Traffic Flow on Freeways: Models, Analysis, Simulations

Reinhard Illner, Victoria, and Michael Herty, Aachen

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Synopsis

1. Types of Traffic Models
   - microscopic, macroscopic
   - kinetic $\rightarrow$ FP
2. Fundamental diagrams
   - Expectations and Observations
   - How to Compute FD from FP
   - Reality: Delays and Nonlocalities
   - Lane-Changing and multivalued FDs
3. From Vlasov- type models to macroscopic models
   - "Jam Equations", Traveling Waves, Stop-and-Go
   - Refining the Models: Individual reaction time
4. Analysis: Reasonable Forces. Maximum Principle.
5. Triggers
6. Simulations
7. What next?

What it's all about

Real observations

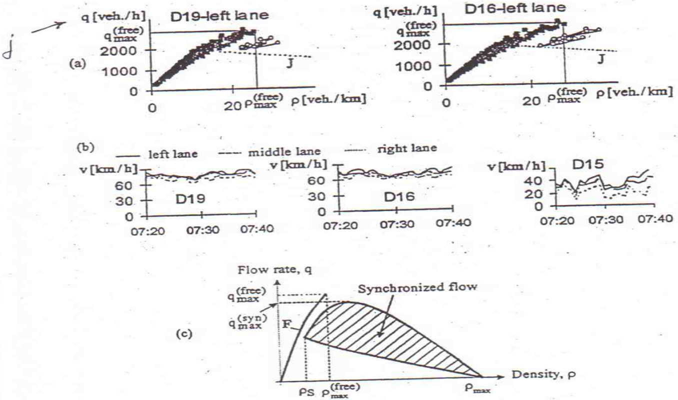

FIGURE 2 Synchronized flow: (*a,b*) observations from March 17, 1997; (*c*) hypothesis about states of free flow (curve *F*) and of homogeneous (steady) states of synchronized flow (hatched region).

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The term $\rho \partial_\rho p(\rho) u_x$ addresses the nonlocality inherent to traffic flow (more later).

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Uses kinetic density $f_i = f_i(x, v, t), i = 1, 2$ (statistical interpretation)...

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From FP models to macroscopic models of “Aw-Rascle” type

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A. moments
...omitted (appeared recently in Quart. Appl. Math.)
B. Assuming denser traffic, the ansatz
$f(x, v, t) = \rho(x, t)\delta(v - u(x, t))$

In dense traffic

B. Consider kinetic model without diffusion
(higher density... recall B. Kerner's observations)

$$\partial_t f + v\partial_x f + \partial_v (B(\rho, v - u^X) f) = 0$$

where $u^X = u(x + H + Tv, t)$.

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This is a Vlasov-type equation. $f = \rho\delta(v - u)$ is a weak solution if and only if

$$\rho_t + (\rho u)_x = 0$$

$$u_t + uu_x - B(\rho, u - u^X) = 0.$$

(where here $u^X = u(x + H + Tu(x, t), t)$).
Later we will define a more general *and more realistic* u^X!

Consider the example

$$B(\rho, w) = \begin{cases} -c_1 \rho w, & w > 0, \text{ i.e., } v - u^X > 0, \text{ "braking"} \\ -c_2(\rho_{max} - \rho) w, & w < 0, \text{ "acceleration scenario"} \end{cases}$$

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It is tempting to expand

$$u - u^X = -u_x(H + Tu) - \frac{1}{2} u_{xx} (H + Tu)^2 + \dots$$

and consider the resulting equations...

To do so we have to replace the (nonlocal) condition $u - u^X > 0$ by a *local* condition:
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To first order (rough approximation): replace $u - u^X > 0$ by $u_x < 0$. Result:

$$u_t - uu_x - g_i(\rho)[(H + Tu)u_x + \frac{1}{2}(H + Tu)^2 u_{xx}] = 0,$$

where

$$i = 1 \text{ for } u_x < 0 \text{ and } g_1(\rho) = -c_1\rho,$$

$$i = 2 \text{ for } u_x > 0 \text{ and } g_2(\rho) = -c_2(\rho_{max} - \rho)$$

(connected by regimes where u is constant).
This "equation" is a diffusive Hamilton-Jacobi type generalization of "Aw-Rascle".

Traveling waves

Let $V > 0$ and let $s := x + Vt$. We seek traveling wave solutions ("moving jams") of the type $\rho(s) = \rho(x + Vt), \; u(s)$.

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$$\frac{d}{ds}(\rho(u + V)) = 0 \quad \Longrightarrow$$

$$\rho(s) = \frac{\rho_{max} V}{u(s) + V},$$

(assuming that $u = 0 \Longleftrightarrow \rho = \rho_{max}$. Reasonable from a "common sense" point of view; we could use other integration constants)

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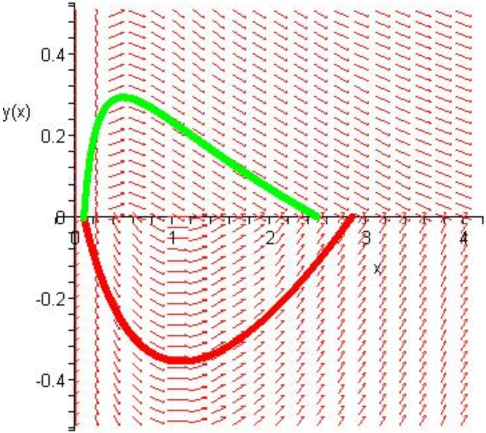
These equations are easily (and best) analysed in phase space (u, u'), with standard ODE methods:

If we assume 1) $c_2 \rho_{max} T > 1$, and 2) $0 < V < c_1 \rho_{max} H$, we assert

THEOREM. If $u_0 > 0$ is small enough (in terms of $V, H, T, \ldots$) then $\exists\ u_{-\infty} > 0$ and a solution of the " braking equation" so that $u(\infty) = u_0, u'(s) < 0$ and $\lim_{s \to -\infty} u(s) = u_{-\infty}$. There is a corresponding "acceleration wave" with $u(\infty) = u_0, \quad u'(s) > 0$ for all s, and $\lim_{s \to \infty} = u_\infty$.

Such a traveling wave deserves being called "moving jam."



$$V = .5, H = 1, T = 2,$$

$$c_1 \rho_{max} = 1.6 \ , \ c_2 \rho_{max} = 1$$

Include individual reaction time $\tau > 0$:

$$
\begin{aligned}
\rho_t + (\rho u)_x &= 0 \\
\rho \left(u_t + u_x u - B(\rho, u - u^X) \right) &= 0
\end{aligned}
$$

where the function $u^X(x,t)$ is now defined by

$$u^X(x,t) := u(x + H + Tu(x,t), t - \tau).$$

A "Jam" equation and braking waves

Focus on a braking regime: $u_x < 0$. Model equations:

$$\rho_t + (\rho u)_x = 0$$

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Traveling wave ansatz (as before)
$\rho(x,t) = \rho(x + Vt), \quad u(x,t) = u(x + Vt)$ and the shorthand $s := x + Vt$ produces ODE

$$(V + u)u'(s) + c_1 \rho(s) \left(u(s) - u(s + (H - \tau V) + Tu(s))\right) = 0.$$

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in a braking scenario there should be a distance of 38 metres from the front of your car to the front of the lead car if traffic moves at 54 km/h. 38 metres are not a small quantity! Truncation error in a Taylor approximation could be significant.

After expanding u to second order

$$u(s + (H - \tau V) + Tu(s)) = u(s) + u'(s)(H - \tau V + Tu(s))$$
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For $\tau = 0$ identical to the braking equation we studied earlier: small change of parameter $(H \to H-\tau V)$. Braking waves ending at a small (positive) residual speed u_0 will exist if the wave speed V satifies

$$0 < V < \frac{c_1\rho_{max}H}{1+c_1\rho_{max}\tau} < H/\tau.$$

These braking waves are best depicted in phase space $\{(u,u')\}$.

Maximum Principles

Assume that traffic flows according to the equations

$$\rho_t + (\rho u)_x = 0$$

$$u_t + u_x u + c_1 \rho (u - u^X) = 0 \quad \text{while } u - u^X \geq 0$$

$$u_t + u_x u + c_2 (\rho_{max} - \rho)(u - u^X) = 0 \quad \text{while } u - u^X < 0$$

Assume that we have a (smooth) solution $\rho(x,t), u(x,t)$ such that $u(x_0, t) = sup_{x, s \leq t} u(x, s)$. A driver at x_0 at time t will be in a braking situation, and: $u_t(x_0, t) \leq 0$.

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$\Longrightarrow$ the model satisfies a maximum principle:

Proposition. Suppose that for all $x \in \Re, s \in [0, \tau]$ we have

$$0 \leq a \leq u(x, s) \leq b.$$

Then for any smooth solution

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for all x and all $t \geq 0$.

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This is not realistic! Traffic jams occur and disappear in steady dense traffic for (sometimes) no apparent reason; such jams usually lead to standing traffic, etc. Our models need refinement in order to account for such effects.

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Reference driver at x, t will act as if he/she is in acceleration scenario, although there are slower vehicles immediately in front of him/her! The nonlocality scale in this case exceeds the monotonicity domain.

Remedies

Redefine

$$\begin{aligned}
\rho^X &= \sup_{\sigma \in (0, H + Tu(x,t))} \rho(x+\sigma, t-\tau), \\
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and

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B(\rho, u, u^X) = \begin{cases} -c_1 \rho^X (u - u^X) & u - u^X > 0. \\ -c_2(\rho_{\max} - \rho^X)(u - \bar{u}^X) & u - u^X \leq 0. \end{cases}
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New braking law uses maximal observed density and minimal observed speed in the relevant window; only if $u-u^X \leq 0$ is the braking case rejected, and then we accelerate according to the old rule.

Triggers

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- High concentration: Reset $B(...) = -c_1 \rho^X u$ if

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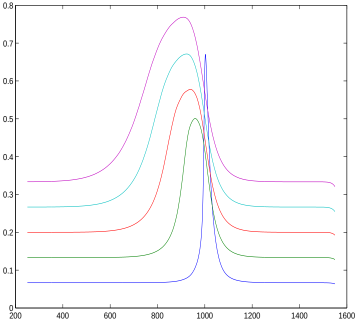
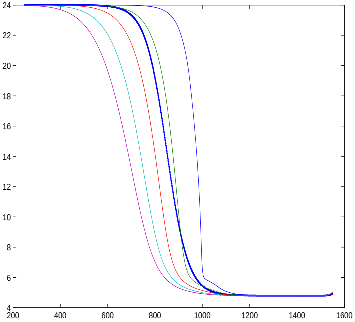
- ...others! Suggestions?

a) Traveling waves obtained for an initial velocity profile as depicted in bold blue in the picture to the right. Density is initially constant. Solution is depicted at time $T = 20s$:


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
200
400
600
800
1000
1200
1400
1600


24
22
20
18
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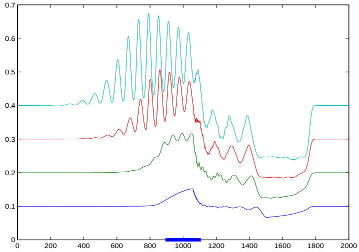
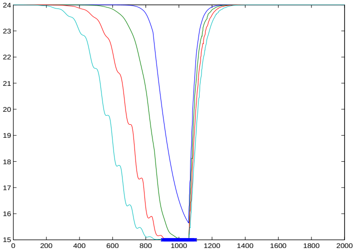
14
12
10
8
6
4
200
400
600
800
1000
1200
1400
1600

b) Density ($T = 30s$) for different initial values. $u_{lim} = 15m/s$. on a strip of $200m$ centered at $x = 1000m$.





That's it. Have a nice day.

Drive safely