Kinetics of Filtration and Clogging

work in collaboration with Somalee Datta

Flows and Networks in Complex Media
IPAM, UCLA, April 27-May 1, 2009

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What is filtration? What is clogging?

Basic Approach:

minimalist modeling; probabilistic descriptions

Two Main Results:

density profiles of trapped & escaped particles

clogging time and its distribution

Summary & Outlook

What is filtration?   What is clogging?

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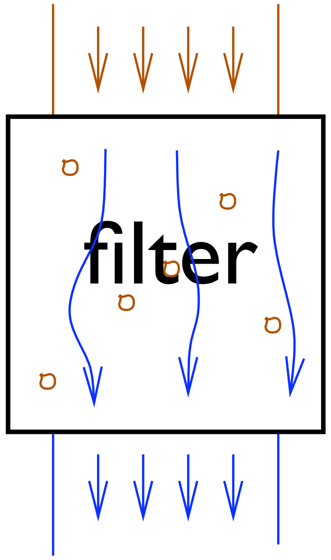

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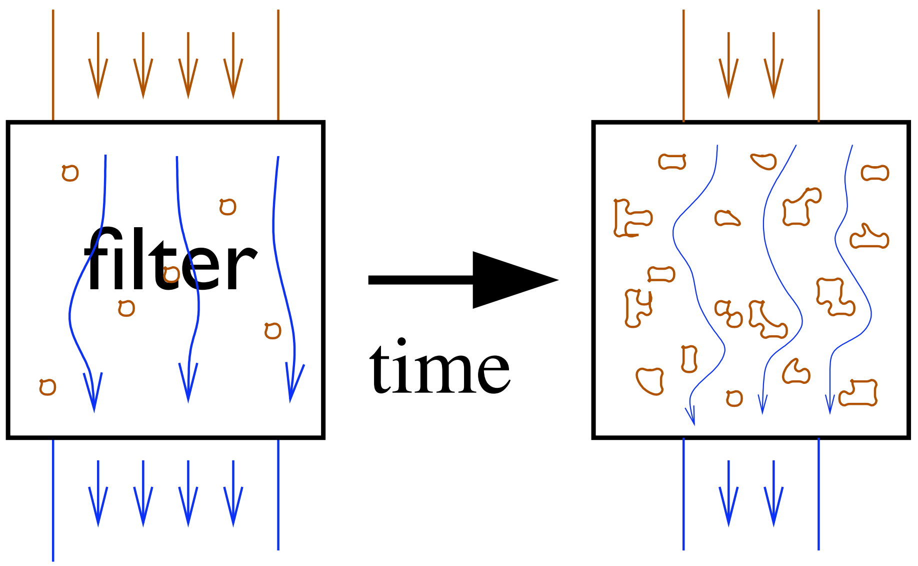

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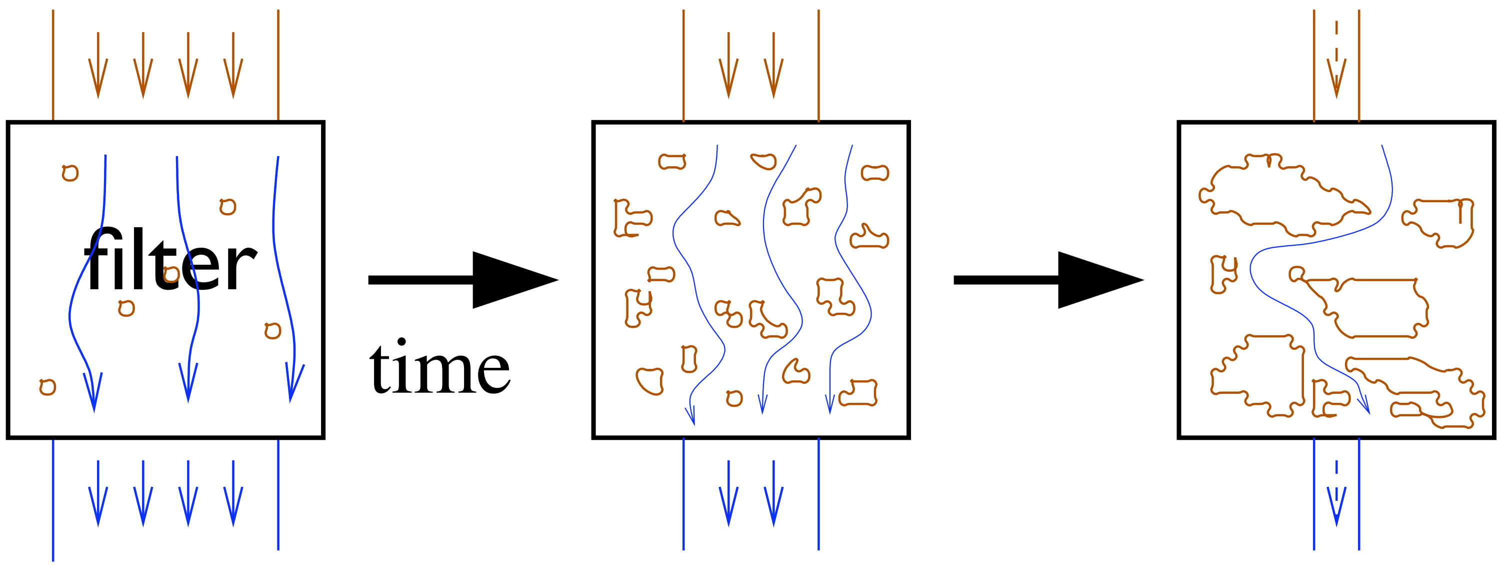

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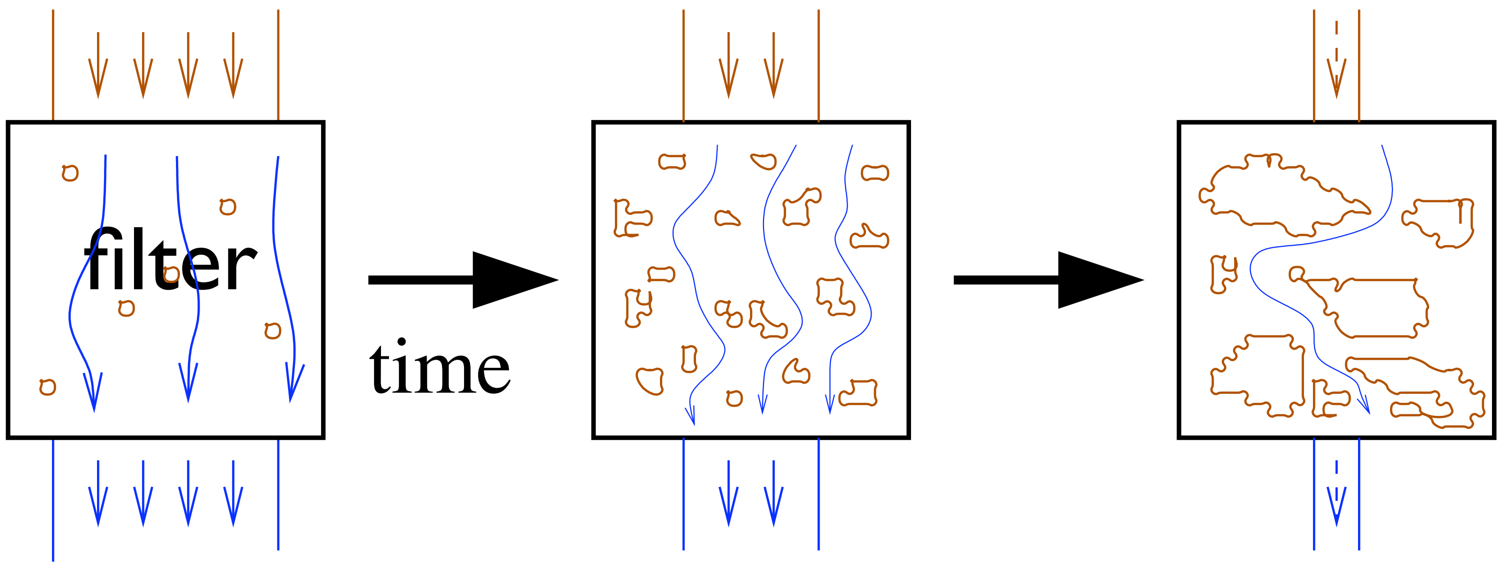

flow ⇔ geometry

no steady state

→*clogging*

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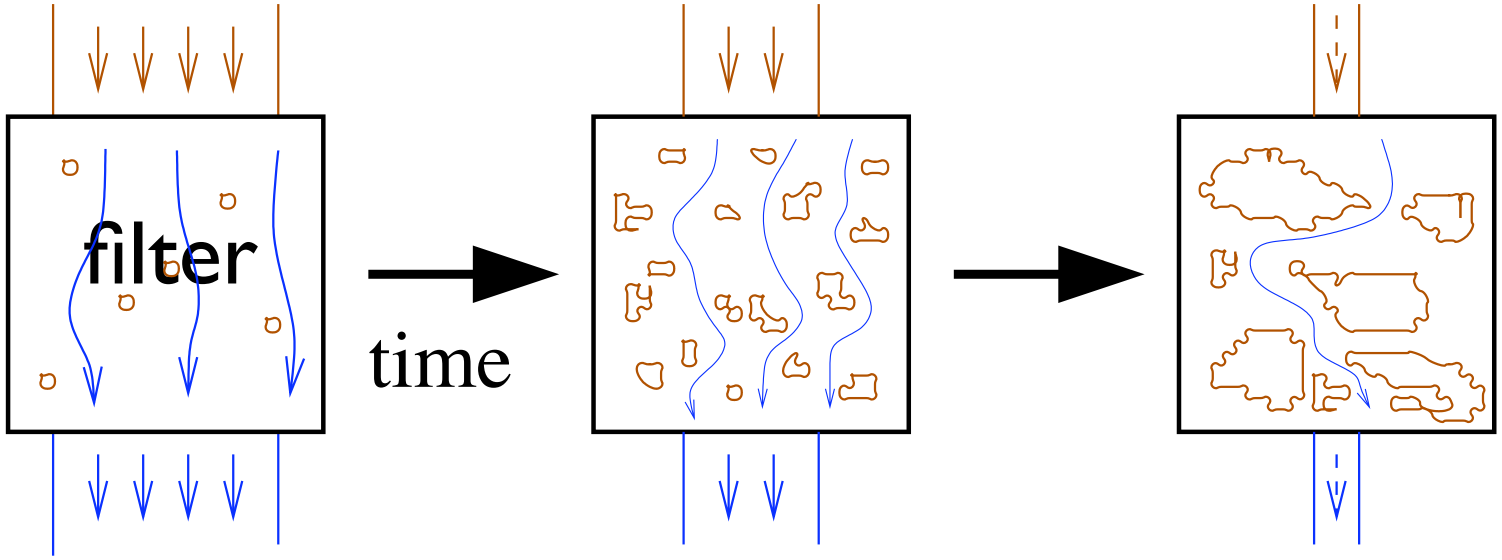

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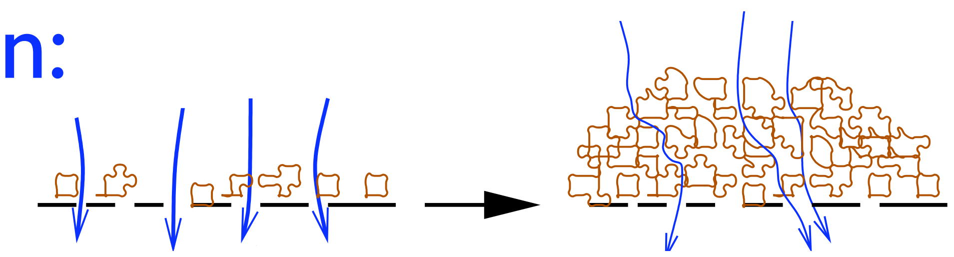
Types of filtration:

cake



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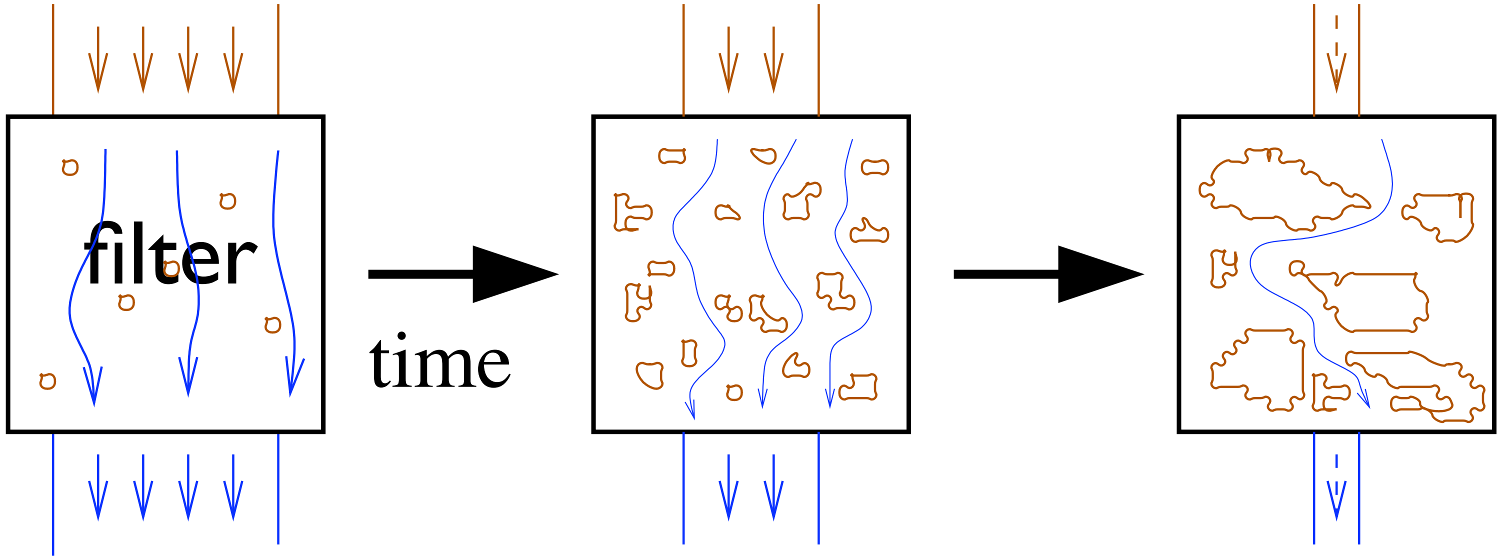

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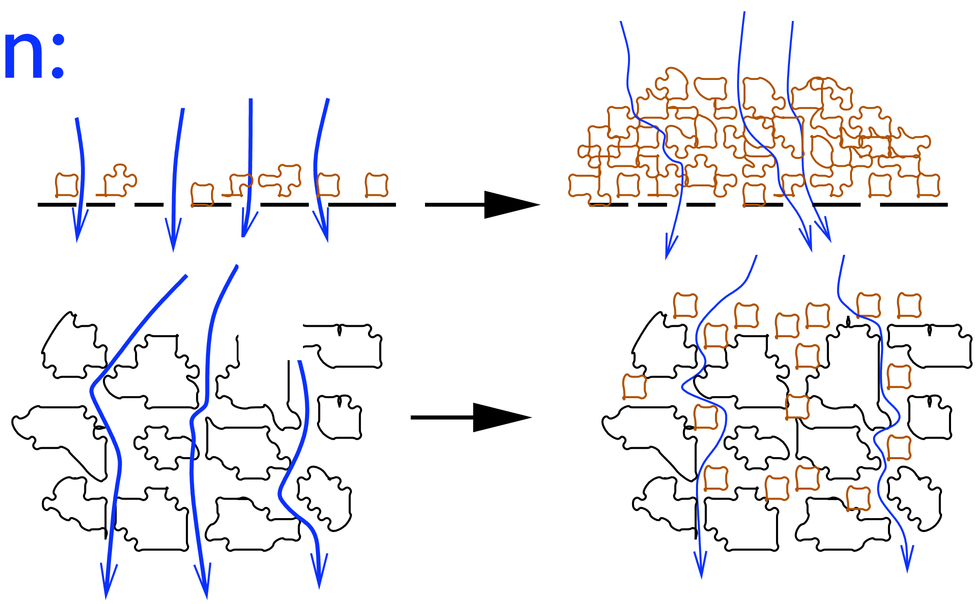

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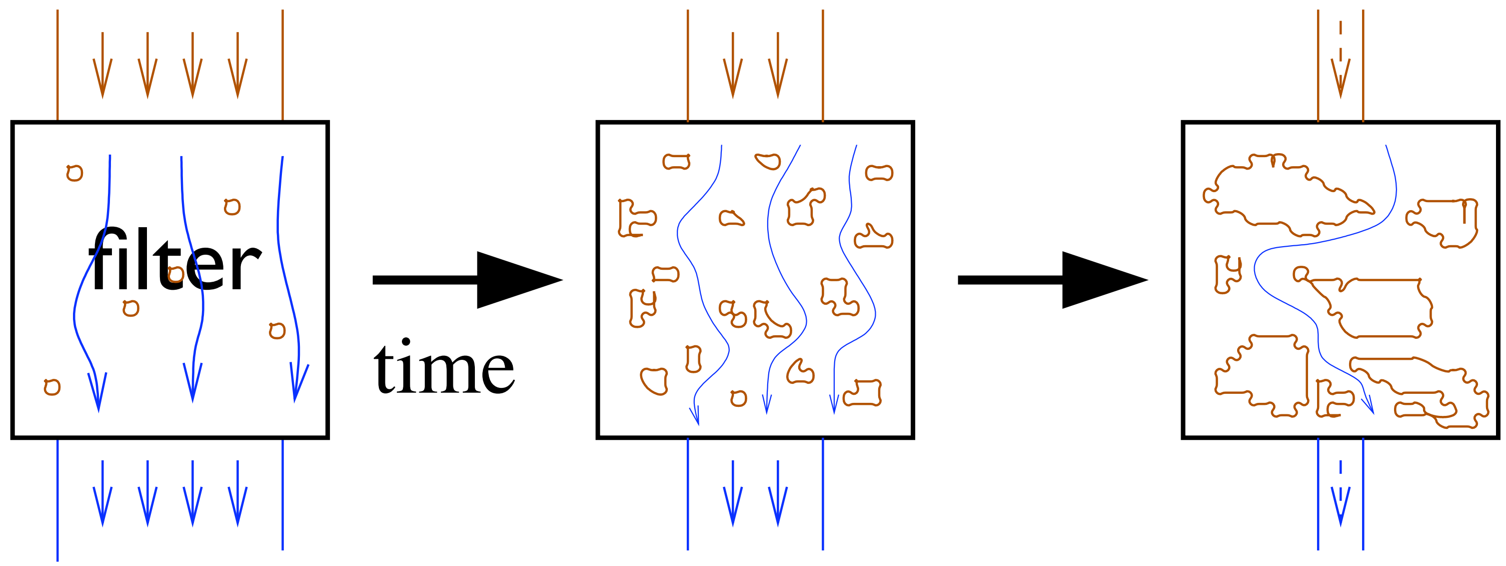

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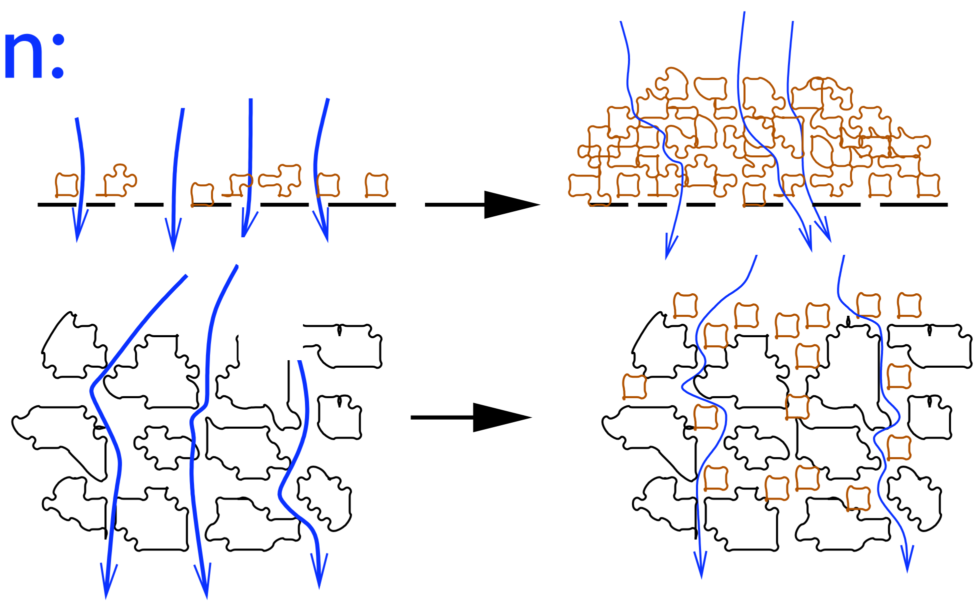

Mechanisms of depth filtration:

hydrodynamic, gravitation

electrochemical

$1\text{–}10\ \mu\mathrm{m}$

Real Cake Filter Clogging: The Aftermath

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Air conditioner filter

Dryer vent

Water filter

Fuel filter

Microscopic Modeling for Depth Filtration

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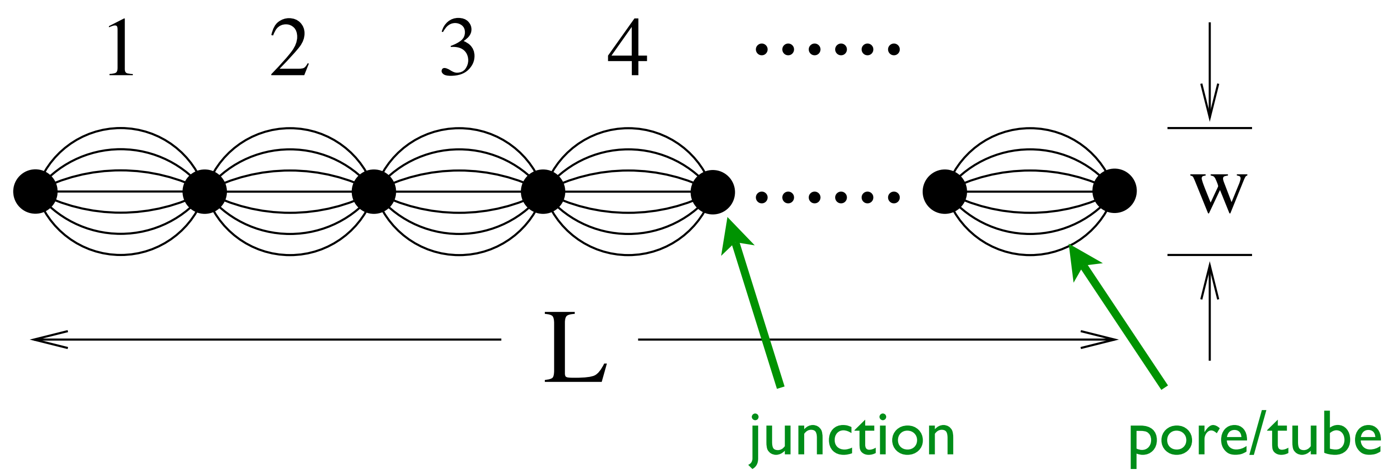
1. Bubble model



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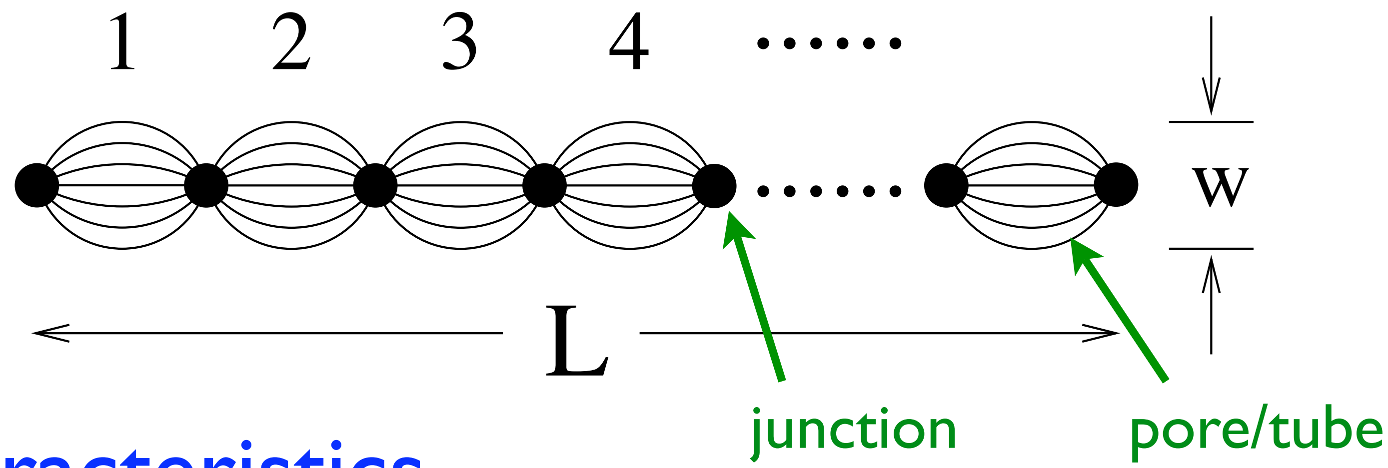

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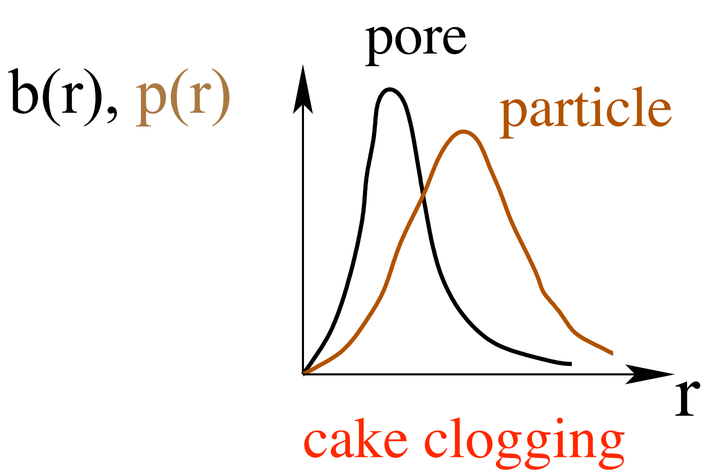

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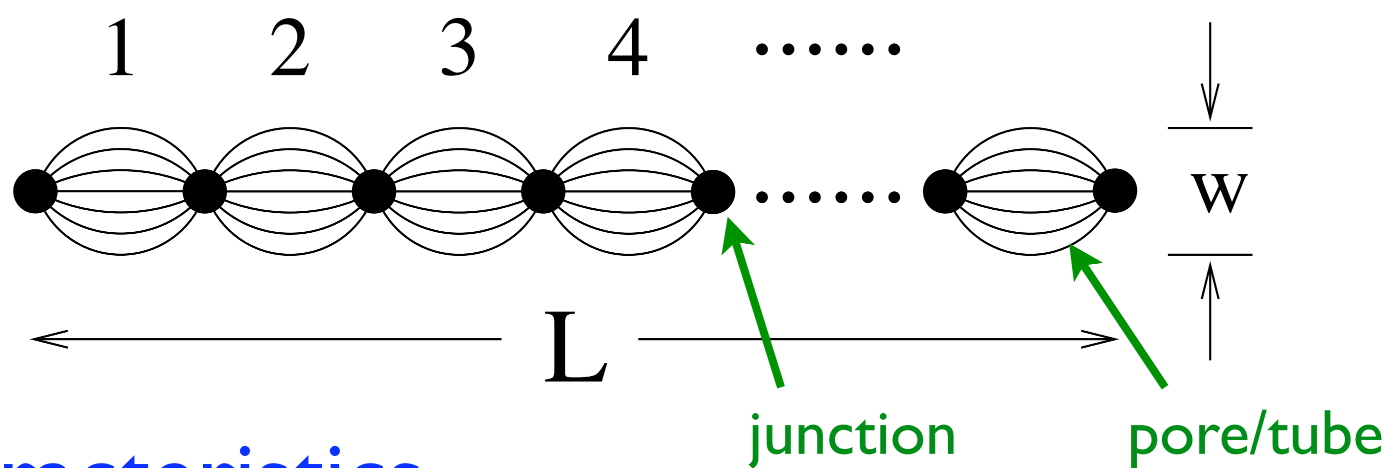

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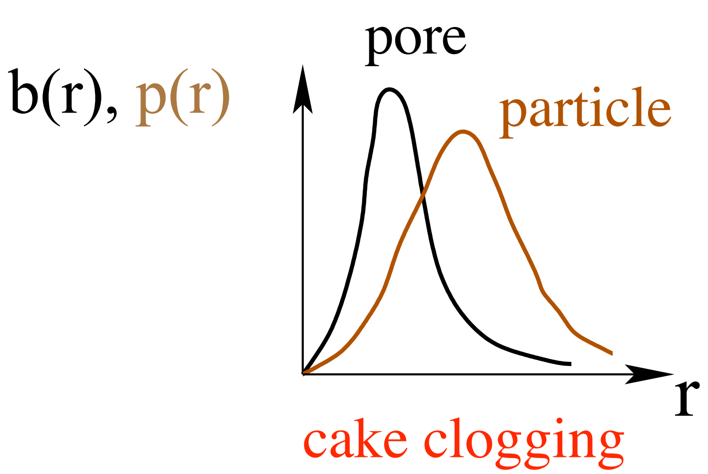

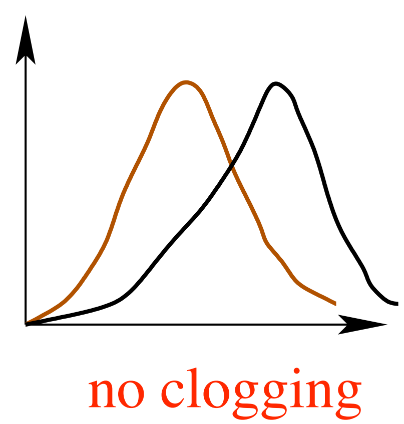

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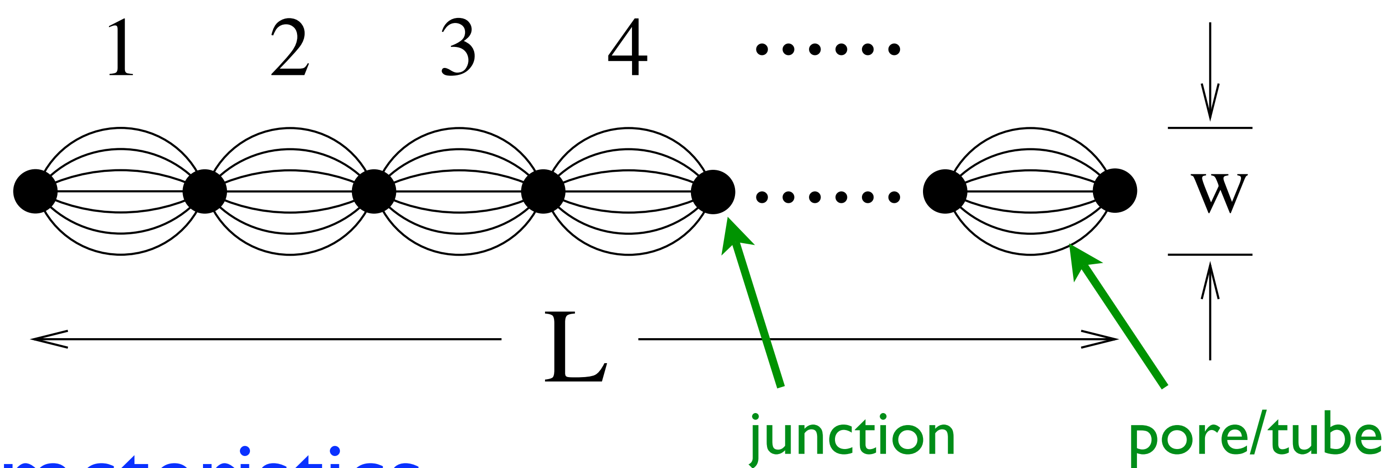

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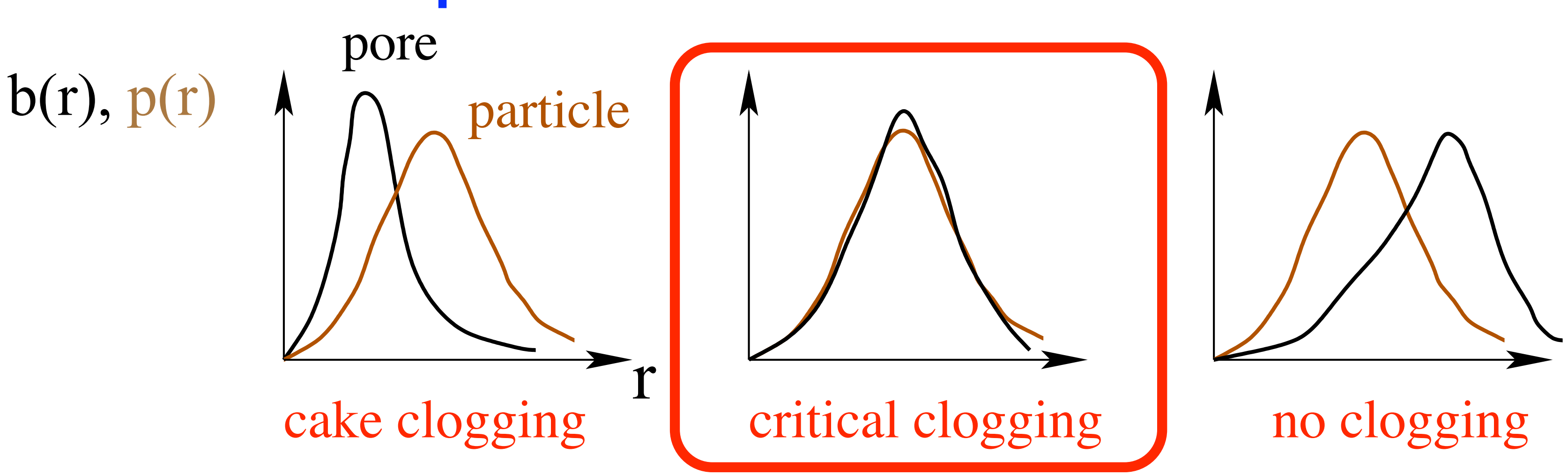

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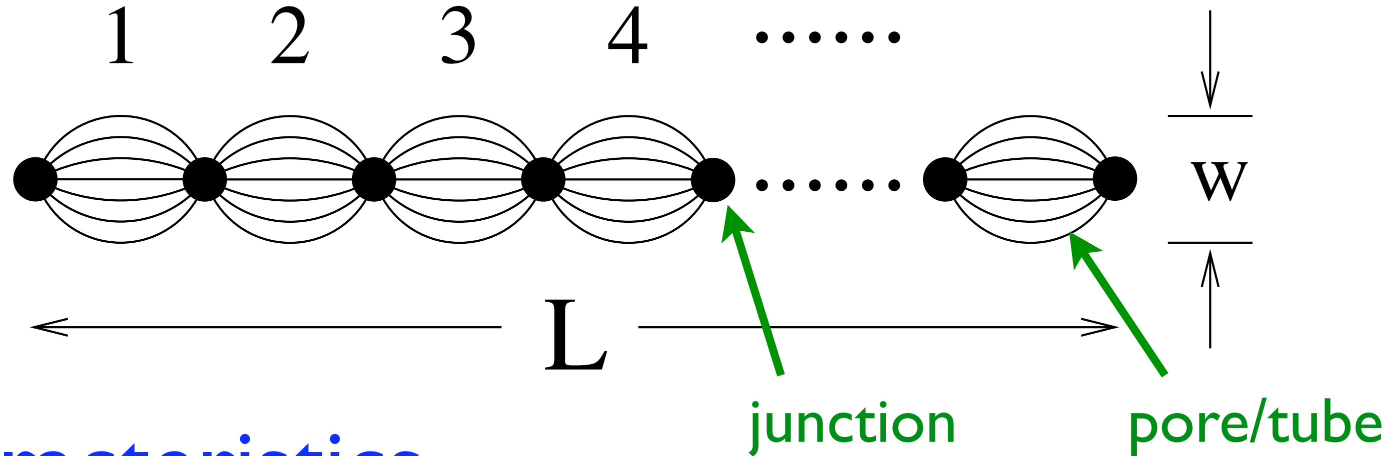

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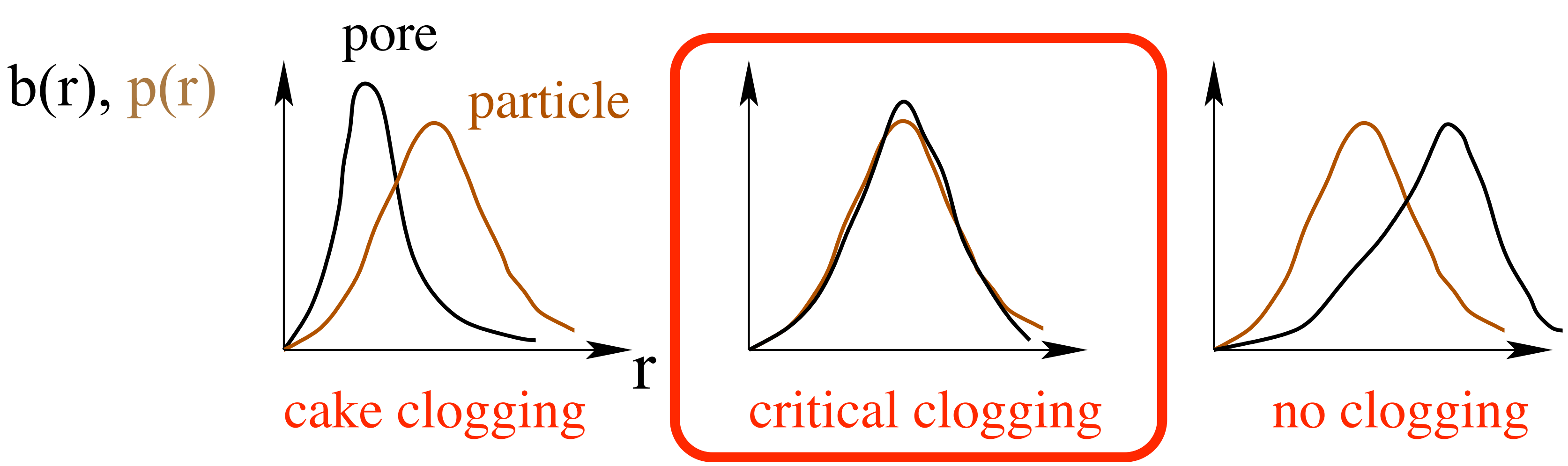

3. Flow characteristics

Poiseuille flow

dynamically neutral particles

perfect mixing at junctions $p_i = \phi_i / \sum \phi_i$, with $\phi_r \propto r_i^4$

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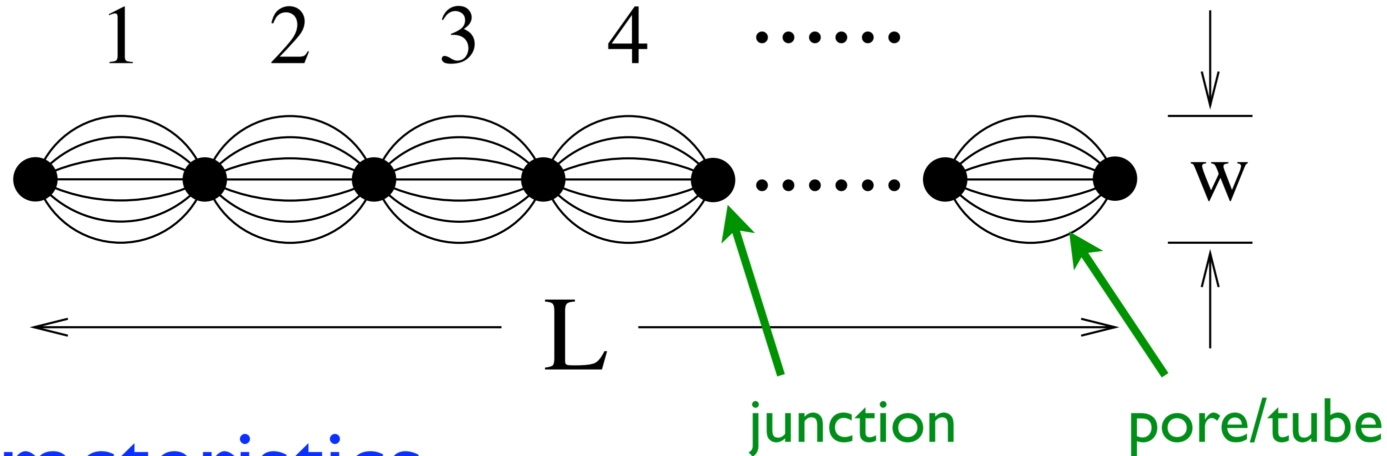

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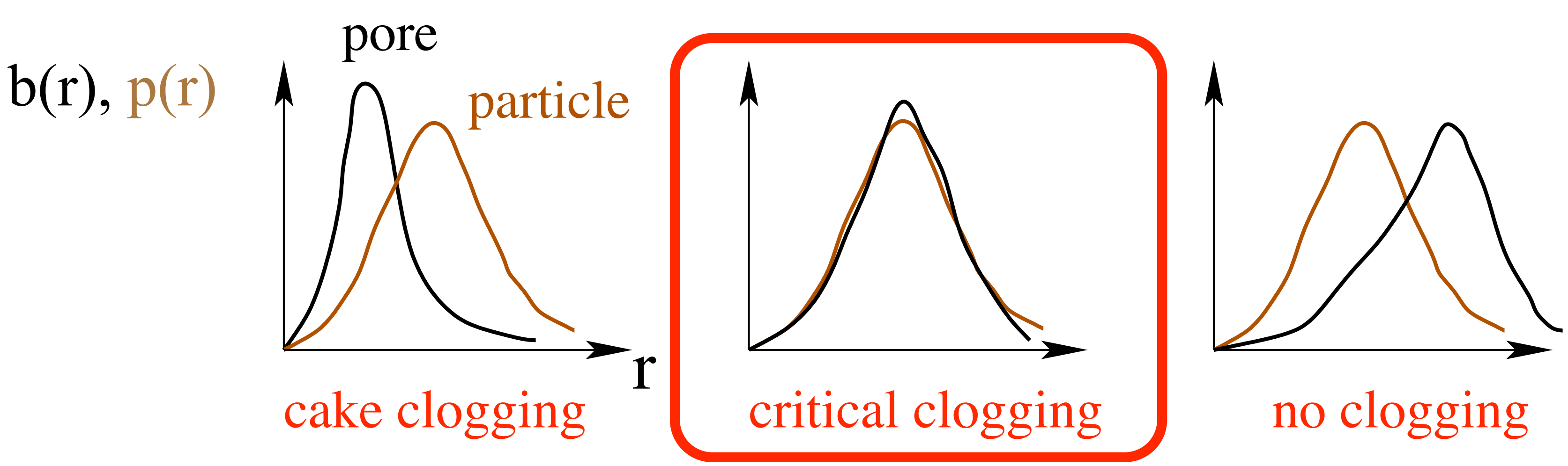

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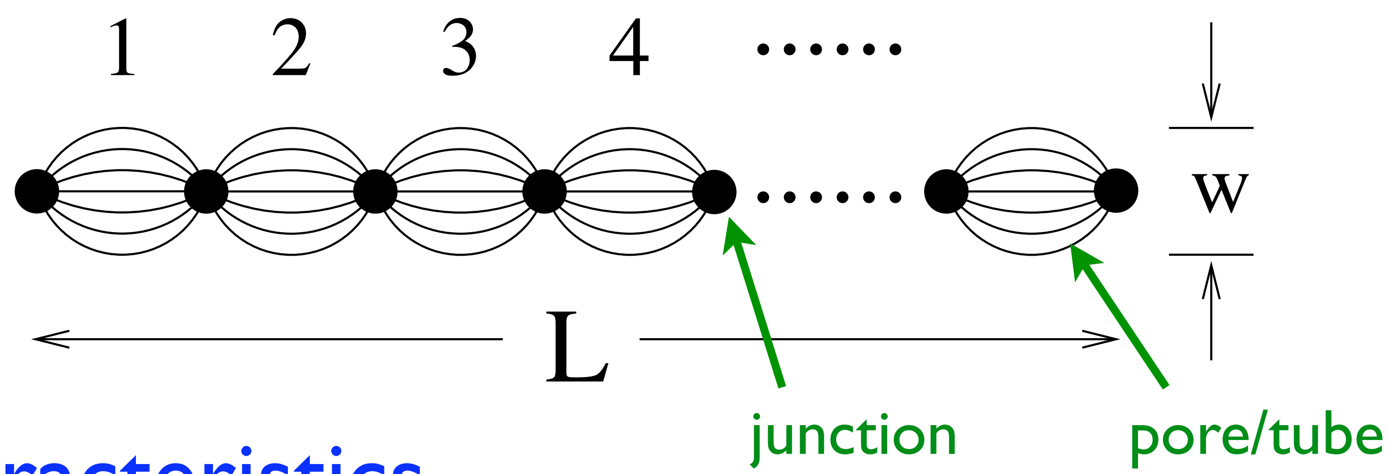

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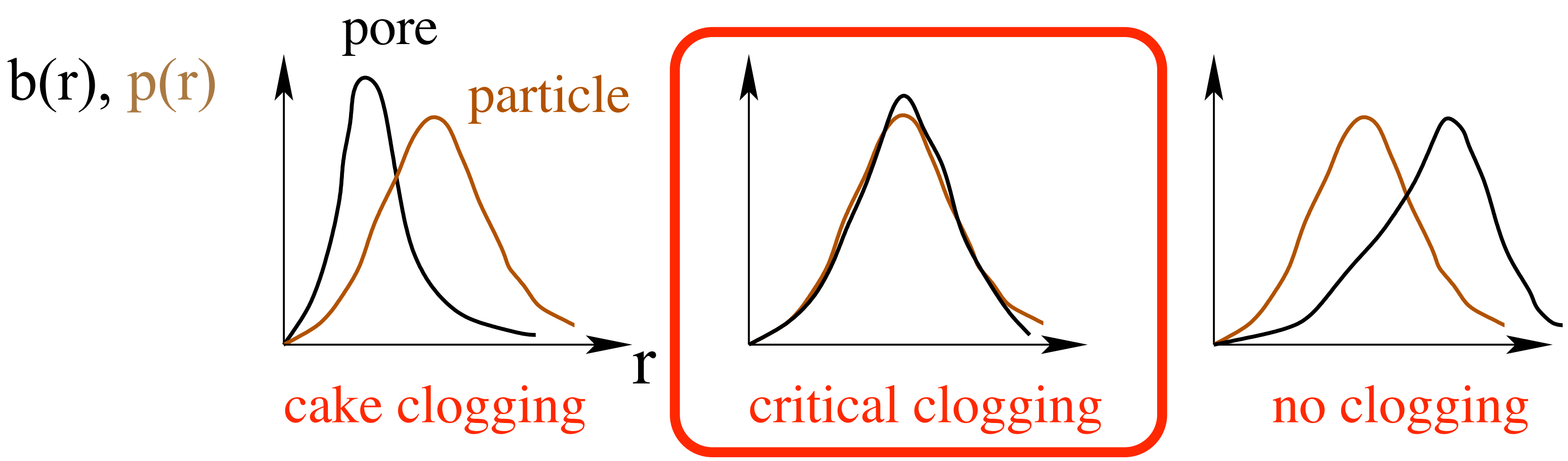

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4. Trapping mechanism:

particle > pore → *permanent, complete blockage*

1. Distribution of Trapped Particles

S. Datta & SR,
PRE **58**, R1203 (1998)

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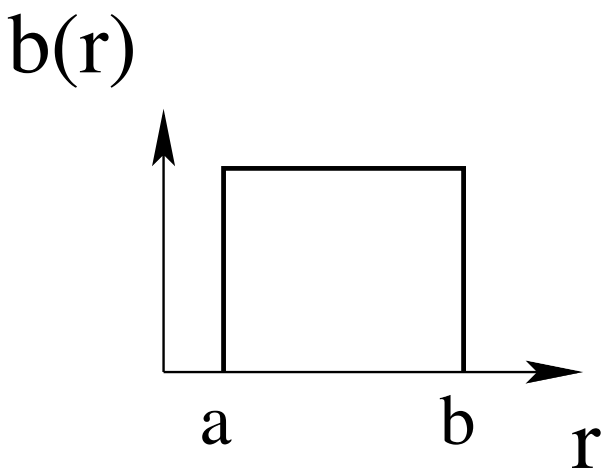

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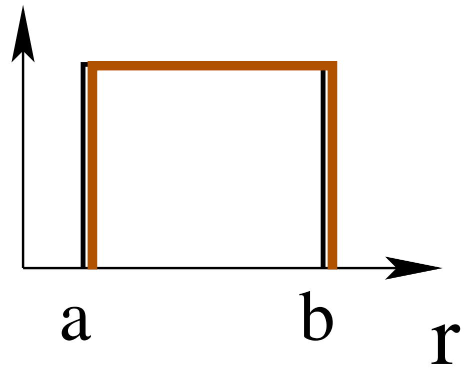
b(r) p(r) (U[a,b])



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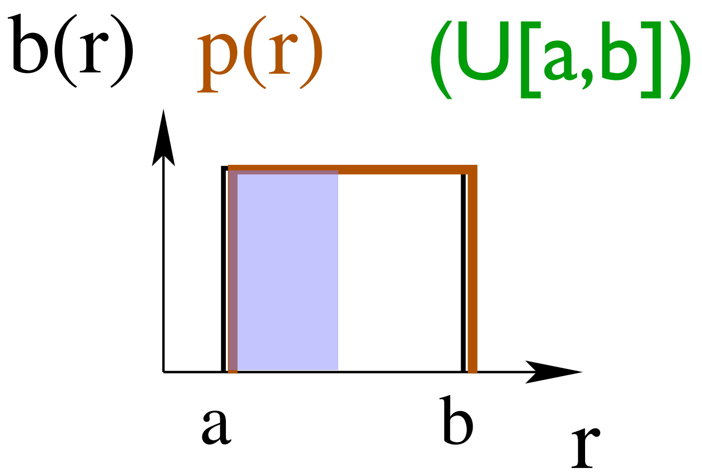

for particle of radius r:

$$
\begin{aligned}
p_< &\equiv \text{prob. of getting stuck in a bubble} \\
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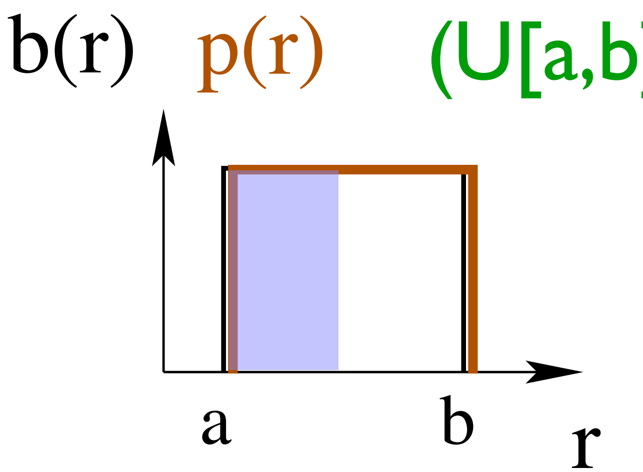

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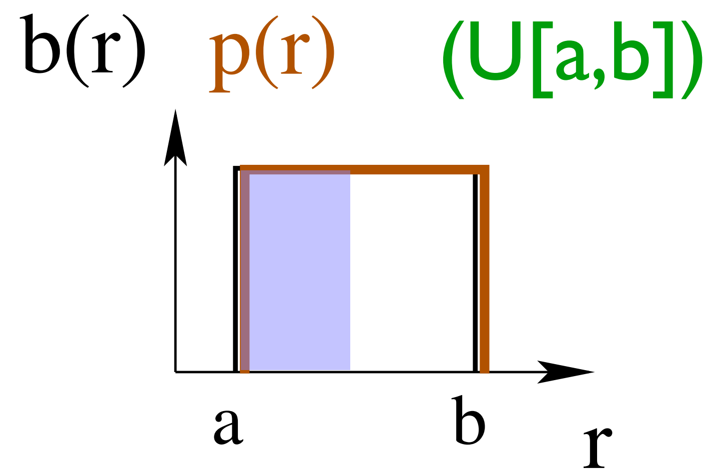

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average over particle radii

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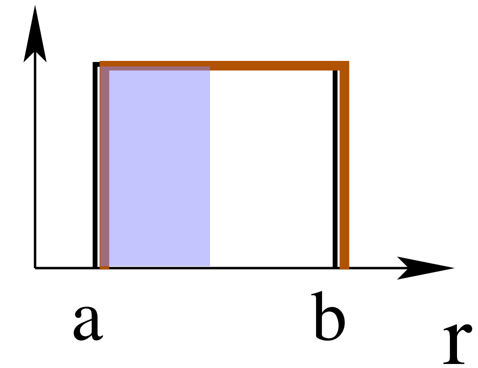

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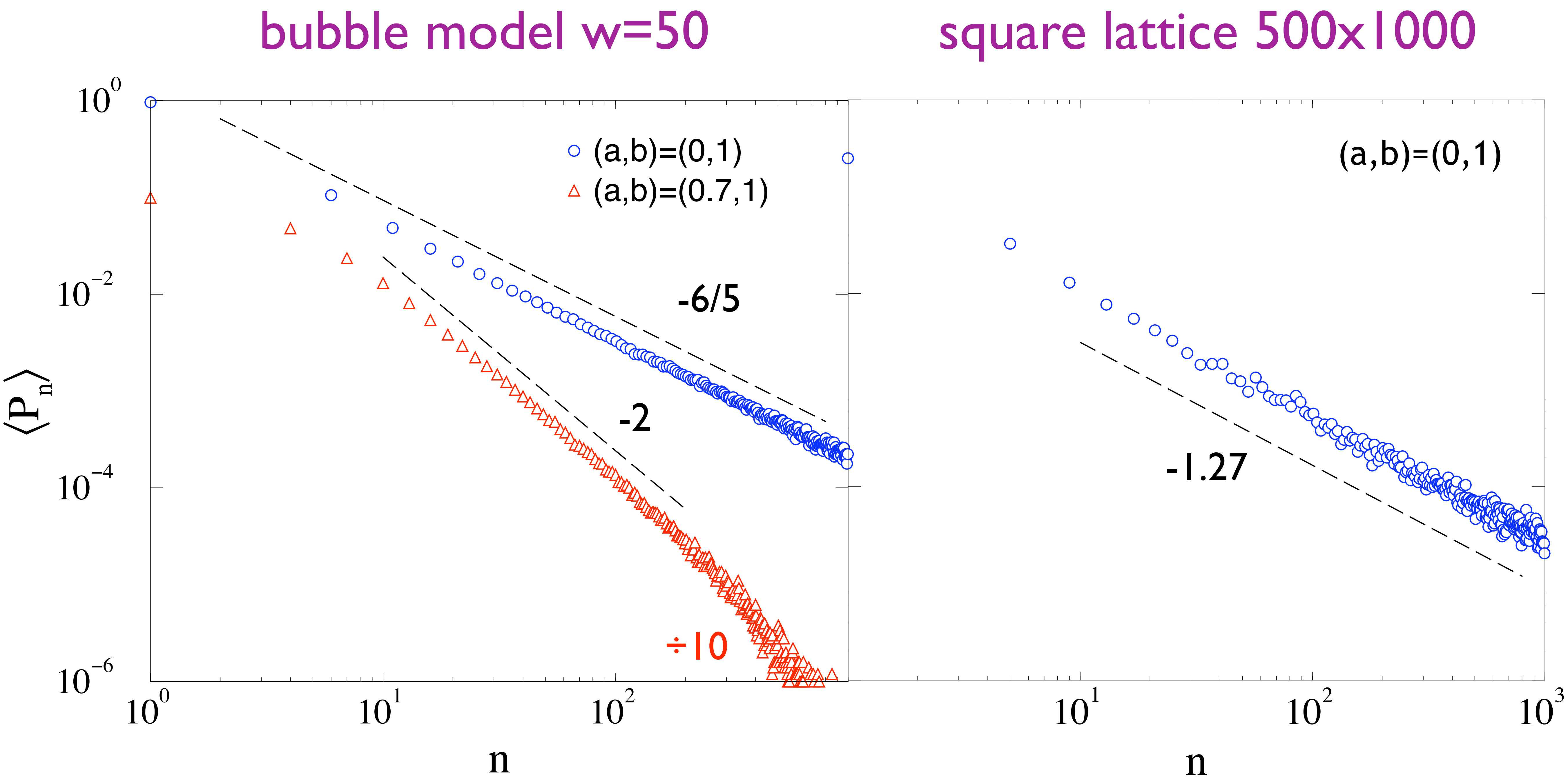
average over particle radii

$$\simeq \begin{cases} \frac{b^5 - a^5}{5a^4(b-a)} n^{-2}, & a \neq 0; \\ 0.1836\ldots n^{-6/5} & a = 0. \end{cases}$$

Trapped Particle Distributions



Large-w Approximation

prob. that particle of radius r is trapped
in a bubble of w bonds:

$$\begin{aligned} p_< &= w\int_0^r dr_1 \int_0^1 dr_2 \ldots \int_0^1 \frac{r_1^4\ dr_w}{r_1^4 + r_2^4 + \ldots + r_w^4} \\ &= w\int_0^r r_1^{w-1} dr_1 \int_0^{1/r_1} du_2 \cdots \int_0^{1/r_1} \frac{du_w}{1 + u_2^4 + \ldots + u_w^4} \\ &\sim w\int_0^r r_1^{w-1} dr_1 \int_{0+}^{1/r_1} \frac{u^{w-2}}{u^4}\, du \end{aligned}$$

$u_i = r_i / r_1$

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$$u_i = r_i/r_1$$

$$p_< \propto \begin{cases} r^w, & w < 5; \\ r^5 \ln r, & w = 5; \\ r^5, & w > 5. \end{cases}$$

2. Size Distribution of Escaping Particles

assume particle and pore sizes (U[0,1])

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$$\sim \exp(-nr^5)$$

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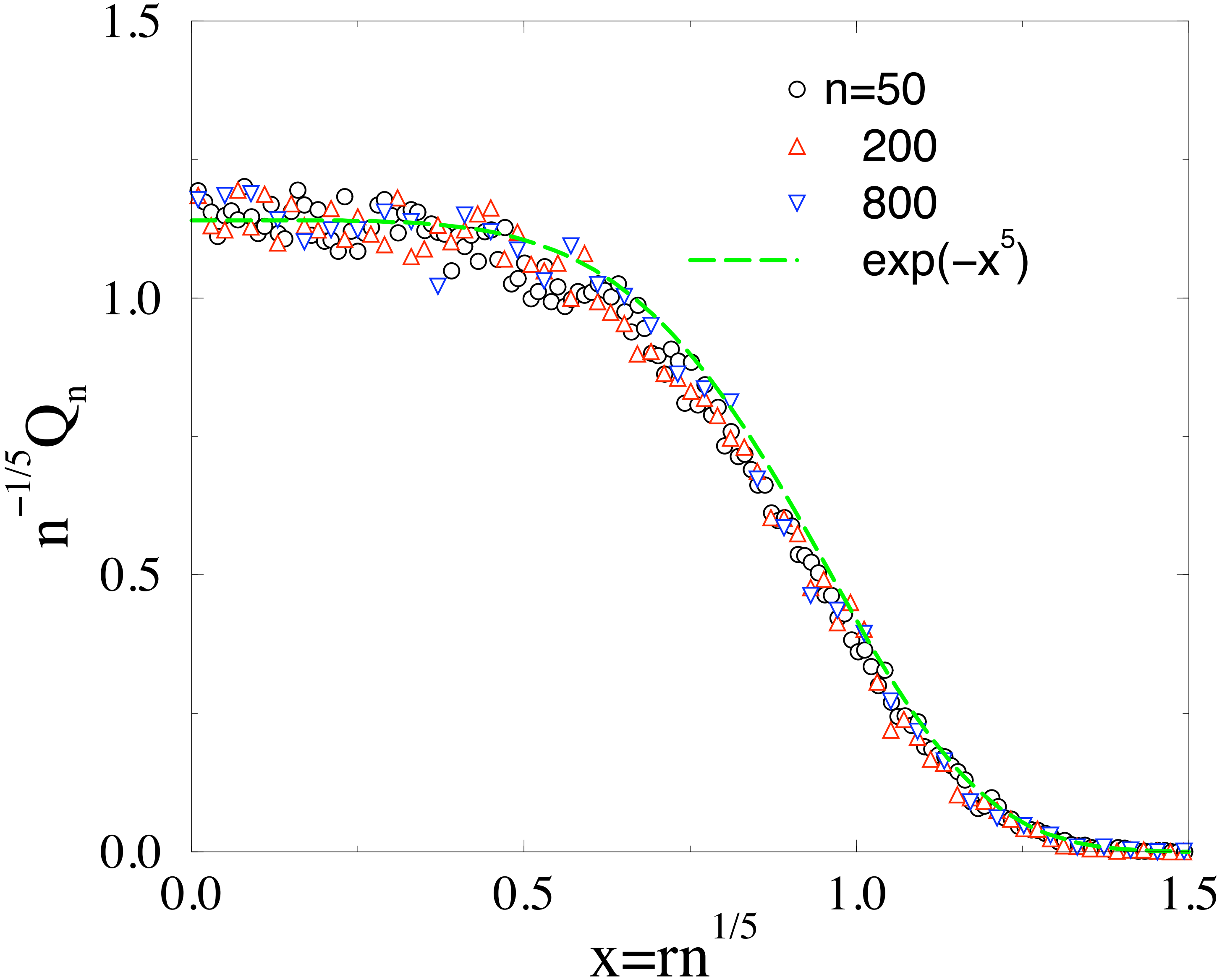
$$Q_n(r) \equiv \text{size distribution of escapees at } n$$
$$\propto n^{1/5}\exp(-nr^5)$$

Escapee Size Distribution

bubble model w=10



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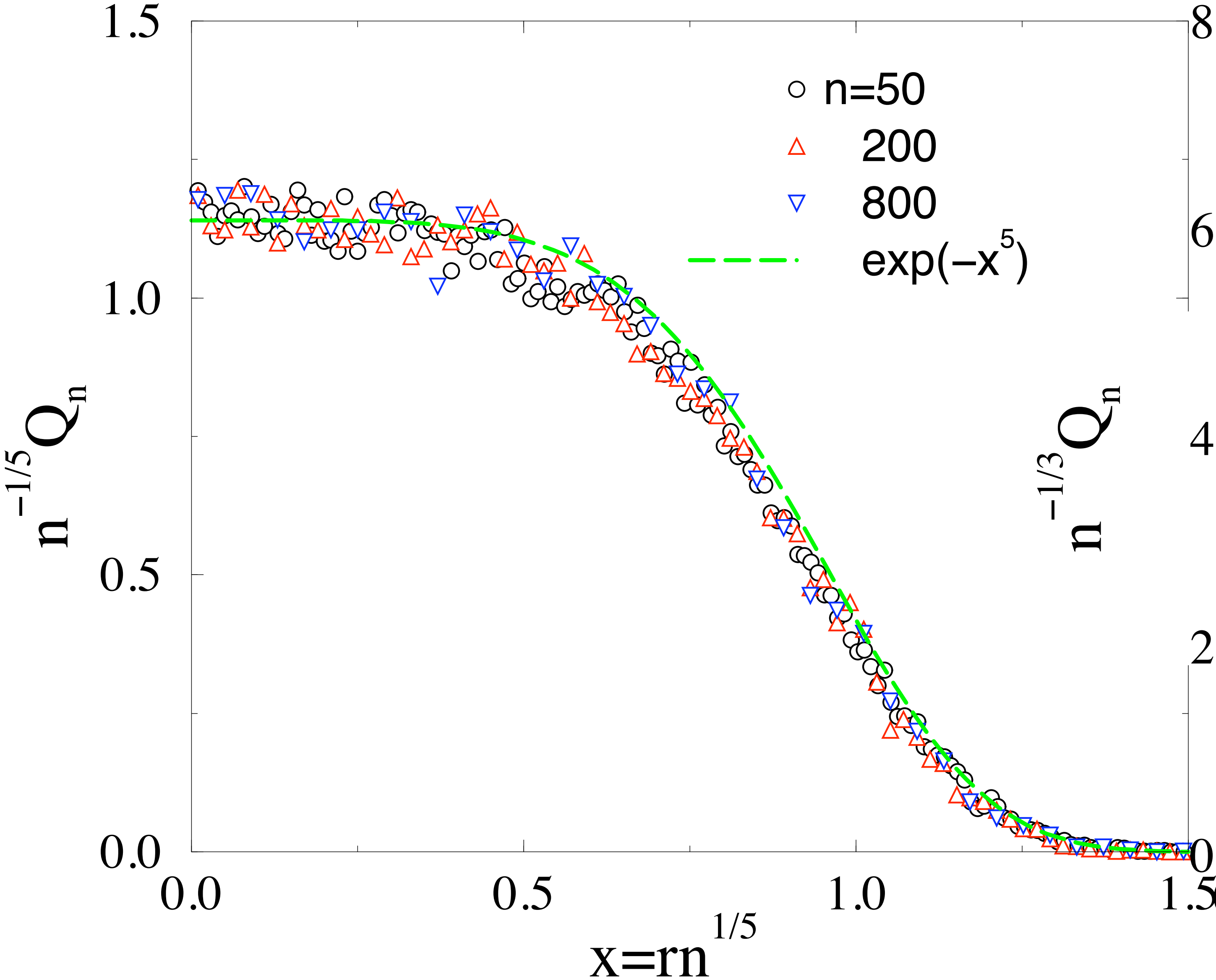
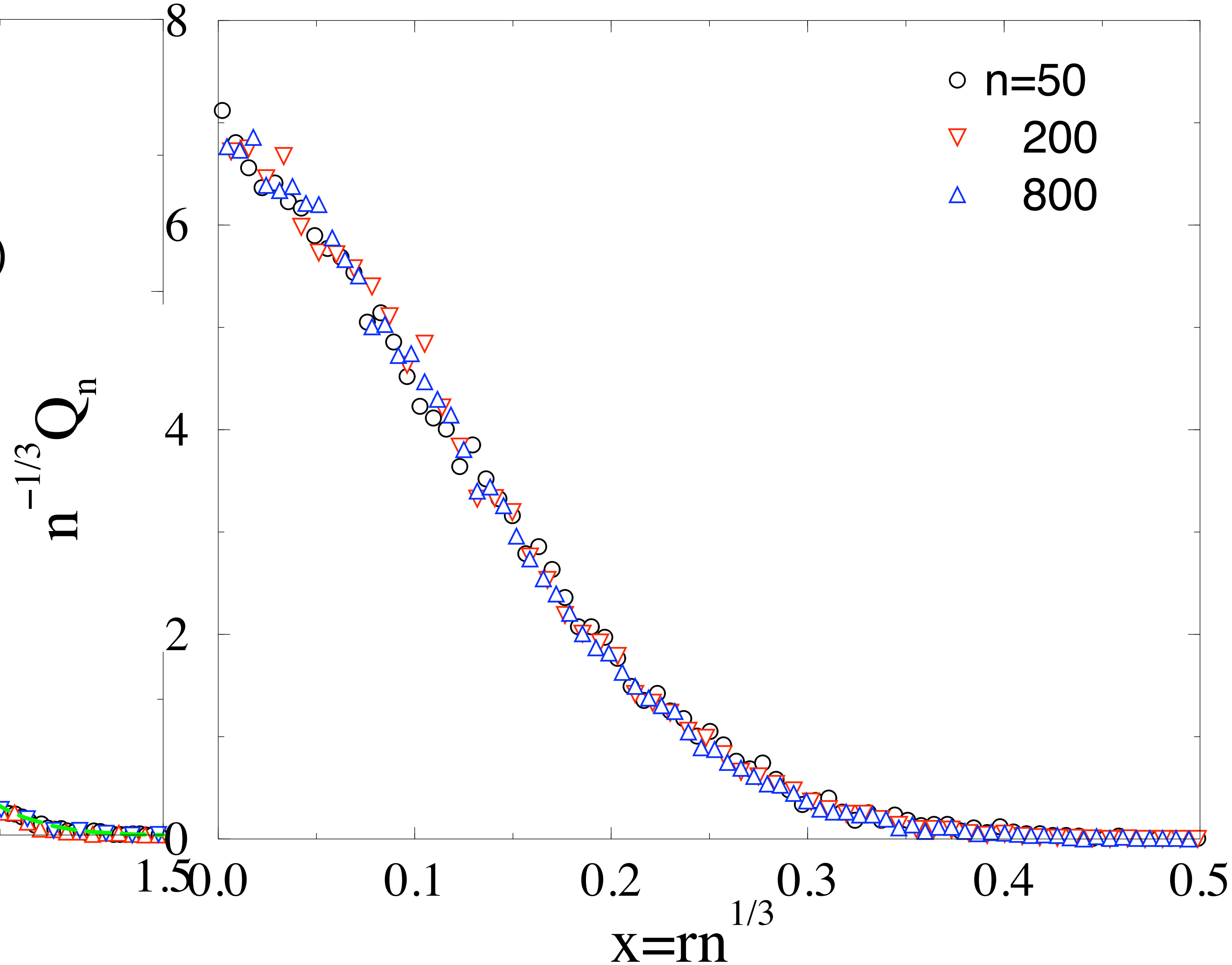


square lattice 50x1000

"dry" flow, entrance prob. $\propto r^2$



Clogging Time and its Distribution

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PRL **84**, 6018 (2000)

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Single bond clogging:

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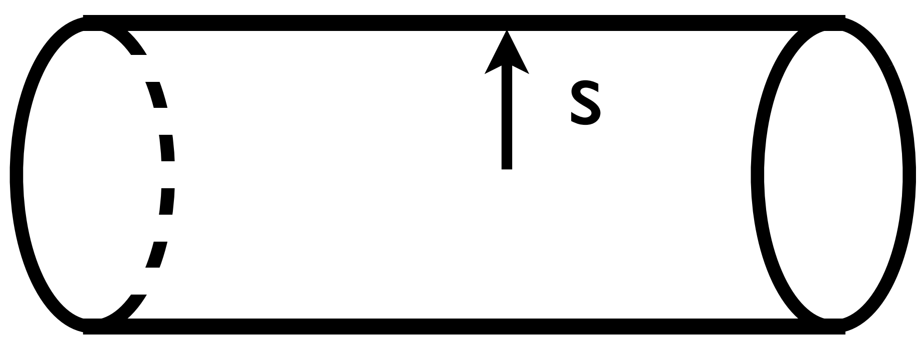
assume $b(r) = 2\alpha r e^{-\alpha r^2} \rightarrow s = 1/\sqrt{\alpha} \quad p(r) = 2re^{-r^2}$



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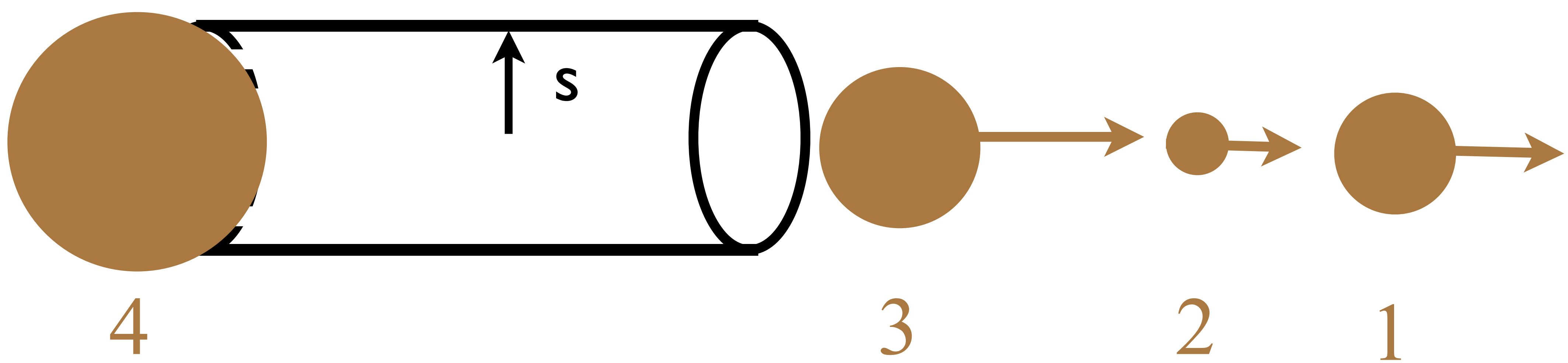

clogged!

Clogging Time and its Distribution

Single bond clogging (cont):

probability that bond of radius R is blocked by Nth particle:

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Single bond clogging (cont):

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$$Q_N(R) = \left[\int_0^R p(r)\,dr\right]^{N-1} \int_R^\infty p(r)\,dr$$

N-1 get through Nth gets stuck

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bond average

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$$\langle Q_N \rangle \quad = \quad \int_0^\infty Q_N(R)\;\; b(R)\,dR$$

bond average

$$p(r) = 2r\,e^{-r^2}$$
$$b(r) = 2\alpha r\,e^{-\alpha r^2}$$

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$$\begin{aligned} \langle Q_N \rangle &= \int_0^\infty Q_N(R)\ b(R)\,dR \\ &\simeq \int_0^\infty (1-e^{-R^2})^{N-1}\, e^{-R^2}\ 2\alpha R e^{-\alpha R^2}\,dR \end{aligned}$$

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&\sim \int e^{-Nx^{1/\alpha}} x^{1/\alpha}\,dx \;\sim N^{-(1+\alpha)}
\end{aligned}$$

bond average

$$p(r) = 2r\,e^{-r^2}$$

$$b(r) = 2\alpha r\,e^{-\alpha r^2}$$

$$x = e^{-\alpha R^2}$$

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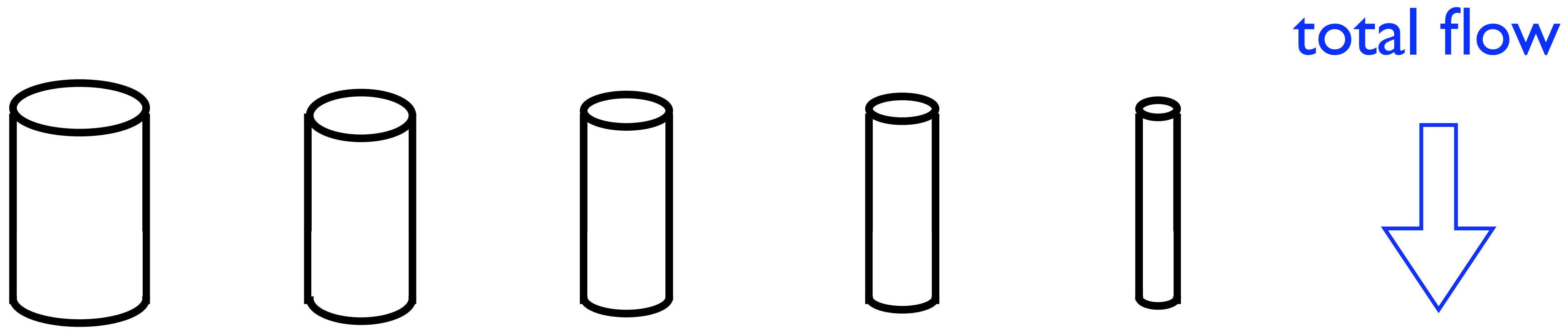
\langle Q_N \rangle &= \int_0^\infty Q_N(R)\;\; b(R)\,dR \quad \textit{bond average} \qquad p(r) = 2r\,e^{-r^2} \quad b(r) = 2\alpha r\,e^{-\alpha r^2} \\
&\simeq \int_0^\infty (1 - e^{-R^2})^{N-1}\, e^{-R^2}\;\; 2\alpha R e^{-\alpha R^2}\, dR \\
&\sim \int e^{-Nx^{1/\alpha}} x^{1/\alpha}\, dx \;\sim N^{-(1+\alpha)} \qquad x = e^{-\alpha R^2}
\end{aligned}$$

$$\langle N \rangle \sim \int_1^\infty \frac{N}{N^{1+\alpha}} dN \sim \begin{cases} \infty & \text{for } s = \frac{1}{\sqrt{\alpha}} \geq 1 \\ (1-\alpha)^{-1} & \text{for } s = \frac{1}{\sqrt{\alpha}} \leq 1 \end{cases}$$

Cartoon for Network Clogging

size-ordered blocking (entrance prob. $r^4 \to r^\infty$)

$t = 0$



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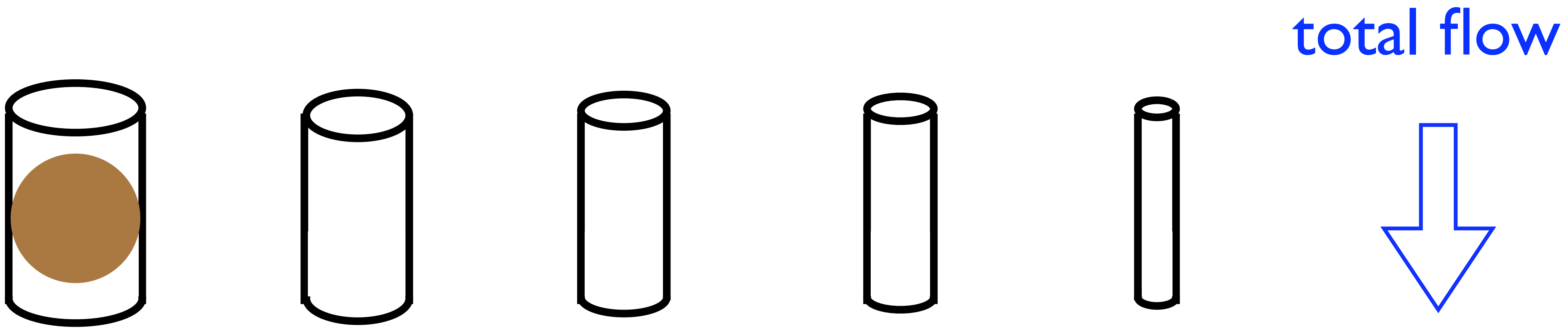

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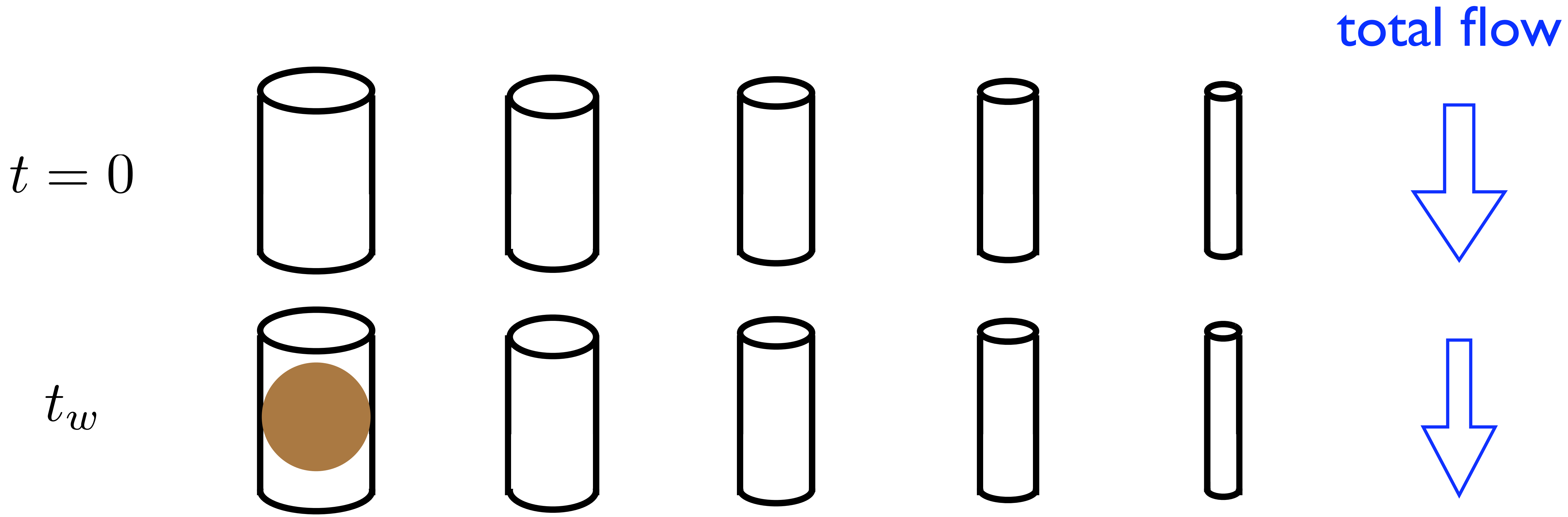

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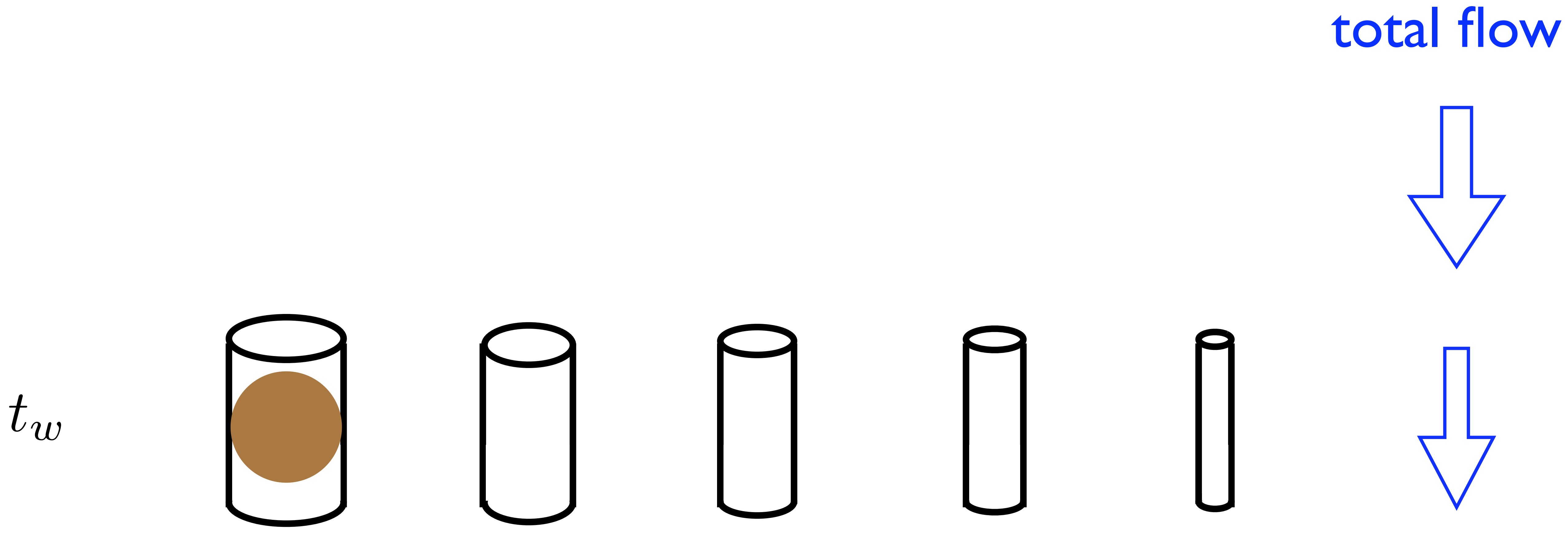

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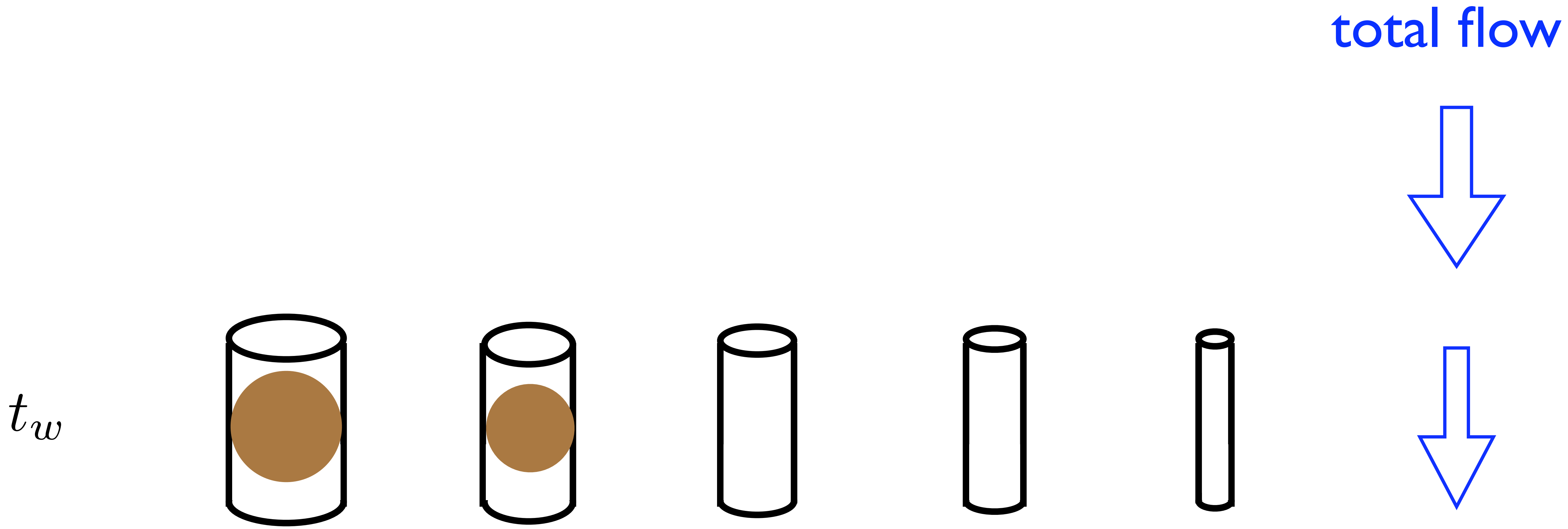

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total flow

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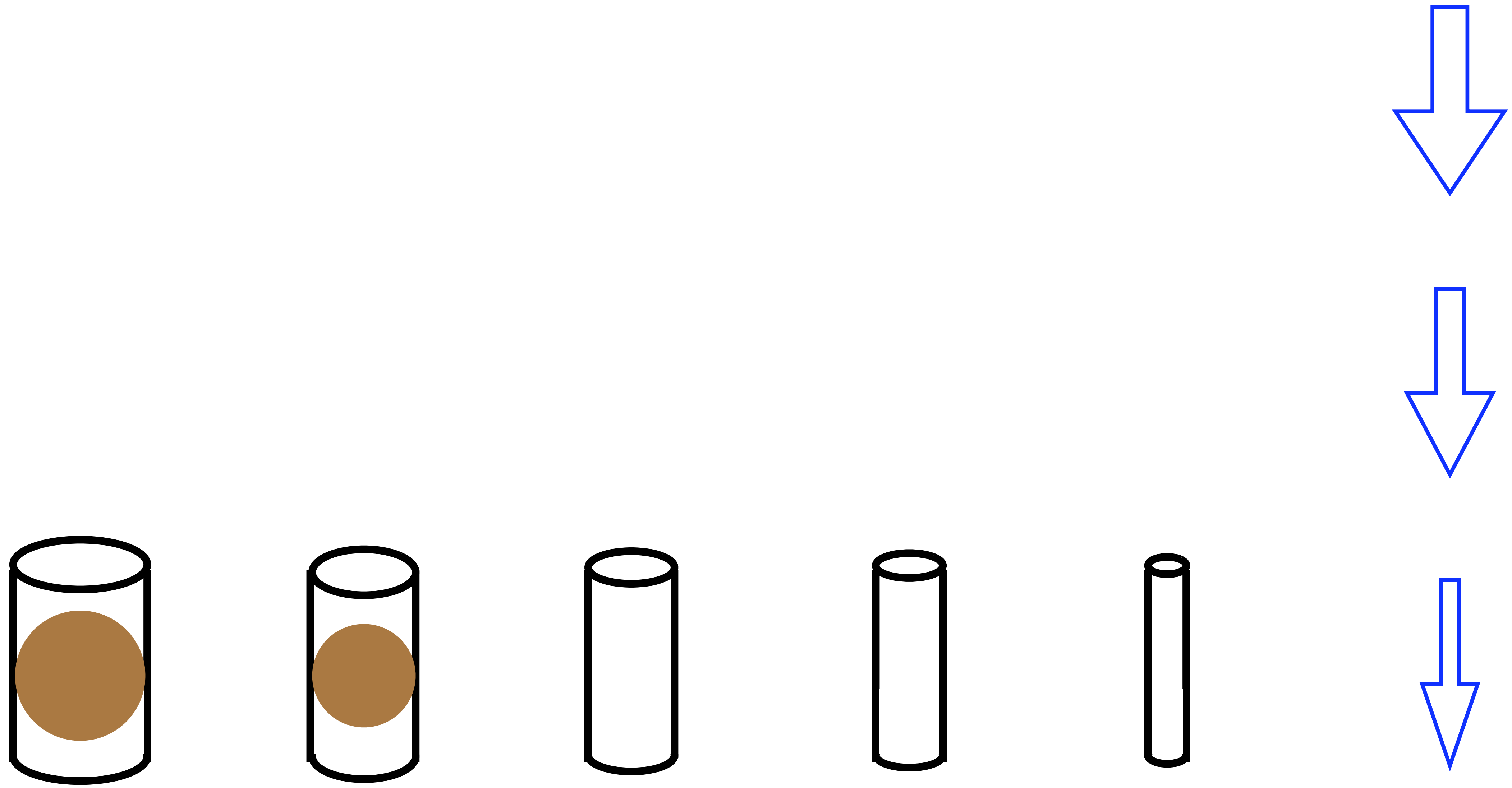

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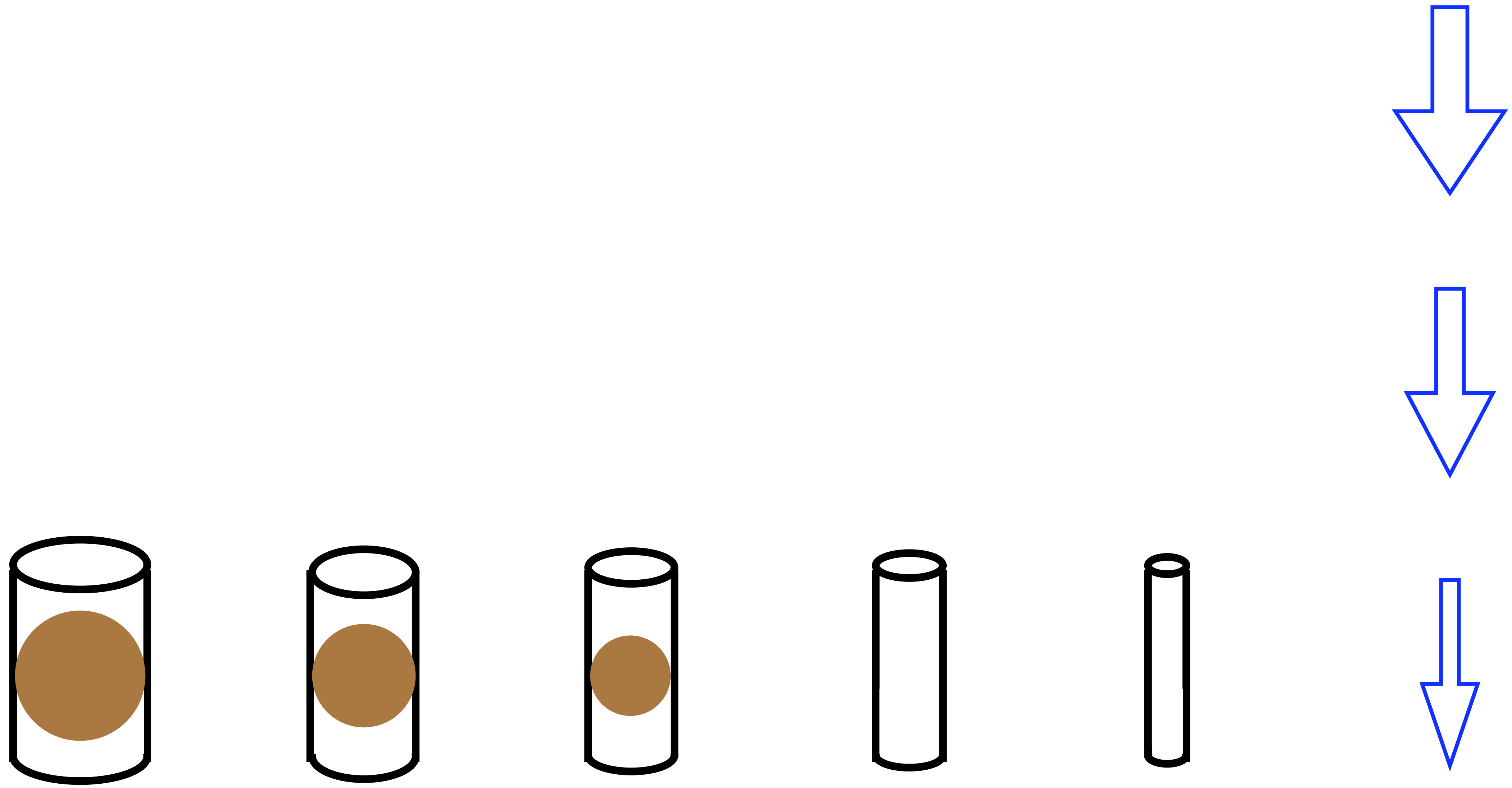

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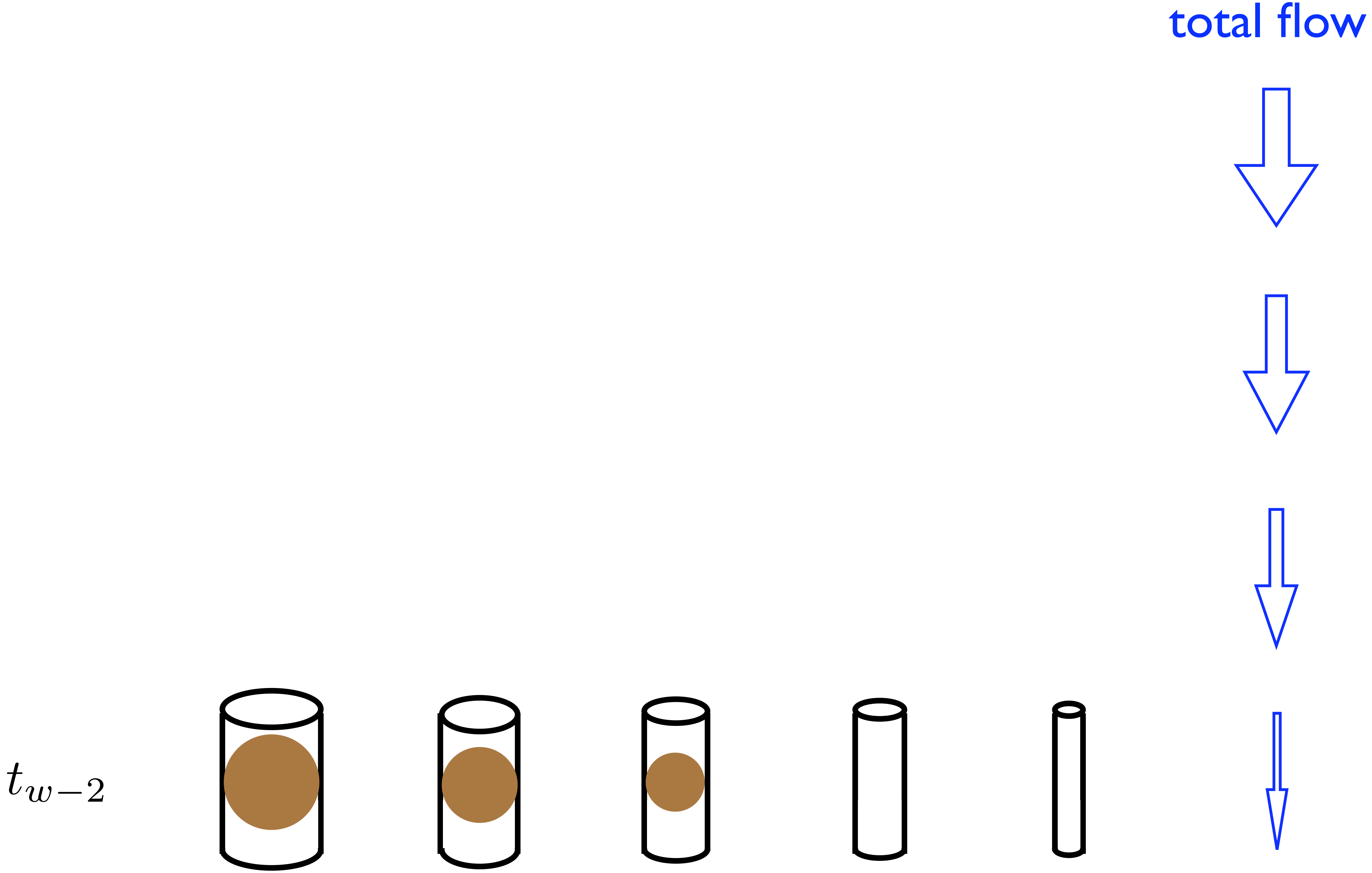

Estimate of the Clogging Time

size-ordered blocking: *biggest first, smallest last*

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clogging time:

$$T = t_1 + t_2 + t_3 + \ldots \approx w^2 \left[1 + \frac{1}{2^3} + \frac{1}{3^3} + \ldots\right] \propto w^2$$

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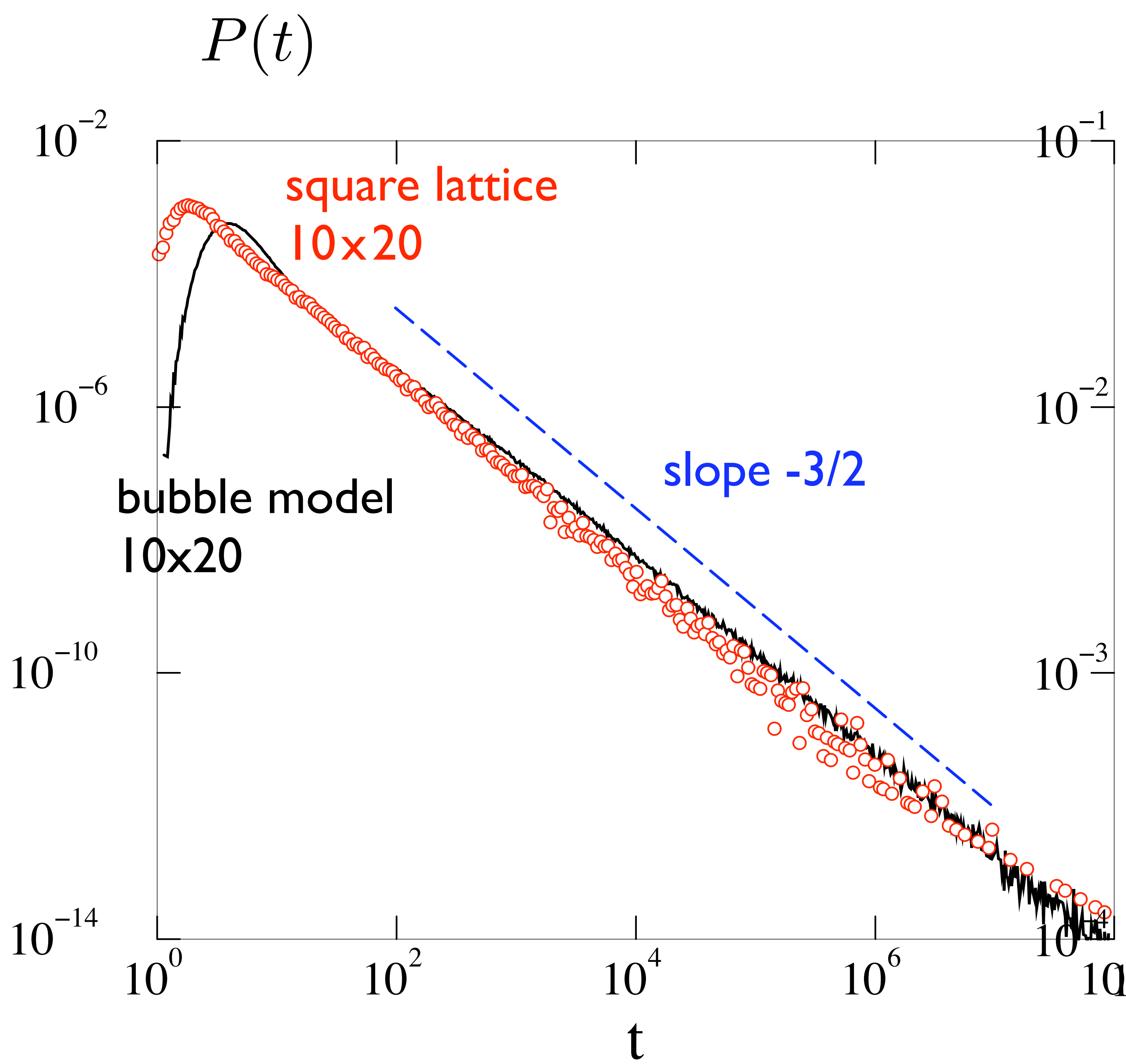
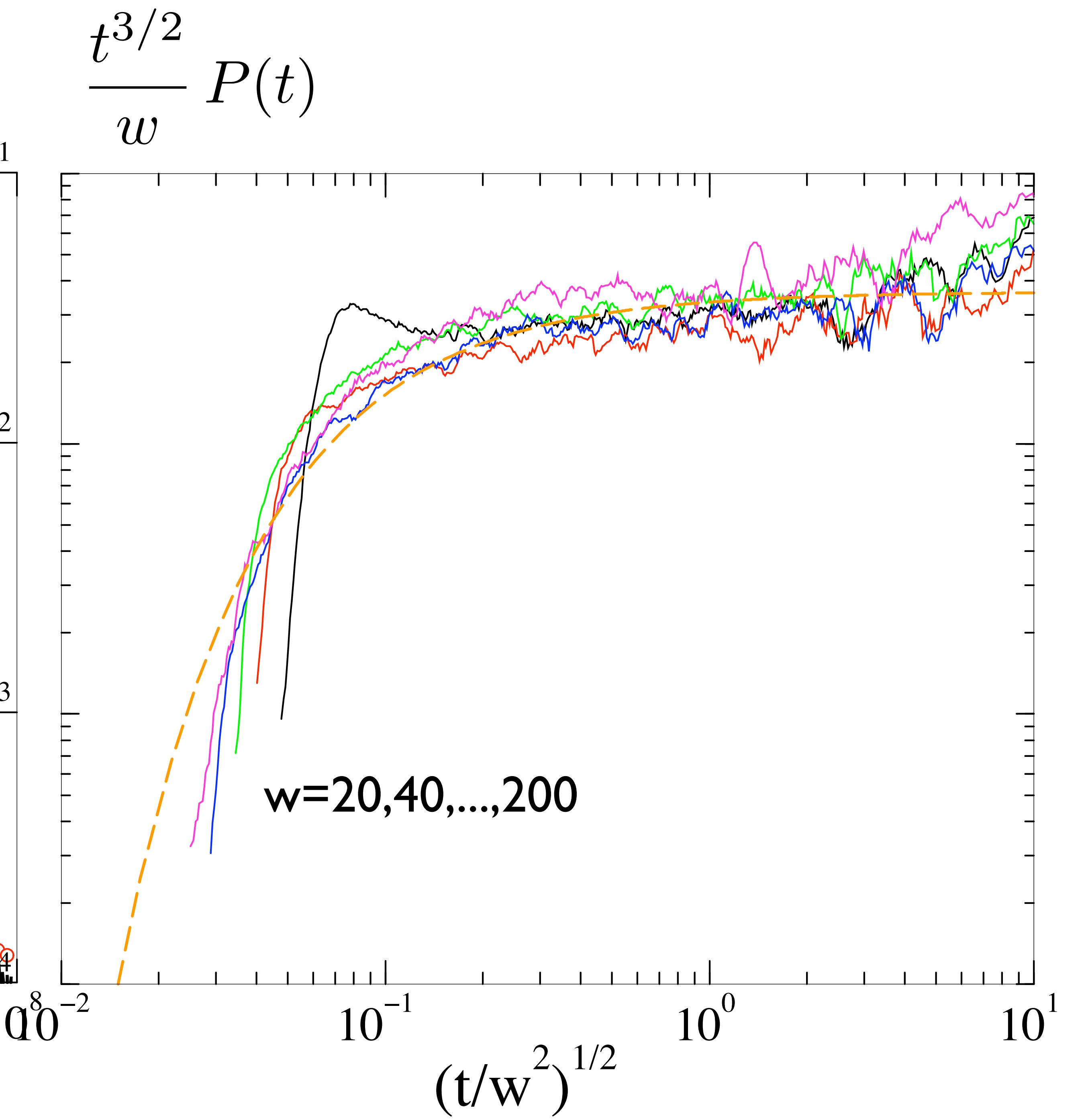
Clogging Time Distribution $P_w(t) \sim \frac{w}{t^{3/2}}\, e^{-w/t^{1/2}}$





Moments of the Clogging Time

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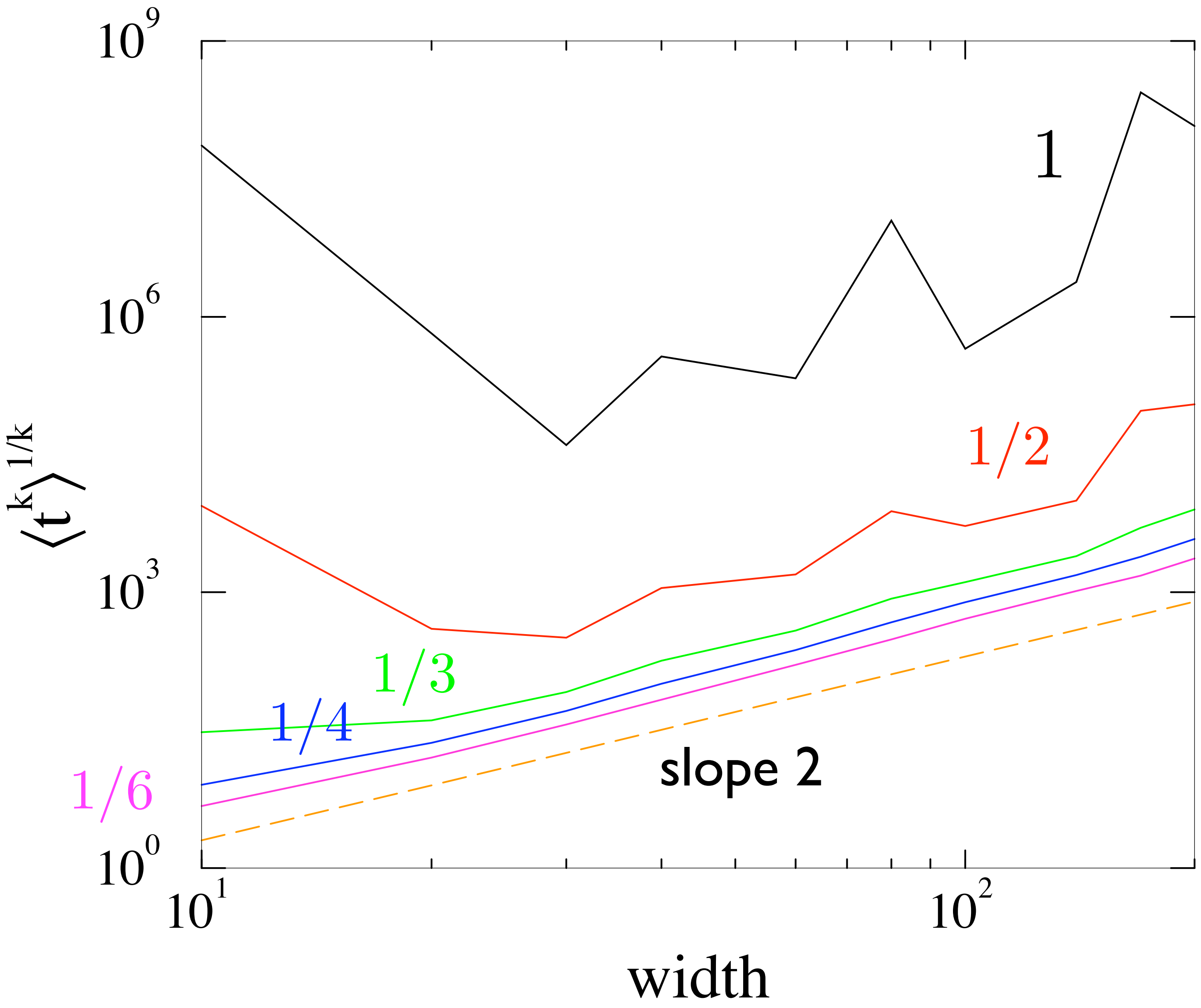
$$\langle t^k \rangle = \int_0^\infty t\, P_w(t)\, dt \approx \int_{w^2}^{N^2 w^2} w\, t^{k-3/2}\, dt$$

$$M_k(w) \equiv \langle t^k \rangle^{1/k} \sim \begin{cases} w^2 N^{2-1/k} & k > 1/2 \\ w^2 (\ln N)^2 & k = 1/2 \\ w^2 & k < 1/2 \end{cases}$$

Measures of Clogging Time

moments of clogging time



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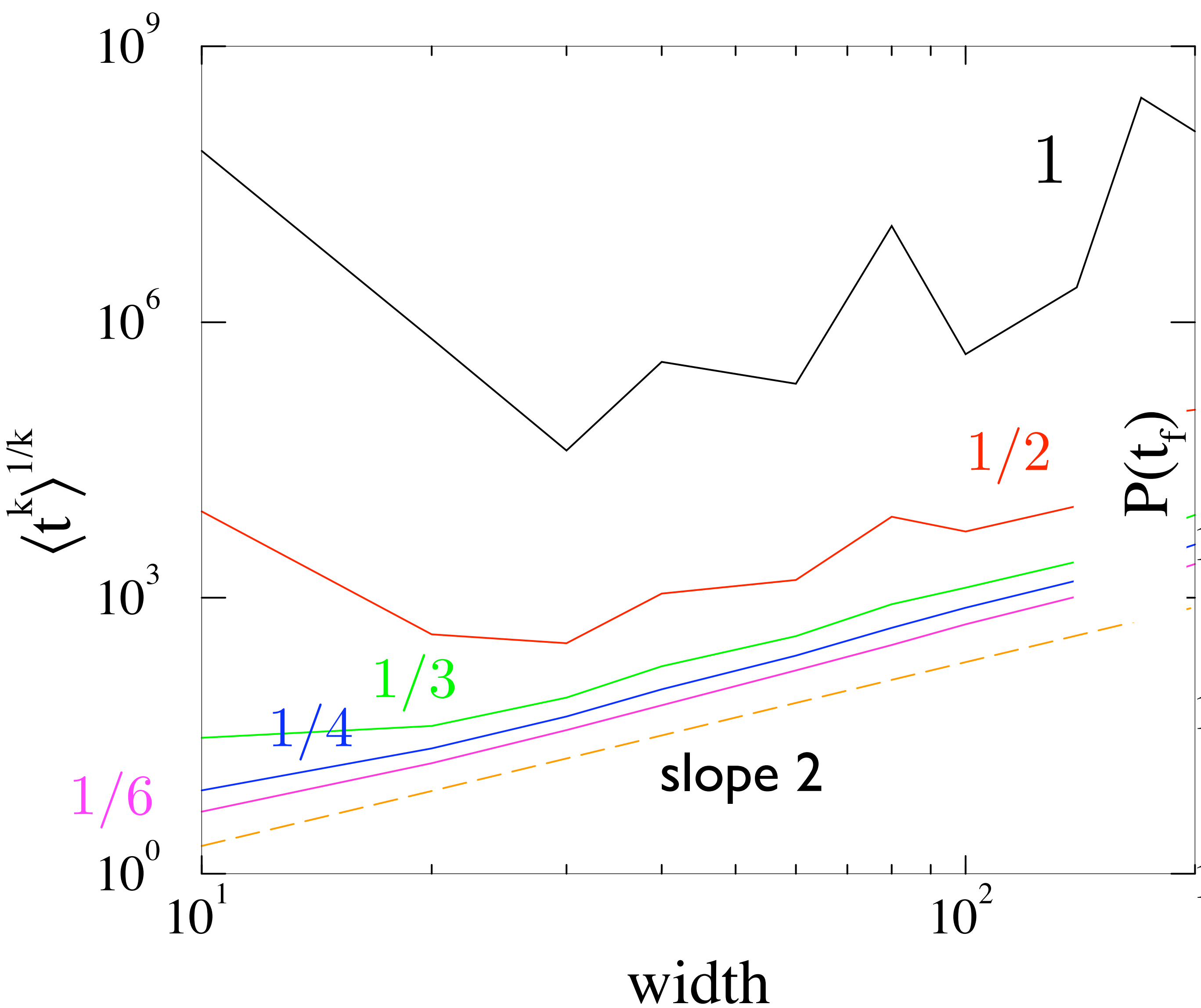
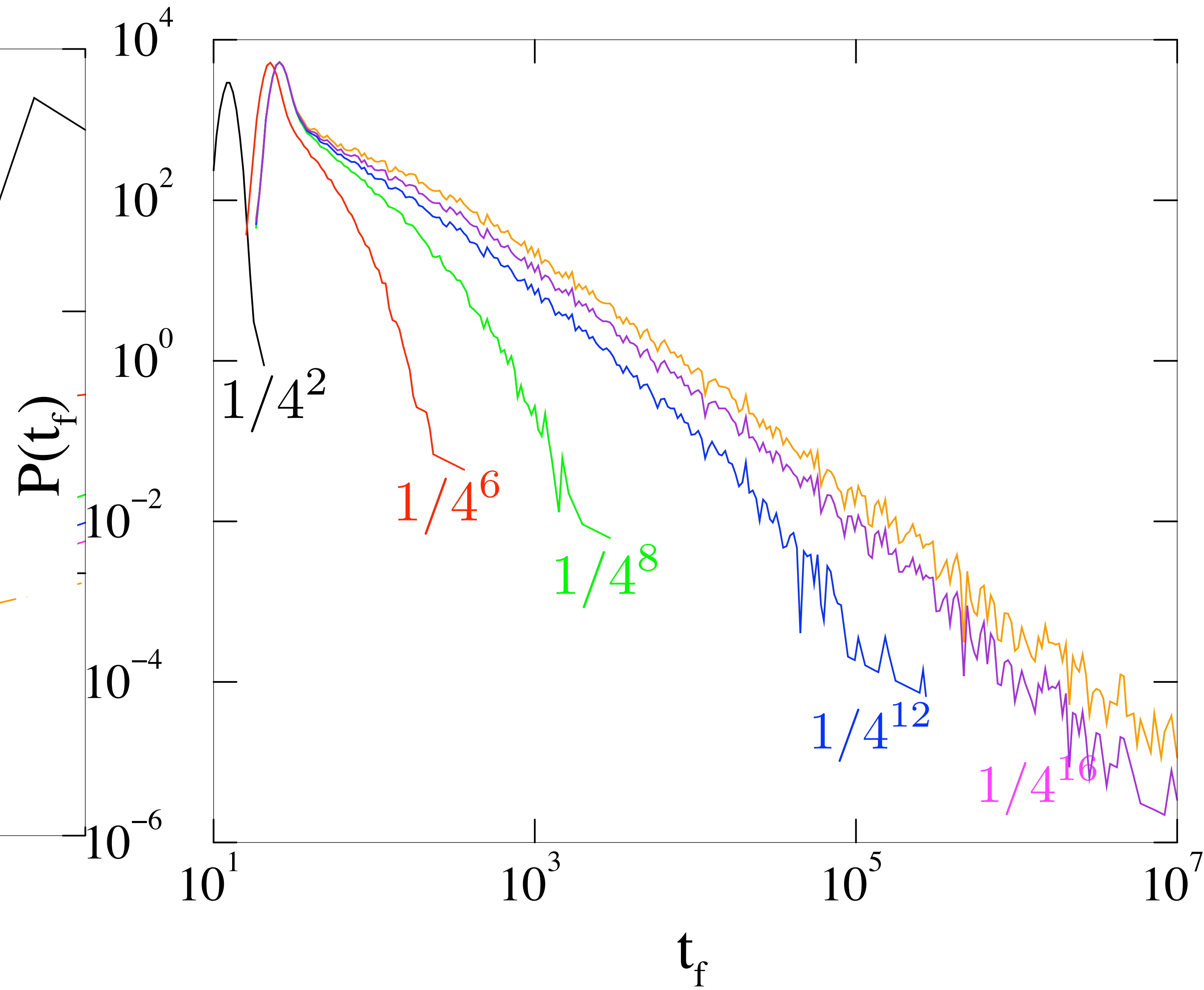


partial clogging time distribution



Summary & Outlook

Gradients drive depth filtration breakdown

Basic contradiction of filters:

A filter should be long for good filtering
short to be active

Clogging time governed by extreme events
$\rightarrow$ *power law clogging time distribution*

Some open questions:

When is a filter "dead"?

How to describe gradient-driven percolation?

Other mechanisms: sclerosis, relaunching, aggregation...