Multiscale control of transport equations

Dieter Armbruster
Department of Mathematics and Statistics,
Arizona State University
& Department of Mechanical Engineering
Eindhoven University of Technology
E-mail: armbruster@asu.edu

April 26, 2009

Collaborators

- **Michael La Marca**, Department of Mathematics and Statistics ASU
- **Christian Ringhofer** Department of Mathematics and Statistics ASU
- **Michael Herty,** RWTH Aachen
- **Erjen Lefeber**, Mechanical Engineering, Eindhoven University of Technology
- **Dominique Perdaen** Mechanical Engineering, Eindhoven University of Technology
- **Karl Kempf**, Intel Corporation

Partial support by NSF grants DMS 0204543 and DMS-0604986 is gratefully acknowledged

Outline

1. Introduction - the output tracking problem
2. Control on the small scales: Discrete event simulations

Outline

1. Introduction - the output tracking problem
2. Control on the small scales: Discrete event simulations
3. Control on the large scales: The continuum model

Assumptions:

- Given a demand for a factory $d(t)$ for a time interval T.
- Given the state of the factory at time $t = 0$ through the distribution of the work in progress (WIP) through the factory.

Task:

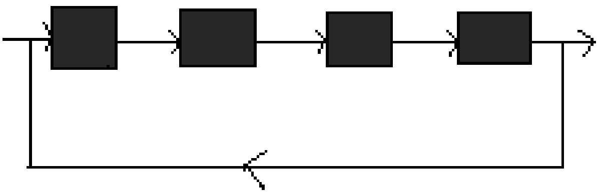
Design a control actuator $u(t)$ for $0 \leq t \leq T$ such that the output $o(t)$ of the factory is as close as possible to the demand over the time interval T.
I.e.

$$Min_{u(t)} ||o(t) - d(t)||$$

in some suitable norm.

Control on the small scales:
Discrete event simulations

Semiconductor production is re-entrant:



Note: If the flow cycles 4 times through this factory then

- Machine 1 produces the steps; 1, 5, 9, 13
- Machine 2 produces the steps 2, 6, 10, 14 etc

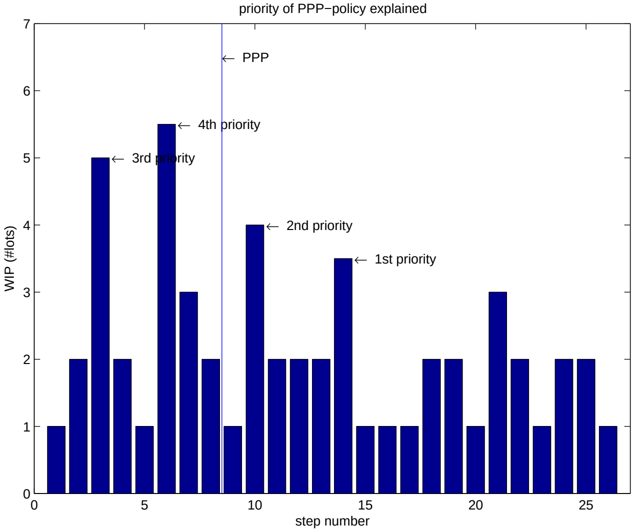
Question: What is the dispatch policy ?

Major policies:

- Push policy - typically at the beginning of the line
- Pull policy - typically at the end of the line

The Push-Pull-Point **PPP** is the point in the factory where the dispatch policy changes from Push to Pull.
The two policies are ordered Pull over Push.

Priority levels



Moving the PPP point

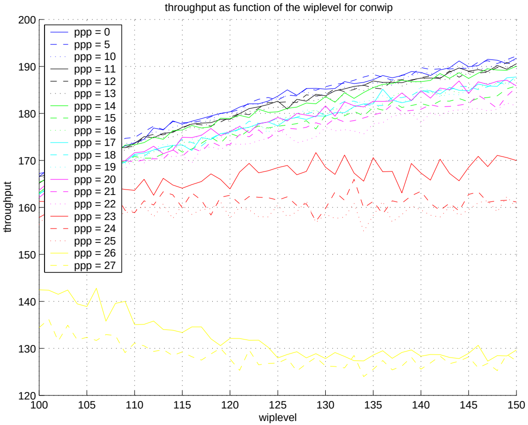
Change priority rules by *moving the PPP point*.
Position PPP such that WIP downstream is equal to the demand in the demand period

Discrete Event Model

- 9 machine sets
- 26 production steps
- between 4 and 21 machines per machine set
- total raw processing time 58 hours
- machine set 1 and 2 are batch machines with 4 parts per batch
- simulation in χ (TU/e)
- machine time in service and time in repair randomly distributed out of Weibull distributions
- actual processing times out of exponential distributions
- nominal capacity about 200 per week, stochastic variation between $160 - 240$.
- demand randomly generated and fixed, average at 180 lots per week.

Observations and conditions:

- The clearing function for Pull policy is significantly higher than for Push policy



Readjustment period and cycle time

have to be related: need to place PPP on average in the middle of the factory to have maximal actuator influence

Experiment

- Simulation time: 144 weeks
- Demand period 2 days
- 500 simulations per data point
- demand is not perishable - keep tally of backlog and overproduction
- Cost:

$$
\begin{aligned}
m(0) &= 0 \\
m(t) &= m(t-1) + d(t) - o(t) \\
cost(T): &= \Sigma_{i=1}^{T} |m(i)|
\end{aligned}
$$

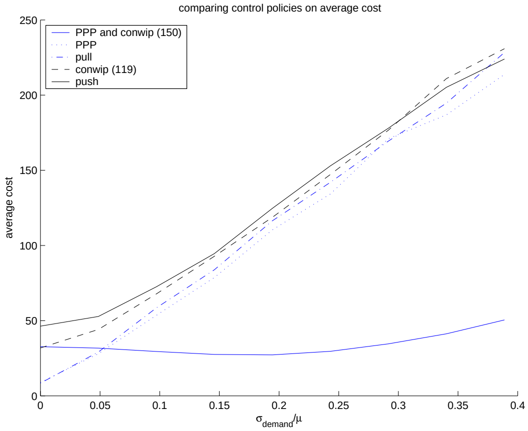
i.e. backlog and overproduction cost the same

Results I

Compare different policies:

- Push
- Pull
- fixed PPP
- CONWIP with WIP $= 119$ and Pull dispatch policy
- CONWIP with WIP $= 150$ and PPP

a policy with PPP and CONWIP has much lower costs for high demand variation.



Explanation I

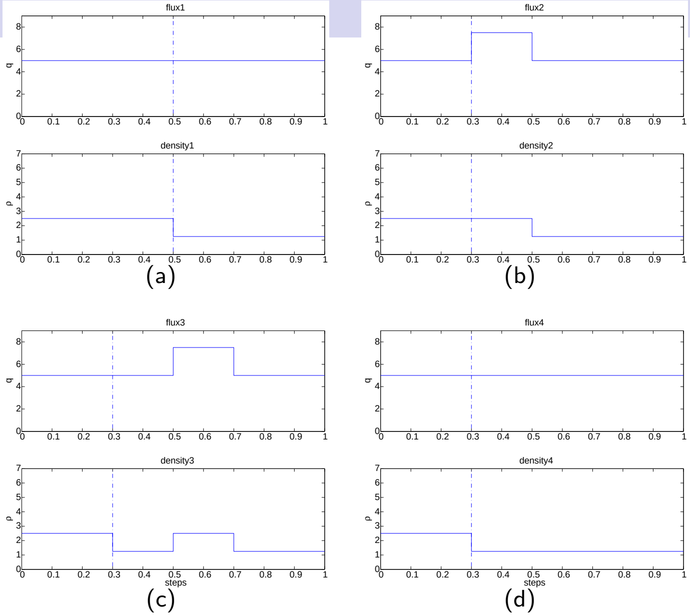


Figure: Moving the PPP upstream

Explanation II

Note:

- Wip is lower after the flux bump has moved out
- If demand stays constant, there is not enough WIP downstream from PPP
- Algorithm is unstable - PPP point will continue to move upstream for constant demand.

Explanation III

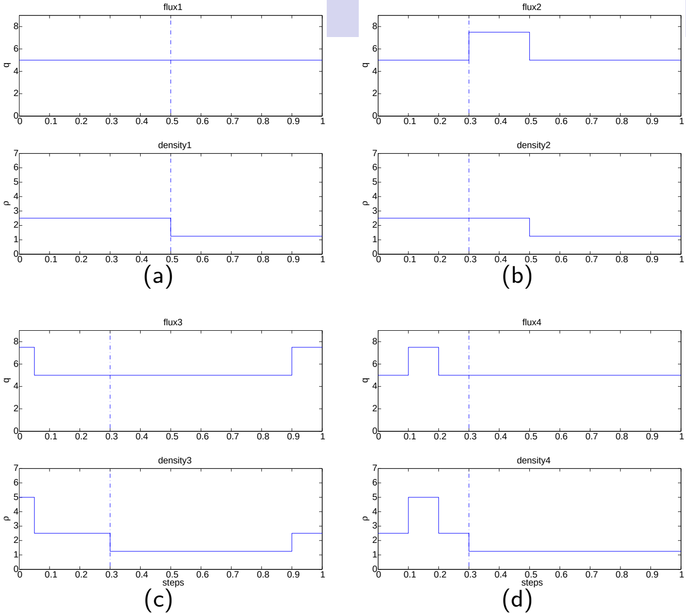


Figure: Moving the PPP upstream with CONWIP

Explanation IV

Note:

- Wip stays constant
- Flux stays high, i.e. total flux is increased
- Algorithm is stable - flux does not change for constant demand.

Control on the large scales:
The continuum model

Usual model: Faithful representation of the factory using *Discrete Event Simulations*, e.g. χ (TU Eindhoven)

Problem:

Simulation of production flows with stochastic demand and stochastic production processes requires Monte Carlo Simulations

Takes too long for a decision tool

Fundamental Idea:
Model high volume, many stages, production via a fluid.

Basic variable

product density (mass density) $\rho(\mathbf{x}, \mathbf{t})$.
x- is the position in the production process, $x \in [0, 1]$.
- degree of completion
- stage of production

Mass conservation and state equations

Mass conservation and state equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial F}{\partial x} = 0$$
$$F = \rho v_{eq}$$

Typical models for the equilibrium velocity v_{eq} (state equation) are

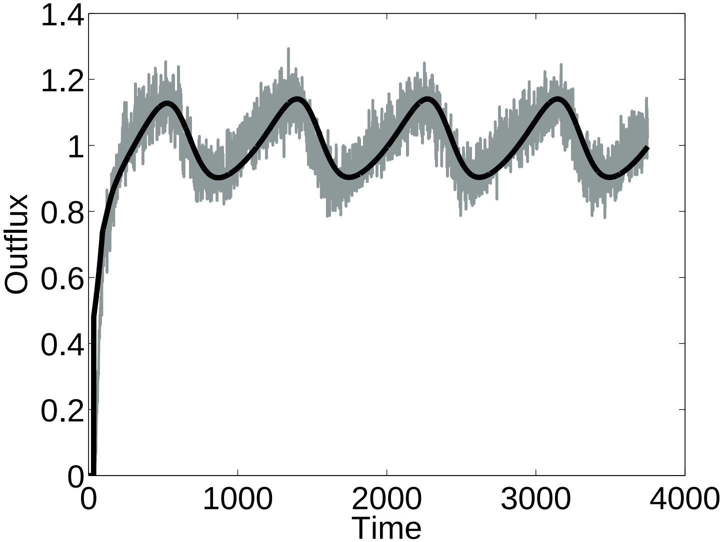
$$v_{eq}^{traffic}(\rho) = v_0(1 - \frac{\rho}{\rho_c}), \quad (1)$$
$$v_{eq}(\rho) = \Phi(L), \quad (2)$$

with L the total load (Work in progress, WIP) given as

$$L(\rho) = \int_0^1 \rho(x,t)dx. \quad (3)$$

Φ maybe determined experimentally or theoretically.

Compare a detailed discrete event simulation with a fluid simulation



Problem: Tracking for a continuum model

Model: PDE model based on a product density $\rho(x,t)$ and a state equation for the velocity:

$$\begin{aligned} v_{eq}(\rho(t)) &= \Phi(L) \\ L(t) &= \int_0^1 \rho(\xi,t)d\xi \end{aligned}$$

specifically - use a queuing model:

$$v_{eq}^Q = \frac{v_{max}}{1+L}$$

Mass conservation

leads to

$$\begin{aligned} \rho_t(x,t) + v_{eq}^{Q} \rho_x(x,t) &= 0, \quad (x,t) \in [0,1] \times [0,\infty) \\ \rho(x,0) &= \rho_0(x), \quad x \in [0,1] \\ \lambda(t) &= v(\rho)\rho(x,t)|_{x=0} \end{aligned}$$

where $\lambda(t)$ is the influx.

Problem setup

- a fixed end time $\tau > 0$.
- an initial profile $\rho_0(x)$.
- $d(t)$ - the demand at time t. $d(t) \in L^2([0,\tau])$.

Find the influx $\lambda(t), t \in [0,\tau]$:s.t.

$$j(\rho,\lambda) = \frac{1}{2}\int_0^\tau \left(v_{eq}^Q(\rho)\rho(1,t) - d(t)\right)^2 dt$$

is minimal

Choose a *test function* $\phi(x,t) \in C^1([0,1] \times [0,\tau])$

Lagrangian

$$L(\rho, \lambda, \phi) = j(\rho, \lambda) + \langle E(\rho), \phi \rangle \tag{4}$$

where

- Equality constraint set

$$E(\rho) = \rho_t + v_{eq}^Q(\rho)\rho_x$$

-

$$\langle u(x,t), v(x,t) \rangle = \int_0^1 \int_0^\tau u(\xi, s) v(\xi, s) ds d\xi$$

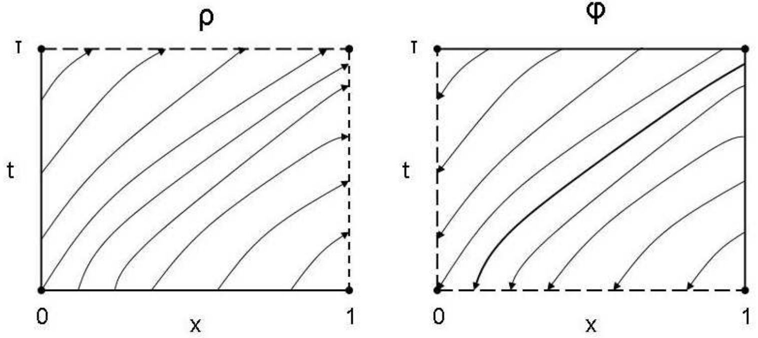
Setting the variational derivatives of $L(\rho(\lambda), \lambda, \phi)$ with respect to λ, ρ, ϕ equal to zero, leads to:

$$\begin{aligned}
0 &= \rho_t(x,t) + v_{eq}^Q(\rho)\rho_x(x,t) && (5)\\
0 &= \phi_t(x,t) + v_{eq}^Q(\rho)\phi_x(x,t) + \frac{v_{eq}^Q(\rho)^2}{v_{max}} && (6)\\
&\quad * \left[v_{eq}^Q(\rho)\rho(1,t)^2 - \rho(1,t)d(t) + \int_0^1 \phi(s,t)\rho_x(s,t) \right] ds &&\\
0 &= \phi(1,t) + v_{eq}^Q(\rho)\rho(1,t) - d(t) && (7)\\
0 &\equiv \phi(x,\tau) && (8)\\
j'(\lambda) &= -\phi(0,t) && (9)
\end{aligned}$$

Structure of the equations:

- (5) is our PDE in ρ
- (6) is our PDE in ϕ
- (7) couples ρ, ϕ and the demand $d(t)$.
- (8) is a terminal condition on ϕ
- (9) links the derivative of j with the solution to the ϕ PDE

Domains of influence



Finding a local minimum:

We have an iteration scheme

$$\lambda \to I(\lambda) = (j(\lambda), j'(\lambda))$$

and we are looking for that input function λ^* s.t. $I(\lambda^*) = (j(\lambda^*), 0)$.

Algorithm

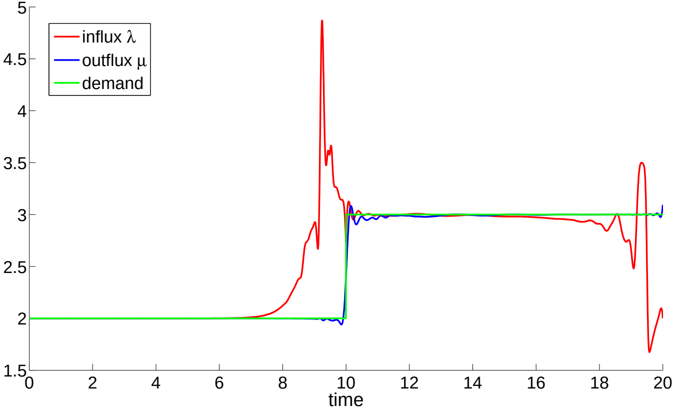
- Pick an inital guess for the control $\lambda(t) = \lambda_0(t) \quad t \in [0, \tau]$.
- Solve PDE for ρ forward in time.
- Solve the adjoint equation for ϕ, using information gained from solving the ρ PDE.
- Use $j'(\lambda)$ to update λ.
- Repeat until suitable stopping criteria is met.

In the following figures:

- The demand is a step function of height 1
- v_{max} is 4
- $\rho_0(x) = 1 \; \forall \qquad x \in [0, 1]$
- $\lambda_0 = 2$.

λ_0 was chosen so that the initial influx maintained the initial WIP profile $\rho_0(x)$.

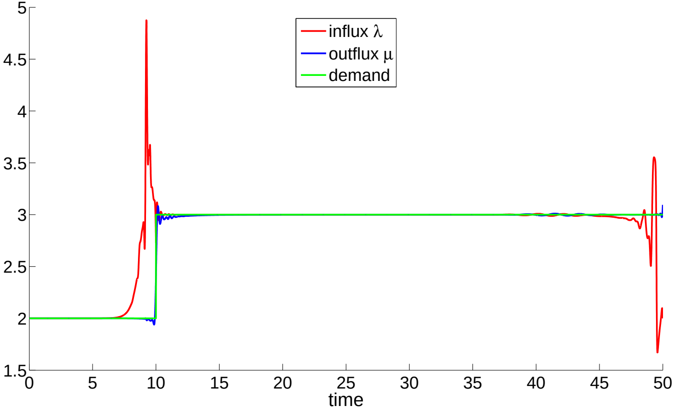
Input/Output/Demand



Note the end effect

A real end effect:

- Does not go away with grid refinement; therefore, must come from optimization routine.
- τ was 20; now 50
- Step occurs in both at $t = 10$.


influx λ
outflux μ
demand
5
4.5
4
3.5
3
2.5
2
1.5
0
5
10
15
20
25
30
35
40
45
50
time

System Reactivity I

The speed

$$v = \frac{v_{max}}{1 + \int_0^1 \rho(s,t)\,ds}$$

is nonlinear, so the system's ability to react to demand depends on:

1. The current system load L
2. v_{max}

The following demands are two sinusoidal waves $d(t) = \sin(\pi t) + 1$ with:

- Same $\tau = 10$
- Same amplitude of 1, height of 1, and frequency of $\frac{1}{2}$
- Same initial condition $\rho_0(x) = 1 \ \forall \quad x \in [0, 1]$

However, the Fig. 4 has a v_{max} of 1 while Fig. 5 has a v_{max} of 3 prior to the jump.

Low Speed

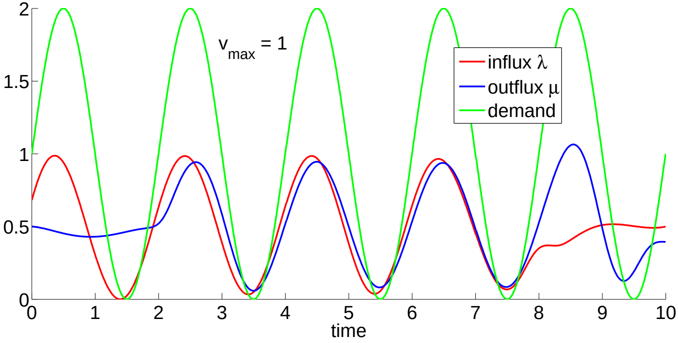


Figure: v_{max} of 1

High Speed

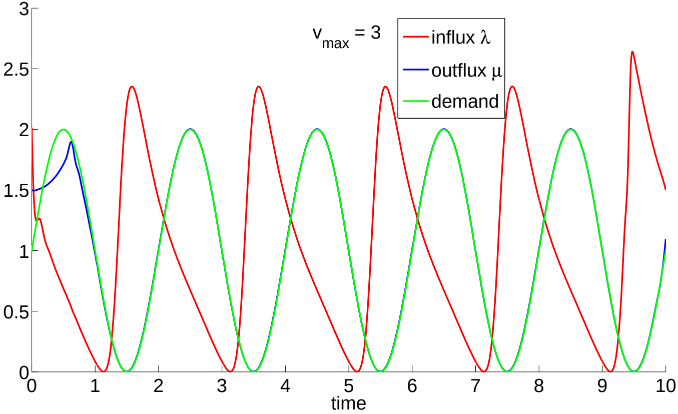


Figure: v_{max} of 3

Heuristic

- uses a new actuator - the Push Pull Point coupled with a CONWIP starts policy..
- is effective to reduce the mismatch between desired output and actual output by a factor of four.
- works at the machine level, i.e. small scales.

Conclusions II

Large scale

We determined a theory to find a local minimum of the mismatch between desired output and actual output as a function of the influx for a large scale in space and time continuum model for factory production.

Extension

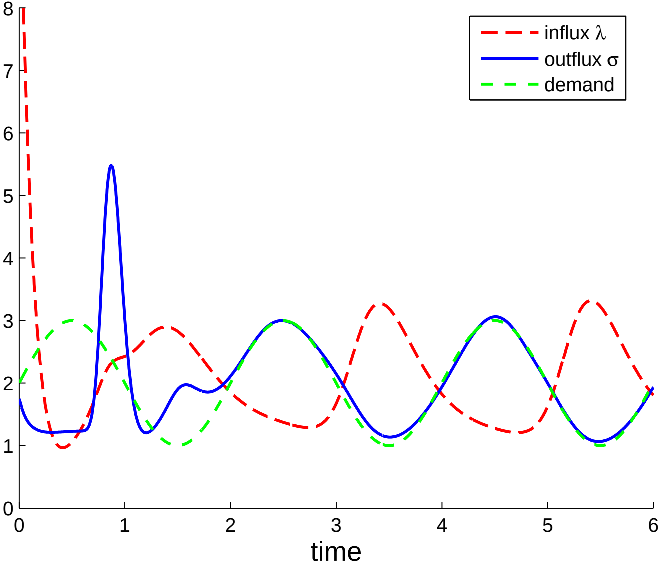
Continuum model can be controlled to minimize backlog $b(t)$, i.e.

$$\begin{aligned} D(t) &:= \int_0^t d(r)\,dr \\ O(t) &:= \int_0^t v(\rho)\rho(1,r)\,dr \\ b(t) &:= D(t) - O(t) \end{aligned}$$

with a cost functional of

$$J(\rho,\lambda) = \frac{1}{2}\int_0^T b(t)^2\,dt$$

Conclusions IV



Further work:

- Integrate both small and large scale optimization. E.g.
  - annual production variation controlled by the influx
  - weakly production variation controlled by the PPP
- Are there any cases where the local minimum is provably global?
- So far Feedforward scheme - can we do a Feedback scheme?