Fundamental aspects of tropical cyclone dynamics. Part II: Buoyancy, size and the boundary layer

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Topics:
1. Buoyancy
2. Boundary layer
3. Size

Motivation

- **Are the eyewall clouds of a hurricane buoyant?**
  - **Recall that the eye is the warmest place in the storm ➔ the eyewall clouds are not buoyant relative to the eye!**
- **Is a hurricane everywhere warm cored?**
  - **Measurements are usually made at constant pressure, but the pressure surfaces descend in the core.**
  - **Is it everywhere warm cored at constant height?**
  - **How large is the "balanced" temperature perturbation at the surface?**

Buoyancy in tropical cyclones and other rapidly rotating atmospheric vortices

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Accurate determination of a balanced axisymmetric vortex in a compressible atmosphere

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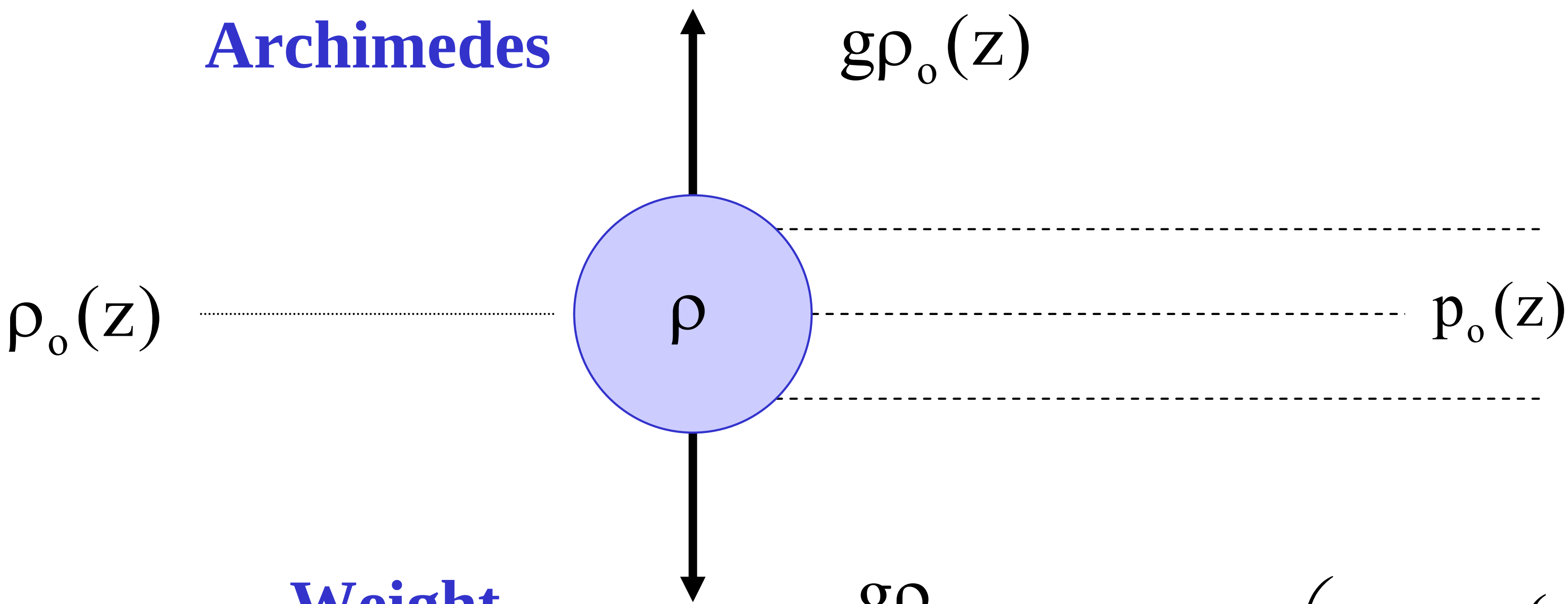
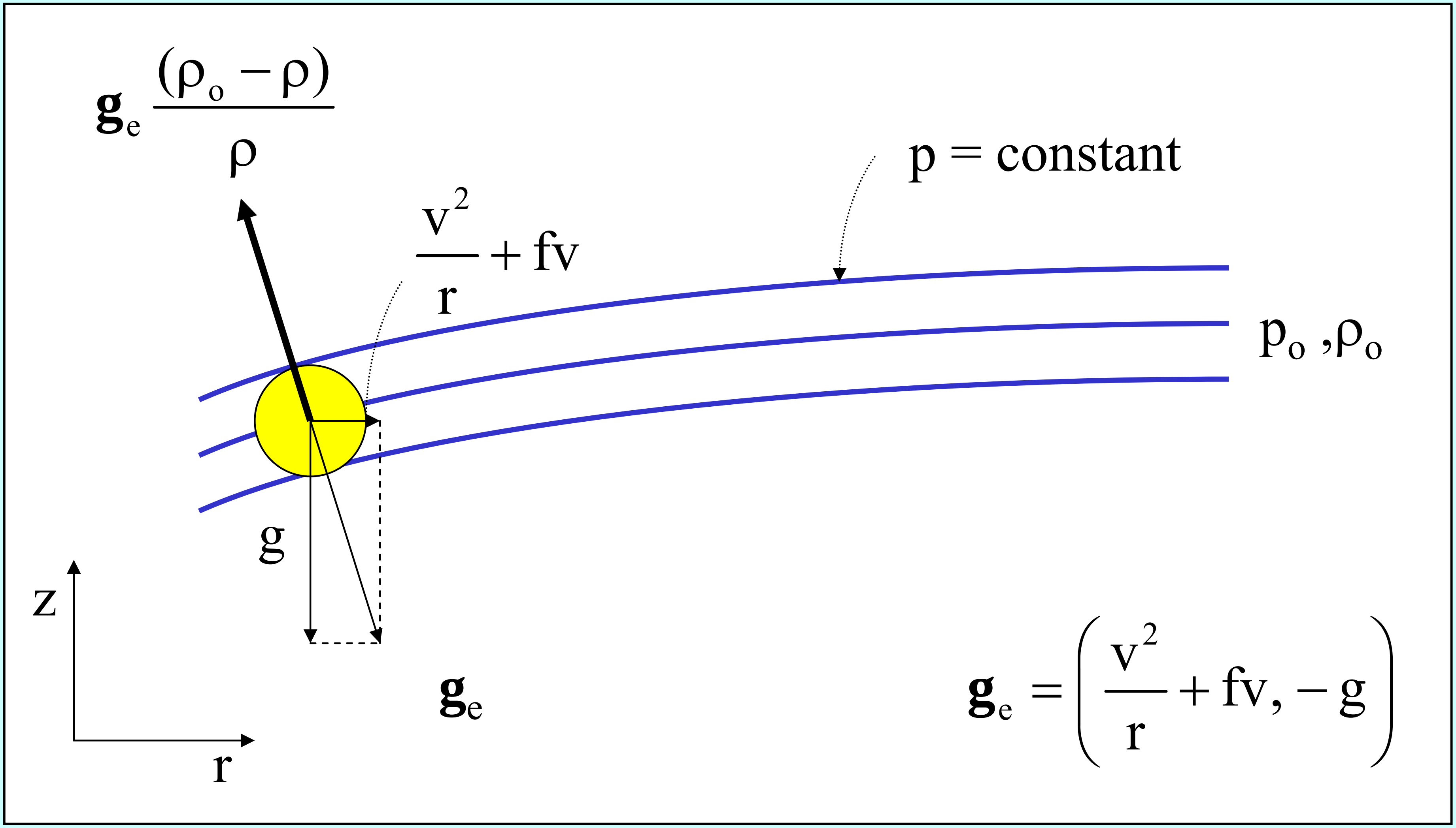
Tellus, 58A, (2006) 98-103

Buoyancy



$$b = -g\left(\frac{\rho - \rho_o(z)}{\rho}\right)$$

Generalized buoyancy in a rapidly-rotating fluid



- Not the same as Emanuel's definition (Emanuel, 1994)
- To calculate generalized buoyancy, see e.g. Tellus paper

Alternative derivation

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

Put $\rho = \rho_o(z) + \rho'$ **and** $p = p_o(z) + p'$ **, where** $\frac{dp_o}{dz} - g\rho_o$

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial z} + b$$

Note that b **is non-unique: it depends on the definition of** $\rho_o(z)$

b **is the system buoyancy**

Alternative derivation for a rapidly-rotating vortex

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

Put $\rho = \rho_o(r,z,t) + \rho'$ **and** $p = p_o(r,z,t) + p'$ **, where** $\frac{dp_o}{dz} - g\rho_o$

$\rho_o(r,z,t)$, $p_o(r,z,t)$ **in thermal wind balance with** $v(r,z,t)$.

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial z} + b_L$$

Now b_L **depends on the definition of** $\rho_o(r,z,t)$

b_L **is the local buoyancy**

Thermal wind equation

Gradient wind balance **Hydrostatic balance**

Write
$$\frac{\partial p}{\partial r} = \rho\left(\frac{v^2}{r} + fv\right) \qquad \frac{\partial p}{\partial z} = -\rho g$$

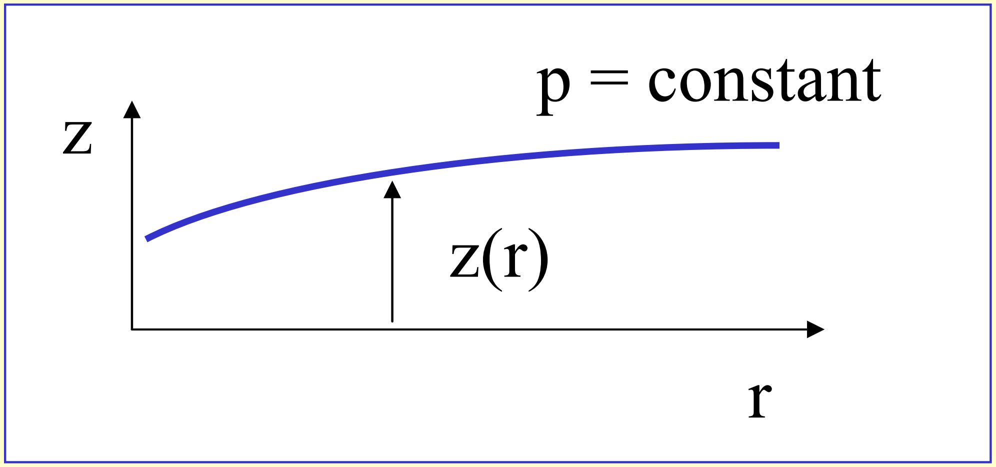
Eliminate p **using**
$$\frac{\partial}{\partial r}\left(\frac{\partial p}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial p}{\partial r}\right)$$

$$\frac{\partial}{\partial r}\ln\rho + \frac{1}{g}\left(\frac{v^2}{r} + fv\right)\frac{\partial}{\partial z}\ln\rho = -\frac{1}{g}\left(\frac{2v}{r} + f\right)\frac{\partial v}{\partial z}$$

Exact form of the thermal wind equation

Mathematical solution

$$\frac{\partial}{\partial r}\ln\rho + \frac{1}{g}\left(\frac{v^2}{r} + fv\right)\frac{\partial}{\partial z}\ln\rho = -\frac{1}{g}\left(\frac{2v}{r} + f\right)\frac{\partial v}{\partial z}$$



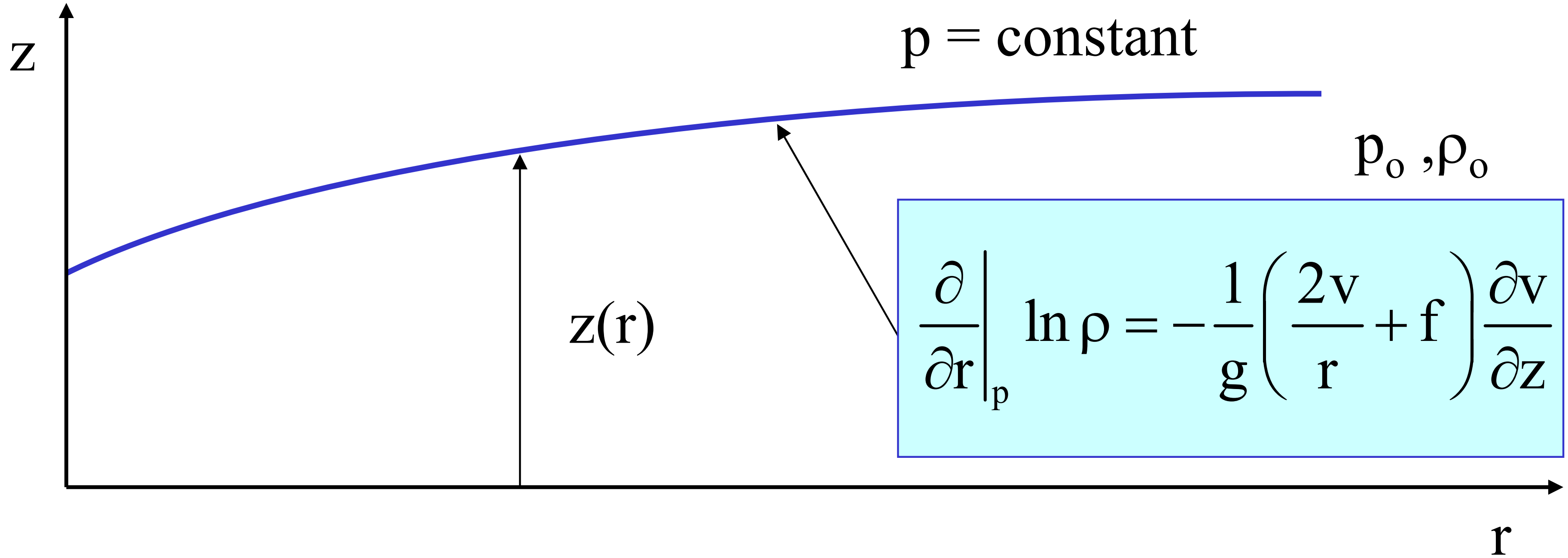
Characteristics

$$\frac{dz}{dr} = \frac{1}{g}\left(\frac{v^2}{r} + fv\right)$$

$$\frac{d}{dr}\ln\rho = -\frac{1}{g}\left(\frac{2v}{r} + f\right)\frac{\partial v}{\partial z}$$

Governs the variation of ρ along characteristics

Inferences



$$\left.\frac{\partial}{\partial r}\right|_p \ln\rho = -\frac{1}{g}\left(\frac{2v}{r}+f\right)\frac{\partial v}{\partial z}$$

Barotropic vortex $\frac{\partial v}{\partial z} = 0 \Rightarrow \left.\frac{\partial \rho}{\partial r}\right|_p = 0 \Rightarrow \left.\frac{\partial T}{\partial r}\right|_p = 0$

Baroclinic vortex $\frac{\partial v}{\partial z} < 0 \Rightarrow \left.\frac{\partial \rho}{\partial r}\right|_p > 0 \Rightarrow \left.\frac{\partial T}{\partial r}\right|_p < 0$

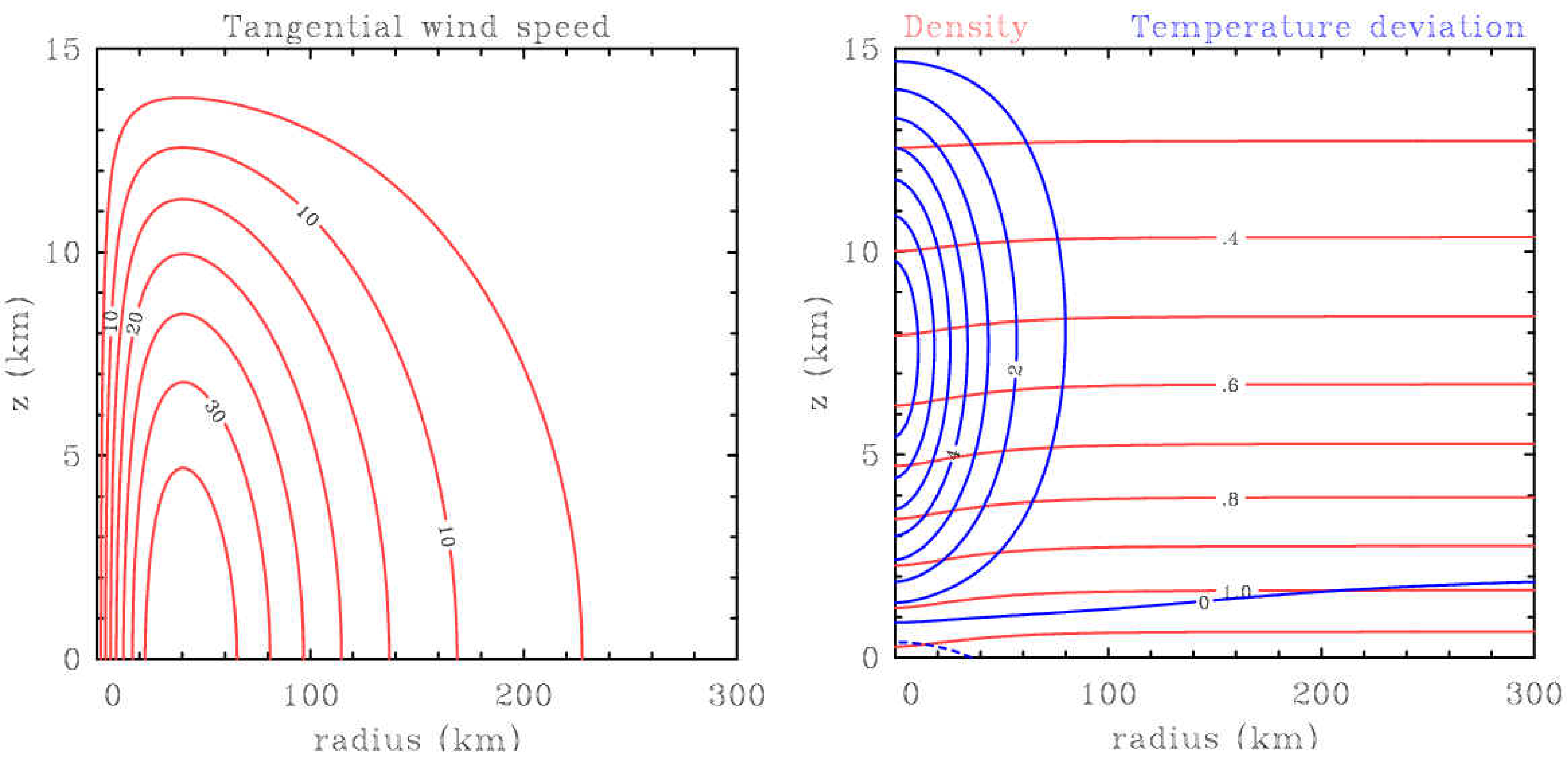
Equation of state $T = p / R\rho$

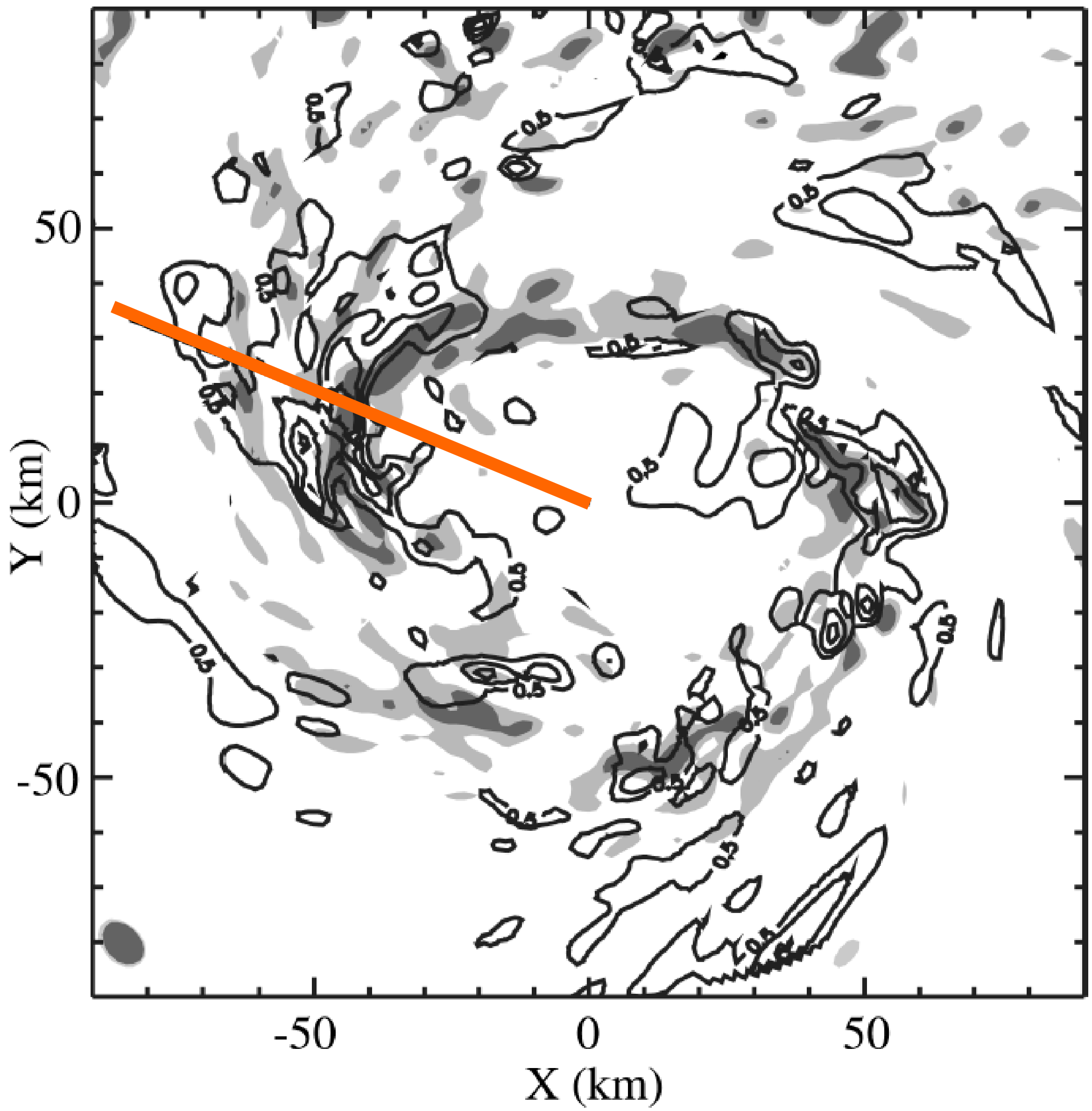
Summary

- **A barotropic vortex is cold cored if temperature contrasts are measured at constant height.**
- **A baroclinic vortex is warm cored if temperature contrasts are measured at constant height, but only if** $-\partial v/\partial z$ **is large enough.**

A sample calculation =>

A sample calculation





Light shading
w > 1 m s $^{-1}$
Dark shading
w > 3 m s $^{-1}$
θ_v contours
0.5 K

Vertical velocity at 3.2 km at 66 h (shading). Perturbation θ_v. Azimuthal wavenumber 2 and higher.

From Braun, 2001

Conclusions Part I

- **In a rapidly-rotating vortex, buoyancy has a radial as well as a vertical component. The former is small in TCs.**
- **The vertical component depends on the definition of the reference density: one can distinguish between system buoyancy and local buoyancy.**
- **Eye-wall clouds may be locally buoyant (Braun 2001).**
- **Barotropic vortices are cold cored because** $T = T(p)$ **and the isobaric surfaces dip down.**
- **Baroclinic vortices are cold cored near the surface (if** $\partial v/\partial z = 0$**) and warm cored aloft if** $-\partial v/\partial z$ **is large enough.**
- **Typical surface** dT's **are a few degrees C in an intense TC.**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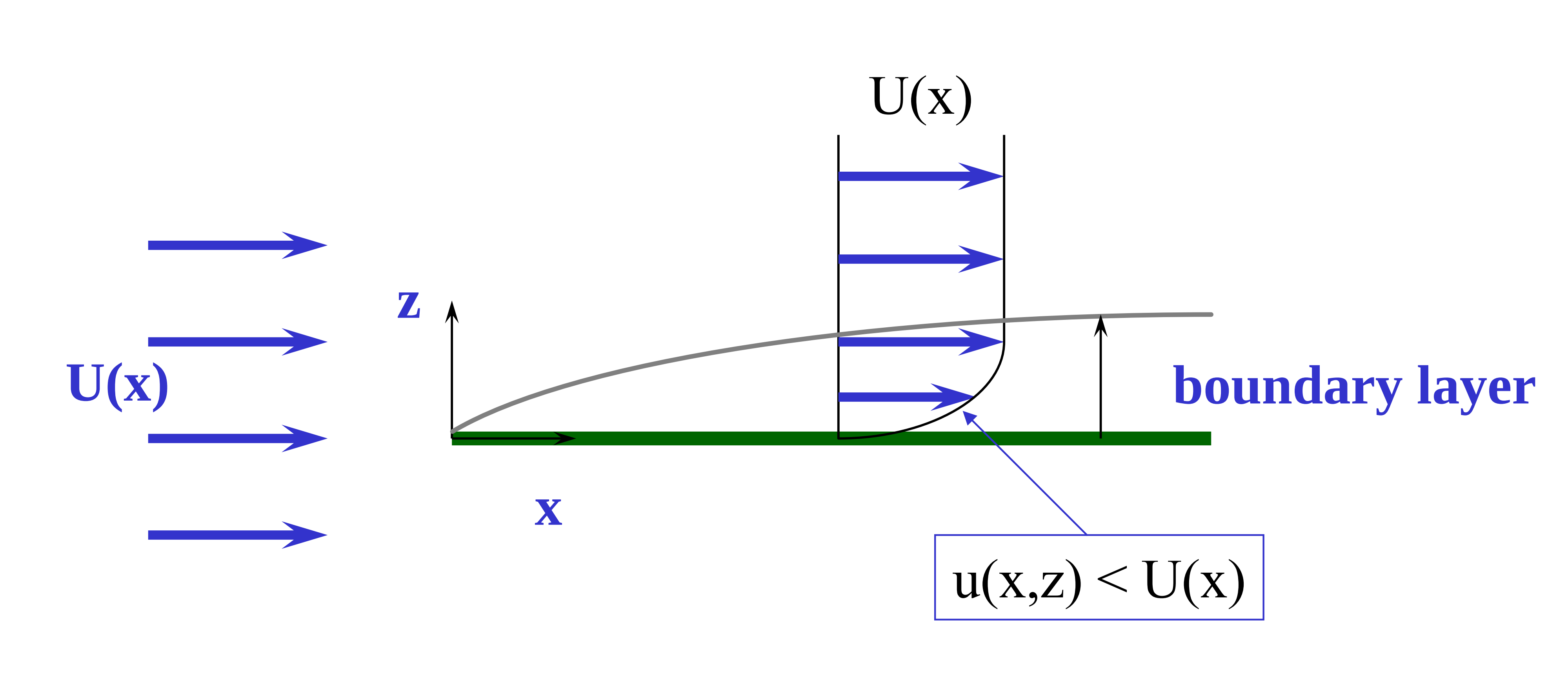
Boundary layers in nonrotating fluids

A simple example: steady two-dimensional boundary layer on a flat plate at normal incidence to an almost parallel stream U(x)**.**



Approximate Euler equation for steady, almost parallel flow above the boundary layer

$$U\frac{dU}{dx} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \qquad w \approx 0$$

The Navier Stokes' equations in the boundary layer:

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial z^2}\right]$$

$$\cancel{u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\cancel{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}}\right]$$

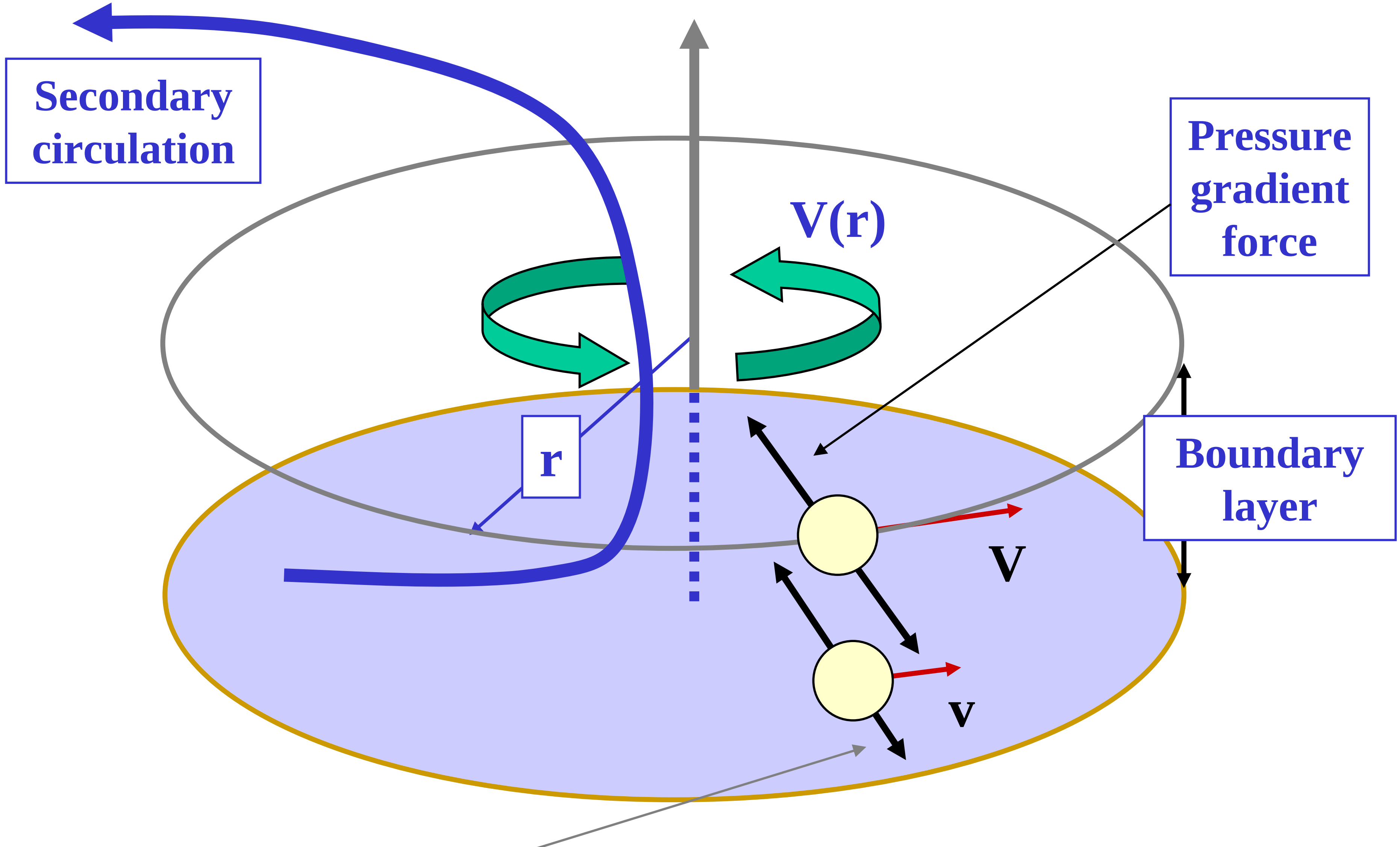
The boundary layer equations:

$$\frac{\partial p}{\partial z} \sim O\left(Re^{-1}\right) \ll 1 \qquad Re = \frac{U_s L}{\nu} \gg 1$$

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial z^2}$$

parabolic!

Frictionally-induced secondary circulation



Centrifugal force and Coriolis force reduced by friction

Importance of the boundary layer

- The boundary layer of a rapidly-rotating vortex provides a powerful coupling between the primary circulation (the azimuthal component) and the secondary circulation (the radial-vertical, or "in-up-and-out" component).
- Moisture enters a hurricane from the sea surface and its radial distribution is strongly influenced by that of the boundary layer winds.
- The vertical transport of moisture and angular momentum out of the boundary layer is determined by the boundary layer dynamics and thermodynamics.
- The radial distribution of these quantities on leaving the layer exerts a strong constraint on the radial distribution of buoyancy

A surprise

- **The maximum wind speed in the boundary layer can exceed that above the boundary layer (or can it?).**
  - **An unusual in feature boundary layers in general.**
  - **A special feature of the termination boundary layer of intense vortices.**
- **Anthes (1972):**
  - **Inward increase of** V **due to large radial displacements in the boundary layer with partial conservation of AAM opposes the fictional loss of AAM to the surface.**

Some consequences

- **The maximum winds in a hurricane occur close to the surface.**

  ⇒ strong surface moisture fluxes

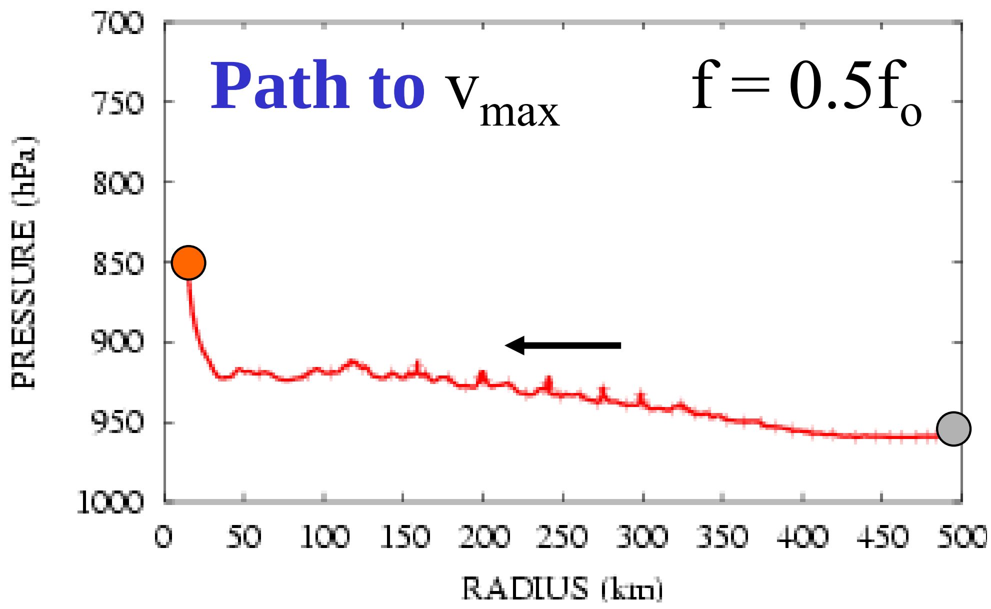
  ⇒ drive large ocean wave fields

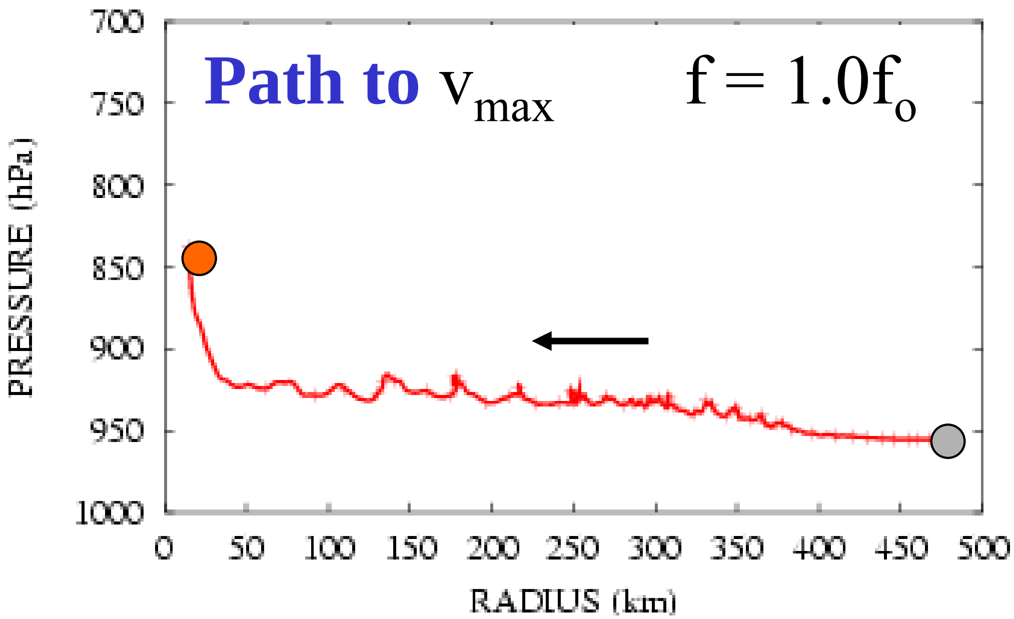
  ⇒ do maximum damage at landfall.

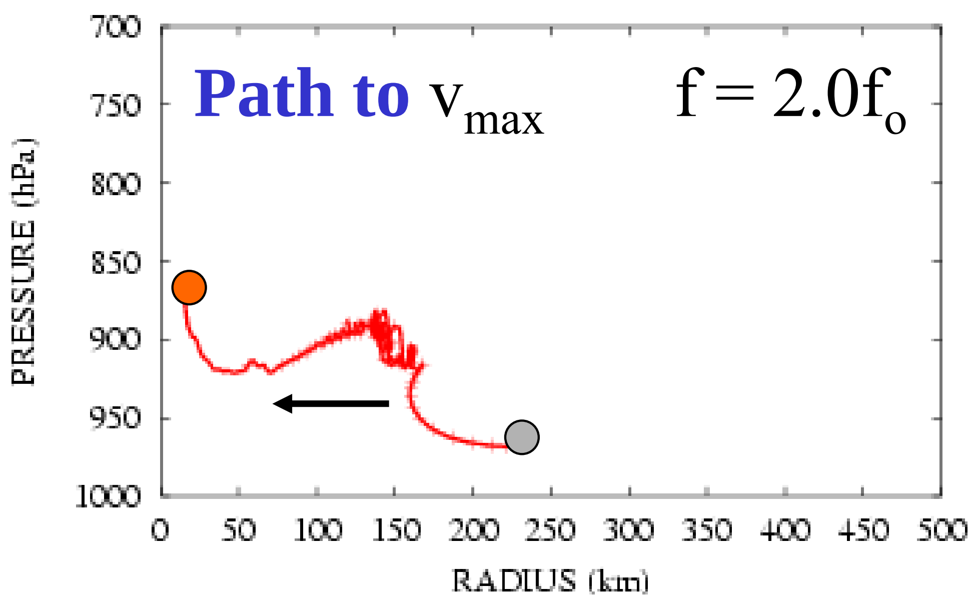
- **Wind reduction factors used to relate maximum flight level winds observed by reconnaissance aircraft to maximum near surface winds may be appreciably in error (Kepert, 2001).**

- **In his PI code Kerry Emanuel reduces** V_m **by** 20% **"… to reflect the typical relationship between gradient winds and actual surface winds".**

A simple axisymmetric hurricane model: back trajectories from v_{max}





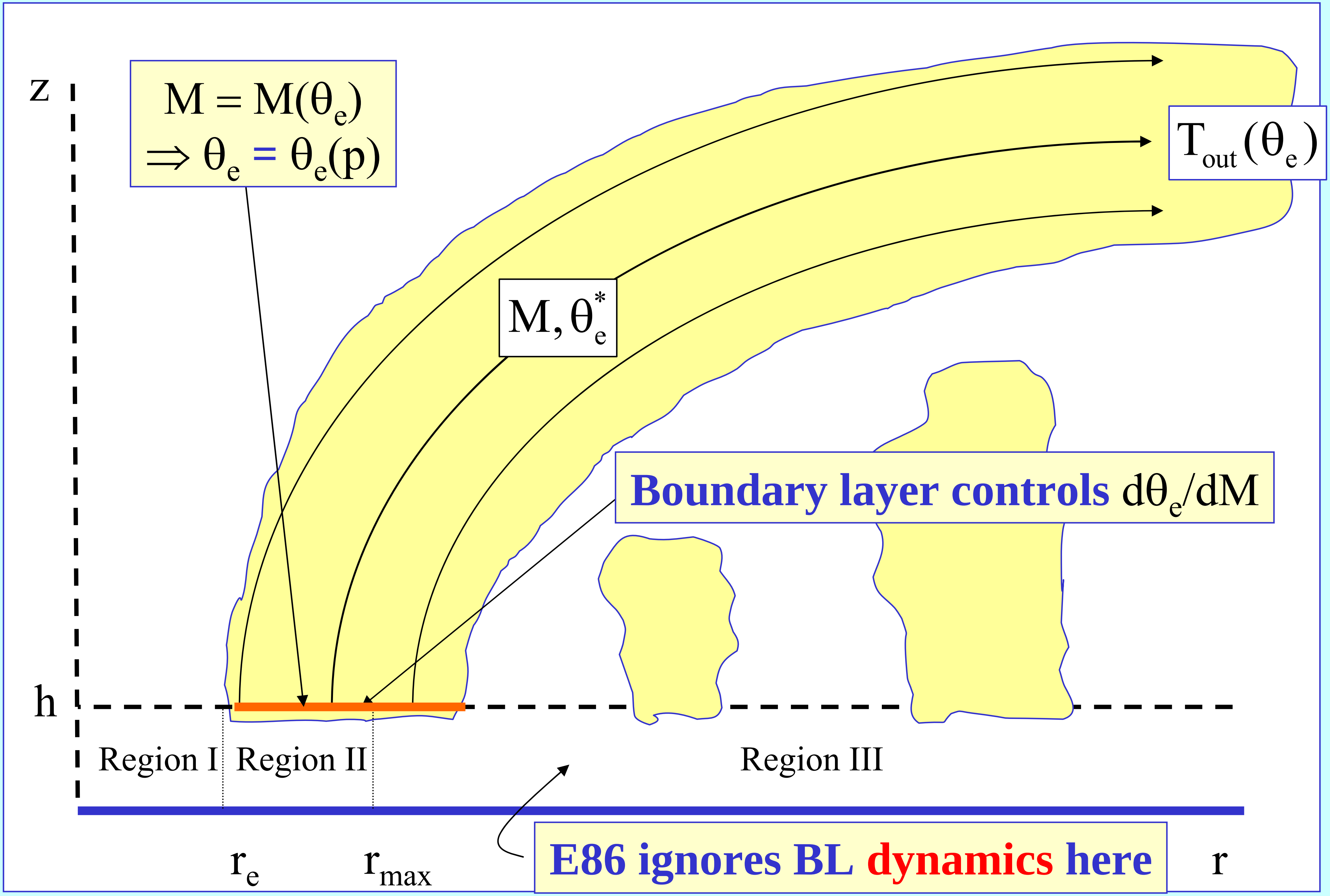


Wolfgang Ulrich
unpublished

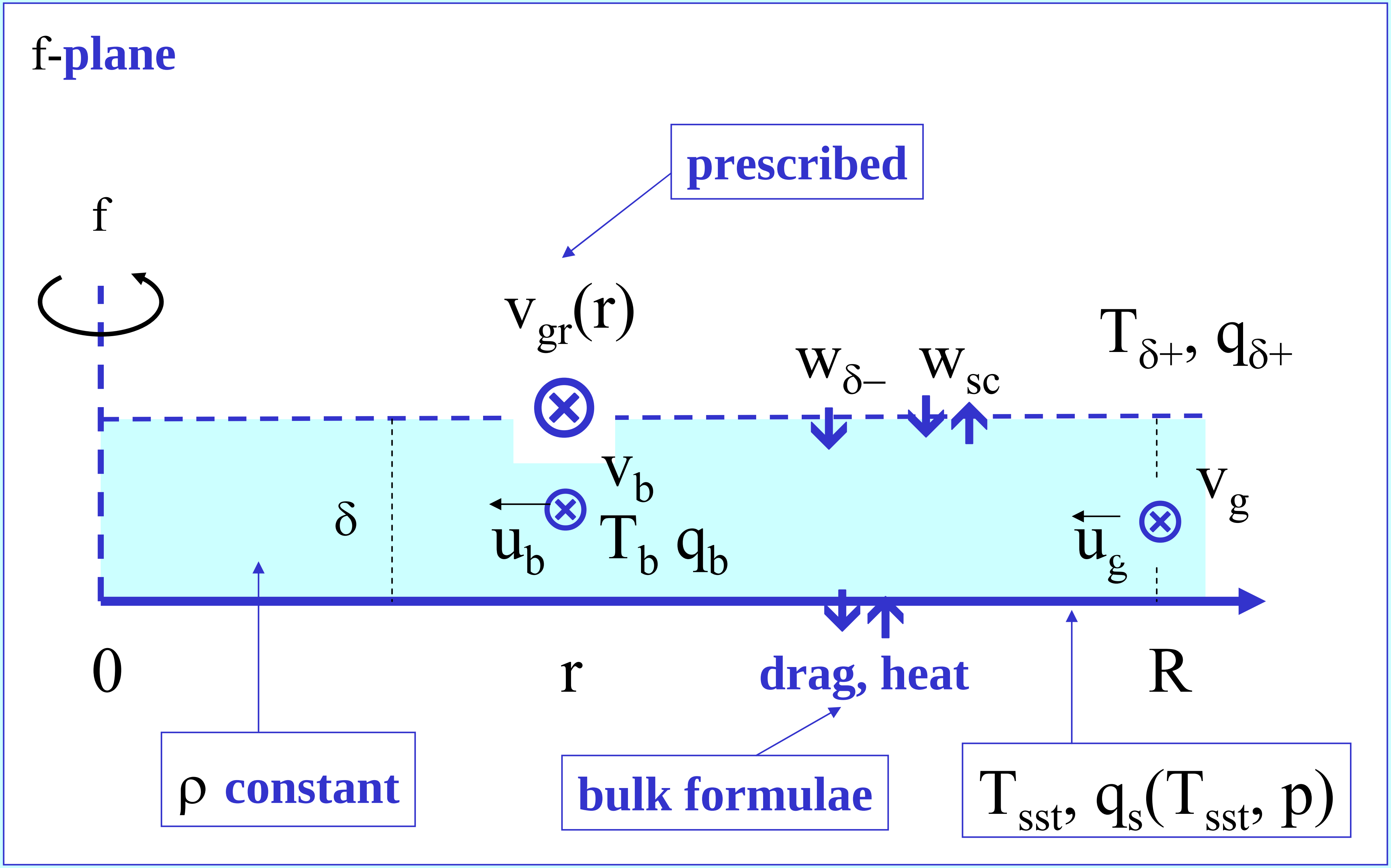
Theoretical literature on the hurricane boundary layer

- **Rosenthal 1962. Quasi-linear theory ($v_{gr}(r)$ prescribed)**
- **Smith 1968, Leslie and Smith 1970, Bode and Smith 1975. Momentum integral method ($v_{gr}(r)$ prescribed)**
- **Shapiro 1983, Slab model ($v_{gr}(x,y)$ prescribed)**
- **Eliassen 1971, Eliassen and Lystad 1977, Montgomery et al. 2001, Jones and Thorncroft (unpublished). Unsteady, focus on vortex spindown (coupled theories: $v_{gr}(r,t)$ determined)**
- **Emanuel 1986, 1995, Bister and Emanuel 1998, Emanuel 2005. Application to PI theory (coupled theories : $v_{gr}(r,t)$ determined)**
- **Kepert 2001, Kepert and Wang 2001, Steady, analytic and numerical, non-axisymmetric ($v_{gr}(x,y)$ prescribed)**
- **Smith 2003, Smith and Vogl 2007. Steady, axisymmetric slab formulation ($v_{gr}(r)$ prescribed)**
- **Vogl and Smith (in preparation) Steady, axisymmetric slab formulation, ($v_{gr}(r)$ determined where $w_{\delta} > 0$.)**

Emanuel's 1986 steady hurricane model



Slab boundary layer model



Boundary-layer equations

$$u_b \frac{du_b}{dr} = u_b \frac{w_{\delta_-}}{\delta} - \frac{(v_{gr}^2 - v_b^2)}{r} - f(v_{gr} - v_b) - \frac{C_D}{\delta}(u_b^2 + v_b^2)^{\frac{1}{2}} u_b - \frac{\overline{(u'w')}_\delta}{\delta}, \tag{1}$$

$$u_b \frac{dv_b}{dr} = \frac{w_{\delta_-}}{\delta}(v_b - v_{gr}) - (\frac{v_b}{r} + f)u_b - \frac{C_D}{\delta}(u_b^2 + v_b^2)^{\frac{1}{2}} v_b - \frac{\overline{(v'w')}_\delta}{\delta}. \tag{2}$$

$$u_b \frac{d\chi_b}{dr} = \frac{w_{\delta_-}}{\delta}(\chi_b - \chi_{\delta+}) + \frac{C_\chi}{\delta}(u_b^2 + v_b^2)^{\frac{1}{2}}(\chi_s - \chi_b) - \frac{\overline{(\chi'w')}_\delta}{\delta} - \dot{\chi}_b, \tag{3}$$

$$\frac{du_b}{dr} = -\frac{u_b}{r} - \frac{w_\delta}{\delta}. \tag{4}$$

$$w_\delta = \frac{\delta}{1+\alpha}\left[\frac{1}{u_b}\left(\frac{(v_{gr}^2 - v_b^2)}{r} + f(v_{gr} - v_b) + \frac{C_D}{\delta}(u_b^2 + v_b^2)^{\frac{1}{2}} u_b\right) - \frac{u_b}{r}\right], \tag{5}$$

Starting conditions at large radius

Ekman-like dynamics

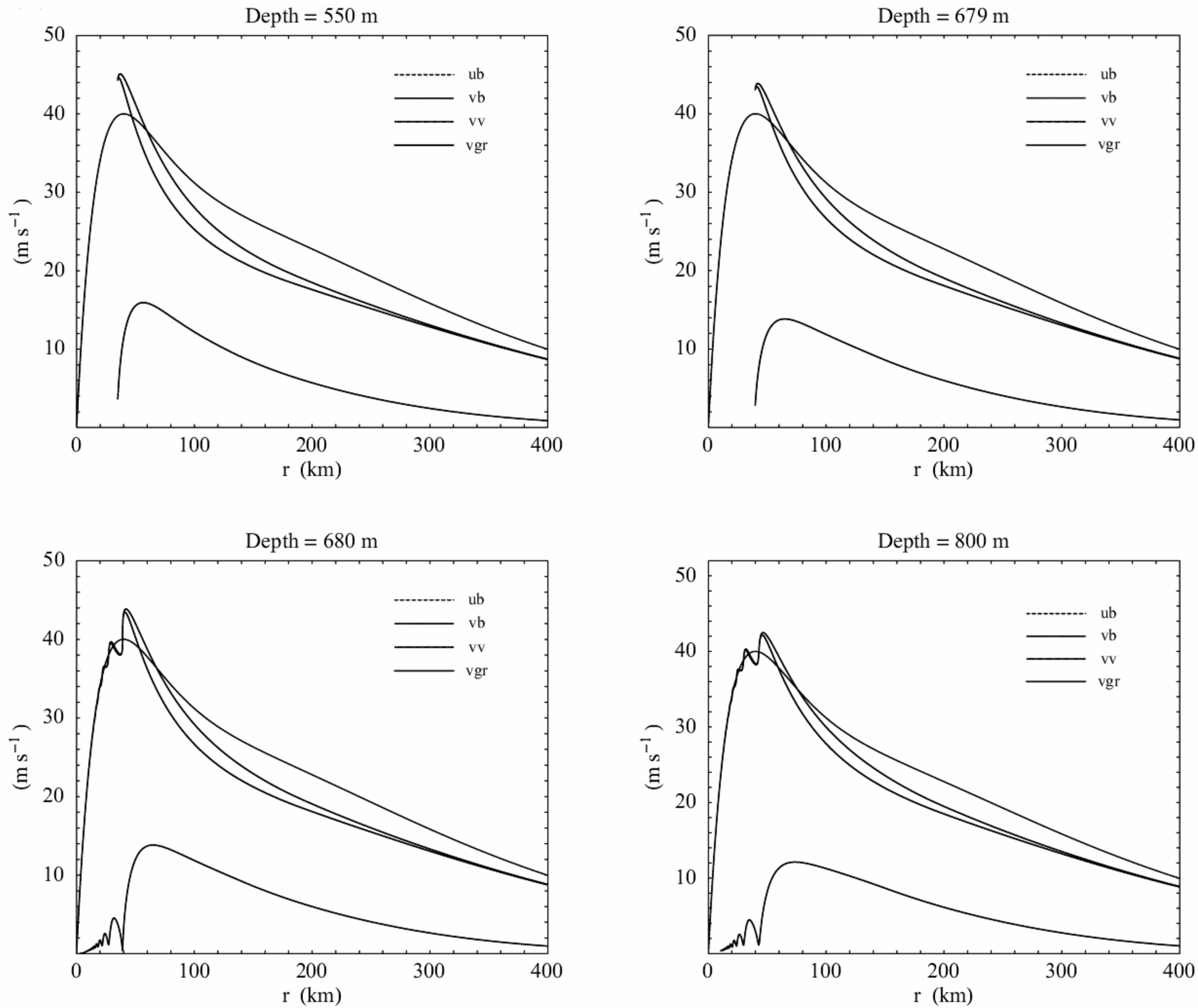
$$f(v_{gr} - v_b) = u_b \frac{w_{\delta_-}}{\delta} - \frac{\overline{(u'w')_\delta}}{\delta} - \frac{C_D}{\delta}(u_b^2 + v_b^2)^{1/2} u_b, \qquad (6)$$

$$f u_b = \frac{w_{\delta_-}}{\delta}(v_b - v_{gr}) - \frac{\overline{(v'w')_\delta}}{\delta} - \frac{C_D}{\delta}(u_b^2 + v_b^2)^{1/2} v_b. \qquad (7)$$

Radiative-convective equilibrium

$$C_D(u_b^2 + v_b^2)^2 (q_s - q_b) = w_{sc}(q_b - q_{\delta_+}), \qquad (8)$$

$$T_{\delta+} - T_b = [-\dot{R}/c_p - C_D(u_b^2 + v_b^2)^{1/2}(T_s - T_{as}) - C_D(u_b^2 + v_b^2)^{3/2}]/w_{sc}, \qquad (9)$$

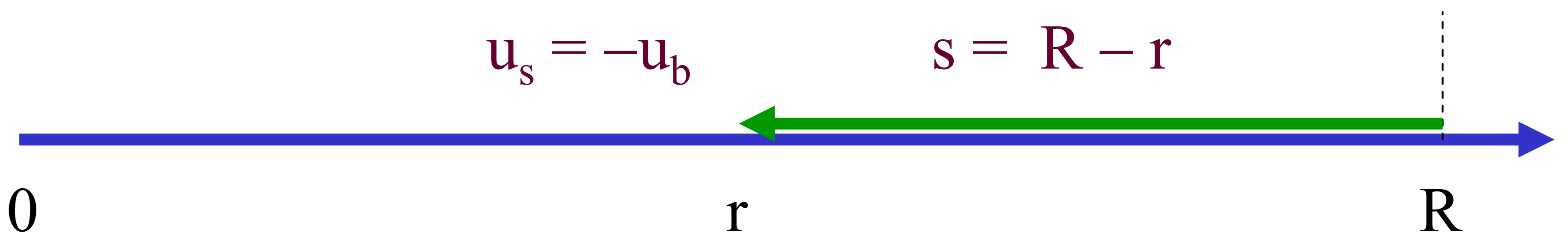


Depth = 550 m
Depth = 679 m
Depth = 680 m
Depth = 800 m
ub
vb
vv
vgr
$(m\,s^{-1})$
r (km)
0
100
200
300
400
10
20
30
40
50

Interpretation

- ❖ **Converging rings of air in the boundary layer spin faster, even though some angular momentum is lost to the surface.**
- ❖ **Supergradient wind speeds require large radial displacements with minimal frictional loss → high radial wind speeds.**
- ❖ **The only effect that can lead to an inward radial acceleration is the net pressure gradient. This increases with the degree to which the BL-flow is subgradient, i.e. as the effective frictional stress increases.**
- ❖ **A shallow boundary layer depth favours larger radial wind speeds, but smaller tangential wind speeds at outer radii.**
- ❖ **At inner radii, large radial wind speeds favour a steep increase in v_b → tendency to supergradient winds.**

Interpretation



$$\frac{du_s}{ds} = \frac{w_{\delta\_} + w_{sc}}{\delta} - \frac{(v_{gr} - v_b)}{u_s}\left(\frac{v_{gr} + v_b}{R - s} + f\right) - \frac{C_D}{\delta}(u_s^2 + v_b^2)^{\frac{1}{2}}, \tag{10}$$

(< 0) → ← (>0) (< 0) →

$$\frac{dv_b}{ds} = \frac{w_{\delta\_} + w_{sc}}{\delta}\frac{v_b - v_{gr}}{u_s} + \frac{v_b}{R - s} + f - \frac{C_D}{\delta}(u_s^2 + v_b^2)^{\frac{1}{2}}\frac{v_b}{u_s}, \tag{11}$$

> 0 > 0 < 0

Interpretation

- **The BL-solution breaks down if $u_b = 0$. This happens if v_b exceeds v_{gr} beyond a certain limit (i.e. if δ is small enough).**
- **Some questions:**
  - **How large is w_{sc}? Not much data!**
  - **How important is the downward mixing compared with the surface flux of a particular quantity?**

Implications and Limitations

- **Ubiquitous tendency to produce supergradient winds.**
- **→ if during the vortex intensification, the radial distribution of v_{gr} is determined by radial convergence above this layer associated with deep moist convection, the BL would tend to produce stronger tangential winds inside r_{max}. These stronger tangential winds would be advected vertically out of the boundary layer, leading to a spin up of the core region.**
- **Idea supported by the simple tropical-cyclone model examined by Emanuel (1997), in which the inner-core spin up appears to be orchestrated by the boundary layer, and also by the calculations of Wolfgang Ulrich.**

Kepert and Wang 2001

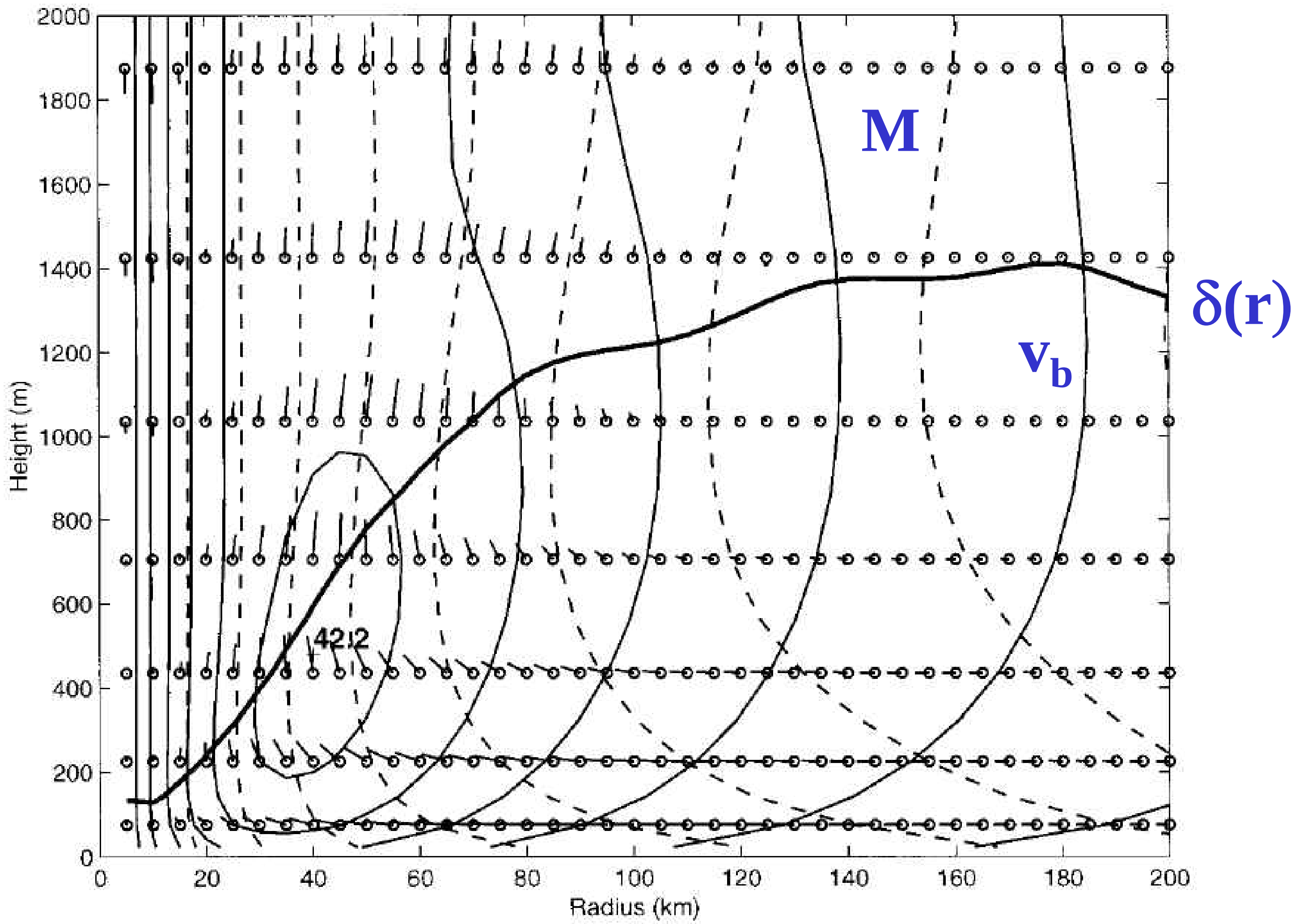


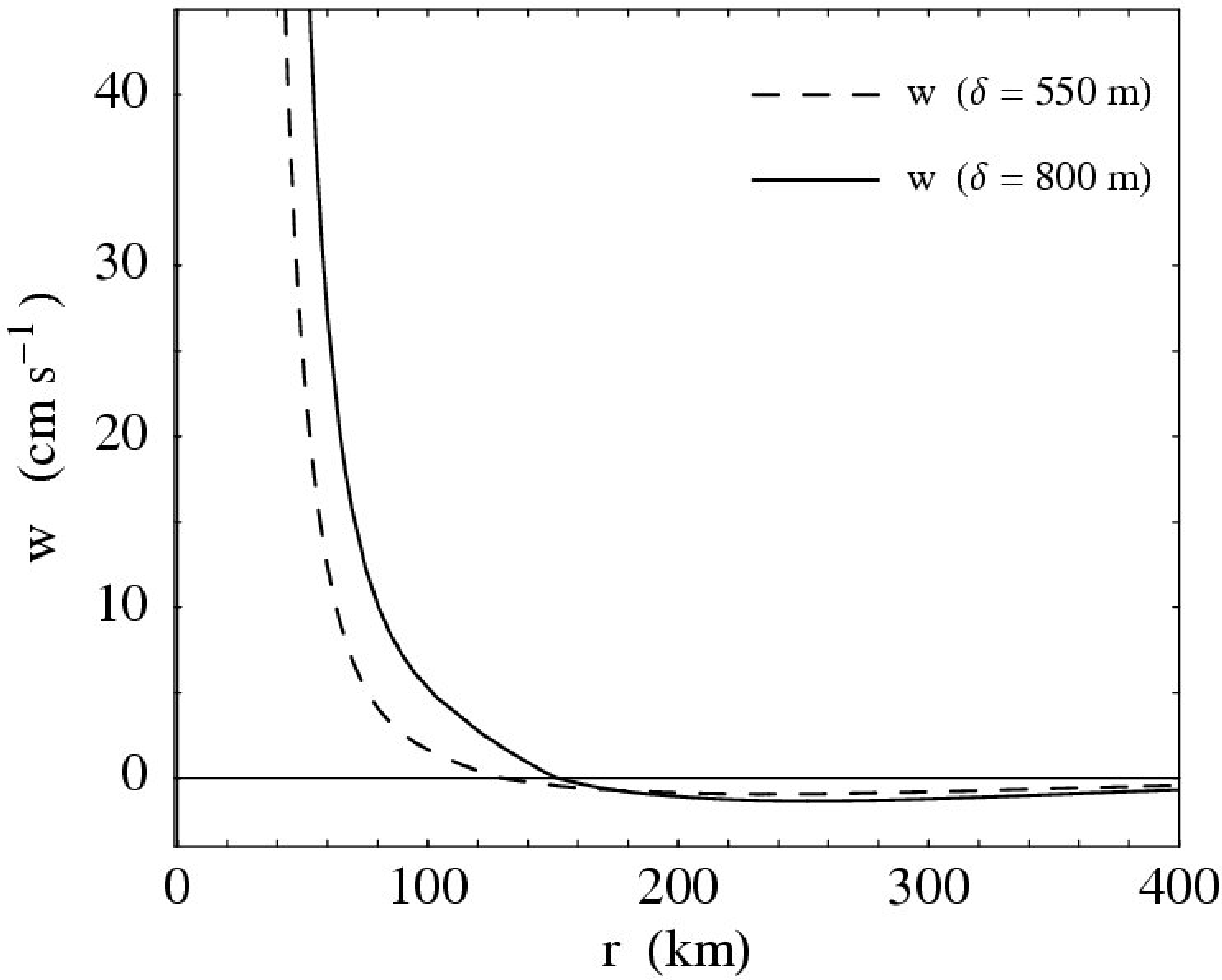
FIG. 2. Radial cross section through storm I. The solid light lines are contours of v, the dashed contours of M_a, the solid heavy line marks the top of the layer in which vertical diffusion plays a marked role in the dynamics, the vectors are of (u, w) with only every second model level shown.

Implications and Limitations

- The calculations suggest that the boundary layer is a fundamental aspect of the spin-up of the inner-core of a tropical cyclone.
- But, is it appropriate to prescribe the radial pressure gradient, above the boundary layer in regions where the flow is out of the boundary layer?
- Most previous boundary models have taken this approach (e.g. Smith 1968, Leslie and Smith 1970, Bode and Smith 1975, Shapiro 1983, Kepert 2001, Kepert and Wang 2001, Smith 2003).
- These calculations provide useful insights, but there is a limit to how far they can be taken.

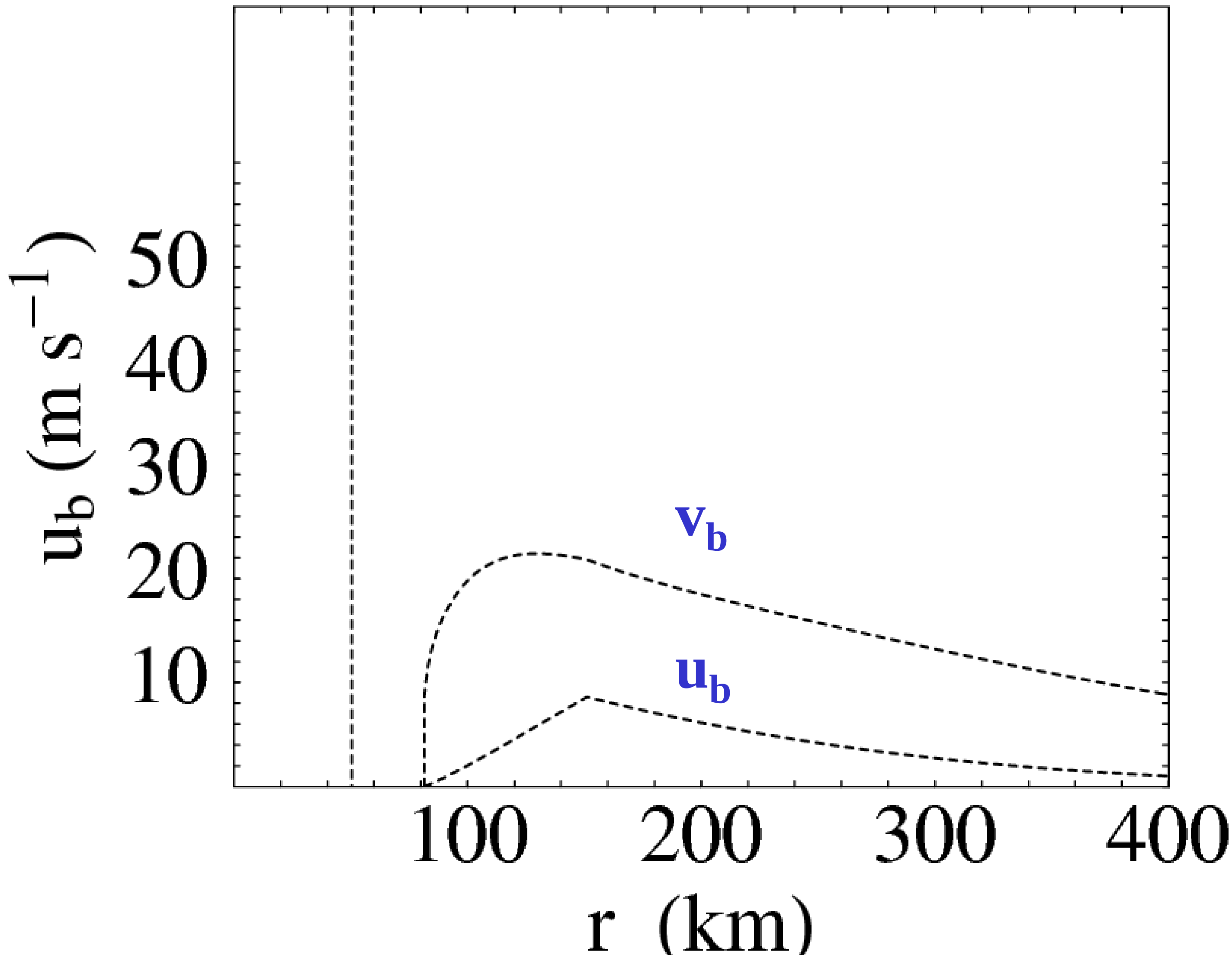
Possible extensions for future work

- **We could take $v_{gr} = v_b$ where $w_\delta > 0$.**
  - **➔ the pressure gradient would be determined by the assumption of gradient wind balance for $z > \delta$.**
  - **In general, this pressure gradient would not be in hydrostatic balance with the thermal field aloft.**
  - **➔ the outer part of the vortex produces an inward jet at the radius where $w_\delta = 0$.**
- **We could use the Emanuel constraint at $z = \delta$ when $w_\delta > 0$.**
  - **Now v_{gr} (and θ_e) would be determined consistently as part of the solution.**
  - **Assumption is that axisymmetric dynamics apply!**



w (δ = 550 m)
w (δ = 800 m)
40
30
20
10
0
w (cm s⁻¹)
0
100
200
300
400
r (km)

Take $v_{gr} = v_b$ where $w_d > 0$

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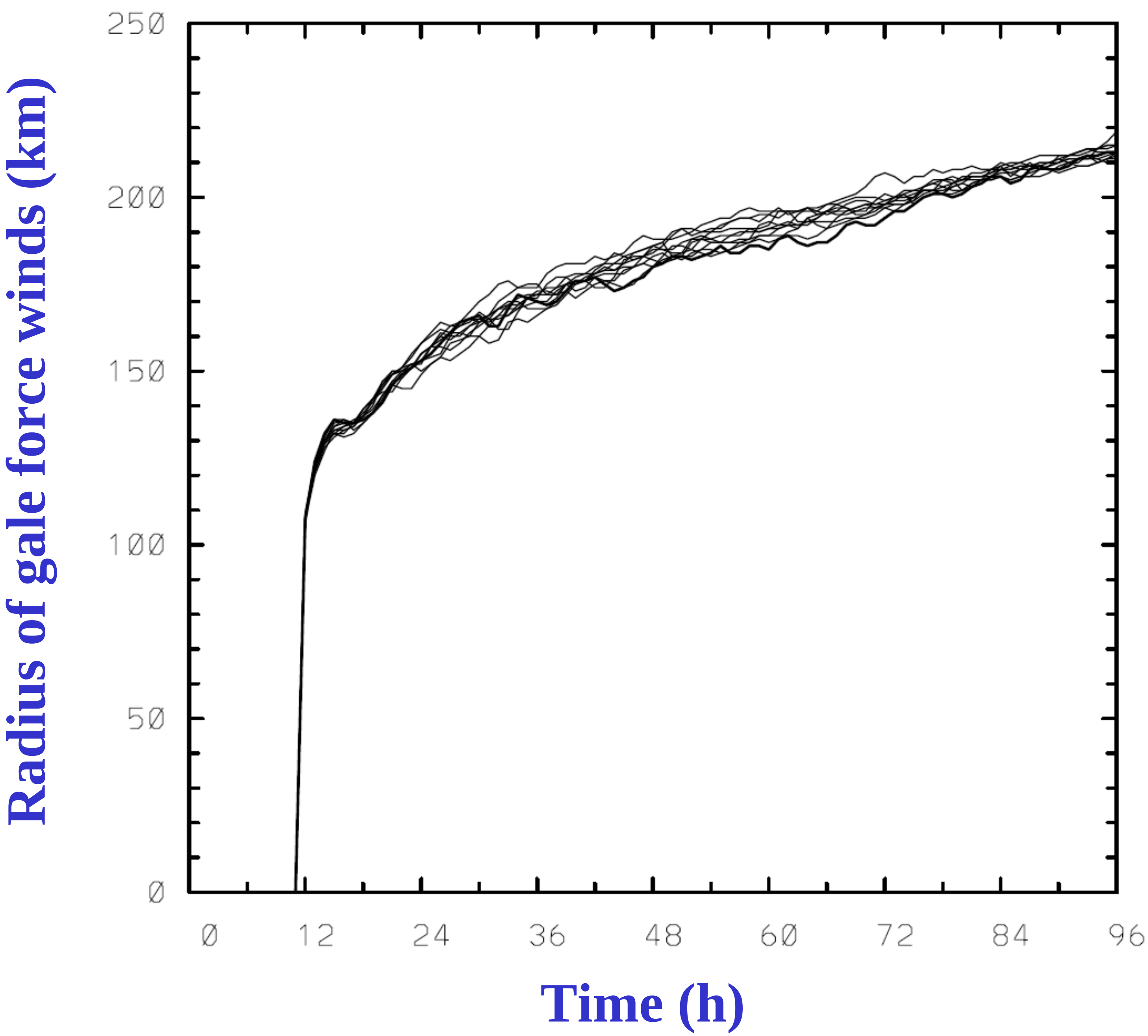
Motivation

- **Observations (e.g. Merrill, 1984) suggest that there is little relationship between the intensity of a tropical cyclone (measured by v_{max}) and its size (measured by $r_{gale\text{-}force\ winds}$).**
- **We know of no theories to account for the range of sizes of tropical cyclones that are observed.**
- **Question: How does the background rotation strength (characterized either by the Coriolis parameter or the width of the initial vortex) influence the intensity and size of a tropical cyclone?**

Work in progress!

Growth in size – Sang Nguyen – Ensemble calculations




The End Thank you!
20
15
10
-140
-145
-150
-155
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