IPAM Workshop II
The Boltzmann Equation: DiPerna-Lions Plus 20 Years
(IPAM-UCLA, April 15 - 17, 2009)

Fluid dynamics for a vapor-gas mixture derived from kinetic theory

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Subject:

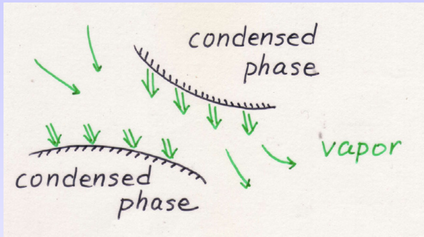
Fluid-dynamic treatment of flows of a mixture of
- a vapor and a noncondensable gas
- caused by surface evaporation/condensation
- near the continuum regime (small Knudsen number)
- based on kinetic theory

Introduction

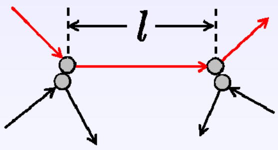
Vapor flows with evaporation/ condensation on interfaces

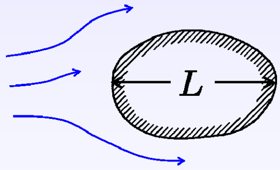
Important subject in RGD
(Boltzmann equation)

Vapor is *not* in equilibrium near the interfaces, even for *small* Knudsen numbers (near continuum regime).



$\mathrm{Kn} = l/L \ll 1$ (Continuum limit $\mathrm{Kn} \to 0$)





l : mean free path L : characteristic length

Fluid-dynamic description

equations ?? BC's ??
not obvious

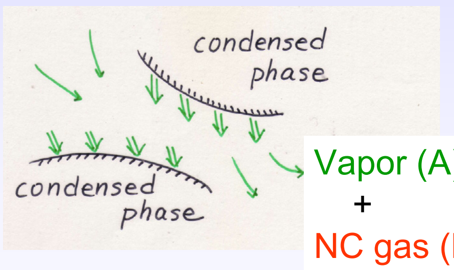
Systematic *asymptotic analysis* (for small Kn) based on kinetic theory

Steady flows

- Pure vapor Sone & Onishi (78, 79), A & Sone (91), …

  Fluid-dynamic equations + BC's in various situations

- Vapor + Noncondensable (NC) gas

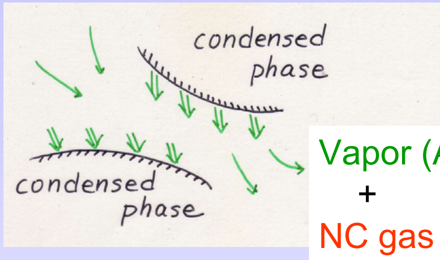


Vapor (A) + NC gas (B)

Fluid-dynamic equations ??
BC's ??

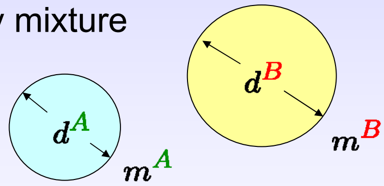
Problem

Steady flows of
vapor and NC gas
at small Kn
for arbitrary geometry
and for large temperature (density) variation



Vapor (A)
+
NC gas (B)

Boltzmann equation for a binary mixture
hard-sphere gases
(and model equation)



B.C. Vapor - Conventional condition
NC gas - Diffuse reflection
Formal discussion: more general B.C.

Dimensionless variables (normalized by $\tilde{T}_r$, $\tilde{n}_r$, L, ...)

Velocity distribution functions

$F^A(\mathbf{x}, \boldsymbol{\zeta})$: Vapor $F^B(\mathbf{x}, \boldsymbol{\zeta})$: NC gas

$\mathbf{x}$: position $\boldsymbol{\zeta}$: molecular velocity

$F^\alpha(\mathbf{x}, \boldsymbol{\zeta})\, d\mathbf{x}\, d\boldsymbol{\zeta}$ $(\alpha = A, B)$

Molecular number of α component in $d\mathbf{x}\, d\boldsymbol{\zeta}$

Boltzmann equations ($\epsilon \sim$ Kn $\ll 1$)

$$\boldsymbol{\zeta}\cdot\frac{\partial F^A}{\partial \mathbf{x}} = \frac{1}{\epsilon}[J^{AA}(F^A, F^A) + J^{BA}(F^B, F^A)]$$

$$\boldsymbol{\zeta}\cdot\frac{\partial F^B}{\partial \mathbf{x}} = \frac{1}{\epsilon}[J^{AB}(F^A, F^B) + J^{BB}(F^B, F^B)]$$

Collision integrals (hard-sphere molecules)

$$J^{\beta\alpha}(f,g) = \frac{1}{4\sqrt{2\pi}} K^{\beta\alpha} \int [f(\zeta'_*)g(\zeta') - f(\zeta_*)g(\zeta)] |\mathbf{e}\cdot\mathbf{V}| d\Omega(\mathbf{e}) d\zeta_*,$$

$$\zeta' = \zeta + \frac{\mu^{\beta\alpha}}{m^\alpha}(\mathbf{e}\cdot\mathbf{V})\mathbf{e}, \quad \zeta'_* = \zeta_* - \frac{\mu^{\beta\alpha}}{m^\beta}(\mathbf{e}\cdot\mathbf{V})\mathbf{e}, \quad \mathbf{V} = \zeta_* - \zeta,$$

$$K^{\beta\alpha} = \frac{(d^\alpha + d^\beta)^2}{4}, \quad \mu^{\beta\alpha} = \frac{2m^\alpha m^\beta}{m^\alpha + m^\beta}.$$

Macroscopic quantities

$$n^\alpha = \int F^\alpha d\zeta, \qquad \mathbf{v}^\alpha = \frac{1}{n^\alpha}\int \zeta F^\alpha d\zeta,$$

$$p^\alpha = n^\alpha T^\alpha = \frac{2}{3}\int m^\alpha |\zeta - \mathbf{v}^\alpha|^2 F^\alpha d\zeta, \qquad (\alpha = A,\ B),$$

$$n = n^A + n^B, \qquad \rho = m^A n^A + m^B n^B,$$

$$\mathbf{v} = \frac{1}{\rho}(m^A n^A \mathbf{v}^A + m^B n^B \mathbf{v}^B),$$

$$p = nT = \sum_{\alpha=A,B}\left[p^\alpha + \frac{2}{3} m^\alpha n^\alpha |\mathbf{v}^\alpha - \mathbf{v}|^2\right].$$

Boundary condition $(\alpha = A,\ B)$

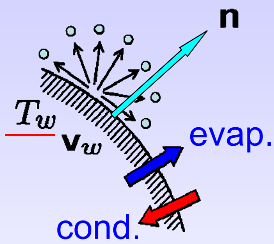
$\mathbf{v}_w \cdot \mathbf{n} = 0$

$$F^\alpha = \sigma^\alpha \left(\frac{m^\alpha}{\pi T_w}\right)^{3/2} \exp\left(-\frac{m^\alpha |\zeta - \mathbf{v}_w|^2}{T_w}\right), \qquad (\zeta\cdot\mathbf{n} > 0)$$

Vapor $(\alpha = A)$

$$\sigma^A = n_w^A = p_w^A / T_w$$

(number density) (pressure) of vapor in saturated equilibrium state at T_w

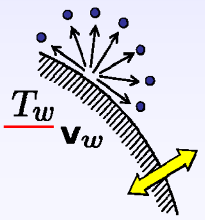


New approach: Frezzotti, Yano, ….

NC gas $(\alpha = B)$

Diffuse reflection (no net mass flux)

$$\sigma^B = -\left(\frac{\pi m^B}{T_w}\right)^{1/2} \int_{\zeta\cdot\mathbf{n}<0} \zeta\cdot\mathbf{n}\, F^B d\zeta$$



Analysis

- small Knudsen number $\epsilon = (\sqrt{\pi}/2)\mathrm{Kn} \ll 1$

$$\mathrm{Kn} = l_r/L \ll 1$$

reference mfp of vapor (l_r) reference length (L)

- amount of NC gas
  - (I) $\sim$ amount of vapor ←
  - (II) $\ll$ amount of vapor

Fluid-dynamic systems are different !!

Asymptotic analysis for $\epsilon \ll 1$

Sone (1969, 1971, … 1991, … 2002, … 2007, …)

- *Kinetic Theory and Fluid Dynamics* (Birkhäuser, 2002)
- *Molecular Gas Dynamics: Theory, Techniques, and Applications* (Birkhäuser, 2007)

(I) amount of NC gas $\sim$ amount of vapor

Fluid-dynamic-type equations

Takata & A (01) TTSP

Hilbert solution (expansion) $[\partial F^\alpha/\partial \mathbf{x} = O(F^\alpha)]$

$$F^\alpha = F^\alpha_{(0)} + F^\alpha_{(1)}\epsilon + F^\alpha_{(2)}\epsilon^2 + \dots, \qquad (\alpha = A, B)$$

Macroscopic quantities $(h = n^\alpha, \mathbf{v}^\alpha, T^\alpha, \; n, \mathbf{v}, T, \dots)$

$$h = h_{(0)} + h_{(1)}\epsilon + h_{(2)}\epsilon^2 + \dots$$

Sequence of integral equations ← Boltzmann eqs.

$$\zeta \cdot \frac{\partial F^A}{\partial \mathbf{x}} = \frac{1}{\epsilon}[J^{AA}(F^A, F^A) + J^{BA}(F^B, F^A)]$$

$$\zeta \cdot \frac{\partial F^B}{\partial \mathbf{x}} = \frac{1}{\epsilon}[J^{AB}(F^A, F^B) + J^{BB}(F^B, F^B)]$$

Sequence of integral equations $(\alpha = A, B)$

$$J^{A\alpha}(F^{A}_{(0)}, F^{\alpha}_{(0)}) + J^{B\alpha}(F^{B}_{(0)}, F^{\alpha}_{(0)}) = 0$$

$$\sum_{\beta=A,B} J^{\beta\alpha}(F^{\beta}_{(1)}, F^{\alpha}_{(0)}) + J^{\beta\alpha}(F^{\beta}_{(0)}, F^{\alpha}_{(1)}) = \zeta\cdot\frac{\partial F^{\alpha}_{(0)}}{\partial \mathbf{x}}$$

$$\cdots$$

Solutions $F^{\alpha}_{(0)}, F^{\alpha}_{(1)}, \cdots$

[$F^{A}_{(0)}$, $F^{B}_{(0)}$: local Maxwellians (*common* flow velocity and temperature)]

Solvability conditions

Constraints for F-D quantities $h_{(0)}, h_{(1)}, \cdots$

Fluid-dynamic-type equations

Assumption

$\mathbf{v}_w = 0 \quad (\mathbf{v}_\infty = 0)$ Boundary at rest (No flow at infinity)

[more generally, $\mathbf{v}_w = O(\epsilon)$, $\mathbf{v}_\infty = O(\epsilon)$]

$\mathbf{v}_{(0)} \; (= \mathbf{v}^{A}_{(0)} = \mathbf{v}^{B}_{(0)}) = 0$ Consistent assumption

(Sone, A, Takata, Sugimoto, Bobylev (96), Phys. Fluids)

Fluid-dynamic-type equations for (finally) $n^{A}_{(0)}, T_{(0)}$, $p_{(2)}, \mathbf{v}_{(1)}$

$$F^{\alpha}_{(0)} = n^{\alpha}_{(0)}\left(\frac{m^{\alpha}}{\pi T_{(0)}}\right)^{3/2} \exp\left(-\frac{m^{\alpha}|\zeta|^2}{T_{(0)}}\right)$$

$$\begin{aligned} n^{A} &= n^{A}_{(0)} + n^{A}_{(1)}\epsilon + n^{A}_{(2)}\epsilon^2 + \cdots \\ T &= T_{(0)} + T_{(1)}\epsilon + T_{(2)}\epsilon^2 + \cdots \\ p &= p_{(0)} + p_{(1)}\epsilon + p_{(2)}\epsilon^2 + \cdots \\ \mathbf{v} &= 0 + \mathbf{v}_{(1)}\epsilon + \mathbf{v}_{(2)}\epsilon^2 + \cdots \end{aligned}$$

$$\begin{cases} p_{(0)} = \text{const} \\ p_{(1)} = \text{const} \end{cases}$$

Fluid-dynamic-type equations

$$\frac{\partial p_{(0)}}{\partial x_i}=0, \qquad \frac{\partial p_{(1)}}{\partial x_i}=0.$$

$$\frac{\partial}{\partial x_j}(n^A_{(0)} v^A_{j(1)})=0, \qquad \frac{\partial}{\partial x_j}(n^B_{(0)} v^B_{j(1)})=0,$$

continuity

$$2\rho_{(0)} v_{j(1)} \frac{\partial v_{i(1)}}{\partial x_j} = -\frac{\partial p_{(2)}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu T_{(0)}^{1/2}\overline{\frac{\partial v_{i(1)}}{\partial x_j}}\right)$$
$$- \frac{\partial}{\partial x_j}\left(\frac{\Upsilon_1}{p_{(0)}}\overline{\frac{\partial T_{(0)}}{\partial x_i}\frac{\partial T_{(0)}}{\partial x_j}}\right) - \frac{\partial}{\partial x_j}\left[\frac{1}{n_{(0)}}\overline{\frac{\partial}{\partial x_j}\left(\Upsilon_2\frac{\partial T_{(0)}}{\partial x_i}\right)}\right]$$
$$- \frac{\partial}{\partial x_j}\left(\Upsilon_3\frac{T_{(0)}}{n_{(0)}}\overline{\frac{\partial X^A}{\partial x_i}\frac{\partial X^A}{\partial x_j}}\right) - \frac{\partial}{\partial x_j}\left(\frac{\Upsilon_4}{n_{(0)}}\overline{\frac{\partial X^A}{\partial x_j}\frac{\partial T_{(0)}}{\partial x_i}}\right)$$
$$- \frac{\partial}{\partial x_j}\left[\frac{T_{(0)}}{n_{(0)}}\overline{\frac{\partial}{\partial x_j}\left(\Upsilon_5\frac{\partial X^A}{\partial x_i}\right)}\right],$$

momentum

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$X^A = n^A_{(0)}/n_{(0)}$, $\quad n_{(0)} = n^A_{(0)} + n^B_{(0)}$, $\quad \overline{A_{ij}} = A_{ij} + A_{ji} - (2/3)A_{kk}\delta_{ij}$

μ, Υ_1, Υ_2, Υ_3, Υ_4, Υ_5 : Functions of X^A $\left(\text{depending on } \frac{m^B}{m^A}, \frac{d^B}{d^A}\right)$

Database: Takata, Yasuda, A, & Shibata (03) RGD23

Fluid-dynamic-type equations

$$\frac{\partial p_{(0)}}{\partial x_i}=0, \qquad \frac{\partial p_{(1)}}{\partial x_i}=0.$$

$$\frac{\partial}{\partial x_j}(n^A_{(0)} v^A_{j(1)})=0, \qquad \frac{\partial}{\partial x_j}(n^B_{(0)} v^B_{j(1)})=0,$$

continuity

$$2\rho_{(0)} v_{j(1)} \frac{\partial v_{i(1)}}{\partial x_j} = -\frac{\partial p_{(2)}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu T_{(0)}^{1/2}\overline{\frac{\partial v_{i(1)}}{\partial x_j}}\right)$$
$$- \frac{\partial}{\partial x_j}\left(\frac{\Upsilon_1}{p_{(0)}}\overline{\frac{\partial T_{(0)}}{\partial x_i}\frac{\partial T_{(0)}}{\partial x_j}}\right) - \frac{\partial}{\partial x_j}\left[\frac{1}{n_{(0)}}\overline{\frac{\partial}{\partial x_j}\left(\Upsilon_2\frac{\partial T_{(0)}}{\partial x_i}\right)}\right]$$
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momentum

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μ, Υ_1, Υ_2, Υ_3, Υ_4, Υ_5 : Functions of X^A $\left(\text{depending on } \frac{m^B}{m^A}, \frac{d^B}{d^A}\right)$

Database: Takata, Yasuda, A, & Shibata (03) RGD23

$$\frac{\partial}{\partial x_j}\left(\lambda T_{(0)}^{1/2}\frac{\partial T_{(0)}}{\partial x_j}\right) - \frac{\partial}{\partial x_j}[k_T\, p_{(0)}\,(v^A_{j(1)} - v^B_{j(1)})]$$
$$-\frac{5}{2}(n^A_{(0)} v^A_{j(1)} + n^B_{(0)} v^B_{j(1)})\frac{\partial T_{(0)}}{\partial x_j} = 0, \qquad \text{energy}$$

$$v^A_{i(1)} - v^B_{i(1)} = -\frac{T_{(0)}^{1/2}}{n_{(0)}}\frac{D_{AB}}{X^A X^B}\left(\frac{\partial X^A}{\partial x_i} + k_T \frac{\partial \ln T_{(0)}}{\partial x_i}\right), \qquad \text{diffusion}$$

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$X^A = n^A_{(0)}/n_{(0)}, \quad n_{(0)} = n^A_{(0)} + n^B_{(0)}$

$\lambda,\ D_{AB},\ k_T = D_T/D_{AB}$: Functions of X^A $\left(\text{depending on } \frac{m^B}{m^A}, \frac{d^B}{d^A}\right)$

Database: Takata, Yasuda, A, & Shibata (03) RGD23

$\rho_{(0)} = m^A n^A_{(0)} + m^B n^B_{(0)}, \qquad p_{(0)} = n_{(0)} T_{(0)},$

$v_{i(1)} = (m^A n^A_{(0)} v^A_{i(1)} + m^B n^B_{(0)} v^B_{i(1)})/\rho_{(0)},$

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Galkin, Kogan, & Fridlender (72)

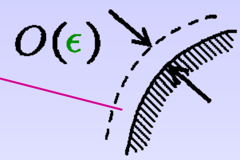
Concentration-stress convection

Knudsen layer and slip boundary conditions

Hilbert solution *does not* satisfy kinetic B.C.

Solution: $F^\alpha = F^\alpha_H + F^\alpha_K \quad (\alpha = A,\ B)$

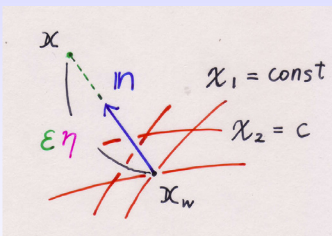
Hilbert solution — Knudsen-layer correction



Stretched normal coordinate η

$\mathbf{x} = \epsilon\,\eta\,\mathbf{n}(\chi_1, \chi_2) + \mathbf{x}_w(\chi_1, \chi_2)$

$F^\alpha_K(\eta, \chi_1, \chi_2, \boldsymbol{\zeta}) \to 0 \quad (\eta \to \infty)$



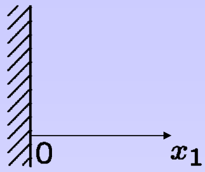
$$\begin{cases} F^\alpha_H = F^\alpha_{H(0)} + F^\alpha_{H(1)}\epsilon + \dots, \\ F^\alpha_K = \qquad\quad F^\alpha_{K(1)}\epsilon + \dots, \end{cases}$$

Eqs. and BC for $F^\alpha_{K(1)}$ ⇒ Half-space problem for linearized Boltzmann eqs.

Knudsen-layer problem

Half-space problem for linearized Boltzmann eqs.

$F^{\alpha}_{K(1)} \Leftrightarrow \phi^{\alpha}$



$$\zeta_1 \frac{\partial \phi^{\alpha}}{\partial x_1} = \mathcal{L}^{A\alpha}(\phi^A, \phi^{\alpha}) + \mathcal{L}^{B\alpha}(\phi^B, \phi^{\alpha}), \quad (\alpha = A,\ B)$$

$$\phi^{\alpha} = c_0^{\alpha} + c_2 m^{\alpha} \zeta_2 + c_3 m^{\alpha} \zeta_3 + c_4 m^{\alpha} |\zeta|^2 + g^{\alpha}(\zeta),$$

$$(\zeta_1 > 0,\ x_1 = 0),$$

Undetermined consts.

$$\phi^{\alpha} \to 0, \qquad (x_1 \to \infty)$$

Solution ϕ^{α} exists uniquely iff $c_0^A,\ c_0^B,\ c_2,\ c_3,\ c_4$ take special values

A, Bardos, & Takata, J. Stat. Phys. (03)

Boundary values of F^{α}_{H} $(h_{(0)}, h_{(1)}, h_{(2)}, \ldots)$

BC for FD-type equations

Knudsen-layer problem

Single-component gas

Grad (69) Conjecture

Bardos, Caflisch, & Nicolaenko (86): CPAM

Maslova (82), Cercignani (86), Golse & Poupaud (89)

Half-space problem for linearized Boltzmann eqs.

Decomposition

Numerical analysis (HS)

Takata, Yasuda, Kosuge, & A (03) Phys. Fluids

- Thermal slip (creep)
- Diffusion slip

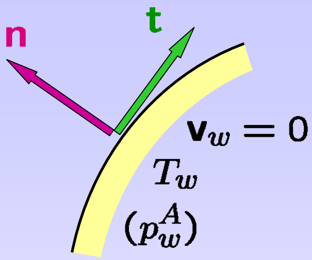
→ BC for FD-type eqs.

- Temperature jump
- Partial pressure jump
- Evaporation and condensation

→ BC for higher-order FD-type eqs.

Takata, Yasuda, & A; Zhdanov & Roldughin; Loyalka; Sharipov & Kalempa; Garcia & Siewert; ….

Boundary conditions



$$n_{(0)}^A = n_w^A \left(= \frac{p_w^A}{T_w} \right),$$

$$T_{(0)} = T_w,$$

$$\mathbf{v}_{(1)}^B \cdot \mathbf{n} = 0,$$

$$\mathbf{v}_{(1)} \cdot \mathbf{t} = -T_{(0)}^{1/2} \left(b_7 \frac{1}{p_{(0)}} \frac{\partial T_{(0)}}{\partial \mathbf{x}} \cdot \mathbf{t} + b_9 \frac{1}{n_{(0)}} \frac{\partial X^A}{\partial \mathbf{x}} \cdot \mathbf{t} \right).$$

Thermal creep   Diffusion slip

b_7, b_9 (Slip coefficients) + Knudsen-layer corrections

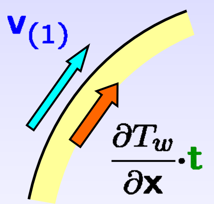
Functions of X^A
$\left(\text{depending on } \frac{m^B}{m^A}, \frac{d^B}{d^A}\right)$

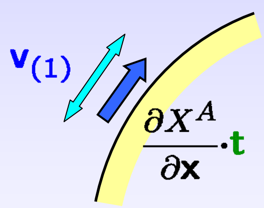
Database: Takata, Yasuda, Kosuge, & A (03) Phys. Fluids

$$\mathbf{v}_{(1)}^B \cdot \mathbf{n} = 0,$$

$$\mathbf{v}_{(1)} \cdot \mathbf{t} = -T_{(0)}^{1/2} \left(b_7 \frac{1}{p_{(0)}} \frac{\partial T_{(0)}}{\partial \mathbf{x}} \cdot \mathbf{t} + b_9 \frac{1}{n_{(0)}} \frac{\partial X^A}{\partial \mathbf{x}} \cdot \mathbf{t} \right).$$

Thermal creep   Diffusion slip





Summary: (I) amount of NC gas $\sim$ amount of vapor

Boltzmann system

↓ Formal asymptotic analysis for small Kn

Takata & A (01) TTSP

Fluid-dynamic-type equations

\+ Database for transport coefficients

Takata, Yasuda, A, & Shibata (03) RGD23

Boundary conditions

Database for slip coefficients (Knudsen-layer correction)

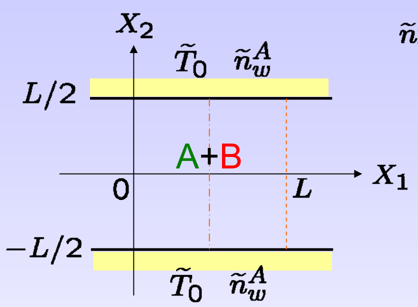
Takata, Yasuda, Kosuge, & A (03) Phys. Fluids

FD system for slow flows of vapor and NC gas with large temperature and density variations

Application | Continuum limit

Application

Laneryd, A, & Takata (07) Phys. Fluids

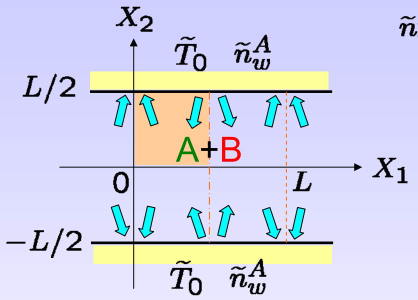


$$\tilde{n}_w^A = \tilde{n}_0[1 - \tilde{n}_c \cos(2\pi X_1/L)]$$

Numerical solution

Finite-volume method

Application Laneryd, A, & Takata (07) Phys. Fluids



$$\tilde{n}_w^A = \tilde{n}_0[1 - \hat{n}_c \cos(2\pi X_1/L)]$$

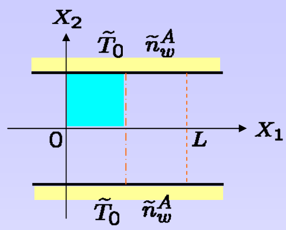
Numerical solution

Finite-volume method

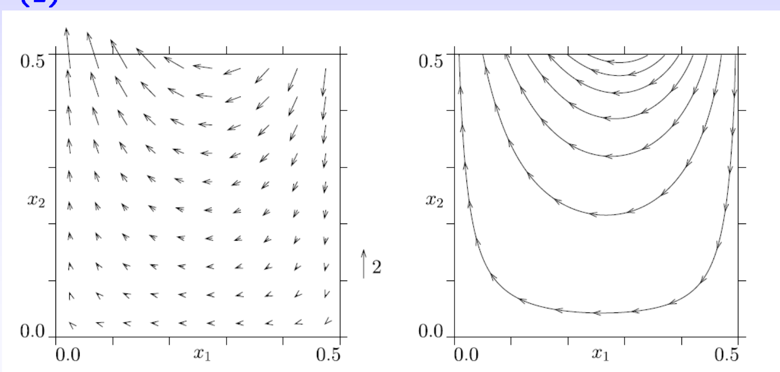
$\dfrac{\tilde{n}_{av}^B}{\tilde{n}_0}$: Amount of NC gas

$$\hat{n}_c = 0.5, \quad \frac{\tilde{n}_{av}^B}{\tilde{n}_0} = 0.8, \quad \frac{\tilde{m}^B}{\tilde{m}^A} = 5, \quad \frac{\tilde{d}^B}{\tilde{d}^A} = 1$$

$$(\ \tilde{n}_w^A = \tilde{n}_0[1 - \hat{n}_c \cos(2\pi X_1/L)] \ )$$

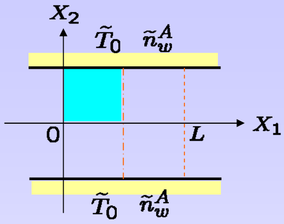


$\mathbf{v}_{(1)}^A$

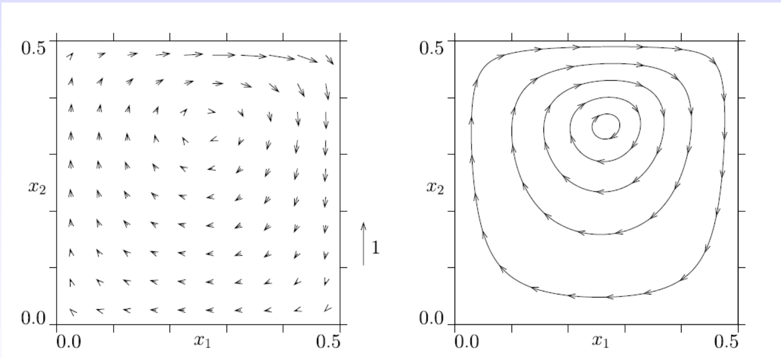


$\hat{n}_c = 0.5, \quad \dfrac{\tilde{n}^B_{av}}{\tilde{n}_0} = 0.8, \quad \dfrac{\tilde{m}^B}{\tilde{m}^A} = 5, \quad \dfrac{\tilde{d}^B}{\tilde{d}^A} = 1$

$(\ \tilde{n}^A_w = \tilde{n}_0[1 - \hat{n}_c \cos(2\pi X_1/L)] \ )$

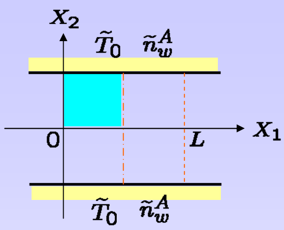


$\mathbf{v}^B_{(1)}$

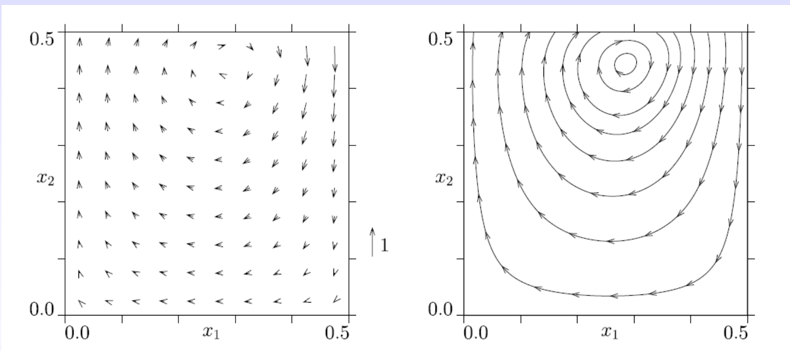


$\hat{n}_c = 0.5, \quad \dfrac{\tilde{n}^B_{av}}{\tilde{n}_0} = 0.8, \quad \dfrac{\tilde{m}^B}{\tilde{m}^A} = 5, \quad \dfrac{\tilde{d}^B}{\tilde{d}^A} = 1$

$(\ \tilde{n}^A_w = \tilde{n}_0[1 - \hat{n}_c \cos(2\pi X_1/L)] \ )$



$\mathbf{v}_{(1)} \quad \left[\tilde{\mathbf{v}} = (2k\tilde{T}_0/m^A)^{1/2} \mathbf{v}_{(1)} \epsilon + \cdots \right]$



Comparison with DSMC

$\hat{n}_c = 0.5, \quad \dfrac{\tilde{n}^B_{av}}{\tilde{n}_0} = 0.8, \quad \dfrac{\tilde{m}^B}{\tilde{m}^A} = 5, \quad \dfrac{\tilde{d}^B}{\tilde{d}^A} = 1$

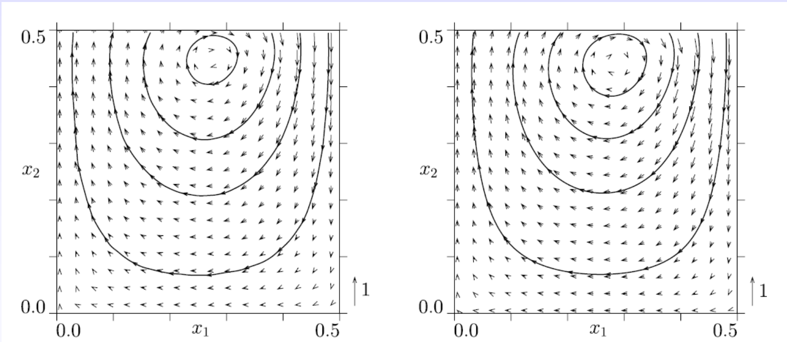
$(\; \tilde{n}^A_w = \tilde{n}_0[1 - \hat{n}_c \cos(2\pi X_1/L)] \;)$

DSMC

$\dfrac{\tilde{\mathbf{v}}}{(2kT_0/m^A)^{1/2}} \dfrac{1}{\epsilon}$

$(\; \epsilon = 0.01108 \;)$

$\mathbf{v}_{(1)} = \lim_{\epsilon \to 0} \dfrac{\tilde{\mathbf{v}}}{(2k\tilde{T}_0/m^A)^{1/2}} \dfrac{1}{\epsilon}$



Continuum limit $\quad \epsilon \to 0$

$$
\begin{aligned}
n^\alpha &= n^\alpha_{(0)} + n^\alpha_{(1)}\epsilon + n^\alpha_{(2)}\epsilon^2 + \dots && \to n^\alpha_{(0)} \\
T &= T_{(0)} + T_{(1)}\epsilon + T_{(2)}\epsilon^2 + \dots && \to T_{(0)} \\
p &= p_{(0)} + p_{(1)}\epsilon + p_{(2)}\epsilon^2 + \dots && \to p_{(0)} = \text{const} \\
\mathbf{v} &= 0 + \mathbf{v}_{(1)}\epsilon + \mathbf{v}_{(2)}\epsilon^2 + \dots && \to 0 \\
\mathbf{v}^\alpha &= 0 + \mathbf{v}^\alpha_{(1)}\epsilon + \mathbf{v}^\alpha_{(2)}\epsilon^2 + \dots && \to 0
\end{aligned}
$$

No evaporation or condensation

The flow vanishes; however, the temperature field is still affected by the invisible flow

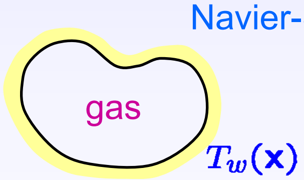
Ghost effect

Navier-Stokes system

$\mathbf{v} = 0, \quad p = \text{const}$

T : steady heat-conduction eq.
+ no-jump cond.

$$\frac{\partial}{\partial x_j}\left(\lambda \frac{\partial T}{\partial x_j}\right) = 0$$



Some references on *ghost effect*

Single gas:

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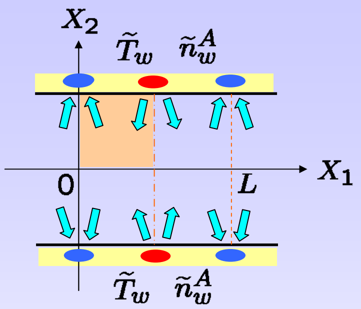
- Sone, *Rarefied Gas Dynamics* (Peking Univ. Press, 1997), *p. 3*
- Sone, Ann. Rev. Fluid Mech. 32, *779* (2000)
- Sone, Doi, Phys. Fluids 15, *1405* (2003)
- Sone, Doi, Phys. Fluids 16, *952* (2004)

Y. Sone, *Molecular Gas Dynamics: Theory, Techniques, and Applications* (Birkhäuser, 2007)

Gas mixture:

- Takata, A, Phys. Fluids 11, *2743* (1999)
- Takata, A, Transp. Theory Stat. Phys. 30, *205* (2001)
- Takata, Phys. Fluids 16, *2182* (2004)



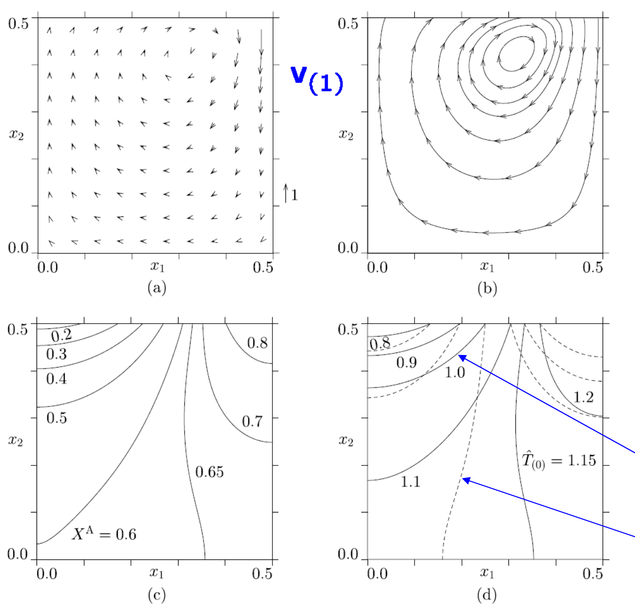
$$\tilde{n}_w^A = \tilde{n}_0[1 - \hat{n}_c \cos(2\pi X_1/L)]$$

$$\tilde{T}_w = \tilde{T}_0[1 - \hat{T}_c \cos(2\pi X_1/L)]$$

Numerical solution of FD system by finite-element method

$$\hat{n}_c = 0.5, \quad \hat{T}_c = 0.3, \quad \frac{\tilde{n}_{av}^B}{\tilde{n}_0} = 0.8, \quad \frac{\tilde{m}^B}{\tilde{m}^A} = 5, \quad \frac{\tilde{d}^B}{\tilde{d}^A} = 1$$

$\hat{n}_c = 0.5,\ \underline{\hat{T}_c = 0.3},\ \dfrac{\tilde{n}^B_{av}}{\tilde{n}_0} = 0.8,\ \dfrac{\tilde{m}^B}{\tilde{m}^A} = 5,\ \dfrac{\tilde{d}^B}{\tilde{d}^A} = 1$



$\mathbf{v} = \mathbf{v}_{(1)}\,\epsilon + \dots$

$\to 0$

$T_{(0)} = \text{const}$

FD system

Heat-cond. eq.

(II) amount of NC gas $\ll$ amount of vapor

A, Taguchi, & Takata (03) Eur. J. Mech. B

- Knudsen number $\quad \mathrm{Kn} = l_r / L \ll 1$

  reference mfp of vapor (l_r)   reference length (L)

- Small amount of NC gas

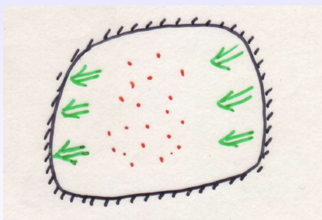
$$\tilde{n}^B_{av} / \tilde{n}_r = O(\mathrm{Kn}) = \text{const} \times \mathrm{Kn} \ll 1$$

$\tilde{n}_r$: reference number density of vapor

$\tilde{n}^B_{av}$: average number density of NC gas

Continuum limit $(\mathrm{Kn} \to 0)$

$$\tilde{n}^B_{av} / \tilde{n}_r = O(\mathrm{Kn}) \to 0$$



Infinitesimal average concentration of NC gas

Fluid-dynamic equations

Hilbert solution (expansion) $[\partial F^\alpha/\partial \mathbf{x} = O(F^\alpha)]$

$$F^\alpha = F^\alpha_{(0)} + F^\alpha_{(1)}\epsilon + F^\alpha_{(2)}\epsilon^2 + \dots, \qquad (\alpha = A,\ B)$$

Macroscopic quantities $(h = n^\alpha,\ \mathbf{v}^\alpha,\ T^\alpha,\ n,\ \mathbf{v},\ T,\ \dots)$

$$h = h_{(0)} + h_{(1)}\epsilon + h_{(2)}\epsilon^2 + \dots$$

Boltzmann eqs. → Sequence of integral equations

$$\zeta_i \frac{\partial F^A}{\partial x_i} = \frac{1}{\epsilon}[J^{AA}(F^A, F^A) + J^{BA}(F^B, F^A)]$$

$$\zeta_i \frac{\partial F^B}{\partial x_i} = \frac{1}{\epsilon}[J^{AB}(F^A, F^B) + J^{BB}(F^B, F^B)]$$

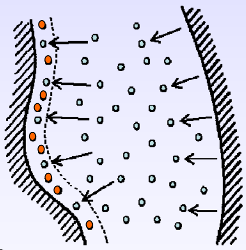
$\tilde{n}^B_{av}/\tilde{n}_r = O(\epsilon)$ [Previous case $\tilde{n}^B_{av}/\tilde{n}_r = O(1)$]

$\tilde{n}^B_{av}/\tilde{n}_r = O(\epsilon)$

- $F^B_{(m)} \equiv 0$
- $F^A_{(0)} = n^A_{(0)} \left(\dfrac{m^A}{\pi T_{(0)}}\right)^{3/2} \exp\left(-\dfrac{m^A|\boldsymbol{\zeta} - \mathbf{v}_{(0)}|^2}{T_{(0)}}\right)$

$$\left(\begin{array}{l} n^A = n^A_{(0)} + n^A_{(1)}\epsilon + \dots, \quad \mathbf{v} = \mathbf{v}_{(0)} + \mathbf{v}_{(1)}\epsilon + \dots, \\ T = T_{(0)} + T_{(1)}\epsilon + \dots \end{array}\right)$$

$n^A_{(0)},\ \mathbf{v}_{(0)},\ T_{(0)}$: Compressible Euler equations (pure vapor)



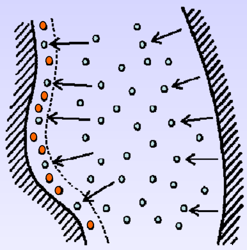
BC

$$F^A = n^A_w \left(\frac{m^A}{\pi T_w}\right)^{3/2} \exp\left(-\frac{m^A|\boldsymbol{\zeta} - \mathbf{v}_w|^2}{T_w}\right), \qquad (\boldsymbol{\zeta}\cdot\mathbf{n} > 0)$$

Compressible Euler equations (pure vapor)

$$\begin{cases} \nabla_x \cdot \left(n^A_{(0)} \mathbf{v}_{(0)} \right) = 0 \\ n^A_{(0)} \mathbf{v}_{(0)} \cdot \nabla_x \mathbf{v}_{(0)} + \frac{1}{2} \nabla_x \left(n^A_{(0)} T_{(0)} \right) = 0 \\ \mathbf{v}_{(0)} \cdot \nabla_x \left(\frac{5}{2} T_{(0)} + |\mathbf{v}_{(0)}|^2 \right) = 0 \end{cases}$$

$n^A_{(0)} T_{(0)} = p^A_{(0)}$



$F^A_{(0)}$ satisfies BC if

$$n^A_{(0)} = n^A_w, \quad \mathbf{v}_{(0)} = \mathbf{v}_w, \quad T_{(0)} = T_w$$

on the boundary

Too many conditions for Euler equations

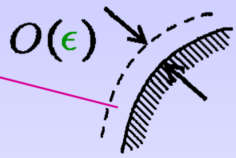
$F^A_{(0)}$ cannot satisfy BC

→ Need of Knudsen-layer correction

Knudsen layer and jump boundary conditions

Solution: $F^\alpha = F^\alpha_H + F^\alpha_K \quad (\alpha = A,\, B)$

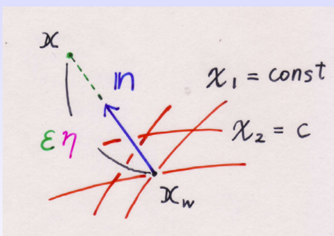
Hilbert solution (F^α_H) — Knudsen-layer correction (F^α_K)



Stretched normal coordinate η

$$\mathbf{x} = \epsilon \eta \mathbf{n}(\chi_1, \chi_2) + \mathbf{x}_w(\chi_1, \chi_2)$$

$$F^\alpha_K(\eta, \chi_1, \chi_2, \boldsymbol{\zeta}) \to 0 \quad (\eta \to \infty)$$



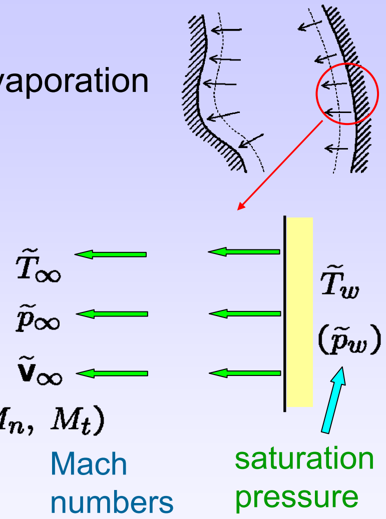
$$\begin{cases} F^A_H = F^A_{H(0)} + F^A_{H(1)}\epsilon + \dots, \\ F^B_H = 0 \\ F^\alpha_K = F^\alpha_{K(0)} + F^\alpha_{K(1)}\epsilon + \dots, \end{cases}$$

Eqs. and BC for $F^\alpha_{K(0)}$ → Half-space problem for nonlinear Boltzmann eqs.

Half-space problems for nonlinear Boltzmann eqs.

- Evaporating surface

  Half-space problem of strong evaporation of pure vapor



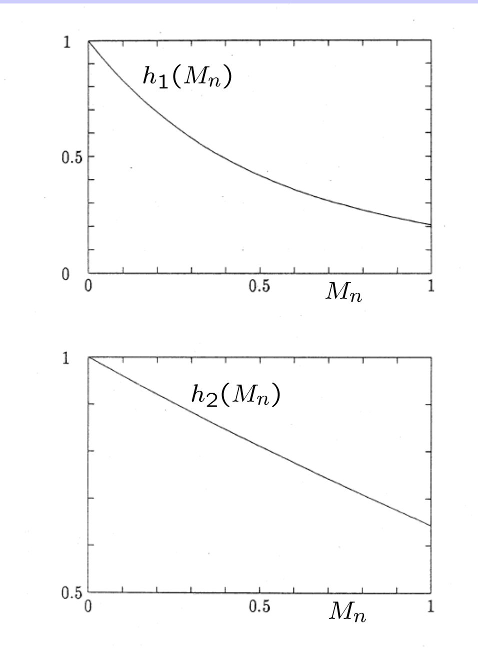
Relations among parameters

$$M_n \le 1, \qquad M_t = 0,$$
$$\tilde{p}_\infty/\tilde{p}_w = h_1(M_n),$$
$$\tilde{T}_\infty/\tilde{T}_w = h_2(M_n).$$

$h_1,\ h_2$: Numerical (BGK)

⇒ B.C. for Euler eqs.

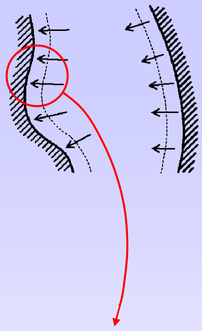
Sone (78), Sone & Sugimoto (90), Sone (00), Sone, Takata, Golse (01), …, Ukai, Yang, & Yu (03), Golse (08), Liu & Yu (09)



Sone & Sugimoto (90)

- Condensing surface

Half-space problem of strong condensation of vapor in the presence of NC gas

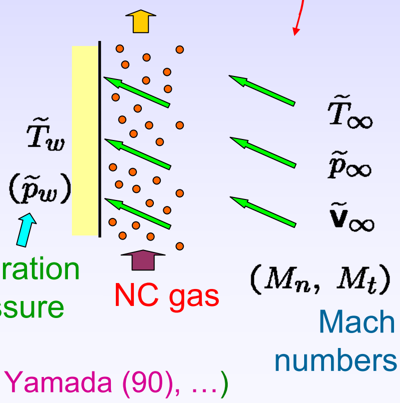


Relations among parameters $[M_n < 1]$

$$\tilde{p}_\infty/\tilde{p}_w = \mathcal{F}_s(M_n,\ M_t,\ \tilde{T}_\infty/\tilde{T}_w,\ \Gamma),$$

numerical

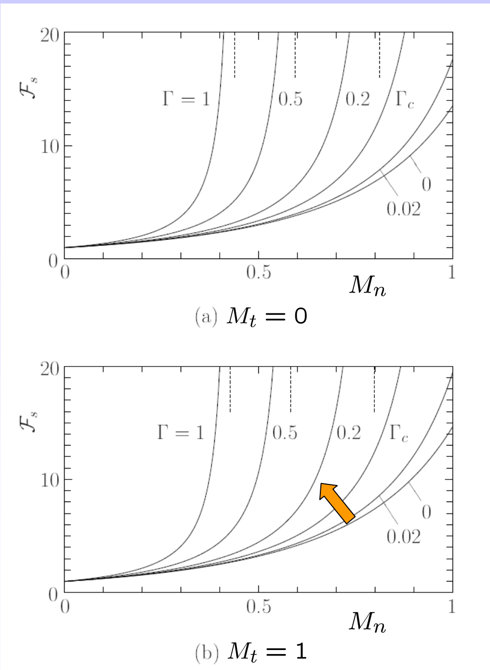
Parameter of $O(1)$ (amount of NC gas)



⇨ Part of B.C. for Euler eqs.

Sone, A, & Doi (92), Taguchi, A, Takata (03, 04)

(Single component: Sone (78), Sone, A, & Yamashita (86), A, Sone, & Yamada (90), …)



$$\mathcal{F}_s\left(M_n,\ M_t,\ \frac{\tilde{T}_\infty}{\tilde{T}_w},\ \Gamma\right) \quad \frac{\tilde{T}_\infty}{\tilde{T}_w} = 1$$

Numerical data:

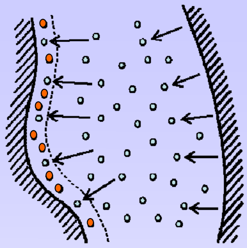
- GSB model
  Garzó, Santos, & Brey (89)
- (molecule of vapor) ≡ (molecule of NC gas)

Analytical form of $\mathcal{F}_s$ for small M_n

hard sphere

Taguchi, A.,& Latocha (06)

Summary (suffix " (0) " omitted)



Euler eqs.

$$\nabla_x \cdot \left(n^A \mathbf{v}\right) = 0$$

$$n^A\, \mathbf{v}\cdot\nabla_x \mathbf{v} + \frac{1}{2}\nabla_x\left(n^A T\right) = 0 \qquad (n^A T = p^A)$$

$$\mathbf{v}\cdot\nabla_x\left(\frac{5}{2}T + |\mathbf{v}|^2\right) = 0$$

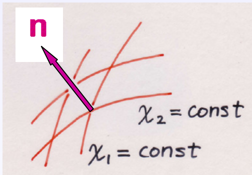
BC $\left(M_t = \sqrt{6/5T}\ |(\mathbf{v}-\mathbf{v}_w)\cdot\mathbf{t}|, \quad M_n = \sqrt{6/5T}\ \mathbf{v}\cdot\mathbf{n} \right)$

Evaporating surface: $M_t = 0, \quad 0 < M_n \le 1$

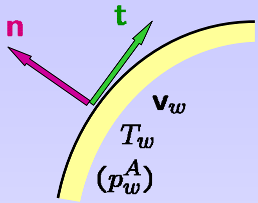
$p^A/p_w^A = h_1(M_n), \quad T/T_w = h_2(M_n)$

Condensing surface: $p^A/p_w^A = \mathcal{F}_s(|M_n|, M_t, T/T_w, \Gamma), \quad (|M_n| < 1)$

Amount of NC gas in Knudsen layer
(function of χ_1, χ_2)



Summary (suffix " (0) " omitted)



Euler eqs.

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$$n^A\, \mathbf{v}\cdot\nabla_x \mathbf{v} + \frac{1}{2}\nabla_x\left(n^A T\right) = 0 \qquad (n^A T = p^A)$$

$$\mathbf{v}\cdot\nabla_x\left(\frac{5}{2}T + |\mathbf{v}|^2\right) = 0$$

BC $\left(M_t = \sqrt{6/5T}\ |(\mathbf{v}-\mathbf{v}_w)\cdot\mathbf{t}|, \quad M_n = \sqrt{6/5T}\ \mathbf{v}\cdot\mathbf{n} \right)$

Evaporating surface: $M_t = 0, \quad 0 < M_n \le 1$

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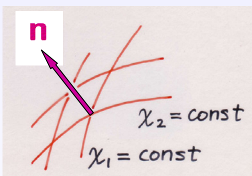
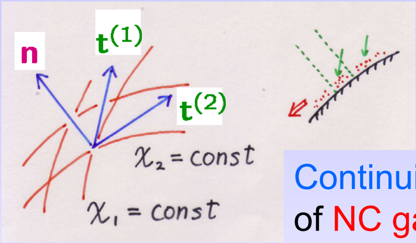
Condensing surface: $p^A/p_w^A = \mathcal{F}_s(|M_n|, M_t, T/T_w, \Gamma), \quad (|M_n| < 1)$

Amount of NC gas in Knudsen layer
(function of χ_1, χ_2)



Condensing surface $\quad p^A/p_w^A = \mathcal{F}_s(|M_n|,\, M_t,\, T/T_w,\, \Gamma),\quad (|M_n| < 1)$



$\Gamma(\chi_1, \chi_2)$ Amount of NC gas per unit area in Knudsen layer

Continuity equation for particle flux $\mathcal{N}$ of NC gas in Knudsen layer

$$\chi_{1,1}\frac{\partial}{\partial\chi_1}(\mathcal{N}\cdot\mathbf{t}^{(1)}) + \chi_{2,2}\frac{\partial}{\partial\chi_2}(\mathcal{N}\cdot\mathbf{t}^{(2)}) + g_2\mathcal{N}\cdot\mathbf{t}^{(1)} + g_1\mathcal{N}\cdot\mathbf{t}^{(2)} = 0$$

1st-order Knudsen-layer eqs.

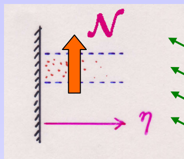
$\chi_{1,1},\ \chi_{2,2},\ g_1,\ g_2$: geometric const

geodesic curvatures

$$\mathcal{N} = \mathcal{G}_s(|M_n|,\, M_t,\, T/T_w,\, \Gamma)\,\mathbf{t},\quad (|M_n| < 1)$$

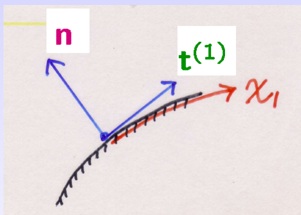
$$\mathcal{N} = \mathcal{G}_s(|M_n|,\, M_t,\, T/T_w,\, \Gamma)\,\mathbf{t},\quad (|M_n| < 1)$$

$\mathcal{N} = 0 \iff M_t = 0$



2D problems

$$\mathcal{N}\cdot\mathbf{t}^{(1)} = \text{const}$$



Summary: (II) amount of NC gas $\ll$ amount of vapor

$n_{av}^{B}/n_r = O(\mathrm{Kn}) \ll 1$

Boltzmann system

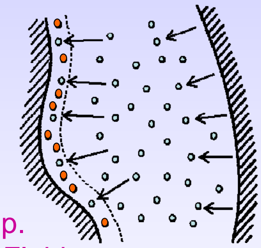
Formal asymptotic analysis for small Kn

A, Taguchi, & Takata (03) Eur. J. Mech. B

Euler Equation for vapor

\+

Boundary conditions



Database for BC

Sone & Sugimoto (90), IUTAM Symp.
Taguchi, A, & Takata (03,04) Phys. Fluids

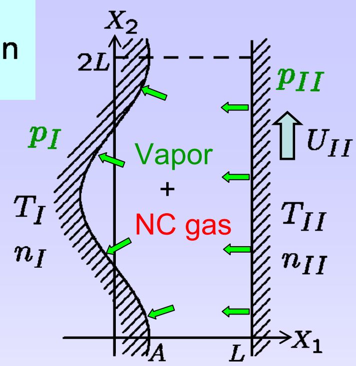
FD system for high-speed flows of vapor in the presence of small amount of NC gas

Application → Continuum limit

Example (subsonic)

Taguchi, A, & Takata (04) Phys. Fluids

$$\frac{n_{av}^{B}}{n_I} = \Delta \mathrm{Kn}$$



Euler eqs. + BC

Evaporating surface

$$M_t = 0, \qquad 0 < M_n \le 1$$

$$\frac{p^A}{p_{II}} = h_1(M_n), \qquad \frac{T}{T_{II}} = h_2(M_n)$$

Condensing surface

$$\frac{p^A}{p_I} = \mathcal{F}_s\left(|M_n|,\, M_t,\, \frac{T}{T_I},\, \Gamma\right)$$

$$\mathcal{N} = \mathrm{const}$$

Pure vapor flow

$\Gamma = 0$

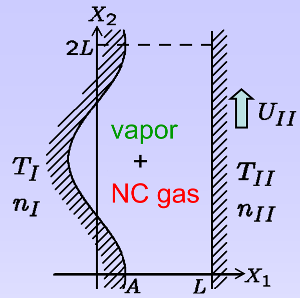
$$\mathrm{Kn} = \frac{l_I}{L} \to 0, \quad \frac{n_{av}^{B}}{n_I} = \Delta \mathrm{Kn} \to 0, \quad \left(\Delta = \mathrm{const} \int \Gamma ds\right)$$

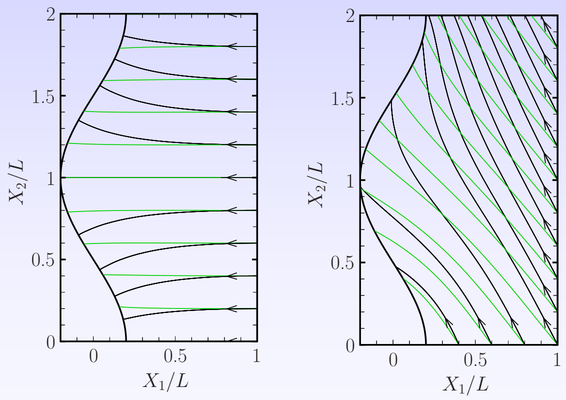
Average concentration of NC gas $\to 0$

Effect of infinitesimal average concentration of NC gas

$T_{II}/T_I = 1, \ \ n_{II}/n_I = 2$

$n^B_{av}/n_I = \Delta\mathrm{Kn} \ \to 0$





Stream lines

pure vapor

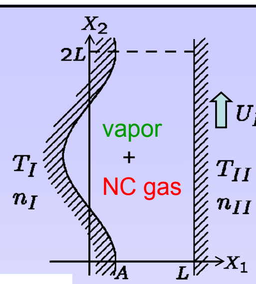
$\Delta = 2$

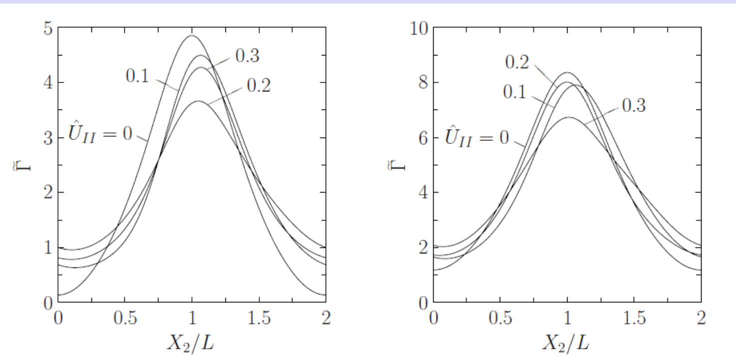
(vapor molecule) = (NC-gas molecule)

$U_{II} = 0, \qquad U_{II}/(2kT_I/m)^{1/2} = 0.2$

$T_{II}/T_I = 1, \ \ n_{II}/n_I = 2$

$n^B_{av}/n_I = \Delta\mathrm{Kn} \ \to 0$





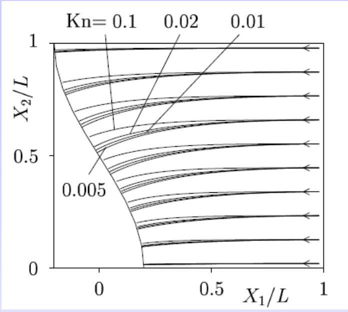
(a) $\Delta = 2$ (b) $\Delta = 4$

DSMC (hard sphere)

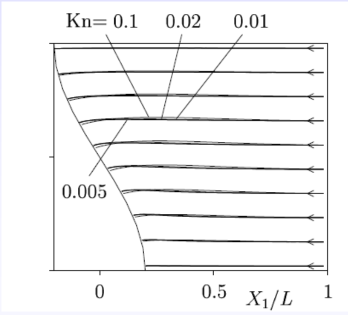
A, Takata, & Suzuki, RGD23 (AIP, 2003)

$T_{II}/T_I = 1, \;\; n_{II}/n_I = 2$

$n^B_{av}/n_I = \Delta \mathrm{Kn}$



$\Delta = 1$



pure vapor

$\mathrm{Kn} = 0.005$