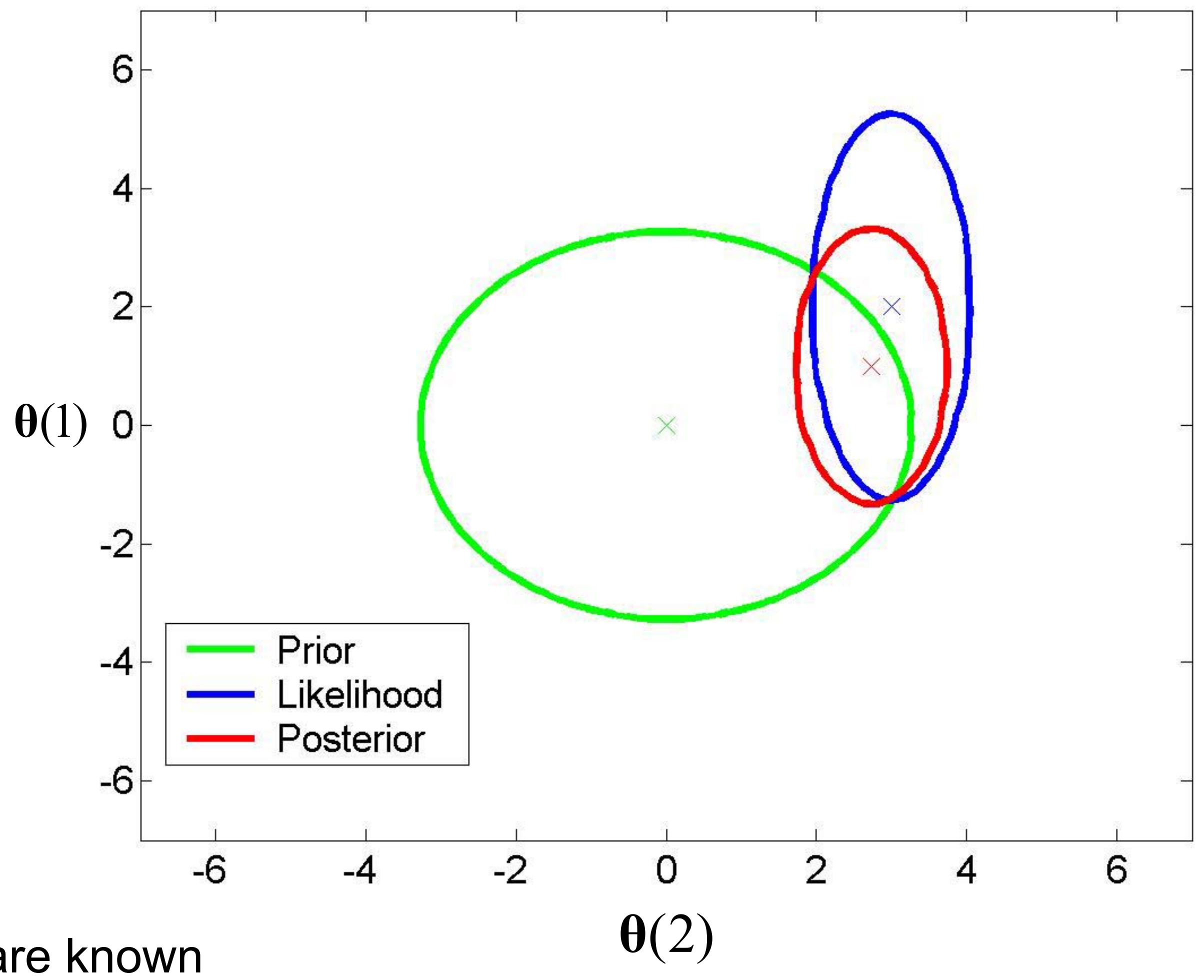
General Linear Model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \mathbf{e}$$

$$p(\boldsymbol{\theta}) = N(\boldsymbol{\theta}; \boldsymbol{\mu}_p, \mathbf{C}_p)$$

$$p(\mathbf{y} \mid \boldsymbol{\theta}) = N(\mathbf{y}; \mathbf{X}\boldsymbol{\theta}, \mathbf{C}_e)$$

$$p(\boldsymbol{\theta} \mid \mathbf{y}) = N(\boldsymbol{\theta}; \boldsymbol{\mu}, \mathbf{C})$$



$$\mathbf{C}^{-1} = \mathbf{X}^T \mathbf{C}_e^{-1} \mathbf{X} + \mathbf{C}_p^{-1}$$

$$\boldsymbol{\mu} = \mathbf{C}\left(\mathbf{X}^T \mathbf{C}_e^{-1} \mathbf{y} + \mathbf{C}_p^{-1} \boldsymbol{\mu}_p\right)$$

One-step if C_e, C_p and μ_p are known

Nonlinear models

$$\boldsymbol{\theta} = \{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{h}\}$$

Current Estimates

$$\boldsymbol{\mu}_i, \mathbf{C}_i$$

$$\mathbf{y} = b(\boldsymbol{\theta}) + \mathbf{e}$$

$$p(\boldsymbol{\theta}) = N(\boldsymbol{\theta}; \boldsymbol{\mu}_p, \mathbf{C}_p)$$

$$p(\Delta\boldsymbol{\theta}) = N(\Delta\boldsymbol{\theta}; \boldsymbol{\mu}_p - \boldsymbol{\mu}_i, \mathbf{C}_p)$$

$$p(\mathbf{y} \mid \boldsymbol{\theta}) = N(\mathbf{y}; b(\boldsymbol{\theta}), \mathbf{C}_e)$$

$$p(\mathbf{r} \mid \Delta\boldsymbol{\theta}) = N(\mathbf{r}; \mathbf{J}\Delta\boldsymbol{\theta}, \mathbf{C}_e)$$

$$\mathbf{C}_{i+1}^{-1} = \mathbf{J}^T \mathbf{C}_e^{-1} \mathbf{J} + \mathbf{C}_p^{-1}$$

$$\boldsymbol{\mu}_{i+1} = \boldsymbol{\mu}_i + \mathbf{C}_{i+1}\left(\mathbf{J}^T \mathbf{C}_e^{-1} \mathbf{r} + \mathbf{C}_p^{-1}\left(\boldsymbol{\mu}_p - \boldsymbol{\mu}_i\right)\right)$$

Gauss-Newton ascent with priors

Linearization

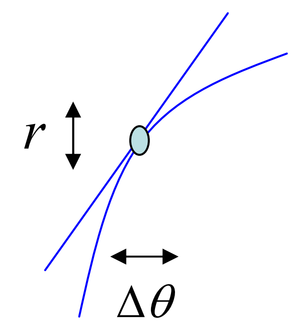
$$b(\boldsymbol{\theta}) = b(\boldsymbol{\mu}_i) + \frac{\partial b(\boldsymbol{\mu}_i)}{\partial \boldsymbol{\theta}}(\boldsymbol{\theta} - \boldsymbol{\mu}_i)$$

$$\mathbf{J} = \frac{\partial b(\boldsymbol{\mu}_i)}{\partial \boldsymbol{\theta}}$$

$$\Delta\boldsymbol{\theta} = \boldsymbol{\theta} - \boldsymbol{\mu}_i$$

$$r = y - b(\boldsymbol{\mu}_i)$$

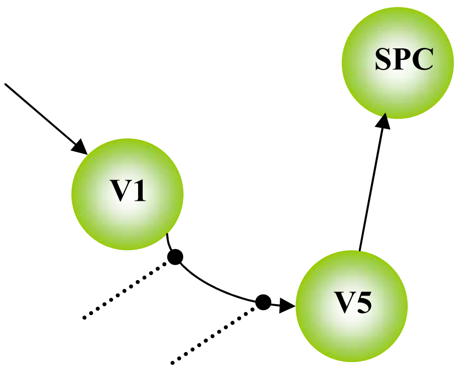
$$r = \mathbf{J}\Delta\boldsymbol{\theta} + \mathbf{e}$$



Friston et al.(2002) NeuroImage, 16 (2), pp. 513-530.

Model Comparison I

Model, m



Parameters: $\boldsymbol{\theta} = \{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{h}\}$

Posterior — Likelihood — Prior

$$p(\boldsymbol{\theta} \mid \mathbf{y}, m) = \frac{p(\mathbf{y} \mid \boldsymbol{\theta}, m)\, p(\boldsymbol{\theta} \mid m)}{p(\mathbf{y} \mid m)}$$

Evidence →

$$p(\mathbf{y} \mid m) = \int p(\mathbf{y} \mid \boldsymbol{\theta}, m)\, p(\boldsymbol{\theta} \mid m)\, d\boldsymbol{\theta}$$

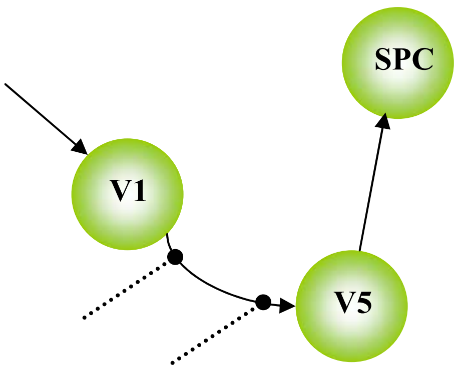
Laplace, AIC, BIC approximations

Model fit + complexity

Penny et al. (2004) NeuroImage, 22 (3), pp. 1157-1172.

Model Comparison II

Model, m



Parameters: $\boldsymbol{\theta} = \{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{h}\}$

Parameter Posterior

Likelihood

Parameter Prior

$$p(\boldsymbol{\theta} \mid \mathbf{y}, m) = \frac{p(\mathbf{y} \mid \boldsymbol{\theta}, m)\, p(\boldsymbol{\theta} \mid m)}{p(\mathbf{y} \mid m)}$$

Model Posterior

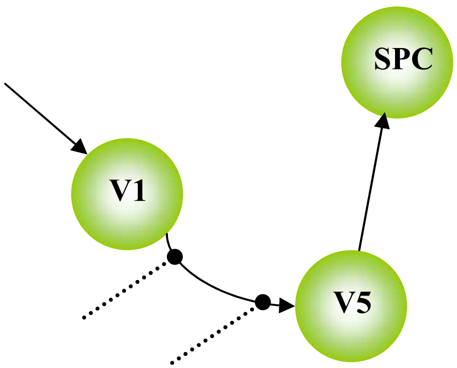
Evidence

Model Prior

$$p(m \mid \mathbf{y}) = \frac{p(\mathbf{y} \mid m)\, p(m)}{p(\mathbf{y})}$$

Model Comparison III

Model, m=i



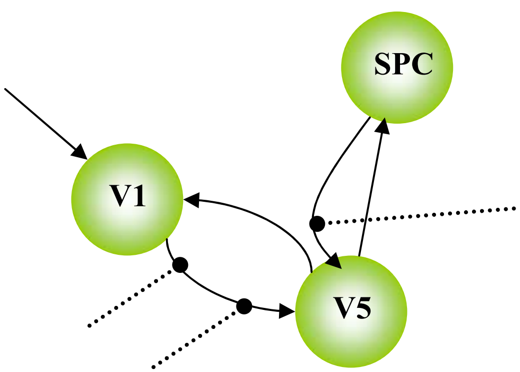
Model Evidences:

$$p(\mathbf{y} \mid m=i) = \int p(\mathbf{y} \mid \boldsymbol{\theta}, m=i)\, p(\boldsymbol{\theta} \mid m=i)\, d\boldsymbol{\theta}$$

$$p(\mathbf{y} \mid m=j) = \int p(\mathbf{y} \mid \boldsymbol{\theta}, m=j)\, p(\boldsymbol{\theta} \mid m=j)\, d\boldsymbol{\theta}$$

Bayes factor:

Model, m=j



$$B_{ij} = \frac{p(\mathbf{y} \mid m=i)}{p(\mathbf{y} \mid m=j)}$$

1 to 3:	Weak
3 to 20:	Positive
20 to 100:	Strong
>100:	Very Strong

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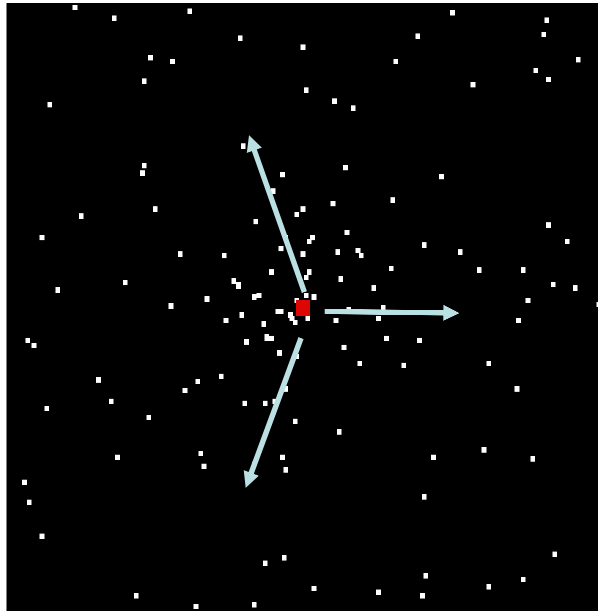
Attention to Visual Motion

Buchel et al. 1997



STIMULI

250 radially moving dots at 4.7 degrees/s

PRE-SCANNING

5 x 30s trials with 5 speed changes (reducing to 1%)

Task - detect change in radial velocity

SCANNING (no speed changes)

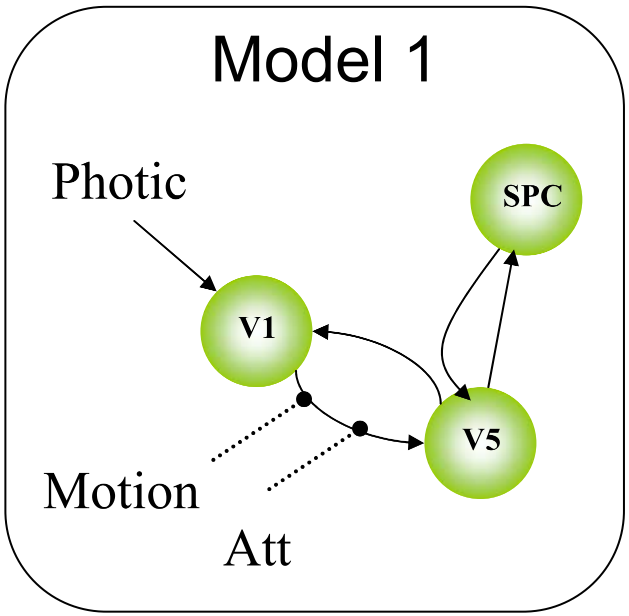
6 normal subjects, 4 100 scan sessions;

each session comprising 10 scans of 4 different condition

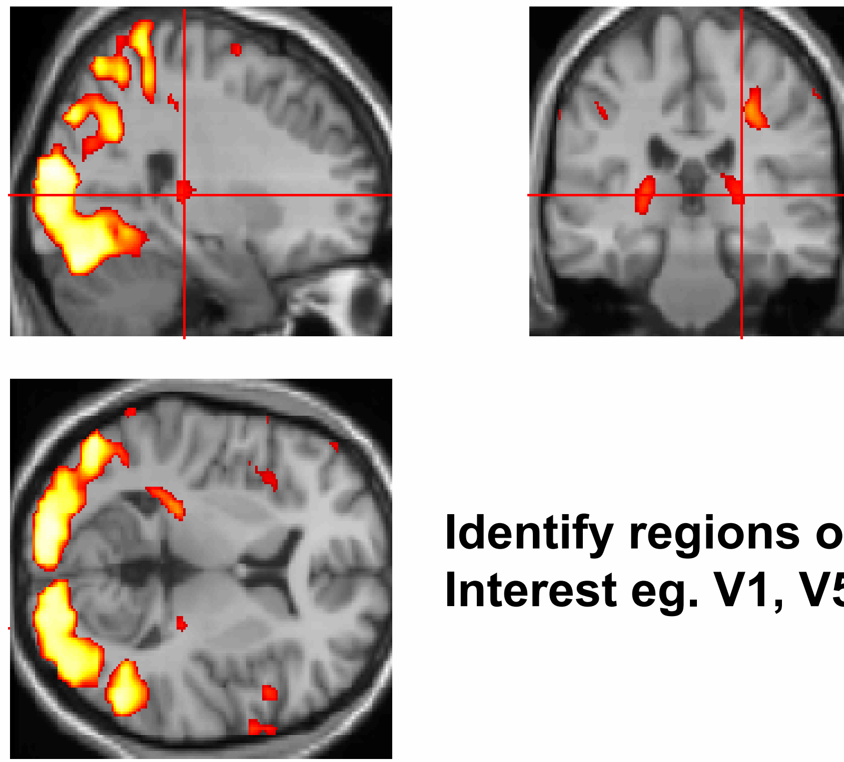
Experimental Factors

1. Photic
2. Motion
3. Attention

Specify regions of interest

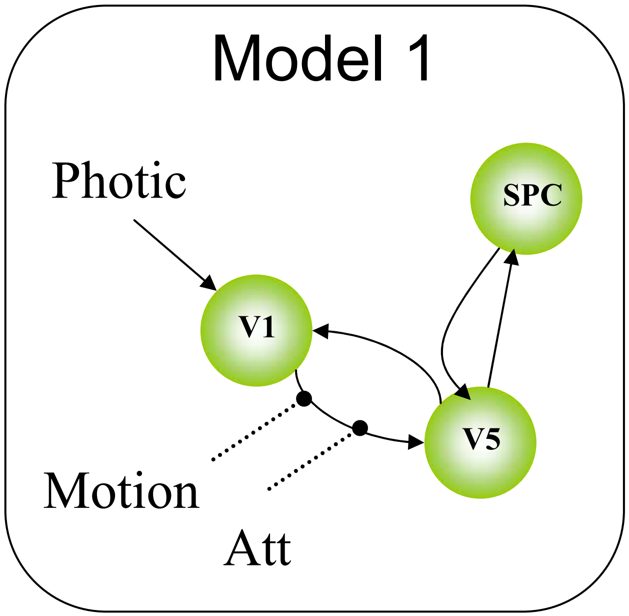


GLM analysis

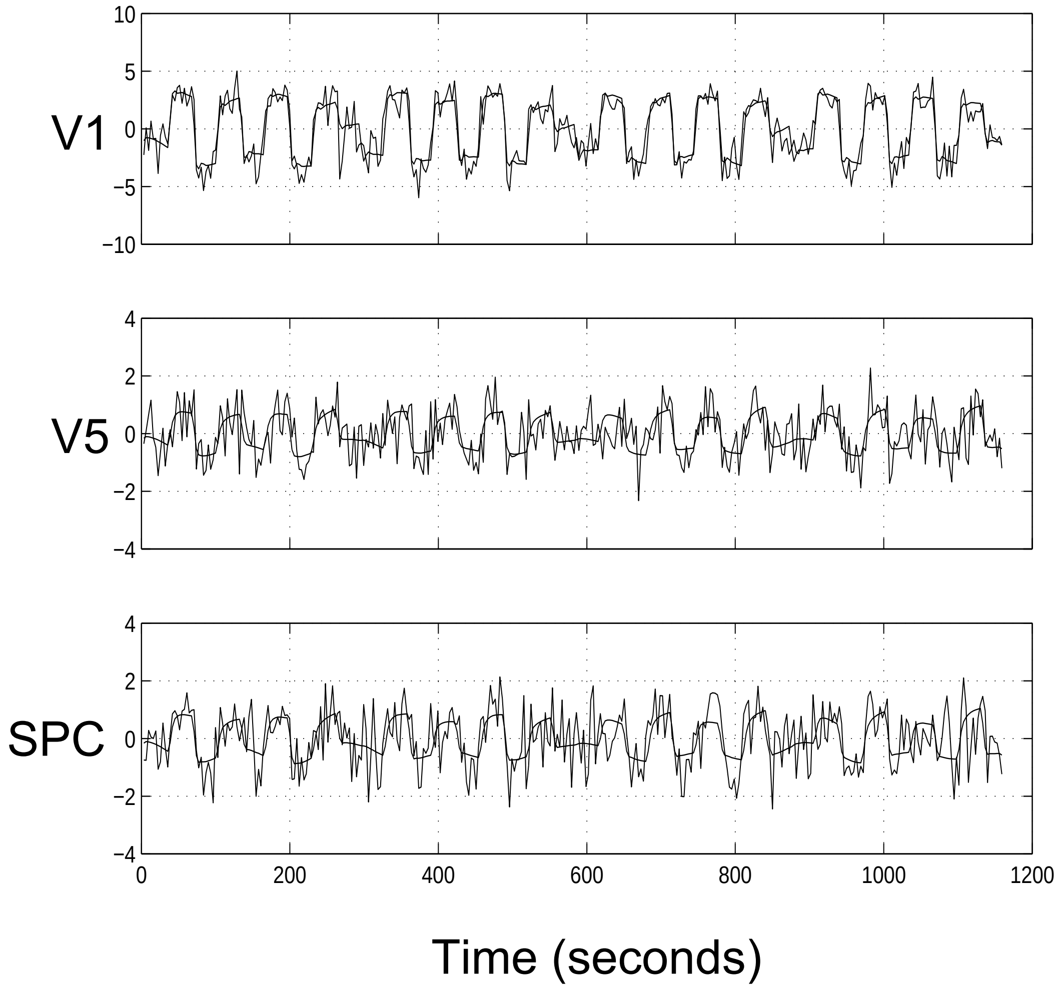


Identify regions of Interest eg. V1, V5, SPC

Estimation

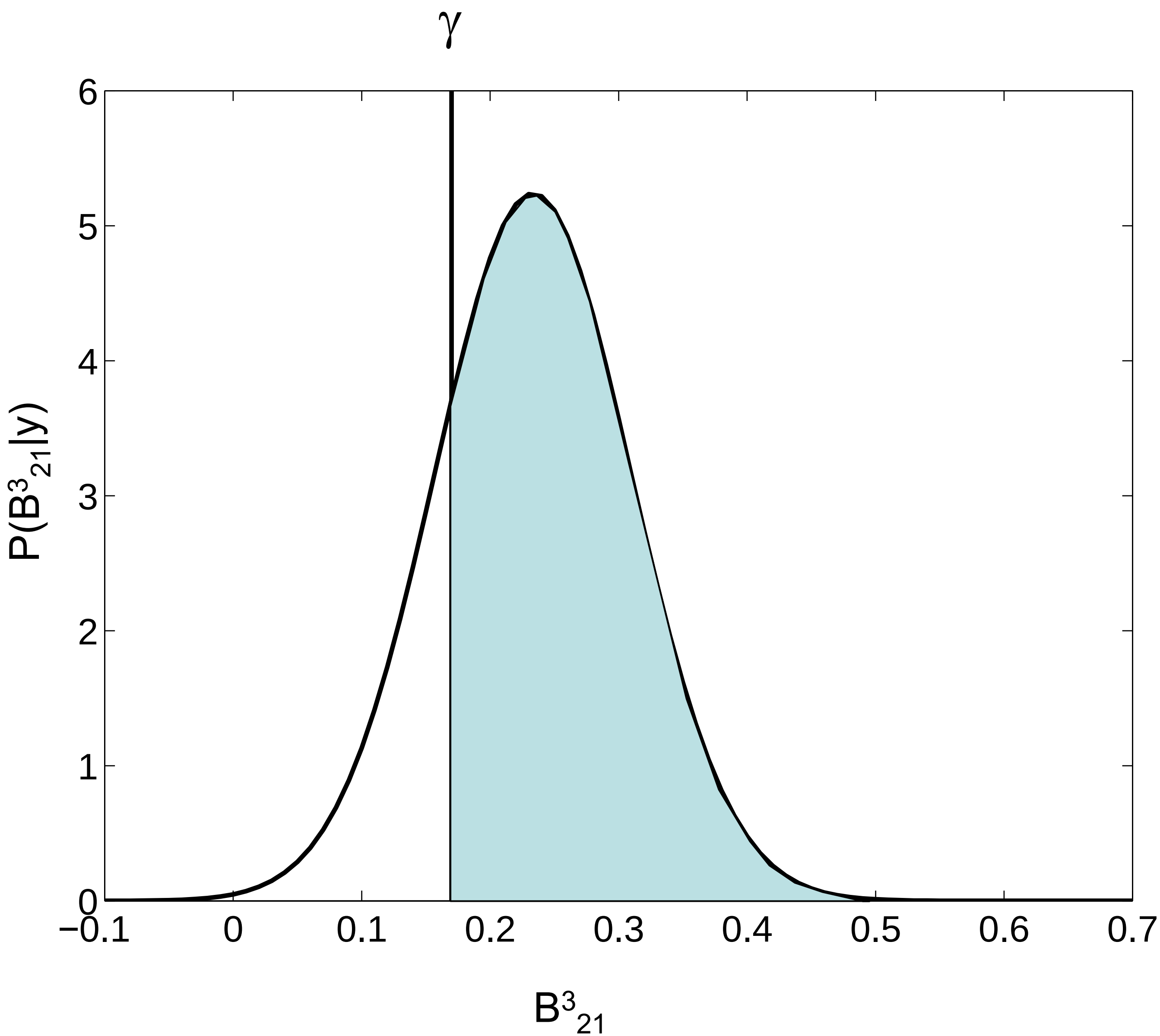


Model 1
Photic
SPC
V1
V5
Motion
Att

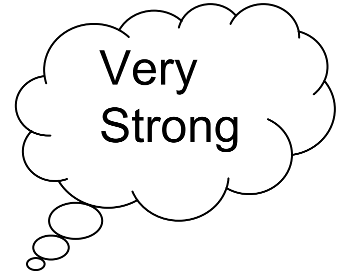


V1
V5
SPC
10
5
0
-5
-10
4
2
0
-2
-4
0
200
400
600
800
1000
1200
Time (seconds)

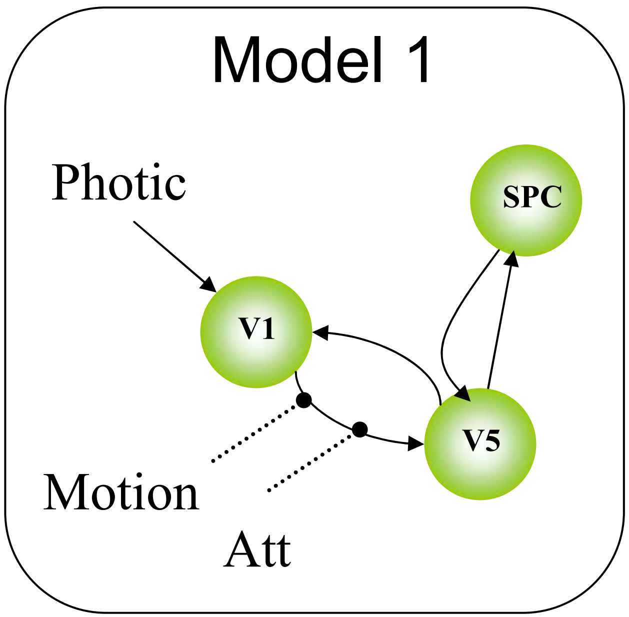
Posterior Inference

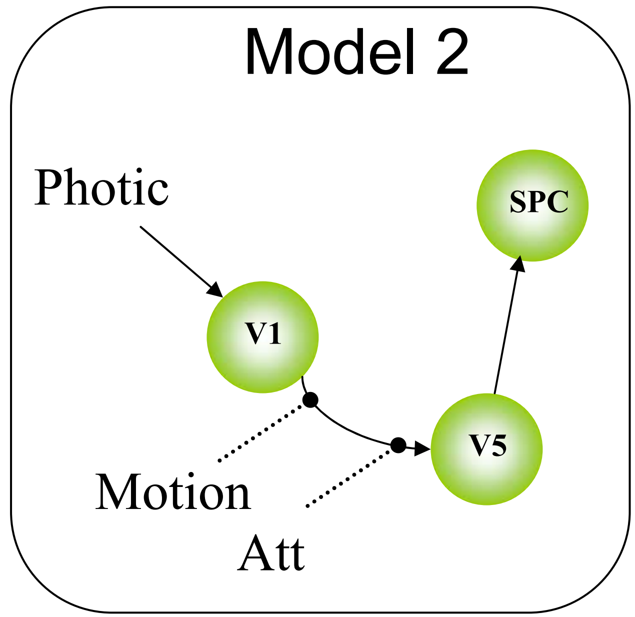


How much attention (input 3) changes connection from V1 (region 1) to V5 (region(2)



Bayes Factor $B_{12} > 10^{19}$

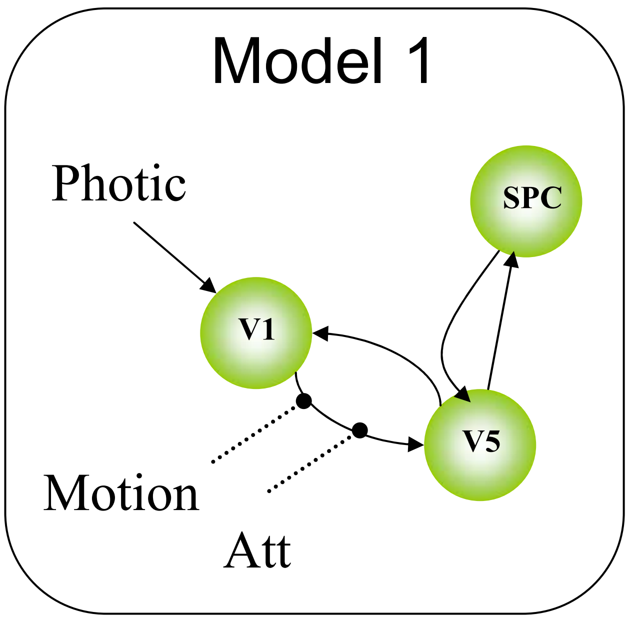


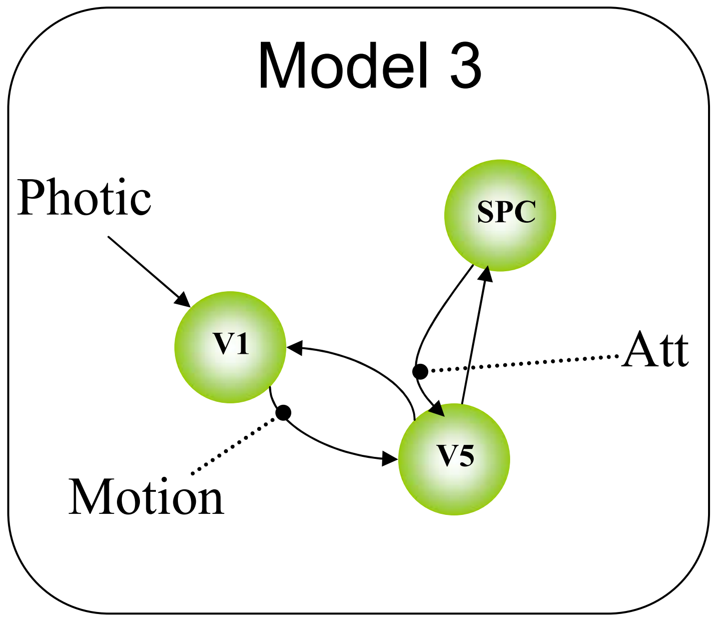


Bayes Factor

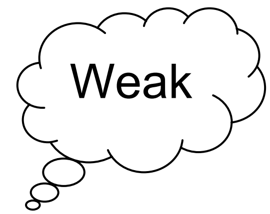
$B_{13}=3.6$

Positive

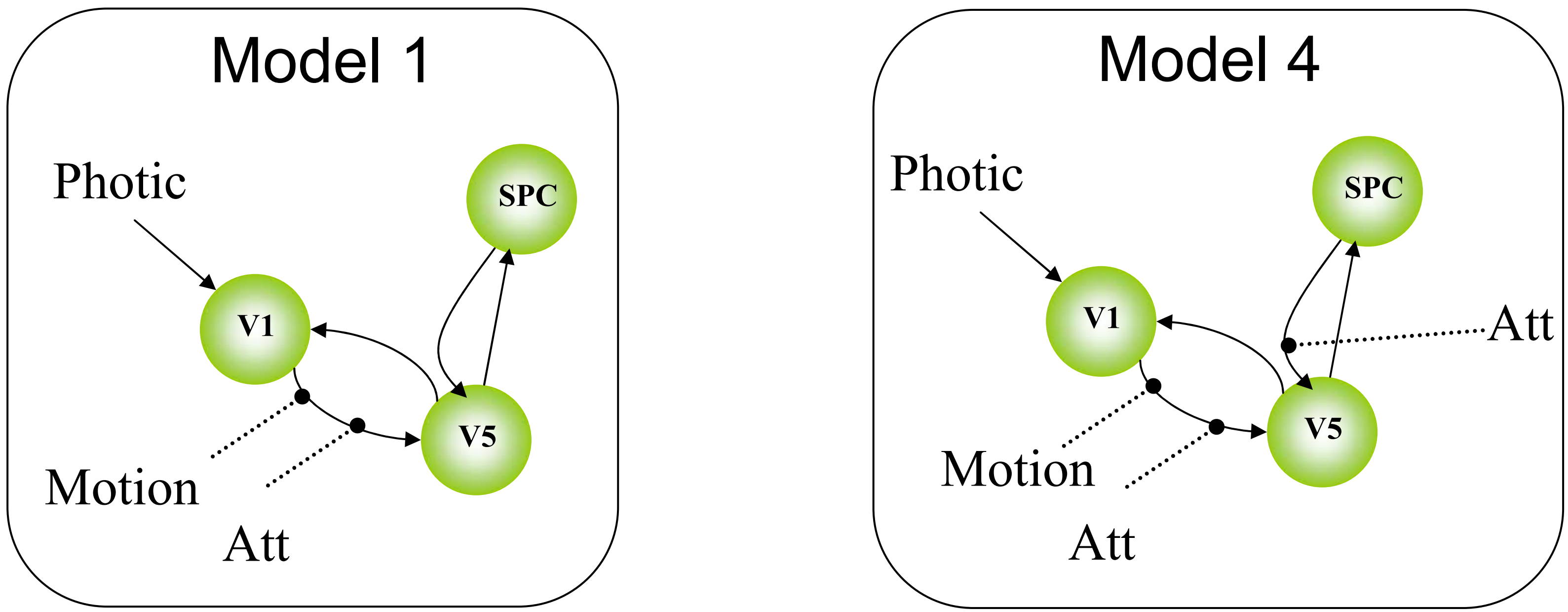




Penny et al. (2004)
NeuroImage, Special Issue.



Bayes Factor $B_{14}=2.8$



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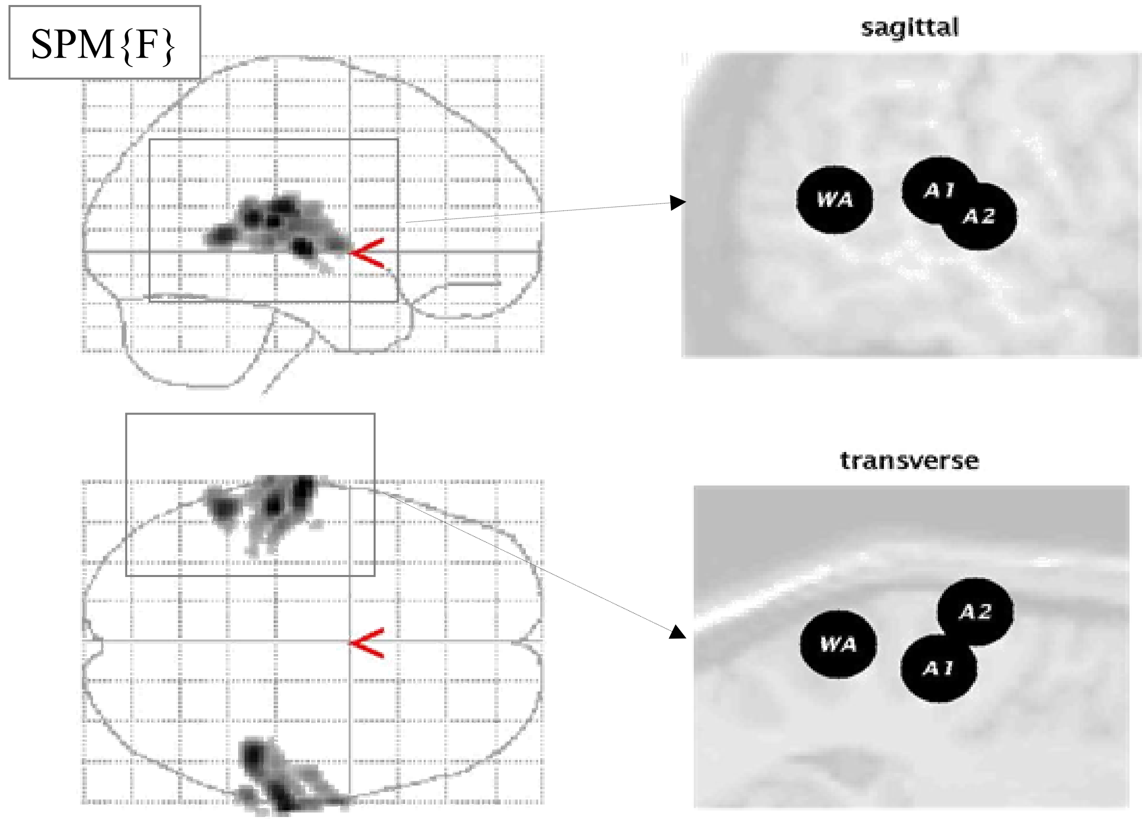
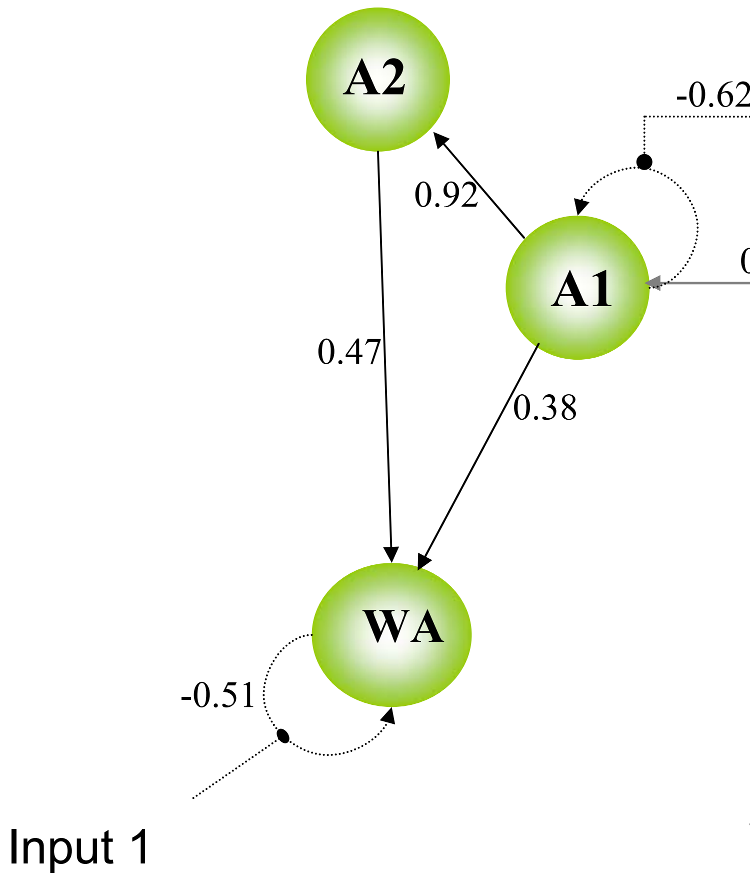
Single word processing

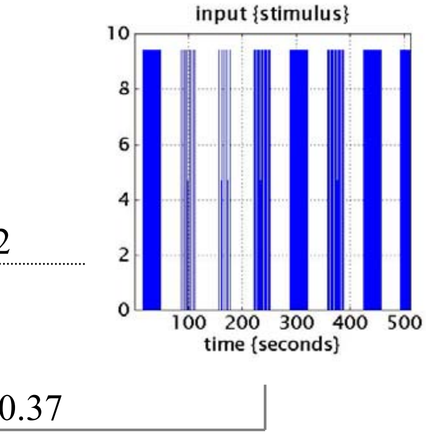
"dog"
"radio"
"gate"
….



10,15,30,
60 and 90
words per
minute

Estimated Model



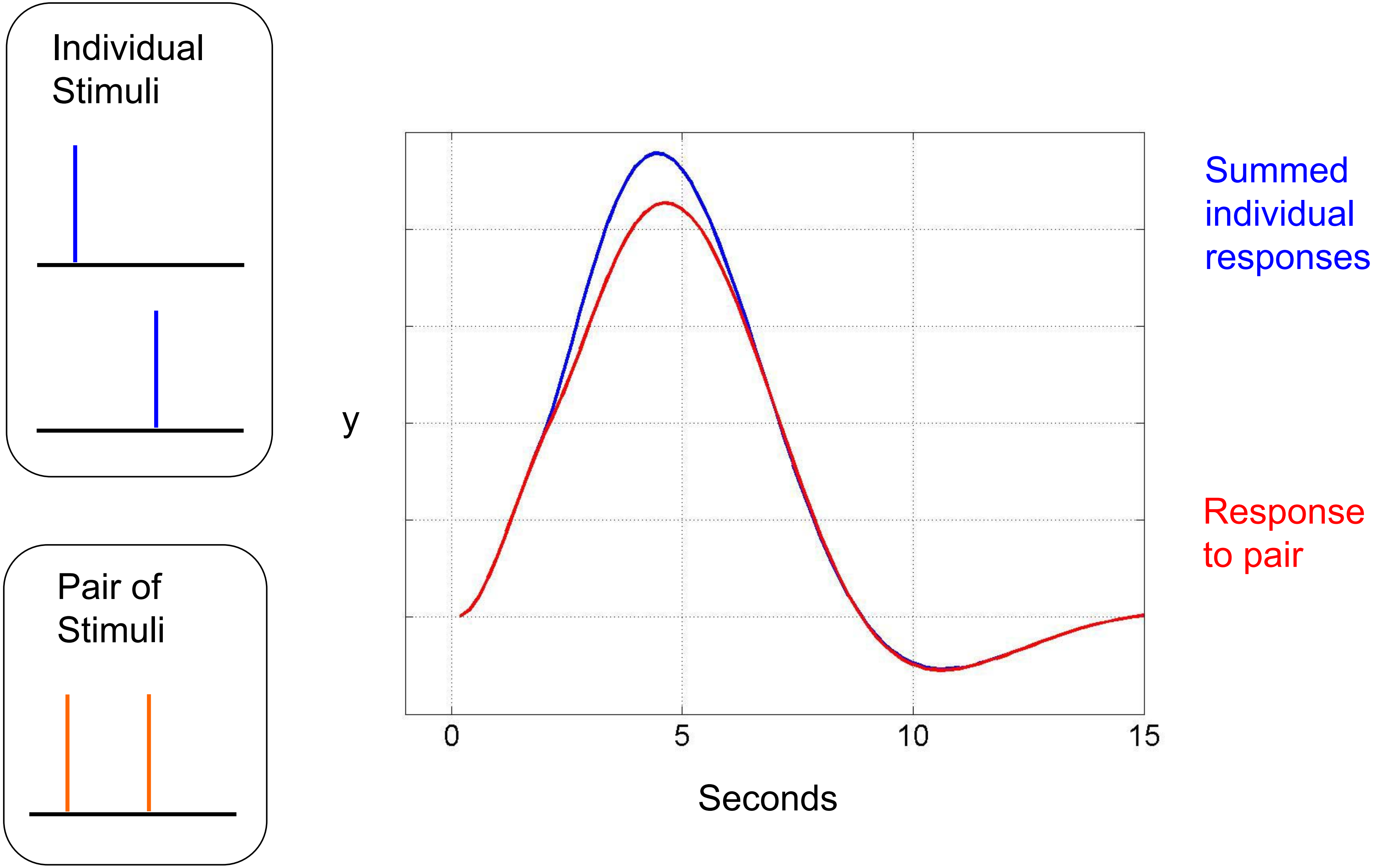


Input 1:
Word Presentation

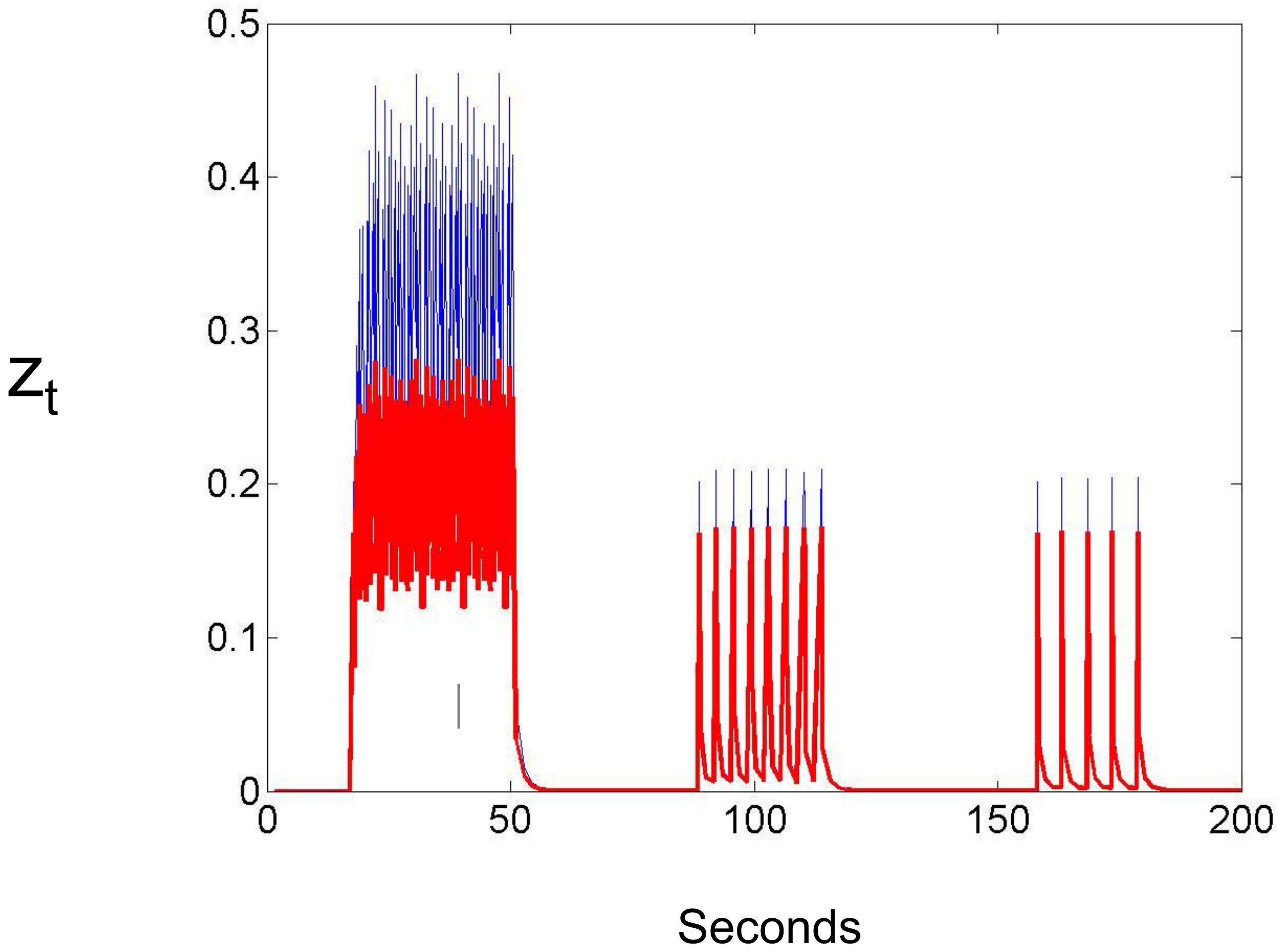
Intrinsic connections: Full connectivity assumed – only significant connections shown.

Modulatory connections: modulation of all self-connections, only A1 and WA significant

Hemodynamic saturation in A1



Neuronal Saturation in A1

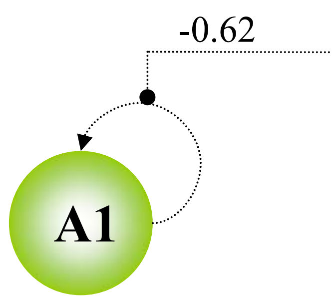


With

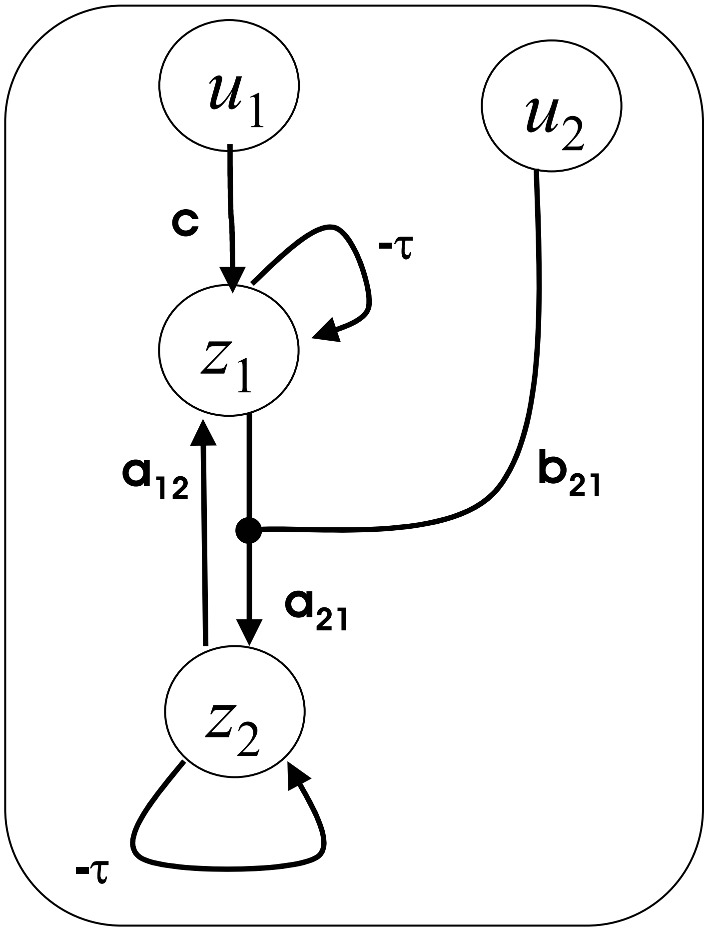
or

without

modulation of A1 self connection



Identifiability



$$A = \tau \begin{bmatrix} -1 & a_{12} & a_{13} \\ a_{21} & -1 & a_{23} \\ a_{31} & a_{32} & -1 \end{bmatrix}$$

Estimation of relative intrinsic connections and modulatory connections is robust to errors in estimation of hemodynamics due to eg. slice timing problems

Indeterminacy in neurodynamic and hemodynamic time constants is soaked up in τ

Summary

- Neurodynamic model
- Hemodynamic model
- Bayesian estimation
- Attention to visual motion
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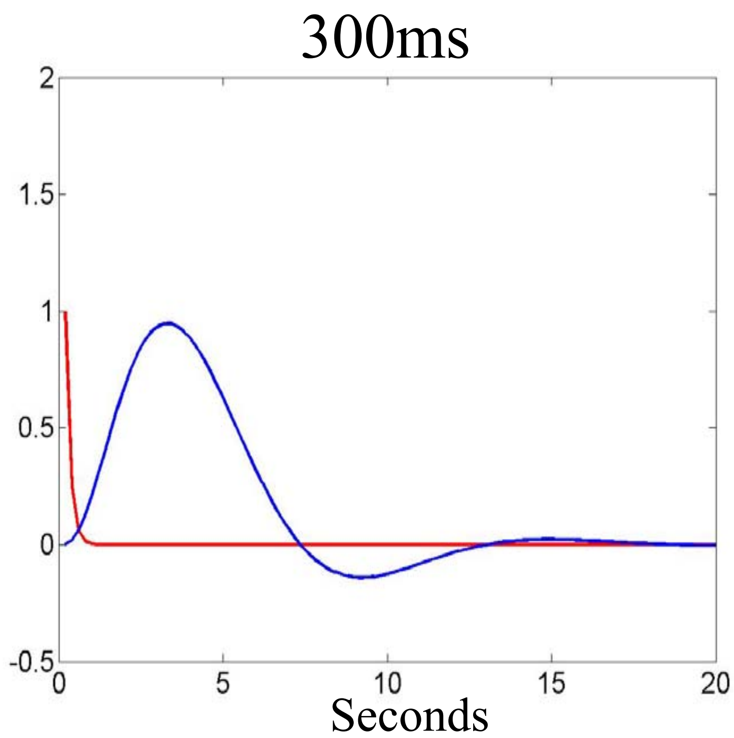
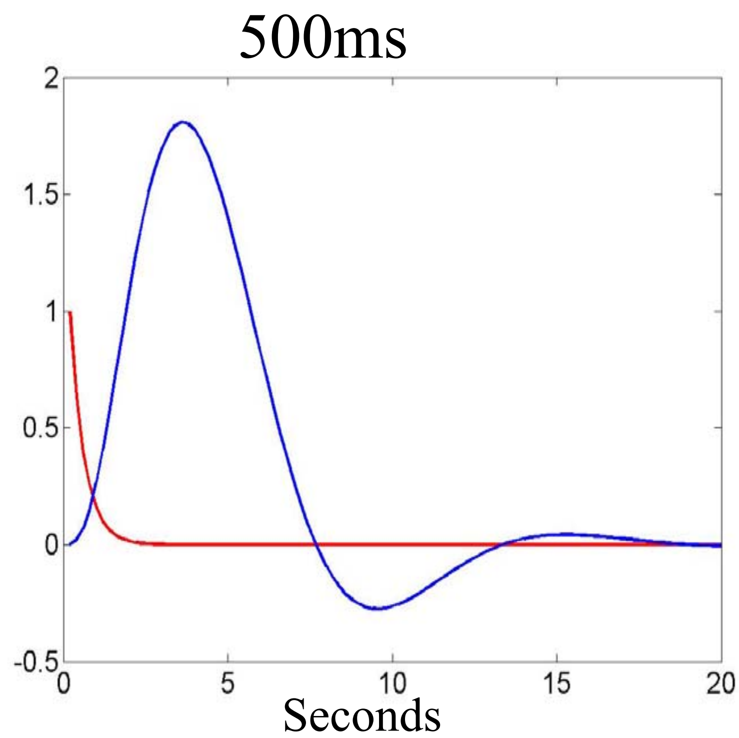
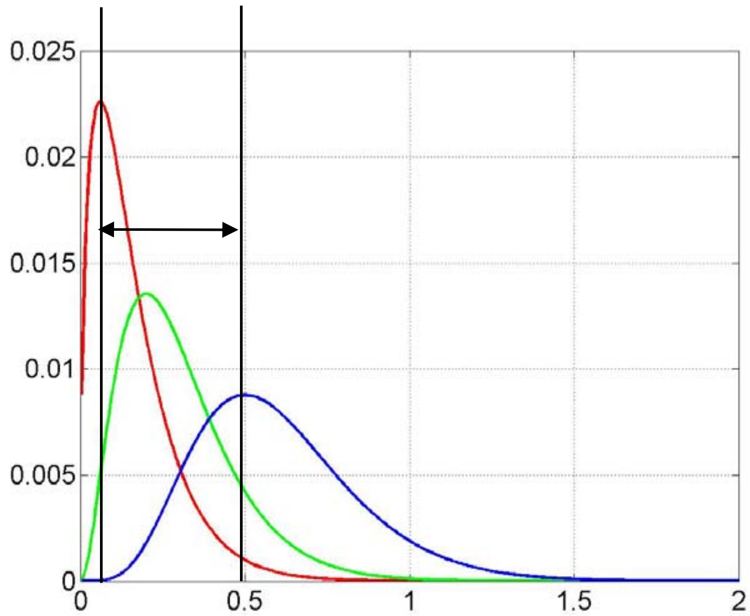
Neuronal Transients and BOLD





More enduring transients produce bigger BOLD signals

Neurodynamics



BOLD is sensitive to frequency content of transients

Relative timings of transients are amplified in BOLD

Hemodynamics