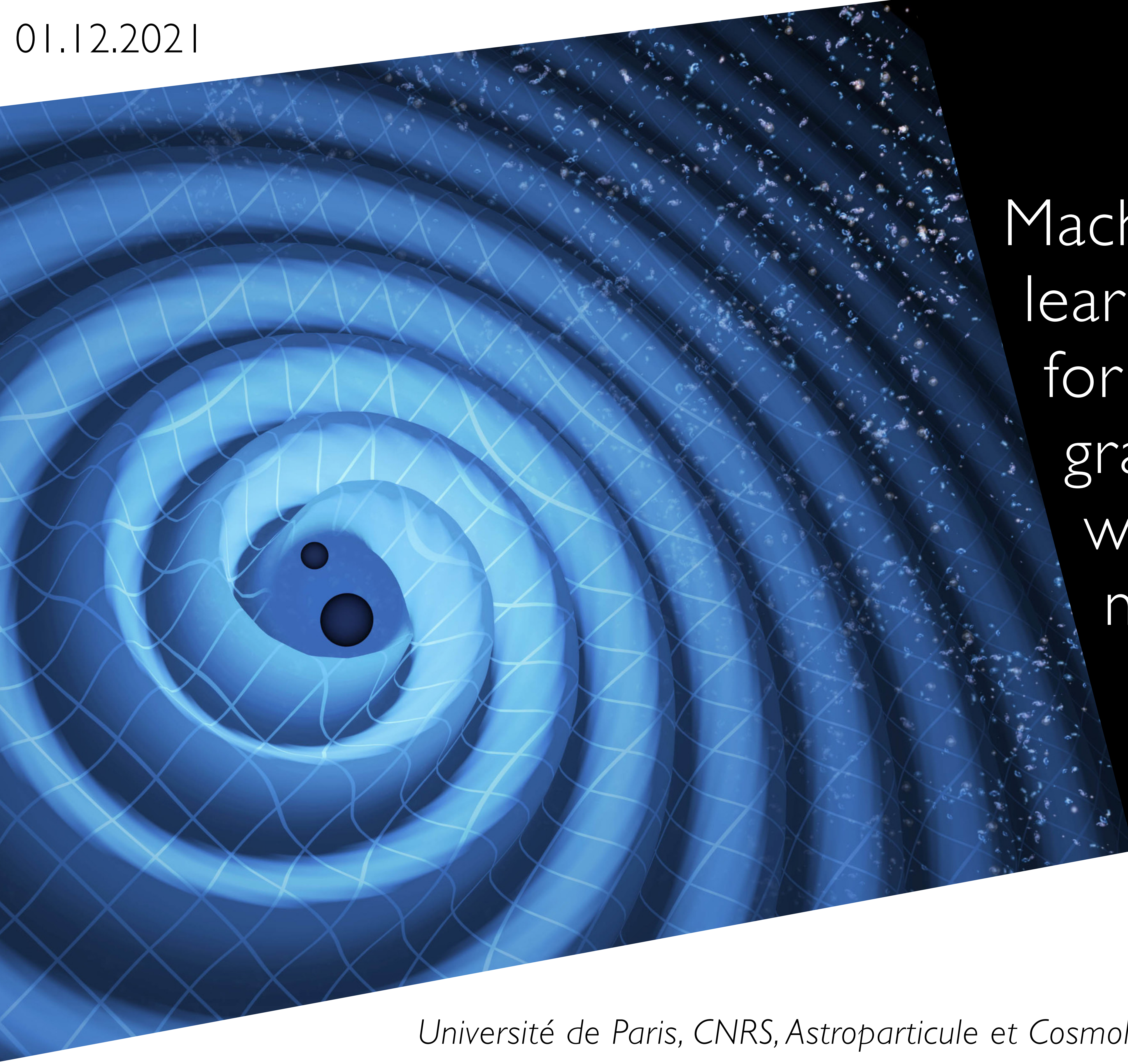
UCLA IPAM GWAWS4 workshop

01.12.2021



Machine learning for accurate gravitational waves modelling

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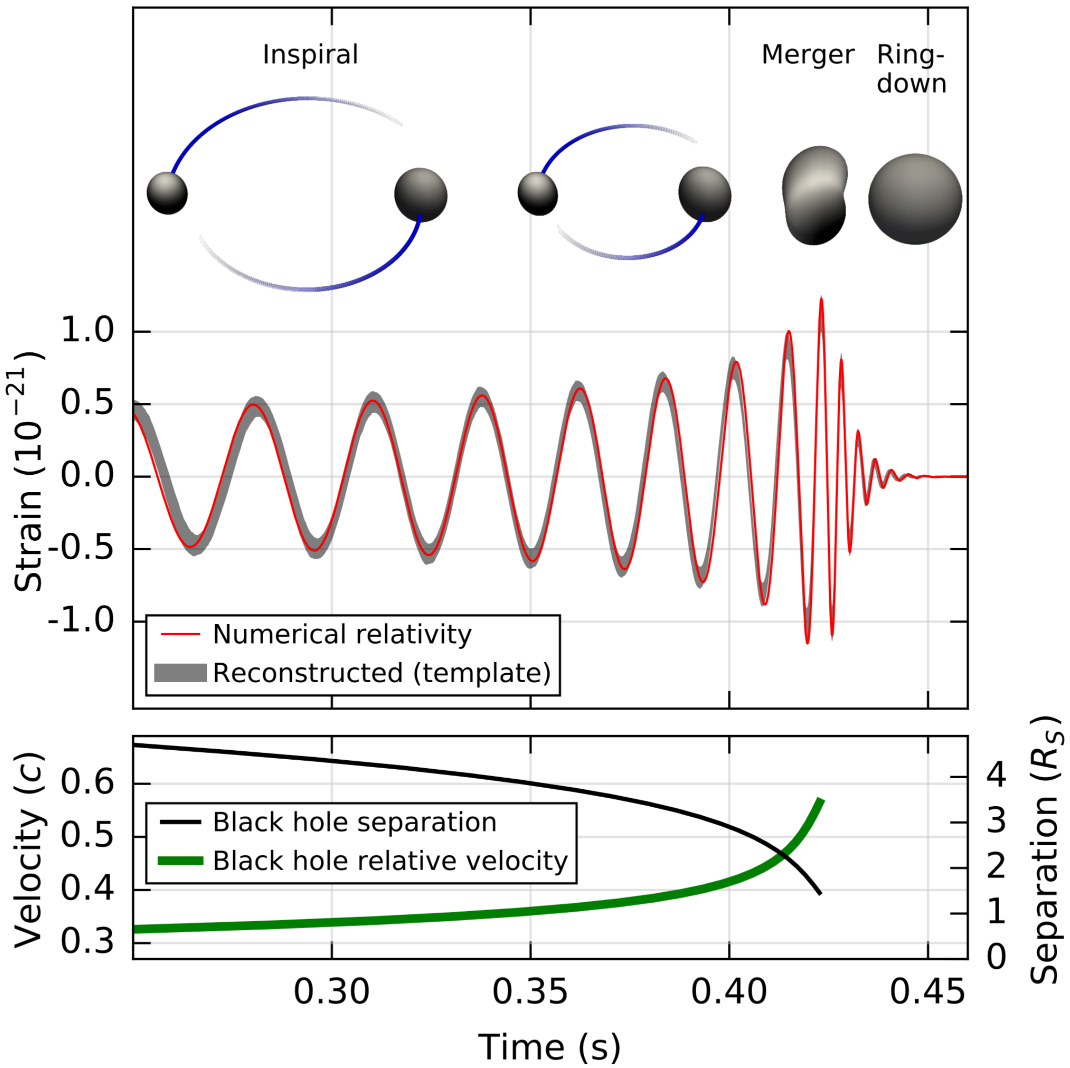
Introduction

- **Gravitational waves (GW) templates are essential**
  - Most sensitive gravitational waves detection pipelines are based on modelled searches
  - Parameter estimation requires template banks (Markov-chain based)

- **Waveform modelling is complex**
  - Detection of binary system: large parameter space (min 15 parameters)
  - Merger: strong-field regime of general relativity (GR), non-linear physics
  - Parameter estimation bottleneck: no propagation of waveform systematics

- **Machine learning has potential**
  - Different applications according to purposes
  - Review in this talk



Content

- **Current modelling approaches**
- Point estimates with machine learning: remnant properties
- Point estimates with machine learning: waveform coefficients
- Waveform generation with machine learning

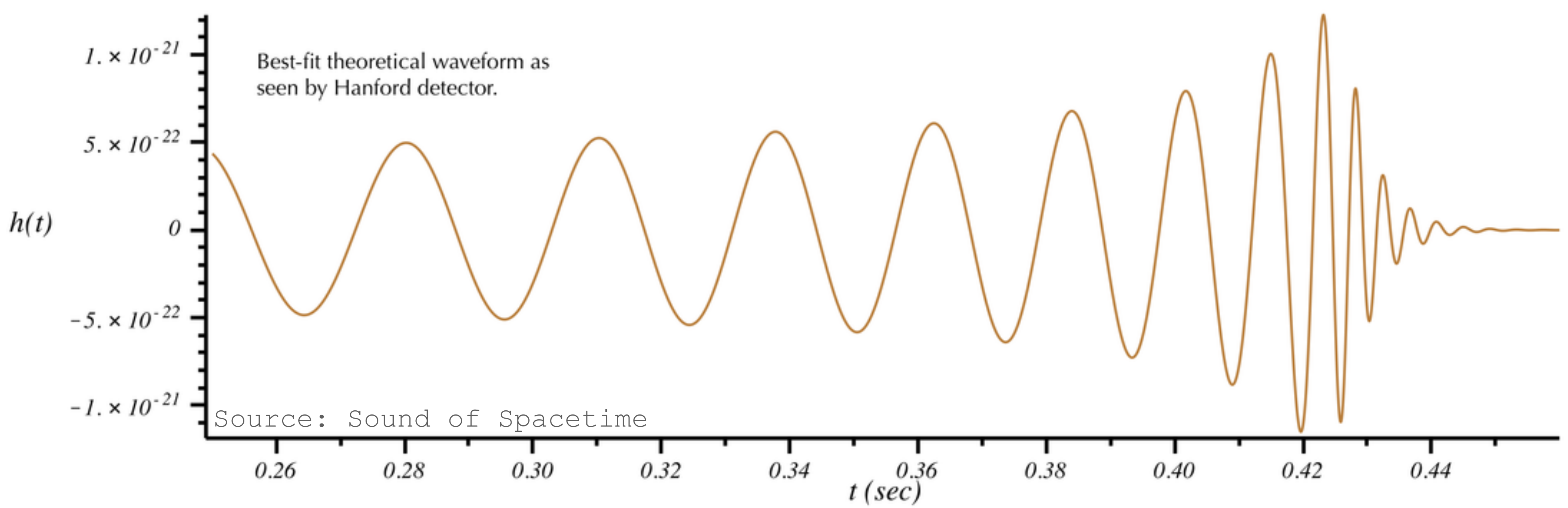
Anatomy of a gravitational waveform

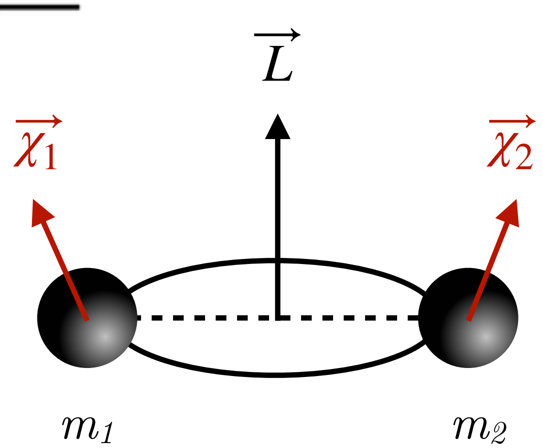
- **GW150914-like signal:**

inspiral, weak field
can be computed with post-Newtonian (pN) expansion
far from innermost stable circular orbit (ISCO)

merger, strong field
requires numerical relativity

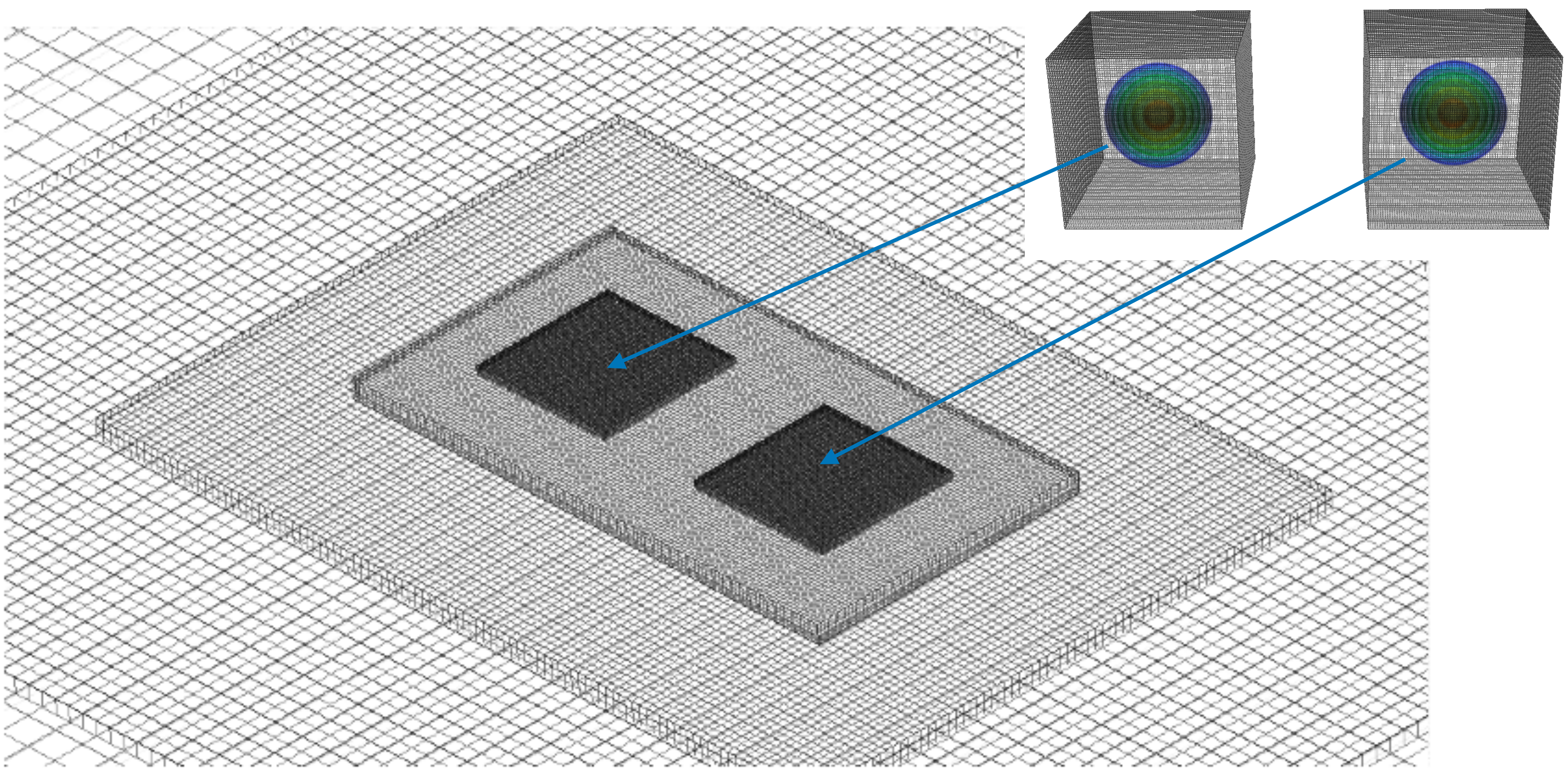
ringdown, strong to weak field
can be modelled as a sum of quasinormal modes





GW computation: numerical relativity simulations

- **Numerical relativity (NR) simulations**
  - Use 3+1 decomposition of spacetime to perform evolution (*à la* ADM)
  - Black hole singularity is excised or analytically factored (puncture)
  - Issue with well-posedness: first GW simulation in 2005
  - Accuracy depends on mesh refinement



Source: S. Caudill

GW computation: numerical relativity catalogs

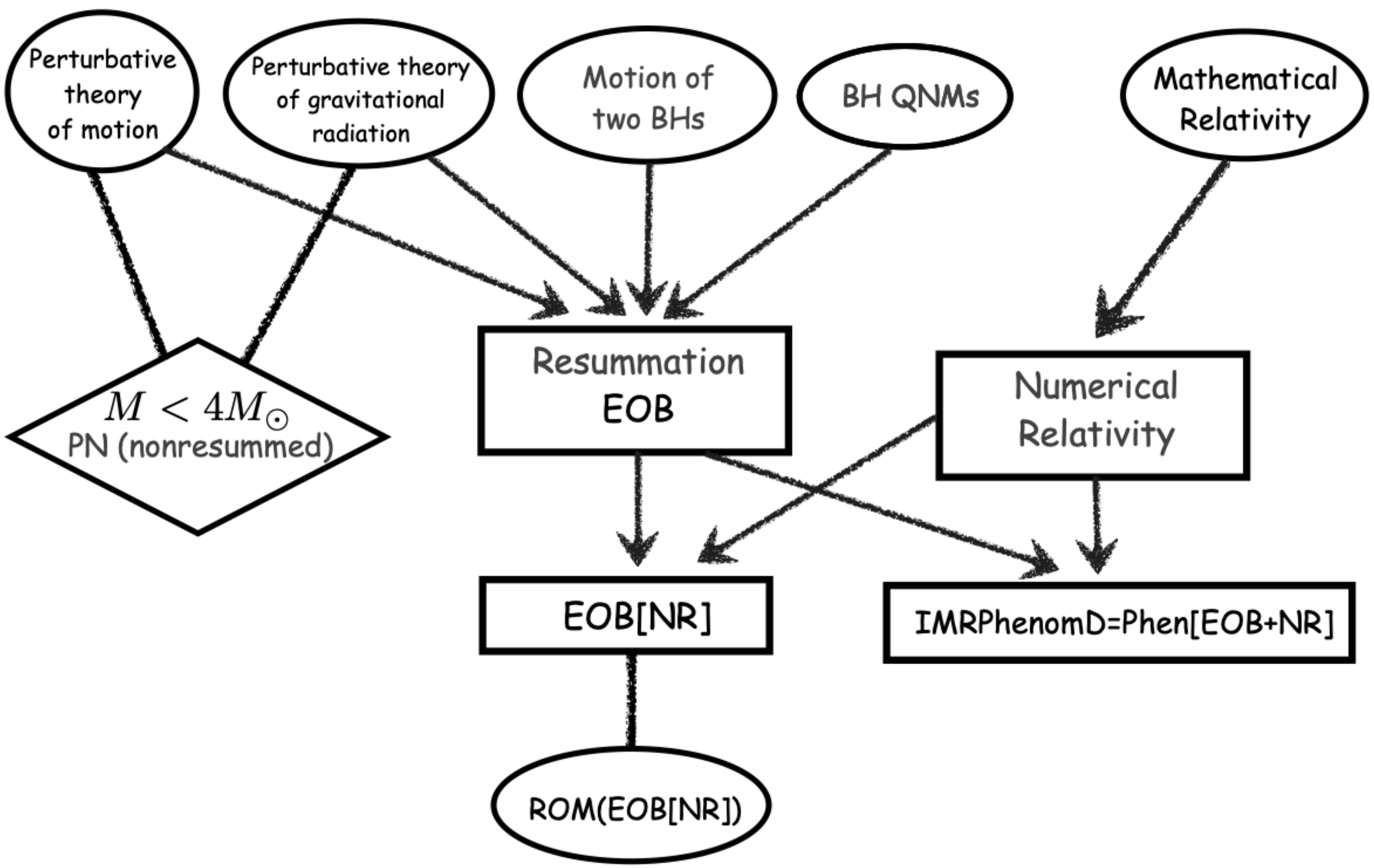
- **Several numerical relativity catalogs**
  - 3 are active and open-access
  - Table from SXS simulation catalog

Catalog	Started	Updating?	Simulations	m_1/m_2 range	$\|\chi_1\|$ range	$\|\chi_2\|$ range	Precessing?	Median $N_{\rm cyc}$	Public?
NINJA [98, 115]	2008	✗	63	1–10	0–0.95	0–0.95	✗	15	✗
NRAR [120]	2013	✗	25	1–10	0–0.8	0–0.6	✓	24	✗
Georgia Tech [122]	2016	✓	452	1–15	0–0.8	0–0.8	✓	4	✓
RIT (2017) [123]	2017	✓	126	1–6	0–0.85	0–0.85	✓	16	✓
RIT (2019) [124]	2017	✓	320	1–6	0–0.95	0–0.95	✓	19	✓
NCSA (2019) [125]	2019	✗	89	1–10	0	0	✗	20	✗
SXS (2018)	2013	✓	337	1–10	0–0.995	0–0.995	✓	23	✓
SXS (2019)	2013	✓	2018	1–10	0–0.998	0–0.998	✓	39	✓

arXiv:1904.04831

GW modelling: effective one-body formalism

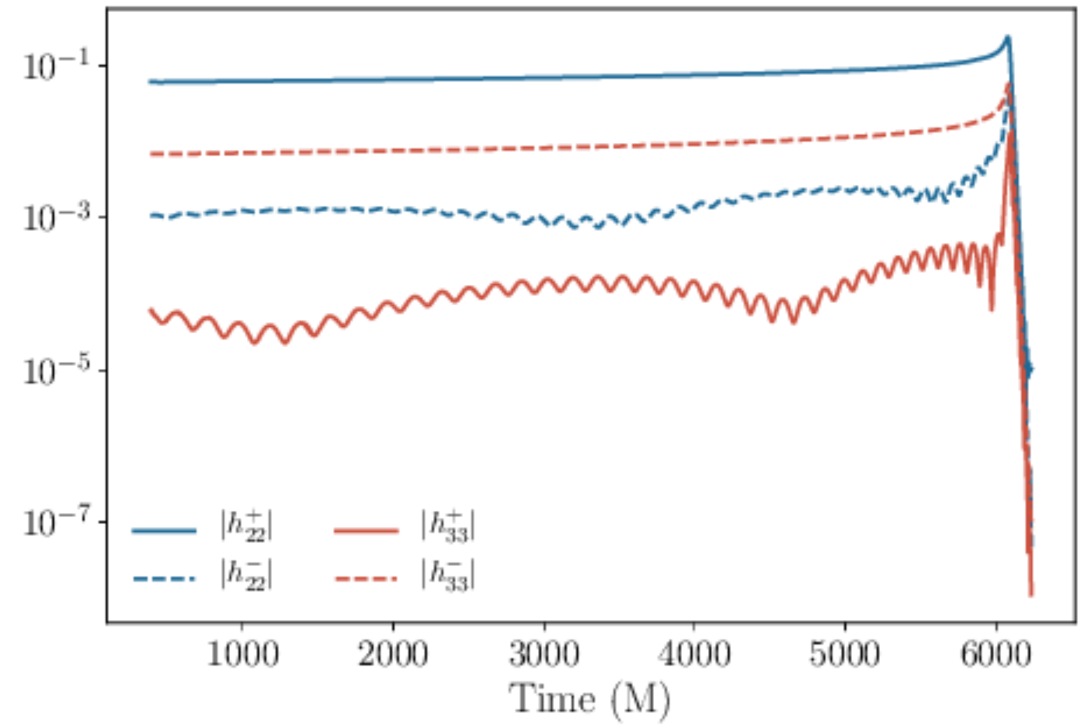
- **Effective one-body (EOB) formalism**
  - 2-body dynamics mapped as 1 body in effective metric
  - Very accurate at inspiral, calibrated to NR simulations for LIGO-Virgo models

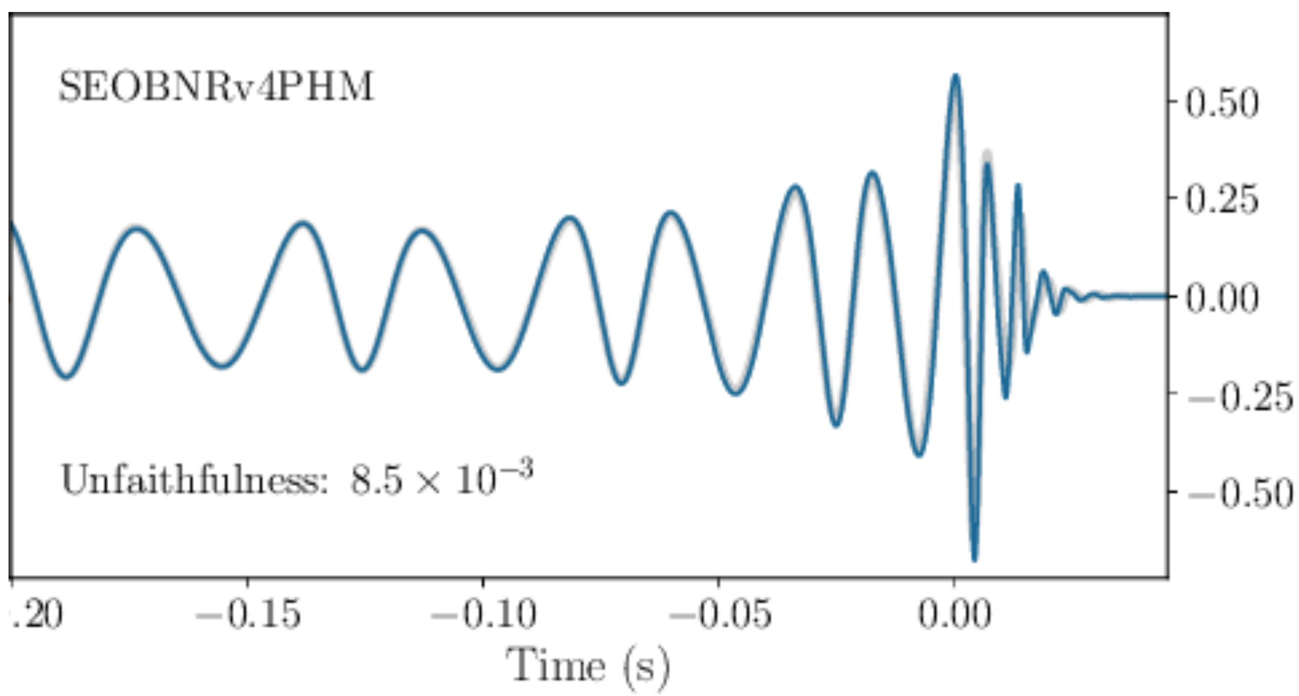


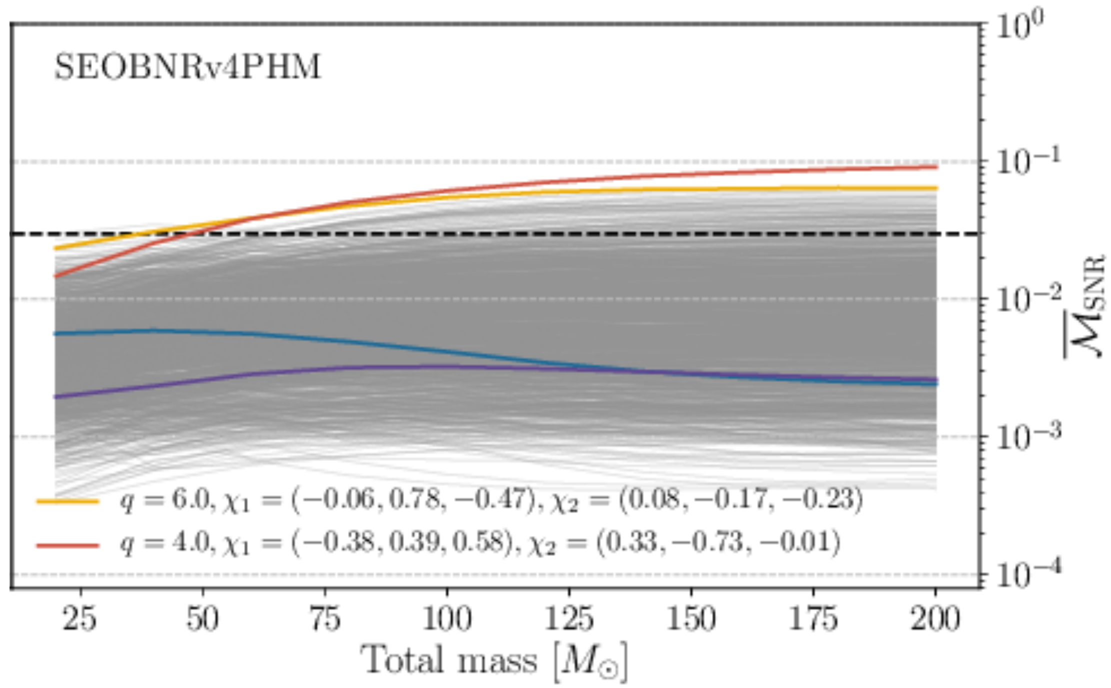
Source: T. Damour

GW modelling: effective one-body formalism

- **SEOBNR models**
  - Includes higher harmonics up to $(l, m) = (5,5)$
  - Calibrated to precessing waveform with $\chi \rightarrow 0.8$



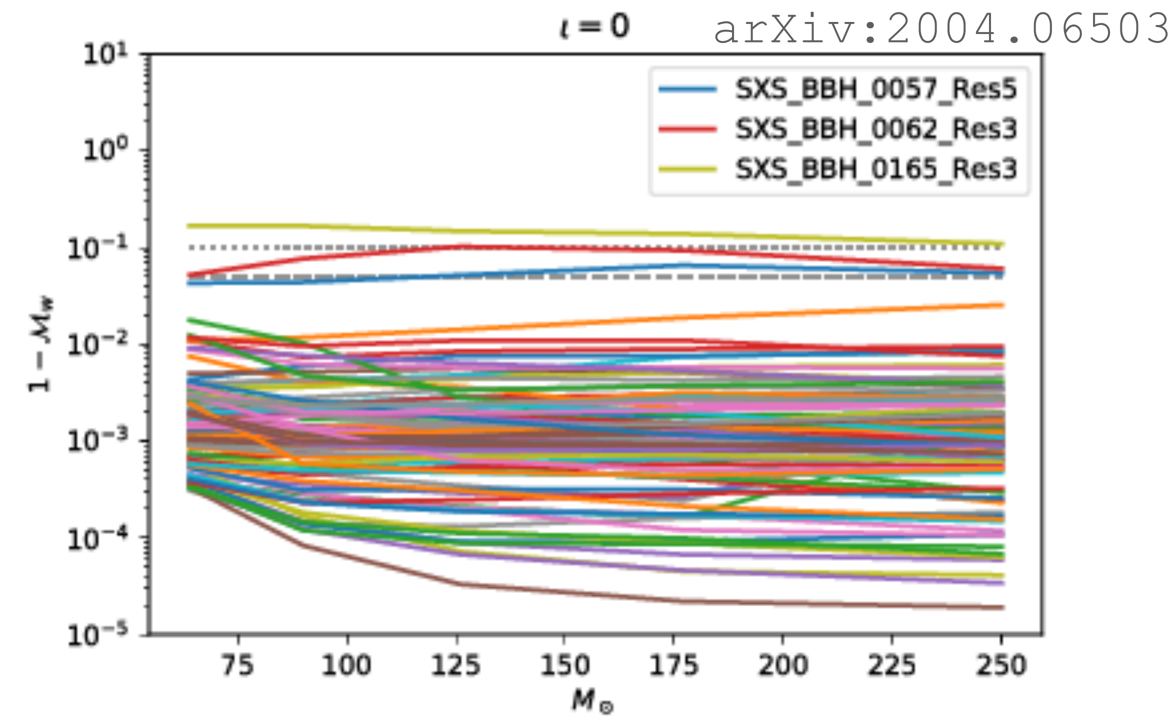


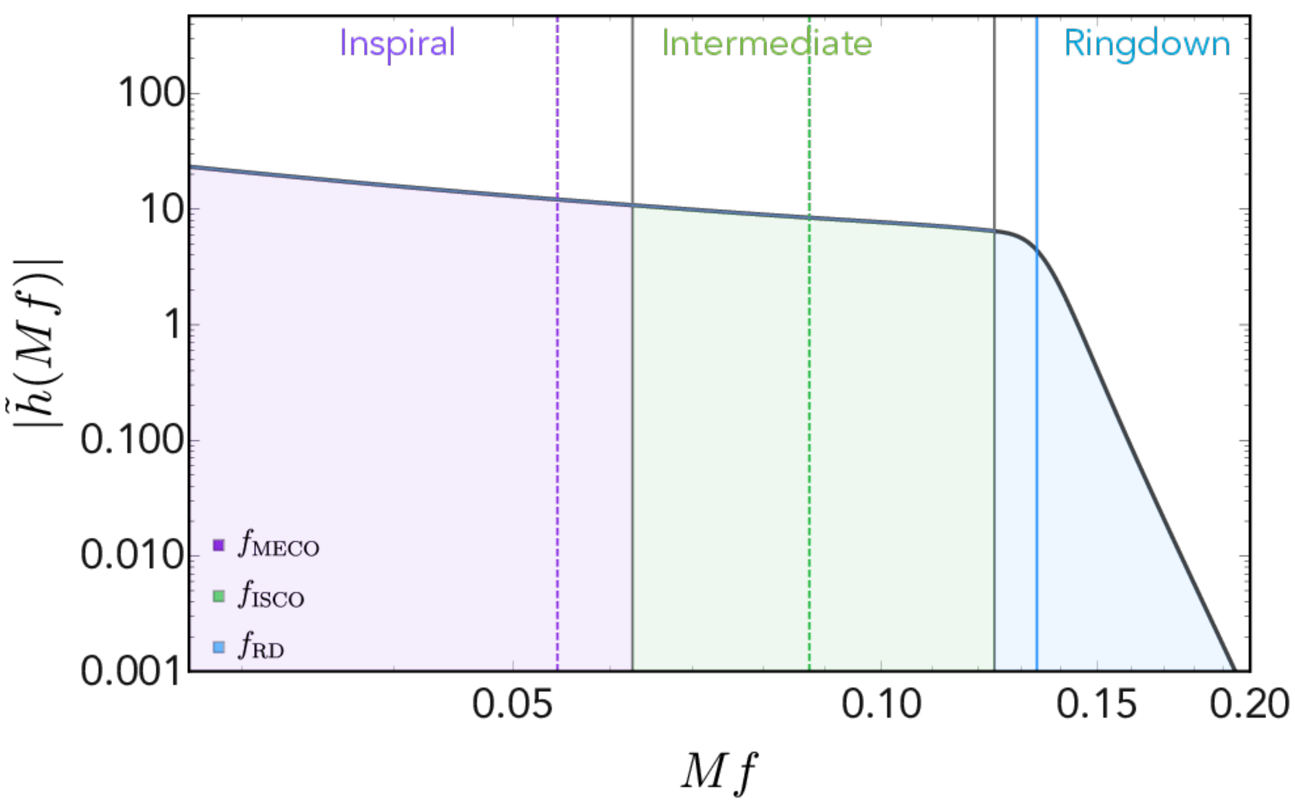


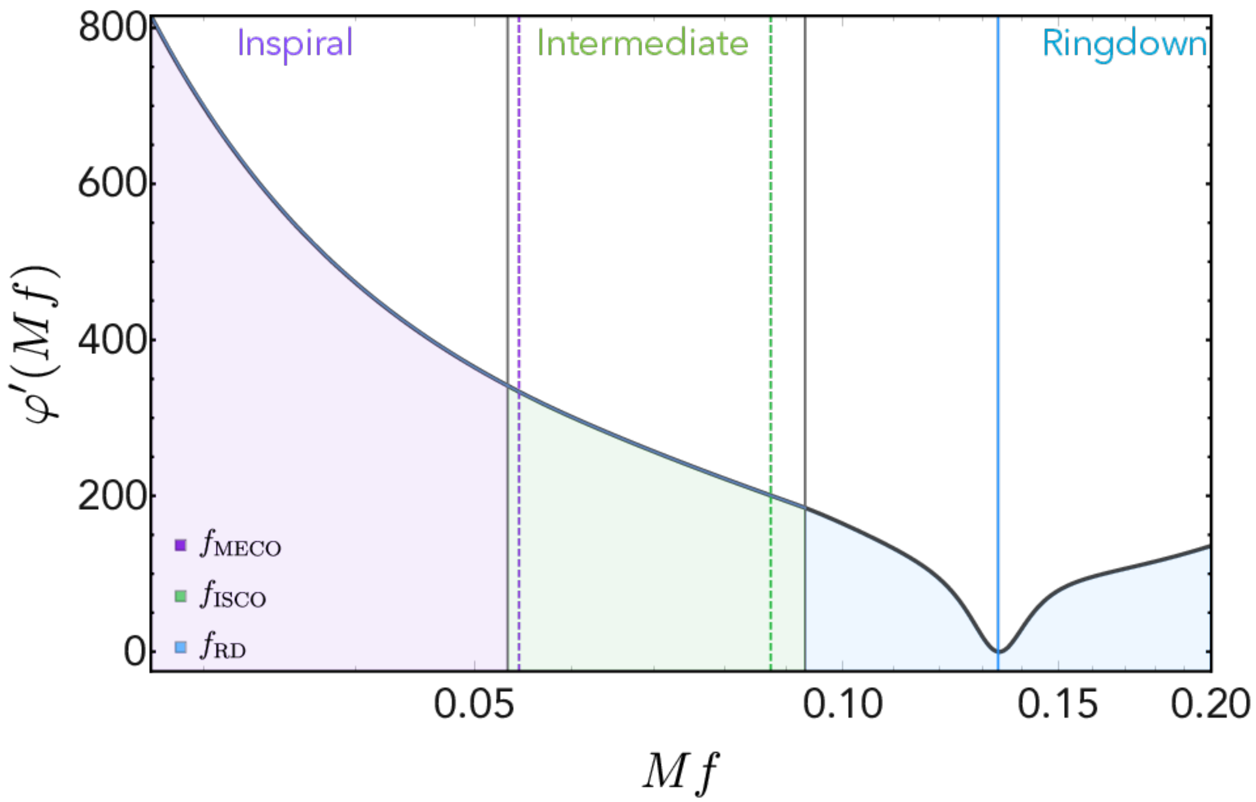
arXiv:2004.09442

GW modelling: phenomenological approaches

- **IMRPhenom approach**
  - Different region of the waveform are modelled by different ansatz
  - Frequency and time domain models
  - Inspiral: pN + pseudo-pN terms
  - Intermediate: phenomenological ansatz
  - Ringdown: quasimornal modes

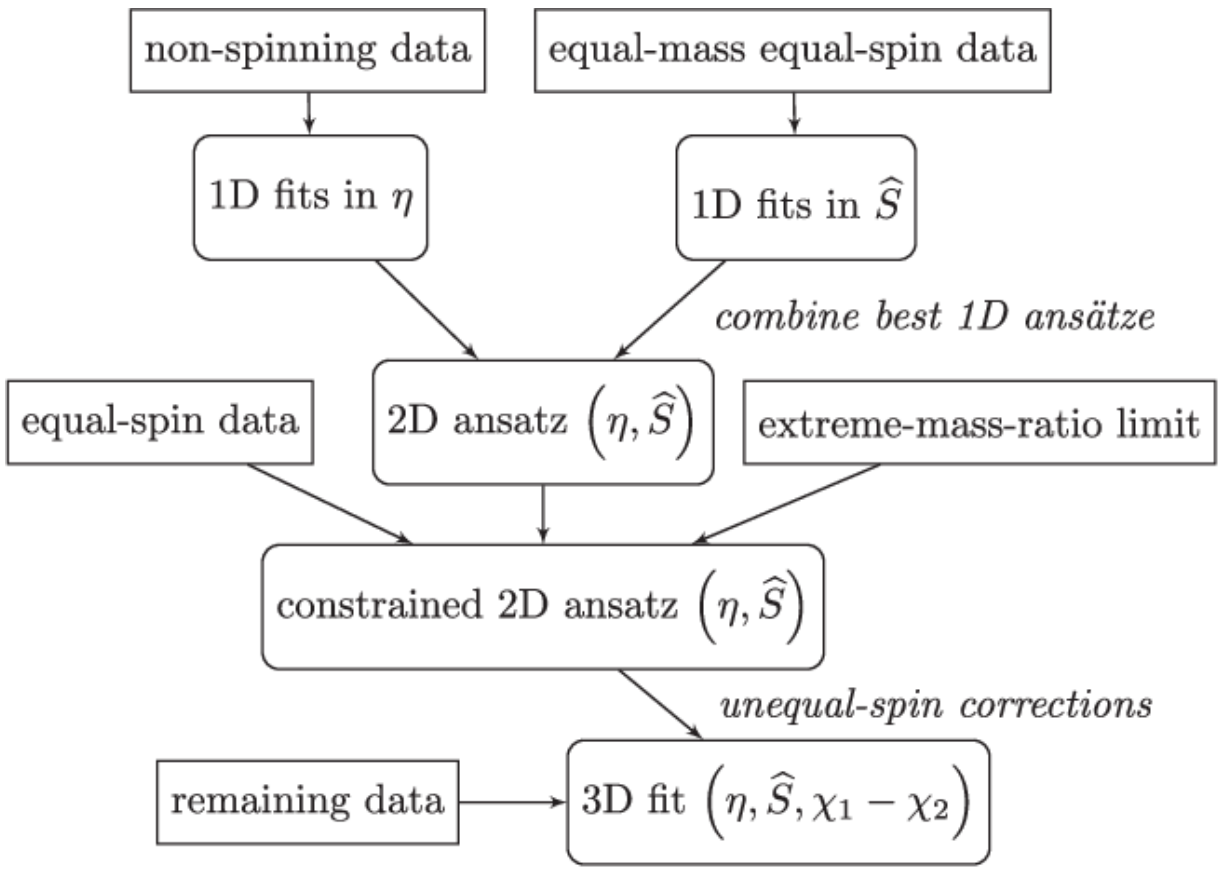




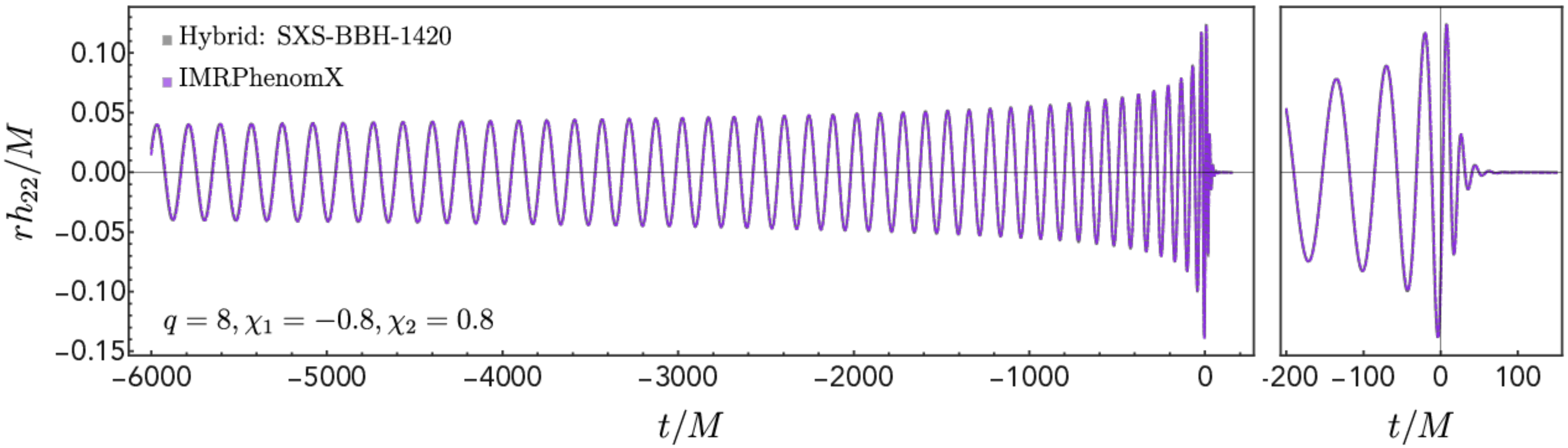


arXiv:2001.11412

- **IMRPhenom models**
  - Hierarchical fit as a function of mass ratio, effective spin and spin difference
  - Mapping from non-precessing to precessing case
  - Higher harmonics up to $(l, m) = (4,4)$



arXiv:2001.11412



Content

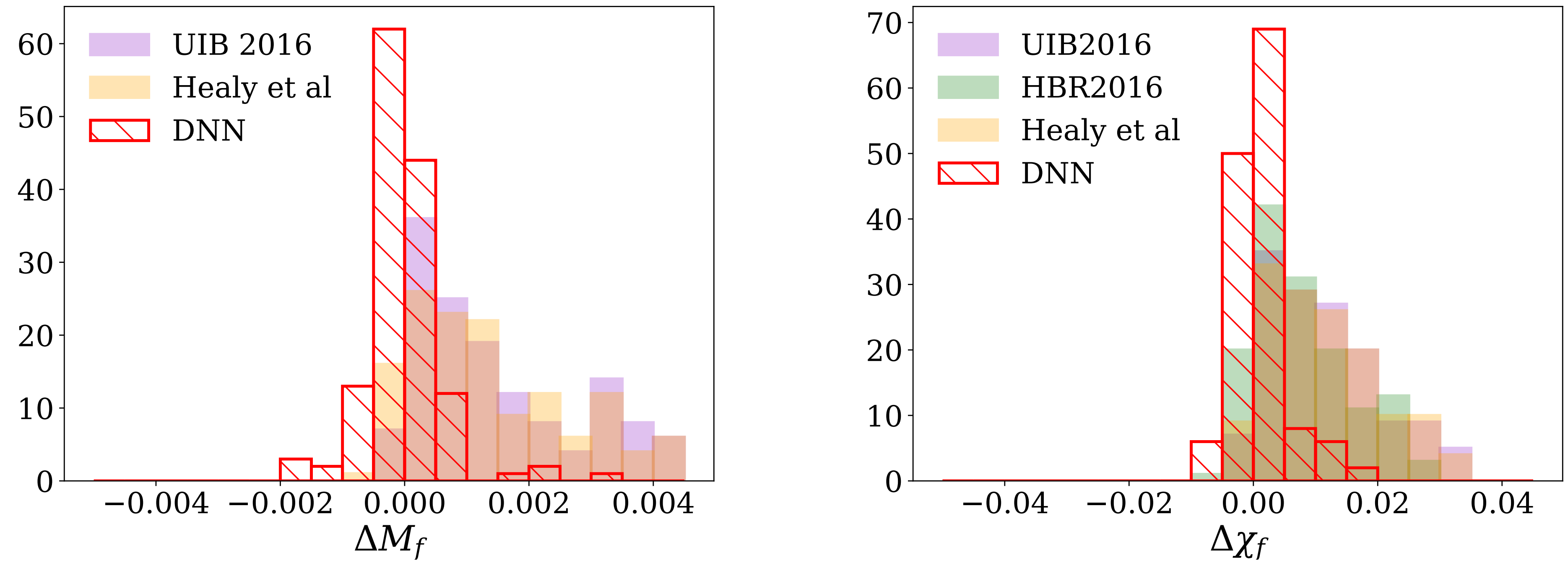
- Current modelling approaches
- **Point estimates with machine learning: remnant properties**
- Point estimates with machine learning: waveform coefficients
- Waveform generation with machine learning

Black holes properties with neural networks

- **Objectives:**
  - Predict the properties of the remnant black hole (BH) resulting from a binary system coalescence with neural networks
  - Improved prediction for precessing system by using the full spin information

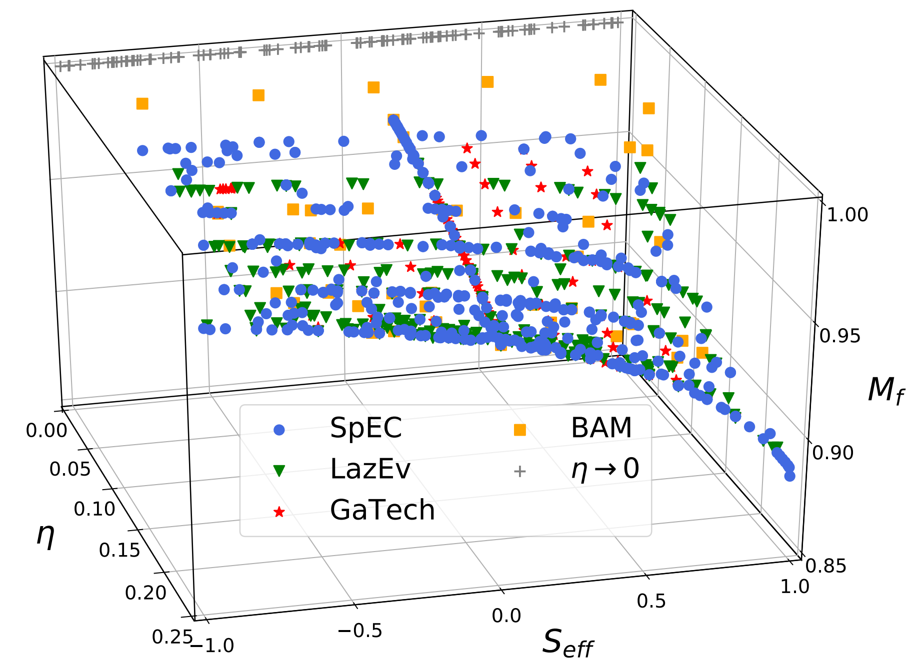
- **Motivation:**
  - Waveform models calibration
  - Remnant properties estimation if only inspiral is available
  - Pre/post-merger consistency tests

arXiv:1911.01496

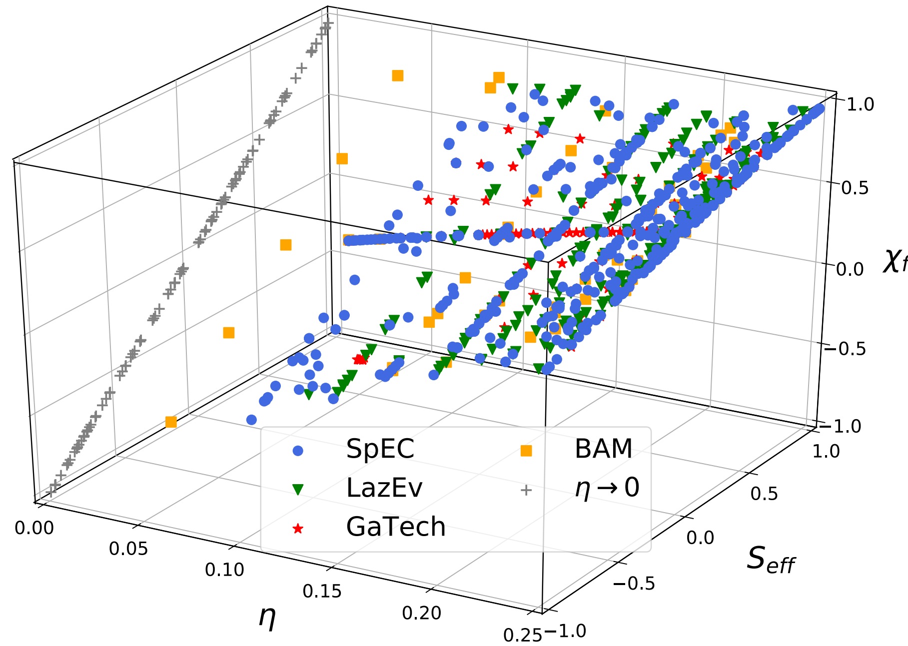


Black holes properties with neural networks

- **Training data:**
  - NR simulations masses and spins
- **Deep NN structure**
  - 4 hidden layers: *4096, 256, 64, 8 nodes*
  - activation function: *rectified linear unit*
  - loss function: *mean absolute error*
  - optimizer: *Adam (adaptative stochastic gradient) with learning rate* $= 10^{-3}$, *decay* $= 10^{-4}$



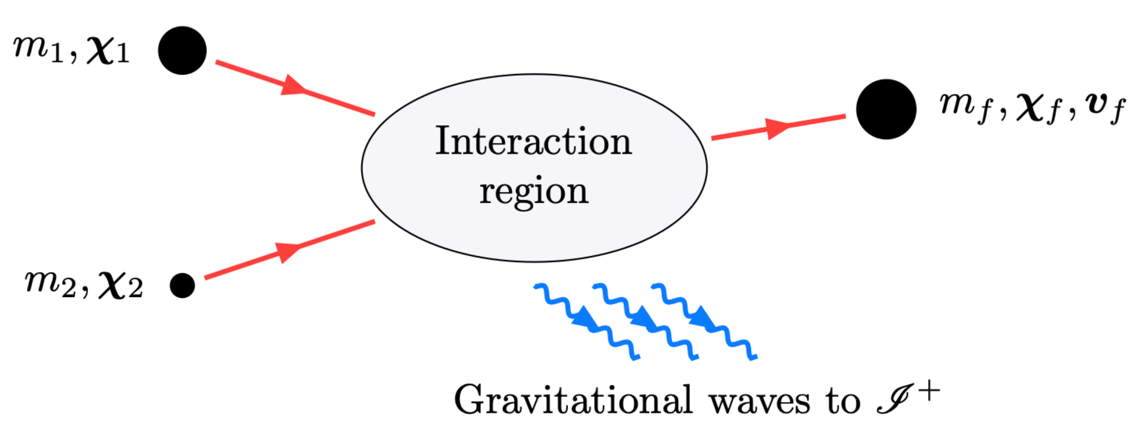
NR code	non-precessing	precessing
SpEC	592	1420
LazEv	280	0
MayaKranc	125	0
BAM	47	0
$\eta \to 0$	300	0
Total	**1344**	**1420**



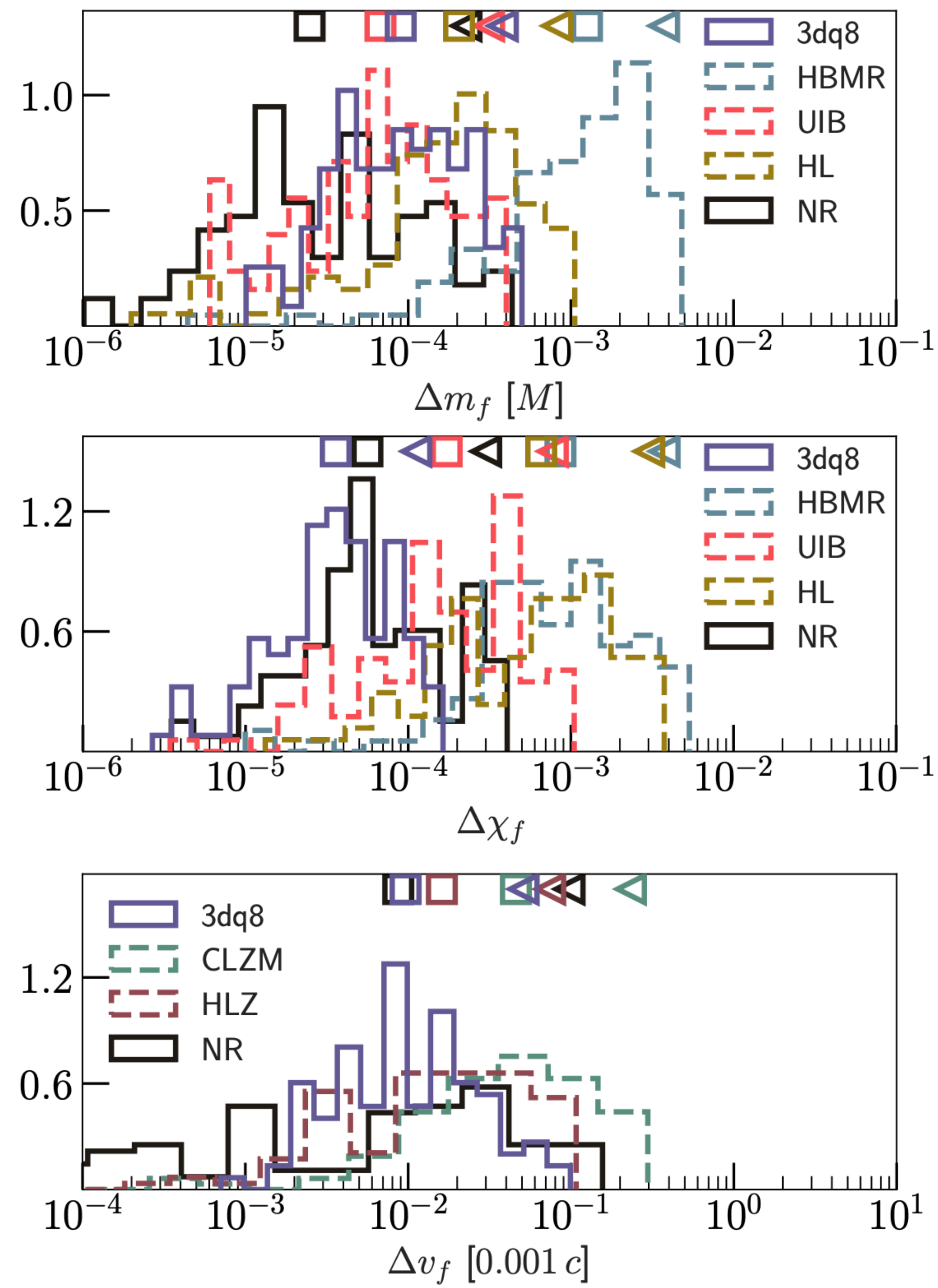
arXiv:1911.01496

Black holes properties with Gaussian processes

- **Similar study with Gaussian processed:**
  - Add full final spin information and recoil velocity for remnant
  - Non-spinning case:
    - 104 SXS simulations for training
    - $q \leq 8$ & $|\chi_i| \leq 0.8$
  - Spinning case:
    - 890 SXS simulations for training
    - $q \leq 2$ & $|\chi_i| \leq 0.8$



arXiv:1809.09125



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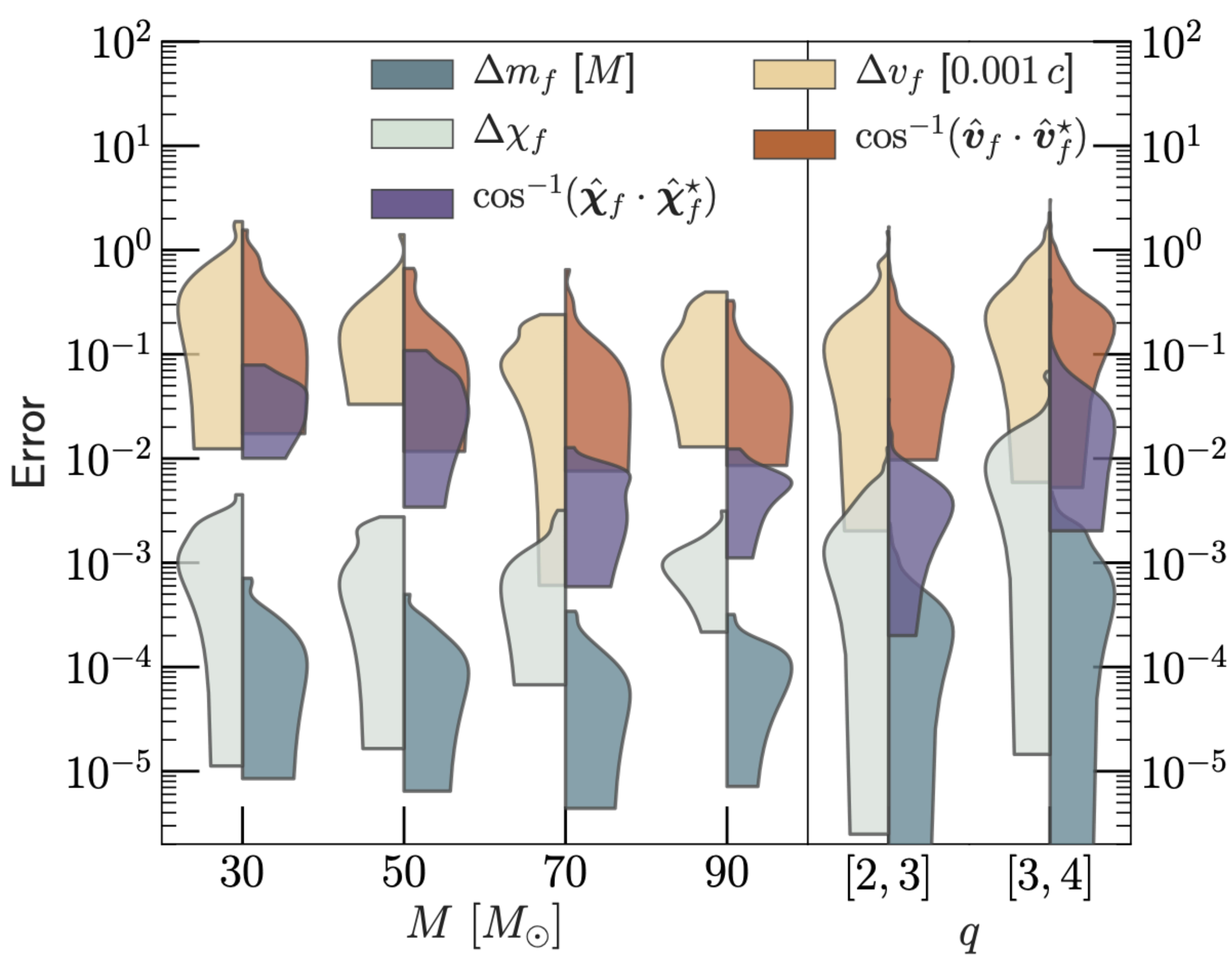

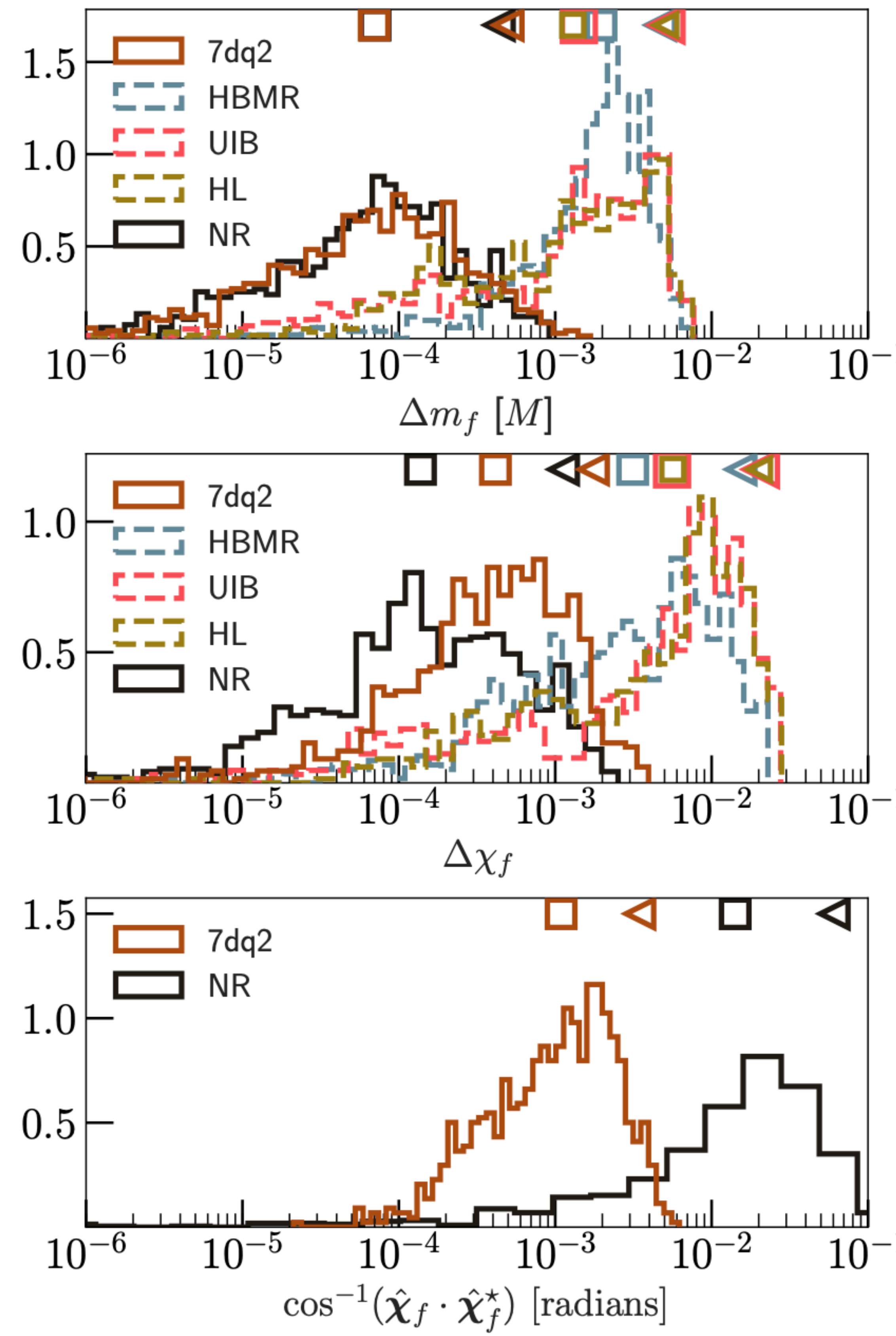

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Waveform models surrogates with neural networks

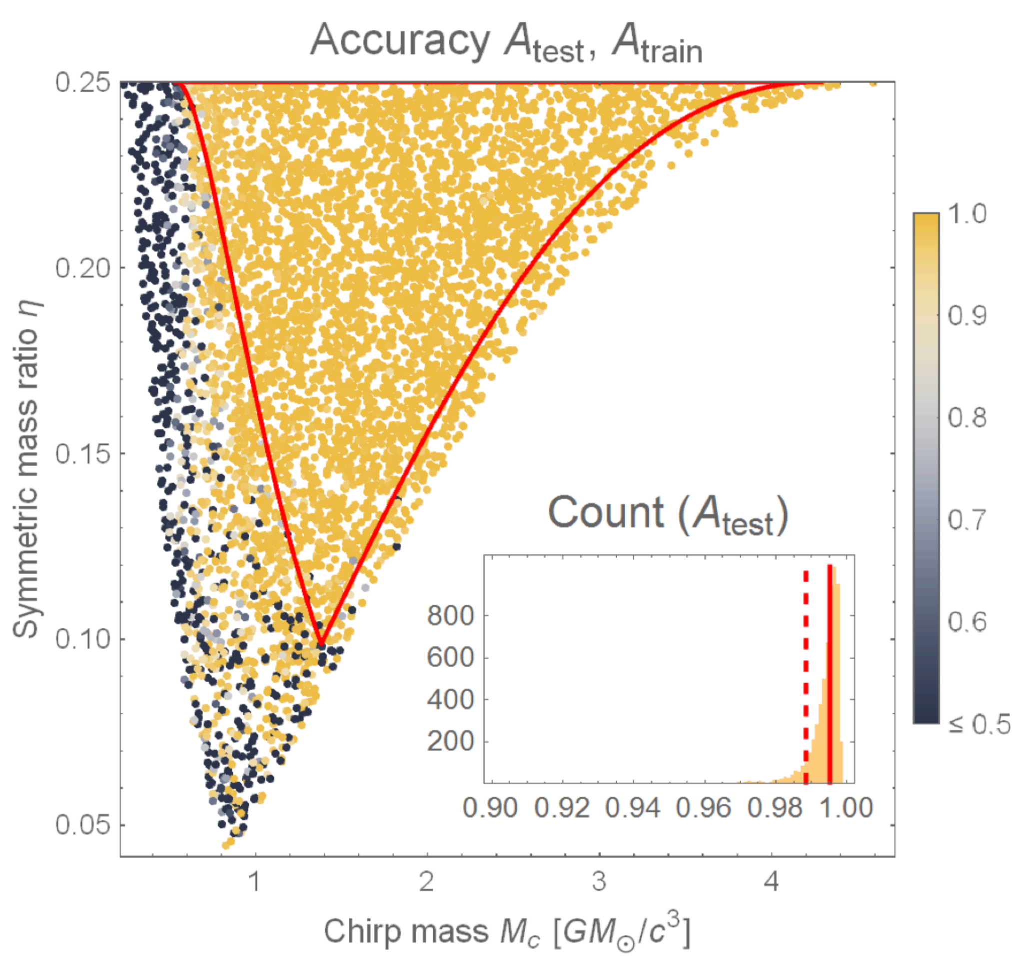
- **GW models surrogates**
  - General expression: $h_{2,2}(t, \overrightarrow{\theta}) = |h_{22}(t, \overrightarrow{\theta})|\ e^{-i\ \phi(t, \overrightarrow{\theta})}$
  - Surrogate model: amplitude and phase are expressed in a reduced basis

$$|h_{22}(t, \overrightarrow{\theta})|_S = \sum_i^n \underbrace{c_i(\overrightarrow{\theta})}_{\text{projection coefficients}}\ \underbrace{e_i(t)}_{\text{basis elements}}$$

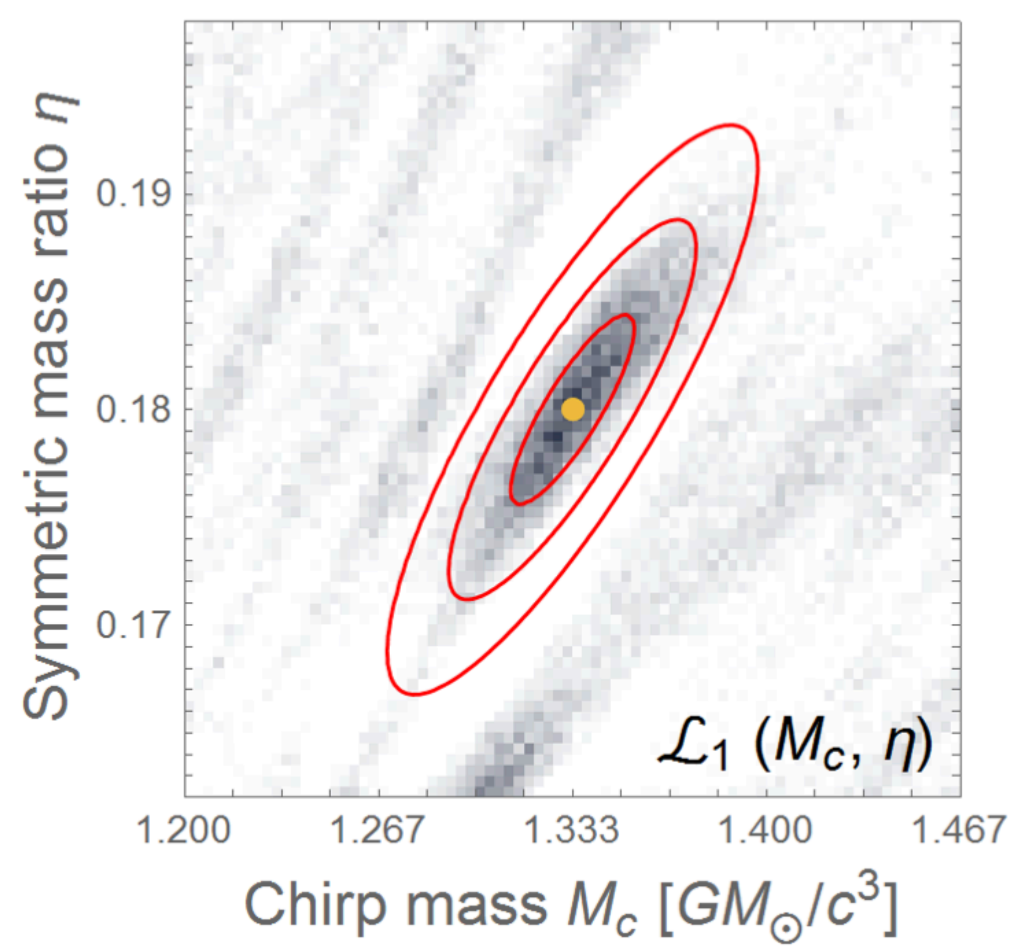
- **Machine learning**
  - Neural networks can be applied to estimate coefficients
  - Suitable for the large parameter space of binary parameters $\overrightarrow{\theta}$
  - Reduction of number of coefficients → faster waveform generation

Waveform models surrogate for inspiral

- **Reproducing TaylorF2 waveforms**
  - Inspiral waveform, suitable for LISA signals
  - Masses ~ $10^5\ M_\odot$
  - Non precessing, aligned spins up to $|\chi_i| = 1$
  - Training on $\sim 10^5$ waveforms
  - Study the impact on the likelihood evaluation for parameter estimation

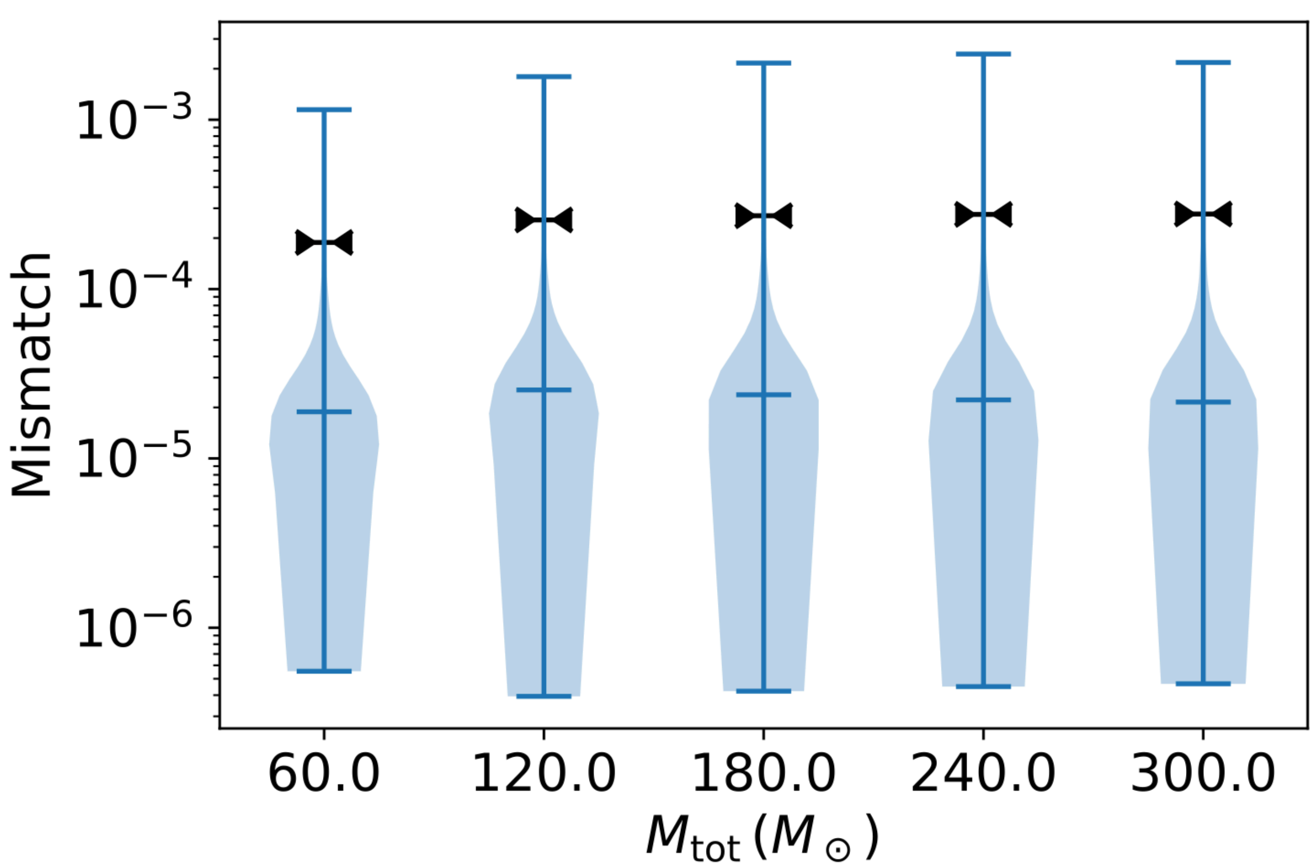


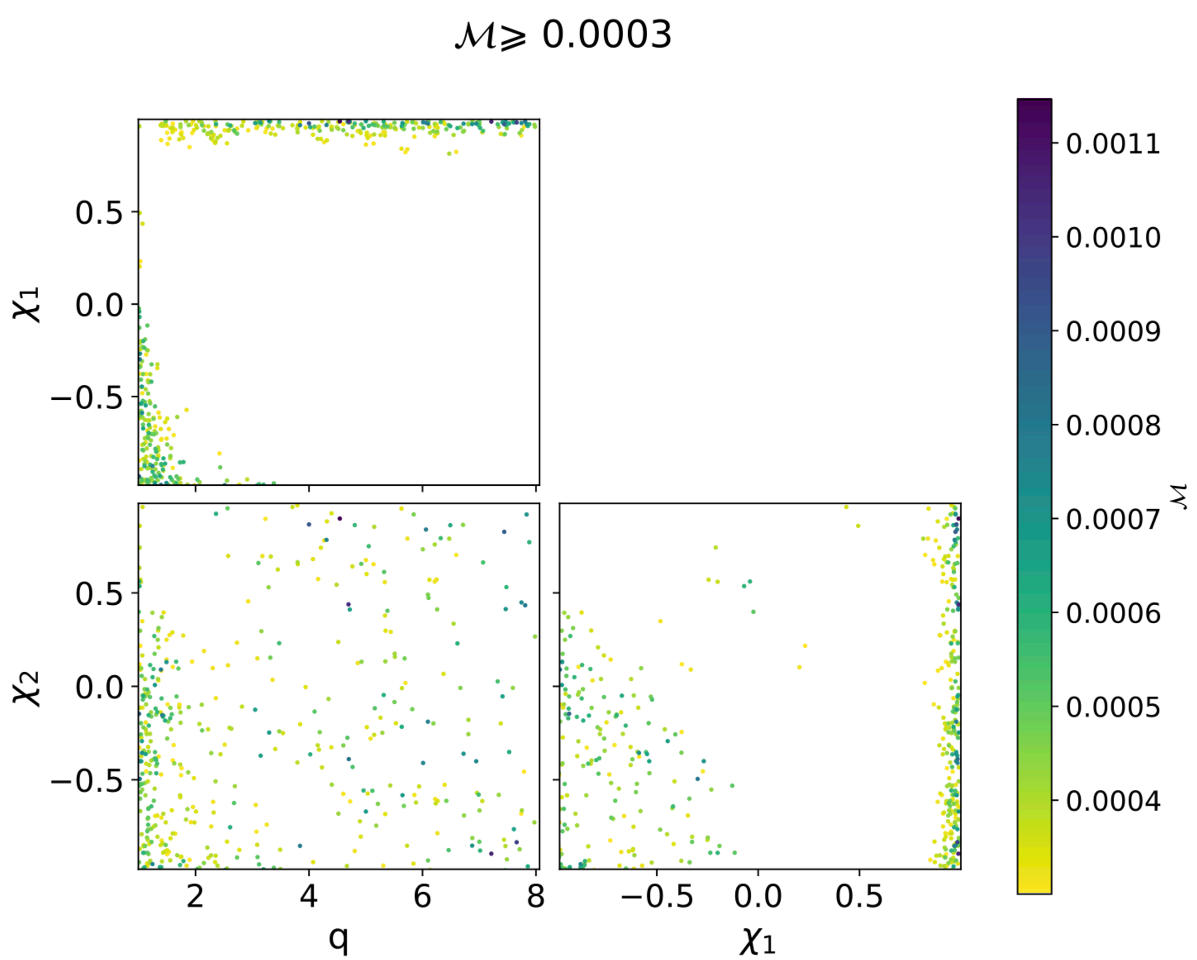
arXiv:1811.05491



- **Reproducing SEOBNR waveforms**
  - Coalescing binaries: inspiral - merger - ringdown as seen in LIGO-Virgo-KAGRA
  - Mass ratio up $q \leq 8$
  - Non precessing, aligned spins up to $|\chi_i| \leq 0.99$
  - Training on $2 \cdot 10^5$ waveforms, testing and validation on $2 \cdot 10^4$
  - ~200 times faster than original model, 15 times faster than ROM model

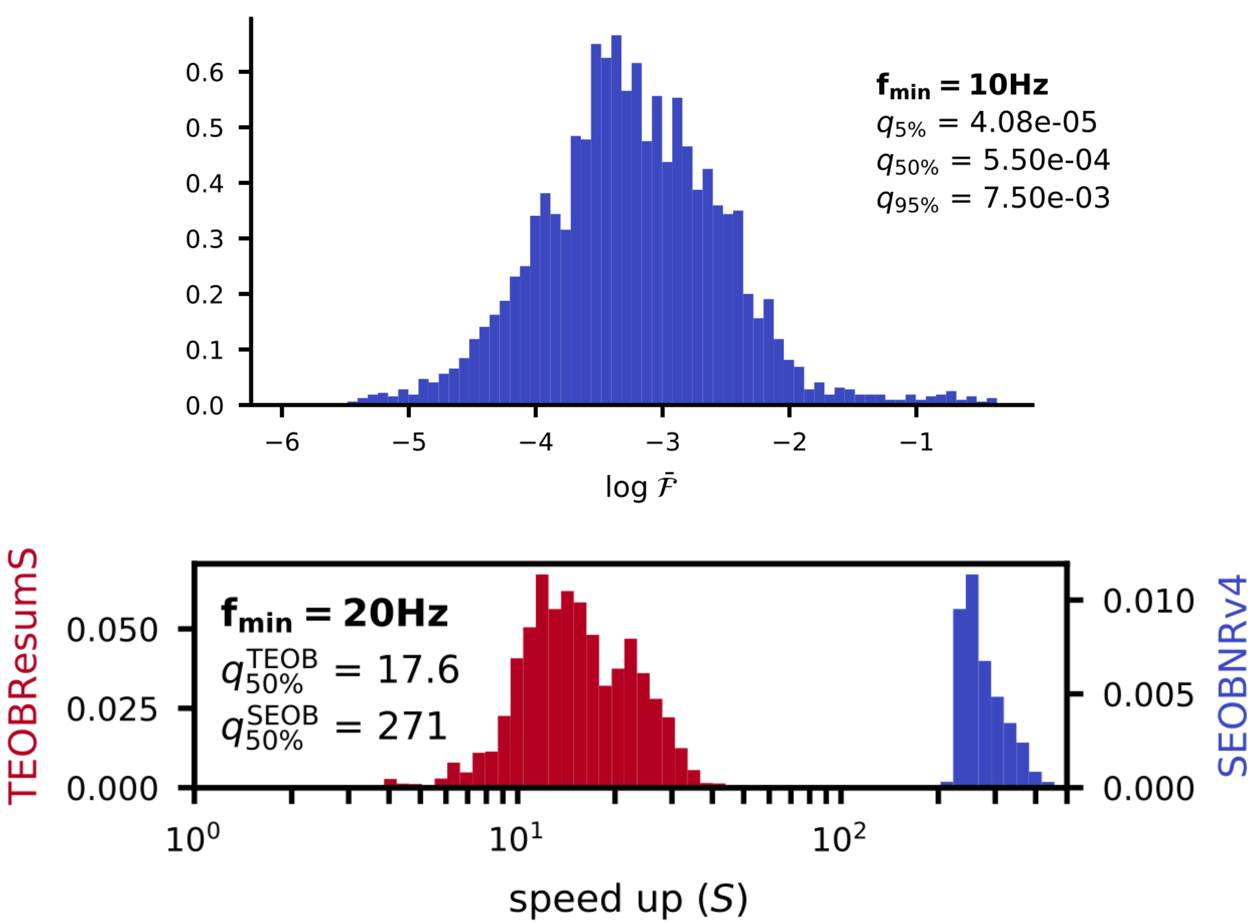
arXiv:2008.12932



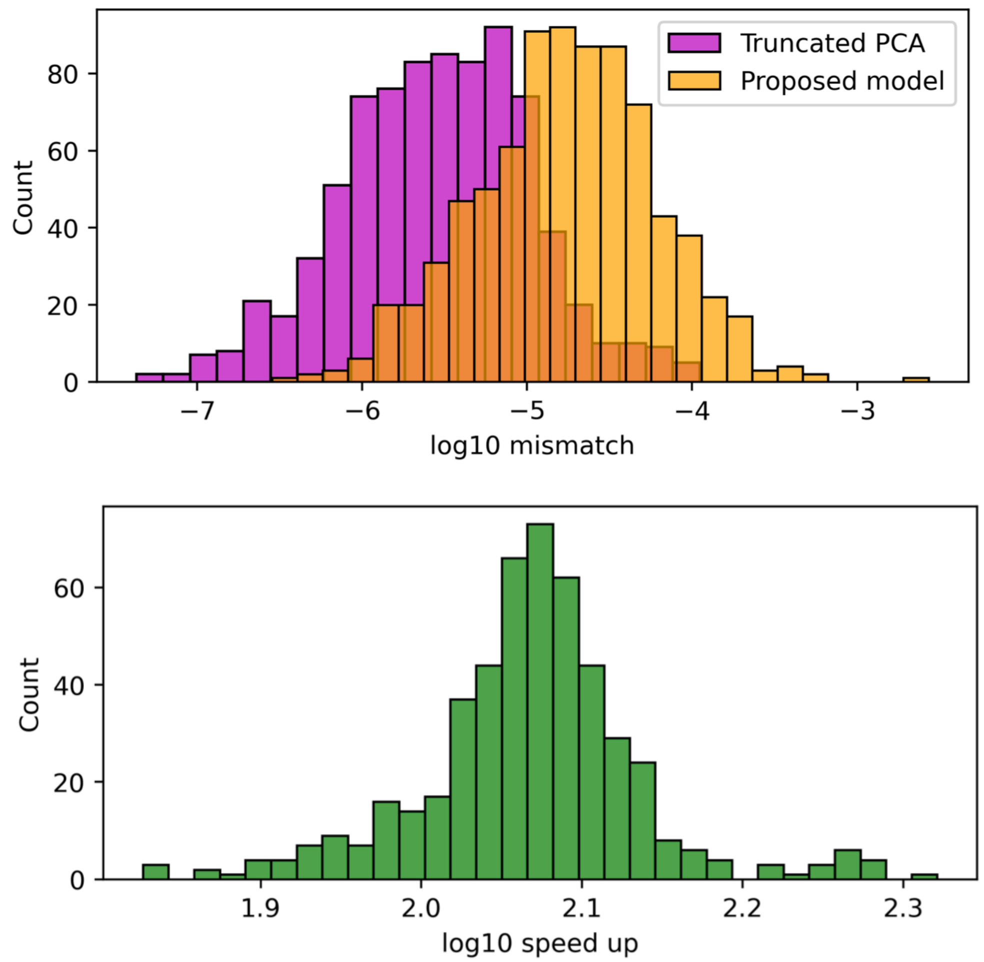


- **Principle Component Analysis (PCA)**
  - Projection of high dimensional data on lower dimension space
  - Linear relation between the two spaces
  - Coefficients can be evaluated with neural networks or Mixture of Experts
  - Trained on non-precessing SEOBNR waveforms

arXiv:2011.01958



Cano et al



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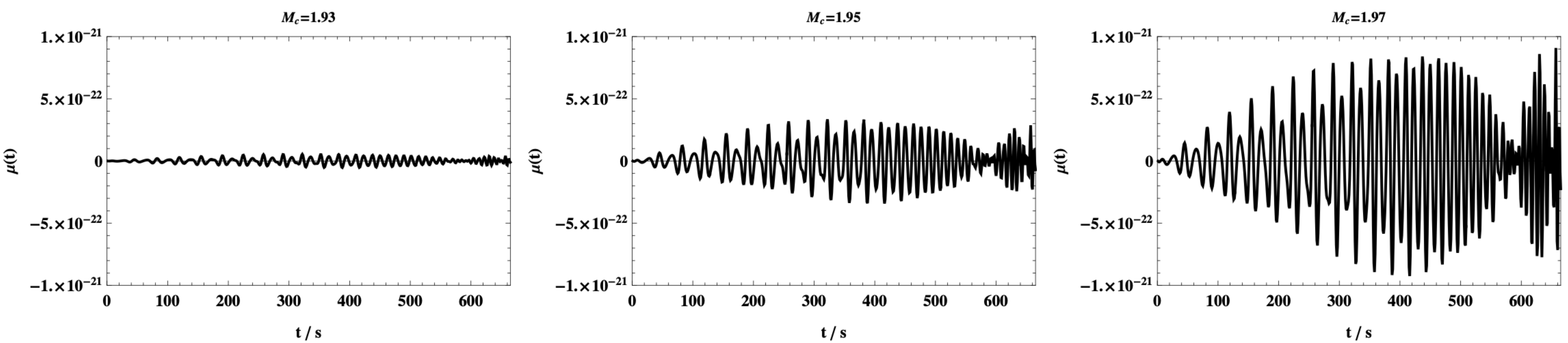
Waveform templates error interpolation

- **Gaussian Processes**
  - Sum of multivariate normal distributions
  - Mean and width fit to reproduce training data
  - Non-parameteric interpolation with error estimation

- **Application to waveform modelling error quantification**
  - No propagation of modelling errors
  - Interpolate the error between 3 pN and 3.5 pN NR waveforms
  - Non-spinning waveforms with different chirp masses

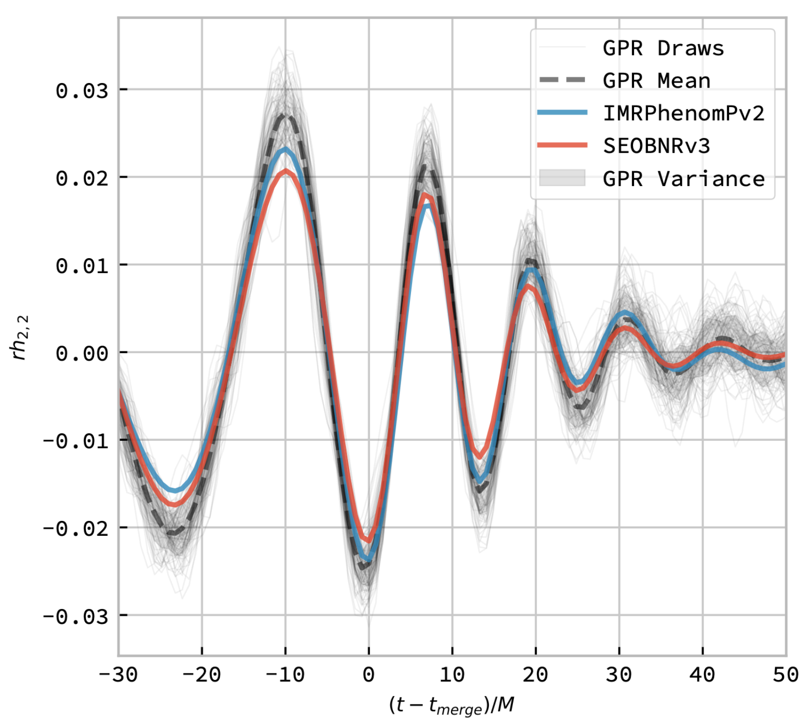
arXiv:1412.3657

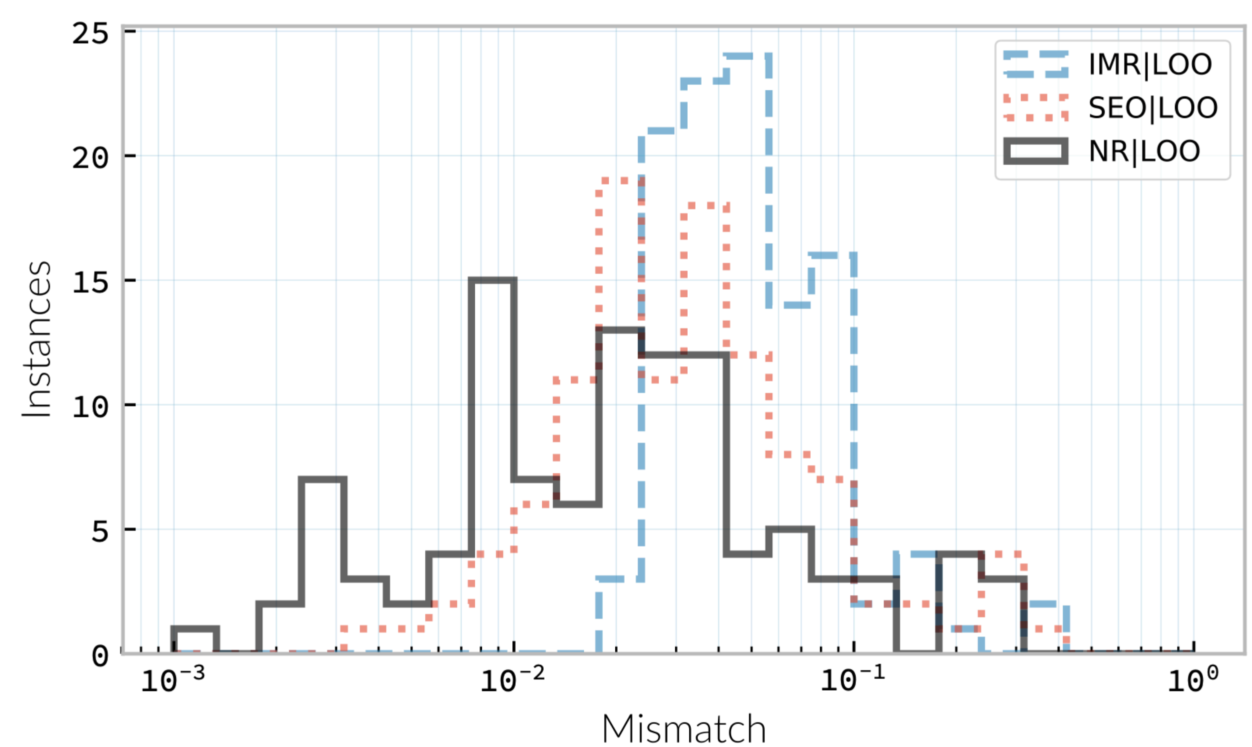


NR interpolation with Gaussian Processes

- **Application to late stages of coalescing binary**
  - Training on 132 waveforms from GeorgiaTech NR catalog
  - Precessing waveforms with $q \leq 10$ & $|\chi_i| \leq 0.9$
  - Main radiation mode: $(l, m) = (2,2)$
  - 2 validations procedures: mismatch with training dataset and leave-one-out

arXiv:1903.09204

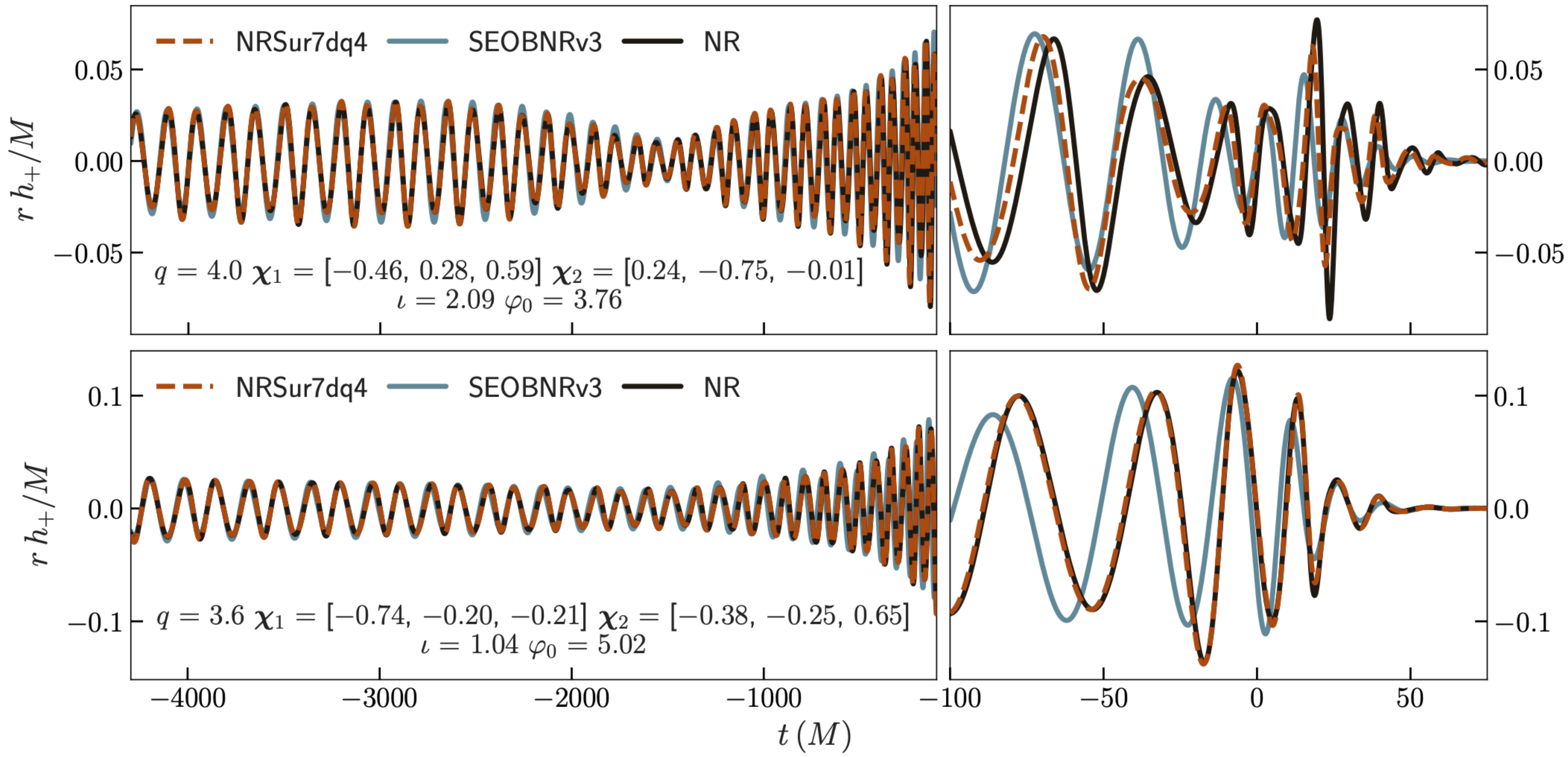




NR interpolation with Gaussian Processes

- **Application to coalescing binary**
  - Hybrids = EOB stitched to NR waveforms to cover all stages of binary coalescence
  - Training on hybridised SXS waveforms
  - Dataset: ~1500 SXS precessing with $q \leq 4$ & $|\chi_i| \leq 0.8$

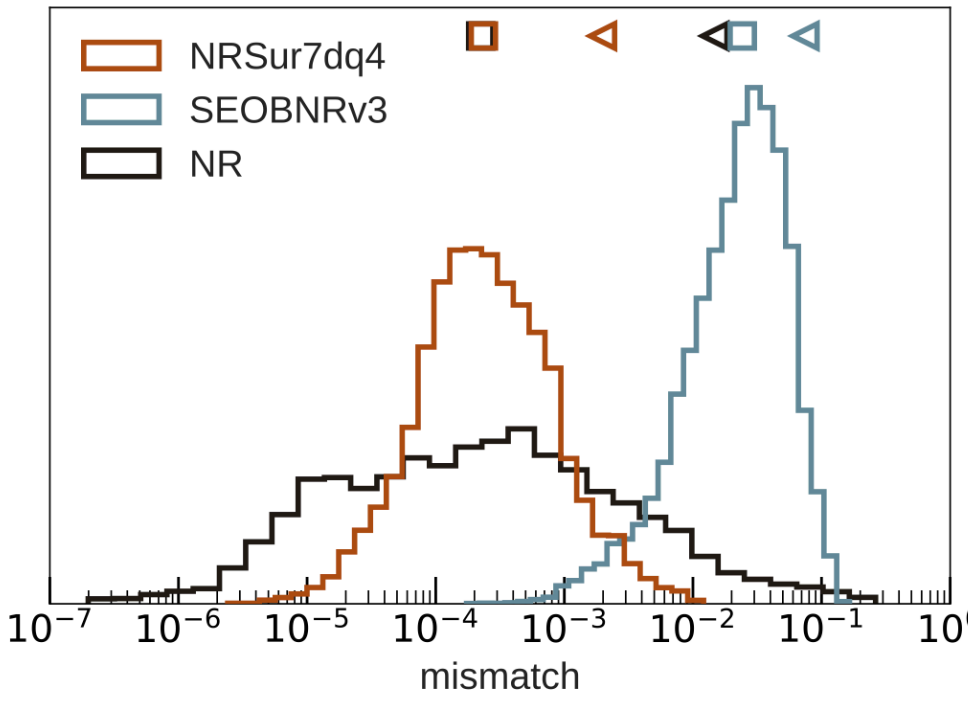
arXiv:1905.09300



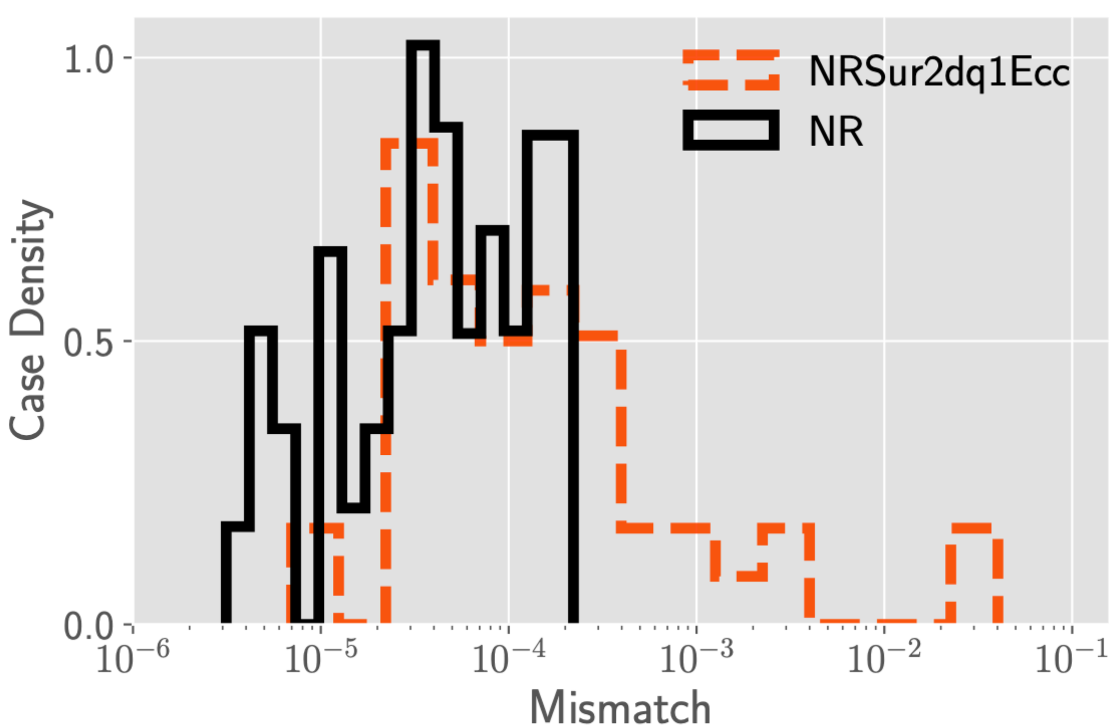
NR interpolation with Gaussian Processes

- **Model extended to include:**
  - Higher harmonics
  - Eccentric binaries
  - Extreme mass ratios

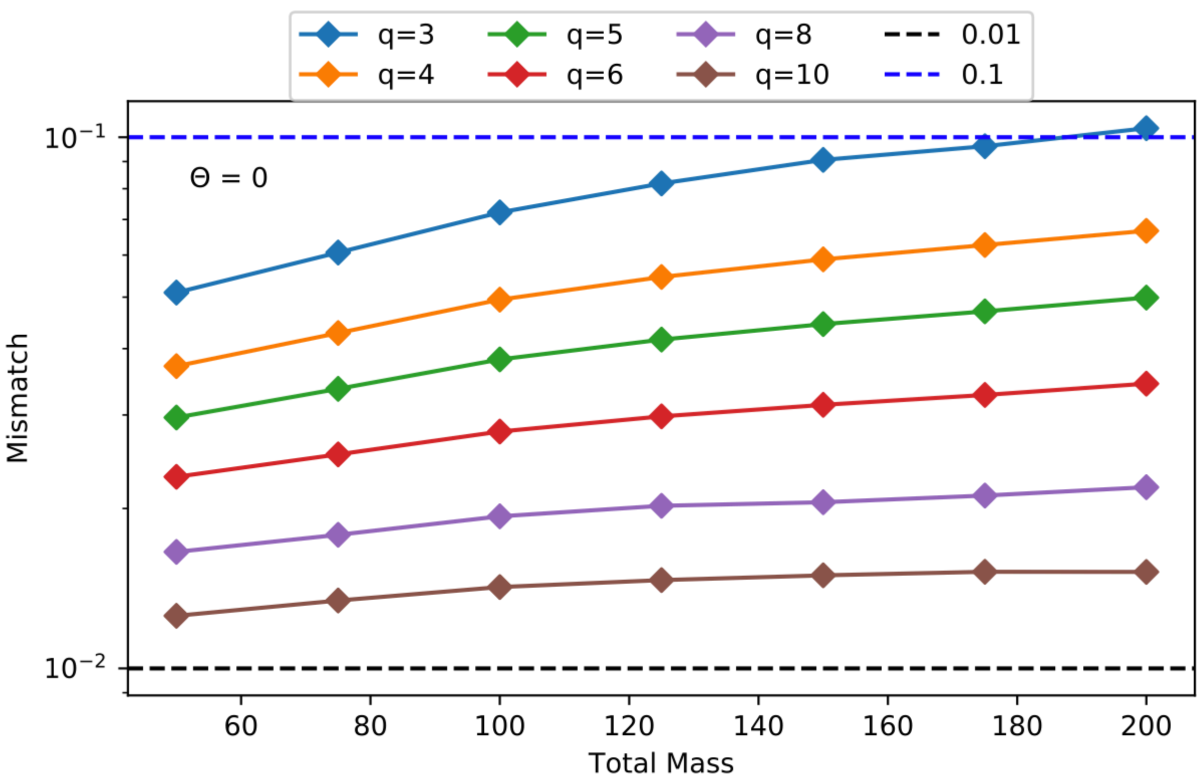
arXiv:1905.09300



arXiv:2101.11798

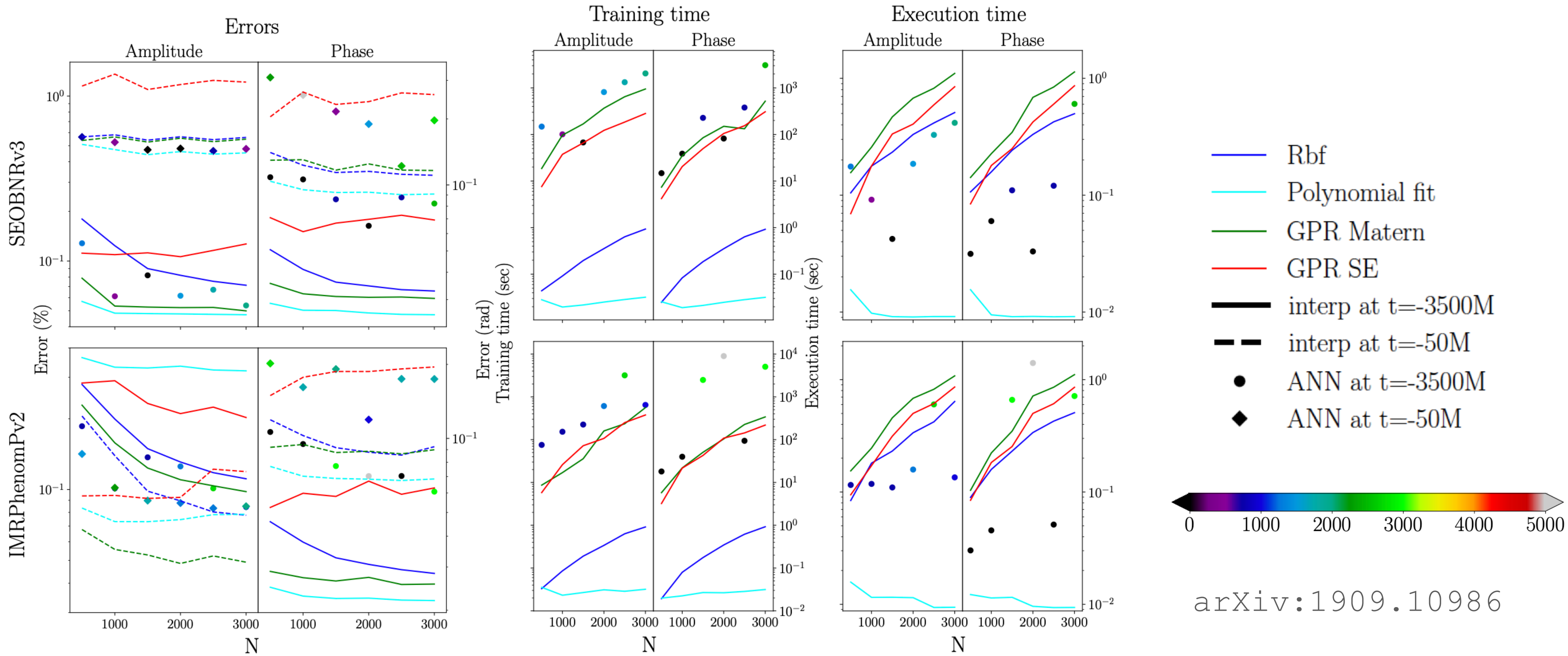


arXiv:1910.10473



Conclusion

- **Machine learning algorithms features are beneficial for waveform modelling**
  - Correlations across large parameter space
  - Provide accurate (or?) fast GW templates, usefulness depends on case
  - The creation of efficient models is built on the knowledge acquired from traditional techniques



arXiv:1909.10986

Thank you for your attention