Brain Shape Analysis & Registration
Contributions from the Epidaure group at INRIA

IPAM-MBI 2004
UCLA
Los Angeles

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Brain Shape Analysis

• In this talk:
  • a survey of registration methods developed in our group
  • combined with methods to detect and quantify anatomical or pathological shape variations

• Medical Applications
  • Patient Follow-Up, Image-Guided Neurosurgery,
  • Atlas Construction from Cross-Sections, Atlas Mapping,
  • Brain Asymmetry, Variability of Sulcal Lines, etc.

Overview

- Geometric Registration
- Iconic Registration
- Hybrid Registration
- Shape Statistics
- Perspectives
Geometric Methods

- Extraction of geometric primitives
  - invariant for the chosen group of transformations (typically rigid)

- Registration then consists of
  - matching homologous primitives
  - estimating the transformation $T$

Crest Lines and Extremal Points

- Intersection of 2 or 3 implicit surfaces

\[ f(x,y,z) = I \quad e_1 = \nabla k_1 \cdot t_1 = 0 \quad e_2 = \nabla k_2 \cdot t_2 = 0 \]
Crest Lines on Cortex (MRI)

Compact description
Invariant by displacement
Rigid Matching

- **Adapted** algorithms from **Computer Vision** (Geometric Hashing, Prediction-Verification, **ICP**) establish correspondences between homologous points and best rigid transformation between 2 images.

- **These** algorithms use additional invariants computed along crest lines and on the underlying anatomical surface.
Multiple Sclerosis Follow-Up

Original Sequence  Rigid Registration  Rigid Registration + Intensity Correction

Patient Followed during 18 months (24 acquisitions)


Image acquisition: R. Kikinis
Geometric Registration

- **PROS:**
  - Automatic, no initialization required
  - Accuracy and Robustness

- **CONS:**
  - Requires High Resolution & Low Noise
  - Invariant Landmarks: Rigid, Mono-modal, Single Patient
  - Possible exception: Skull Images
Skull (CT Scan)

Through Aging or Through Ages

1 month  8 months  4 years

• G. Subsol-Meline-Mafart- De Lumley 2002

Content

- Geometric
- **Iconic (Monomodal, Multimodal)**
- Hybrid
- Rule Based
- Shape Statistics
- Perspectives
Demons Algorithm (Thirion)

- Inspired by the work of Christensen, Miller, et al.
- O(N) Algorithm.
- 2 main iterated stages
  - Image forces which create displacements $u_n$ (normalized optical flow)
  - Regularization of $u_n$ by Gaussian Filtering

Demons Algorithm

- \( T_0 = \text{Identity} \)
- Correction field \( C_{n+1} \)
  
  \[
  C_{n+1} = \frac{I - J \circ T_n}{\| \nabla I \|^2 + (I - J \circ T_n)^2} \nabla I
  \]

- Regularization: by Gaussian Filtering
  
  \( \hat{C}_{n+1} = U_n \circ C_{n+1} \)  
  \( U_{n+1} = G_\sigma \ast \hat{C}_{n+1} \)

- Fluid  
  \( \tilde{C}_{n+1} = G_\sigma \ast C_{n+1} \)  
  \( U_{n+1} = U_n \circ \tilde{C}_{n+1} \)

- Remark:  
  \( \| C_{n+1} \| \leq 1/2 \)
Interpretation of Demons

- Pennec-Cachier and then Modersitzki put the demons algorithm in a variational framework to show that it minimizes a global energy.
- Modersitzki: Min $E$ under Neumann BC:

$$E = SSD \ast + \int \|\nabla u\|^2$$


PASHA Algorithm (1/2)

- P. Cachier introduces auxiliary variables (cf. L. Cohen 1996) to formalize the alternate minimization of the Demon’s Algorithm while preserving its efficiency.

\[ E(C, U) = E_s(I, J, C) + \sigma \int \| C - U \|^2 + \lambda \int \| \nabla U \|^2 \]

PASHA Algorithm (2/2)

\[ E(C,U) = E_s(I,J,C) + \sigma \int \| C - U \|^2 + \lambda \int \| \nabla U \|^2 \]

- Minimization on \( C \) by differentiation of the similarity criterion (gradient descent, 1st or 2nd order)
- Minimization on \( U \), explicit solution by Gaussian convolution
Mixed Elastic/Fluid Regularization

\[ E(C,U,\dot{U}) = E_s(I,J,C) + \sigma \int \| C - U \|^2 + \lambda \int \| \nabla U \|^2 + \mu \int \| \nabla \dot{U} \|^2 \]

- **Advantage:**
  - regularization still by convolution
  - can handle larger displacements

Nice Properties of PASHA

- Alternate minimization of a single positive criterion : Convergence
- Algorithmic Complexity O(N)
- Smooth deformation field:
  - Careful gradient descent \( \|C_{n+1}\| \leq 1/2 \)
  - Eulerian scheme for interpolation
  - Regularization by “low pass” filters
Symmetric energies

- **Similarity**

\[ SSD(I, J, C) = \int (I - J \cdot C)^2 \neq SSD(J, I, C^{-1}) \]

\[
E^*_S(I, J, C) = \frac{1}{2} \left[ E_s(I, J, C) + E_s(J, I, C^{-1}) \right]
\]

\[
SSD^*(I, J, C) = \frac{1}{2} \int (1 + |\nabla C|)(I - J \cdot C)^2
\]

- **Regularization**

\[ E_\nabla (I, J, U) = \int \| \nabla U \|^2 \neq E_\nabla (J, I, U^{-1}) \]

\[
E^*_\text{reg}(U) = \frac{1}{2} \left[ E_{\text{reg}}(U) + E_{\text{reg}}(U^{-1}) \right]
\]

\[
E^*_{\nabla}(U) = \frac{1}{2} \int \left( 1 + \frac{1}{|\nabla U|} \right) \| \nabla U \|^2 \]

[ P. Cachier, D. Rey, MICCAI’00 ]

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Epidaure
Quantifying Apparent Brain Variations

• Introduce Differential Operators to the deformation field to detect non-rigid brain variations

• Exemple : Jacobian of deformation field:
  • J = 1 for rigid transformation
  • J > 1 for local expansion
  • J < 1 for local contraction
1. Multiple Sclerosis Evolution

Time $i$  
Time $i+1$

Apparent Deformation Field
Apparent Residual Deformations

Time $i$  
Aligned Time $i+1$
Isovalues of Log\([\text{Jacobian}]\)
Detection of evolving lesions

\[ \ln(\text{jac}) \geq 0.4 \]
Symmetric Energies (1/2)

axial  coronal  sagittal

T2- MRI 0.89x0.89x5.5mm

DIRECT T

log(Jac)= +1

[ P. Cachier, D. Rey, MICCAI’00 ]

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Symmetric Energies (2/2)

axial  coronal  sagittal

Time i  log(Jac) = -1  Time i+1

T2- MRI 0.89x0.89x5.5mm

[ P. Cachier, D. Rey, MICCAI’00 ]
Temporal Evolution of Lesions

2. Brain Asymmetry

- Local and quantitative measure of cerebral asymmetry

PhD Thesis of S. Prima
Biomorph Project
A. Colchester, G. Gerig, M. Brady, T. Crow, et al.
Stage 1: Find Mid-Sagittal Plane

- Find Plane which minimizes SSD criterion between homologous points
- Rotate image to place MSP in a reference position

Mid-Sagittal Plane

\[ R = \left( S_K \circ S_P \right)^{1/2} \]
Estimation of coronal mid-sagittal plane

Stage 2: Quantify Asymmetry

- Compute deformation field $\mathbf{F}$ between each hemisphere and its symmetrized version

- Quantify “deviation” from rigid transformations
  - several differential operators including Jacobian
  - good experimental results with $||\mathbf{F}|| \cdot \text{div} (\mathbf{F})$

Asymmetry Field (Synthetic Example)
Asymmetry Field (Real Example)
Controls vs. Schizophrenics

Reference Image | 10 Controls | 10 Patients | Statistically meaningful differences

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Content

- Geometric
- Iconic (Monomodal, Multimodal)
- Hybrid
- Shape Statistics
- Perspectives
Comparing Multimodal images?

• Which Similarity Criterion?

  • Numerous criterions available:
    • SSD, Correlation, Mutual information,...?

  • Variable costs and performances

  • Which one is optimal?

Maintz & Viergever, Survey of Registration Methods, Medical Image Analysis 1997
A general framework

- A. Roche proposed a unifying maximum likelihood framework
- Based on a physical and statistical modeling of the image acquisition process
- Creates a hierarchy of criteria, introduces new ones (correlation ratio)


- Following the pioneering work of (Costa et al, 1993), (Viola, 1995), (Leventon & Grimson, 1998), (Bansal et al, 1998)
Maximum Likelihood Formulation

- General dependence model (Roche et al.)

\[
\theta_S \quad \text{Statistical model} \quad \rightarrow \quad \text{Scene } S
\]

\[
T \quad \text{Spatial transfo.} \quad \rightarrow \quad \theta_I \quad \text{Statistical model} \quad \rightarrow \quad \text{Image } I
\]

\[
\theta_J \quad \text{Statistical model} \quad \rightarrow \quad \text{Image } J
\]

- Maximum Likelihood

\[
\max_{T, \theta_I, \theta_J, \theta_S} P(I, J | T, \theta_I, \theta_J, \theta_S)
\]

Auxiliary parameter: \( \theta \)
Optimal Criterion for Intensity Similarity

Identity: Sum of Square Differences

\[ ssd^2 = \sum_k (I(x_k) - J(T(x_k)))^2 \]

Affine: Correlation Coefficient

\[ \rho^2 = \frac{\text{Cov}^2(I, J(T))}{\text{Var}(I)} \]

Functional: Correlation Ratio

\[ \eta^2 = 1 - \frac{\text{Var}(E(I / J(T)))}{\text{Var}(I)} \]

Statistical: Mutual Information

\[ I(I,J) = H(I) + H(J) - H(I,J) \]
Roboscope: Quantify Brain Deformation during Neurosurgery

Image-Guided Manipulator-Assisted Neuro-Endoscopy

Courtesy Brian Davies
MR-US Images

Pre - Operative MR Image

Per - Operative US Image

Acquisition of images: L. & D. Auer, M. Rudolf

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Physics

- Physics of ultrasound and MRI show that as a first approximation, it is reasonable to assume a dependence of the US signal as a function of MR intensity and gradient.

\[ = \text{function} ( , , ) \]
Bivariate Correlation Ratio

- **I function of 2 variables**
  \[ I = f(J, |\nabla J|) \]

- **2 iterated stages**
  - Robust polyn. approx. of \( f \)
  - Estimation of \( T \):
    \[
    \hat{T} = \arg\min_T \sum_k (I(x_k) - \hat{f}(J(T(x_k)), |\nabla J(T(x_k))|))^2
    \]

A. Roche, X. Pennec, G. Malandain, and N. Ayache
Rigid Registration of 3D Ultrasound with MR Images: a New Approach Combining Intensity and Gradient Information.
Typical Registration Result
with Bivariate Correlation Ratio

Pre - Operative MR Image

Per - Operative US Image

Axial

Coronal

Sagittal

Registered

Acquisition of images: L. & D. Auer, M. Rudolf

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Accuracy/Robustness

Sensitivity to initialization

200 registration with

- 15 deg random rotation
- 20 mm random translation

Bronze standard

- registration loops

<table>
<thead>
<tr>
<th></th>
<th>Failures</th>
<th>Mean accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biv. CR</td>
<td>6%</td>
<td>0.9 mm / 0.9 deg</td>
</tr>
<tr>
<td>Corr. ratio</td>
<td>12%</td>
<td>0.9 mm / 0.7 deg</td>
</tr>
<tr>
<td>Mutual info.</td>
<td>28%</td>
<td>1.4 mm / 0.8 deg</td>
</tr>
</tbody>
</table>

[Roche, Pennec, Malandain, Ayache, IEEE TMI 20(10), 1038-1049, Oct. 2001]
Tracking US Images

- Parallel version of Pasha
Metamorphosis

t=0

\[ t = 0 \]

\[ t = 0.5 \]

\[ t = 1 \]
Metamorphosis
Interpolation and Extrapolation

\begin{array}{cccc}
\text{t=0} & \text{t=0,5} & \text{t=1} & \text{t=1,5} \\
\text{neutral} & \text{attenuated expression} & \text{expression (smile)} & \text{exagerrated expression} \\
\end{array}
Interpolation of 2 images

- Slow down motion in video sequences

Original sequence of 7 images
Slow down 10 times, 61 images

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Content

- Geometric
- Iconic (Monomodal, Multimodal)
- Hybrid
  - Geometric & Iconic;
  - Bloc Matching:
  - Piecewise parametric
Geometric-Iconic-Semantic

JF. Mangin, D. Rivièrè, SHFJ-CEA
Concerted action : CEA-Epidaure-Robotvis-Salpêtrière-Vista

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Intersubject Brain Registration

- Geometric approach to match homologous sulci
- Iconic approach otherwise

Cortex 1

Cortex 2
Extension of Pasha Algorithm

- Add geometrical constraints $C_2$ between homologous sulci

$$E(C_1, C_2, U) = E_s(I, J, C_1) + \sigma_1 \int \| C_1 - U \|^2 + \sigma_2 \int \| C_2 - U \|^2 + \lambda \int \| \nabla U \|^2$$

[ P. Cachier et al, MICCAI 2001 ]

Min. $C_1$: gradient descent
Min. $C_2$: closest point
Min. $U$: explicit solution (convolution+spline)
Three Sulcal Lines

[Precentral]

[Central]

[Lateral]

[Rivière et al. 00]
Results with 5 subjects

Affine Initialization

Iconic

Iconic + Geometric

Multisubject Non-Rigid Registration of Brain MRI using Intensity and Geometric Features.
MICCAI'01, LNCS vol 2208, 734-742, 2001.
Results with 5 subjects

A Iconic + Geometric

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  - Bloc Matching:
  - Piecewise parametric
Histological Atlases

- Built from histological 2-D cross-sections
  - microscopic, macroscopic optical images
  - autoradiographies

- Fusion with 3-D medical images
  - for localisation or validation purposes

Mapawamo Project

- **Objectives**
  - Mapping visual cortical regions in awake, behaving monkey using functional MRI
  - Compare fMRI results with standard metabolic mapping (ground truth): double label 2deoxyglucose - 2DG

- Coordinator: G. Orban (Louvain), partners involve Odyssée, Epidaure,...
- Work of S. Ourselin, E. Bardinet, G. Malandain (Epidaure/INRIA)
Two registration problems

2-D --> 3D autoradiographies

3-D Autoradiographies & 3-D Anatomy Fusion
Registration by Block Matching

Displacement field

Floating image $I_1$ Reference image $I_2$

Robust & Multiscale Estimate of Rigid/Affine Global transformation

Following Z. Zhang et al.

Sébastien Ourselin Thesis
Ourselin et al., IVC 2000

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65
Alignment results
Posterior part

14C
Anterior part

14C
MRI / Autoradiography Registration

E. Bardinet, G. Malandain
Mapawamo Project
New Atlas of Deep Nuclei

- Built from histological cross-sections
- Fused with post-mortem MRI

(INRIA & Inserm U 289, Pitié-Salpêtrière)

S. Ourselin, E. Bardinet, J. Yelnik, D. Dormont et al., MICCAI’01
Content

- Geometric
- Iconic (Monomodal, Multimodal)
- Hybrid
  - Geometric & Iconic;
  - Bloc Matching:
  - Piecewise parametric
Piecewise Affine Registration

- Hierarchical Clustering
- …Not a diffeomorphism

Pitiot, Bardinet
Thompson, Malandain
WBIR’03, Philadelphia
Polyrigid Transformations

- N Components:
  - Local Rigid transformation: \( T_i(x) = R_i.x + t_i \)
  - Gaussian spatial influence
    - anchor point: \( a_i \)
    - local weight: \( p_i \)
    - influence distance: \( \sigma_i \)

- Direct averaging of transformations is not invertible
  \[
  T(x) = \frac{\sum_i w_i(x)T_i(x)}{\sum_i w_i(x)}
  \]

Polyrigid Transformations

- for each rigid component, the trajectory satisfies the following ODE ($A_i = \log(R_i)$):

$$\dot{x}(s) = V_i(x, s) = t_i + A_i(x - s \ t_i) \text{ for } s \in [0, 1]$$

- Idea: averaging speed vectors:

$$\dot{x}(s) = V(x, s) = \frac{\sum_i w_i(x) V_i(x, s)}{\sum_i w_i(x)}$$

- Global transformation found by integration:
  - diffeomorphism regular with respect to each parameter
Polyrigid Transformations

- 4 Rigid Components
- Optimized by Gradient Descent (ITK)

Rigid registration   Affine registration   PRT registration   PRT deformed grid

Difference images

Special Issue of Medical Image Analysis Journal on ITK, 2005

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Content

- Geometric
- Iconic (Monomodal, Multimodal)
- Hybrid
- Shape Statistics
  - Revisiting Regularization
  - Statistics on Sulcal Lines
Revisiting Regularization

\[
E(C,U,\dot{U}) = E_S(I,J,C) + \sigma \int \| C - U \|^2 \\
+ \lambda \int \| \nabla U \|^2 + \mu \int \| \nabla \dot{U} \|^2
\]

- Modulate regularization as a function of
  - 1- local information (presence of texture/edges)
  - 2- local variability (statistics on anatomy)

R. Stefanescu, X. Pennec, N. Ayache, *Grid Powered Nonlinear Image Registration with Locally Adaptive Regularization*, Medical Image Analysis, Sept 2004 (also MICCAI’03)
1. Non Stationary Fluid Regularization

Inspired from non-stationary image diffusion
- Weickert 1997, 2000

Confidence in the correction field
- $k \sim 1$ for edges (driving forces)
- $k \sim 0$ for uniform regions (interpolation)

- Used to model pathologies (e.g. tumors)

\[
\frac{\partial \dot{U}}{\partial t} = (1 - k) \Delta \dot{U}
\]
Patient with Pathology

Fuzzy segmentation of the resection

Confidence

Low confidence values in the resection region

Patient T1-MRI
Atlas and Patient with Pathology

Initialization: affine registration maximizing the correlation ratio

Atlas
Non-Rigid Atlas to Subject Registration with Pathologies for Conformal Brain Radiotherapy.

Patient T1-MRI

Data courtesy of Dr. Pierre-Yves Bondiau, M.D., Centre Antoine Lacassagne, Nice, France

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INRIA
Epidaure
Registration Result

Resection is “preserved”

Atlas

Patient T1-MRI

Data courtesy of Dr. Pierre-Yves Bondiau, M.D., Centre Antoine Lacassagne, Nice, France

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Epidaure
Classical (wrong) Registration

Wrong registration

Atlas
Non-Rigid Atlas to Subject Registration with Pathologies for Conformal Brain Radiotherapy.

Data courtesy of Dr. Pierre-Yves Bondiau, M.D., Centre Antoine Lacassagne, Nice, France

Patient T1-MRI

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2- Non Stationary Elastic Regularization

\[ \frac{\partial U}{\partial t} = \text{div}(D \nabla (U)) \]

\( D \) encodes a priori variability
Algorithm Complexity

- Parallel implementation on cluster of PC’s
  - Efficient parallel AOS scheme for diffusion PDEs:
  - Image size: 256x256x124
  - 15 PC’s: 2GHz, Pentium IV processors
  - Execution time: 5 minutes

Content

- Geometric
- Iconic (Monomodal, Multimodal)
- Hybrid
- Shape Statistics
  - Revisiting Regularization
  - Statistics on Sulcal Lines
Statistics on Sugal Lines

- **Goal:**
  - Learn local brain variability from sulci
  - Better constrain inter-subject registration
  - Correlate this variability with age, pathologies

Collaborative work between Epidaure (INRIA) and LONI (UCLA)
V. Arsigny, N. Ayache, P. Fillard, X. Pennec and P. Thompson
Computation of Average Sulci

- Alternate minimization of global variance
  - Dynamic programming to match the mean to instances
  - Gradient descent to compute the mean curve position

Arsigny et al. 2004, to appear

Sylvius Fissure

red : mean curve
green et yellow : ~80 instances of 72 sulci
Covariance Tensors

Currently:
80 instances of 72 sulci
About 1250 tensors

Fillard, Pennec, Ayache, Thompson, 2004, to appear

Color codes Trace

Covariance Tensors along Sylvius Fissure
Tensor Computing

- Tensors = Symmetric Definite Positive Matrices
- Various operations
  - regularization, interpolation, compression, extrapolation
  - statistical comparisons
- Previous work includes Statistics on Manifolds and Tensor Computations
  - Skovgaard84, Pennec96&99&04, Pennec-Ayache98, Forstner-Moonen99, Poupon00, Alexander01, Tschumperlé02, Chef’d’hôtel02&04, Lenglet04, Coulon04, Fletcher-Joshi04, etc.
Affine Invariant Metric

- Action of Linear Group $\text{GL}_n$

$$\forall A \in \text{GL}_n, A \ast \Sigma = A \Sigma A^T$$

- Affine Invariant Distance

$$\text{dist}(A \ast \Sigma_1, A \ast \Sigma_2) = \text{dist}(\Sigma_1, \Sigma_2), \forall A \in \text{GL}_n$$

Scalar product on $T_{Id}M$ :

$$\langle W_1 \mid W_2 \rangle_{Id} \overset{\text{def}}{=} \text{Tr}(W_1^T W_2) = \text{Tr}(W_1 W_2)$$

$W_1, W_2 \in T_{Id}M$

Scalar product on $T_{\Sigma}M$ :

$$\langle W_1 \mid W_2 \rangle_{\Sigma} \overset{\text{def}}{=} \langle \Sigma^{-1/2} \ast W_1, \Sigma^{-1/2} \ast W_2 \rangle_{Id}$$

Exponential and Logarithmic Maps

- Geodesics
  \[ \Gamma_{Id,W}(t) = \exp(tW) \]

- Exponential Map:
  \[ \exp_\Sigma(\Sigma\Psi) = \Sigma^{1/2} \exp(\Sigma^{-1/2} \Sigma\Psi \Sigma^{-1/2}) \Sigma^{1/2} \]

- Logarithmic Map:
  \[ \log_\Sigma(\Psi) = \Sigma^{1/2} \log(\Sigma^{-1/2} \Psi \Sigma^{-1/2}) \Sigma^{1/2} \]

\[ \text{dist}(\Sigma, \Psi)^2 = \left( \frac{\Sigma\Psi}{\Sigma} \right)^2 = \left\| \log(\Sigma^{-1/2} \Psi \Sigma^{-1/2}) \right\|_{L^2}^2 \]
Linear vs. Riemannian Interpolation

Interpolation of the coefficients:

Interpolation achieved with the Riemannian metric:
Compressed Tensor Representation

- Mean sulcal line + 4 covariance matrices
  - optimize for the 4 most representative tensors
  - Interpolation in-between, extrapolation outside (removes outliers)

Sylvian fissure
Compressed Tensor Representation

Representative Tensors (250)  Reconstructed Tensors (1250)
(Riemannian Interpolation)

Fillard-Pennec-Thompson-Ayache 2004, to appear
Variability Tensors

Color codes tensor trace

Fillard-Pennec-Thompson-Ayache 2004, to appear
Asymmetry Measure

Color Codes Distance between “symmetric” tensors

$$dist(\Sigma, \Sigma')^2 = \langle \Sigma \Sigma' | \Sigma \Sigma' \rangle_{\Sigma} = \| \log(\Sigma^{-1/2} . \Sigma' . \Sigma^{-1/2}) \|_{L_2}^2$$
Extrapolation by Diffusion

- sources = tensors at given positions
- smooth extrapolation
Extrapolation by Diffusion

• Minimize

\[ C(\Sigma) = C_A(\Sigma) + C_D(\Sigma) \]

\[ = \frac{1}{2} \int_{\Omega} \sum_{i=1}^{n} G_\sigma(x - x_i) \text{dist}(\Sigma(x), \Sigma_i)^2 \, dx + \frac{\lambda}{2} \int_{\Omega} \|\nabla \Sigma\|^2 \]

• Evolution Equation:

\[ \Sigma_{t+1}(x) = \exp_{\Sigma_i(x)} \left( -\varepsilon \nabla C(\Sigma)(x) \right) \]

\[ \nabla C(\Sigma)(x) = -\sum_{i=1}^{n} G_\sigma(x - x_i) \Sigma(x) \Sigma_i - \lambda (\Delta \Sigma)(x) \]
Extrapolation by Diffusion

\[ C(\Sigma) = \frac{1}{2} \int_{\Omega} \sum_{i=1}^{n} G_{\sigma}(x-x_i) \text{dist}(\Sigma(x), \Sigma_i)^2 \, dx + \frac{\lambda}{2} \int_{\Omega} \| \nabla \Sigma \|^2 \]

Original Tensor Data  | Diffusion Without data attachment  | Diffusion with data attachment

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Full Brain Interpolation

Color code: Principal Eigenvector
- red: left-right,
- green: posterior-anterior,
- blue: inferior-superior

Color Code: Trace

Fillard-Pennec-Thompson- Ayache 2004, to appear
Full Brain Interpolation

Principal Eigenvector

left-right,

post.-anterior,

inf.-superior

temporal

Parietal

Trace
Full Brain Interpolation

Principal Eigenvector

Trace

Fillard-Pennec-Thompson- Ayache 2004, to appear
Anisotropic Filtering

Original Tensor Field

Noisy Tensor Field

Filtered Tensor Field
Anisotropic Filtering

- Raw tensors
- Gaussian Flat Metric
- Gaussian Riemann Metric
- Anisotropic Riemann Metric
Next Stages

- Learn Variability from Large Group Studies
- Statistical Comparisons between Groups
- Exploit Learned Variability to Improve Inter-Subject Registration
Content

- Geometric
- Iconic (Monomodal, Multimodal)
- Hybrid
- Shape Statistics
- Perspectives
Registration and Shape Variations

- Registration tools, based on geometrical and/or physical models
- Differential operators on deformation fields to detect and quantify local shape variations
- Tensor fields to encode local variability and adapted tensor metric to compare tensors
- Possibility to learn variability and improve non-rigid registration
Some Remaining Challenges

- **Shape Statistics**: Avoid any registration?
- **Validating non-rigid registration**
  - intra-subject: e.g. Truth Cube at Harvard,
  - inter-subject: e.g. Bronze Standard (Pennec et al.)
- **Microscopic imaging**:
  - Detect shape variations at microscopic level
  - Correlate with macroscopic changes
In Vivo “Endoscopic” Observations of Rat Bladder

- 1 mm probe introduced via catheter
- real-time dynamic images of tissue surface with cellular resolution (cf. movie below)

Images obtained with the team of Prof. Guillemin at the Centre Alexis Vautrin, National Cancer Center in Nancy, France

http://www.maunakeatech.com/
Neurobiology

Transgenic mice expressing EGFP in neurons and dendrites

Field: 400 x 280 μm
Proflex diameter: 800μm

In VITRO visualization of lateral dendrites from the basal pyramidal neuron layer

Images obtained by Mauna Kea Technology with the team of Prof. Changeux at Institut Pasteur, Paris.

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Epidaure
Micro-circulation

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- Live images of micro-vessels

Leukocytes:
Size = 7-11 microns

Study of
- Angiogenesis,
- drug delivery
- cardiovascular diseases
- etc.

Images obtained by Mauna Kea Technology with the team of Prof. Eric Vicaut at Hôpital Lariboisière, Paris.
Dynamic images of microcirculation

- Live images of micro-vessels
- Angiogenesis, drug delivery, cardiovascular disease management.

Images obtained with the team of Prof. Eric Vicaut at Hôpital Lariboisière, Paris.

http://www.maunakeatech.com/
Real-Time Blood Flow Measurement

- Average speed: 7.18 mm/s; Std Deviation: 0.7 mm/s

Brain Deformation & Tumor Growth

- Coupling physiological model of tumor growth with geometrical and physical (biomechanical) models

- In Vivo Cellular Imaging to Calibrate the model

Brain Tumor Growth Simulation.
March         September  September
Simulation

O. Clatz, P.Y. Bondiau, H. Delingette,
M. Sermesant, S. K. Warfield,
G. Malandain, N. Ayache.
Brain Tumor Growth Simulation.

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Credits

• Epidaure Team *past/current members*


• Academic & Clinical partners

  • L. Auer, D. Dormont, R. Kikinis, C. Lebrun, J.F. Mangin, D. Rivière, P. Thompson, J. Yelnik, S. Warfield etc.
Thank You

Publications available on line  http://wwwsop.inria.fr/epidaure/