Machine learning for accurate gravitational waves modelling

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Introduction

‣ Gravitational waves (GW) templates are essential
  • Most sensitive gravitational waves detection pipelines are based on modelled searches
  • Parameter estimation requires template banks (Markov-chain based)

‣ Waveform modelling is complex
  • Detection of binary system: large parameter space (min 15 parameters)
  • Merger: strong-field regime of general relativity (GR), non-linear physics
  • Parameter estimation bottleneck: no propagation of waveform systematics

‣ Machine learning has potential
  • Different applications according to purposes
  • Review in this talk
Current modelling approaches

- Point estimates with machine learning: remnant properties
- Point estimates with machine learning: waveform coefficients
- Waveform generation with machine learning
Anatomy of a gravitational waveform

- GW150914-like signal:

  inspiral, weak field
  can be computed with post-Newtonian (pN) expansion
  far from innermost stable circular orbit (ISCO)

  merger, strong field
  requires numerical relativity

  ringdown, strong to weak field
  can be modelled as a sum of quasinormal modes

Source: Sound of Spacetime

GW150914-like signal: $m_1 \quad m_2 \quad \vec{L} \quad \vec{\chi}_1 \quad \vec{\chi}_2$
GW computation: numerical relativity simulations

- **Numerical relativity (NR) simulations**
  - Use 3+1 decomposition of spacetime to perform evolution (à la ADM)
  - Black hole singularity is excised or analytically factored (puncture)
  - Issue with well-posedness: first GW simulation in 2005
  - Accuracy depends on mesh refinement

Source: S. Caudill
### GW computation: numerical relativity catalogs

- **Several numerical relativity catalogs**
  - 3 are active and open-access
  - Table from SXS simulation catalog

| Catalog          | Started | Updating? | Simulations | $m_1/m_2$ range | $|\chi_1|$ range | $|\chi_2|$ range | Precessing? | Median $N_{\text{ec}}$ | Public? |
|------------------|---------|-----------|-------------|-----------------|-----------------|-----------------|-------------|-----------------------|---------|
| NINJA [98,115]   | 2008    | x         | 63          | 1–10            | 0–0.95          | 0–0.95          | x           | 15                    | x       |
| NRAR [120]       | 2013    | x         | 25          | 1–10            | 0–0.8           | 0–0.6           | ✓           | 24                    | ✓       |
| Georgia Tech [122]| 2016    | ✓         | 452         | 1–15            | 0–0.8           | 0–0.8           | ✓           | 4                     | ✓       |
| RIT (2017) [123] | 2017    | ✓         | 126         | 1–6             | 0–0.85          | 0–0.85          | ✓           | 16                    | ✓       |
| RIT (2019) [124] | 2017    | ✓         | 320         | 1–6             | 0–0.95          | 0–0.95          | ✓           | 19                    | ✓       |
| NCSA (2019) [125]| 2019    | x         | 89          | 1–10            | 0               | 0               | x           | 20                    | x       |
| SXS (2018)       | 2013    | ✓         | 337         | 1–10            | 0–0.995         | 0–0.995         | ✓           | 23                    | ✓       |
| SXS (2019)       | 2013    | ✓         | 2018        | 1–10            | 0–0.998         | 0–0.998         | ✓           | 39                    | ✓       |

arXiv:1904.04831
GW modelling: effective one-body formalism

- Effective one-body (EOB) formalism
  - 2-body dynamics mapped as 1 body in effective metric
  - Very accurate at inspiral, calibrated to NR simulations for LIGO-Virgo models

Source: T. Damour
GW modelling: effective one-body formalism

- SEOBNR models
  - Includes higher harmonics up to $(l, m) = (5, 5)$
  - Calibrated to precessing waveform with $\chi \rightarrow 0.8$

GW modelling: phenomenological approaches

- IMRPhenom approach
  - Different region of the waveform are modelled by different ansatz
  - Frequency and time domain models
  - Inspiral: pN + pseudo-pN terms
  - Intermediate: phenomenological ansatz
  - Ringdown: quasimormal modes

\[ h(M_f) \]

\[ \phi(M_f) \]


GW modelling: phenomenological approaches

- **IMRPhenom models**
  - Hierarchical fit as a function of mass ratio, effective spin and spin difference
  - Mapping from non-precessing to precessing case
  - Higher harmonics up to \((l, m) = (4,4)\)

Content

- Current modelling approaches
- **Point estimates with machine learning:** remnant properties
- Point estimates with machine learning: waveform coefficients
- Waveform generation with machine learning
Black holes properties with neural networks

- **Objectives:**
  - Predict the properties of the remnant black hole (BH) resulting from a binary system coalescence with neural networks
  - Improved prediction for precessing system by using the full spin information

- **Motivation:**
  - Waveform models calibration
  - Remnant properties estimation if only inspiral is available
  - Pre/post-merger consistency tests

arXiv:1911.01496
Black holes properties with neural networks

- Training data:
  - NR simulations masses and spins

- Deep NN structure
  - 4 hidden layers: 4096, 256, 64, 8 nodes
  - activation function: rectified linear unit
  - loss function: mean absolute error
  - optimizer: Adam (adaptative stochastic gradient) with learning rate $= 10^{-3}$, decay $= 10^{-4}$

<table>
<thead>
<tr>
<th>NR code</th>
<th>non-precessing</th>
<th>precessing</th>
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<tbody>
<tr>
<td>SpEC</td>
<td>592</td>
<td>1420</td>
</tr>
<tr>
<td>LazEv</td>
<td>280</td>
<td>0</td>
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<tr>
<td>MayaKranc</td>
<td>125</td>
<td>0</td>
</tr>
<tr>
<td>BAM</td>
<td>47</td>
<td>0</td>
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<tr>
<td>$\eta \to 0$</td>
<td>300</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>1344</td>
<td>1420</td>
</tr>
</tbody>
</table>

arXiv:1911.01496
Black holes properties with Gaussian processes

- Similar study with Gaussian processed:
  - Add full final spin information and recoil velocity for remnant
  - Non-spinning case:
    - 104 SXS simulations for training
    - $q \leq 8$ & $|\chi_i| \leq 0.8$
  - Spinning case:
    - 890 SXS simulations for training
    - $q \leq 2$ & $|\chi_i| \leq 0.8$

arXiv:1809.09125
Black holes properties with Gaussian processes

- Similar study with Gaussian processed:
  - Add full final spin information and recoil velocity for remnant

arXiv:1809.09125
Content

- Current modelling approaches
- **Point estimates with machine learning: remnant properties**
- Point estimates with machine learning: waveform coefficients
- Waveform generation with machine learning
Waveform models surrogates with neural networks

- **GW models surrogates**
  - General expression: \( h_{2,2}(t, \vec{\theta}) = |h_{22}(t, \vec{\theta})| e^{-i \phi(t, \vec{\theta})} \)
  - Surrogate model: amplitude and phase are expressed in a reduced basis
    \[
    |h_{22}(t, \vec{\theta})|_S = \sum_{i}^{n} c_i(\vec{\theta}) e_i(t)
    \]
  - Machine learning
    - Neural networks can be applied to estimate coefficients
    - Suitable for the large parameter space of binary parameters \( \vec{\theta} \)
    - Reduction of number of coefficients \( \rightarrow \) faster waveform generation
Waveform models surrogate for inspiral

- **Reproducing TaylorF2 waveforms**
  - Inspiral waveform, suitable for LISA signals
  - Masses \( \sim 10^5 \, M_\odot \)
  - Non precessing, aligned spins up to \( |\chi_i| = 1 \)
  - Training on \( \sim 10^5 \) waveforms
  - Study the impact on the likelihood evaluation for parameter estimation

\[ \text{arXiv:1811.05491} \]
Waveform models surrogate for coalescence

- **Reproducing SEOBNR waveforms**
  - Coalescing binaries: inspiral - merger - ringdown as seen in LIGO-Virgo-KAGRA
  - Mass ratio up $q \leq 8$
  - Non precessing, aligned spins up to $|\chi_i| \leq 0.99$
  - Training on $2 \cdot 10^5$ waveforms, testing and validation on $2 \cdot 10^4$
  - ~200 times faster than original model, 15 times faster than ROM model

Waveform generation with PCA

- **Principle Component Analysis (PCA)**
  - Projection of high dimensional data on lower dimension space
  - Linear relation between the two spaces
  - Coefficients can be evaluated with neural networks or Mixture of Experts
  - Trained on non-precessing SEOBNR waveforms

\[ \text{arXiv:2011.01958} \]

\[ f_{\text{min}} = 10 \text{Hz} \]
\[ q_{50\%} = 4.08 \times 10^{-5} \]
\[ q_{95\%} = 5.50 \times 10^{-4} \]
\[ q_{99\%} = 7.50 \times 10^{-3} \]

\[ f_{\text{min}} = 20 \text{Hz} \]
\[ q_{50\%}^{\text{TEOB}} = 17.6 \]
\[ q_{95\%}^{\text{SEOBNRv4}} = 271 \]
Current modelling approaches

Point estimates with machine learning: remnant properties

Point estimates with machine learning: waveform coefficients

Waveform generation with machine learning
Waveform templates error interpolation

- **Gaussian Processes**
  - Sum of multivariate normal distributions
  - Mean and width fit to reproduce training data
  - Non-parameteric interpolation with error estimation

- **Application to waveform modelling error quantification**
  - No propagation of modelling errors
  - Interpolate the error between 3 pN and 3.5 pN NR waveforms
  - Non-spinning waveforms with different chirp masses

arXiv:1412.3657
NR interpolation with Gaussian Processes

- **Application to late stages of coalescing binary**
  - Training on 132 waveforms from GeorgiaTech NR catalog
  - Precessing waveforms with $q \leq 10$ & $|\chi_i| \leq 0.9$
  - Main radiation mode: $(l, m) = (2, 2)$
  - 2 validations procedures: mismatch with training dataset and leave-one-out

arXiv:1903.09204
NR interpolation with Gaussian Processes

- Application to coalescing binary
  - Hybrids = EOB stitched to NR waveforms to cover all stages of binary coalescence
  - Training on hybridised SXS waveforms
  - Dataset: ~1500 SXS precessing with $q \leq 4$ & $\chi_i \leq 0.8$

![Graphs showing NR interpolation with Gaussian Processes](arXiv:1905.09300)
NR interpolation with Gaussian Processes

- **Model extended to include:**
  - Higher harmonics
  - Eccentric binaries
  - Extreme mass ratios

**arXiv:** 2101.11798

**arXiv:** 1905.09300

**arXiv:** 1910.10473
Conclusion

- **Machine learning algorithms features are beneficial for waveform modelling**
  - Correlations across large parameter space
  - Provide accurate (or?) fast GW templates, usefulness depends on case
  - The creation of efficient models is built on the knowledge acquired from traditional techniques

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**arXiv:1909.10986**
Thank you for your attention