

The cost of brain state transitions

Kate Brynildsen



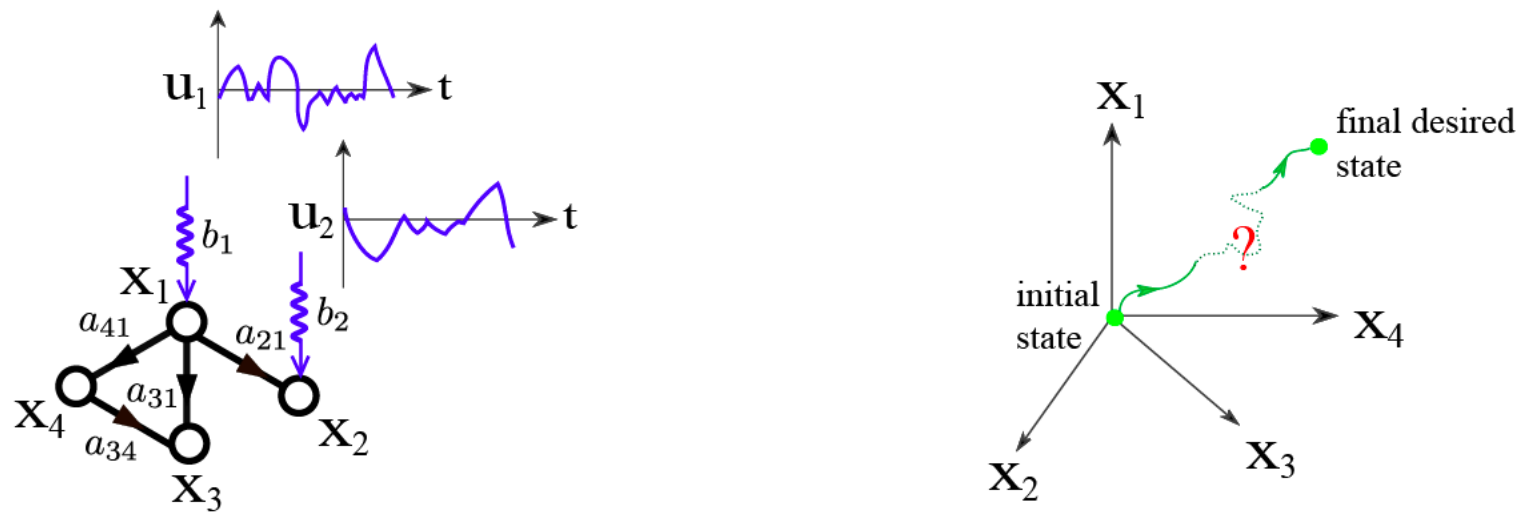
Department of Bioengineering
University of Pennsylvania

Complex
Systems

Overview

- I. Network control theory and its application to neural systems
- II. Examining cognitive and neurobiological correlates of control energy
- III. Extension: multiplex network control

The network control model



Network control theory is a mathematical framework that determines which perturbations can drive the whole system to a desired state. It is typically applied to the study of the power grid, mechanical systems, air traffic control systems, & robotics.

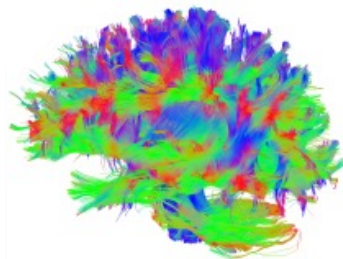
A network control model for the brain

The model stipulates how activity flows along structural connections.

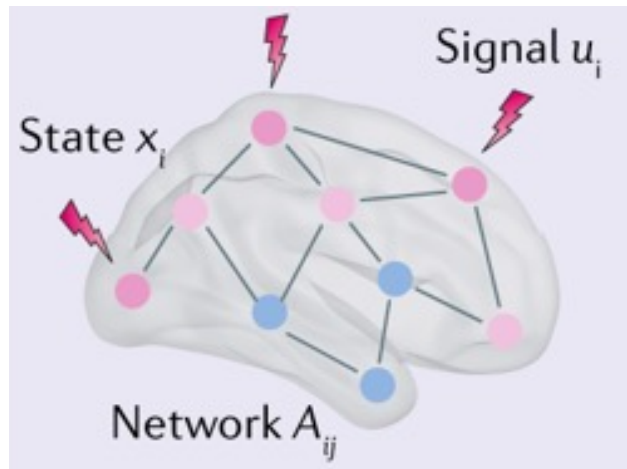
$$x(t + 1) = Ax(t) + B\kappa u_{\kappa}(t)$$

Diagram illustrating the network control model equation:

- $x(t + 1)$: State of brain regions over time
- A : Weighted adjacency matrix
- $B\kappa u_{\kappa}(t)$: Control energy
- κ : Number of regions being controlled



What are the inputs and outputs of NCT?



Network control theory

Inputs:

- Mapping of network
- Model of dynamics

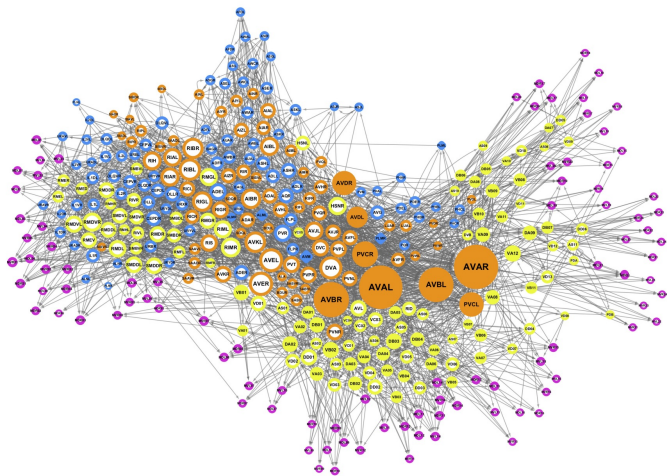
Outputs:

- Design of perturbation

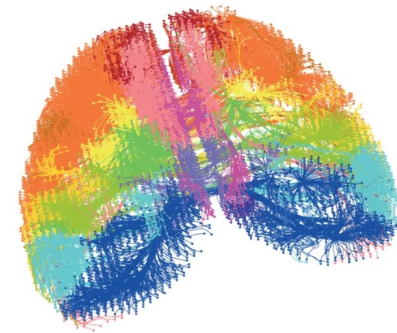
Input: A map of the network



Diffusion tractography

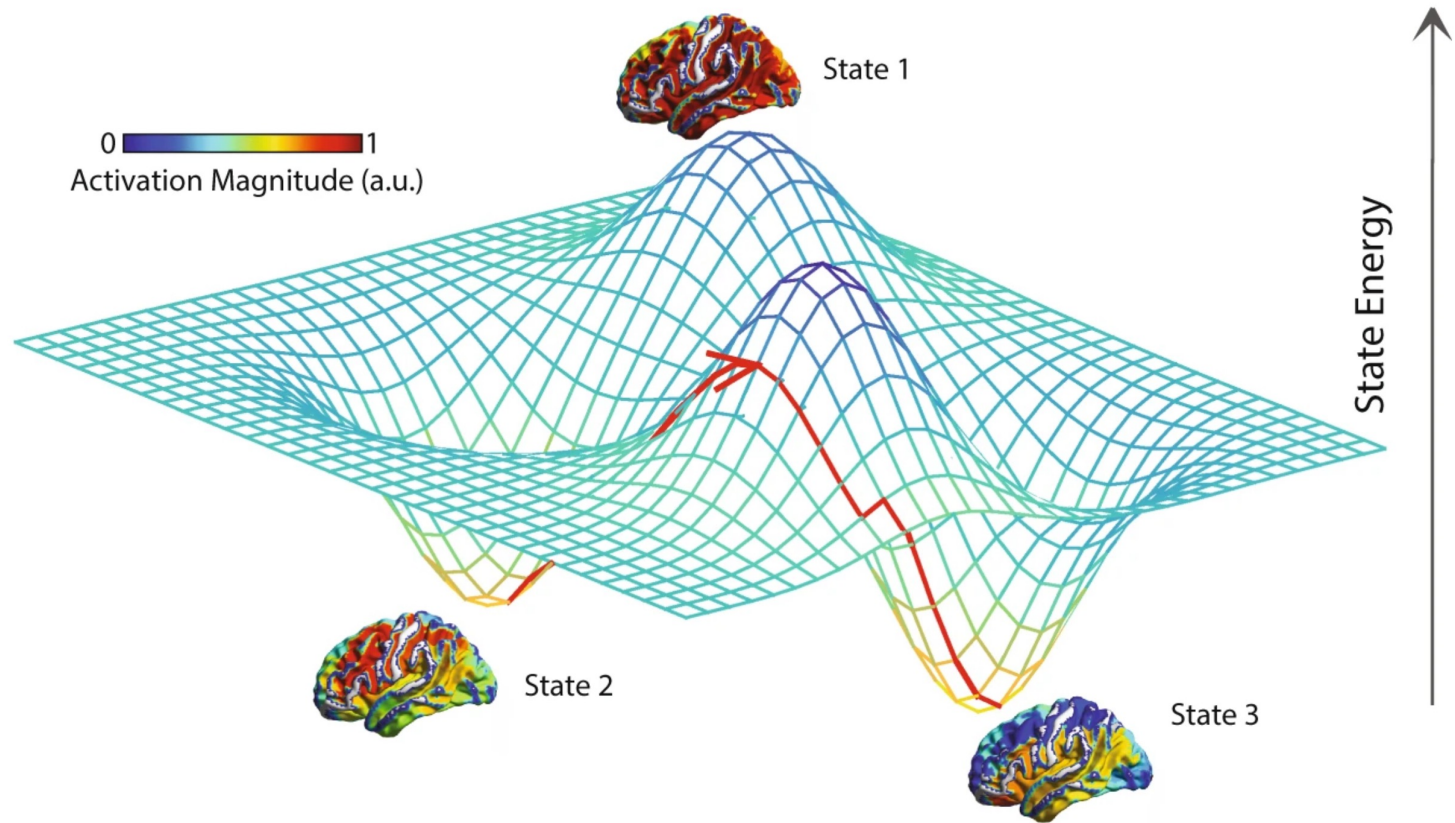


Electron microscopy



Tract tracing

Input: A model of the dynamics



Input: A model of the dynamics

complexity

- Linear, time-invariant models

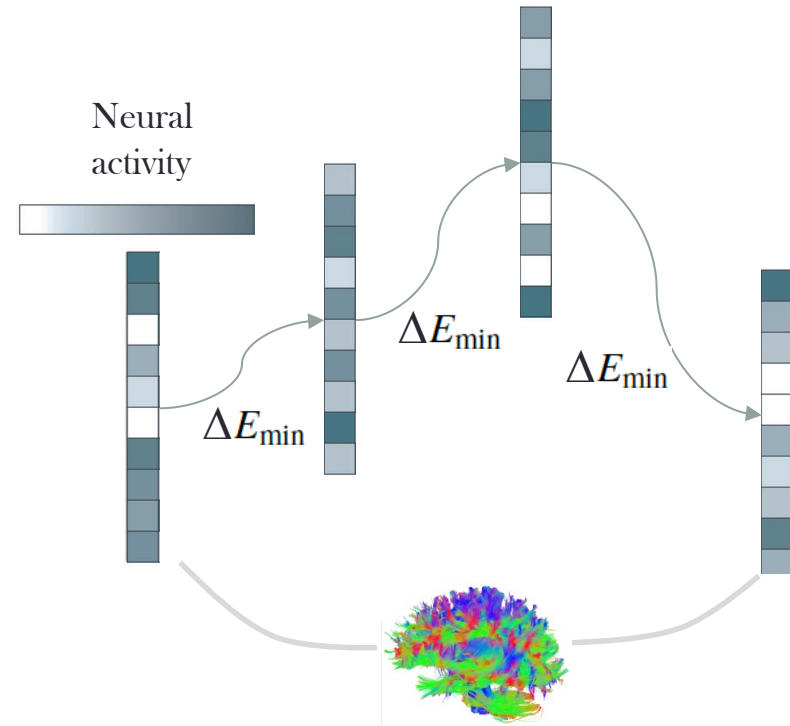
$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t).$$

- Linear, time-varying models

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t).$$

- Nonlinear models

$$\frac{d}{dt}\mathbf{x}(t) = f(\mathbf{x}(t), \mathbf{u}(t), t)$$
$$\mathbf{y}(t) = h(\mathbf{x}(t), t).$$



Input: A model of the dynamics

nature biomedical engineering




Article

<https://doi.org/10.1038/s41551-023-01117-y>

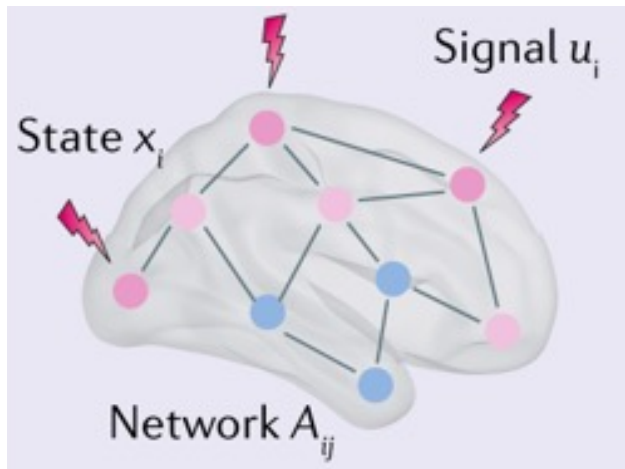
Macroscopic resting-state brain dynamics are best described by linear models

Received: 23 December 2020

Accepted: 26 September 2023

Erfan Nozari ^{1,2,3}, Maxwell A. Bertolero⁴, Jennifer Stiso ^{4,5}, Lorenzo Caciagli⁴,
Eli J. Cornblath ^{4,5}, Xiaosong He ⁴, Arun S. Mahadevan⁴, George J. Pappas ⁶
& Dani S. Bassett ^{4,6,7,8,9,10} 

What are the inputs and outputs of NCT?



Network control theory

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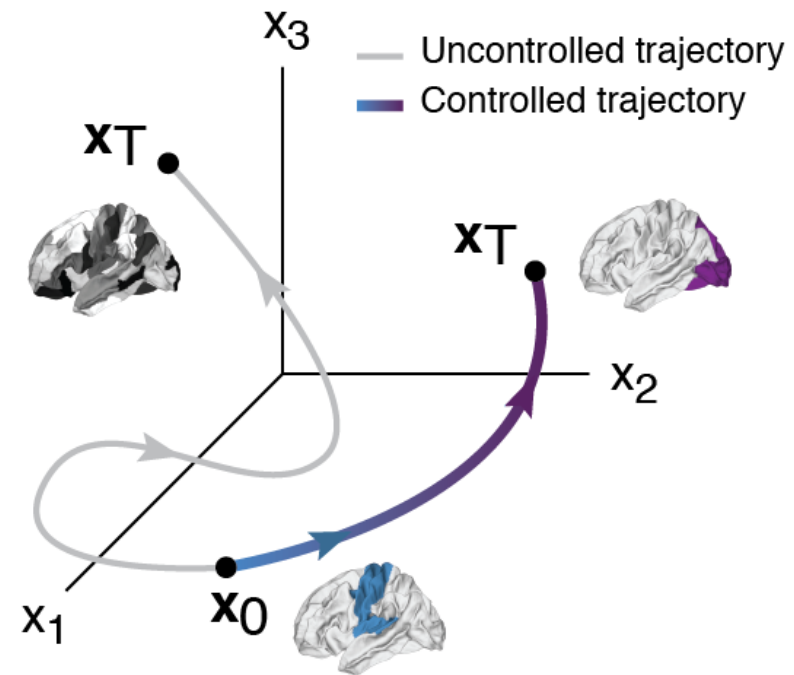
Outputs:

- Design of perturbation

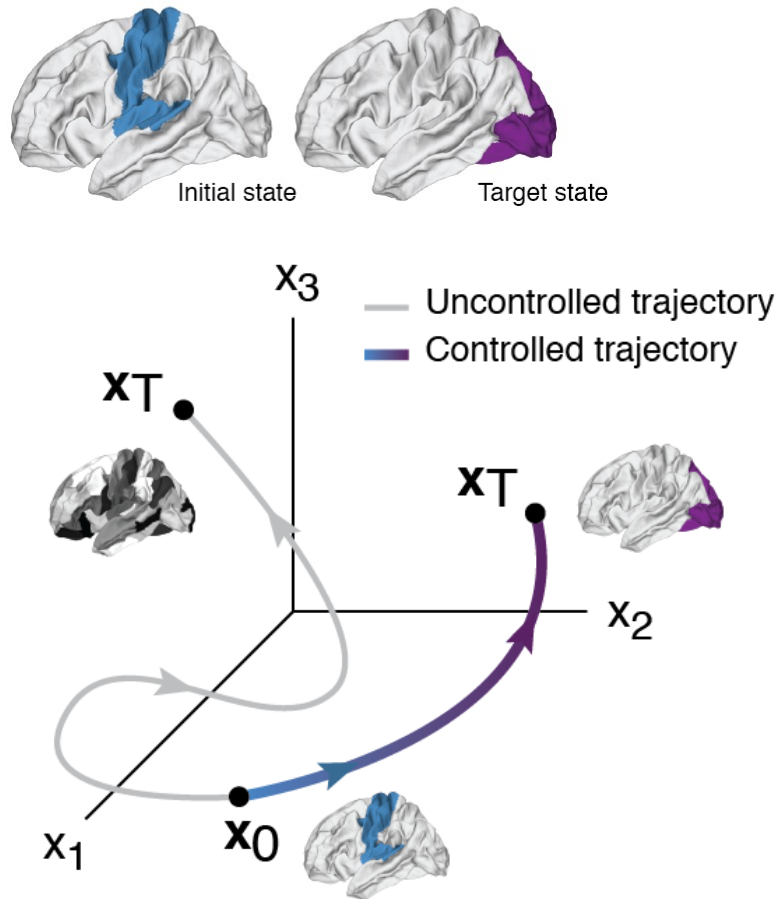


Outputs of NCT

- **Controlled response:** the system's response to some controlling input $u(t)$ from some initial state x_0 .
- **Controllability:** A system is controllable if there is a control input that brings our system from any initial state to any final state in finite time.
- **Achieving desired state transitions through minimum energy control:** Designing the control input $u(t)$ to minimize the control energy E (and possibly other factors) to drive the desired response.



Quantifying ease and difficulty of state transitions



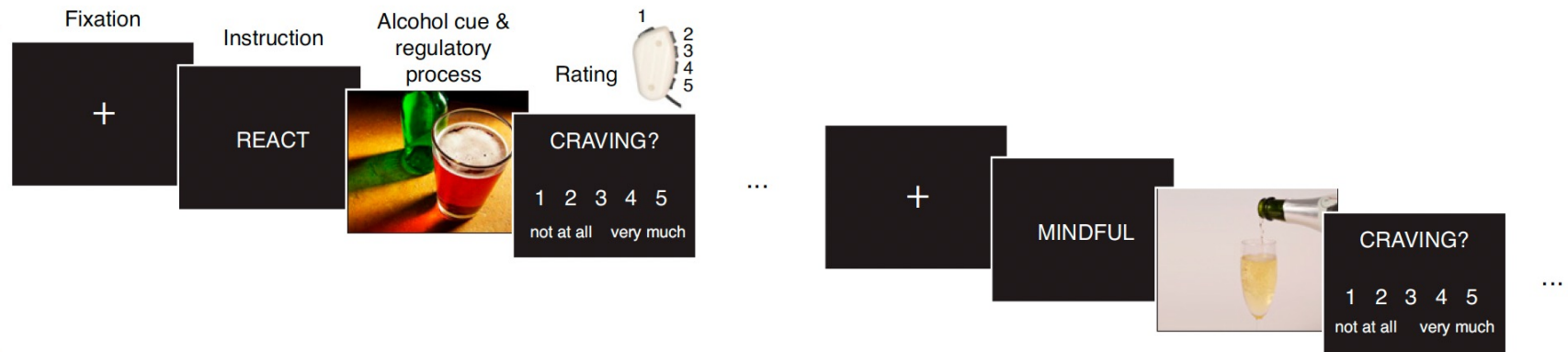
Given the model of network dynamics, we can define a cost function that penalizes control energy required and distance of $x(t)$ from the target state

$$\min_{\mathbf{u}} \int_0^T \underbrace{(\mathbf{x}_T - \mathbf{x})^T (\mathbf{x}_T - \mathbf{x})}_{\text{distance}} + \underbrace{\rho \mathbf{u}_K^T \mathbf{u}_K}_{\text{energy}}$$

II. Examining cognitive and neurobiological correlates of control energy

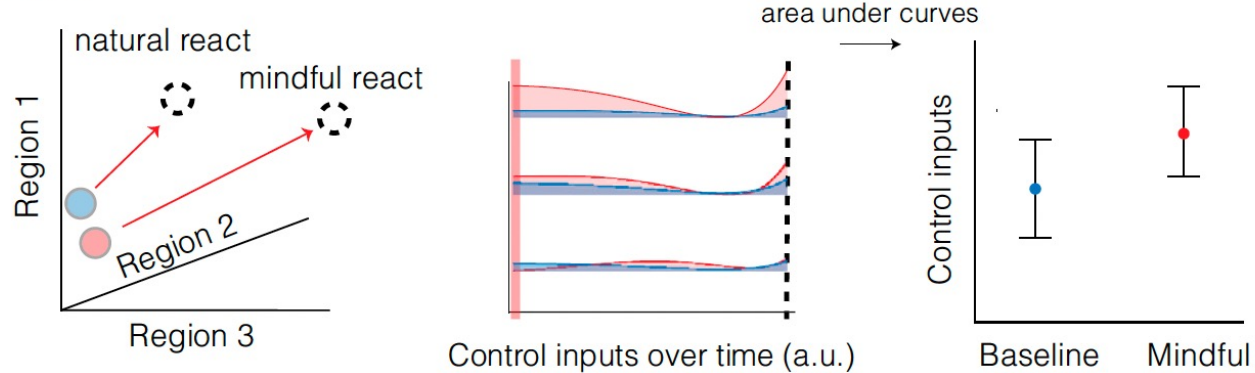
Relating control energy and cognitive effort

Does a brain state associated with higher cognitive effort require more control input to reach and to maintain?

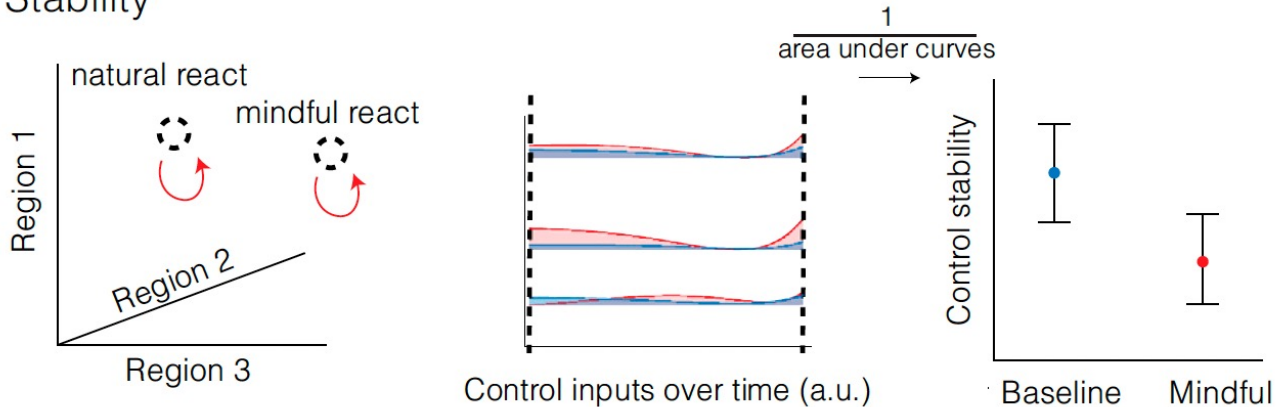


Relating control energy and cognitive effort

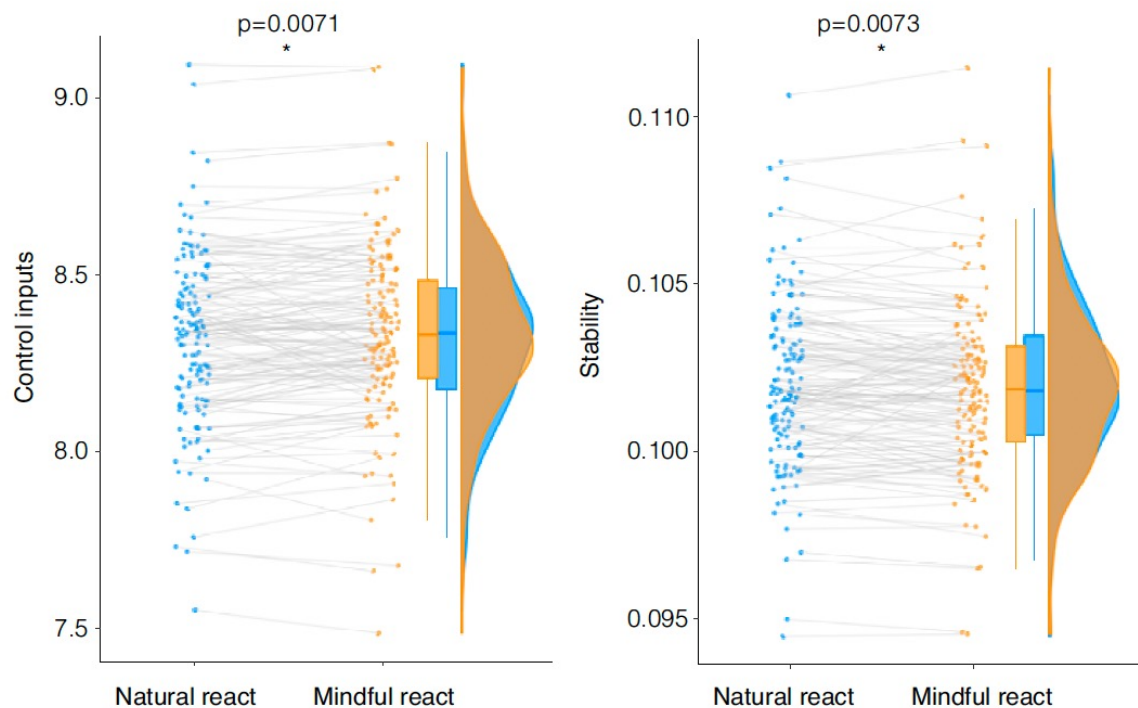
Effort



Stability

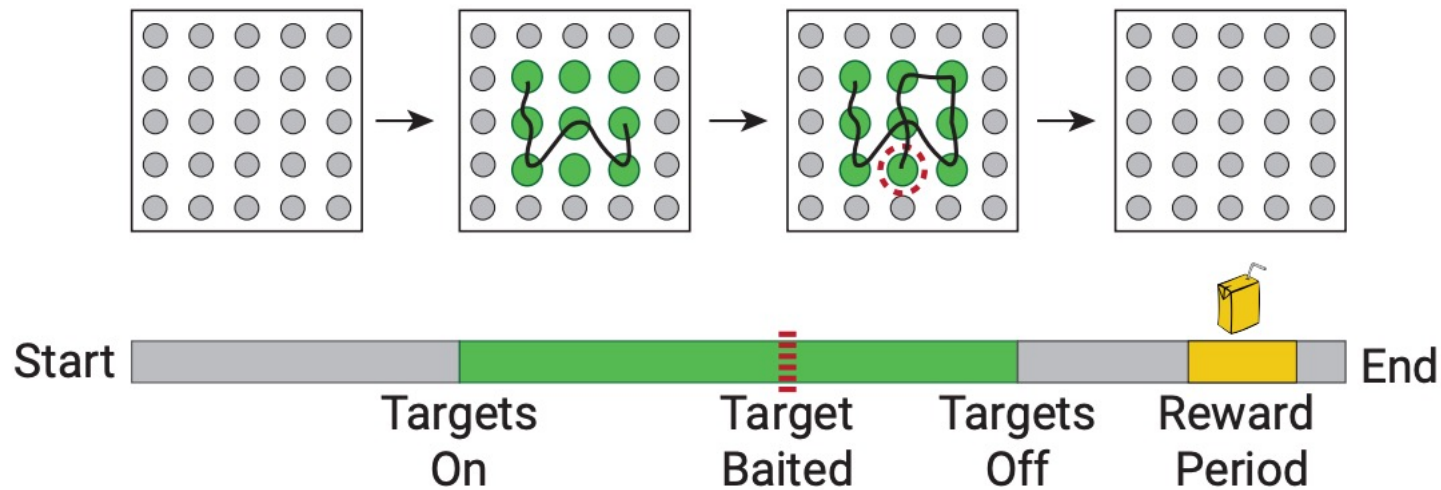


Relating control energy and cognitive effort



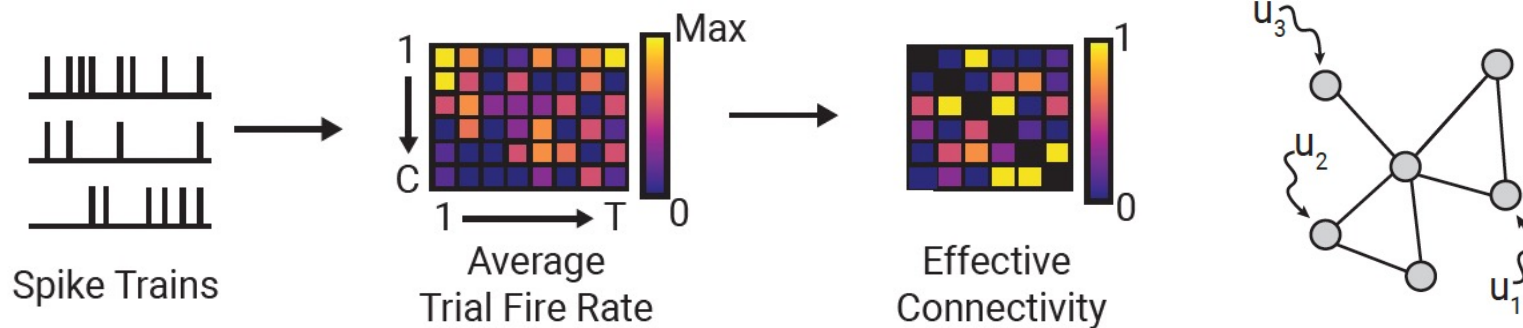
Relating control energy and cognitive effort

Do task-associated brain state transitions become less costly over the course of habit learning?

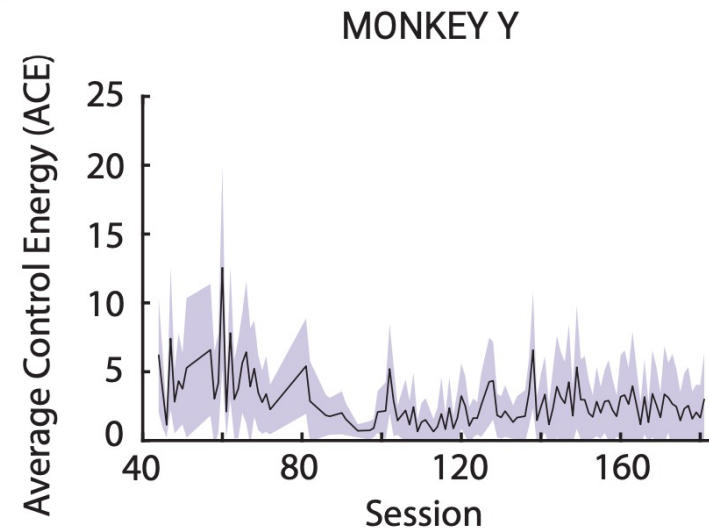
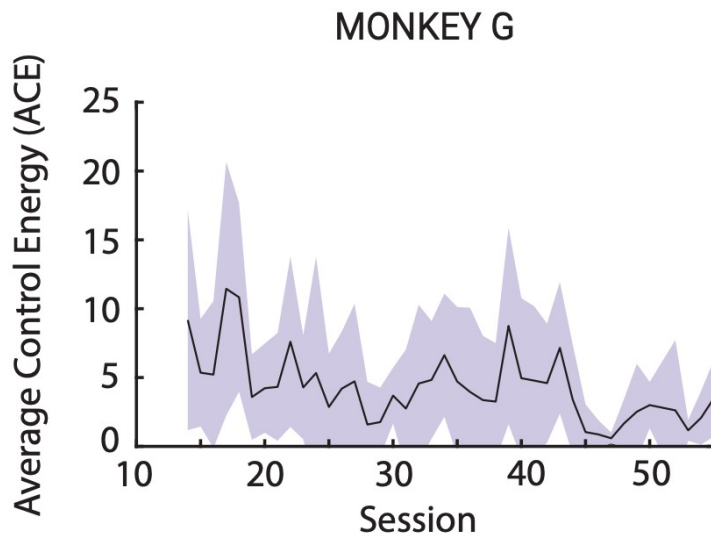


Relating control energy and cognitive effort

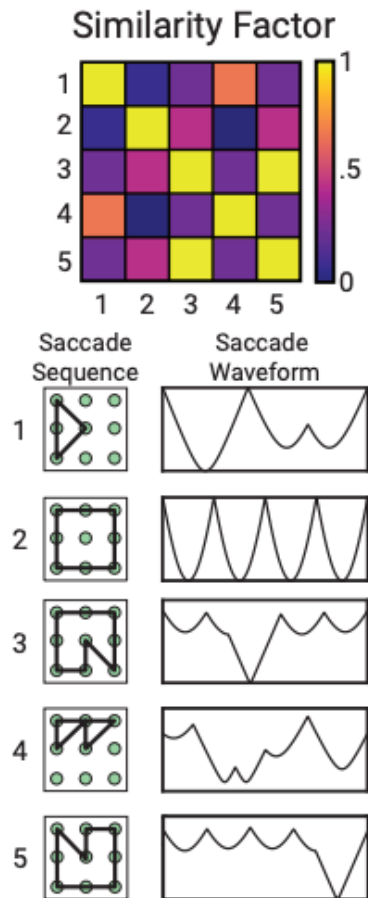
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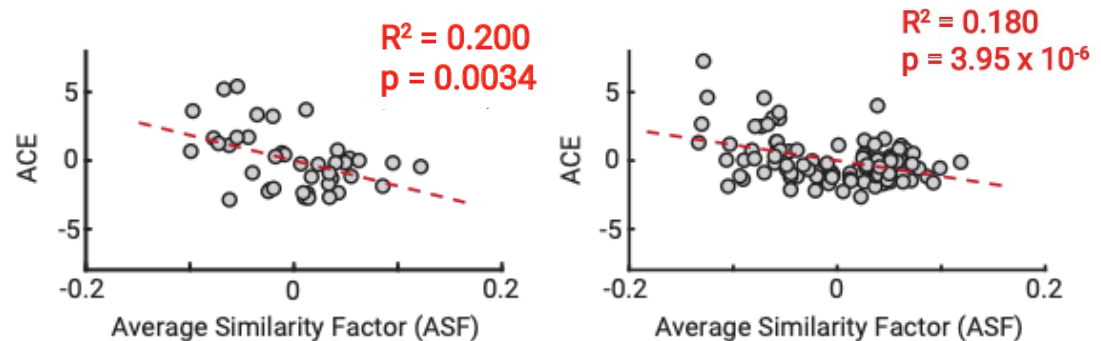
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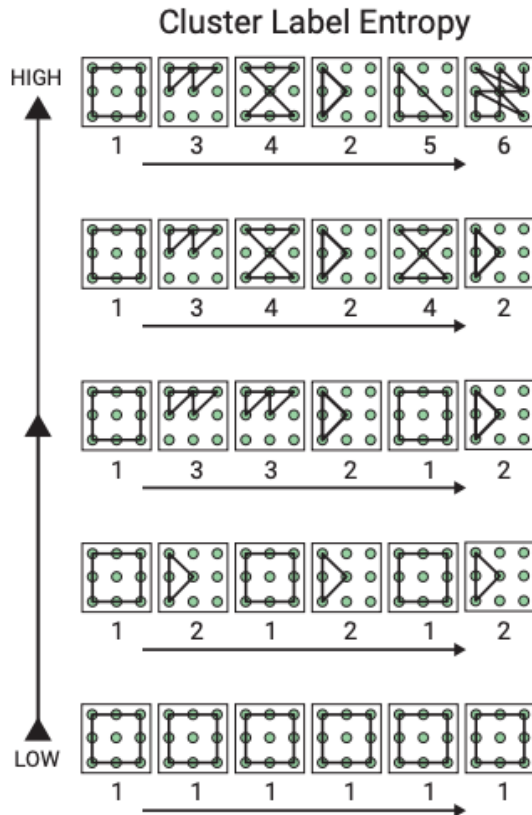
Relating control energy and cognitive effort



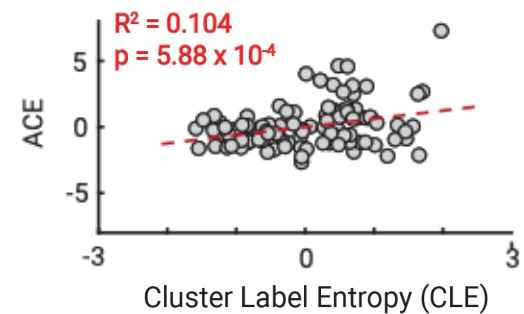
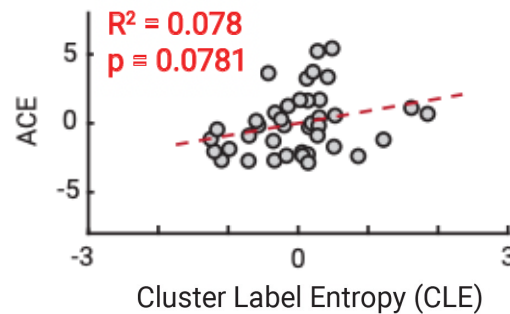
Is the monkey performing increasingly similar patterns the longer she engages in the task?



Relating control energy and cognitive effort

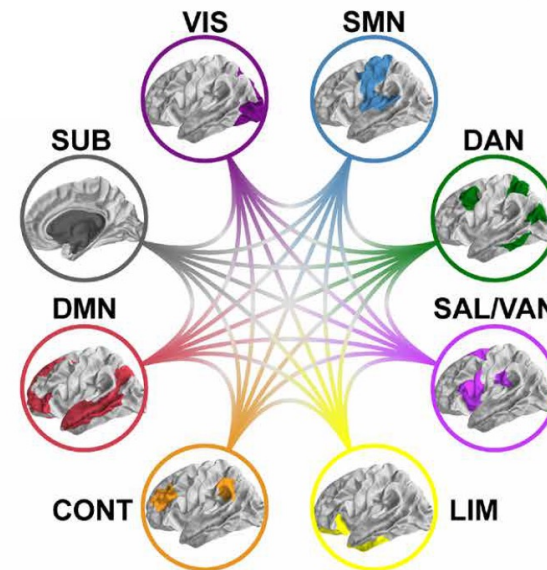
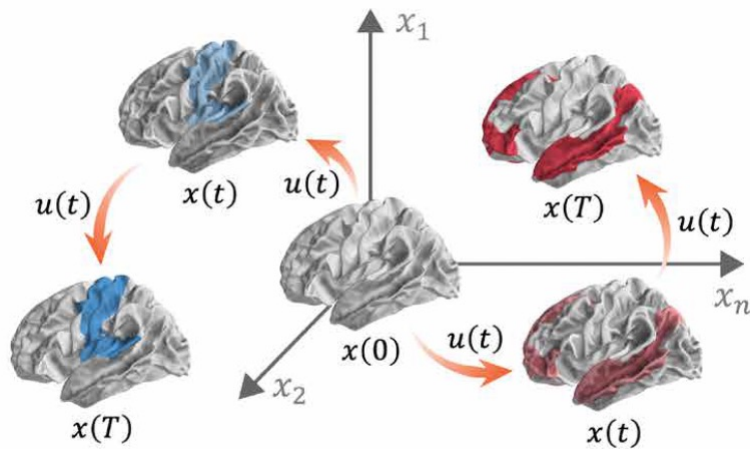


Is the monkey choosing to explore many different saccade patterns across trials or does she continuously exploit a select few?

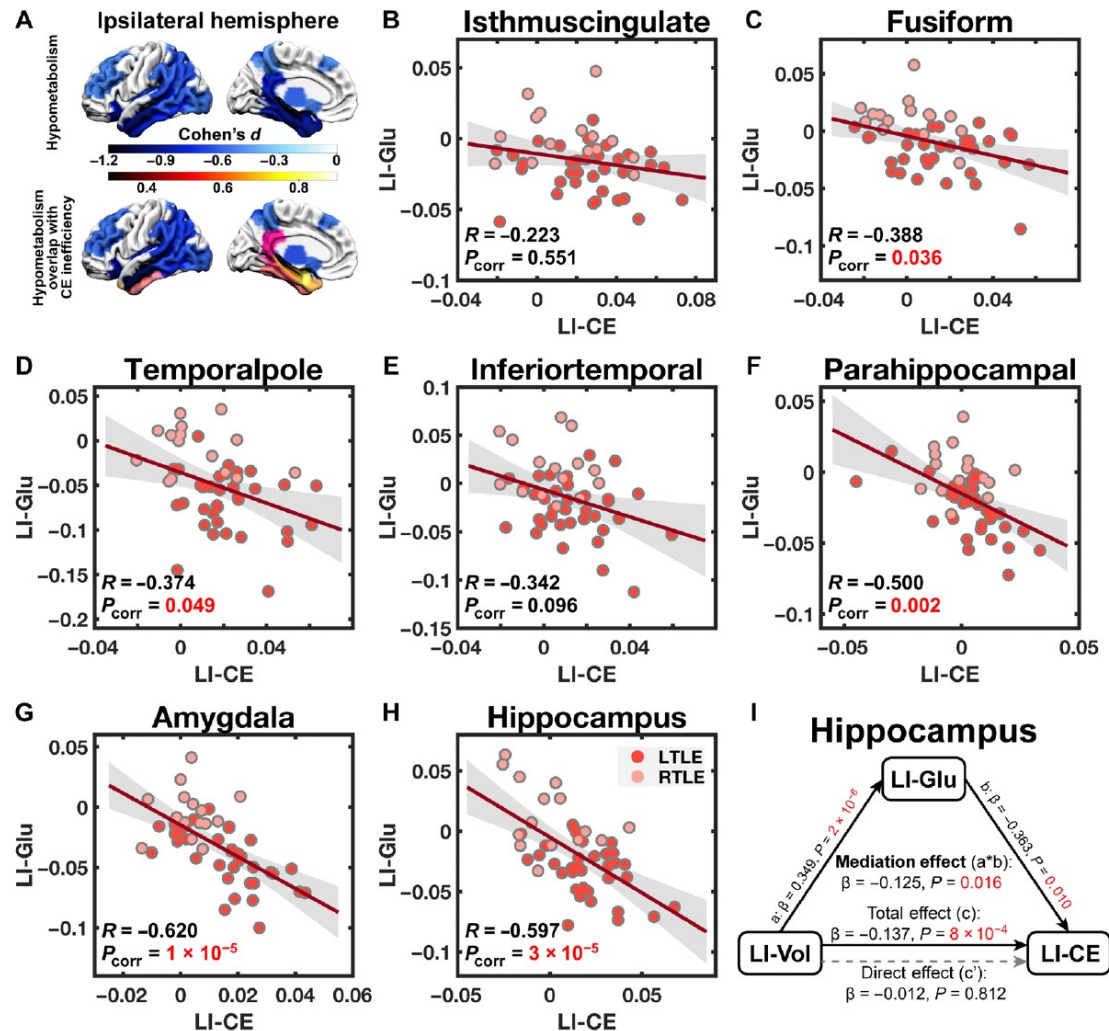


The biological basis of control energy

Does control energy reflect glucose utilization in the brain?



The biological basis of control energy



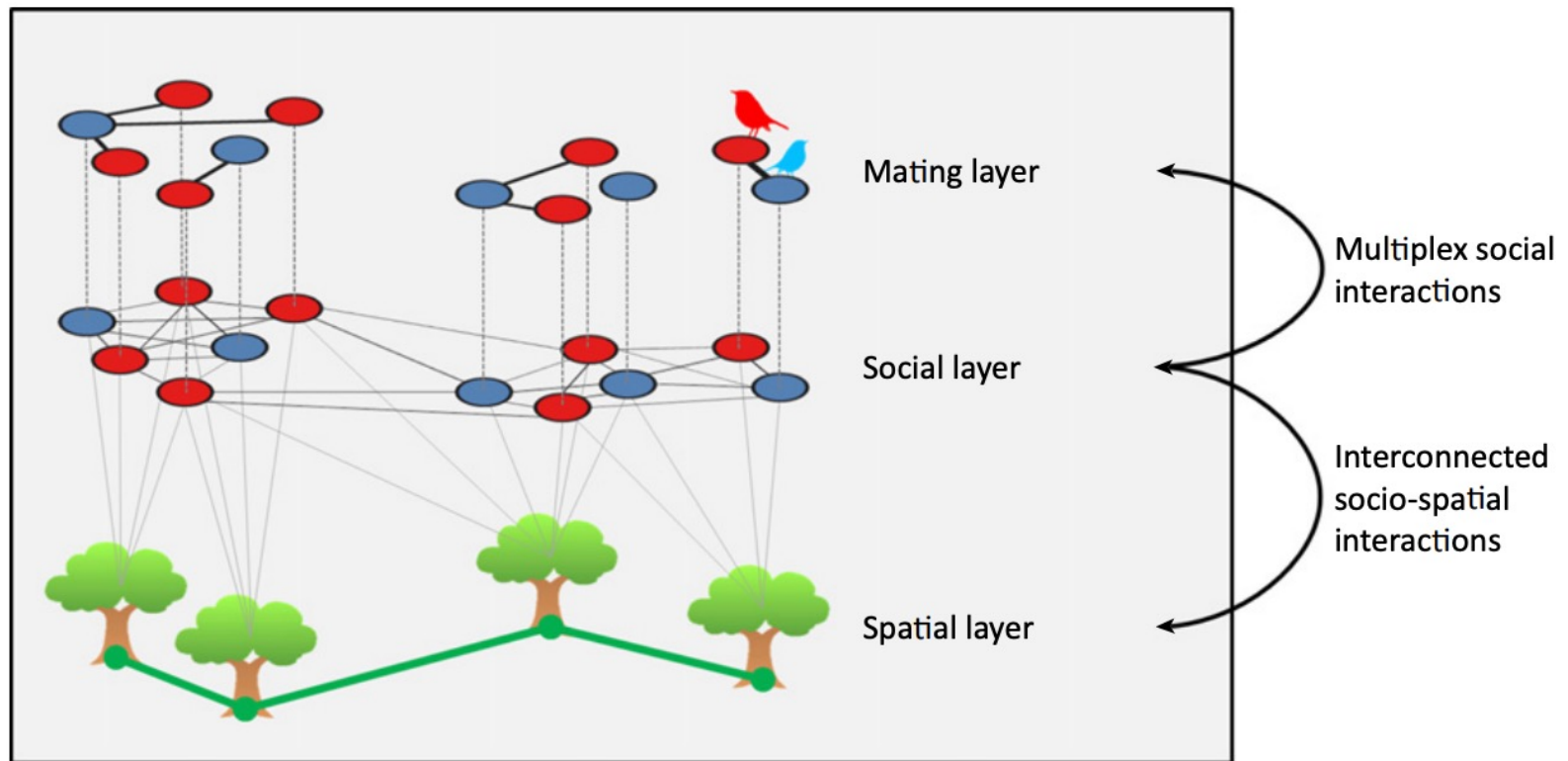
Summary

- Engaging mindful attention requires more control input into cognitive-control associated regions than reacting naturally to cues (Zhou et al., *PNAS* 2023)
- Control energy for state transitions decreases over the course of repeated task trials (Szymula, Brynildsen, Fotiadis et al.)
 - Changes in behavior associated with habit learning are correlated with control energy
- Regional glucose hypometabolism in TLE is associated with greater control energy requirements (He et al., *Sci Adv* 2022)

III. A multilayer network control framework for modeling interactions between neural activity and gene expression

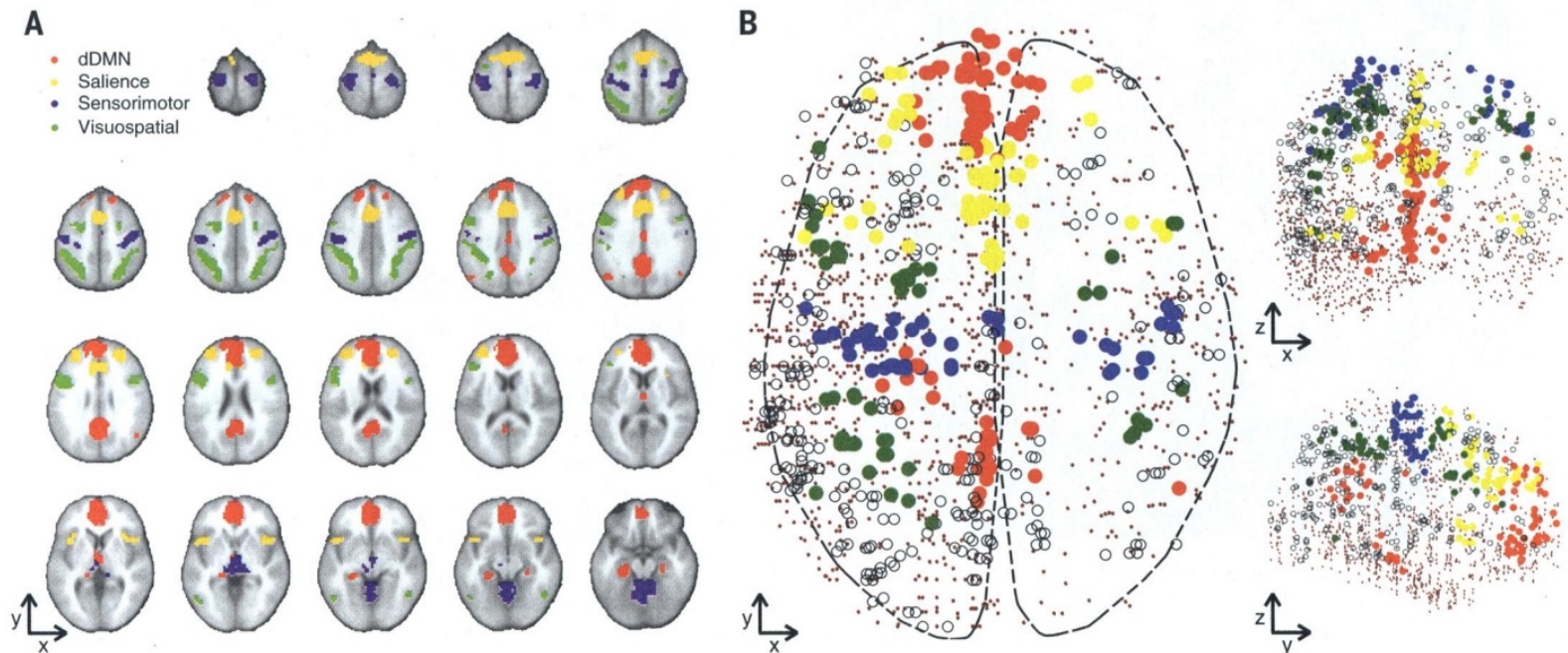
Control of multilayer networks

Multilayer networks model multiple types of interactions between components of a system.



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Multilayer networks model multiple types of interactions between components of a system.



Applying NCT to multilayer networks

Given access to only one layer, how can we drive a multilayered system to a desired state?

Applying NCT to multilayer networks

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Addressing this question is relevant to considering:

- How the brain transitions between health and disease states in which disease pathology impacts multiple layers

Applying NCT to multilayer networks

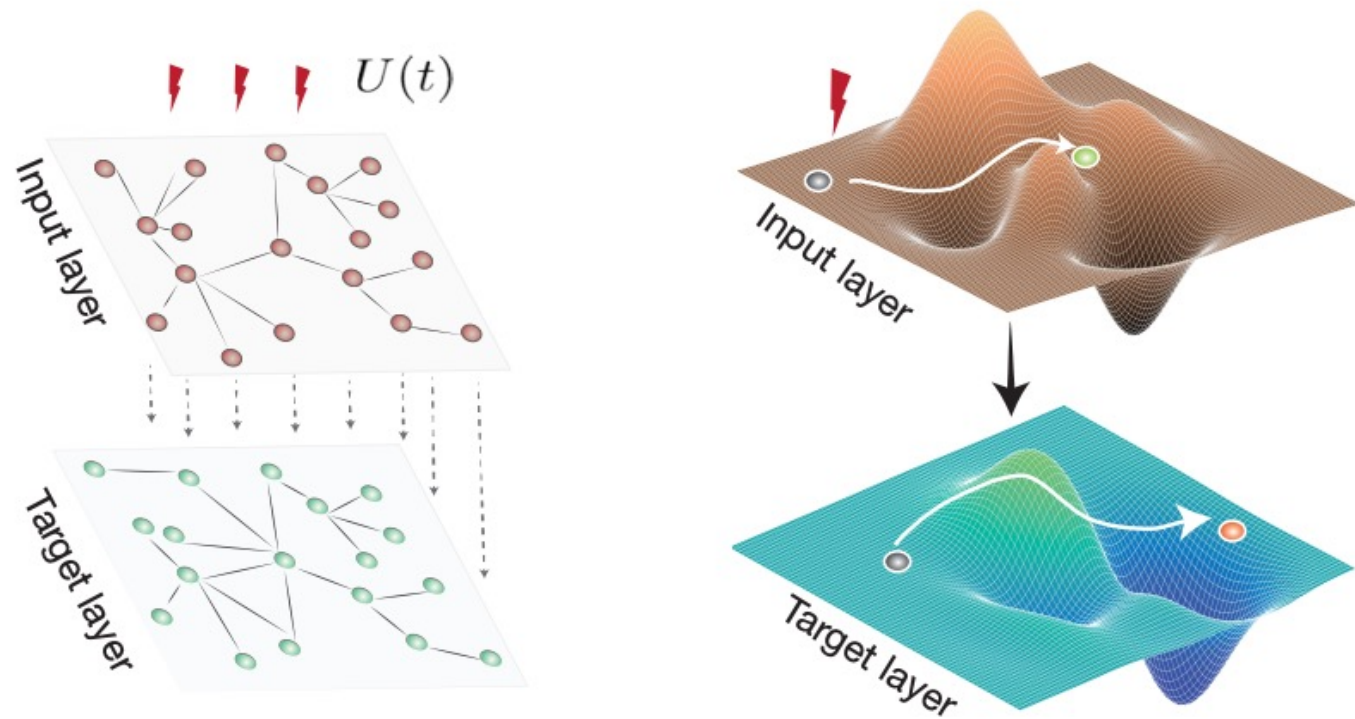
Given access to only one layer, how can we drive a multilayered system to a desired state?

Addressing this question is relevant to considering:

- How the brain transitions between health and disease states in which disease pathology impacts multiple layers
- How to target an intervention to impact multiple layers when only one layer is accessible

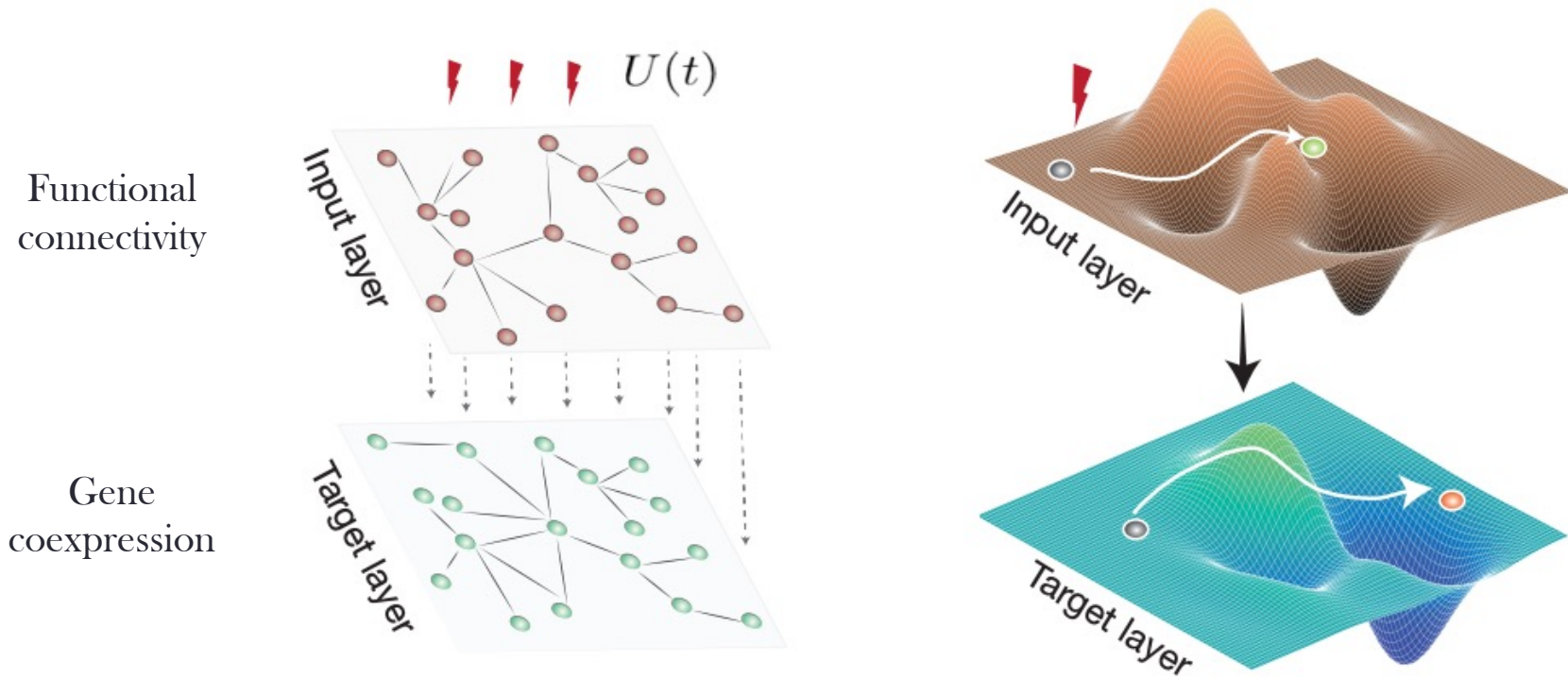
Applying NCT to multilayer networks

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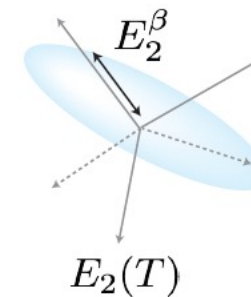
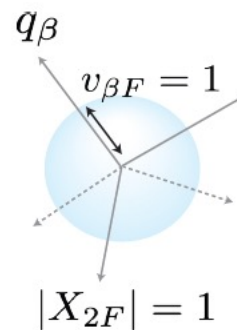
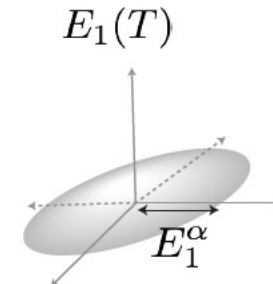
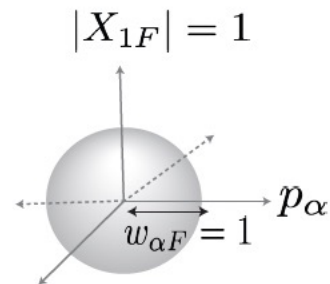
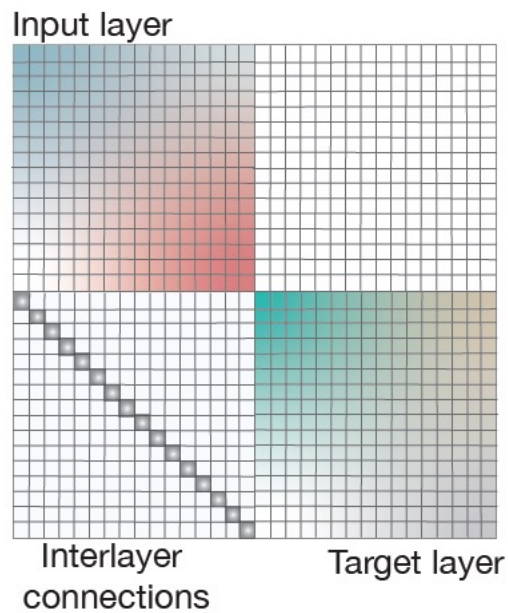


Applying NCT to multilayer networks

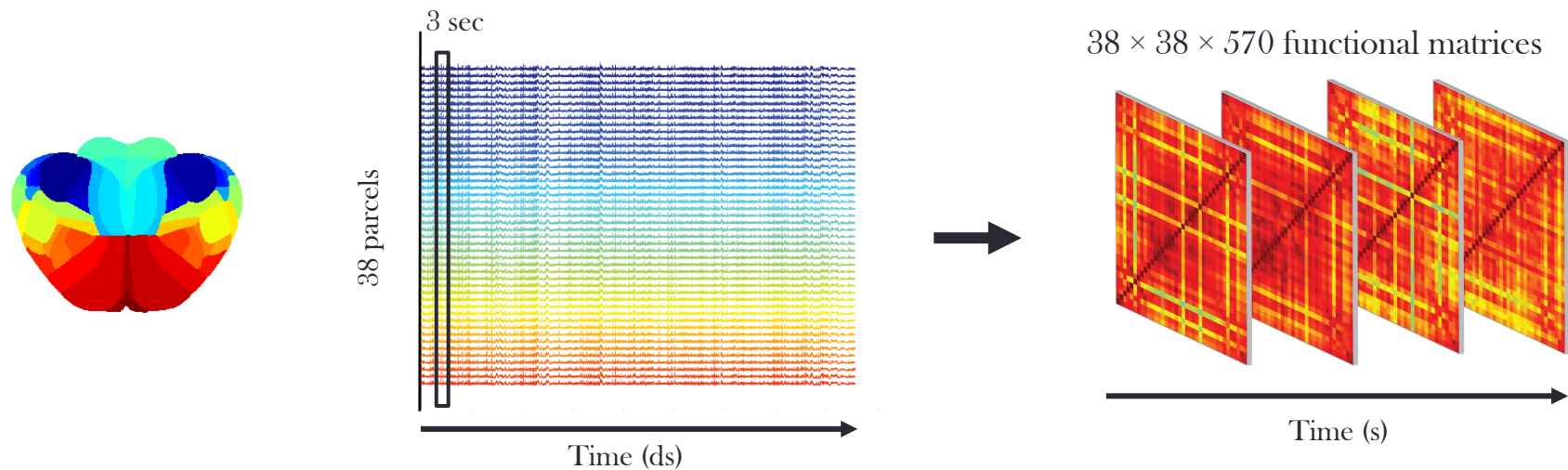
Given access to only one layer, how can we drive a multilayered system to a desired state?



Applying NCT to multilayer networks

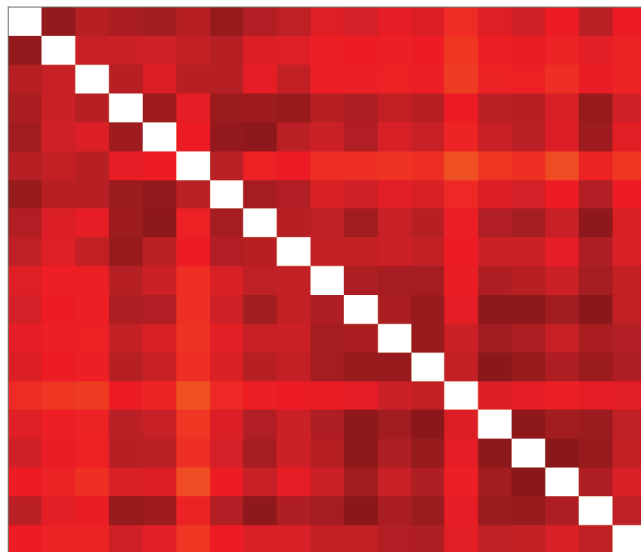


Overview of functional data

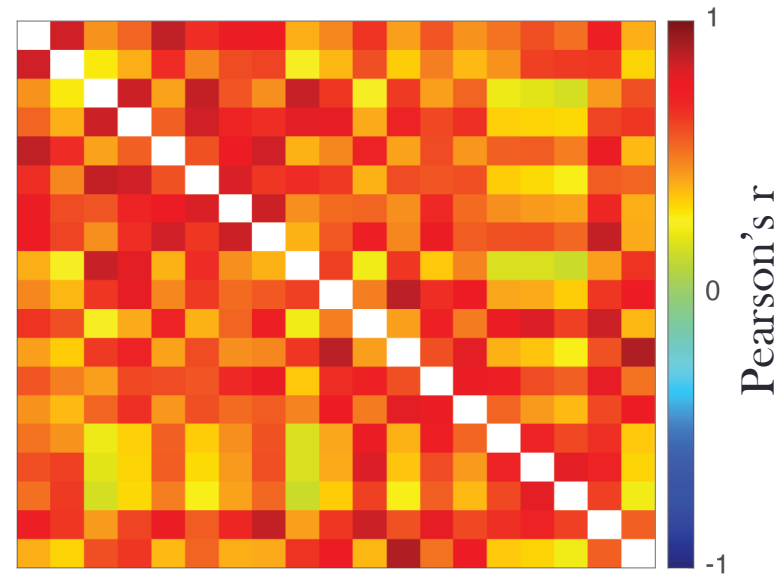


Construction of the duplex network

Function layer



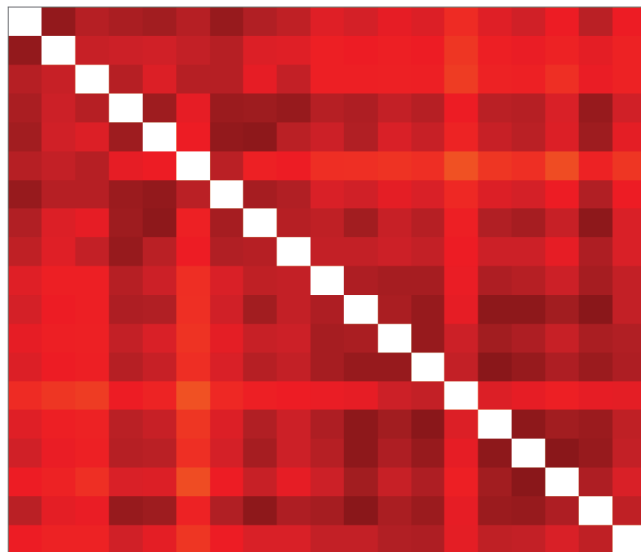
Gene expression layer



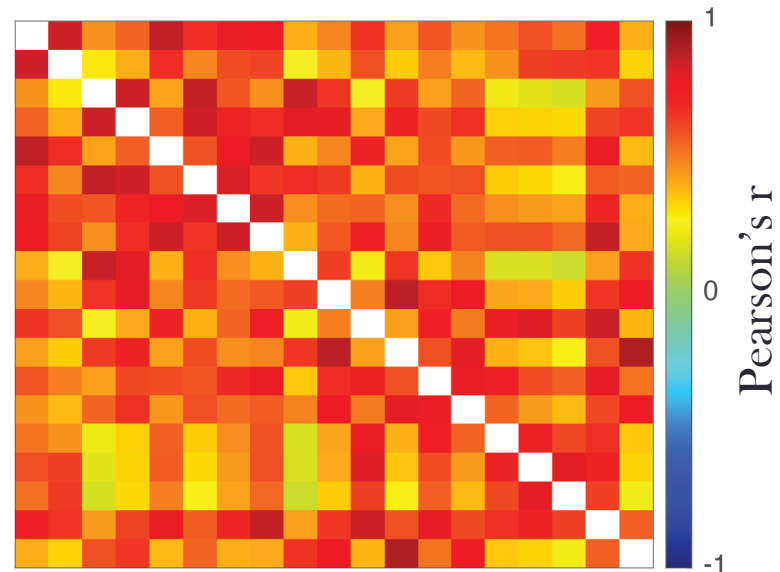
Interregional correlations across $\sim 7,000$
genes

Construction of the duplex network

Function layer



Gene expression layer

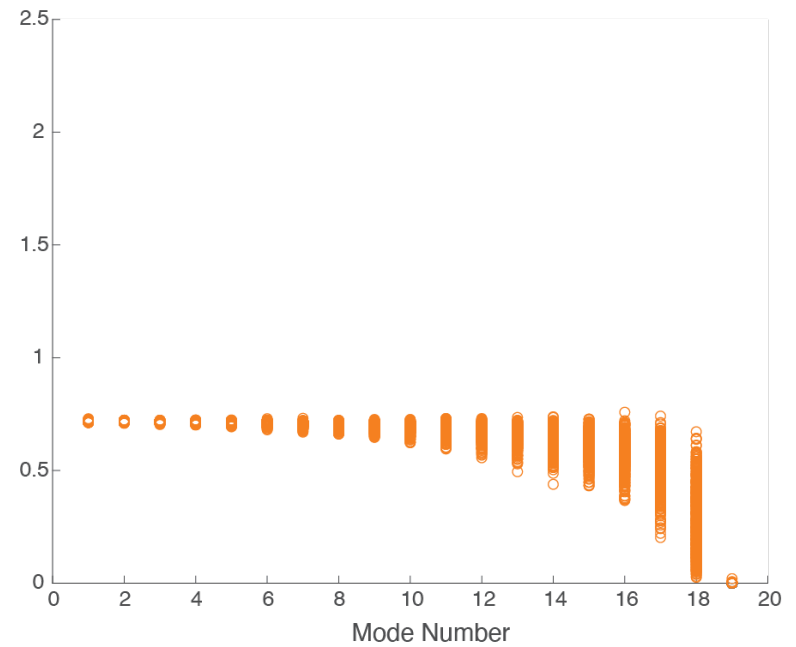
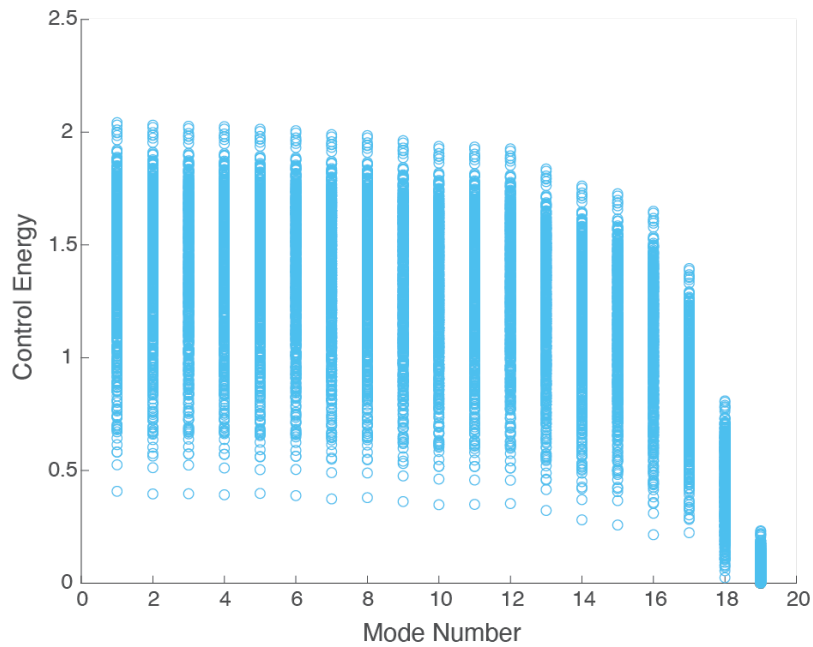


Into which layer is it more energetically favorable to give control input?

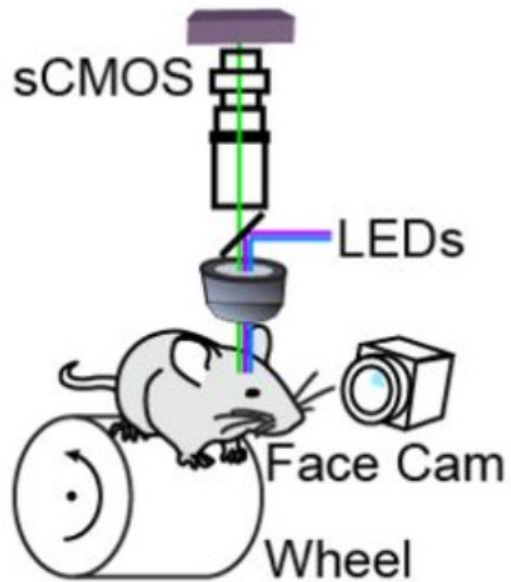
Duplex network control in mouse

Control spectra
Function to gene:

Gene to function:



Duplex network control in mouse



Pupil size

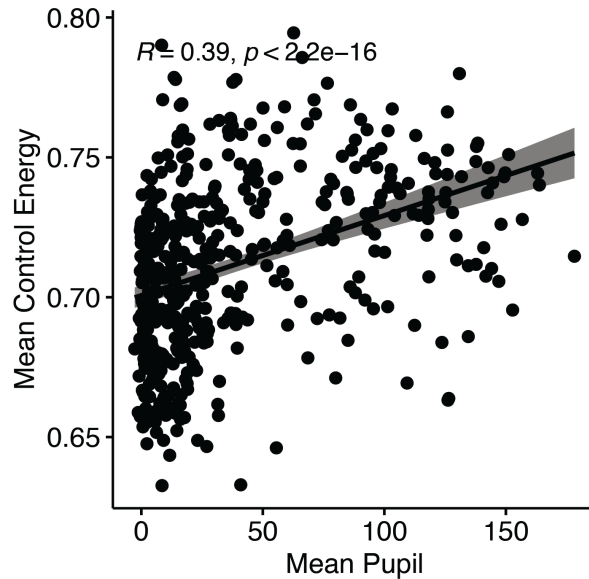
Facial movement

Running speed

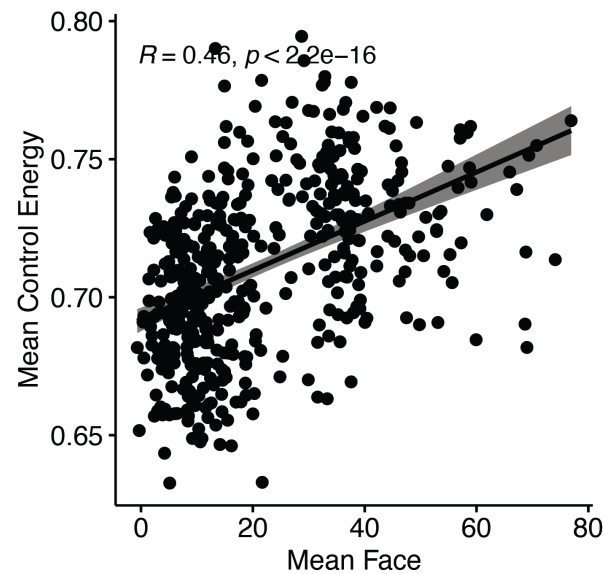
Duplex network control in mouse

Hypothesis: Less control input is required for the functional layer to control the gene expression layer during rest as compared to active states

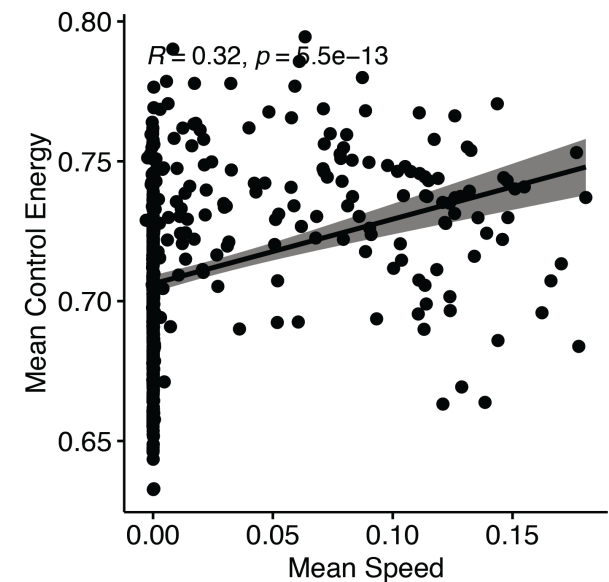
Pupil size



Facial movement

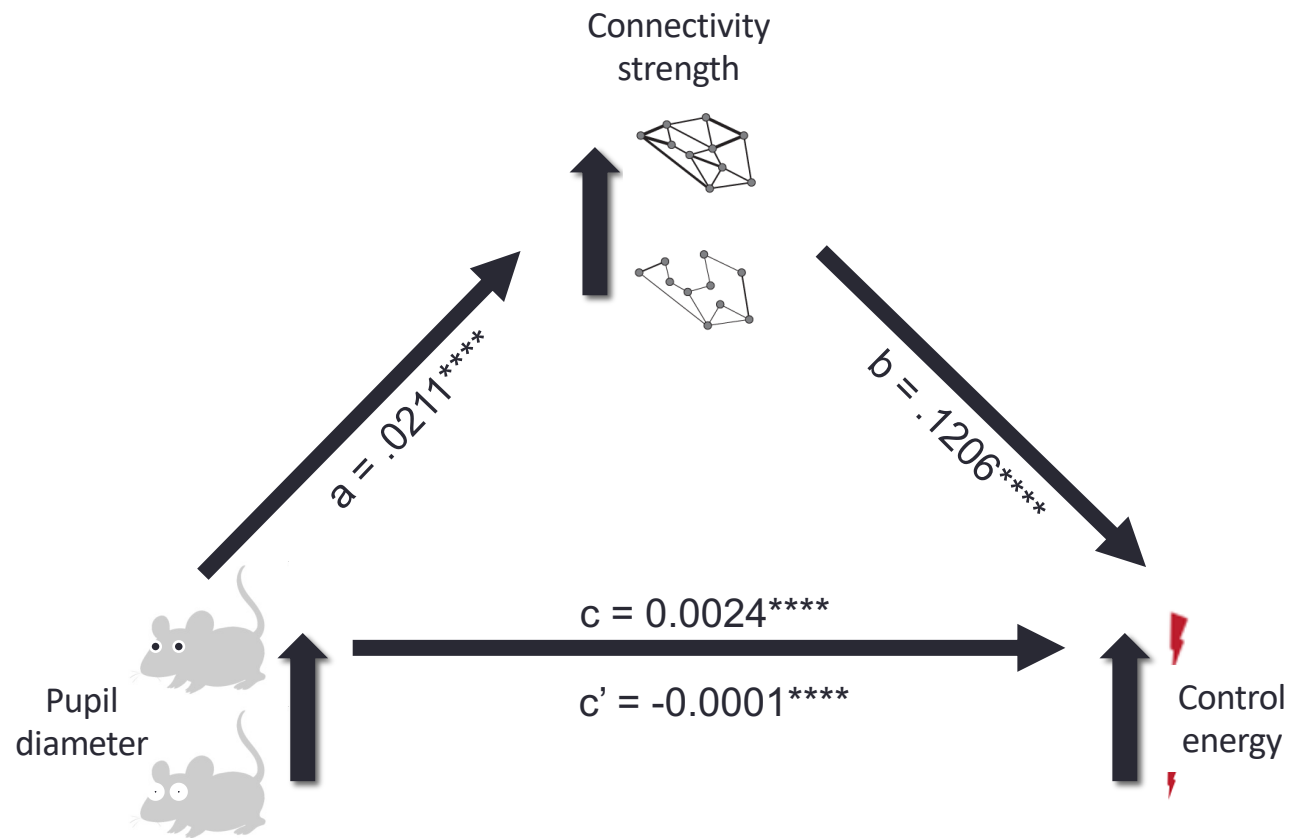
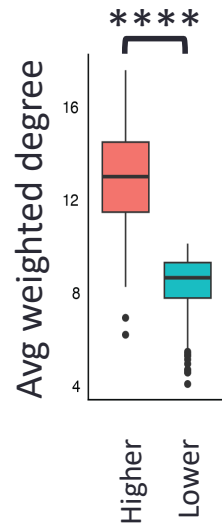


Running speed



Linking duplex network control with behavioral states

Relative cost of function to gene control:



Open questions

- How can we examine changes in gene expression across time, and their relation to changes in neural activity?
- Which genes matter?

And more...

Network control theory protocol



Confirmatory Results

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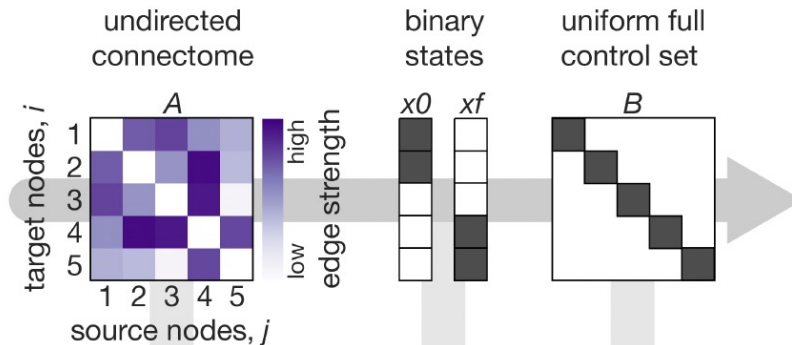
Using network control theory to study the dynamics of the structural connectome

 Linden Parkes,  Jason Z. Kim,  Jennifer Stiso,  Julia K. Brynildsen,  Matthew Cieslak,  Sydney Covitz,  Raquel E. Gur,  Ruben C. Gur,  Fabio Pasqualetti,  Russell T. Shinohara,  Dale Zhou,  Theodore D. Satterthwaite,  Dani S. Bassett

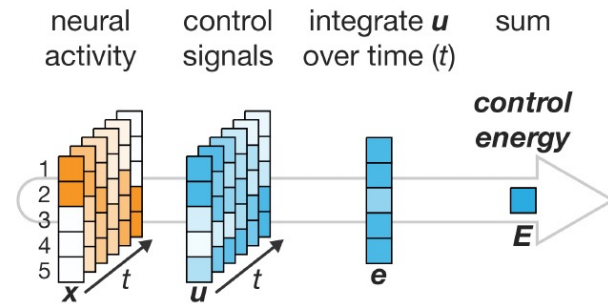
doi: <https://doi.org/10.1101/2023.08.23.554519>

Network control theory protocol

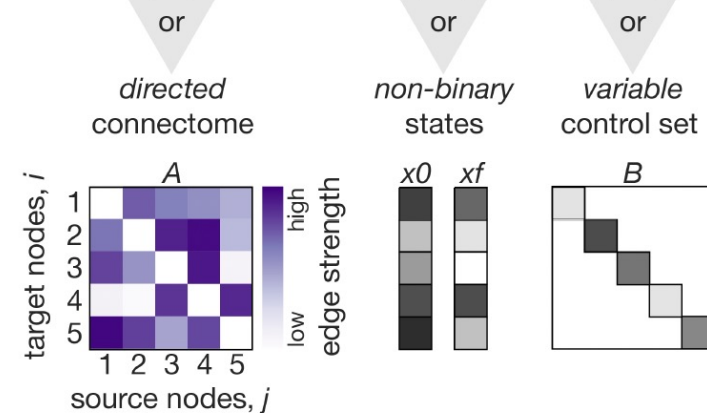
A | pathway A: model inputs



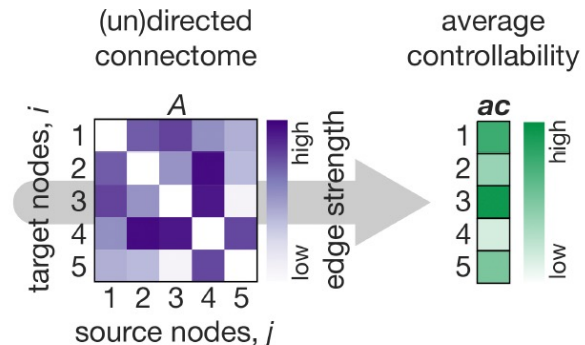
B | pathway A: model outputs



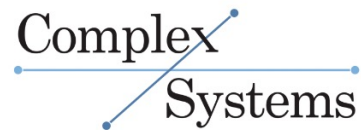
C | pathway A: variations



D | pathway B: average controllability



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