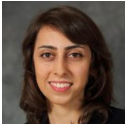
Optimal Regulated Firm Behaviour in Renewable Energy Certificate (REC) Markets



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Climate change

- Large-scale threat to society that must be counteracted
- Major contribution is burning of **dirty** energy sources, releasing CO_2 into atmosphere
- Dirty energy cheap relative to clean energy (historically), but has an **externality** (CO_2 emissions)
- As a society, how can we make clean energy relatively cheaper?
- One idea: enforce penalties for dirty energy generation

Means to price carbon emissions

- Carbon tax
- Emissions markets / **Renewable Energy Certificate** markets
- Market based method designed to incentivise clean energy
- For power generators, a regulator wants to impose a **floor** on the proportion of the energy mix sold to grid from renewables
- **Renewable Energy Certificates (RECs)** used to enforce this

REC markets

- Firms obtain a REC by **generating** electricity from renewables
- This **decouples** the electricity from its ‘green-ness’
- Floor set on the **RECs inventory** of power generator at T
- RECs submitted to regulator annually - firms face a **monetary penalty** for each lacking certificate
- Firms can also **trade** RECs on a market - price derived through supply and demand

SREC Markets

- REC markets are **generic** with respect to the renewable energy sources used
- Can also design markets with a **'carve-out'** for a specific energy type, such as solar, hydroelectric, wind, etc.
- In the case of solar, this results in an **SREC market**
- SREC markets exist in many of the northeastern states in the US; New Jersey's is the largest and most mature

SREC markets

"On January 18, 2019, the mayor of the District of Columbia signed the CleanEnergy DC Omnibus Bill Amendment Act of 2018, increasing the District's Renewable Portfolio Standard to 100% by 2032 and the solar carve-out to 10% by 2041."

– source: SRECTrade.com

What problem do we address?

- We consider firms regulated by a REC market
- Is there set of **strategies** across **all** regulated firms such that we achieve a **Nash equilibrium** and an **endogenous price**?
  - Strategies correspond to REC generation and trading behaviour
- Accounting for:
  - **Generation** costs
  - **Trading** costs
  - **Behaviours** of other agents
  - Agent **heterogeneity**

Relationship to the Literature

Non-Exhaustive list...

- C&T Literature
  - central planner: Seifert, Uhrig-Homburg, & Wagner (2008)
  - multi-period: Hitzemann & Uhrig-Homburg (2018)
  - Functional analysis: Carmona, Fehr, Hin (2009); Carmona, Fehr, Hinz, & Porchet (2010)
- REC Literature
  - Price Modeling: Coulon, Khazaei, & Powell (2015)
  - Single Agent: Shrivats & J (2019)
- MFGs
  - Price formation: Gomes & Saúde (2019); Fujii & Takahashi (2020)
  - Fracking: Chan & Sircar (2017)

Our setup

- Formulate optimal behaviour as a **stochastic game**
- Firm $i \in \{1, ..., N\}$ has **controls** (α_t^i), **state variables** (X_t^i), and a **performance criterion** (J^i) they seek to minimize, representing costs
- As other firms' states matter, the **empirical distribution** $(\mu^{[N]})$ of **states** play a key role
- Since REC price depends on all agents' states, denote by $S_t^{\mu_t^{[N]}}$

Some global notation

- P - penalty per unit of non-compliance
- R - REC requirement to submit to regulator
- ζ - generation cost parameter
- γ - trading cost parameter

Our setup, continued

- $\alpha_t^i = (g_t^i, \Gamma_t^i)$ - planned REC generation and trading at t
- X_t^i - REC inventory at t
- $\mu_t^{[N]}$ - empirical distribution across firms of REC inventory at t
- State dynamics and costs:

$$dX_t^i = (g_t^i + \Gamma_t^i)dt + \sigma dW_t^i$$

$$f(t, X_t^i, \mu_t^{[N]}, \alpha_t^i) = \underbrace{\tfrac{\zeta}{2}(g_t^i - h_t)^2}_{\text{Generation cost}} + \underbrace{\tfrac{\gamma}{2}(\Gamma_t^i)^2}_{\text{Transaction cost}} + \underbrace{S_t^{\mu_t^{[N]}} \Gamma_t^i}_{\text{Trading Loss}}$$

$$g(X_T^i, \mu_t^{[N]}) = \underbrace{P(R - X_T^i)_+}_{\text{Non-compliance cost}} \xrightarrow[\text{regularize}]{} P\, F_\delta(R - X_T^i)$$

$$J^i(\alpha^i, \alpha^{-i}) := \mathbb{E}\left[\int_0^T f(t, X_t^i, \mu_t^{[N]}, \alpha_t^i)dt + g(X_T^i, \mu_T^{[N]})\right]$$

Information restriction

- Agents' adapt their strategy to their **personal filtration**

$$\mathcal{G}_t^i := \sigma\left((\boldsymbol{X_u^i})_{u\in[0,t]}\right) \vee \sigma\left(\left(S_t^{\boldsymbol{\mu_t^{[N]}}}\right)_{u\in[0,t]}\right)$$

- $\mathcal{A}^i$ is the **admissible set** of controls:
  - Square-integrable
  - $\mathbf{g}_t^i \geq 0$ for all t
  - $\mathbf{g}^i, \boldsymbol{\Gamma}^i$ are $\mathcal{G}^i$-adapted

Our goal

- We seek a **collection of controls** $\{\boldsymbol{\alpha}^{\boldsymbol{i}} = (\mathbf{g}^{\boldsymbol{i}}, \boldsymbol{\Gamma}^{\boldsymbol{i}}) \in \mathcal{A}^i : i \in \mathfrak{N}\}$ that from the Nash equilibrium:

$$\boldsymbol{\alpha}^{\boldsymbol{i}} = \arg\inf_{\boldsymbol{\alpha} \in \mathcal{A}^i} J^i(\boldsymbol{\alpha}, \alpha^{-i}), \qquad \forall i \in \mathfrak{N}.$$

- and impose the **clearing condition**: $\frac{1}{N} \sum_{i=1}^{N} \boldsymbol{\Gamma}_{\boldsymbol{t}}^{\boldsymbol{i}} = 0$
- This is is a difficult problem, due to dimensionality and information restriction
[heterogeneity adds another complexity layer]

The MFG limit

- Taking $N \to \infty$, each agent becomes **negligible** - as such, agents interact with the **population distribution of states**, rather than directly
- The empirical distribution converges to the mean field distribution $\boldsymbol{\mu_t^{[N]}} \to \boldsymbol{\mu_t}$
  - $\boldsymbol{\mu_t}(A) = \mathbb{P}(\boldsymbol{X_t^{\cdot}} \in A)$
  - $S_t^{\boldsymbol{\mu_t^{[N]}}} \to S_t^{\boldsymbol{\mu_t}}$

The MFG limit

- Firm i aims to minimise the **performance criterion** *[now with infinite population dynamics]*

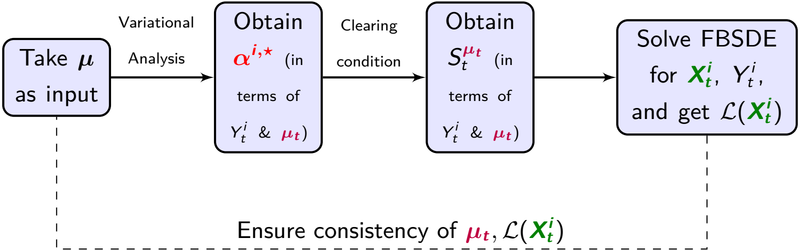
$$\inf_{\alpha^i \in \mathcal{A}^i} \overline{J^i}(\alpha^i; \mu) := \mathbb{E}\left[\int_0^T f(t, X_t^i, \mu_t, \alpha_t^i) dt + g(X_T^i, \mu_T)\right]$$

- And we seek the **Nash-Equlibirum**: $\{\alpha^{i,\star} \in \mathcal{A}^i\}_{i=1}^{\infty}$ s.t.

$$\alpha^{i,\star} = \arg\inf_{\alpha \in \mathcal{A}^i} \overline{J^i}(\alpha; \mu), \qquad \forall i = 1, 2, \ldots$$

- As well as the **mean field** that is **consistent with** $\mathcal{L}(X_t^{\cdot})$

Our strategy



Optimal controls

Proposition (Convexity of Cost Functional)

The functional $\overline{J^i}$ is strictly convex *in $\mathcal{A}^i$, given a mean-field distribution $\boldsymbol{\mu}$.*

Proposition (Existence of Gâteaux derivative)

The functional $\overline{J^i}(\cdot,\cdot;\boldsymbol{\mu})$, for $i \in \mathfrak{N}_k$, is ***everywhere Gâteaux differentiable*** *in $\mathcal{A}^i$ in all directions. The Gâteaux derivative at $(g,\Gamma) \in \mathcal{A}^i$ in the direction of $(\omega^g,\omega^\Gamma) \in \mathcal{A}^i$ is*

$$\langle \mathcal{D}\overline{J^i}(g^i,\Gamma^i;\boldsymbol{\mu}),\omega^g\rangle = \mathbb{E}\left[\int_0^T \omega_t^g\left(\zeta^k(g_t^i - h_t^k) - PY_t^i\right)dt\right],$$

$$\langle \mathcal{D}\overline{J^i}(g^i,\Gamma^i;\boldsymbol{\mu}),\omega^\Gamma\rangle = \mathbb{E}\left[\int_0^T \omega_t^\Gamma\left(\gamma^k\Gamma_t + S_t^{\boldsymbol{\mu}} - PY_t^i\right)dt\right],$$

where

$$Y_t^i := \mathbb{E}\left[\, F_\delta'(R^k - X_T^i) \,\middle|\, \mathcal{G}_t^i\right], \qquad \textbf{\textit{Non-compliance Probability}}$$

Optimal Controls

Proposition (Optimal Controls)

Given a mean-field flow $\boldsymbol{\mu}$, the controls $\{g_t^{i,\star}, \Gamma_t^{i,\star}\}_{i=1,2,\ldots}$ is the ***unique Nash-equilibrium*** *iff $\forall\, i \in \mathfrak{N}_k$, $(g_t^{i,\star}, \Gamma_t^{i,\star}) \in \mathcal{A}^i$, where*

$$
\begin{aligned}
\boldsymbol{g_t^{i,\star}} &= h_t^k + \tfrac{P}{\zeta^k} Y_t^i, \quad \textit{and} \\
\boldsymbol{\Gamma_t^{i,\star}} &= \tfrac{1}{\gamma^k}\left(P Y_t^i - S_t^{\boldsymbol{\mu}}\right).
\end{aligned}
$$

Proposition (Equilibrium Price.)

The ***equilibrium SREC price*** *is given by*

$$
S_t^{\boldsymbol{\mu}} = P \sum_{k \in \mathcal{K}} \eta_k \, \mathbb{E}^{\mu_t^{(k)}}[Y_t^{\cdot}],
$$

where $\eta_k = \frac{\pi_k}{\gamma^k} \Big/ \sum_{k' \in \mathcal{K}} \frac{\pi_{k'}}{\gamma^{k'}} \in (0,1), \qquad \forall\, k \in \mathcal{K}.$

An FBSDE for Optimality

Corollary (FBSDE Formulation)

The triple (X_t^i, Y_t^i, Z_t^i)*, given* $(\boldsymbol{\mu}_t)_{t\in\mathfrak{T}}$*, for* $i \in \mathfrak{N}_k$*, satisfies the FBSDE*

$$
\begin{aligned}
d\boldsymbol{X_t^i} &= \left(h_t^k + \upsilon^k\, P Y_t^i - P \tfrac{1}{\tilde{\gamma}^k} \sum_{k\in\mathcal{K}} \tfrac{\pi_k}{\gamma^k}\, \mathbb{E}^{\boldsymbol{\mu_t^{(k)}}}[Y_t^\cdot] \right) dt + \sigma_t^k dW_t^i, \\
X_0^i &= \xi^i, \\
dY_t^i &= Z_t^i\, dW_t^i, \\
Y_T^i &= F_\delta'(R^k - X_T^i),
\end{aligned}
$$

where

$$
\tilde{\gamma}^k := \gamma^k \sum_{k'\in\mathcal{K}} \frac{\pi_{k'}}{\gamma^{k'}}, \qquad \upsilon^k := \frac{1}{\gamma^k} + \frac{1}{\zeta^k},
$$

and $\xi^i \sim \mu_0^{(k)}$.

REC price process

- Recall that we find

$$S_t^{\boldsymbol{\mu}} = P \sum_{k \in \mathcal{K}} \eta_k \, \mathbb{E}^{\mu_t^{(k)}}[Y_t^{\cdot}], \approx P \sum_{k \in \mathcal{K}} \eta_k \, \mathbb{P}^{\mu_t^{(k)}}(X_t^{\cdot} < R^k),$$

- Contrasts with previous work (Hitzemann & Uhrig-Homburg 2012, among others)
  - Others find $S_t \propto \mathbb{P}_t(\sum_{i=1}^N X_t^i < N\,R)$
  - Difference stems from **trading frictions** and **heterogeneity**

MV-FBSDE

- We have assumed $\boldsymbol{\mu}^{(\boldsymbol{k})}$ is given exogenously, when it is in fact emergent from the firm's behaviour
- We must rather solve the MV-FBSDE for $\boldsymbol{\mu}^{(\boldsymbol{k})}$, Y_t^i, and $\boldsymbol{X_t^i}$, s.t. $\mathcal{L}(X_t^{i_k}) = \boldsymbol{\mu}^{(\boldsymbol{k})}$

Proposition (Existence of a Fixed Point)

There exists a mean-field distribution $\boldsymbol{\mu}$ and a progressively measurable triple (X^i, Y^i, Z^i) that satisfies the MV-FBSDE, such that $\mu_t^{(k)}$ coincides with $\mathcal{L}(X_t^i)$ for all $i \in \mathfrak{N}_k,\ k \in \mathcal{K}$.

MV-FBSDE

- Introduce an auxiliary MFG optimization problem (without clearing) that generates the previous MV-FBSDE

$$\inf_{\tilde{g}^i, \tilde{\Gamma}^i \in \mathcal{A}^i} \mathbb{E}\left[\int_0^T \left(\frac{\zeta^k}{2}(\tilde{g}_t^i - h_t^k)^2 dt + \frac{\gamma^k}{2}(\tilde{\Gamma}_t^i)^2 + \tilde{\Gamma}_t^i \, S(t, \tilde{\boldsymbol{\mu}})\right) dt + P\, F_\delta'(R^k - \tilde{X}_T^i)\right],$$

$$\text{subject to} \qquad d\tilde{X}_t^i = (\tilde{g}_t^i + \tilde{\Gamma}_t^i)dt + \sigma\, dW_t^i,$$

$$S(t, \tilde{\boldsymbol{\mu}}) = P \sum_{k \in \mathcal{K}} \eta_k \, \mathbb{E}^{\tilde{\mu}_t^{(k)}}\left[\mathbb{E}_t\left[F_\delta'(R^k - \tilde{X}_T^i)\right]\right],$$

- Write out the corresponding MV-FBSDE
- From Delarue (2002), $\exists$ a Markov representation $\tilde{Y}_t^i = \tilde{Y}^{(k)}(t, \tilde{X}_t^i; \tilde{\mu}_t)$
- This allows us to use Carmona & Delarue (2013) to argue the existence of a solution to the auxiliary problem
- The auxiliary problem is equivalent to the original problem with clearing conditions

MV-FBSDE

Proposition (Uniqueness of a Fixed Point)

There is at most one solution to the MV-FBSDE.

- suppose we have $\boldsymbol{\mu}$ and $\hat{\boldsymbol{\mu}}$ that solve the MV-FBSDE and corresponding optimal controls $\alpha^*, \hat{\alpha}^*$
- Prop 2.5 in Carmona & Delarue (2013) gives

$$\begin{aligned}
&\bar{J}^i(\alpha_t^\star; \boldsymbol{\mu}) + \lambda^k \, \mathbb{E}\left[\int_0^T \|\alpha_t^\star - \hat{\alpha}_t^\star\|^2 \, dt\right] \\
&\quad \leq \bar{J}^i([\hat{\alpha}_t^\star, \hat{\boldsymbol{\mu}}]; \boldsymbol{\mu}) \\
&\quad = \mathbb{E}\left[\int_0^T \left(\tfrac{\zeta^k}{2}(\hat{g}_t^\star - h_t^k)^2 + \tfrac{\gamma^k}{2}(\hat{\Gamma}_t^{i,\star})^2 + S_t^{\boldsymbol{\mu}} \hat{\Gamma}_t^\star\right) dt + PF_\delta'(R^k - \hat{X}_T)\right]
\end{aligned}$$

- $[\hat{\alpha}_t^\star, \hat{\boldsymbol{\mu}}]$ indicates the measure flow in the drift of $\hat{X}^\star$ is $\hat{\boldsymbol{\mu}}$, whereas the measure flow in the cost functional is $\boldsymbol{\mu}$

MV-FBSDE

- Interchanging the role of $\boldsymbol{\mu}$ and $\hat{\boldsymbol{\mu}}$, summing and then summing over sub-populations:

$$\sum_{k\in\mathcal{K}} \pi_k \, 2\lambda^k \mathbb{E}\left[\int_0^T \left\|\alpha^{i_k,\star} - \hat{\alpha}^{i_k,\star}\right\|^2 dt\right]$$

$$\leq \int_0^T \sum_{k\in\mathcal{K}} \pi_k \left(\int_{\mathbb{R}} \hat{\Gamma}_t^{i_k,\star} \hat{\mu}_t^{(k)}(dx) - \int_{\mathbb{R}} \Gamma_t^{i_k,\star} \mu_t^{(k)}(dx)\right) (S_t^{\boldsymbol{\mu}} - S_t^{\hat{\boldsymbol{\mu}}})\, dt$$

$$= 0 \qquad \textbf{due to clearing conditions}$$

- Hence, we must have $\alpha^{i_k} = \hat{\alpha}^{i_k}$.

Numerical results - parameter choice

n	Δt	T	P (\$/SREC)	R (SREC)	K
1	$\frac{1}{52}$	1	1	1	2

Table: Compliance parameters.

Sub-population	π_k	h^k	σ^k	ζ^k	γ^k	ν_0^k	ψ_0^k
$k=1$	0.25	0.2	0.1	1.75	1.25	0.6	0.1
$k=2$	0.75	0.5	0.15	1.25	1.75	0.2	0.1

Table: Model Parameters.

Numerical Scheme

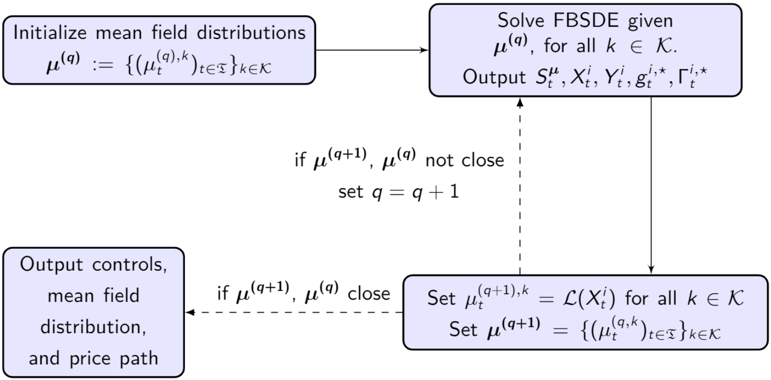
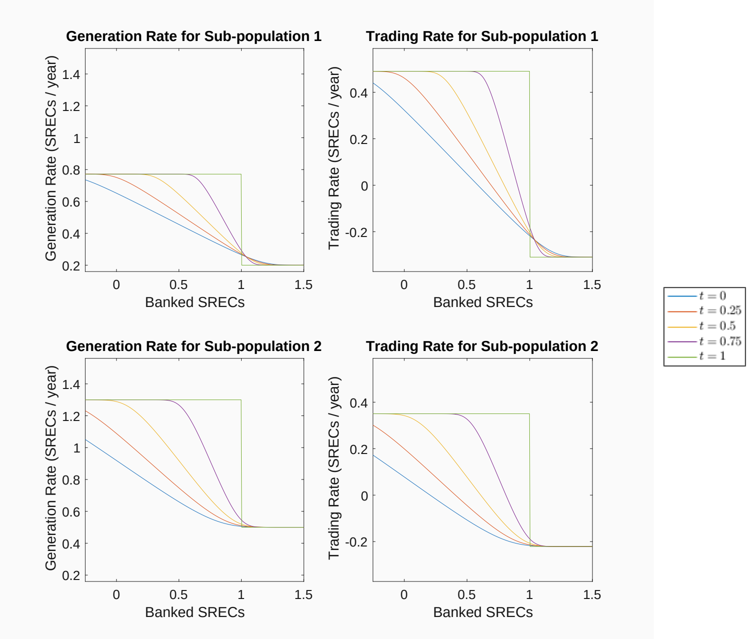


Figure: Numerical Scheme Diagram

Optimal behaviour

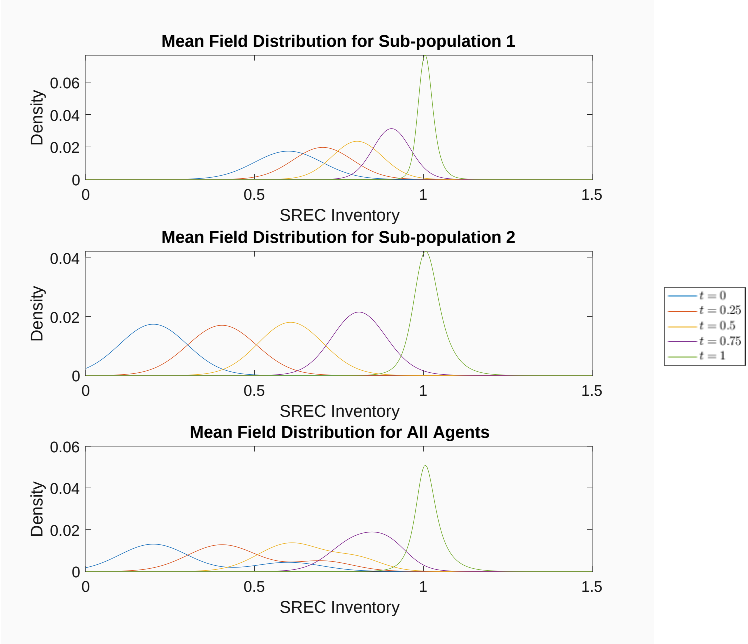


Intuition of optimal behaviour

Three regimes (heuristically)

- Marginal benefit of additional SREC is P
- Marginal benefit of additional SREC is between 0 and P
- Marginal benefit of additional SREC is 0

Mean field distribution



Finite Player

- For the finite player game, we assume agents use **MFG optimal strategies** applied to their **observable states**

$$g_t^{i,\star} = h_t^k + \tfrac{P}{\zeta^k}\, \boldsymbol{Y}^{(\boldsymbol{k})}(t, \boldsymbol{X}_{\boldsymbol{t}}^{\boldsymbol{i},[\boldsymbol{N}]}; \boldsymbol{\mu}), \quad \text{and}$$

$$\Gamma_t^{i,\star} = \tfrac{1}{\gamma^k}\left(P\, \boldsymbol{Y}^{(\boldsymbol{k})}(t, \boldsymbol{X}_{\boldsymbol{t}}^{\boldsymbol{i},[\boldsymbol{N}]}; \boldsymbol{\mu}) - S_t^{\mu^{[N]}}\right).$$

- The equilibrium price is

$$S_t^{\mu^{[N]}} = P\frac{\displaystyle\sum_{k\in\mathcal{K}} \frac{N_k}{N\gamma^k}\, \frac{1}{N_k} \sum_{i\in\mathfrak{N}_k} \boldsymbol{Y}^{(\boldsymbol{k})}(t, \boldsymbol{X}_{\boldsymbol{t}}^{\boldsymbol{i},[\boldsymbol{N}]}; \boldsymbol{\mu})}{\displaystyle\sum_{k\in\mathcal{K}} \frac{N_k}{N\gamma^k}}.$$

Finite Player

Definition

(ϵ-Nash equilibrium) A set of controls $\mathcal{U}^\star = \{(g^{i,\star}, \Gamma^{i,\star}) \in \mathcal{A}^i, i \in \mathfrak{N}\}$ forms an ϵ-Nash equilibrium if there exists an $\epsilon > 0$ s.t.

$$J^i(g^{i,\star}, \Gamma^{i,\star}, \boldsymbol{g}^{-i,\star}, \boldsymbol{\Gamma}^{-i,\star}) \geq \inf_{(g^i,\Gamma^i)\in\mathcal{A}^i} J^i(g^i, \Gamma^i, \boldsymbol{g}^{-i,\star}, \boldsymbol{\Gamma}^{-i,\star})$$
$$\geq J^i(g^{i,\star}, \Gamma^{i,\star}, \boldsymbol{g}^{-i,\star}, \boldsymbol{\Gamma}^{-i,\star}) - \epsilon, \qquad \forall\, i \in \mathfrak{N}_k, k \in \mathcal{K}.$$

Proposition

Take an arbitrary admissible controls $(g, \Gamma) \in \mathcal{A}^i$ and $(\boldsymbol{g}^{-i,\star}, \boldsymbol{\Gamma}^{-i,\star})$ for agents $j \in \mathfrak{N}$, $j \neq i$. Suppose there exists a sequence $\{\delta_N\}_{N=1}^{\infty}$ such that $\delta_N \to 0$ and $\left|\frac{N_k}{N} - \pi_k\right| = o(\delta_N)$, for all $k \in \mathcal{K}$. Then

$$\left|J^i(g, \Gamma, \boldsymbol{g}^{-i,\star}, \boldsymbol{\Gamma}^{-i,\star}) - \overline{J^i}(g, \Gamma; \boldsymbol{\mu})\right| = o\left(\delta_N\right) + o\left(\tfrac{1}{\sqrt{N}}\right),$$

Theorem (ϵ-Nash Property)

Under some mild assumptions, the optimal controls $\{\boldsymbol{g}^\star, \boldsymbol{\Gamma}^\star\}$ form an ϵ-Nash equilibria.

Finite-player simulation

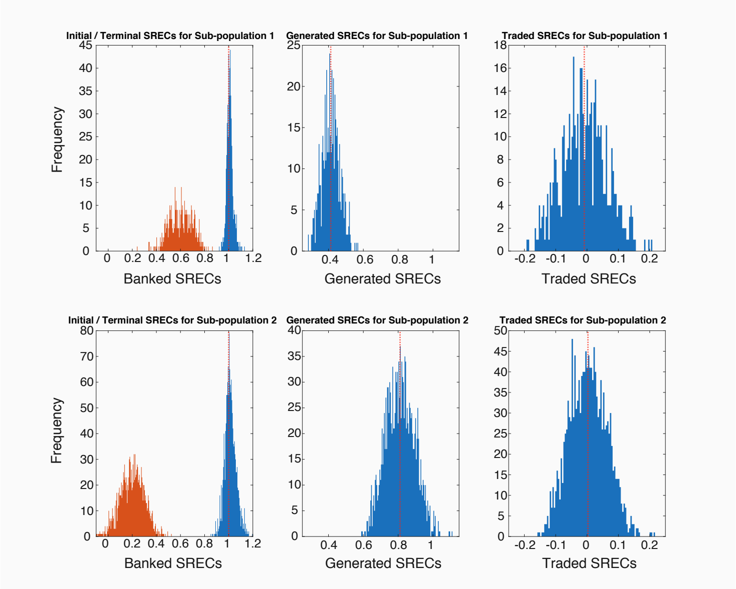


Figure: Histograms of initial (red) and terminal (blue) SRECs (left panel), generation rate (middle panel), trading rate (right panel).

Finite-player REC price

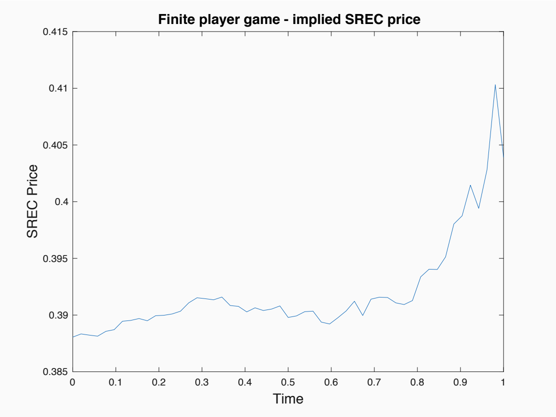
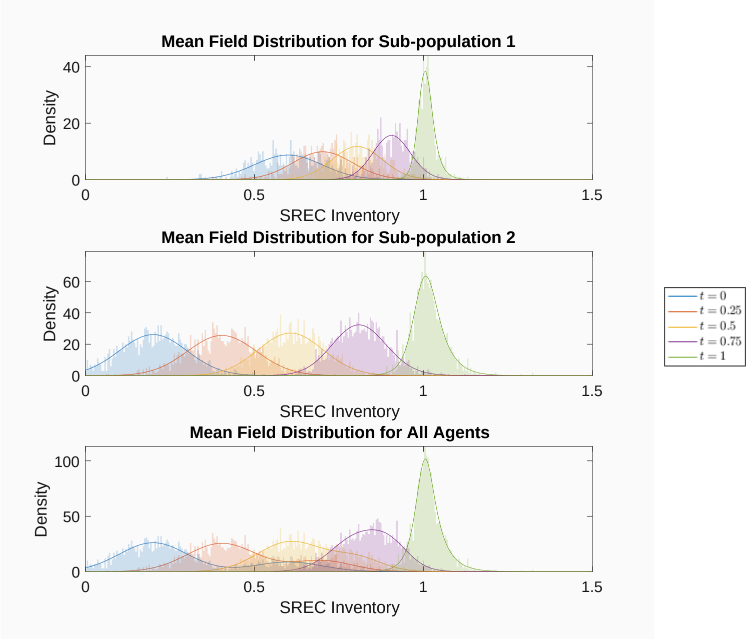


Figure: Equilibrium SREC price in finite-player setting.

MFG vs empirical distributions



Main Contributions

- Endogenonize Price Formation in REC markets
- Incorporated
  - Heterogeneous Agents
  - Trading Frictions
  - Generation Costs
- Main Theoretical Results:
  - Uniqueness/Existence of MV-FBSDE with Equilibrium Pricing
  - ϵ-Nash property
- Future Work:
  - More realistic cost structure
  - Common noise components
  - Major-minor players
  - Generalized Equilibrium pricing with MFGs

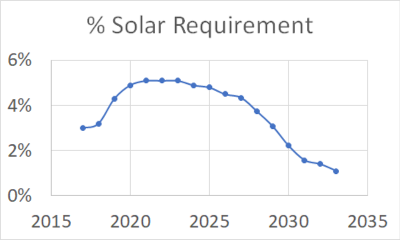
Thank you for your attention!

http://sebastian.statsistics.utoronto.ca

based on **A Mean-Field Game Approach to Equilibrium Pricing, Optimal Generation, and Trading in Solar Renewable Energy Certificate Markets**

https://arxiv.org/abs/2003.04938

SREC markets



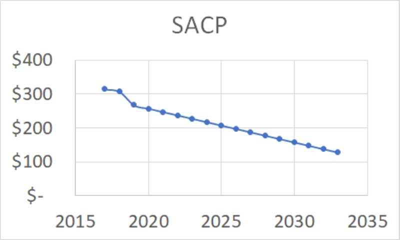


Figure: New Jersey SREC Market Requirements set by State Legislation. Allow 5 banking years. Data from SRECTrade.com

- 1 SREC = 1 MWh of solar electricity
- A 10 kW facility generates around 12 SRECs annually

New Jersey SREC market

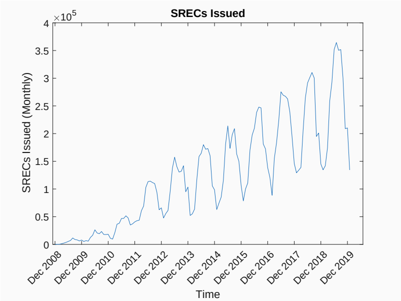


Figure: Issued SRECs from 2008 - 2019

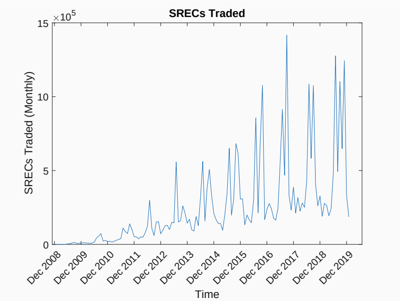


Figure: Traded SRECs (all vintages) from 2008 - 2019

New Jersey SREC market

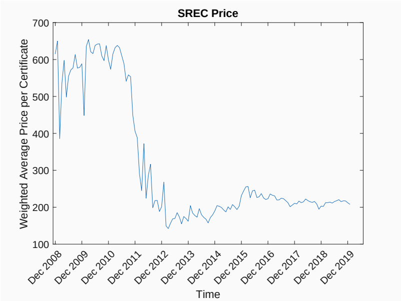


Figure: New Jersey SREC prices from 2008 - 2019