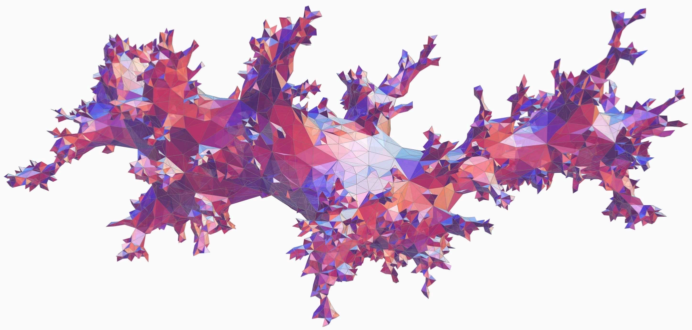
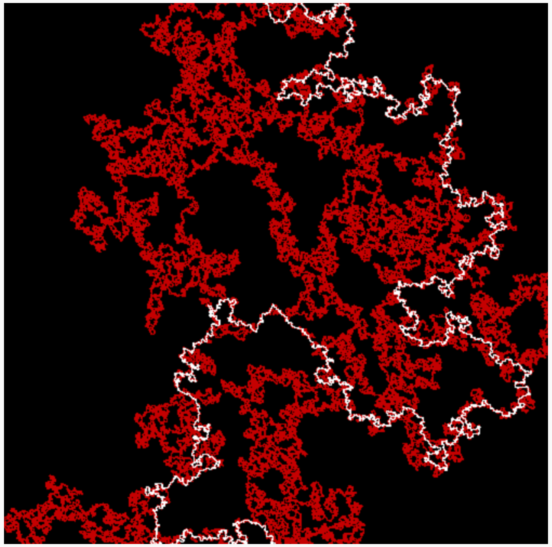
Scaling limit of loop-erased percolation interface on the uniform infinite triangulation

Yuyang Feng

January 29, 2026

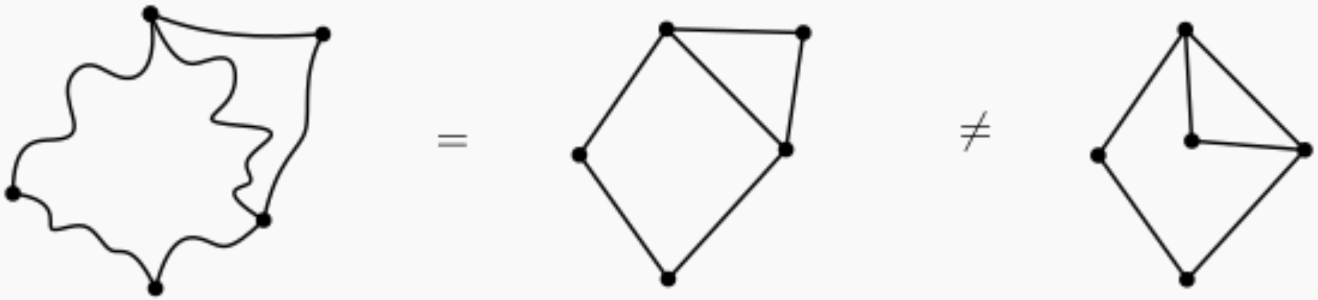
University of Chicago





Random planar map

- A planar map is a graph embedded in the Riemann surface such that no two edges cross, viewed modulo orientation-preserving homeomorphism.



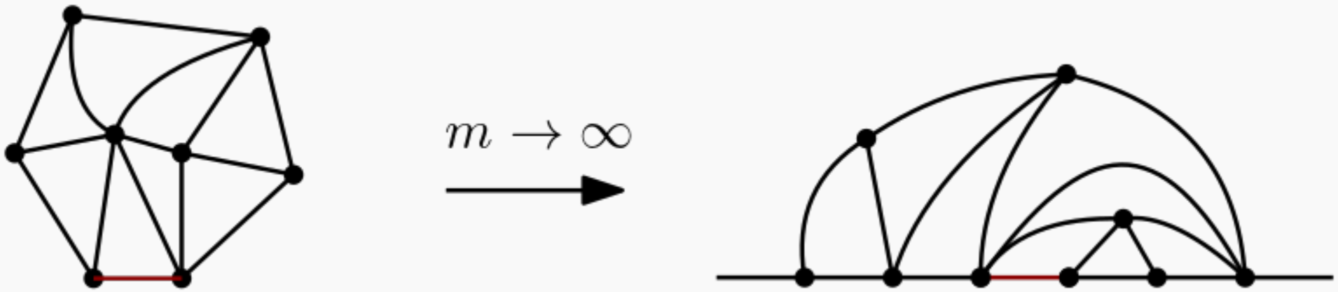
- Planar maps as metric-measure spaces: graph distance + counting measure
- What is *uniform* random planar map?

Boltzmann triangulation

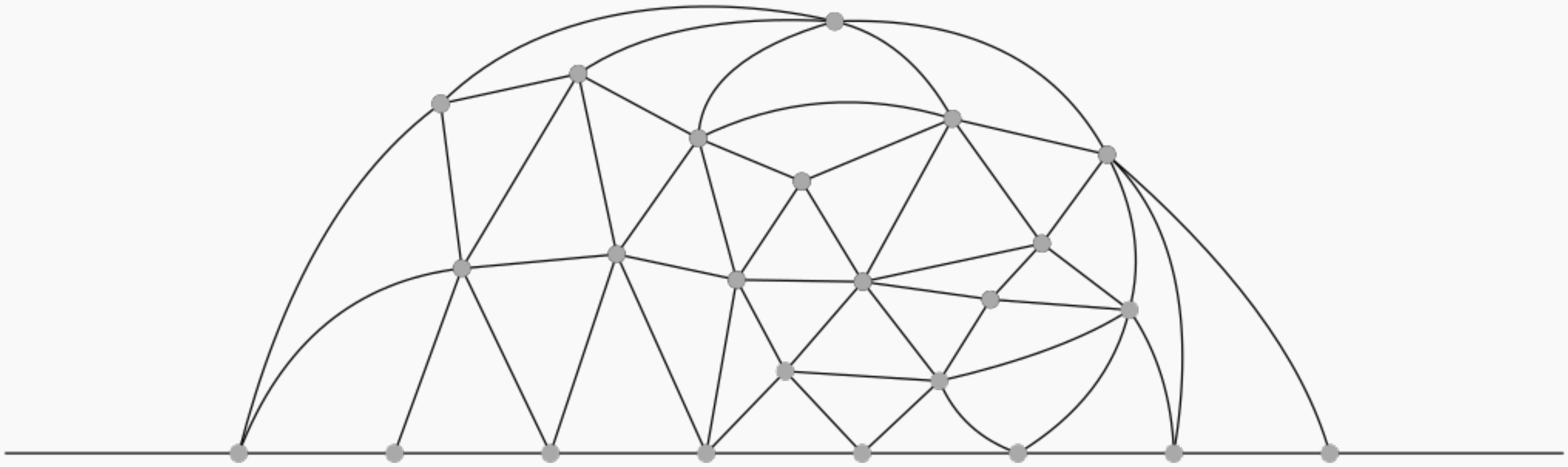
- Consider rooted triangulations in m-polygon with n vertices, there are only finitely many choices.
- (Tutte '62) Number of such maps $\varphi_{n,m} \sim c_m \alpha^n n^{-5/2}$.
- **Boltzmann measure** on rooted triangulations of m-polygon: For rooted triangulation T in m-polygon with n vertices,

$$\mu_m(T) = \frac{\alpha^{-n}}{Z_m}.$$

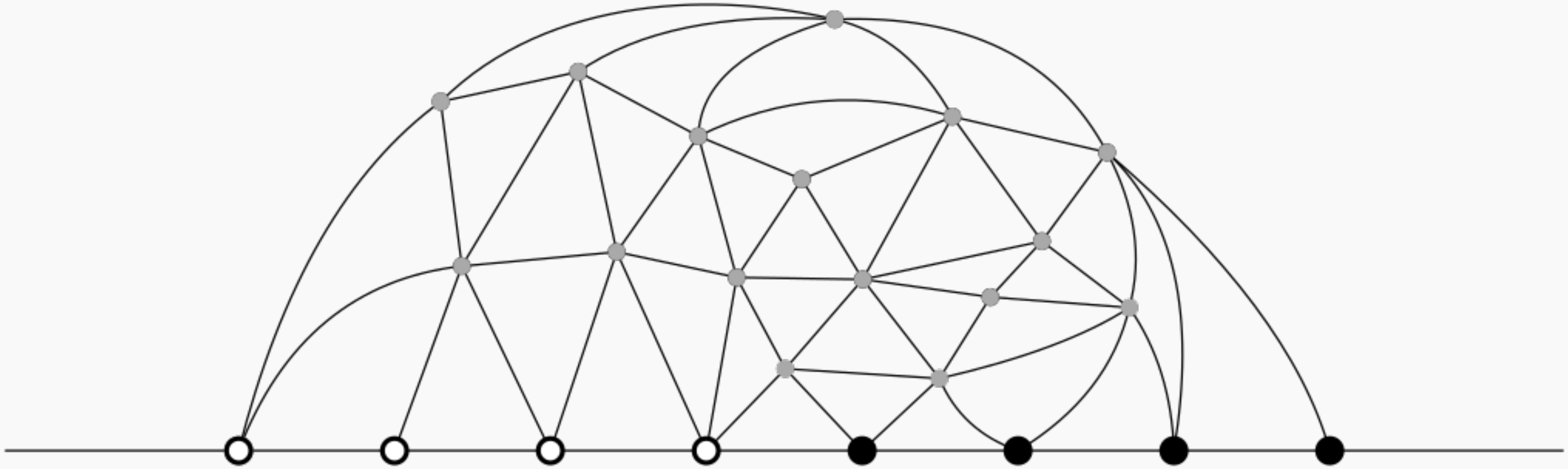
  Here, $Z_m = \sum_n \varphi_{n,m} \alpha^{-n}$ is the partition function.
- (Angel '04) The Benjamini-Schramm local limit of μ_m is called Uniform Infinite Half-Planar Triangulation (**UIHPT**).



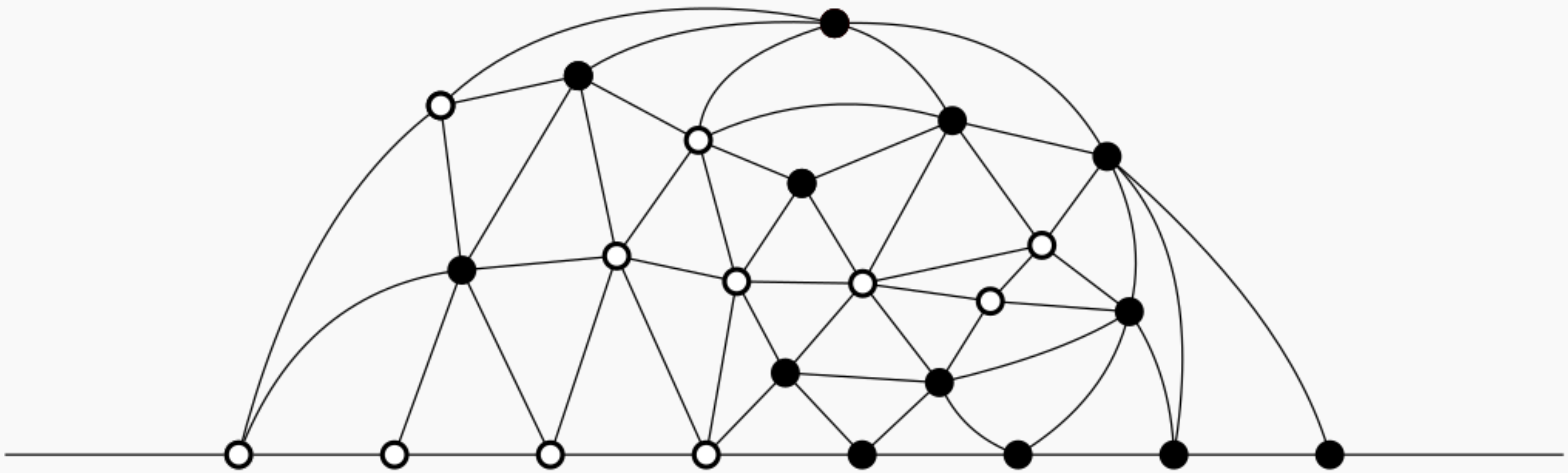
Uniform infinite half-planar triangulation (UIHPT)



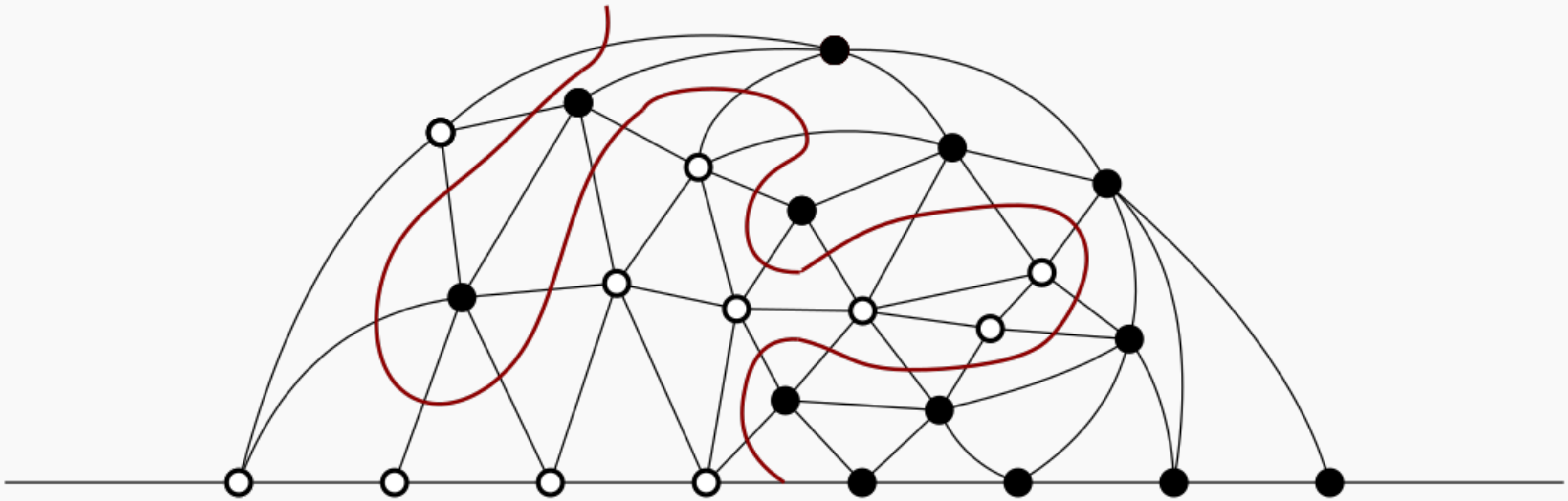
Percolation on UIHPT



Percolation on UIHPT



Percolation interface

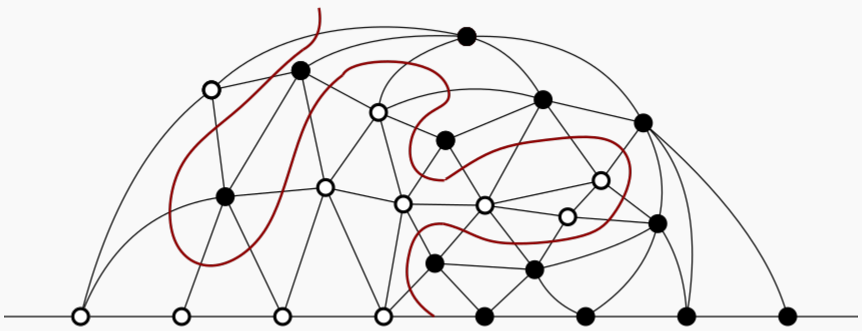


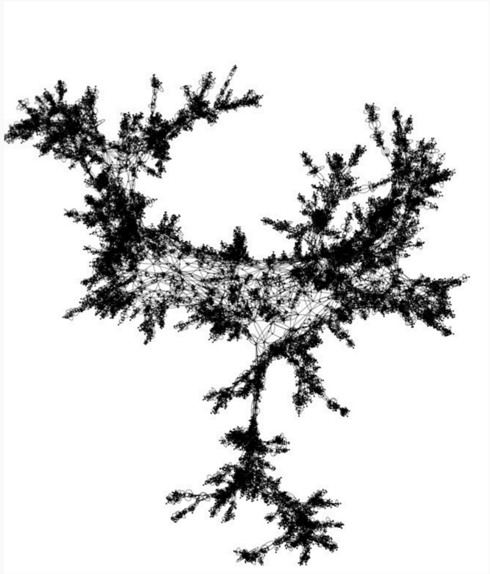
Known result: Scaling limit

Theorem

After appropriate scaling, as curve-decorated metric-measure spaces,

$$\text{UIHPT} + \text{percolation interface} \to \sqrt{8/3} - \text{LQG} \quad + \quad \text{SLE}_6$$





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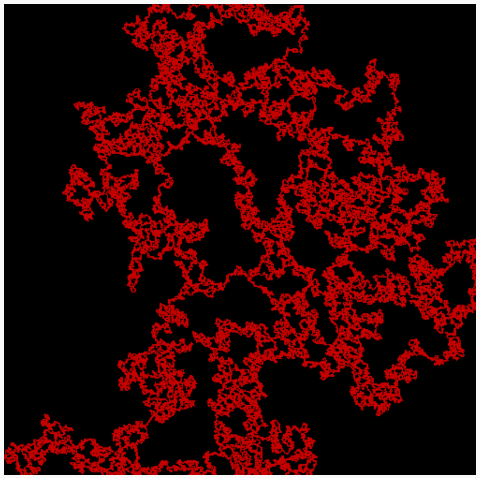

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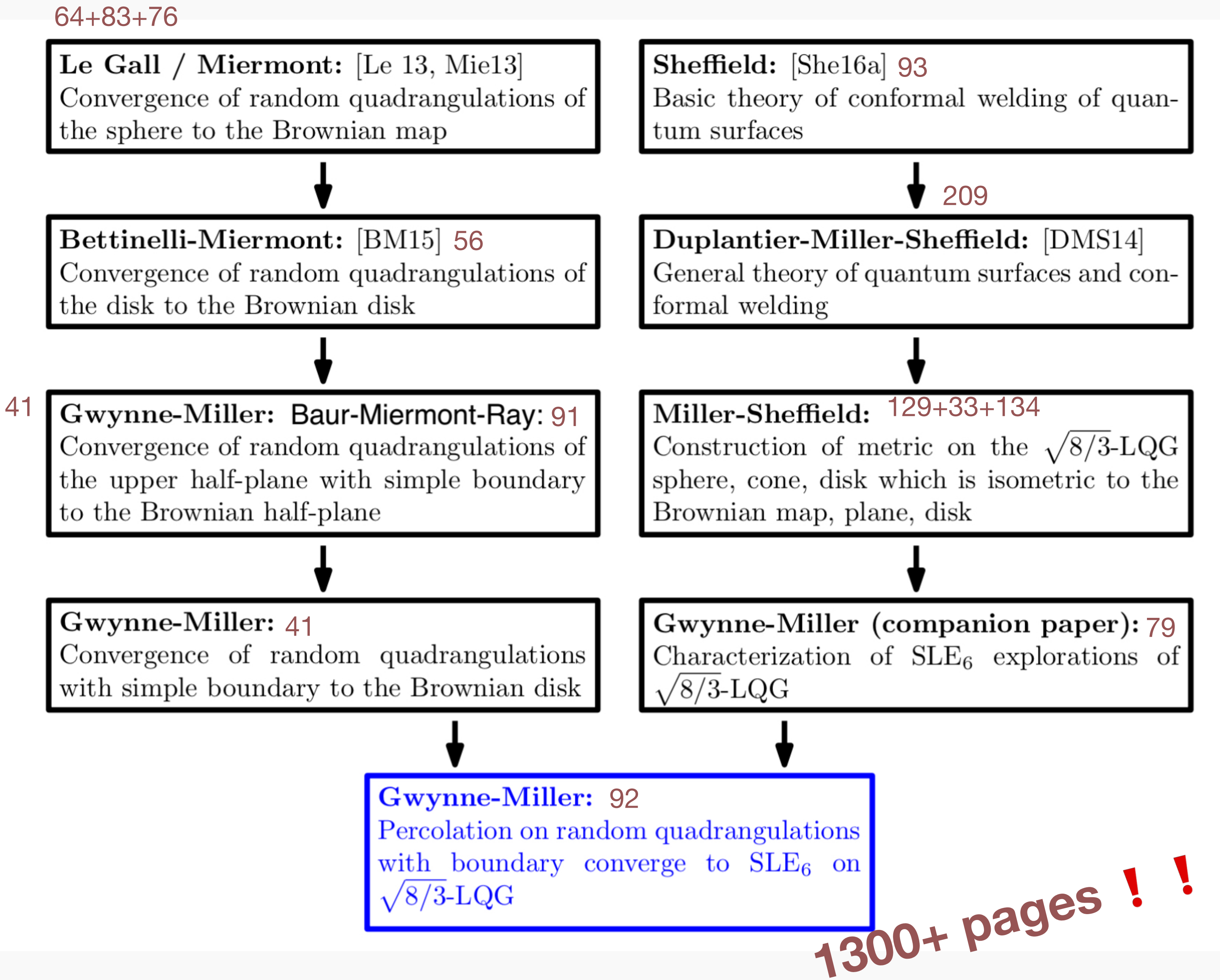
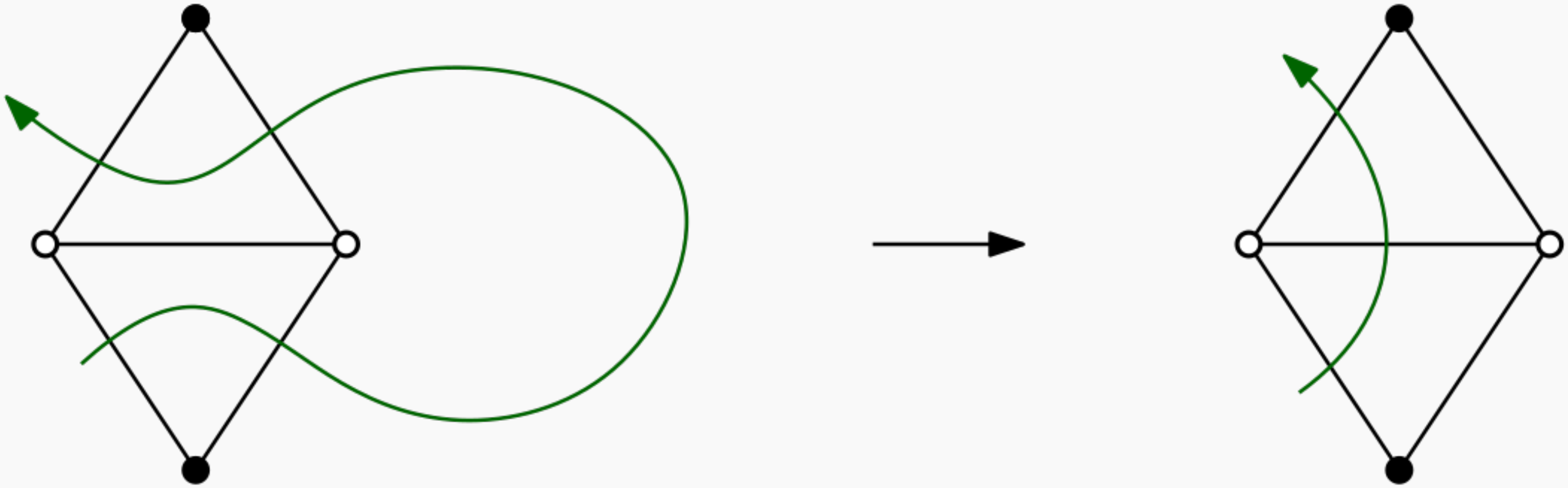

64+83+76
Le Gall / Miermont: [Le 13, Mie13]
Convergence of random quadrangulations of the sphere to the Brownian map
Sheffield: [She16a] 93
Basic theory of conformal welding of quantum surfaces
209
Bettinelli-Miermont: [BM15] 56
Convergence of random quadrangulations of the disk to the Brownian disk
Duplantier-Miller-Sheffield: [DMS14]
General theory of quantum surfaces and conformal welding
41
Gwynne-Miller: Baur-Miermont-Ray: 91
Convergence of random quadrangulations of the upper half-plane with simple boundary to the Brownian half-plane
Miller-Sheffield: 129+33+134
Construction of metric on the $\sqrt{8/3}$-LQG sphere, cone, disk which is isometric to the Brownian map, plane, disk
Gwynne-Miller: 41
Convergence of random quadrangulations with simple boundary to the Brownian disk
Gwynne-Miller (companion paper): 79
Characterization of SLE_6 explorations of $\sqrt{8/3}$-LQG
Gwynne-Miller: 92
Percolation on random quadrangulations with boundary converge to SLE_6 on $\sqrt{8/3}$-LQG
1300+ pages ! !

Loop erasure of percolation interface



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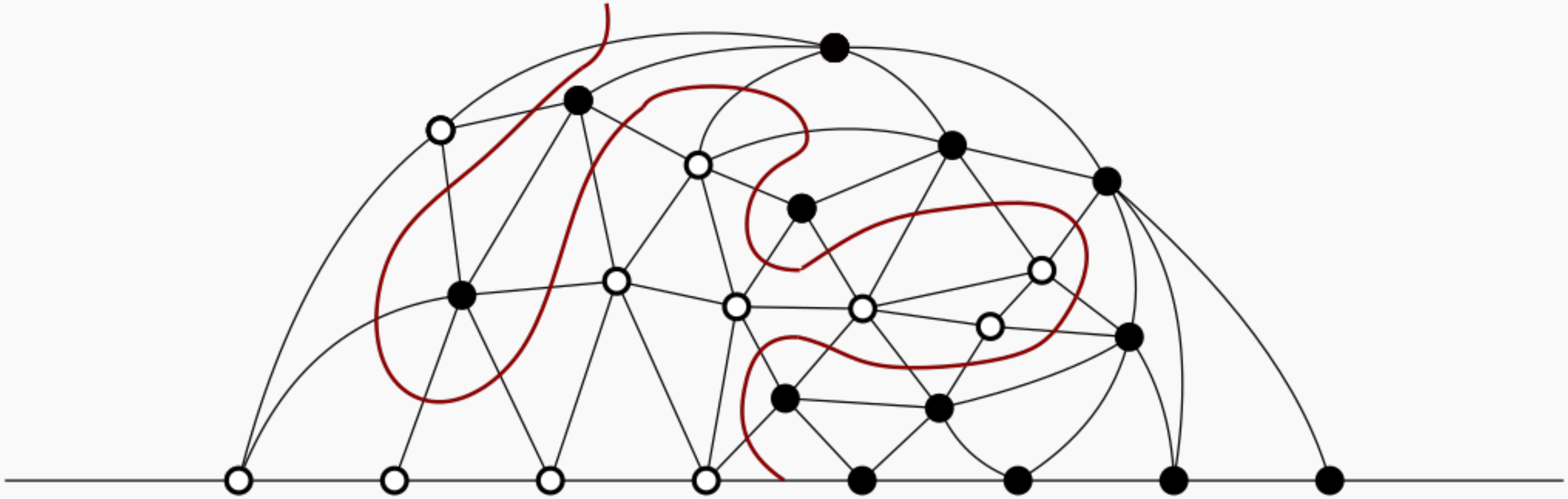

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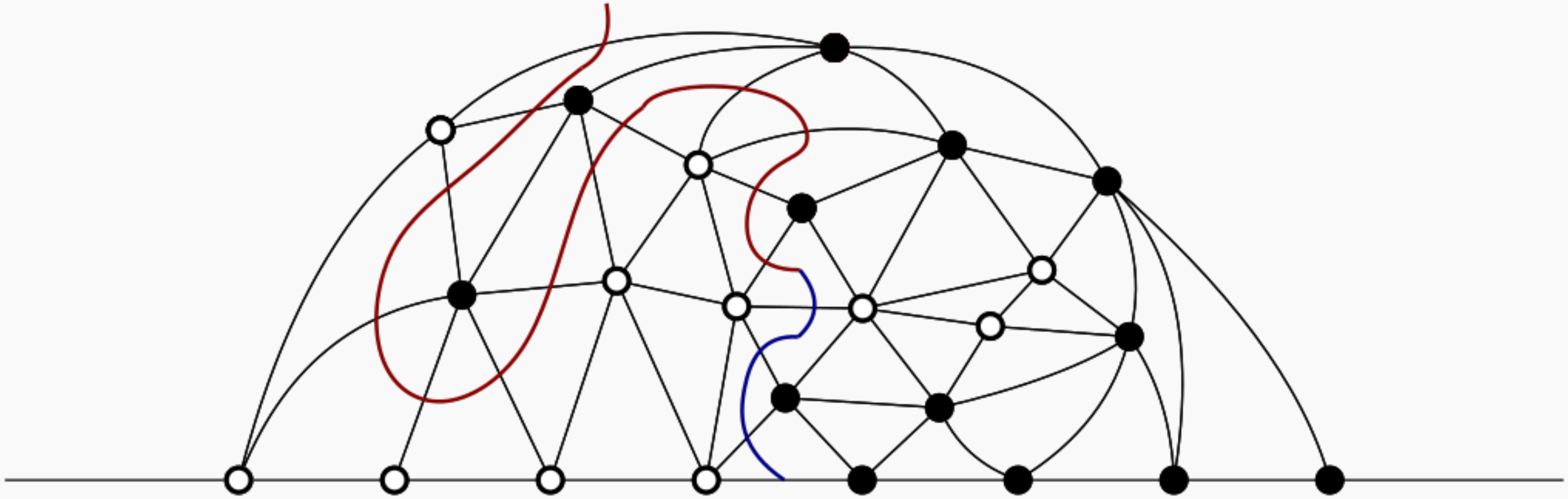

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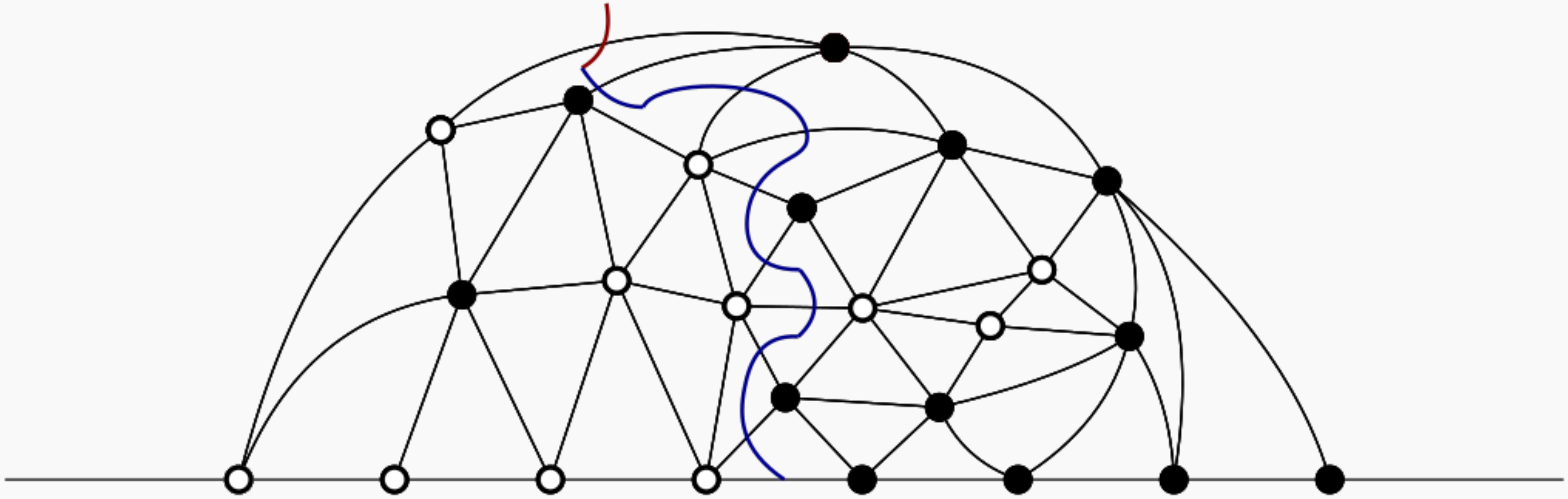
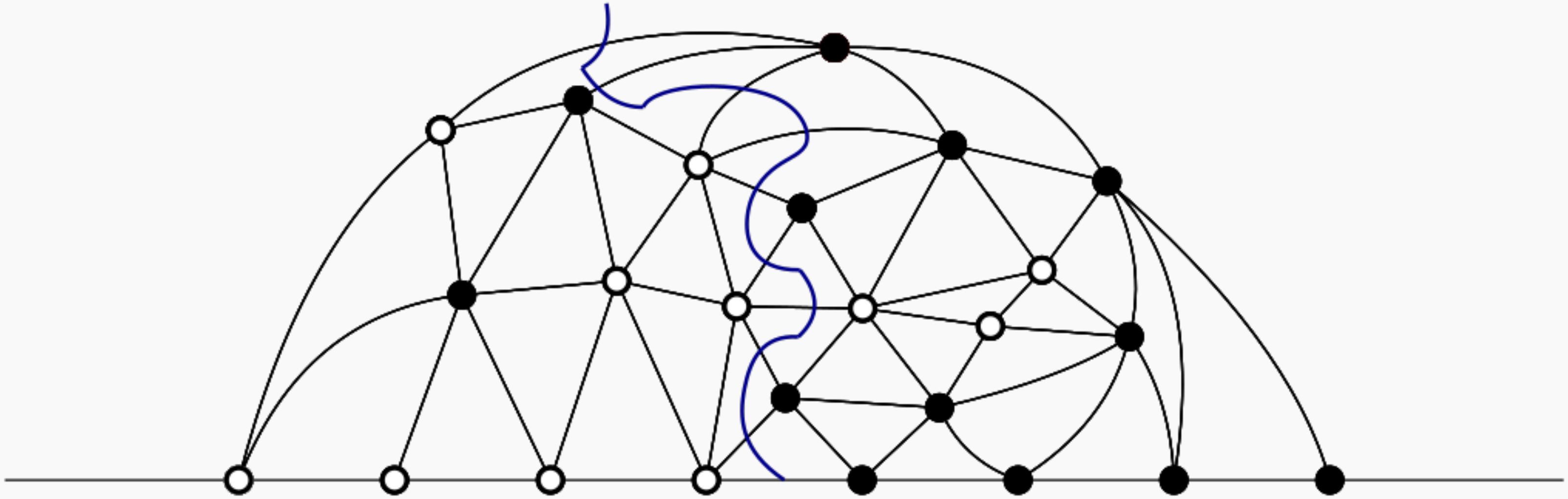
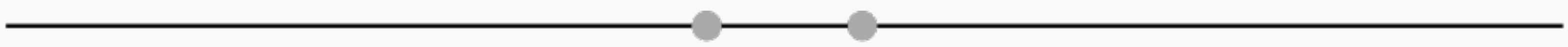


Loop erasure of percolation interface



Main tool: Peeling



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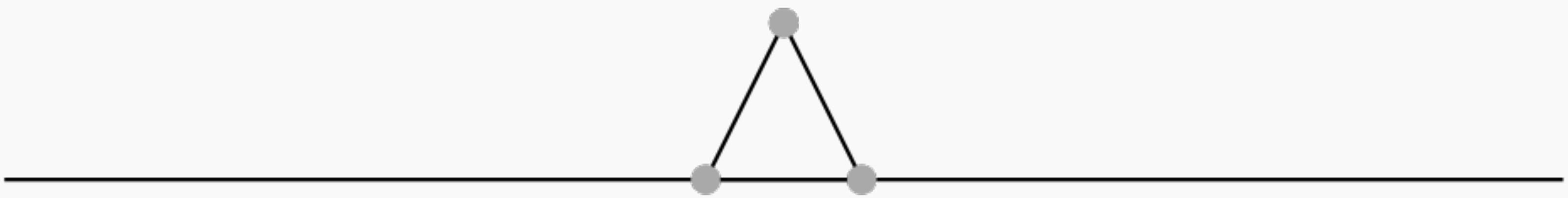

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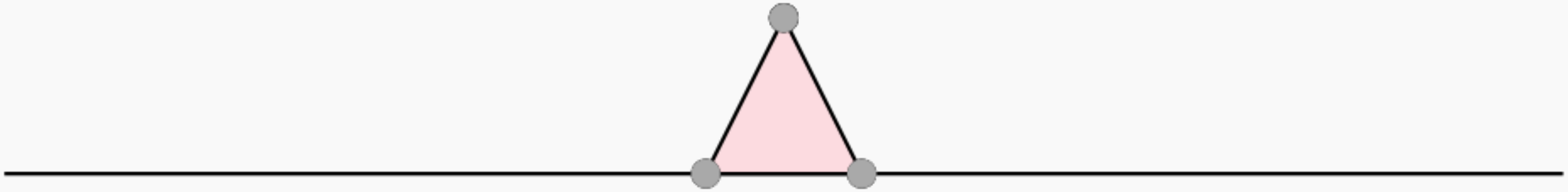

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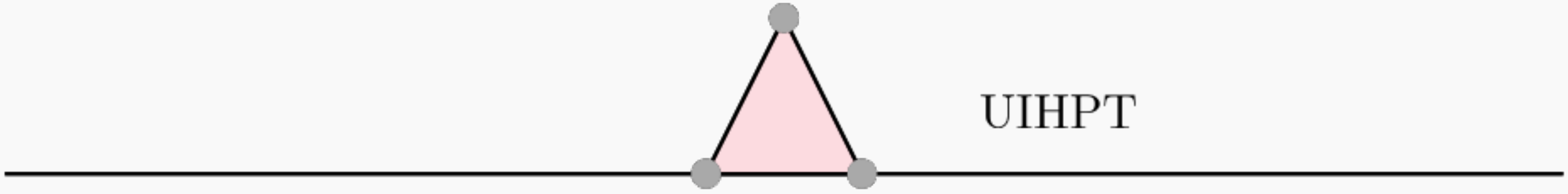

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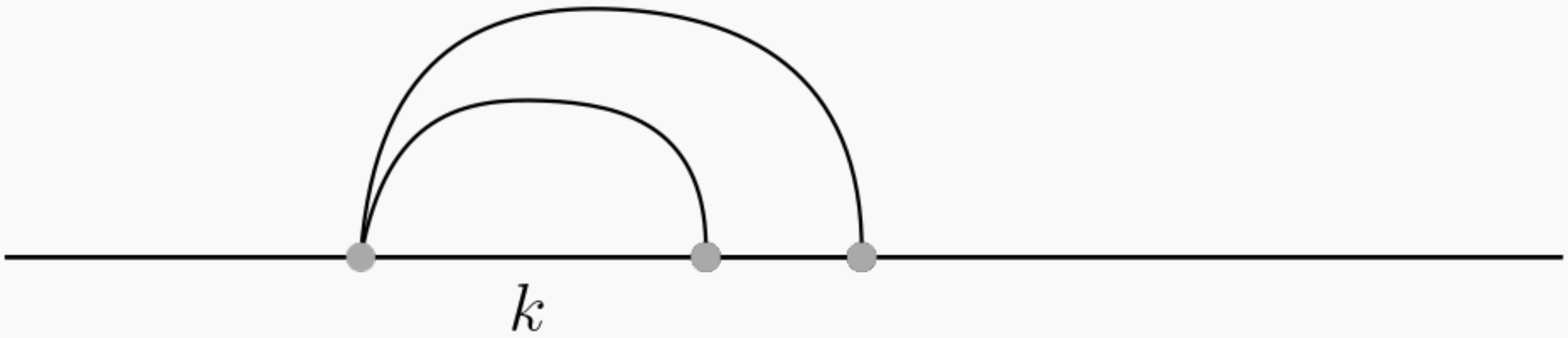

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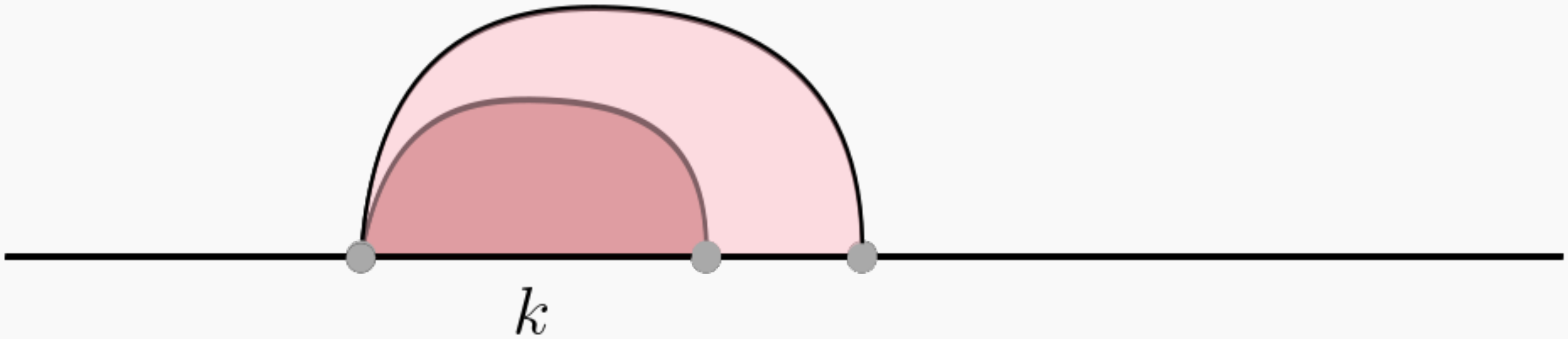

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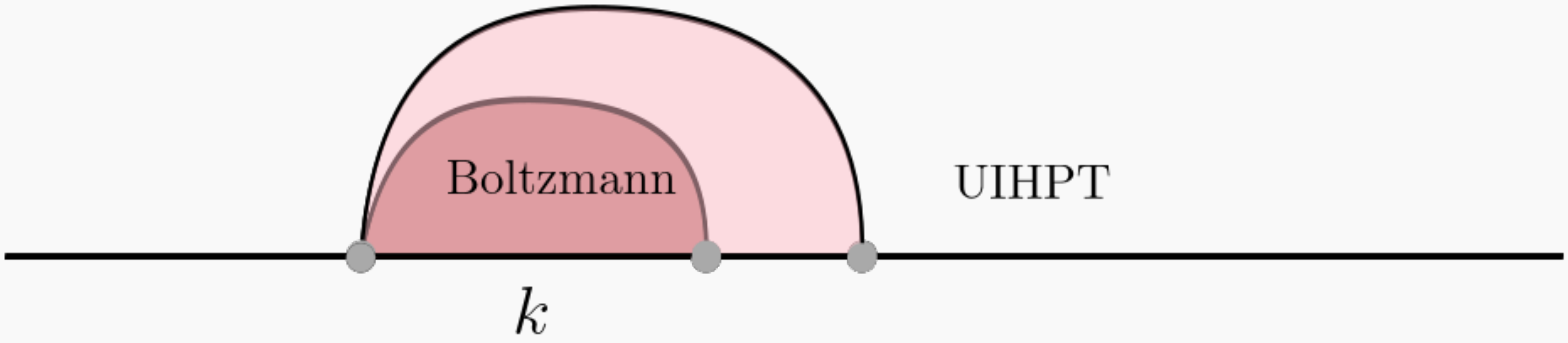

Peeling algorithm for loop erasure

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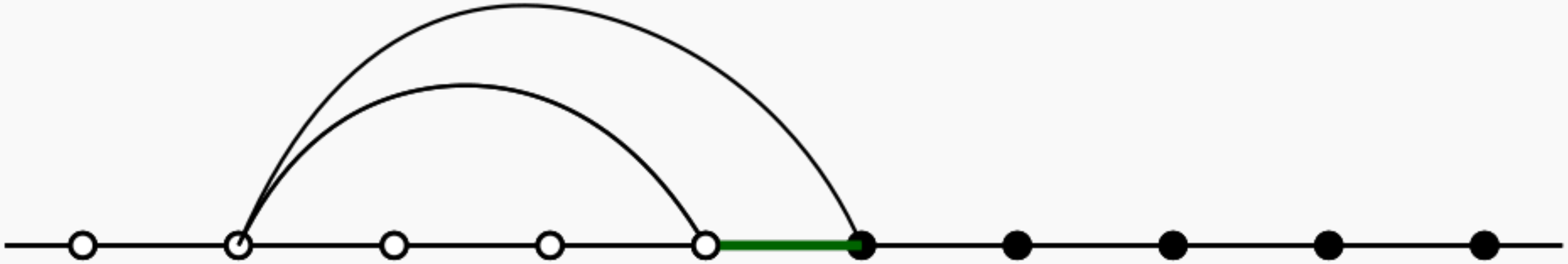

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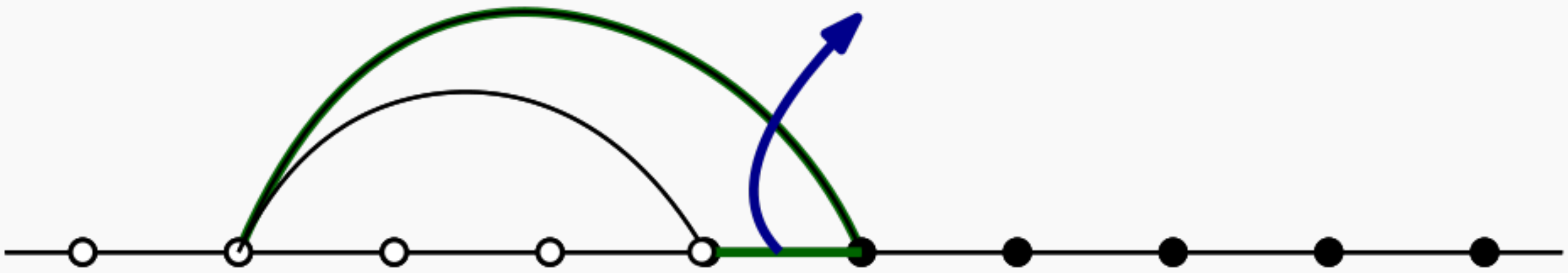

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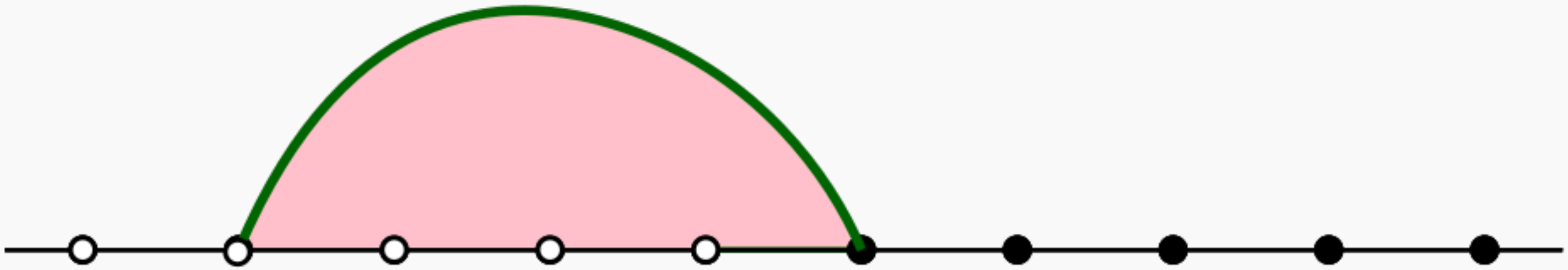

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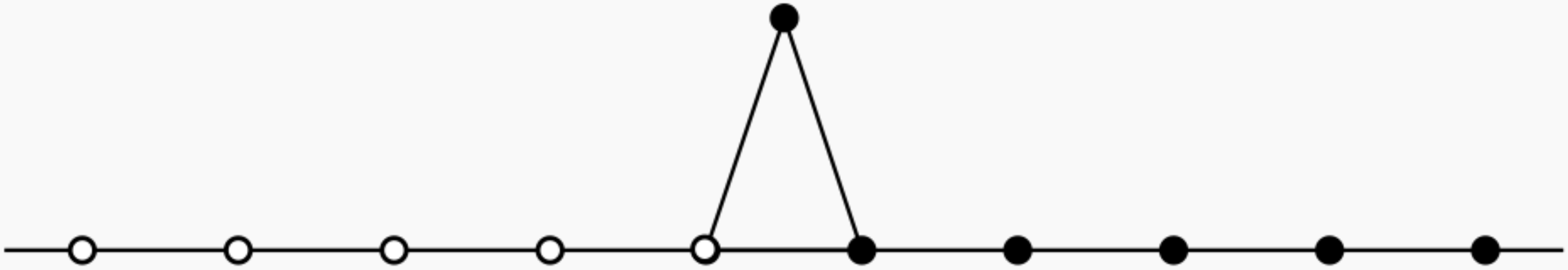

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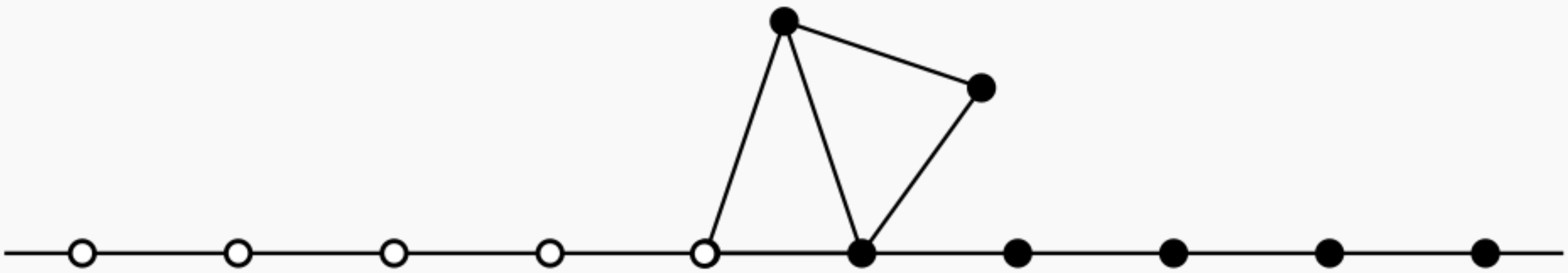

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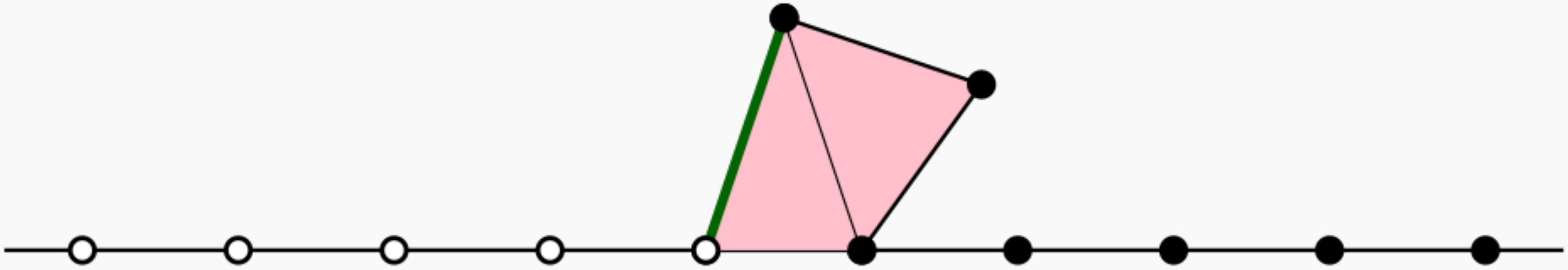

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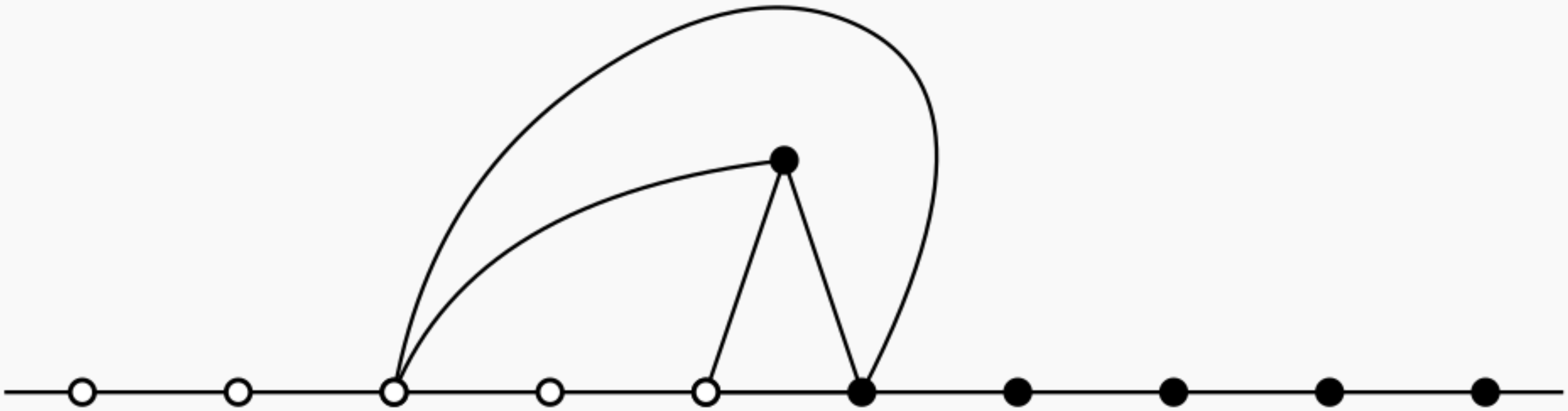

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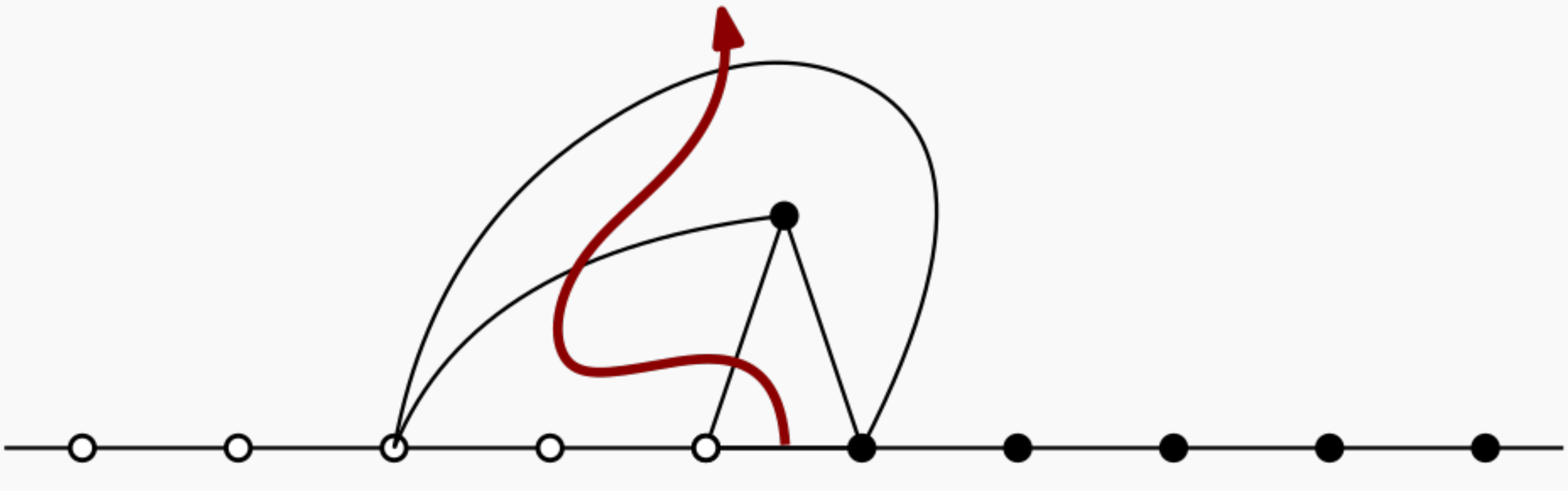

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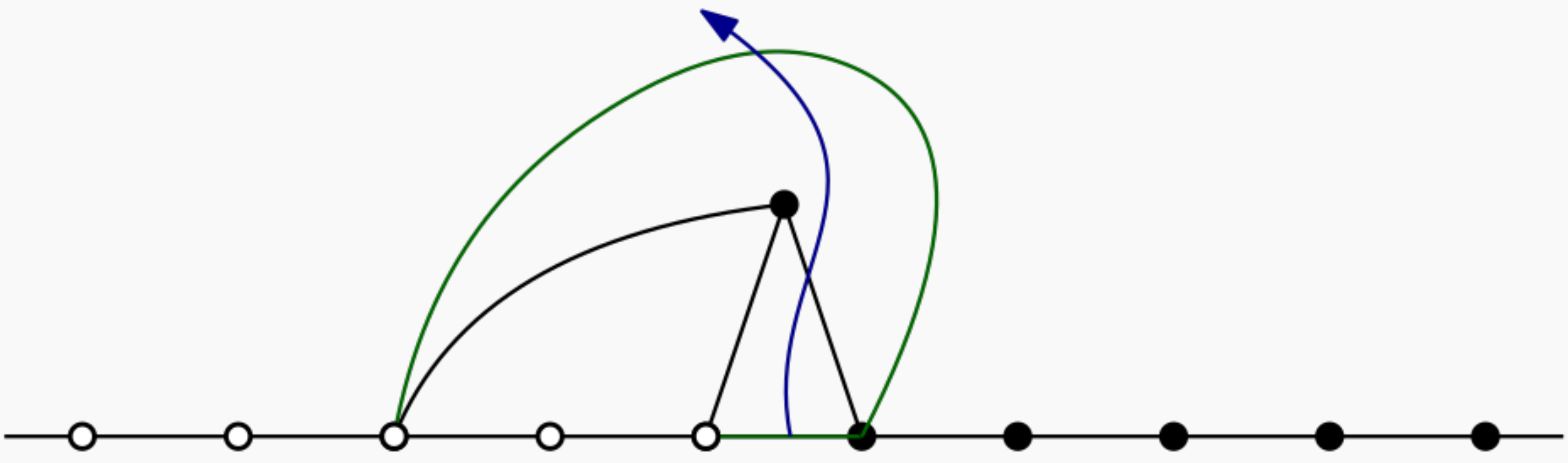

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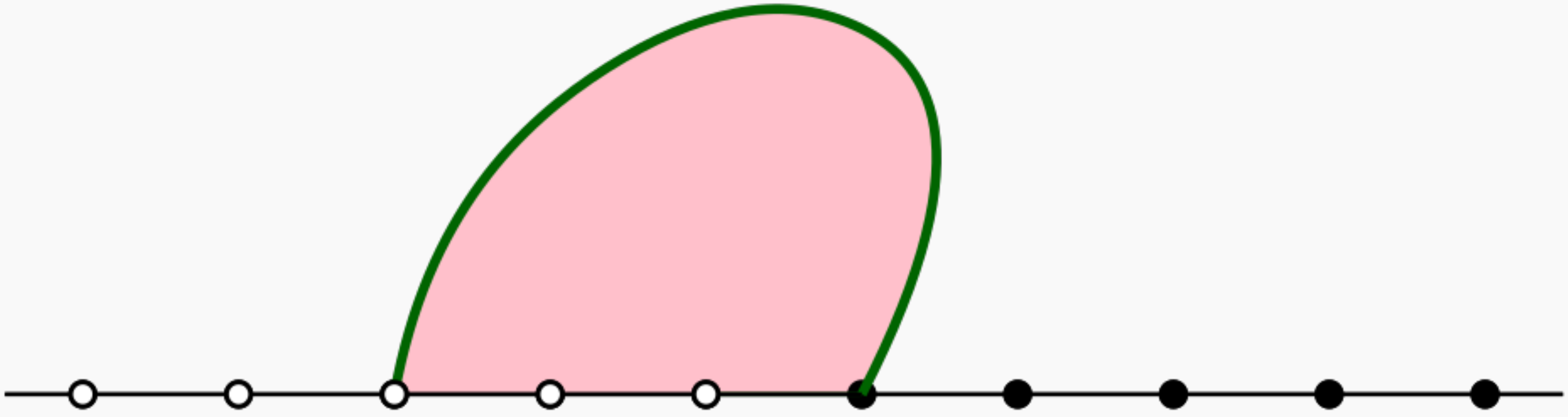

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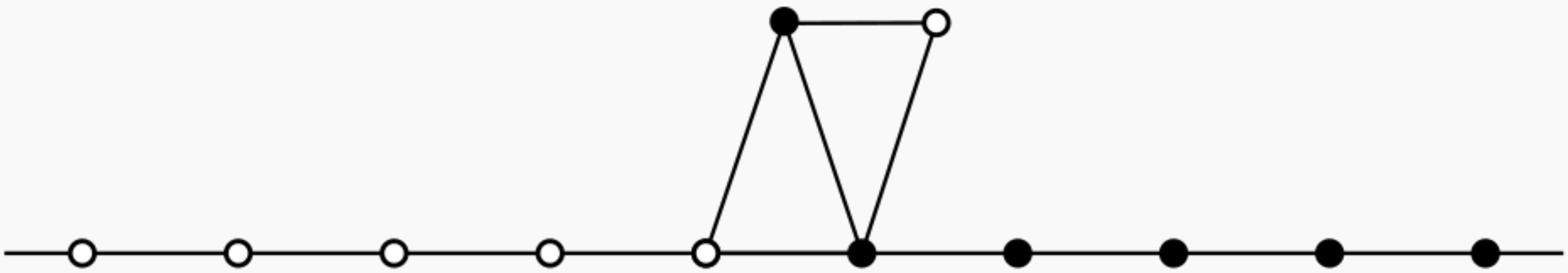

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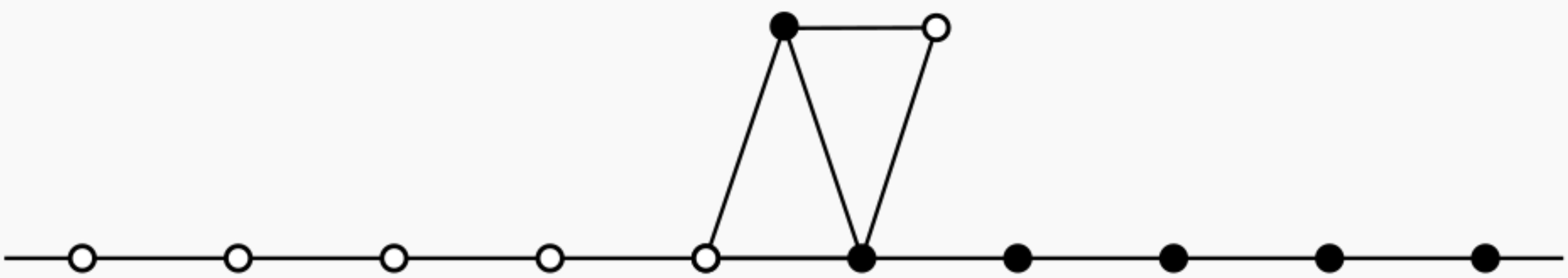

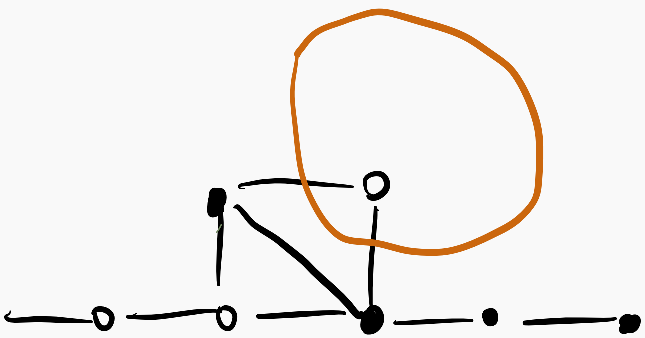

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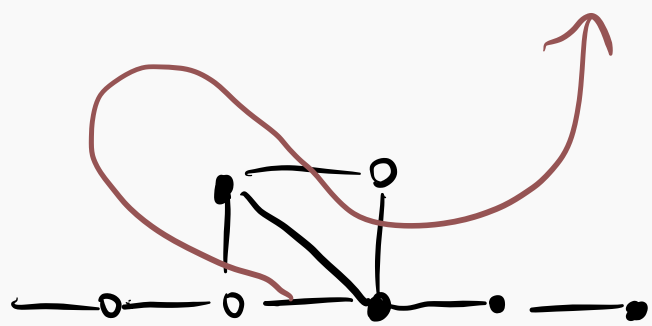

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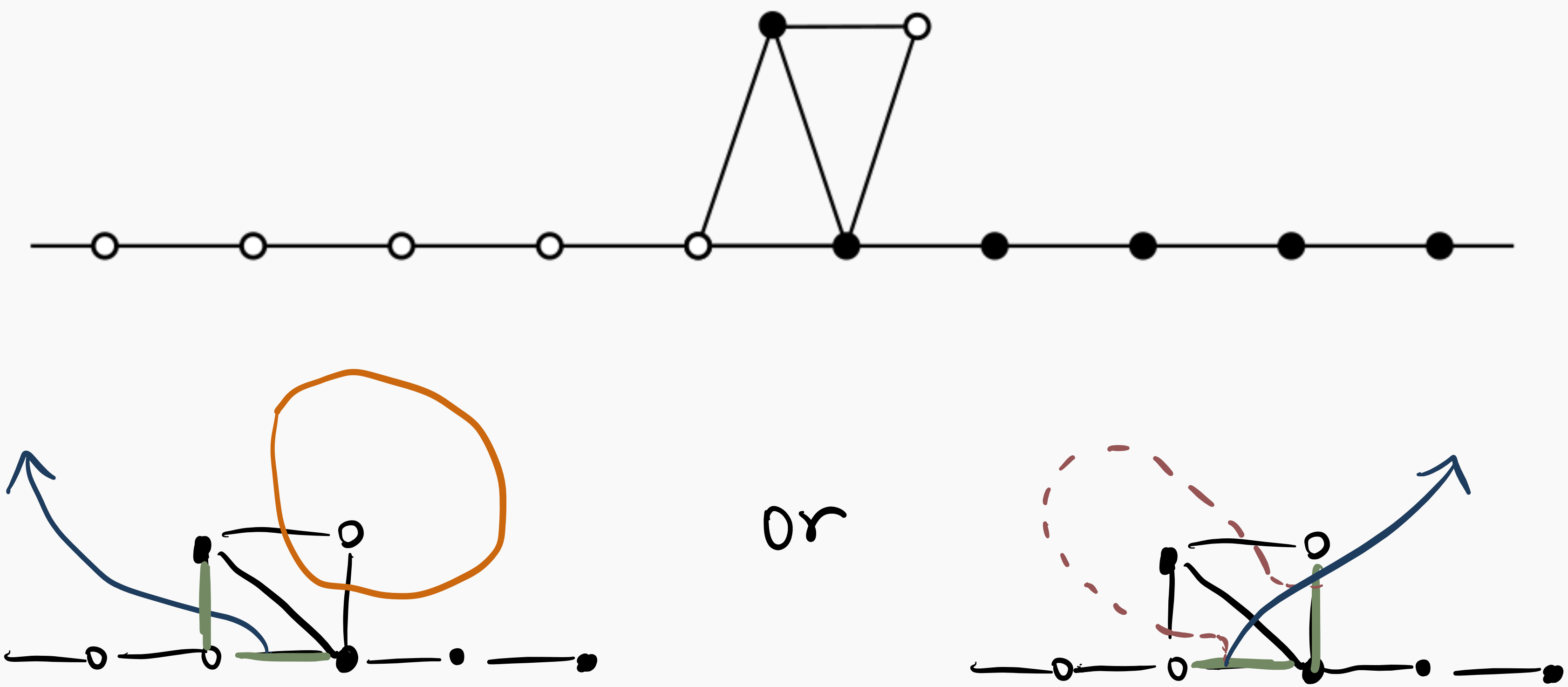

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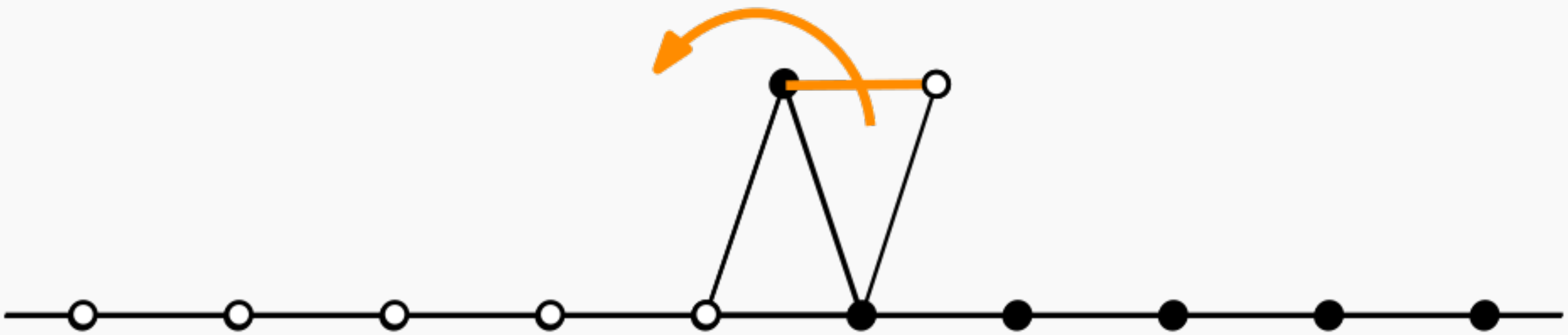

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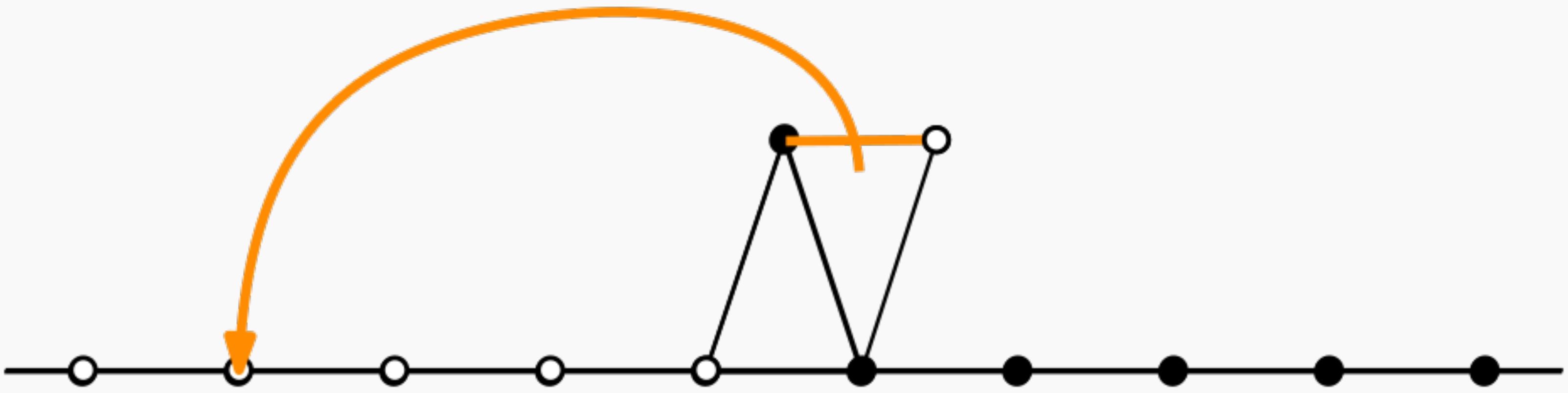

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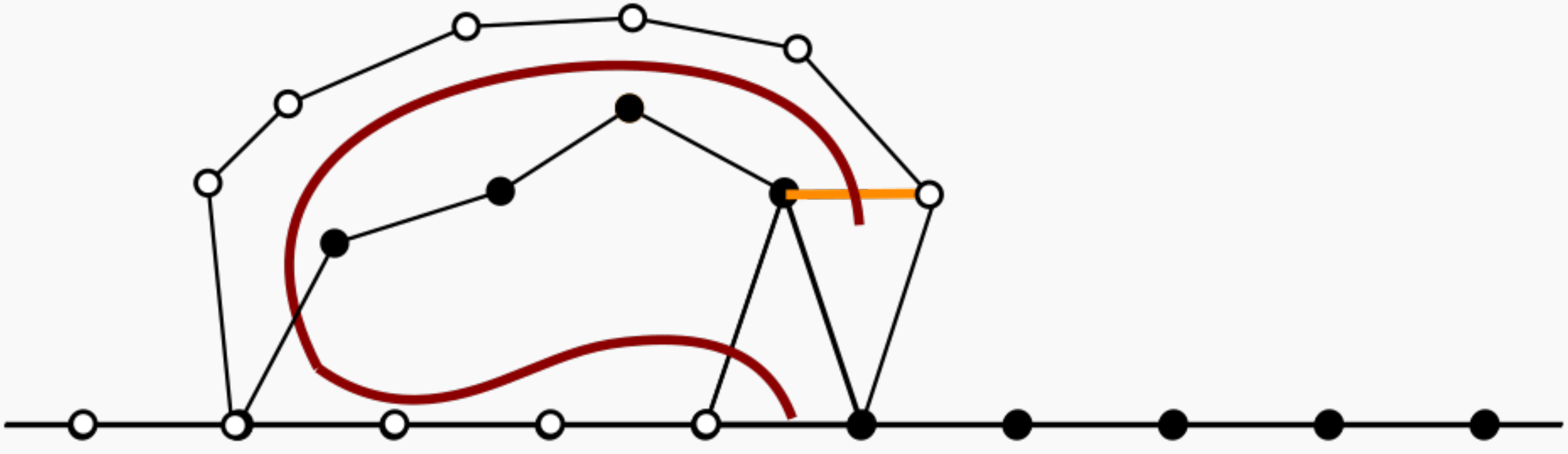

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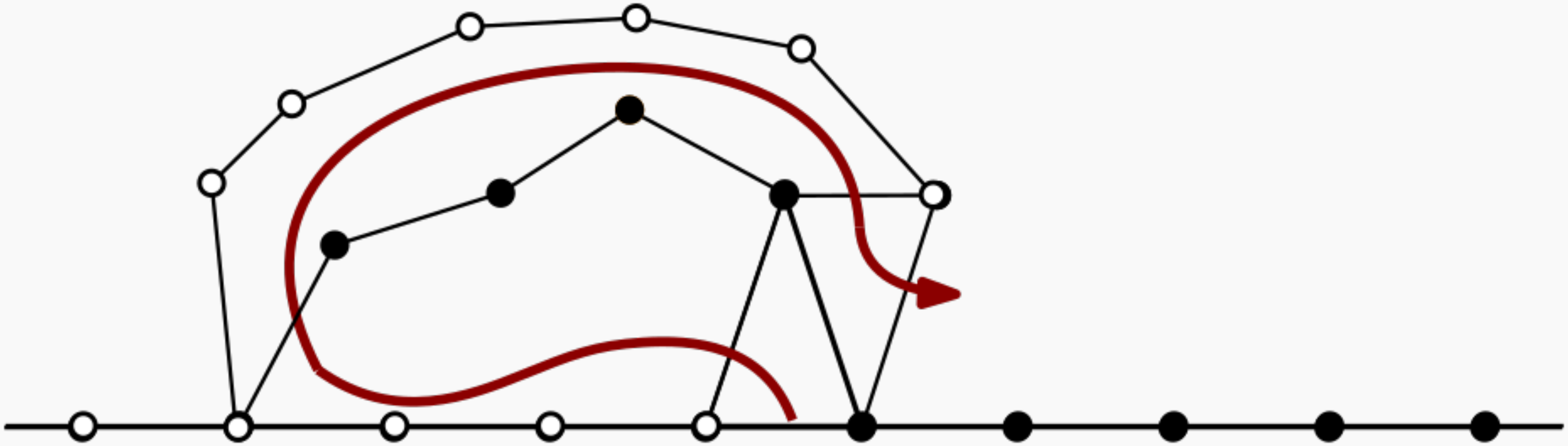

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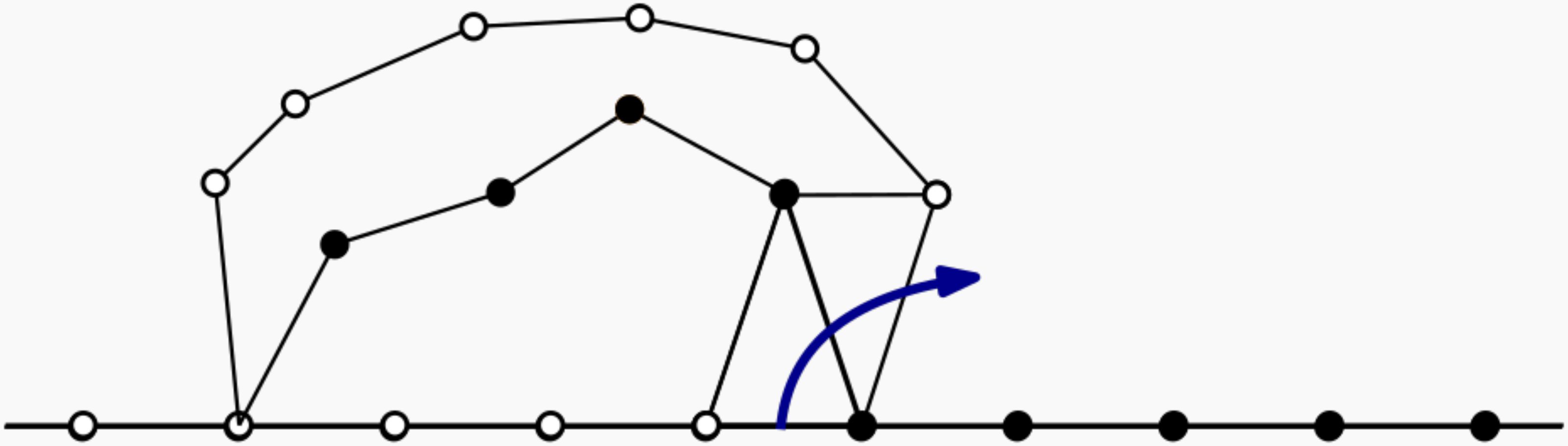

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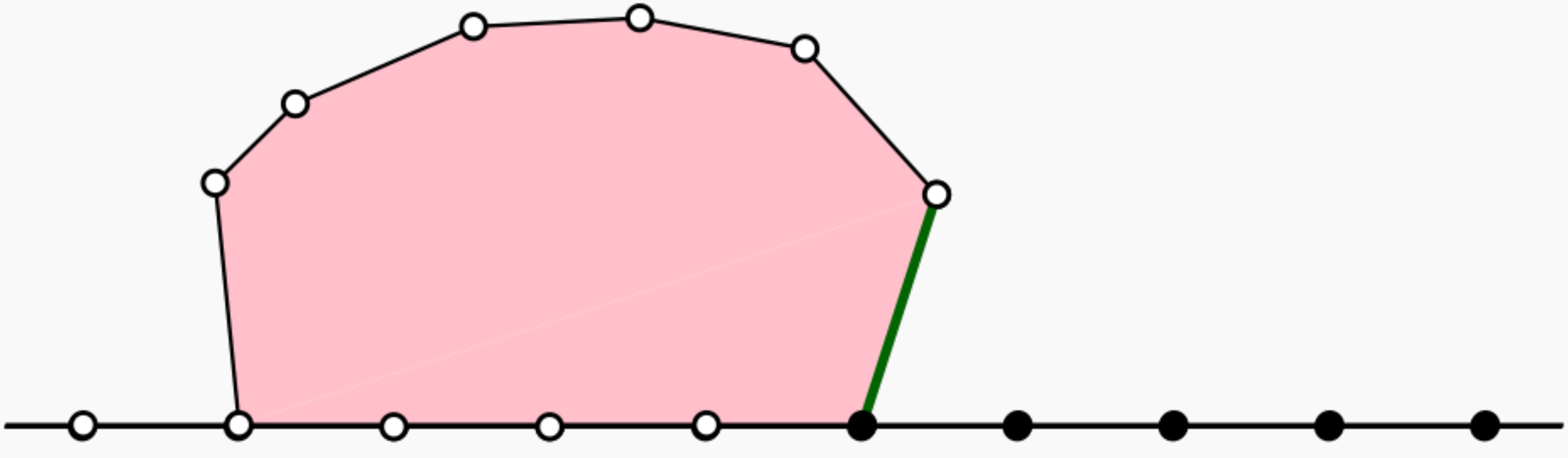

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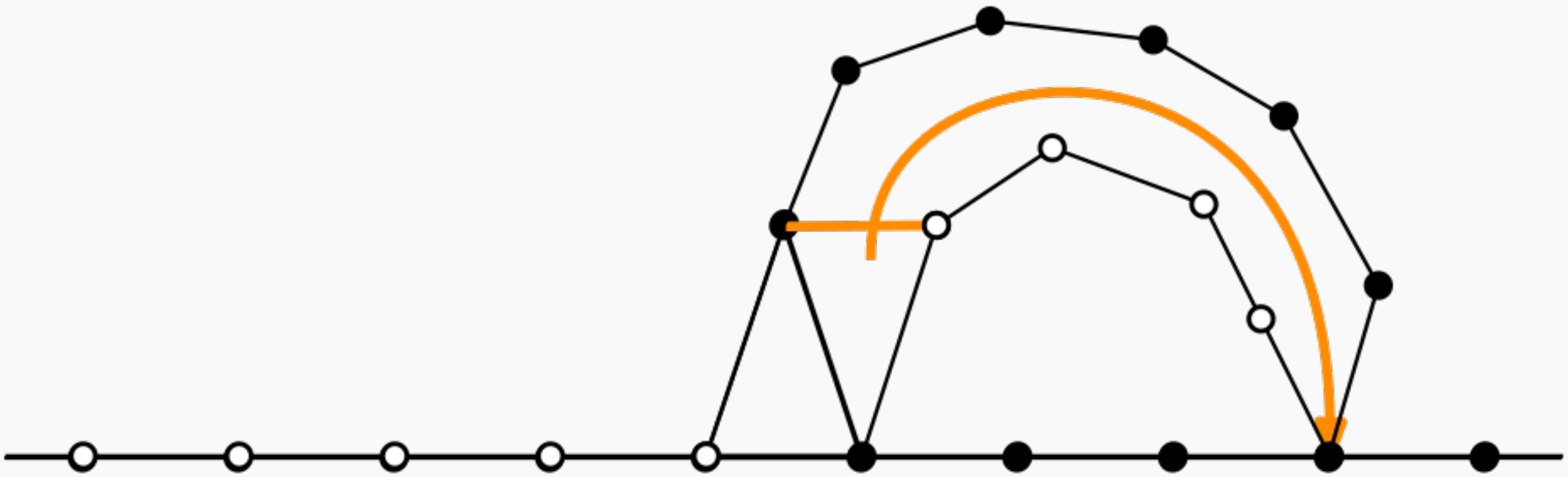

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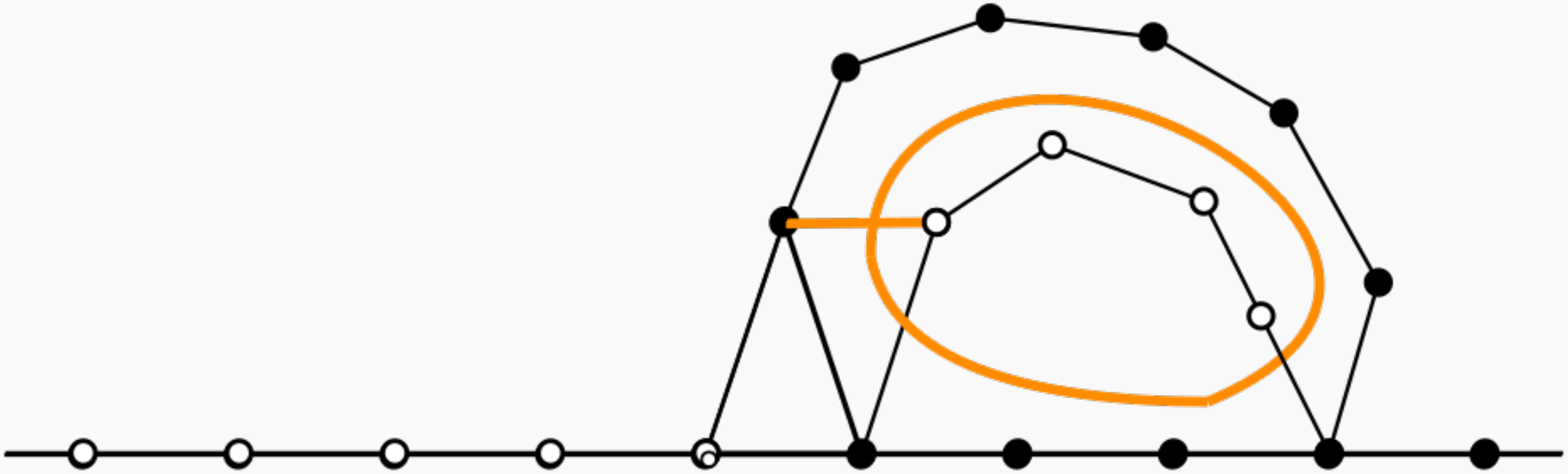

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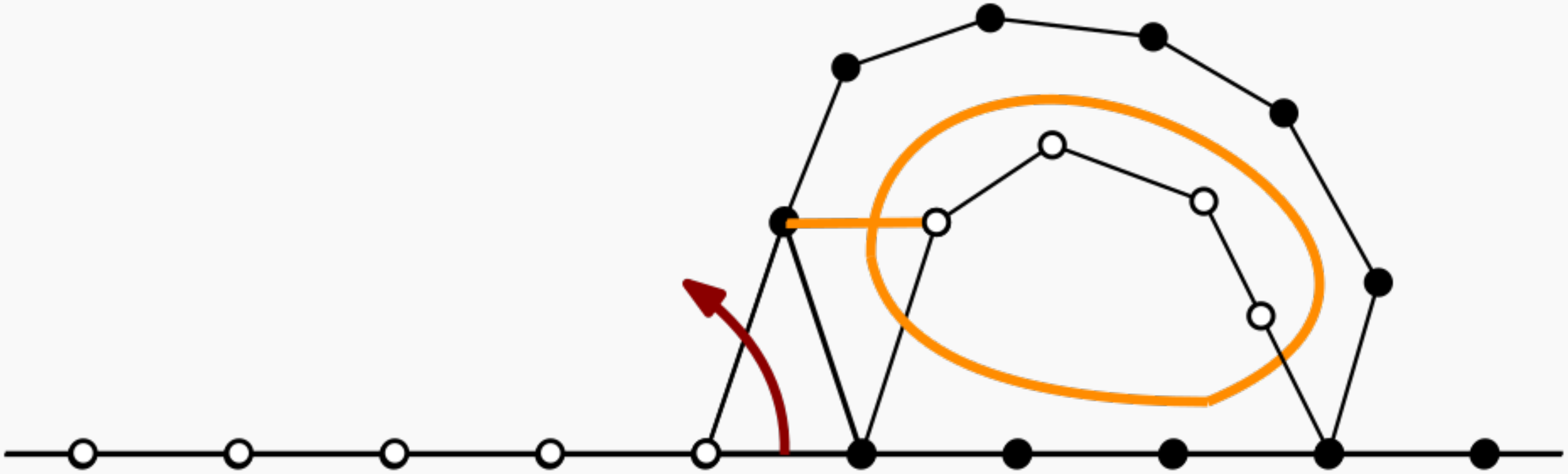

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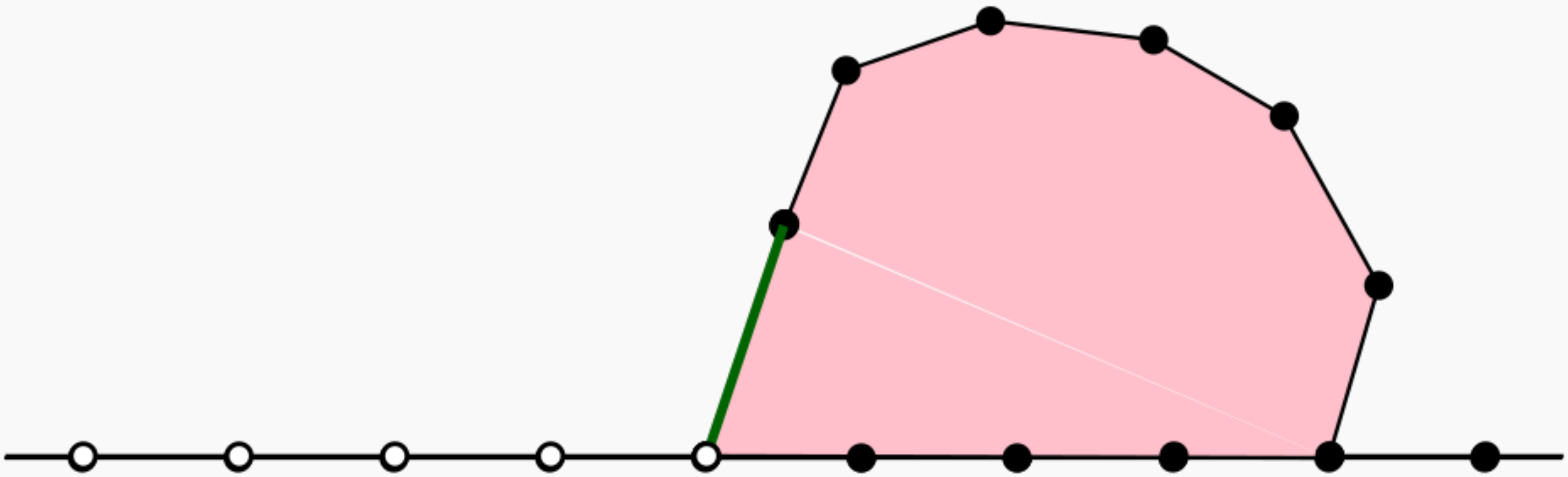
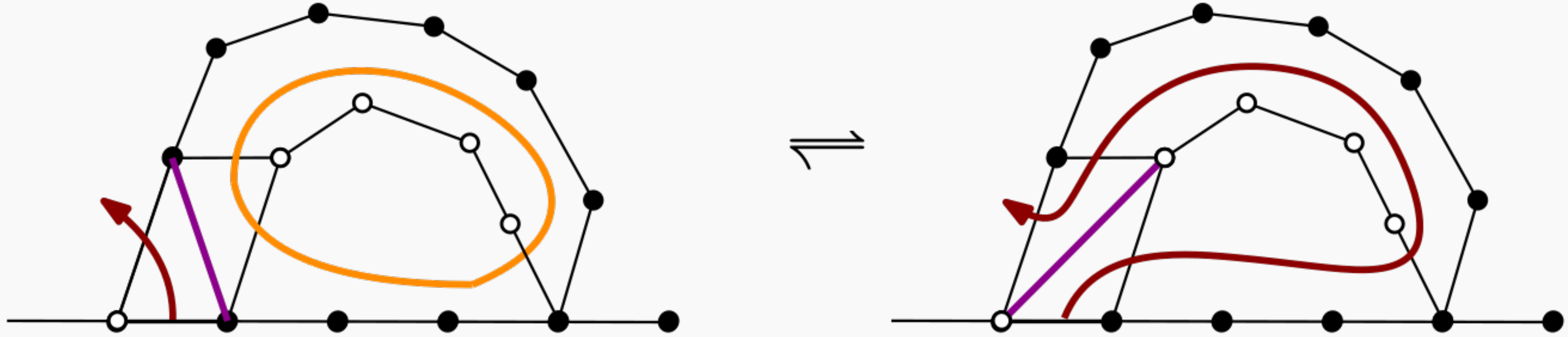


Switch property

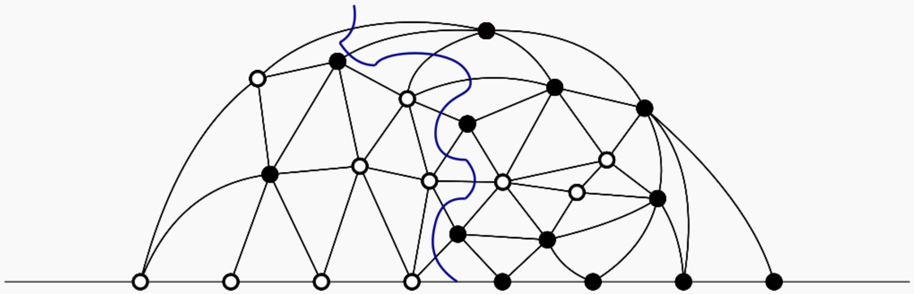


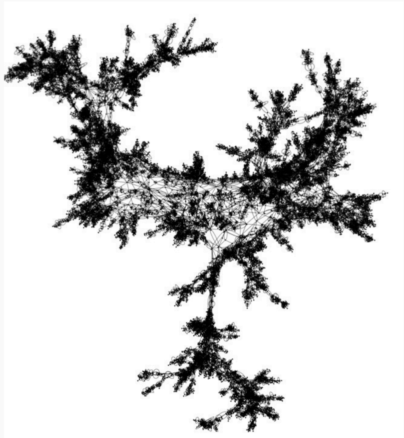
Main result

Theorem (F. 26+)

The scaling limit of loop erasure of percolation interface exists.

$$\text{UIHPT} \; + \; \text{loop erasure} \to \sqrt{8/3} - \text{LQG} \quad + \quad \text{“LE”}$$





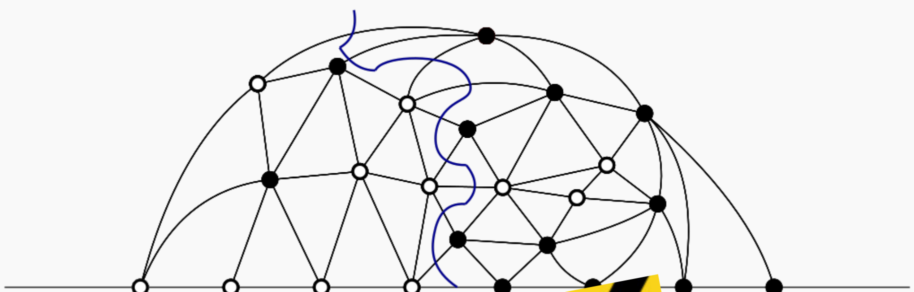
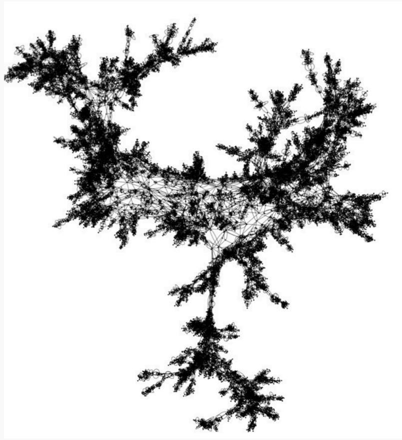
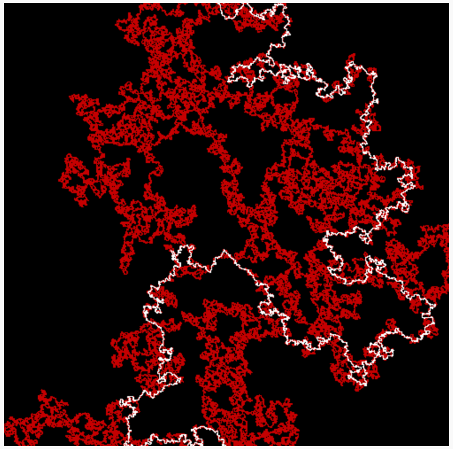
- Tightness: peeling estimates.
- Uniqueness: Poissonian structure and conformal welding.

Main result

Theorem (F. 26+)

The scaling limit of loop erasure of percolation interface exists.

$$\text{UIHPT} \;+\; \text{loop erasure} \to \sqrt{8/3} - \text{LQG} \quad + \quad \text{“LE”}$$

LE has the law of $\mathrm{SLE}_{8/3}(-1;-1)$.

- … peeling estimates.
- Uniqueness: Poissonian structure and conformal welding.

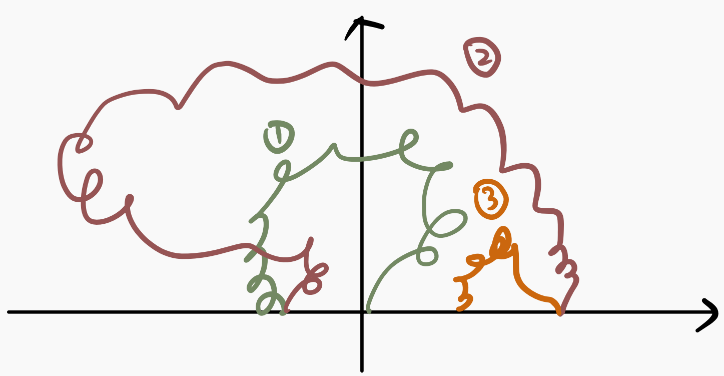
Mating of trees theory for 1-wedge?

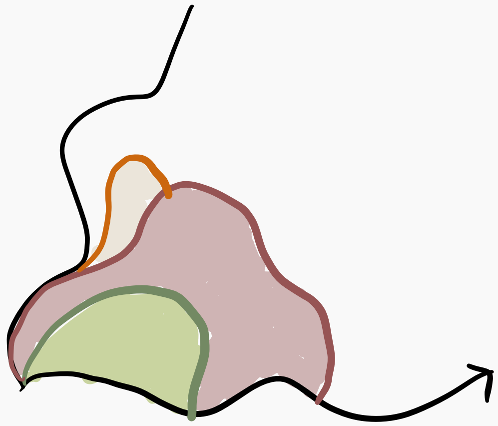
- LE is an $\mathrm{SLE}_{8/3}(-1;-1) \Leftrightarrow$ the two submaps separated by LE are independent **quantum wedges**.
- How to characterize quantum wedges?
- **Work in progress** Boundary process of a *space-filling* SLE in a quantum wedge is a planar Brownian motion reflected at x-axis.

  (Borga-Das-Gwynne '26+) Proved for thick quantum wedges.

Work in progress

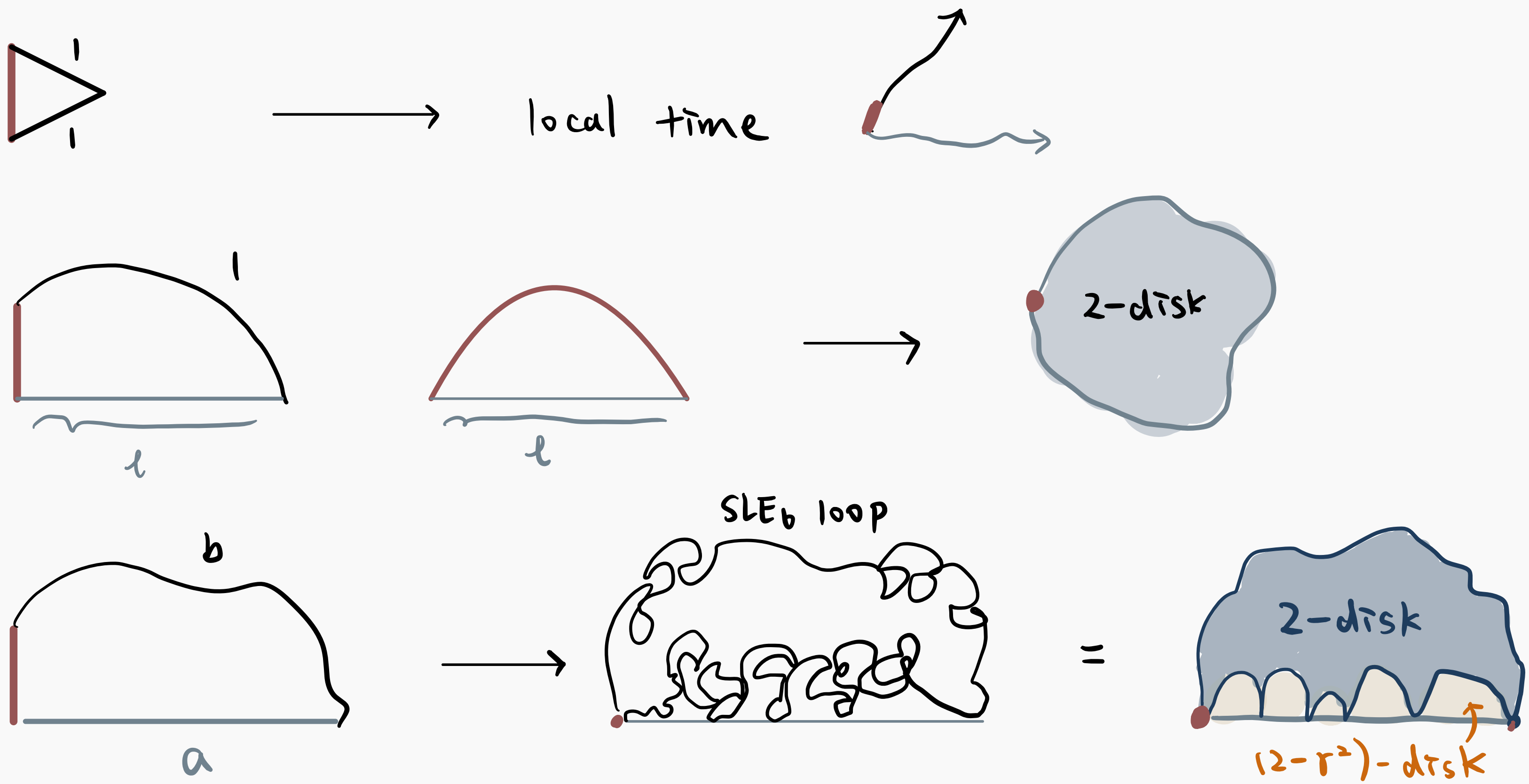
Gluing LQG disks together according to a reflected Brownian motion yields a quantum wedge.





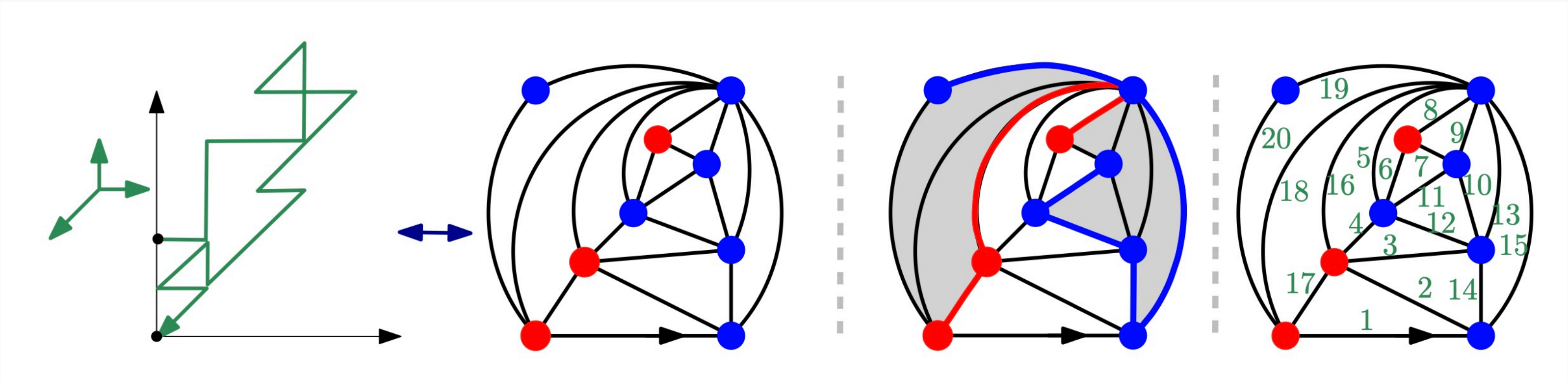
Identify the limit?

Types of patches from Markovian exploration:



We can define a space-filling exploration on each patch and then concatenate them. We expect its boundary process converges to a reflected Brownian motion.

Space-filling percolation exploration



Space-filling exploration bijective with the Kreweras walk

Theorem (Bernardi-Holden-Sun '18)

After appropriate scaling, as curve-decorated metric-measure spaces,

$$\text{UIPT} + \begin{matrix}\text{Space-filling} \\ \text{exploration}\end{matrix} \to \sqrt{8/3}\text{-LQG} + \begin{matrix}\text{Space-filling} \\ \text{SLE}_6.\end{matrix}$$

Loop erasure of space-filling percolation exploration

Theorem (F. 26+)

After appropriate scaling, as curve-decorated metric-measure spaces,

$$\mathrm{UIPT} \; + \; \begin{array}{c}\text{Loop erasure of} \\ \text{space-filling exploration}\end{array} \; \to \sqrt{8/3} - \mathrm{LQG} \; + \; \mathrm{SLE}_{8/3}(-5/3; -5/3).$$

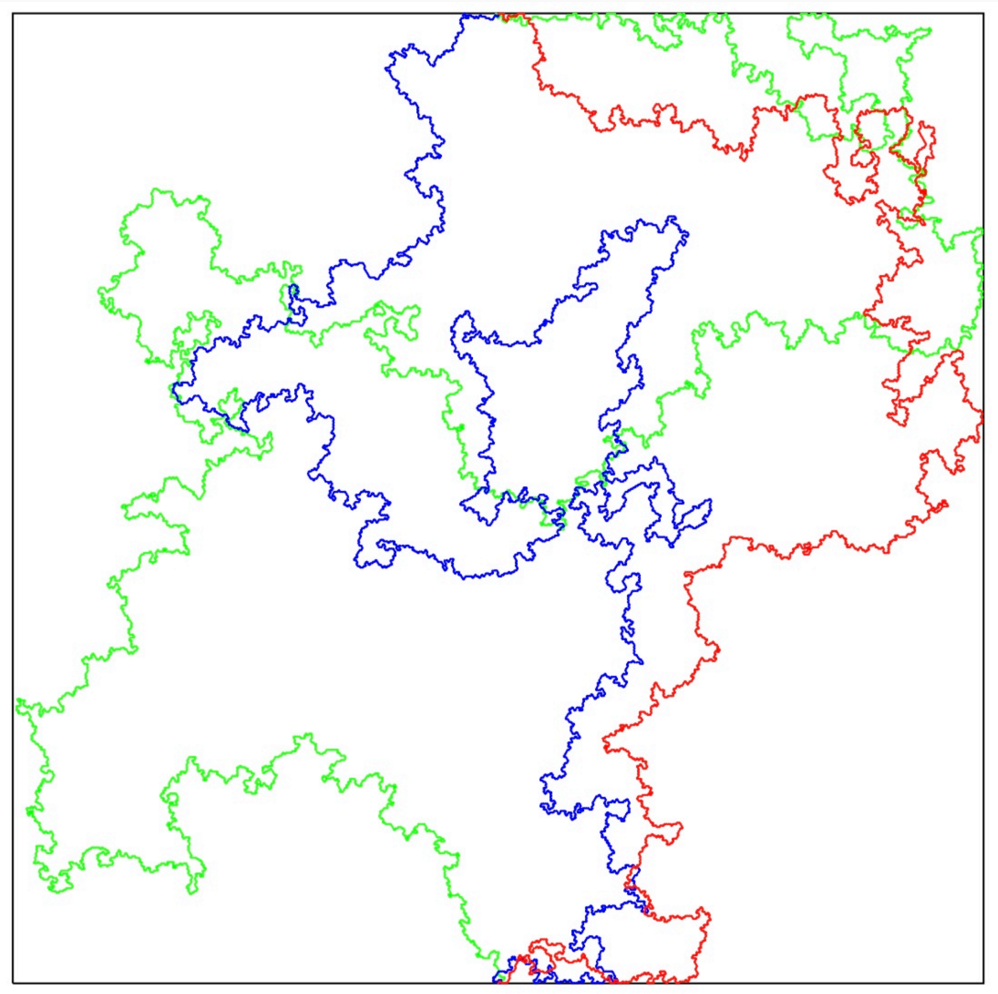
- It is a different curve with the loop erasure of percolation interface.
- The proof relies on **mating of trees theory** (Duplantier-Miller-Sheffield '14).
- The limiting curve is the 0-angle flow line of the GFF corresponding to the space-filling SLE_6. That is, the "solution" to

$$\xi'(t) = e^{ih(\xi(t))/\chi} \quad \text{for} \quad t > 0, \quad \xi(0) = 0, \quad \chi = \sqrt{6}/6.$$

- By putting some bias on the selection of loop erasure, we can get flow lines of all angles.

Open questions

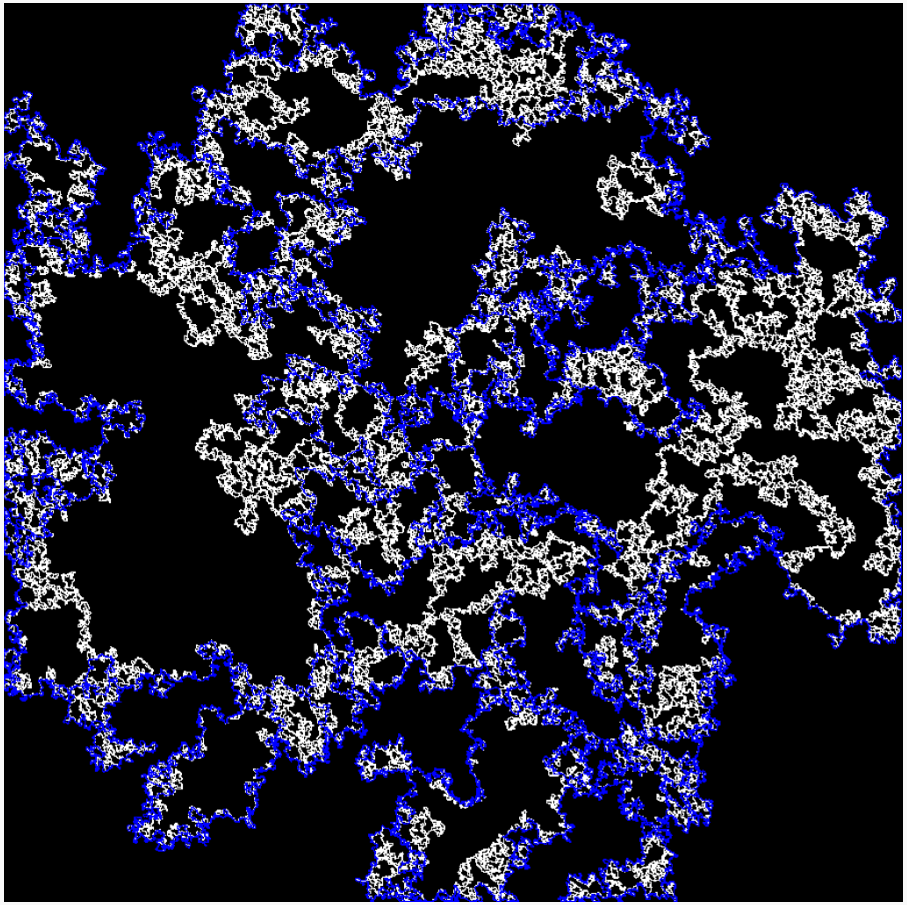
- Euclidean case?



3 samples of the loop-erased percolation explorer in hexagonal lattice.

Open questions

- Euclidean case?
- Uniqueness of loop erasure for general SLE_κ ($\kappa \in (4, 8)$)?
  - Open: Is SLE_2 the unique loop erasure for planar Brownian motion?



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SLE_κ (counterflow line) contains infinitely many $\mathrm{SLE}_{16/\kappa}$-type curves (flow lines).