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Title: The Structure of Radiative Shocks

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The Structure of Radiative Shocks

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Motivation

- Non-trivial, analytic solutions do not exist for radiation hydrodynamics.
  - Compromise: Generate *semi-analytic* solutions; solutions from numerically integrating nonlinear ODEs.
  - Seek traveling wave solutions $\Rightarrow$ *radiative shocks*.
- In this talk, use solutions to
  - verify code correctness
  - test AMR (rad-shocks are multiscale problems)
- Gives physics insight that would be *very* difficult to obtain by a series of runs from a computational physics code.

Outline

1. Overview of radiative shocks
2. Equations of radiation hydrodynamics
   - Grey nonequilibrium diffusion
   - Euler coupled with grey nonequilibrium diffusion
   - Equilibrium diffusion limit
3. Overview of Semi-analytic Approach
4. Sample Solutions
5. Code Comparison

Outline

Physics Regime

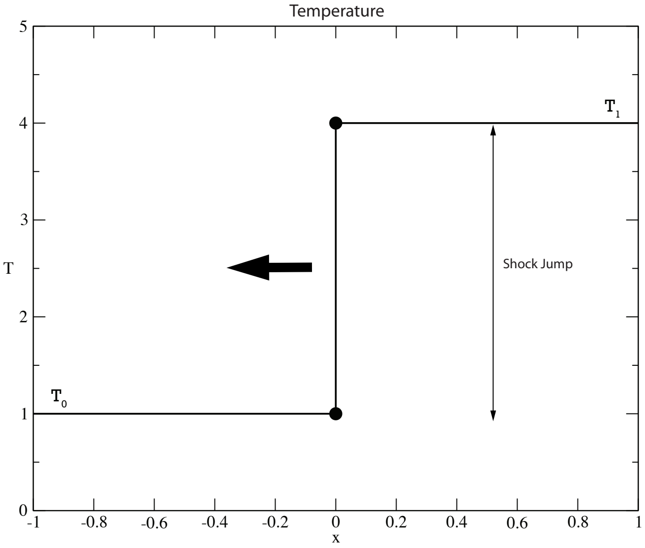
As one example, regimes in astrophysics governed by

- Inviscid hydrodynamics (Euler equations; non-relativistic)
- Thermal radiation (X-ray) transport
- High-energy density:
  - Material temperatures $O(1\text{ keV} = 11.6 \times 10^6\text{ Kelvin})$
  - Radiation *pressure* affects hydrodynamics

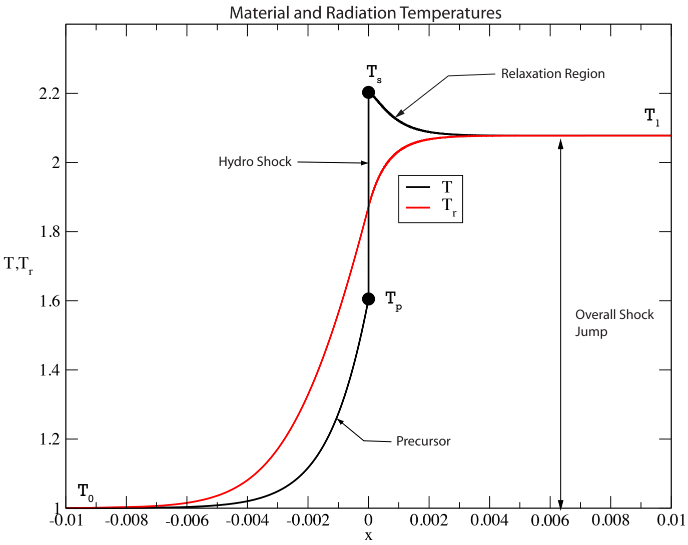
Past Work on Radiative Shocks

- Most theory on "thick – thick" shocks: equilibrates on either side of shock.
- **Overview of Theory:**
  - Y. B. ZEL'DOVICH and Y. P. RAIZER, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena*, 1966 (Dover 2002).
  - D. MIHALAS and B. W. MIHALAS, *Foundations of Radiation Hydrodynamics*, 1984 (Dover 1999).
  - R.P. DRAKE, *High-Energy-Density Physics*, Springer, 2006.
- Grey VEF/AMR, ion/electron, calculations:
  - M. W. SINCELL, M. GEHMEYR, and D. MIHALAS, two articles in *Shock Waves*, 1999.
- **Theory and Solutions:**
  - **Equilibrium:** S. BOUQUET, R. TEYSSIER, and J. P. CHIEZE, *Astrophysical Journal Supplement Series*, 2000.
  - **Equilibrium:** R. B. LOWRIE and R. M. RAUENZAHN, *Shock Waves*, 2007.
  - **Nonequilibrium:** R. B. LOWRIE and J. D. EDWARDS, *Shock Waves*, 2008.
- **Extended thermodynamics:**
  - W. Weiss, "Structure of Shock Waves," in *Rational Extended Thermodynamics*, Müller & Ruggeri (eds.), Springer, 1998.

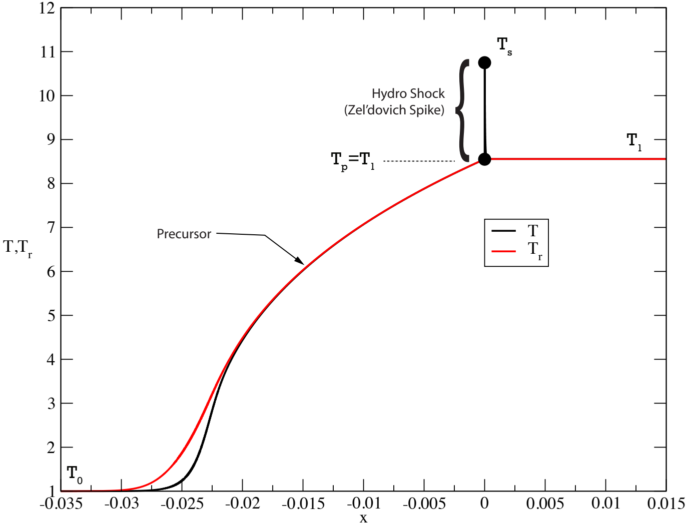
Inviscid Hydrodynamic Shocks



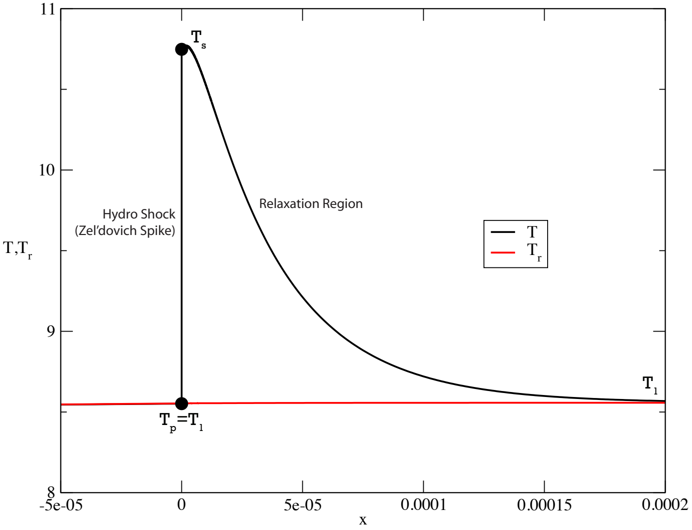
Subcritical Radiative Shocks ($T_p < T_1$)



Supercritical Radiative Shocks ($T_p = T_1$)



Zel'dovich Spike



Outline

Grey nonequilibrium diffusion in 1 slide

1-D planar (slab) geometry

First two moments of radiative transfer equation:

$$\partial_t E + \partial_x F = c\sigma_a[B(T) - E] - \frac{\sigma_t}{c}\left(F - \frac{4}{3}vE\right)v$$

$$\frac{1}{c^2}\partial_t F + \partial_x P = -\frac{\sigma_t}{c}\left(F - \frac{4}{3}vE\right)$$

Nonequilibrium diffusion sets $P = E/3$ and drops $\partial_t F$ to yield

$$\partial_t E + \underline{\frac{4}{3}\partial_x(vE)} - \partial_x\left(\frac{c}{3\sigma_t}\partial_x E\right) = c\sigma_a[B(T) - E] + \underline{\frac{1}{3}v\partial_x E}$$

Underlined terms often dropped in “low-energy density” approximation.

Euler Coupled with Grey Nonequilibrium Diffusion

Nondimensional form, 1-D planar (slab) geometry

Most of this talk will be about solutions of

$$\partial_t \rho + \partial_x(\rho v) = 0,$$

$$\partial_t(\rho v) + \partial_x\left(\rho v^2 + p + \underline{\frac{1}{3}\mathcal{P}_0 {T_r}^4}\right) = 0,$$

$$\partial_t(\rho\mathcal{E}) + \partial_x\left[v(\rho\mathcal{E} + p)\right] = \mathcal{P}_0\sigma_a({T_r}^4 - T^4) - \underline{\frac{1}{3}\mathcal{P}_0 v \partial_x {T_r}^4},$$

$$\partial_t(\rho\mathcal{E} + \mathcal{P}_0 {T_r}^4) + \partial_x\left[v\left(\rho\mathcal{E} + p + \underline{\frac{4}{3}\mathcal{P}_0 {T_r}^4}\right)\right] = \mathcal{P}_0\partial_x(\kappa\partial_x {T_r}^4),$$

where $\mathcal{E} = e + \frac{1}{2}v^2$ and for a γ-law EOS

$$p = \frac{\rho T}{\gamma}, \quad e = \frac{T}{\gamma(\gamma - 1)}, \quad \mathcal{P}_0 = \frac{\tilde{a}_R \tilde{T}_0^4}{\gamma \tilde{p}_0} \approx \frac{\text{rad. pressure}}{\text{mat. pressure}}.$$

Embedded discontinuities are hydrodynamic shocks.

Equilibrium Diffusion Limit

We'll also discuss "1T" solutions. Optically thick limit, $T_r \to T$, and our system reduces to

$$\partial_t \rho + \partial_x(\rho v) = 0,$$

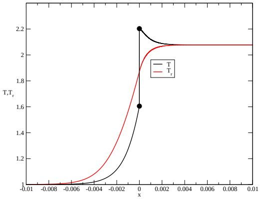
$$\partial_t(\rho v) + \partial_x \left(\rho v^2 + p + \frac{1}{3}\mathcal{P}_0 T^4\right) = 0,$$

$$\partial_t(\rho E + \mathcal{P}_0 T^4) + \partial_x \left[v\left(\rho E + p + \frac{4}{3}\mathcal{P}_0 T^4\right)\right] = \mathcal{P}_0 \partial_x(\kappa \partial_x T^4).$$

- Euler equations with nonlinear heat conduction and a modified equation-of-state.
- For small-$\mathcal{P}_0$, heat conduction may dominate.
- Embedded discontinuities are isothermal shocks.

Problem statement

- Assume that far from shock, $T_r = T$ ("thick–thick").
- Galilean invariant, so use steady frame.



- *Given:* The value γ and
  - Pre-shock state ($x \to -\infty$): ρ_0, T_0, $T_{r0} = T_0$.
  - Non-dimensional constants: $\mathcal{P}_0$ and $\mathcal{M}_0$ ($\equiv \tilde{v}_0/\tilde{a}_0$, Mach number).
  - Functions $\sigma_a(\rho, T)$ and $\kappa(\rho, T)$ ($\tilde{\kappa} = \tilde{c}/3\tilde{\sigma}_t$).
- *Calculate:* $\rho(x)$, $v(x)$, $T(x)$, and $T_r(x)$.
- *Optional:* Transform back to a frame where shock is moving.

Outline

Euler Coupled with Grey Nonequilibrium Diffusion

Nondimensional form, 1-D planar (slab) geometry

Seek traveling-wave solutions of

$$\partial_t \rho + \partial_x(\rho v) = 0,$$

$$\partial_t(\rho v) + \partial_x\left(\rho v^2 + p + \frac{1}{3}\mathcal{P}_0 {T_r}^4\right) = 0,$$

$$\partial_t(\rho E) + \partial_x\left[v(\rho E + p)\right] = \mathcal{P}_0 \sigma_a({T_r}^4 - T^4) - \frac{1}{3}\mathcal{P}_0 v \partial_x {T_r}^4,$$

$$\partial_t(\rho E + \mathcal{P}_0 {T_r}^4) + \partial_x\left[v\left(\rho E + p + \frac{4}{3}\mathcal{P}_0 {T_r}^4\right)\right] = \mathcal{P}_0 \partial_x(\kappa \partial_x {T_r}^4),$$

Overall Jump Relation

Integrate conservation equations from $-\infty < x < \infty$ to give:

$$\begin{pmatrix} \rho v \\ \rho v^2 + p^* \\ (\rho E^* + p^*)v \end{pmatrix}_0 = \begin{pmatrix} \rho v \\ \rho v^2 + p^* \\ (\rho E^* + p^*)v \end{pmatrix}_1 ,$$

where

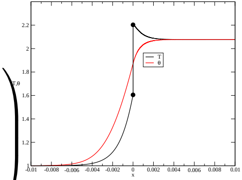
$$p^* = p + \frac{1}{3}\mathcal{P}_0 T^4, \quad e^* = e + \frac{1}{\rho}\mathcal{P}_0 T^4, \quad E^* = e^* + \frac{1}{2}v^2.$$

- Get ninth-order polynomial in T_1; see Bouquet *et al* (2000).
- Same procedure used for any radiation model.

Hydro Shock Relations

At a discontinuity separating state-$_p$ and state-$_s$:

$$\begin{pmatrix} \rho v \\ \rho v^2 + p \\ (\rho E + p) v \\ -\kappa \partial_x {T_r}^4 + \frac{4}{3} v {T_r}^4 \end{pmatrix}_p = \begin{pmatrix} \rho v \\ \rho v^2 + p \\ (\rho E + p) v \\ -\kappa \partial_x {T_r}^4 + \frac{4}{3} v {T_r}^4 \end{pmatrix}_s$$



- First 3 equations: Standard hydro jump conditions. Holds for any radiation model.
- Last equation: Continuity of *Eulerian frame* radiation flux.

Reduced Equations

4 PDEs reduce to 2 ODEs. We use Mach number ($\mathcal{M}$) as the independent variable:

$$\frac{dx}{d\mathcal{M}} = \frac{3\mathcal{M}_0(\mathcal{M}^2 - 1)\rho\beta}{\mathcal{P}_0},$$

$$\frac{dT}{d\mathcal{M}} = (\gamma - 1)\beta \left[4\mathcal{M}_0 {T_r}^3 {T_r}' + (\gamma\mathcal{M}^2 - 1)r\right],$$

where $\beta(T, \mathcal{M})$ and $r(T, \mathcal{M})$ are known functions, and

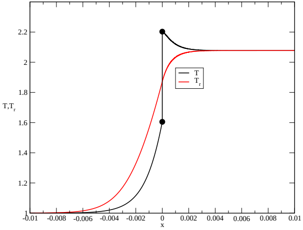
$$\rho(T, \mathcal{M}) = \frac{\mathcal{M}_0}{\mathcal{M}\sqrt{T}}, \quad T_r(T, \mathcal{M}) = \frac{1}{\gamma\mathcal{P}_0}\left[K_m - 3\gamma\frac{\mathcal{M}_0^2}{\rho(T, \mathcal{M})} - 3T\rho(T, \mathcal{M})\right]$$

- If $T_r \approx$ const., then $\mathcal{M} = 1/\sqrt{\gamma}$ corresponds to maximum T (isothermal sonic point – ISP).

Overview of Solution Procedure

1. Compute post-shock state ($x \to \infty$): Find root of a ninth-order polynomial in T_1.
2. End states are typically *saddle* points. Solve two IVPs:
   - Find precursor region: Integrate ODEs from $\mathcal{M} = \mathcal{M}_0$ to $\mathcal{M} = 1$.
   - Find relaxation region: Integrate ODEs from $\mathcal{M} = \mathcal{M}_1$ to $\mathcal{M} = 1$.
3. If the two ODE solutions do not match at $\mathcal{M} = 1$, then shift the solutions such that they are connected with a hydro shock.

See Lowrie & Edwards (*Shock Waves*, 2008) for the details.



Outline

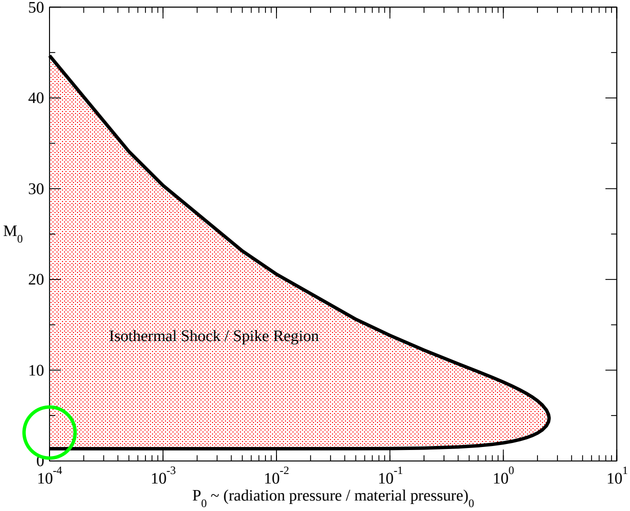
Solution Regimes

Radiative shock solutions may be characterized by the following:

- Is there an embedded hydro shock?
  - If no, all variables continuous.
  - If yes,
    - Subcritical ($T_p < T_1$).
    - Supercritical ($T_p \approx T_1$).
- Is there an isothermal sonic point (ISP; $\mathcal{M} = 1/\sqrt{\gamma}$)?
  - If yes, then there is a temperature (Zel'dovich) spike $\Longrightarrow T_{\max} > T_1$.
  - Lowrie & Edwards (2008) derive an algebraic expression for $T_{\max}$ that is very accurate.
- May have a hydro shock without a spike.
- May have a spike without a hydro shock.

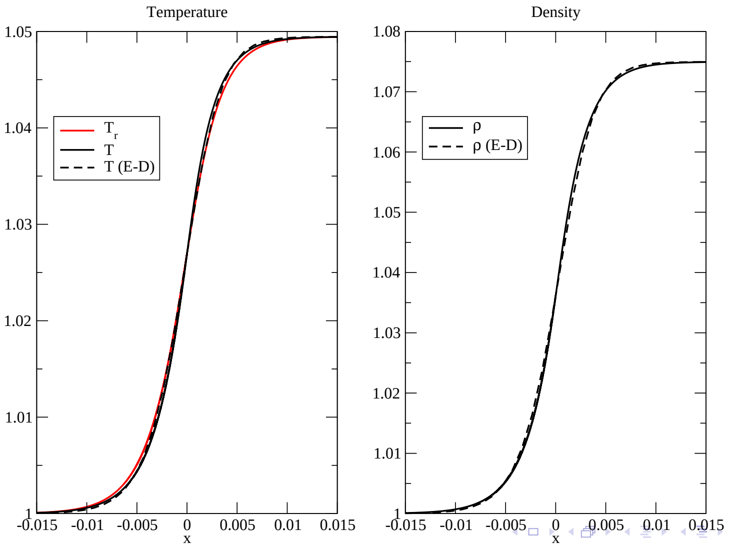
Isothermal Shock (Equilib.) ⇔ Spike (Non-equilib.)

$\gamma = 5/3$. See Lowrie & Rauenzahn (2007).



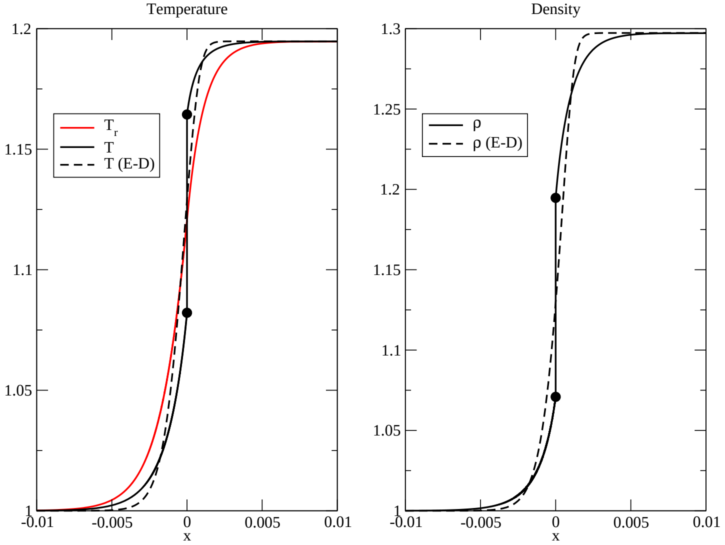
$\mathcal{M}_0 = 1.05$ solution

$\mathcal{P}_0 = 10^{-4}$, $\sigma_a = 10^6$, $\kappa = 1$, $\gamma = 5/3$; subcritical; no embedded shock or isothermal sonic point (ISP).



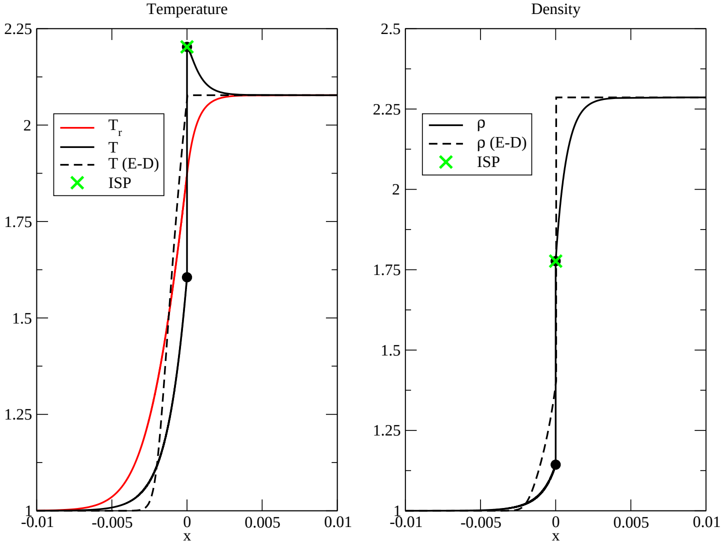
$\mathcal{M}_0 = 1.2$ solution

Subcritical; shock, but no ISP.



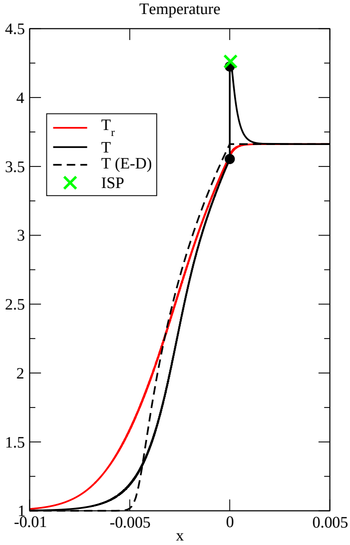
$\mathcal{M}_0 = 2$ solution

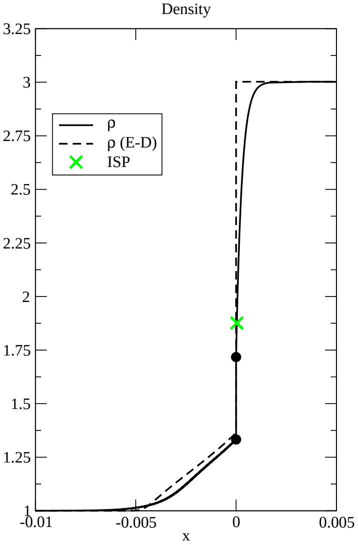
Subcritical; ISP coincident with shock.



$\mathcal{M}_0 = 3$ solution

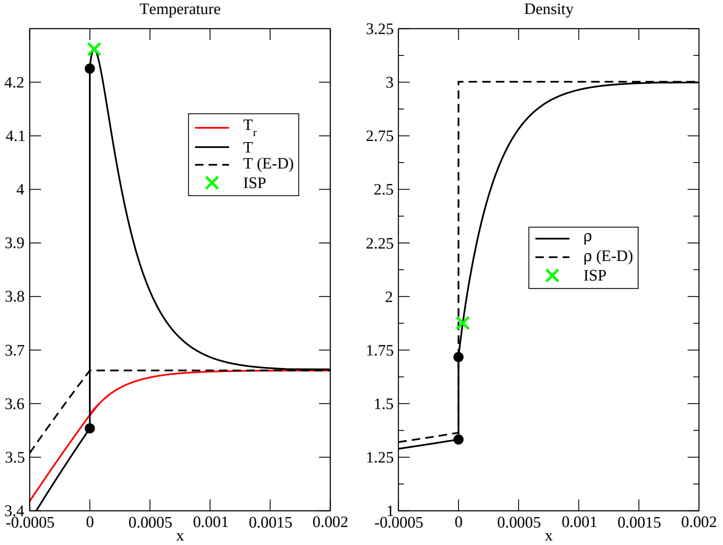
Subcritical; ISP downstream of shock.





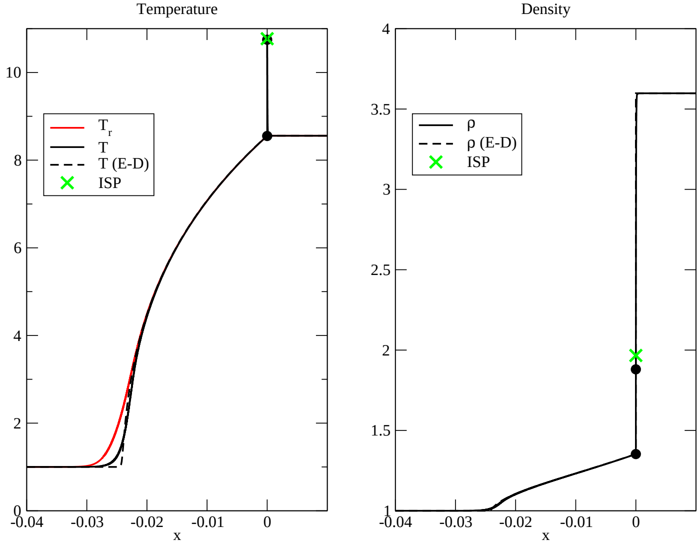
$\mathcal{M}_0 = 3$ solution

Spike region.



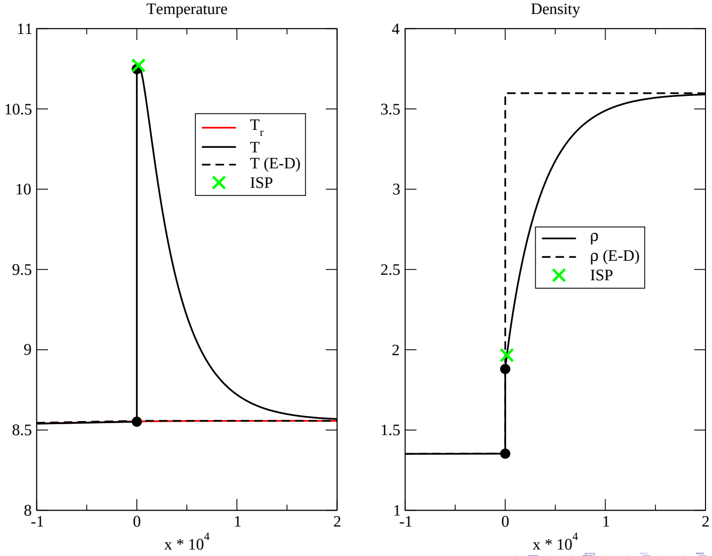
$\mathcal{M}_0 = 5$ solution

Supercritical; ISP downstream of shock.



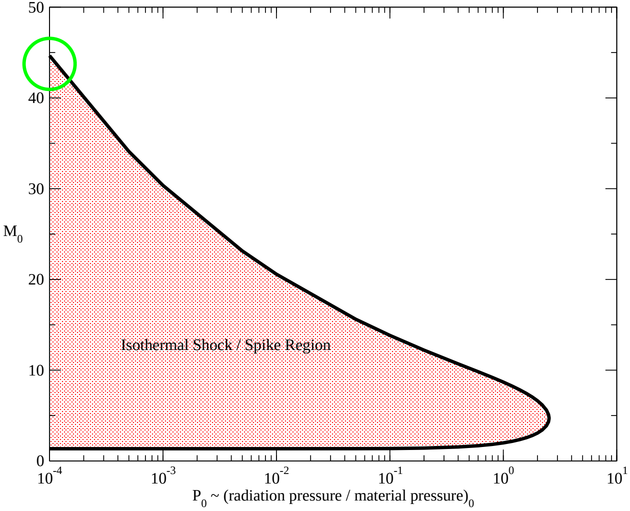
$\mathcal{M}_0 = 5$ solution (continued)

Zel'dovich spike region



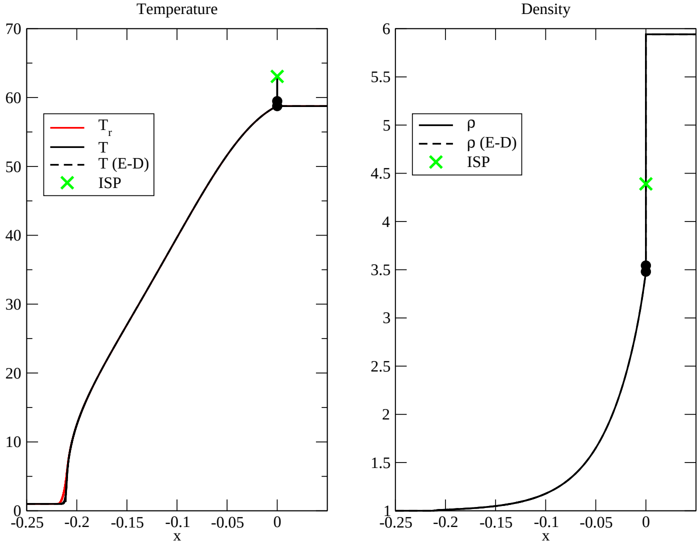
Isothermal Shock (Equilib.) $\Leftrightarrow$ Spike (Non-equilib.)

$\gamma = 5/3$. See Lowrie & Rauenzahn (2007).



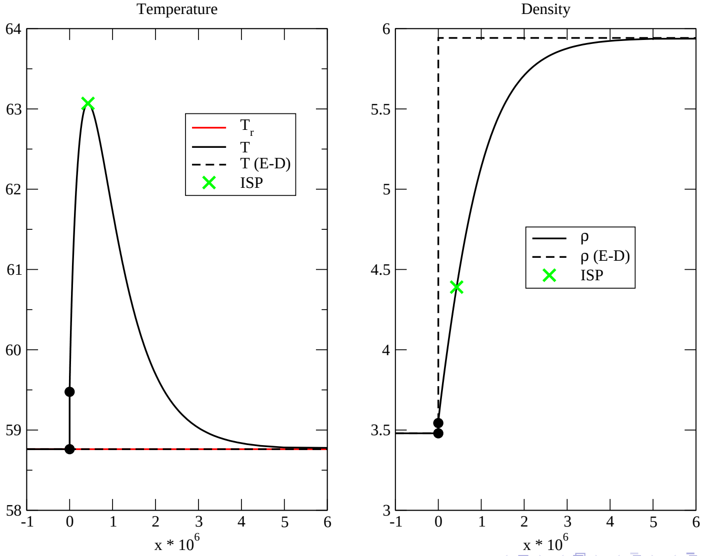
$\mathcal{M}_0 = 27$ solution

Supercritical; ISP downstream of shock. NOTE OVER COMPRESSION.



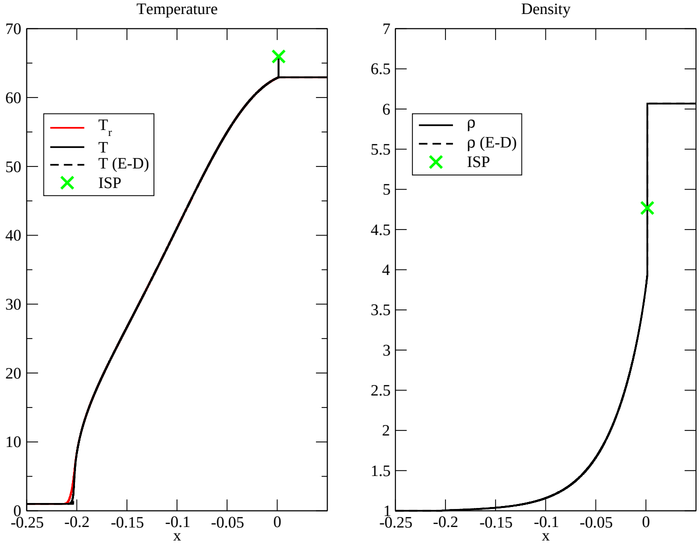
$\mathcal{M}_0 = 27$ solution (continued)

Zel'dovich spike region



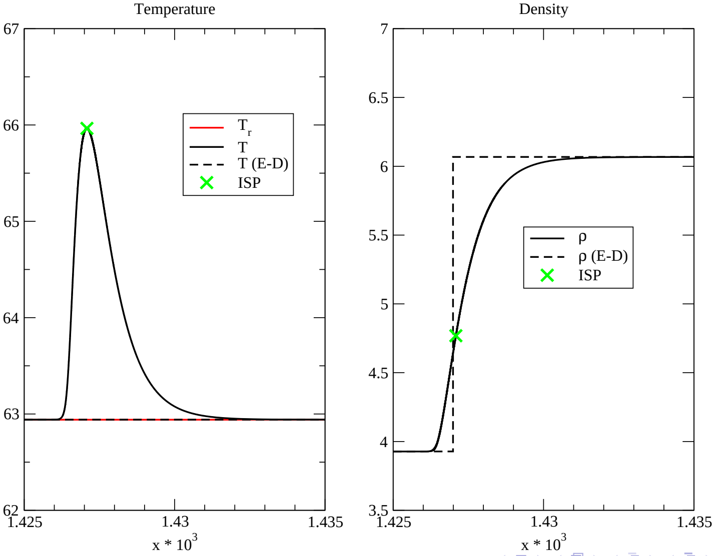
$\mathcal{M}_0 = 30$ solution

No shock, but still an ISP!



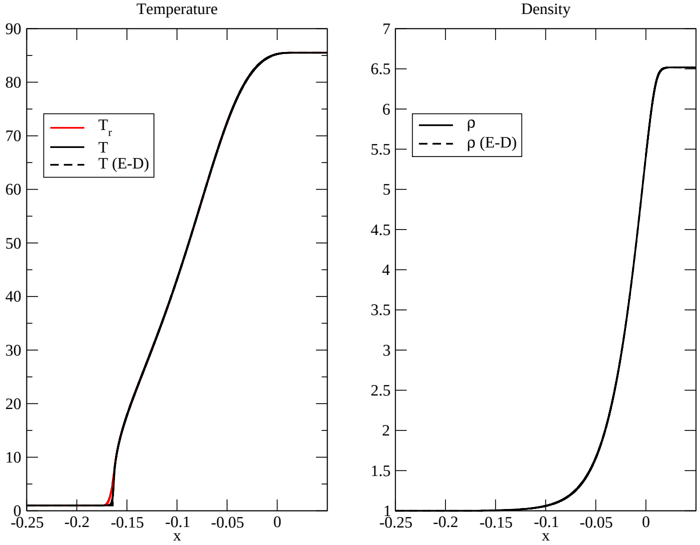
$\mathcal{M}_0 = 30$ solution (continued)

Spike region



$\mathcal{M}_0 = 50$ solution

No shock or ISP



Outline

Code Comparison Test Cases

- Hydrogen gas, $\gamma = 5/3$
- Bremsstrahlung absorption model (Zel'dovich & Razier; assume fully ionized):

$$\sigma_a(\rho, T) = \sigma_{a,0}\rho^2 T^{-7/2}$$

- Thomson scattering:

$$\sigma_s(\rho) = \sigma_{s,0}\rho$$

- $\rho_0 = 1$ g/cc
- Cases:
  1. $T_0 = 10$ eV, $\mathcal{M}_0 = 10$, subcritical
  2. $T_0 = 100$ eV, $\mathcal{M}_0 = 5$, supercritical
  3. $T_0 = 100$ eV, $\mathcal{M}_0 = 45$, no embedded hydro shock
- Comparison of relaxation rates shows that 3T effects should be small.
- Compare with a finite-volume, Godunov-based AMR code (RAGE).

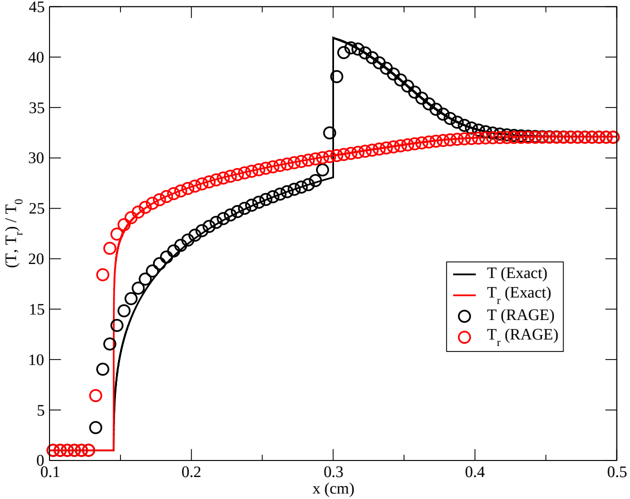
Two Initialization Methods

1. Initialize with exact shock profile.
   - How well can the code propagate the shock and maintain the profile?
   - Pros: Boundary conditions not an issue.
   - Cons: Requires code initialize from exact solution.
   - This method was used for all error-norm calculations.
2. Piston problem.
   - Black-body piston moving at v_1, radiating at $T_r = T_1$.
   - Given enough time, the resulting shock should match the semi-analytic solution.
   - Pros: Easy setup; better test.
   - Cons: Requires "large" solution domain; boundary conditions become an issue.

With either method, you should propagate the shock long enough to establish a self-similar profile.

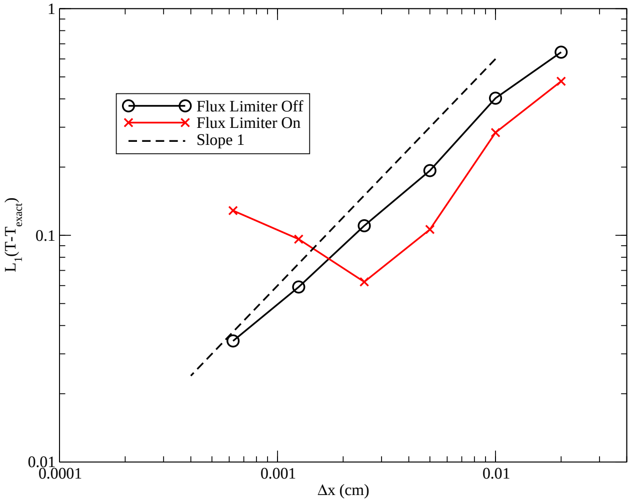
$T_0 = 10$ eV, $\mathcal{M}_0 = 10$ Sample Results

$\Delta x = 50\ \mu$m, propagated 1 cm, $T_1 \approx 321$ eV, $T_{max} \approx 419$ eV



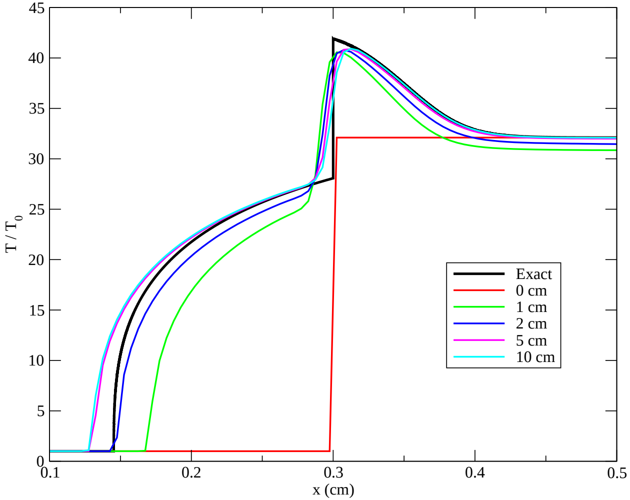
$T_0 = 10$ eV, $\mathcal{M}_0 = 10$ Error Convergence

Exact solution computed without flux limiter.

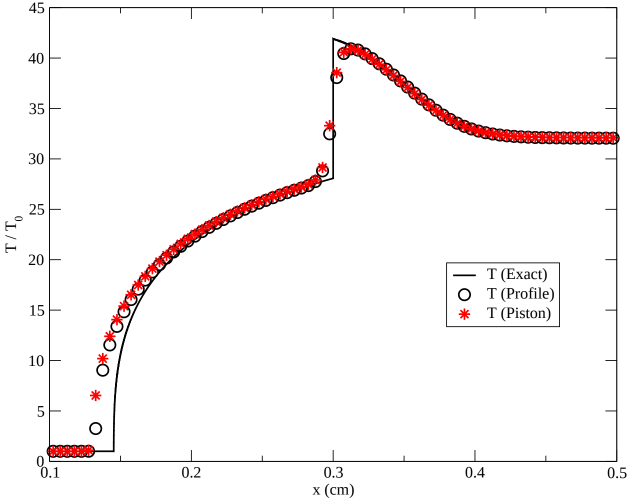


$T_0 = 10$ eV, $\mathcal{M}_0 = 10$, Piston Problem Initialization

Legend indicates distance exact profile has moved. Solutions shifted to align.
$\Delta x = 50\ \mu$ m.

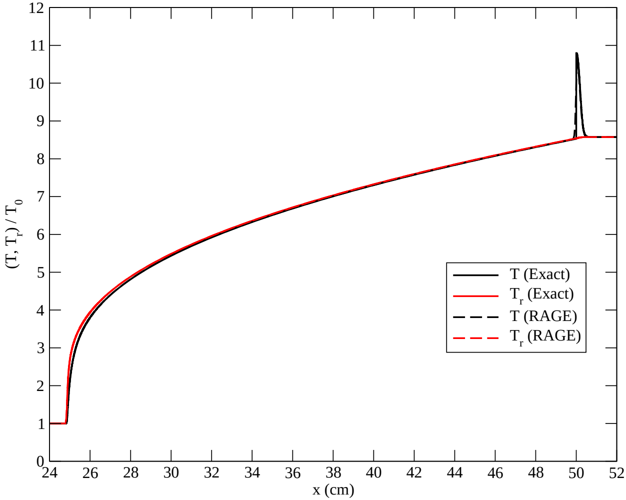


Profile vs. Piston Initialization

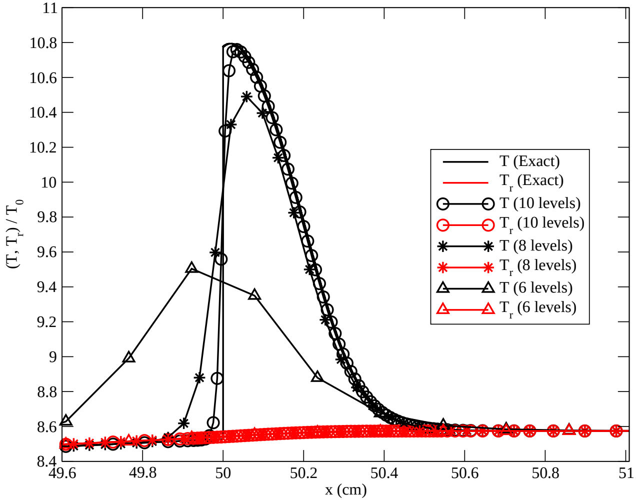


$T_0 = 100$ eV, $\mathcal{M}_0 = 5$ Sample Results

AMR (8 levels): $\Delta x_{max} = 5$ cm, $\Delta x_{min} \approx 391\ \mu$m, propagated 230 cm, $T_1 \approx 857$ eV, $T_{max} \approx 1.08$ keV

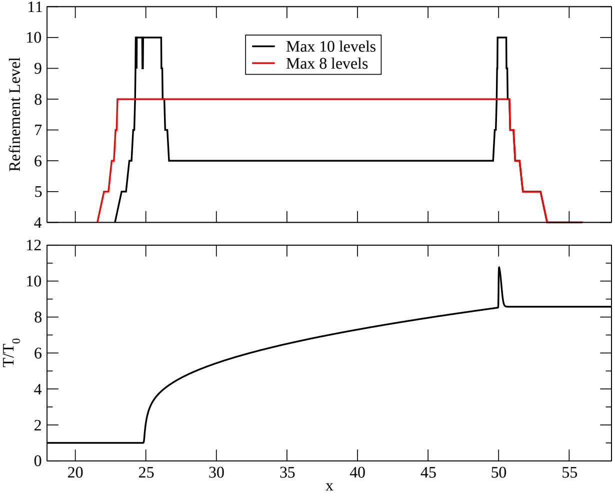


$T_0 = 100$ eV, $\mathcal{M}_0 = 5$ Spike Region



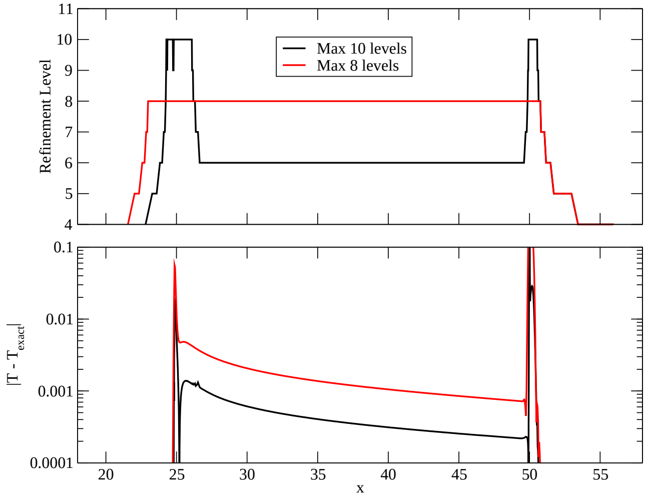
$T_0 = 100$ eV, $\mathcal{M}_0 = 5$ Refinement Distribution

8 levels $\Longrightarrow$ 814 cells; 10 levels $\Longrightarrow$ 528 cells (10-level mesh derefined in precursor).



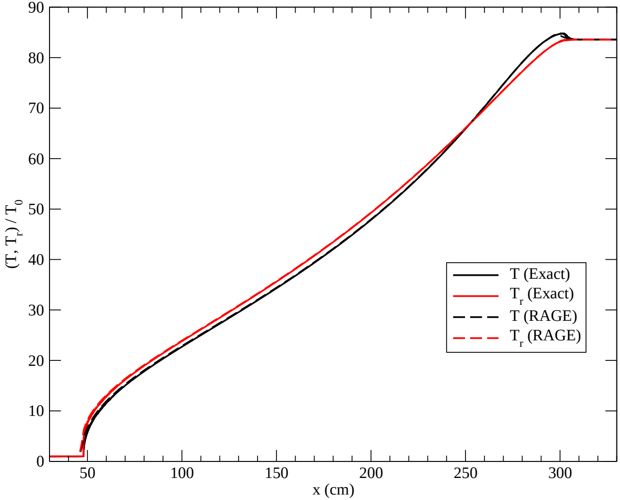
Refinement and Error Distribution

Even though 10-level mesh derefined in $27 \lessapprox x \lessapprox 49$, it is more accurate.



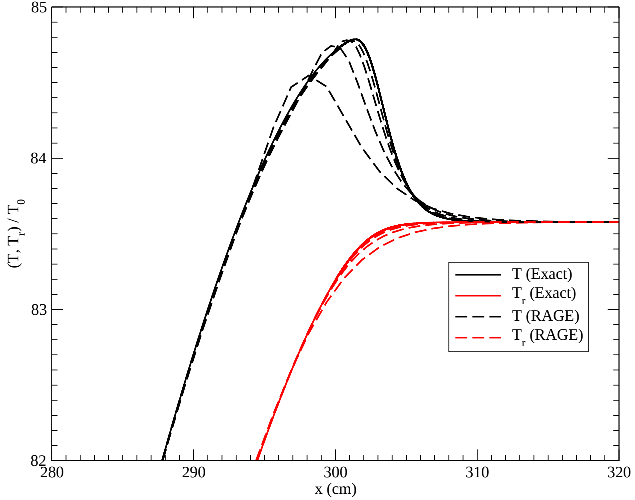
$T_0 = 100$ eV, $\mathcal{M}_0 = 45$ Sample Results

$\Delta x = 1.25$ cm, propagated 20 m, $T_1 \approx 8.36$ keV, $T_{max} \approx 8.48$ keV



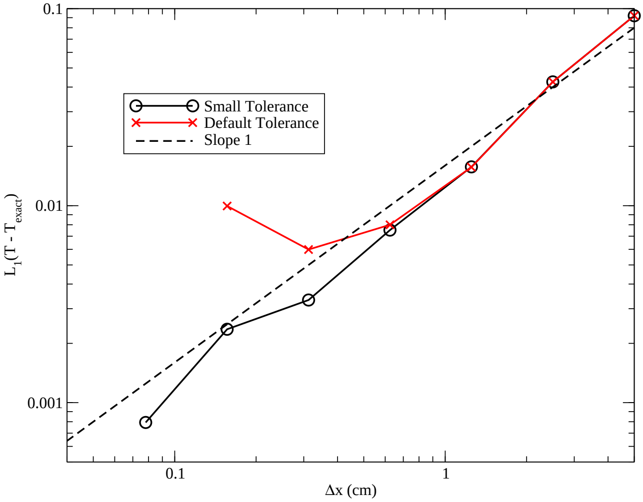
$T_0 = 100$ eV, $\mathcal{M}_0 = 45$ Spike Region

RAGE results with $\Delta x = 1.25 \text{ cm}/2^N$, with $N = 0, 1, 2, 3$



$T_0 = 100$ eV, $\mathcal{M}_0 = 45$ Convergence

A hard-coded tolerance was limiting the convergence rate.



Summary

- The radiative shock solutions test several issues:
  - Fully-coupled radiation hydrodynamics
  - Embedded shock problems $\Longrightarrow$ ability to capture hydro shocks
  - Smooth problems $\Longrightarrow$ ability to attain theoretical convergence rate
  - Refinement criteria for AMR (solutions are multiscale)
  - Other radiation models
- Future work
  - Separate ion/electron temperatures
  - More advanced radiation models