Building flexible models of gravitational wave data

…but not *too* flexible…

Tyson B. Littenberg

$$p(A \mid B) = \frac{p(B \mid A)p(A)}{p(B)}$$

Probability (density) of *A* given *B*

$$p(\theta \mid d, M) = \frac{p(d \mid \theta, M)\, p(\theta \mid M)}{p(d \mid M)}$$

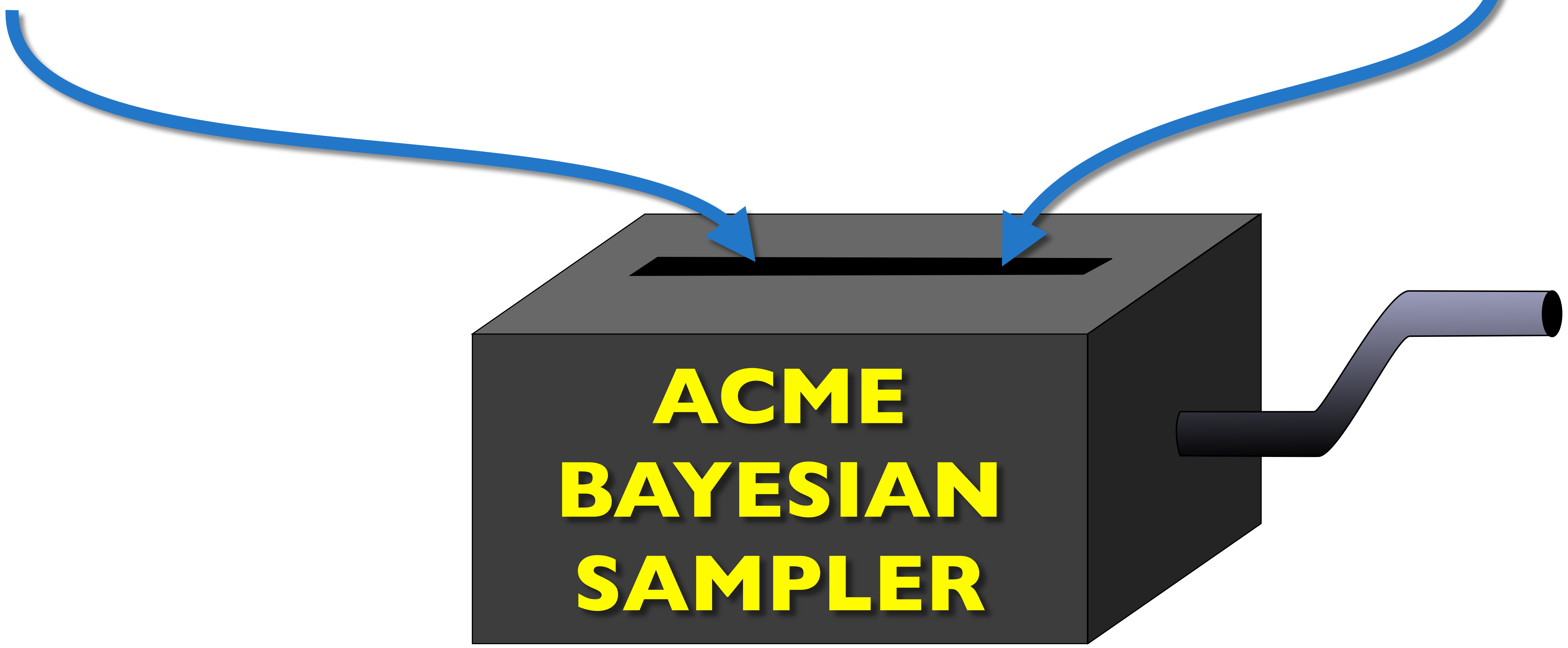
Probability (density) of *parameters*
given *data && model*

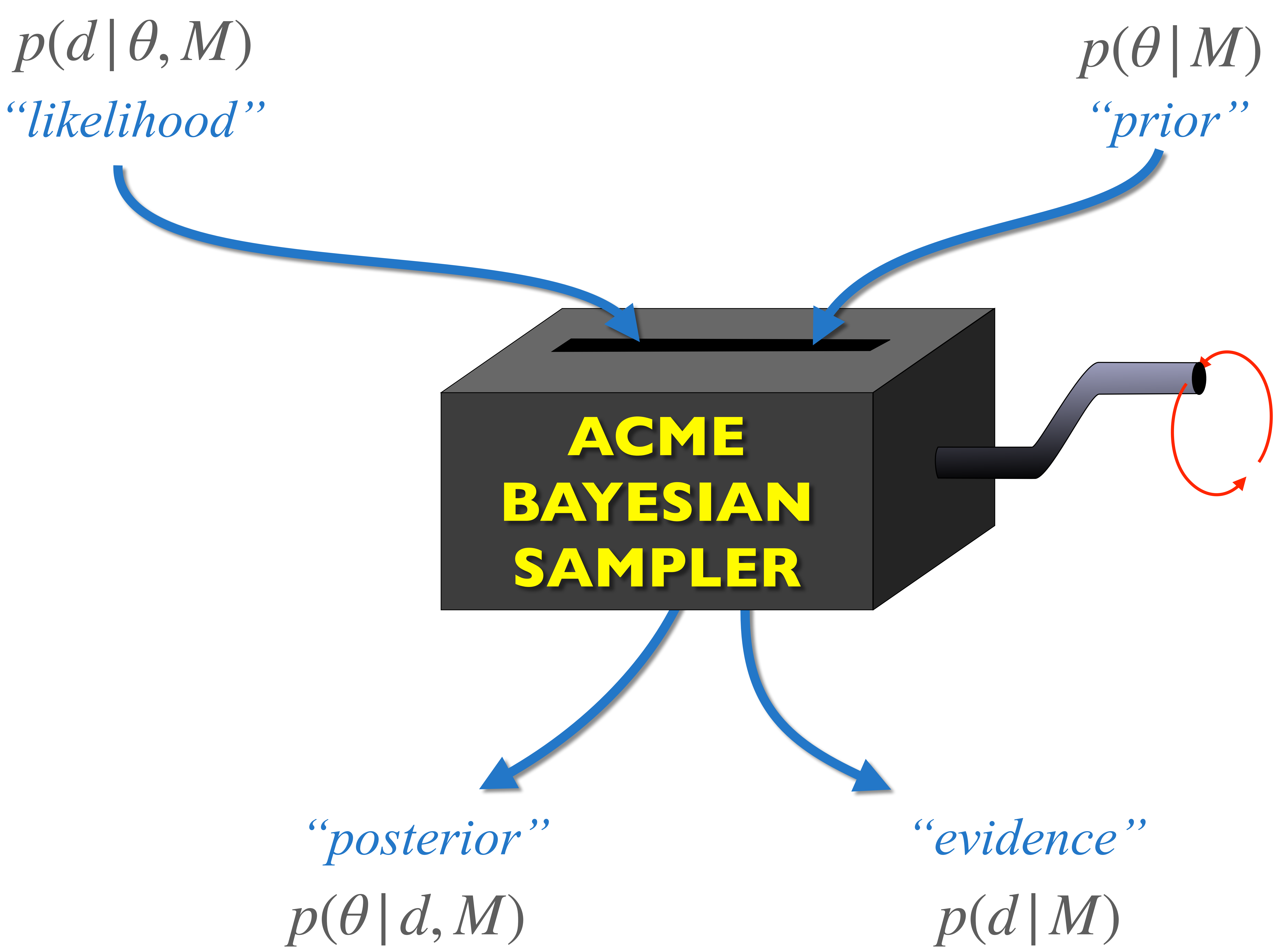
$p(d \mid \theta, M)$

"likelihood"

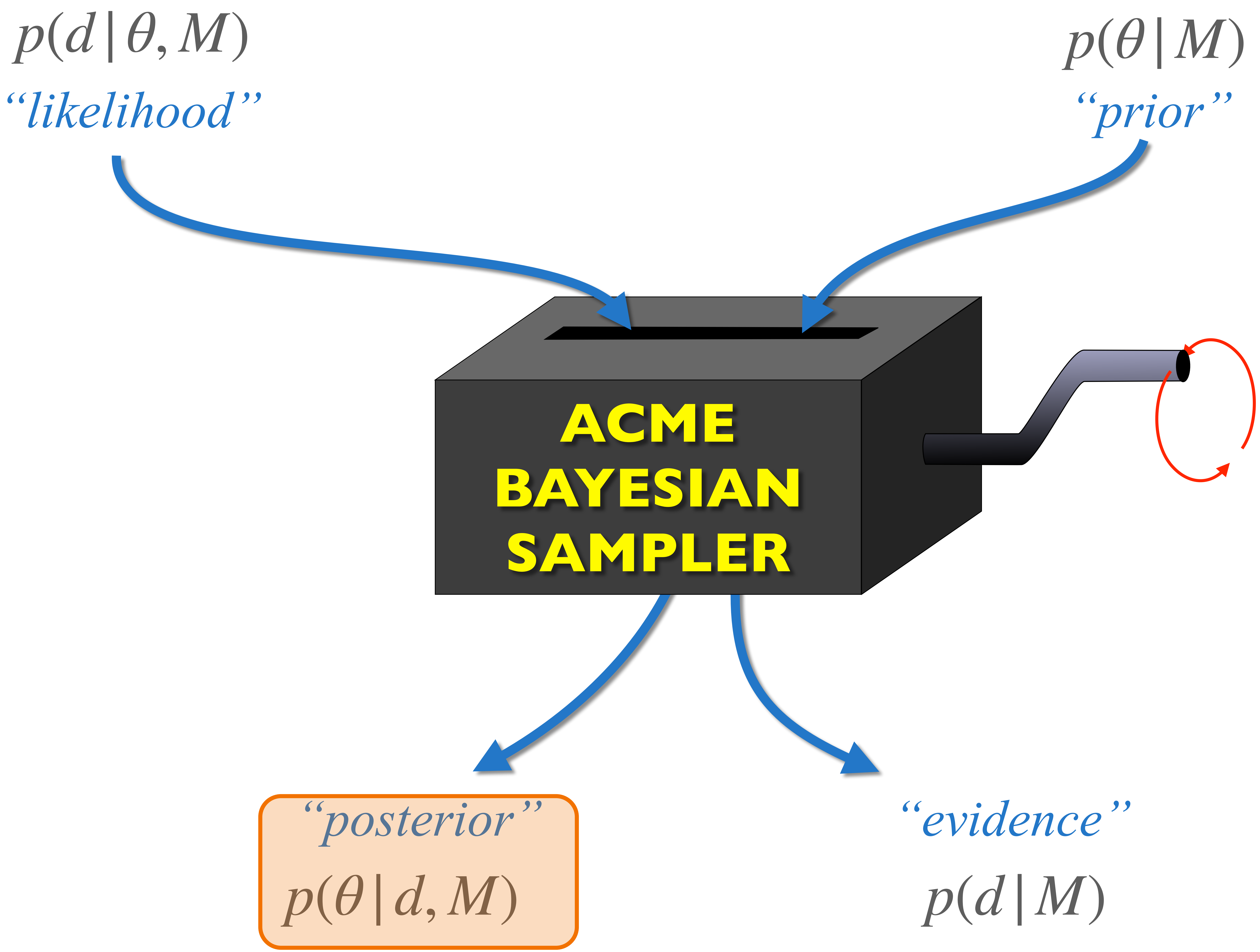
$p(\theta \mid M)$

"prior"





$p(d|\theta, M)$
"likelihood"
$p(\theta|M)$
"prior"
ACME
BAYESIAN
SAMPLER
"posterior"
$p(\theta|d, M)$
"evidence"
$p(d|M)$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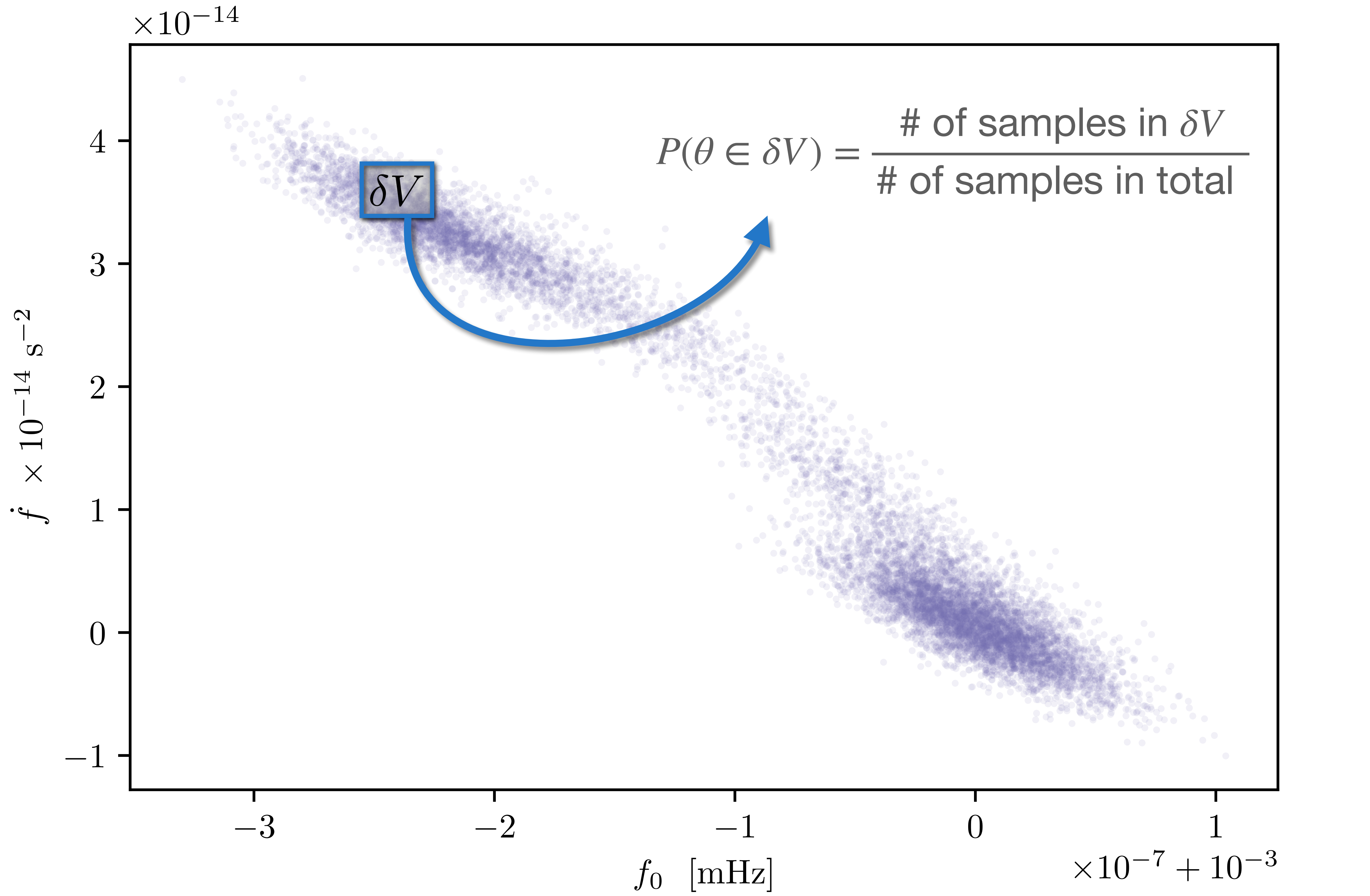
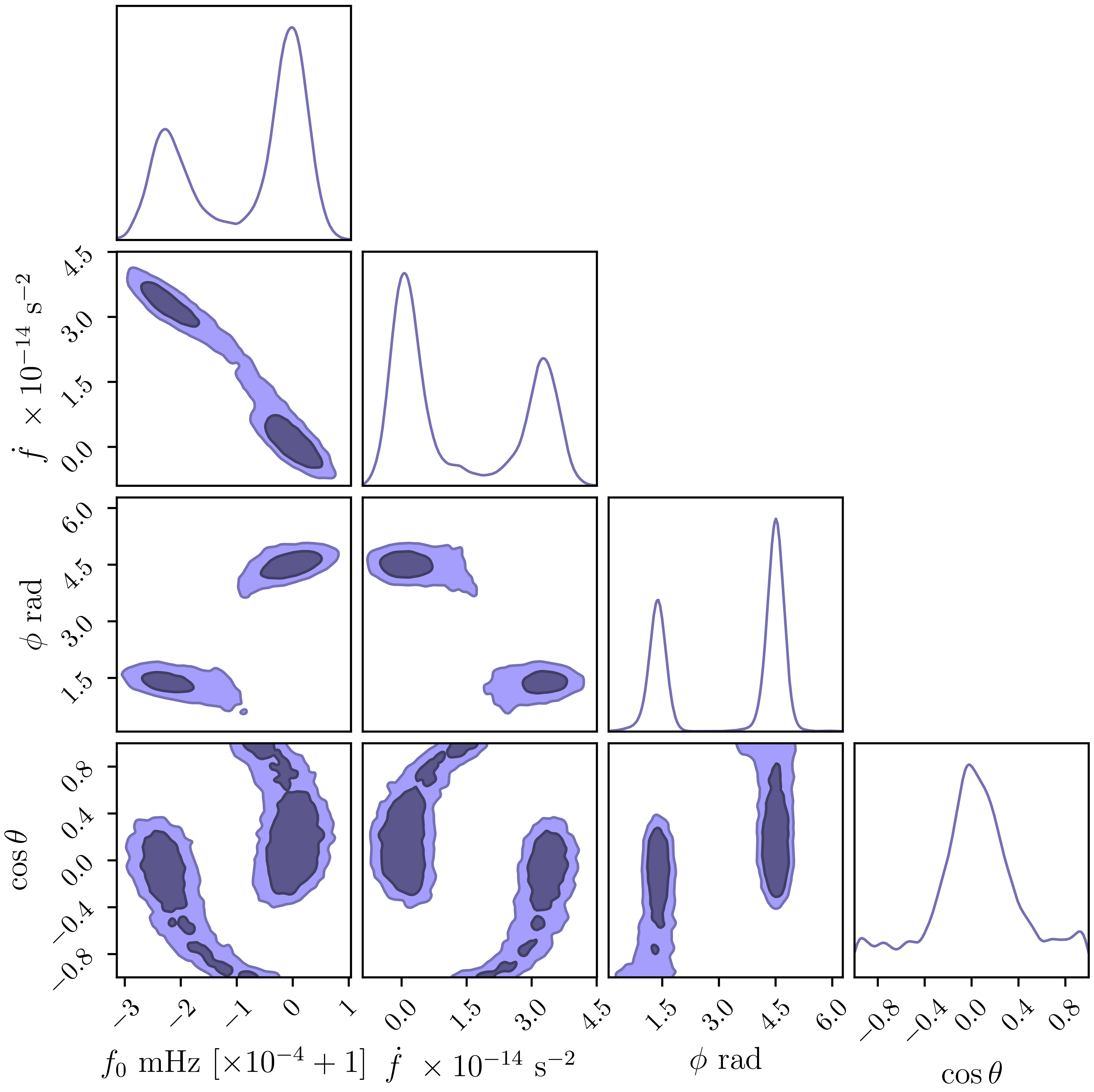
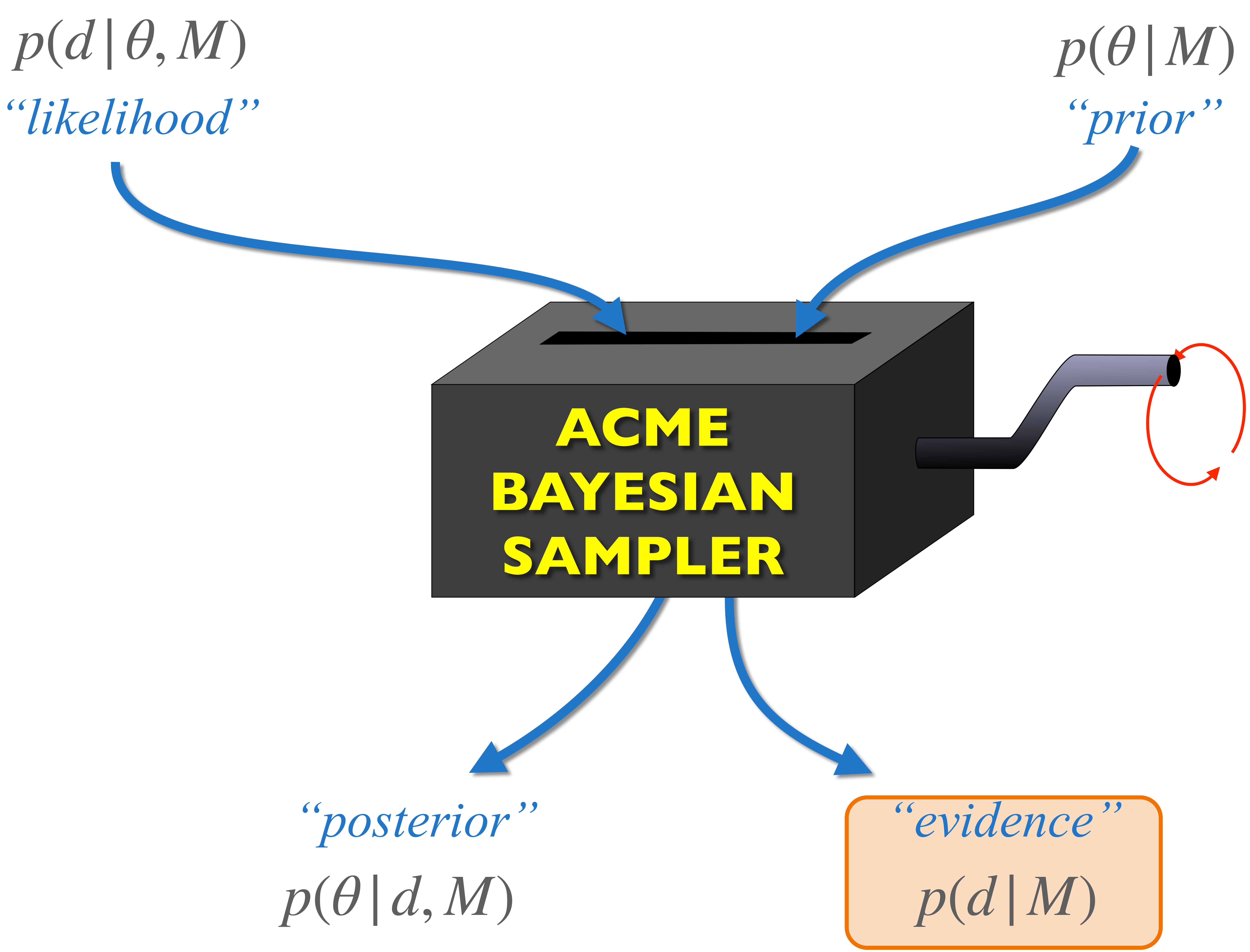


$$p(x\,|\,d, M) = \int p(x, y, z\,|\,d, M)\; dy\; dz$$

"nuisance parameters"



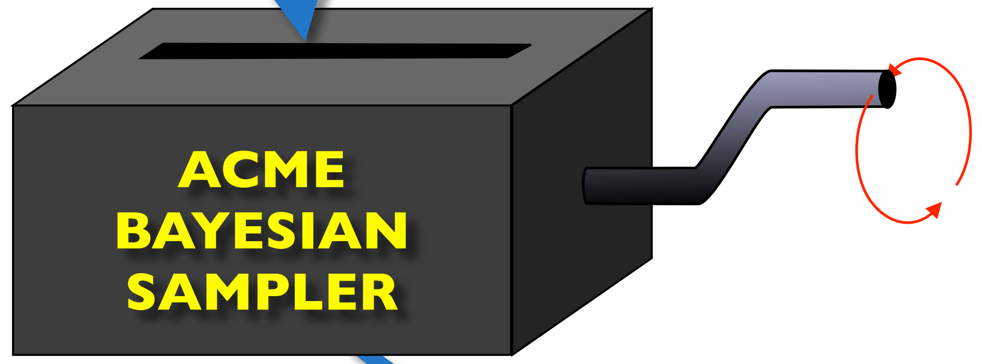
$\dot{f} \times 10^{-14}$ s^{-2}
ϕ rad
$\cos\theta$
f_0 mHz $[\times 10^{-4} + 1]$
$\dot{f} \times 10^{-14}$ s^{-2}
ϕ rad
$\cos\theta$


$p(d|\theta, M)$
"likelihood"
$p(\theta|M)$
"prior"
ACME BAYESIAN SAMPLER
"posterior"
$p(\theta|d, M)$
"evidence"
$p(d|M)$

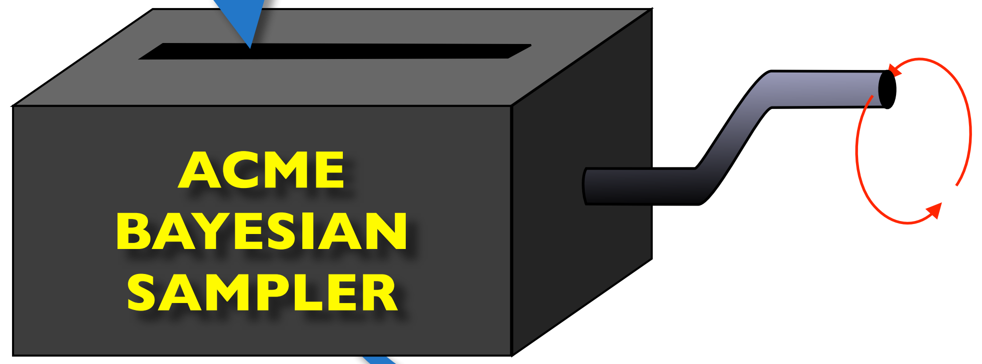
$$p(d\,|\,M) = \int d\theta\; p(d\,|\,\theta, M)\; p(\theta\,|\,M)$$

$$p(d\,|\,M) = \int d\theta\; p(d\,|\,\theta, M)\; p(\theta\,|\,M)$$

"I've detected Gravitational Waves!"



"I've measured lots of noise!"



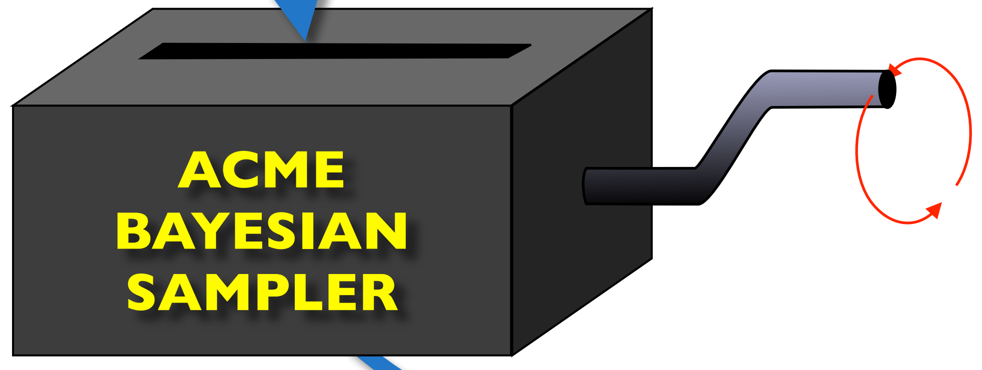
$$\mathcal{O}_{A,B} = \frac{p(M_A)}{p(M_B)} \times \frac{p(d\,|\,M_A)}{p(d\,|\,M_B)}$$

"odds ratio"

"Bayes factor"

$$p(d\,|\,M) = \int d\theta \; p(d\,|\,\theta, M)\; p(\theta\,|\,M)$$

"I've detected Gravitational Waves!"



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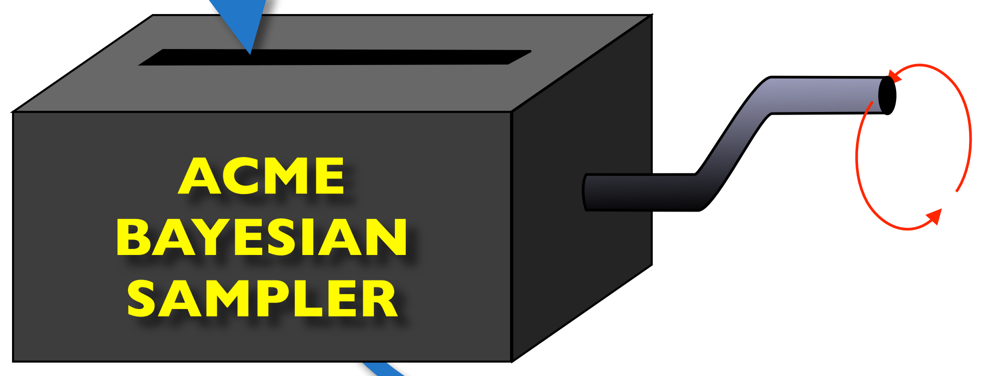


$$\mathcal{O}_{A,B} = \frac{p(M_A)}{p(M_B)} \times \frac{p(d\,|\,M_A)}{p(d\,|\,M_B)}$$

"odds ratio"

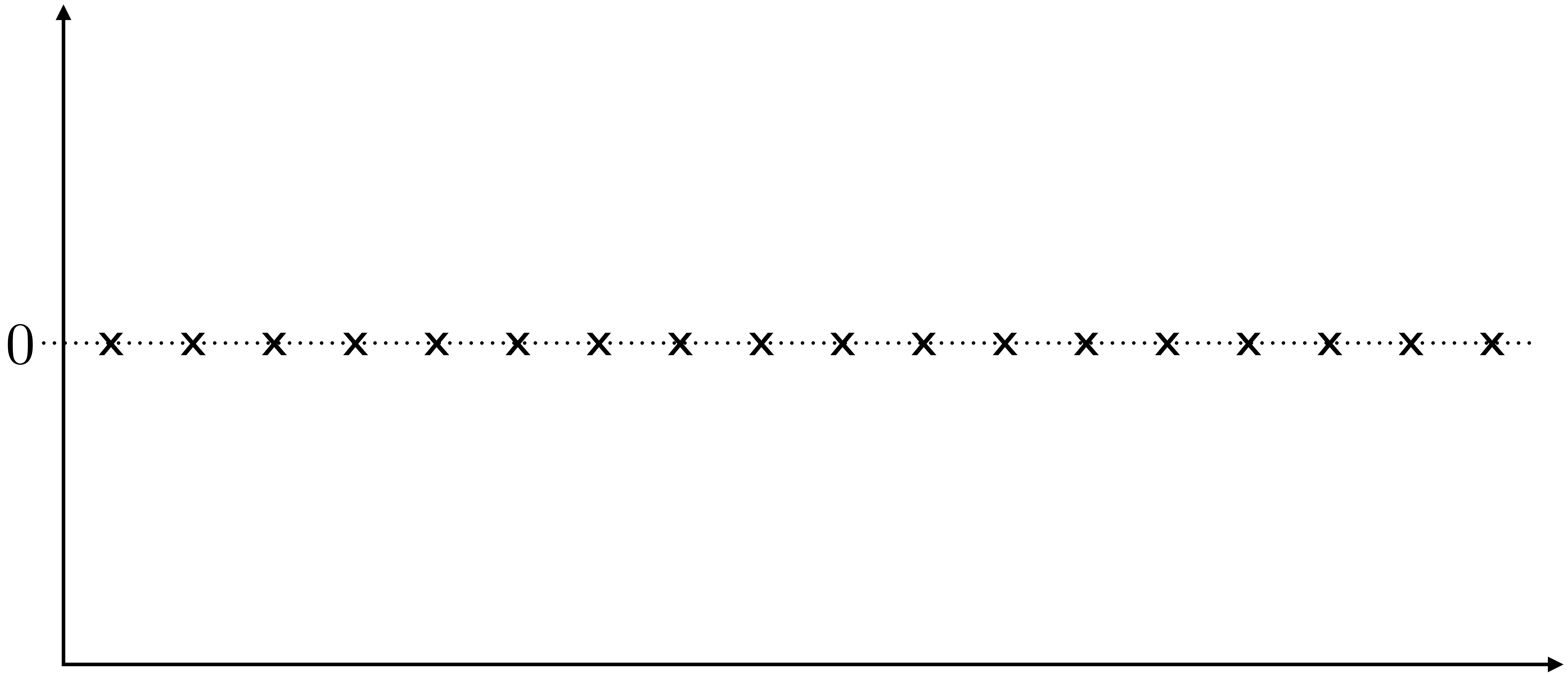
"Bayes factor"

$\mathcal{O}_{A,B} = X \equiv$ Model A is preferred over model B with $X : 1$ odds

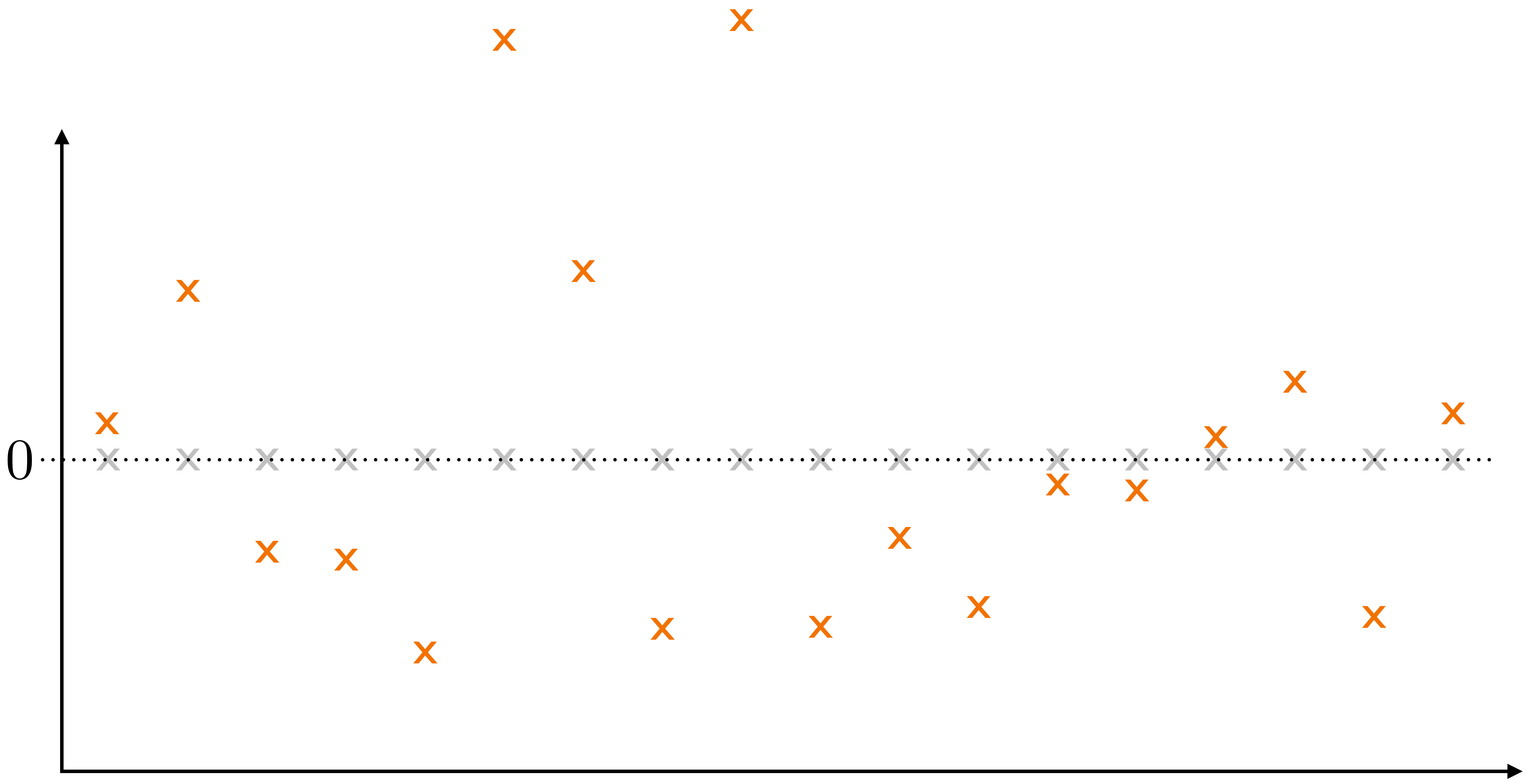
Bayesian Analyses: Not magic.

Let’s Build a Likelihood Function

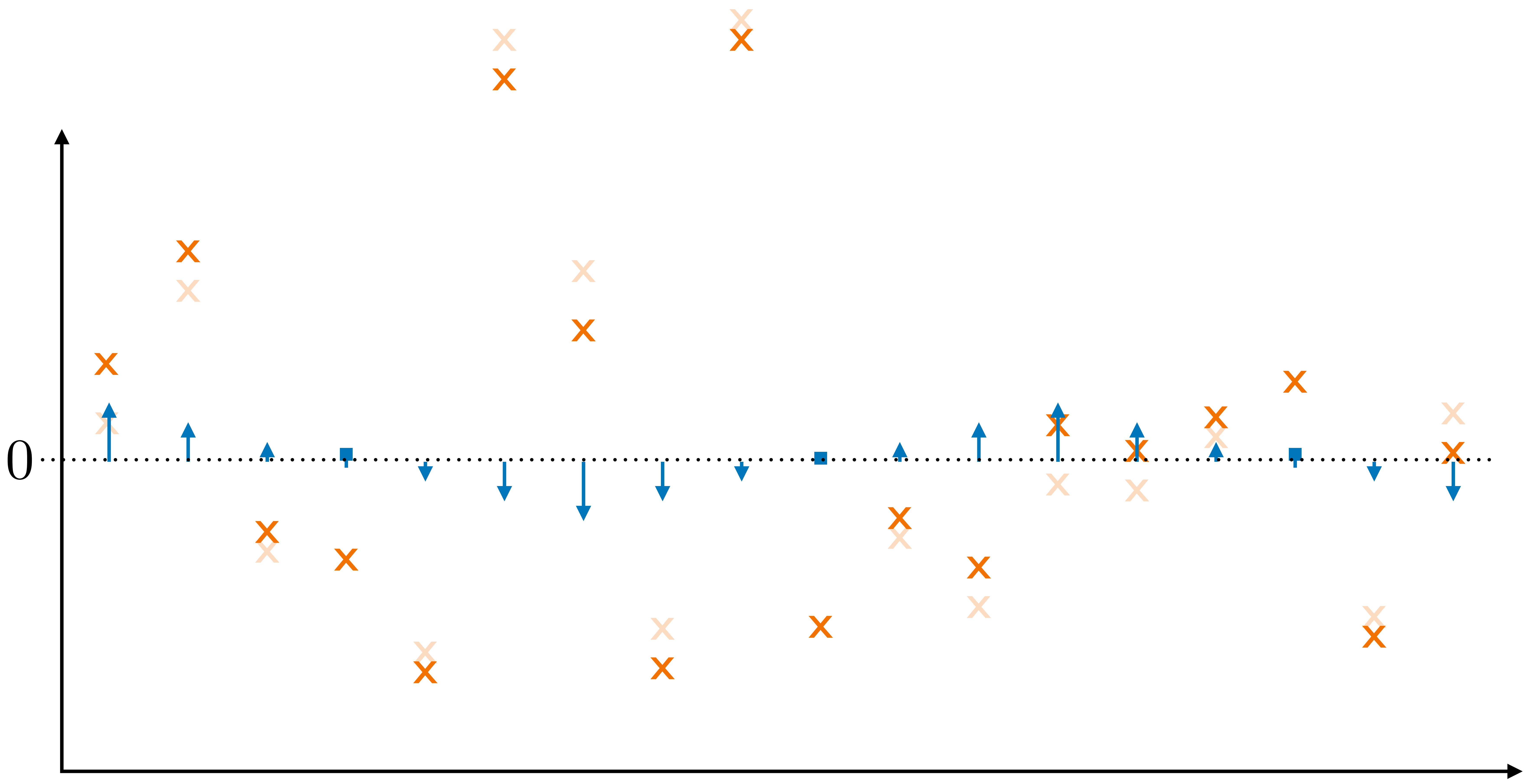
d



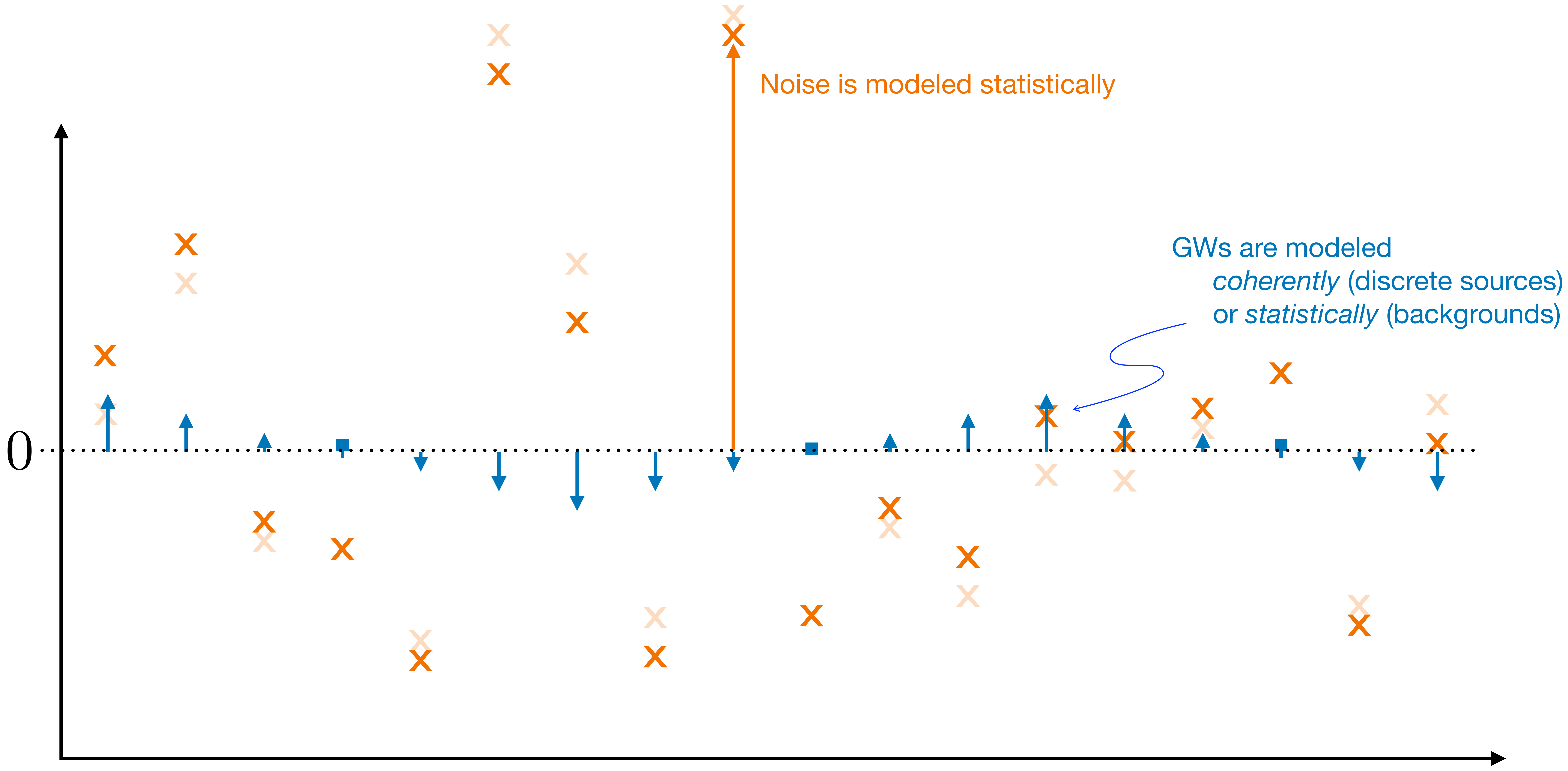
$d = n$



$$d = n + h$$



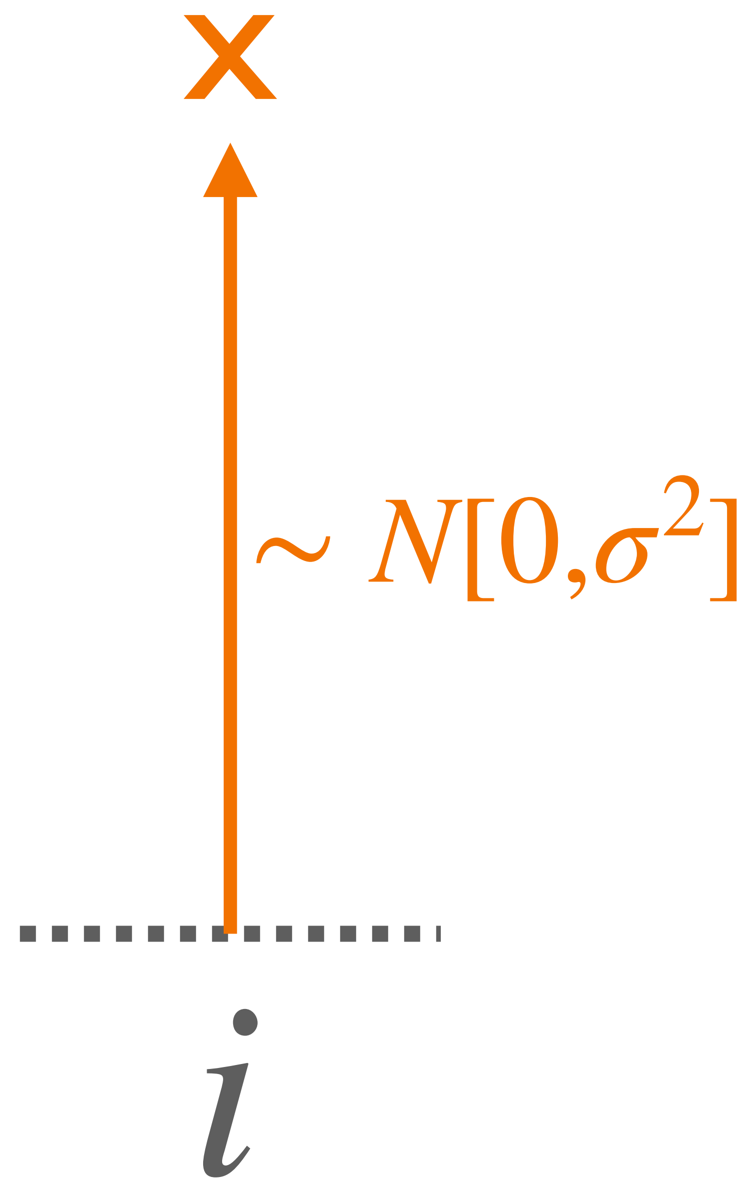
$$d = n + h$$


Noise is modeled statistically
GWs are modeled
coherently (discrete sources)
or *statistically* (backgrounds)
0

$$d = n + h$$

Noise is zero-mean Gaussian

Noise has known variance



Probability of measuring noise n_i

$$p(n_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{|n_i|^2}{2\sigma^2}}$$

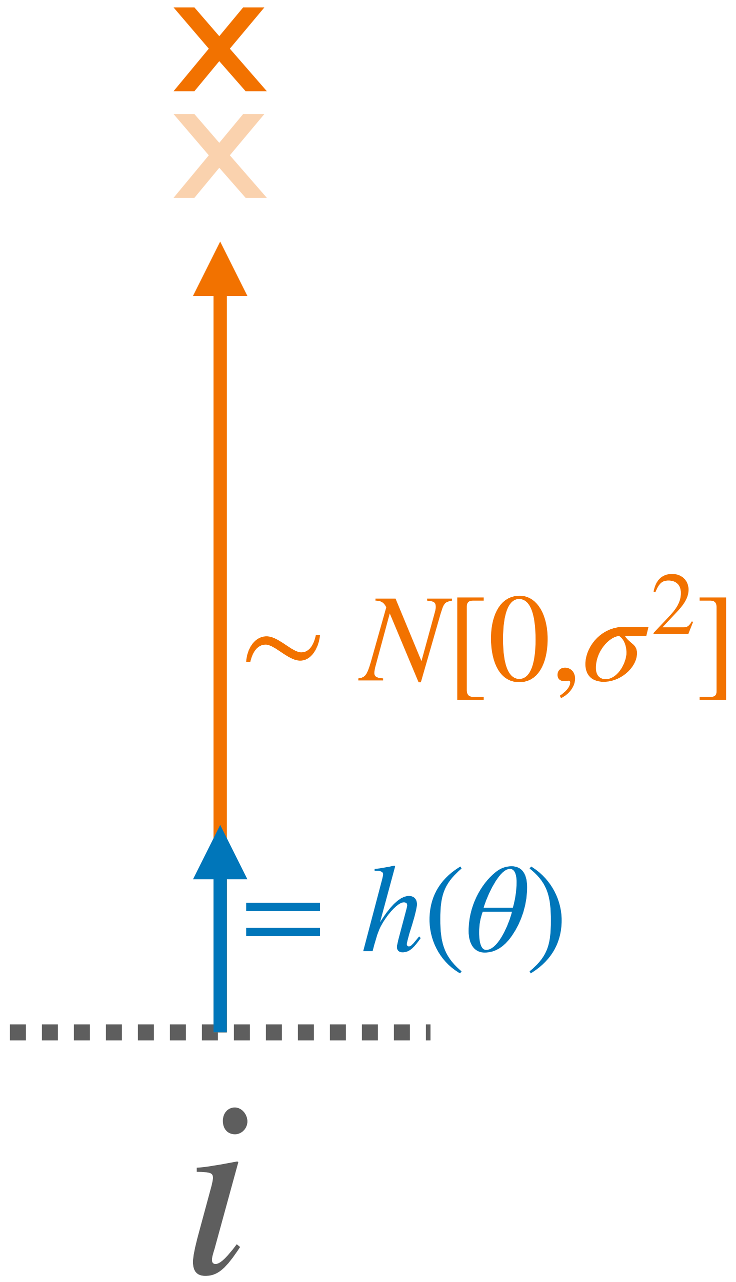
$$d = n + h$$

Noise is zero-mean Gaussian

Noise has known variance

Data are perfectly calibrated

Waveform model is perfect



Probability of measuring data d_i

$$p(d_i \,|\, \theta) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{|d_i - h_i|^2}{2\sigma^2}}$$

$$d = n + h$$

Noise is zero-mean Gaussian

Noise has known variance

Data are perfectly calibrated

Waveform model is perfect

Probability of measuring set of data **d** with *k* samples:

$$p(\mathbf{d}\,|\,\theta) = \frac{1}{\sqrt{(2\pi)^k \det C}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^T C^{-1}(\mathbf{d}-\mathbf{h})}$$

$$d = n + h$$

Noise is zero-mean Gaussian

Noise has known variance

Data are perfectly calibrated

Waveform model is perfect

Noise variance is *stationary*

$$\langle \tilde{n}_i \tilde{n}_j \rangle = \sigma_i^2 \delta_{i,j} \equiv \frac{T}{2} S_n(f_i)$$

Probability of measuring set of data **d** with *k* samples:

$$p(\mathbf{d} \,|\, \theta) = \frac{1}{\sqrt{(2\pi)^k \det C}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^T C^{-1} (\mathbf{d}-\mathbf{h})}$$

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$$\Downarrow$$

$$p(\mathbf{d}\,|\,\theta) = \prod_k \frac{2}{\pi T S_{n,k}} e^{-\frac{2|\tilde{d}_k - \tilde{h}_k|^2}{T S_{n,k}}}$$

$$d = n + h$$

Noise is zero-mean Gaussian

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$$\langle \tilde{n}_i \tilde{n}_j \rangle = \sigma_i^2 \delta_{i,j} \equiv \frac{T}{2} S_n(f_i)$$

Probability of measuring set of data **d** with *k* samples:

$$p(\mathbf{d} \,|\, \theta) = \frac{1}{\sqrt{(2\pi)^k \det C}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{h})^T C^{-1} (\mathbf{d}-\mathbf{h})}$$

$$\Downarrow$$

$$p(\mathbf{d} \,|\, \theta) \propto e^{-\frac{2}{T} \sum_k \frac{|\tilde{d}_k - \tilde{h}_k|^2}{S_{n,k}}}$$

Noise is zero-mean Gaussian

Noise has known variance

Data are perfectly calibrated

Waveform model is perfect

Noise variance is *stationary*

~~Noise is zero-mean Gaussian~~

~~Noise has known variance~~

~~Data are perfectly calibrated~~

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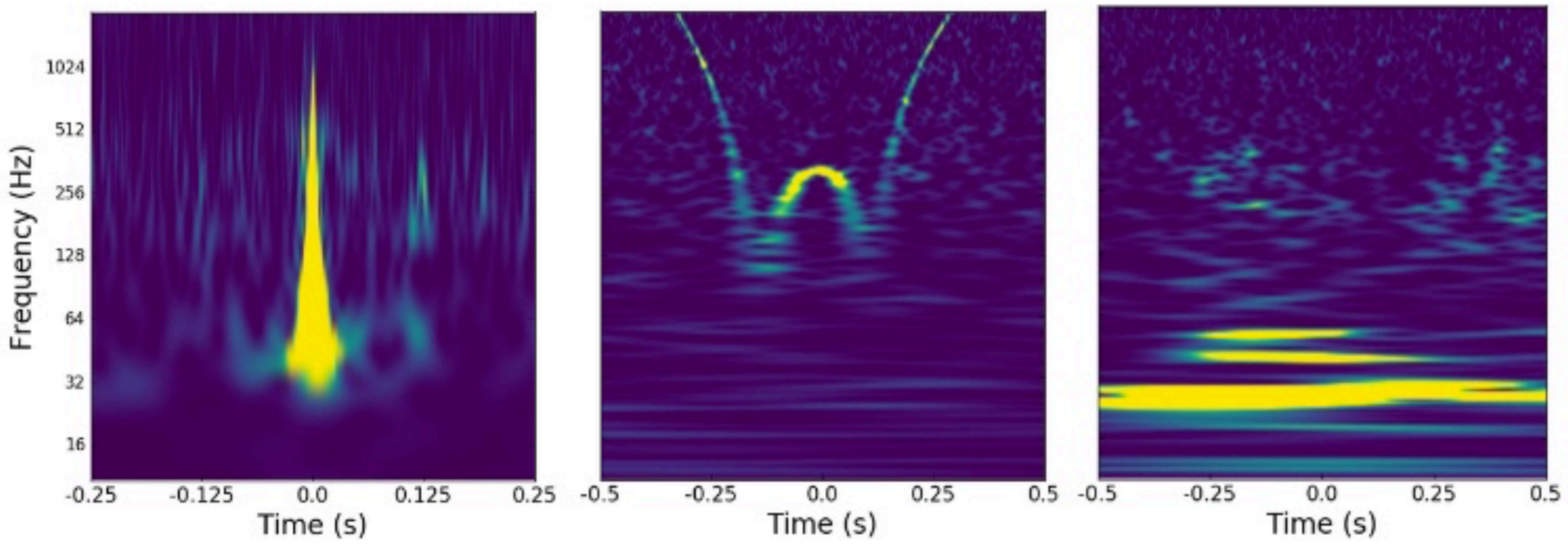
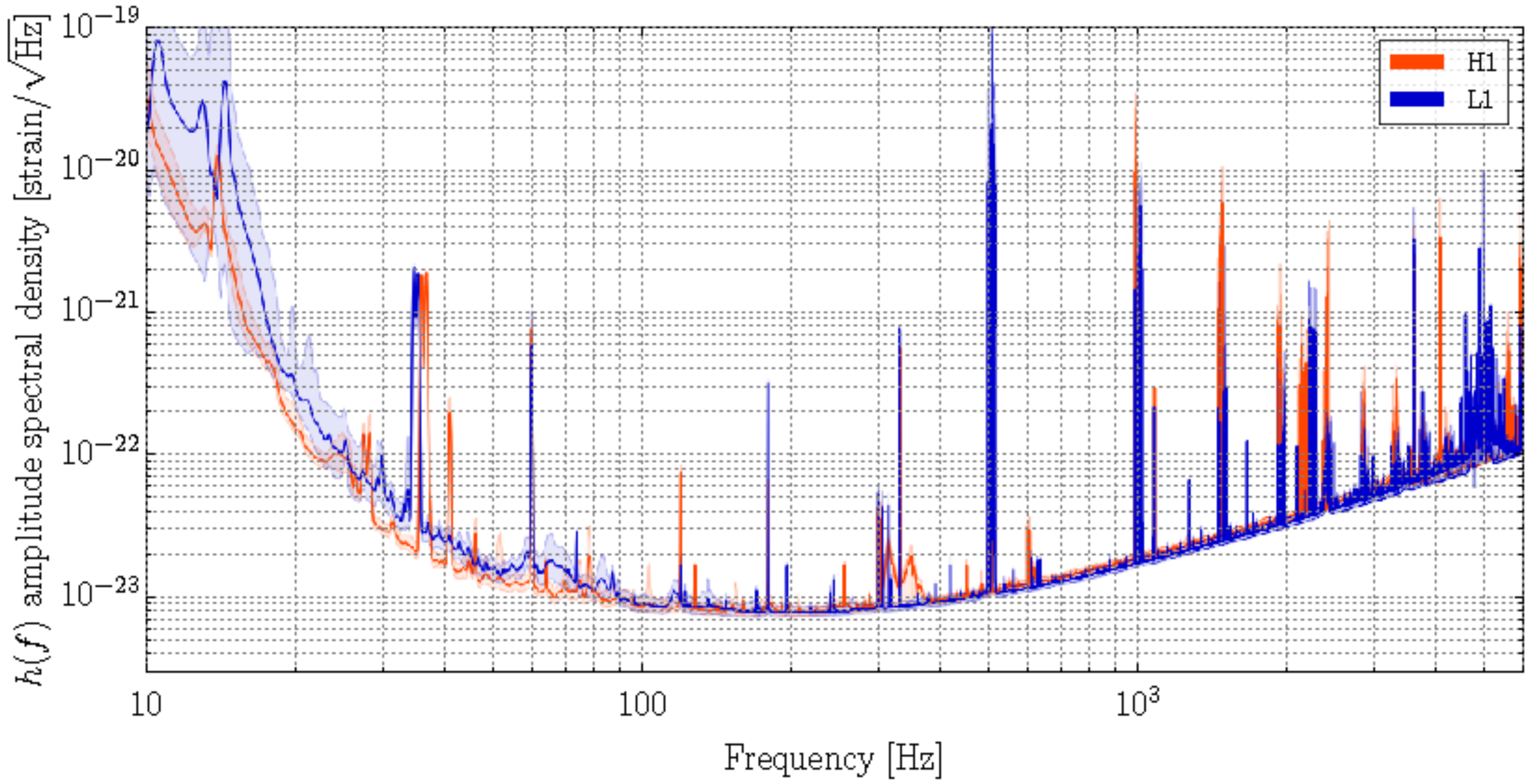
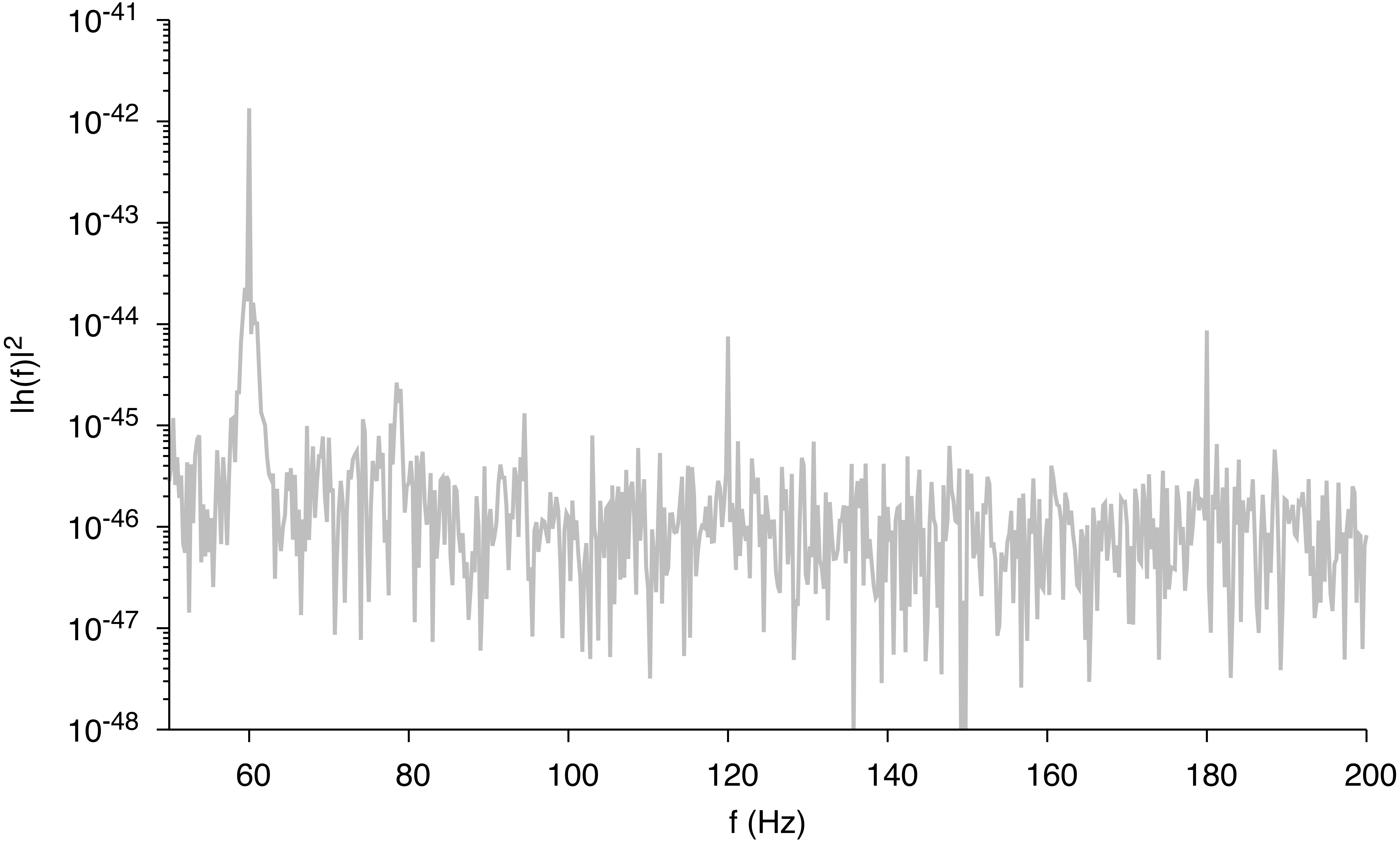




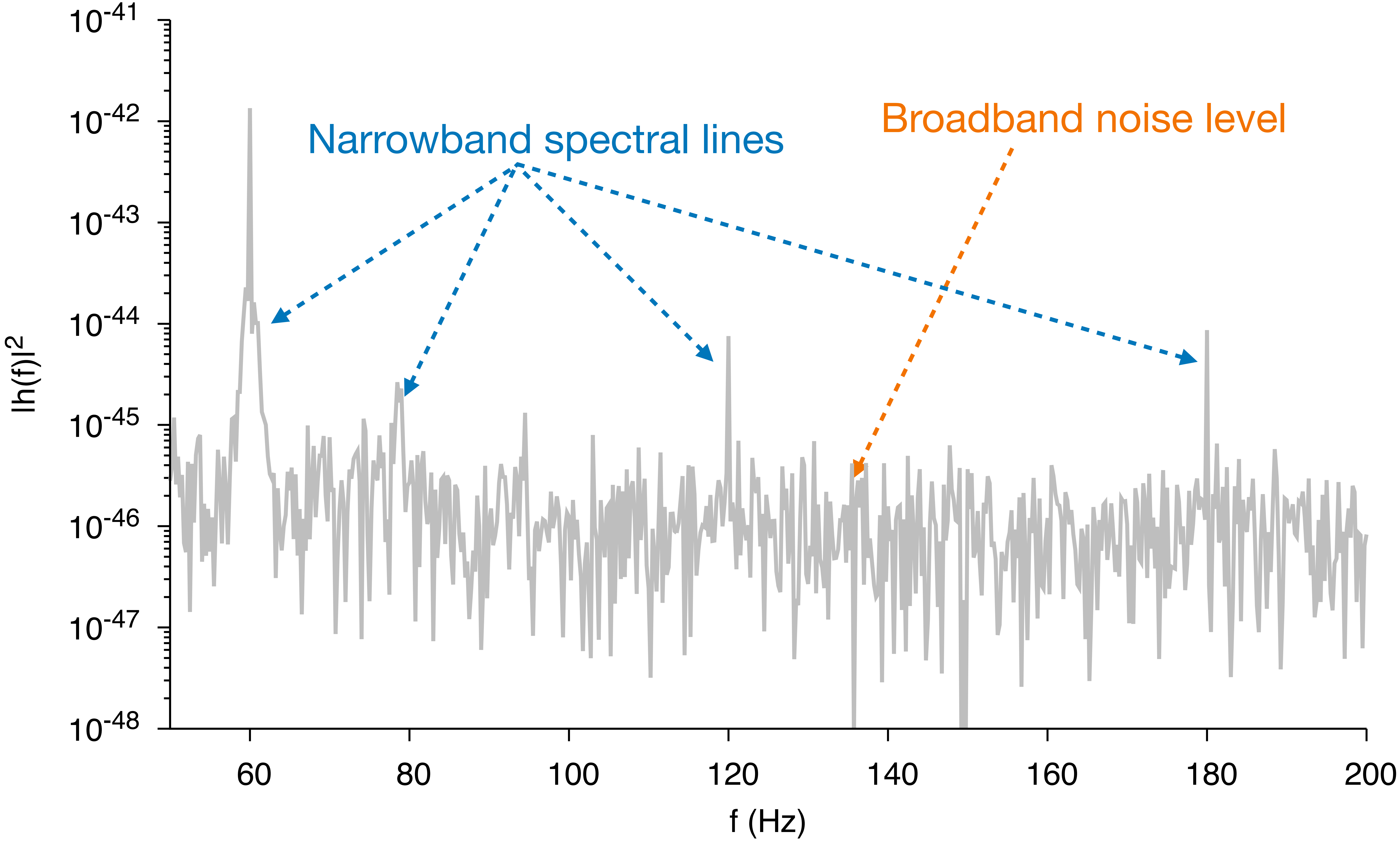
Model everything and let the data sort it out

Choose a convenient “basis set” to phenomenologically model features in data

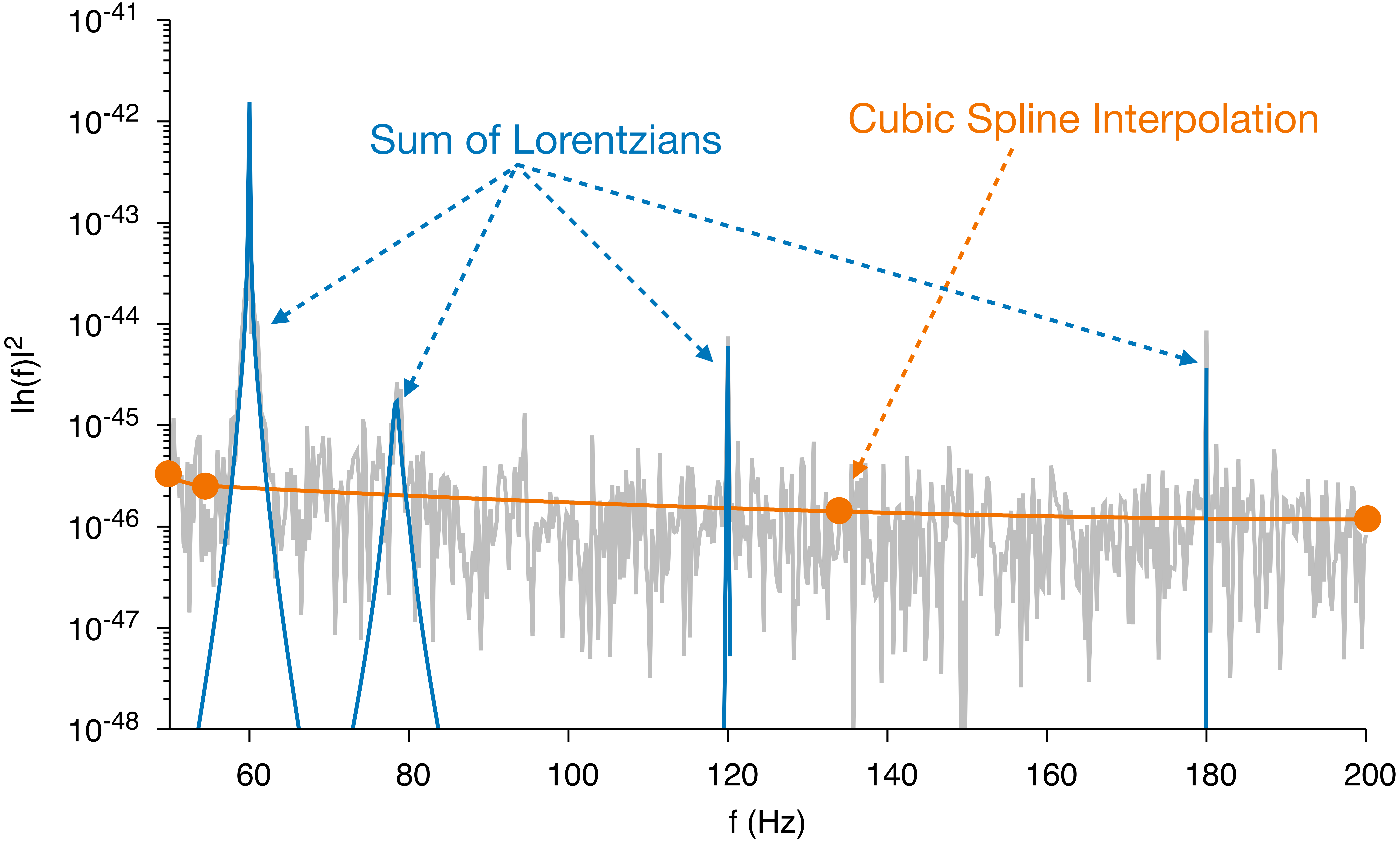
Use *evidence* to determine the number of “basis functions” to use in the model



10^{-41}
10^{-42}
10^{-43}
10^{-44}
10^{-45}
10^{-46}
10^{-47}
10^{-48}
$|h(f)|^2$
60
80
100
120
140
160
180
200
f (Hz)



Narrowband spectral lines
Broadband noise level
10^-41
10^-42
10^-43
10^-44
10^-45
10^-46
10^-47
10^-48
|h(f)|^2
60
80
100
120
140
160
180
200
f (Hz)



Sum of Lorentzians
Cubic Spline Interpolation
10^{-41}
10^{-42}
10^{-43}
10^{-44}
10^{-45}
10^{-46}
10^{-47}
10^{-48}
$|h(f)|^2$
60
80
100
120
140
160
180
200
f (Hz)

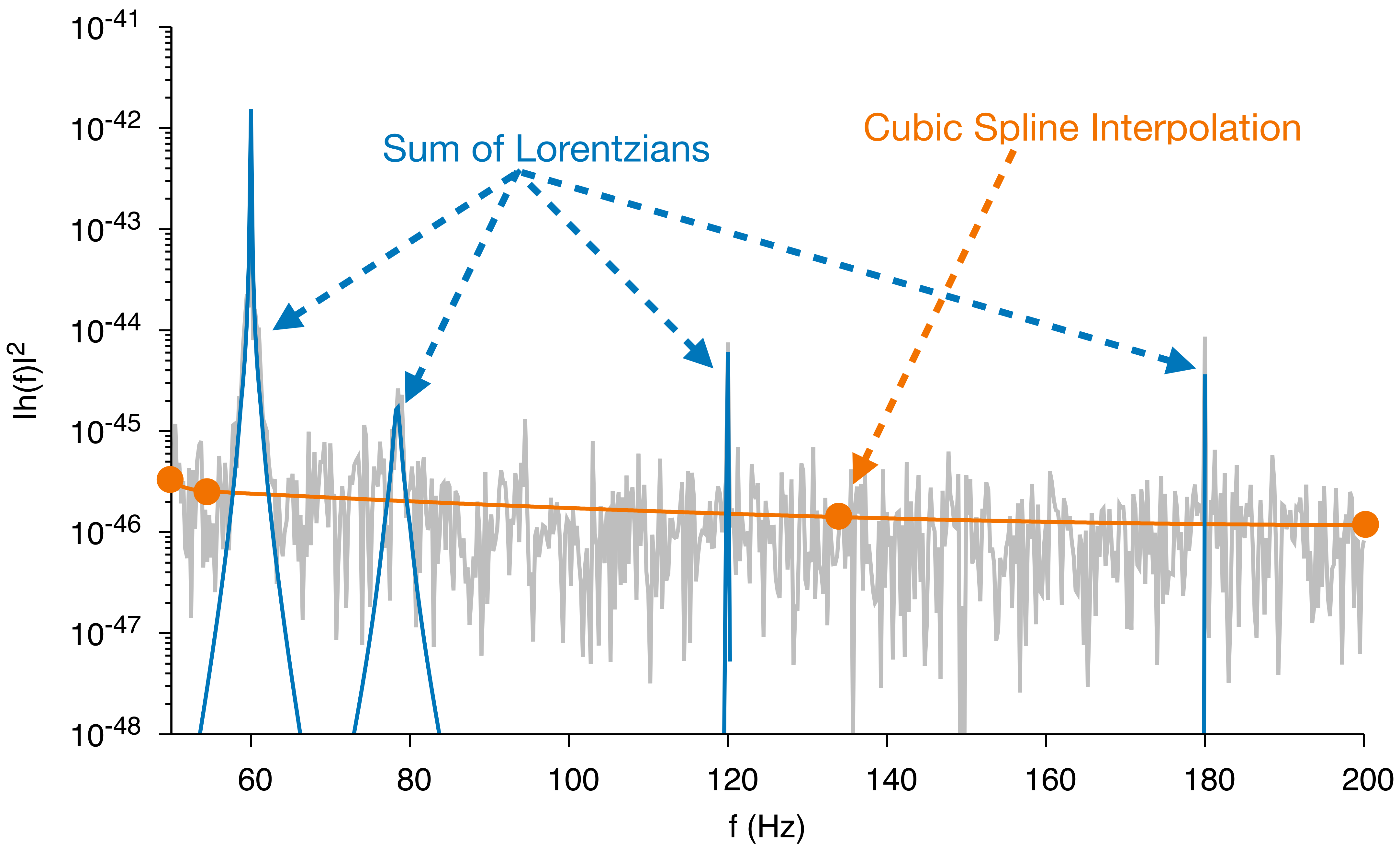
~~Noise is zero-mean Gaussian~~

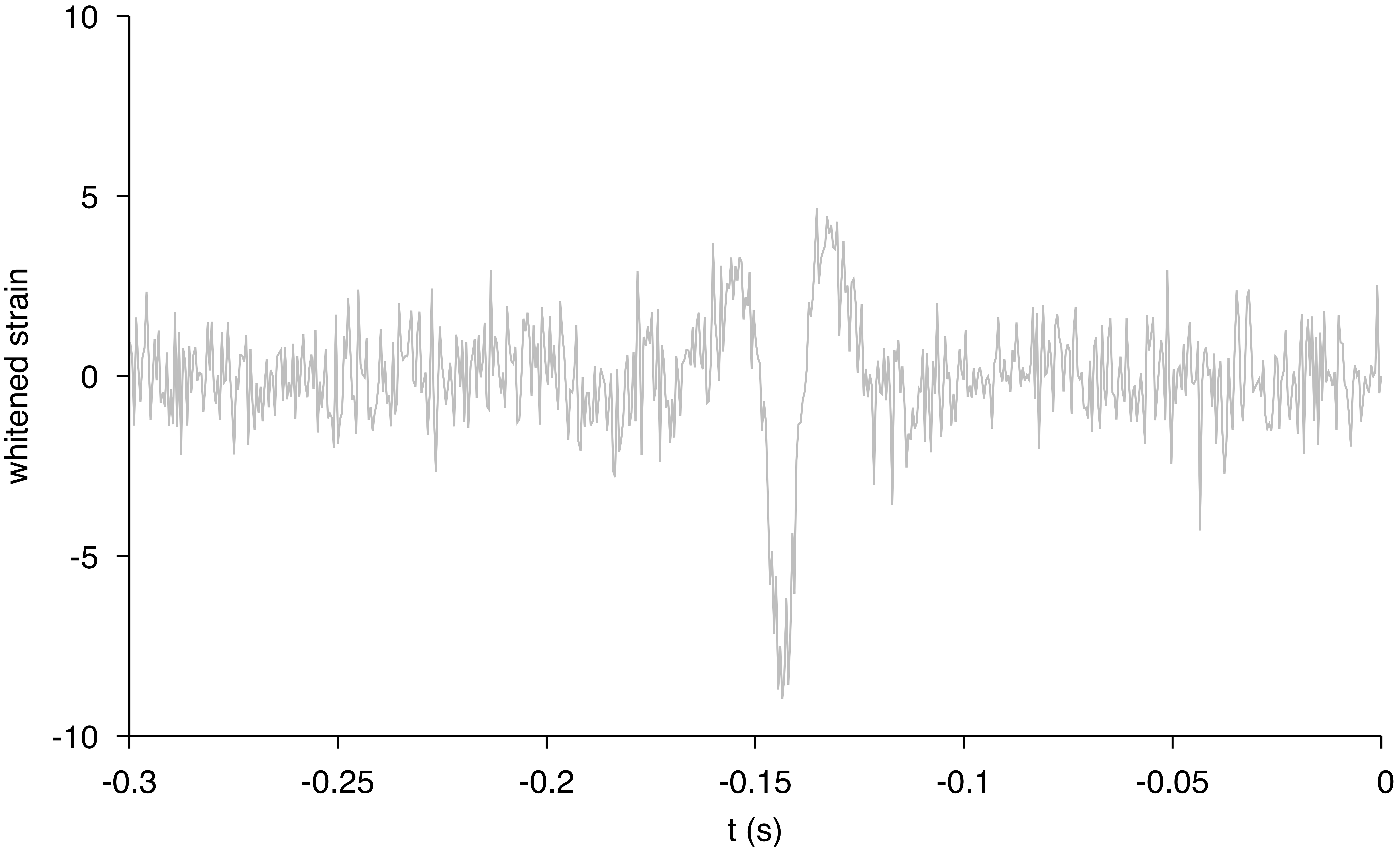
Noise has known variance ✔

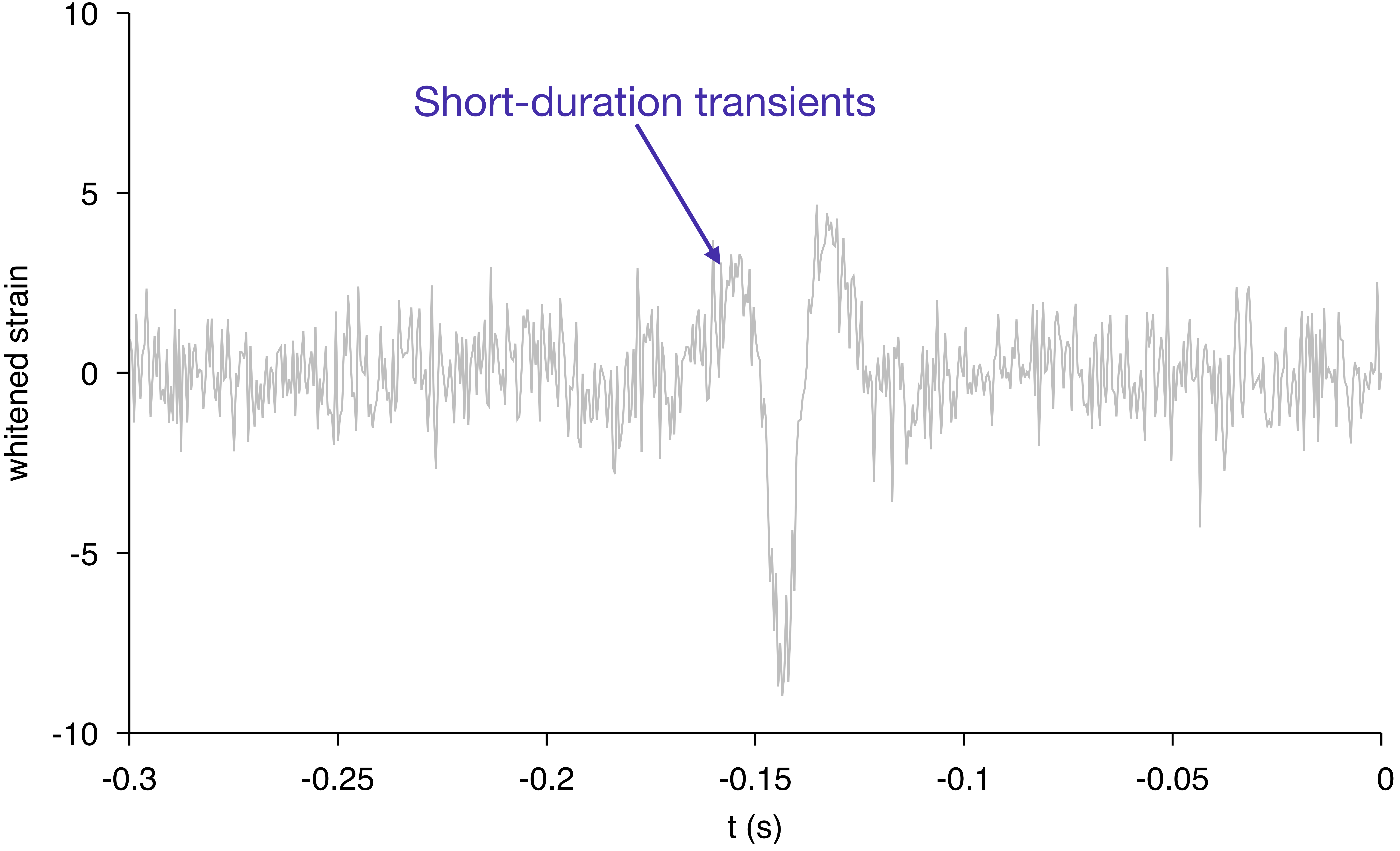
~~Data are perfectly calibrated~~

~~Waveform model is perfect~~

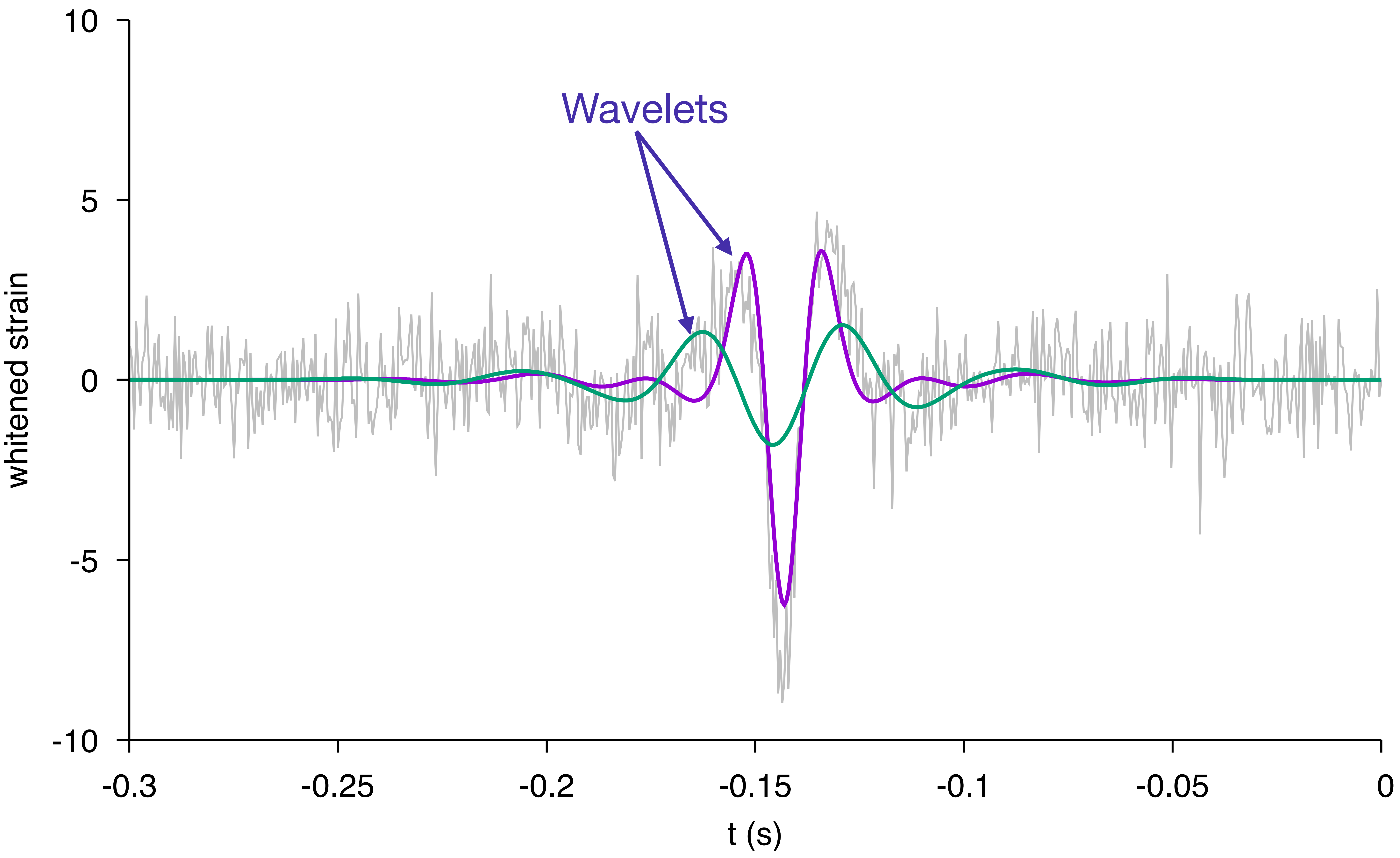
~~Noise variance is *stationary*~~ **OK**



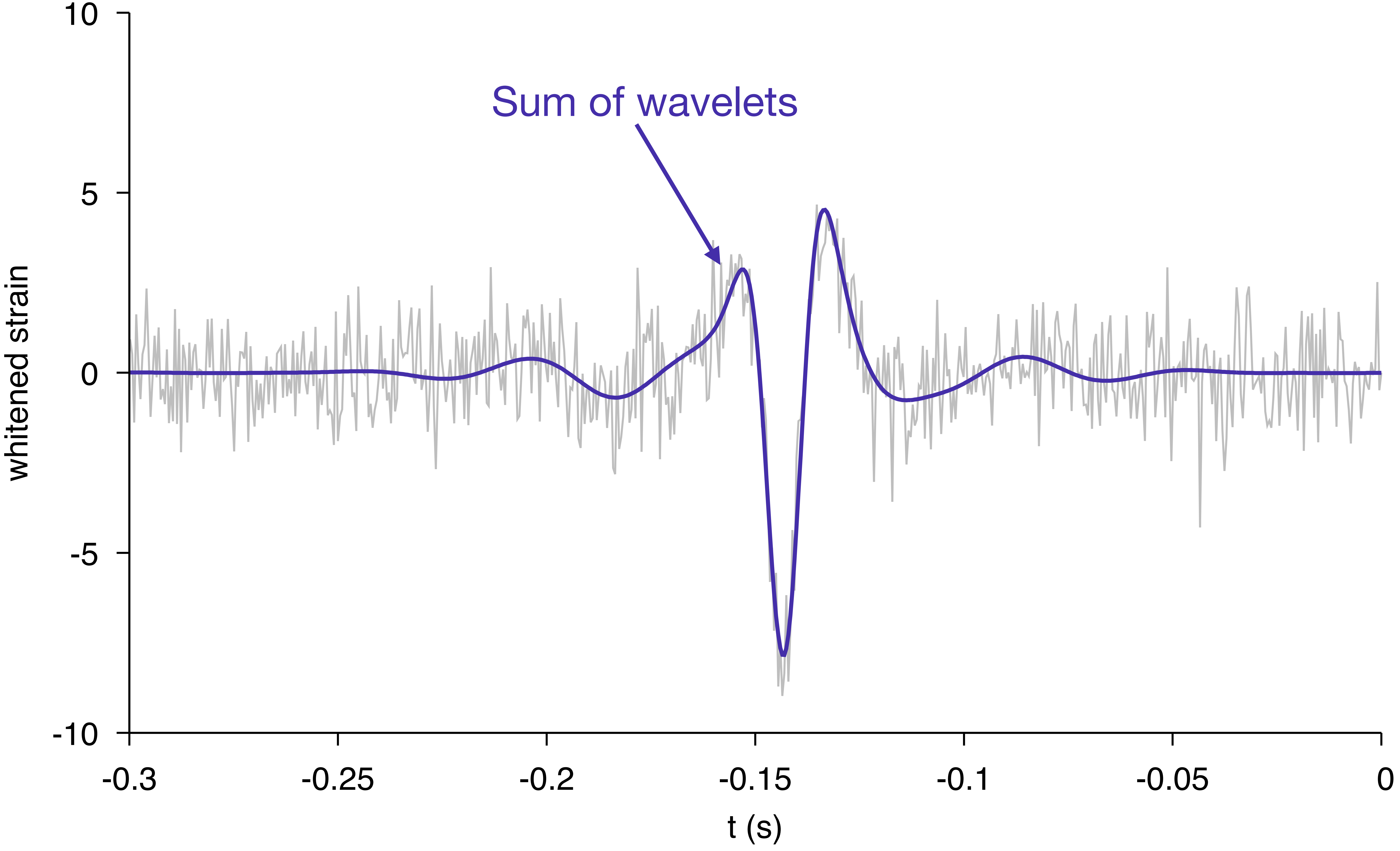

10
5
0
-5
-10
whitened strain
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
t (s)



Short-duration transients
whitened strain
10
5
0
-5
-10
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
t (s)



Wavelets
10
5
0
-5
-10
whitened strain
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
t (s)


Sum of wavelets
whitened strain
10
5
0
-5
-10
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
t (s)

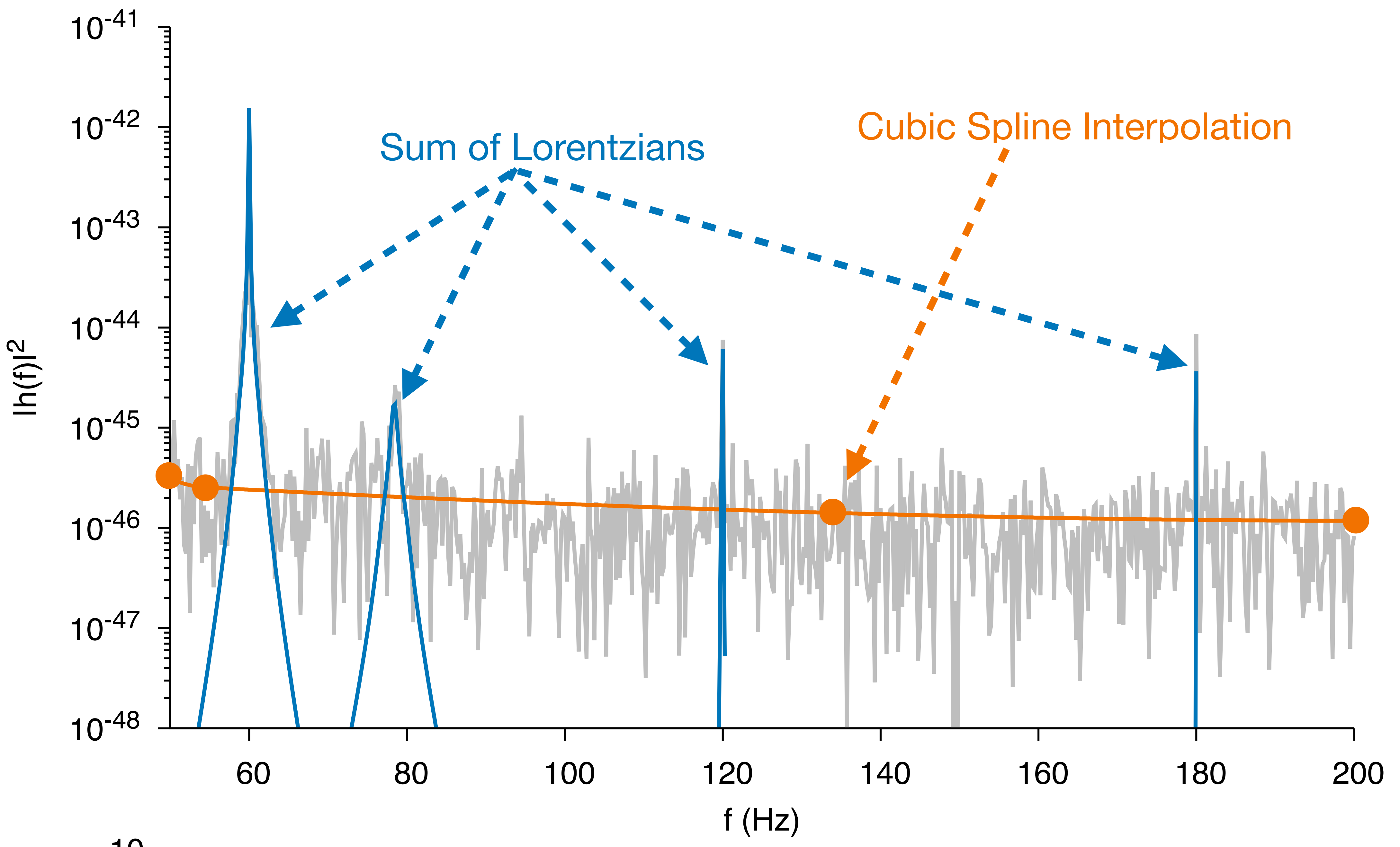
Noise is zero-mean Gaussian ✔

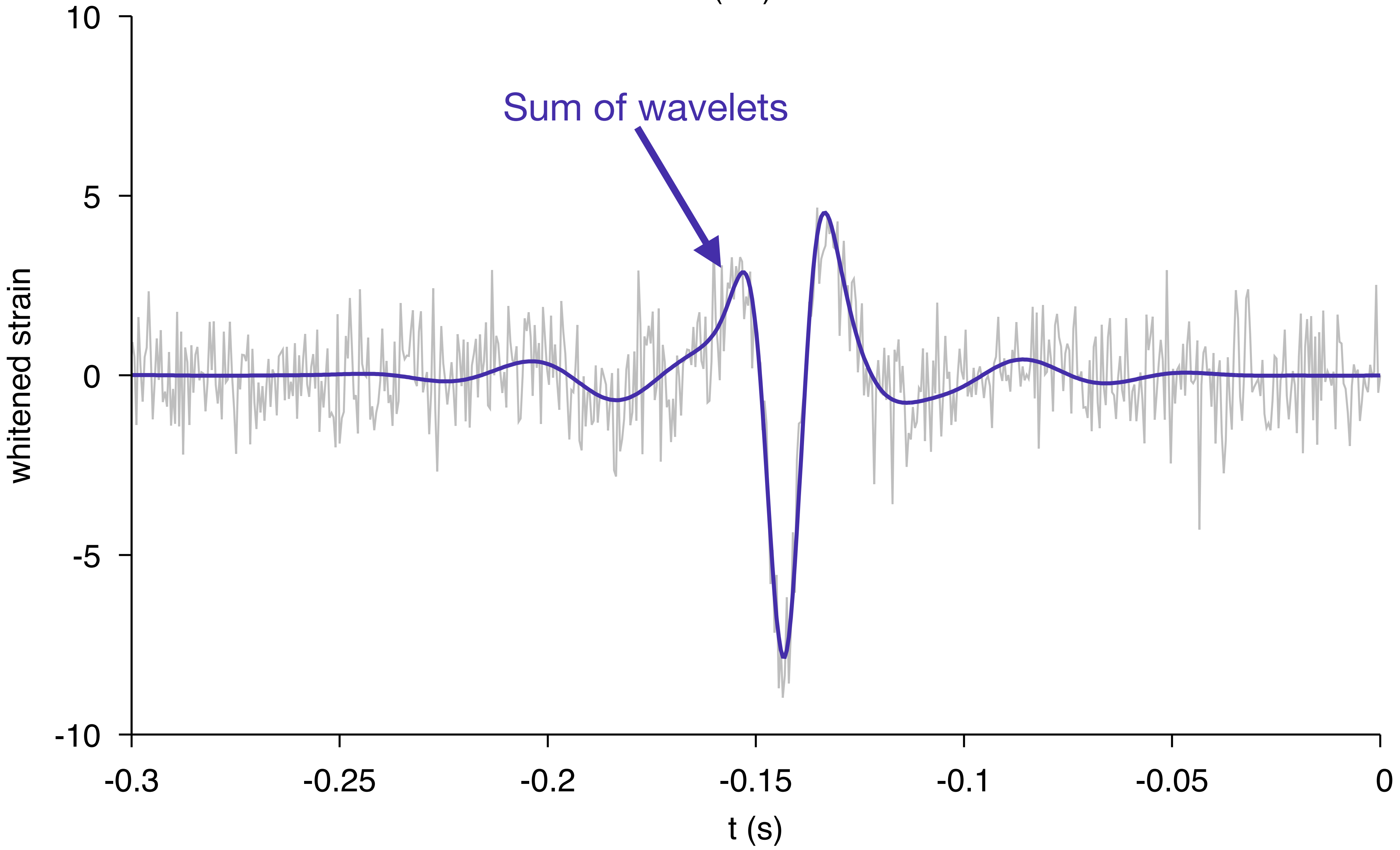
Noise has known variance ✔

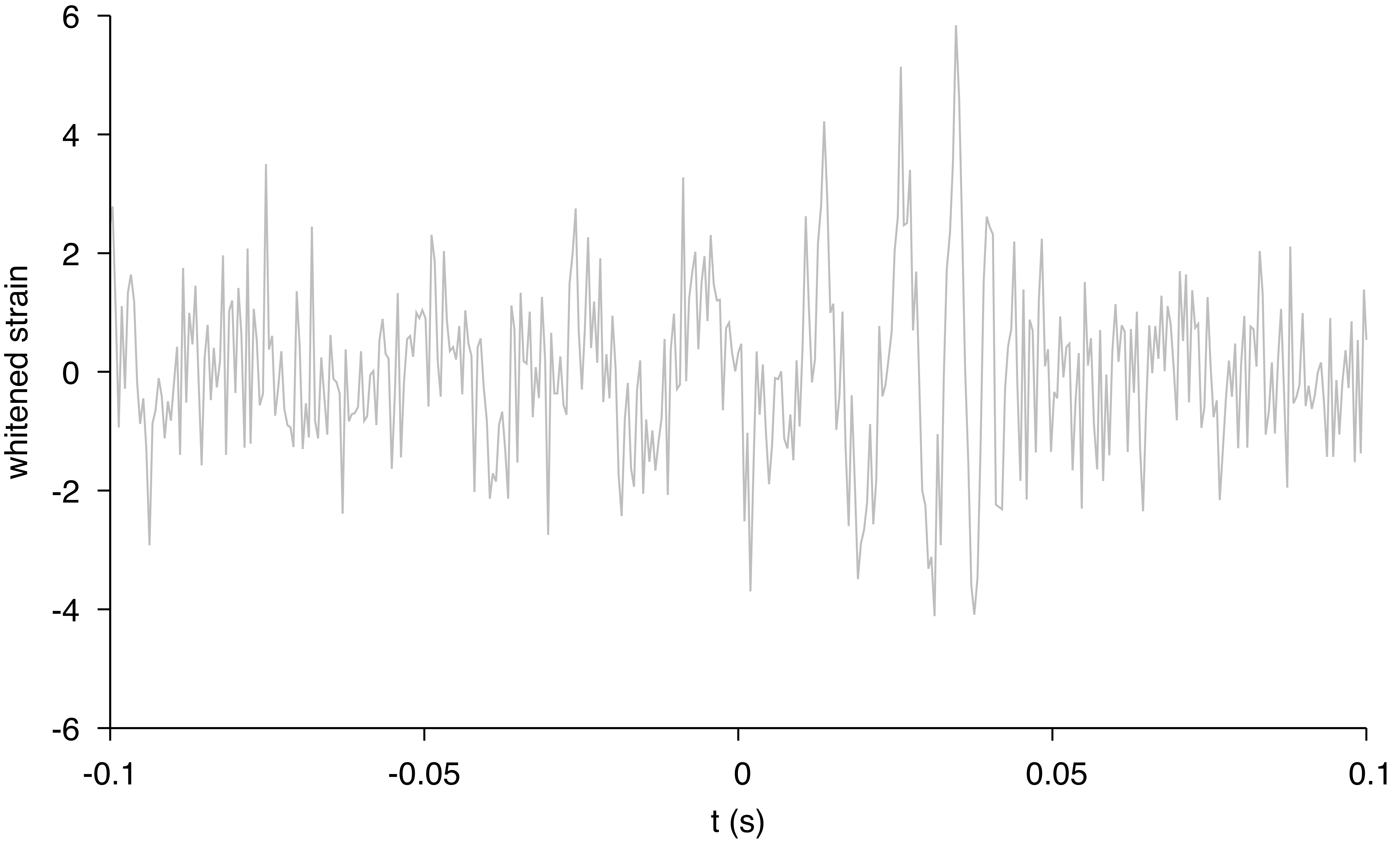
~~Data are perfectly calibrated~~

~~Waveform model is perfect~~

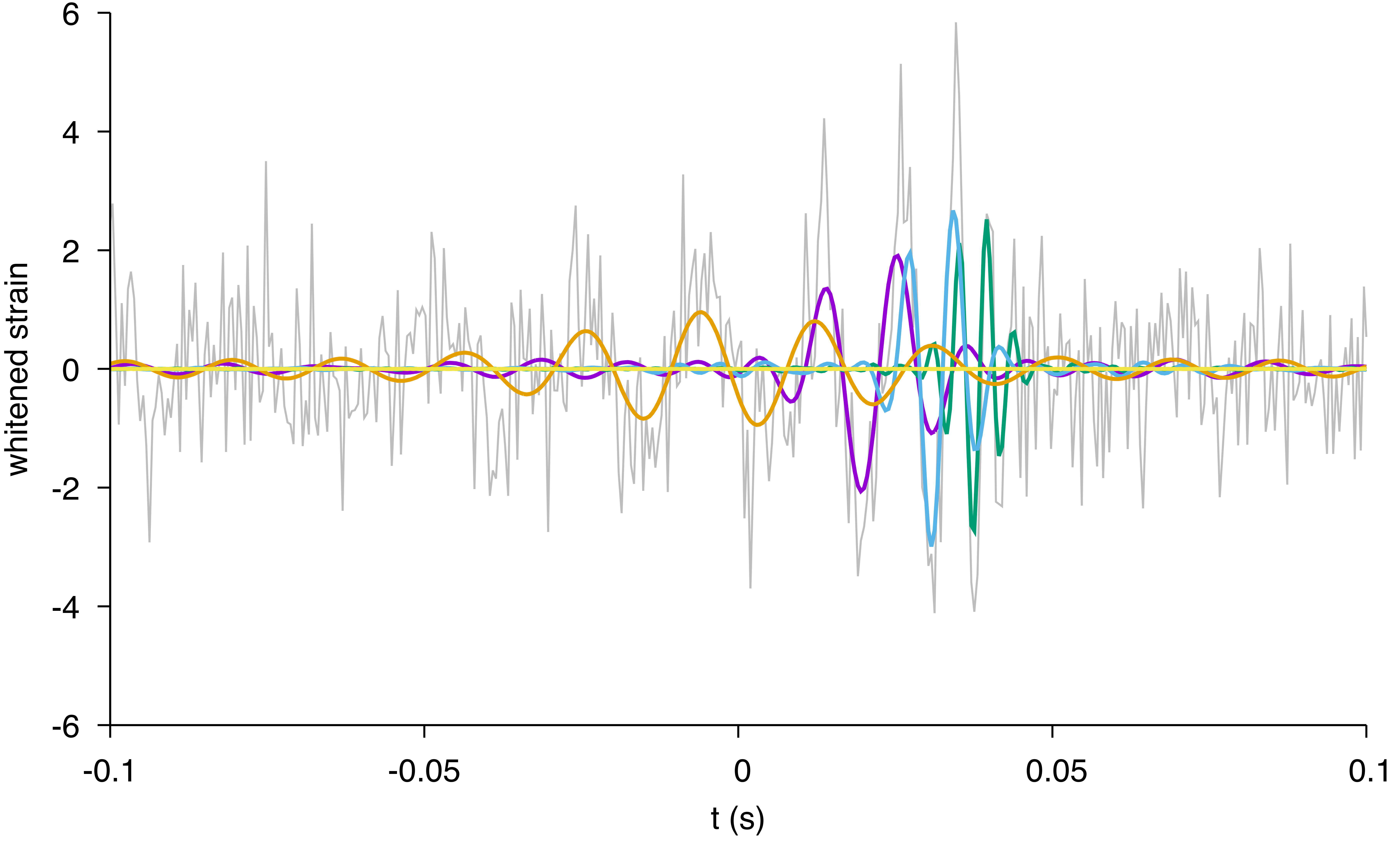
~~Noise variance is *stationary*~~ OK

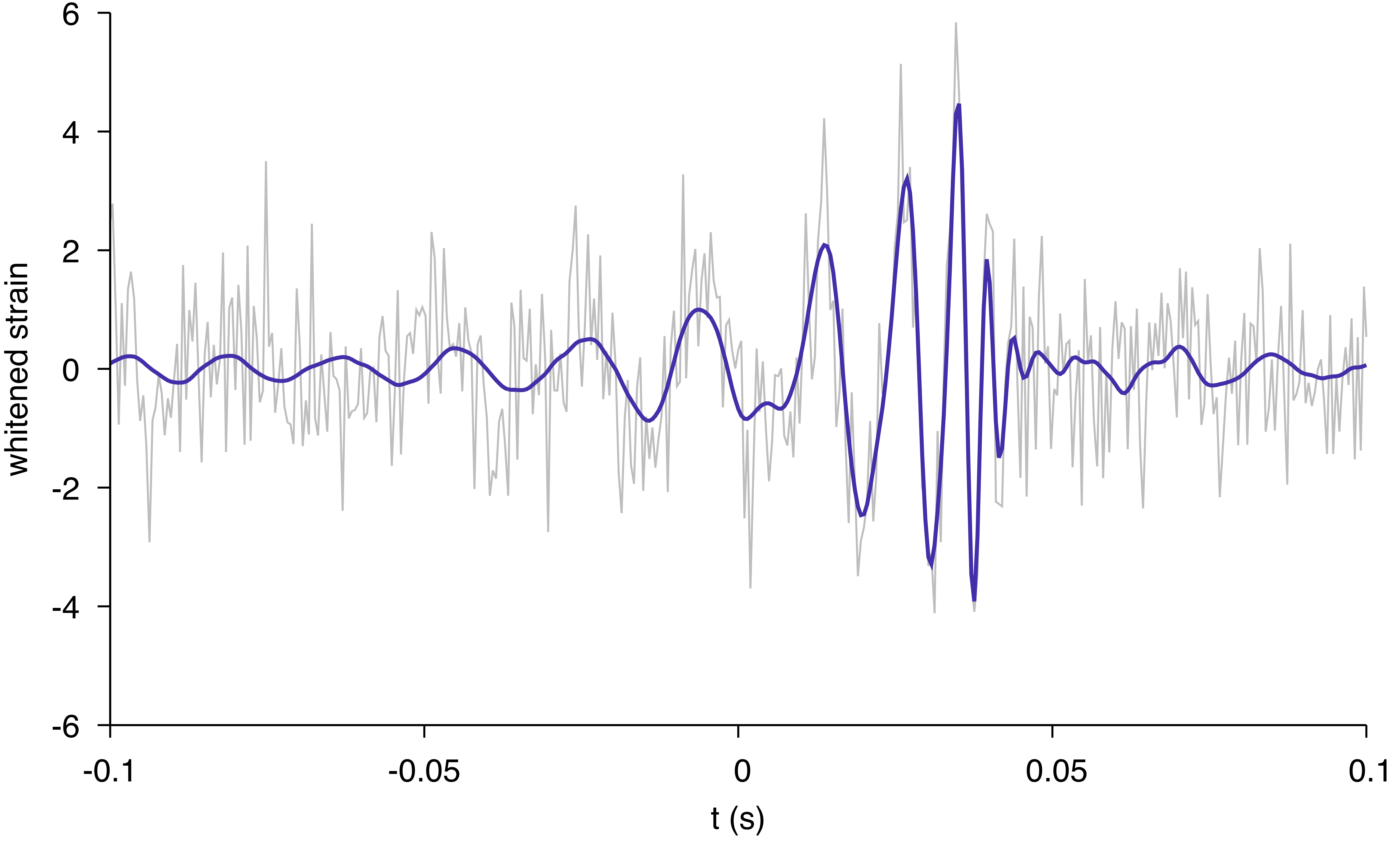




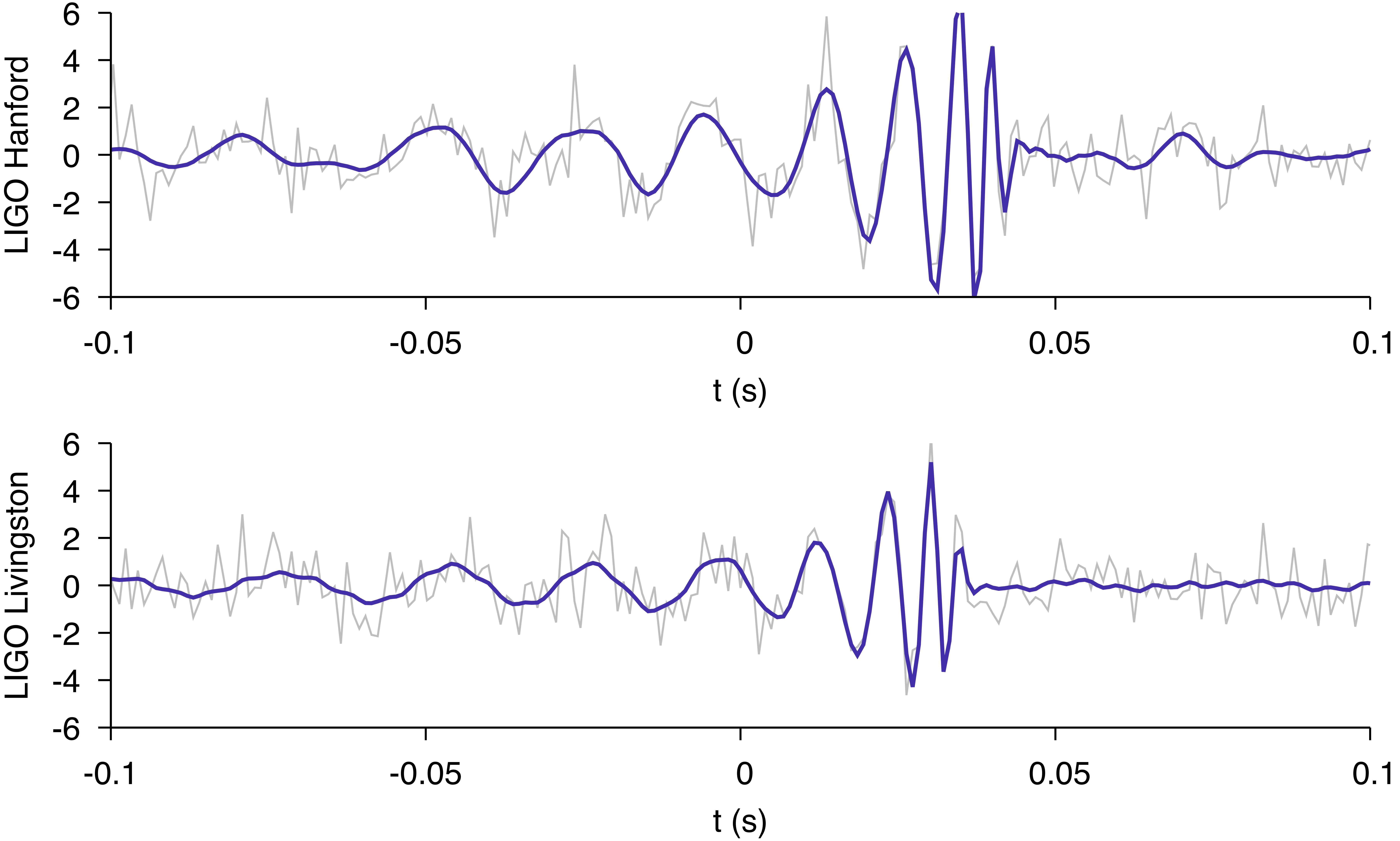


whitened strain
6
4
2
0
-2
-4
-6
-0.1
-0.05
0
0.05
0.1
t (s)


6
4
2
0
-2
-4
-6
whitened strain
-0.1
-0.05
0
0.05
0.1
t (s)



6
4
2
0
-2
-4
-6
whitened strain
-0.1
-0.05
0
0.05
0.1
t (s)



LIGO Hanford
6
4
2
0
-2
-4
-6
-0.1
-0.05
0
0.05
0.1
t (s)
LIGO Livingston
6
4
2
0
-2
-4
-6
-0.1
-0.05
0
0.05
0.1
t (s)

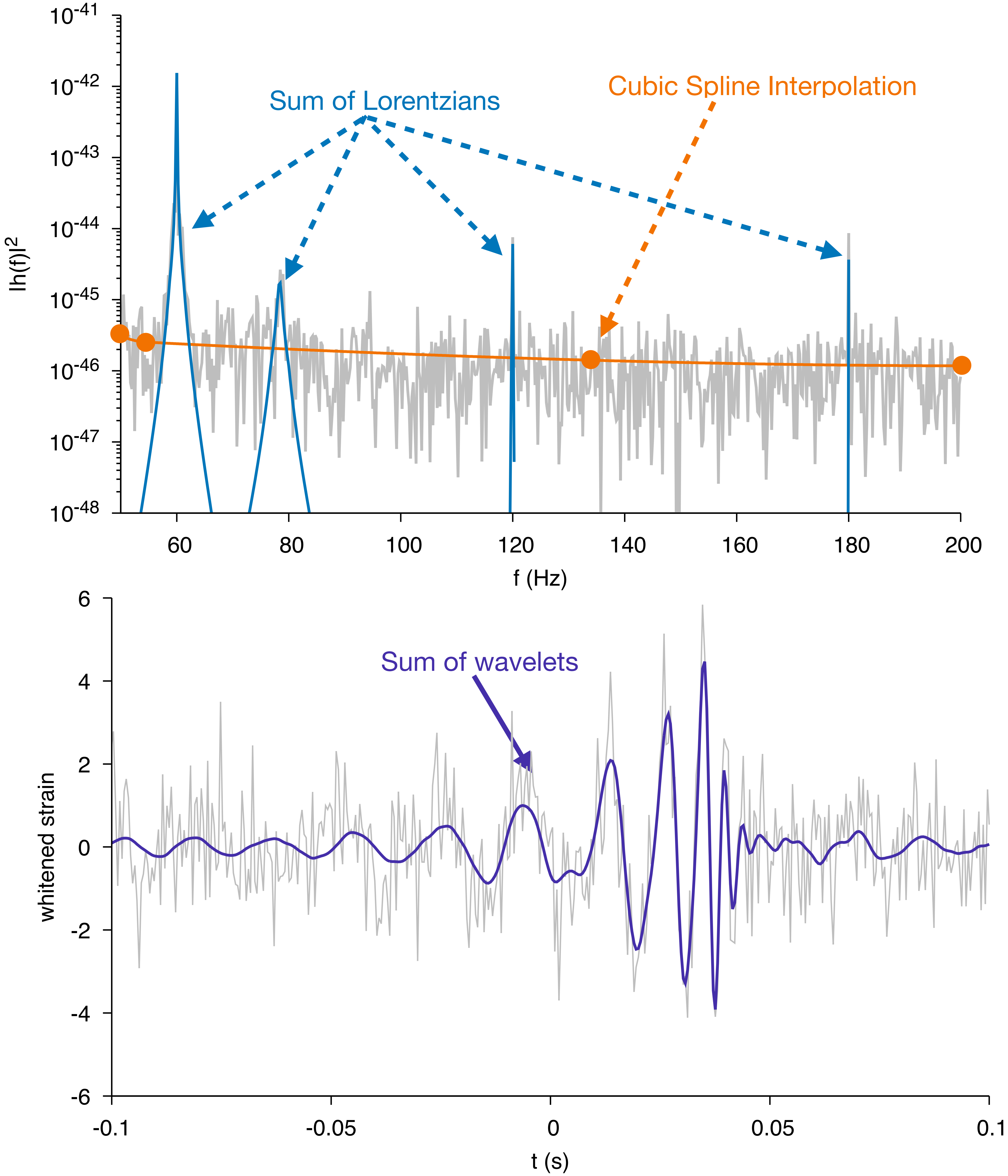
Noise is zero-mean Gaussian ✓

Noise has known variance ✓

~~Data are perfectly calibrated~~

~~Waveform model is perfect~~ **OK**

~~Noise variance is *stationary*~~ **OK**



$N_{\mathrm{spline}} \times \{f_0, S_n\}$: Spline Model

$N_{\mathrm{lines}} \times \{f_0, A, Q\}$: Line Model

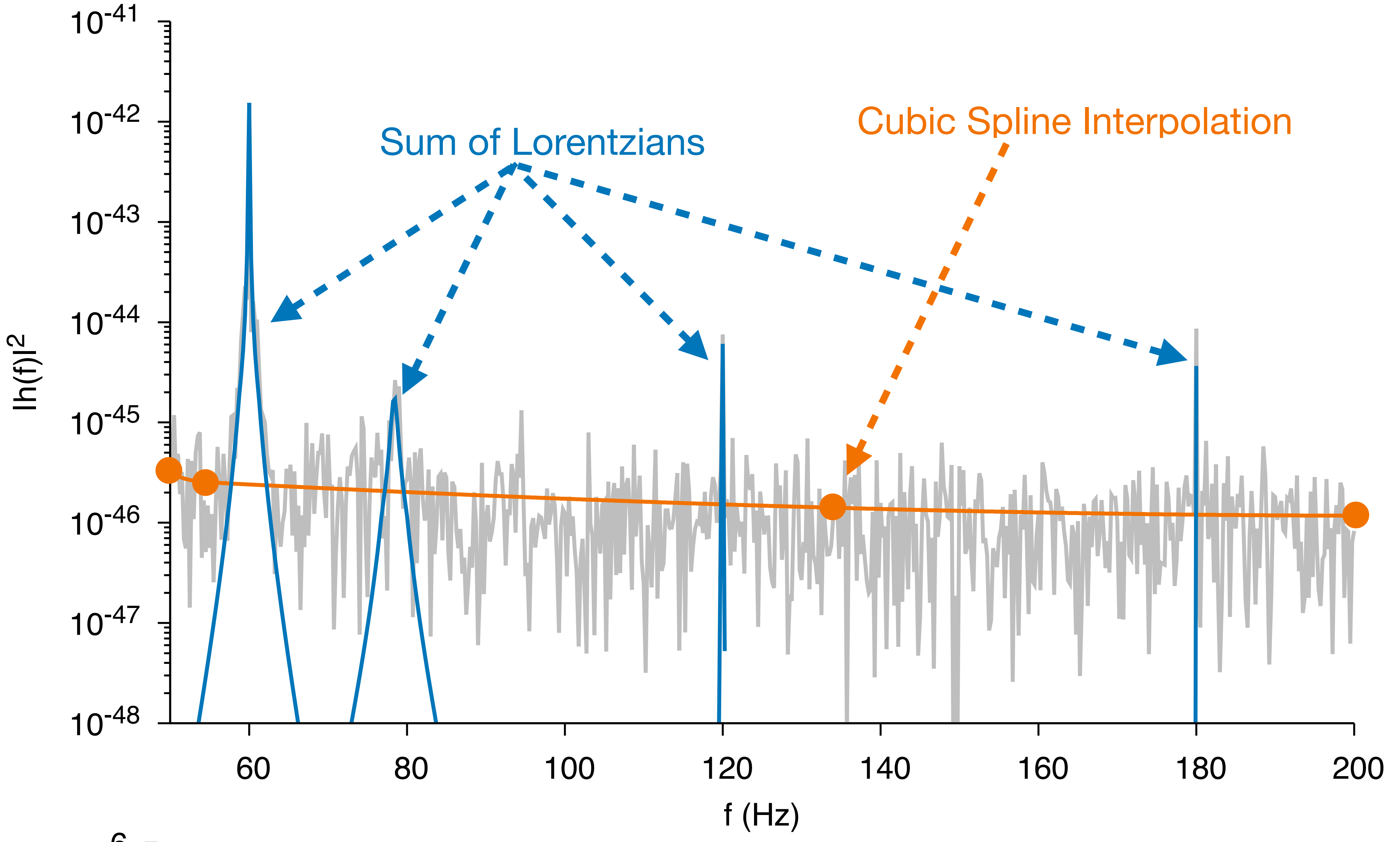
$N_{\mathrm{G,I}} \times \{f_0, t_0, A, Q, \phi_0\}$: Glitch Model

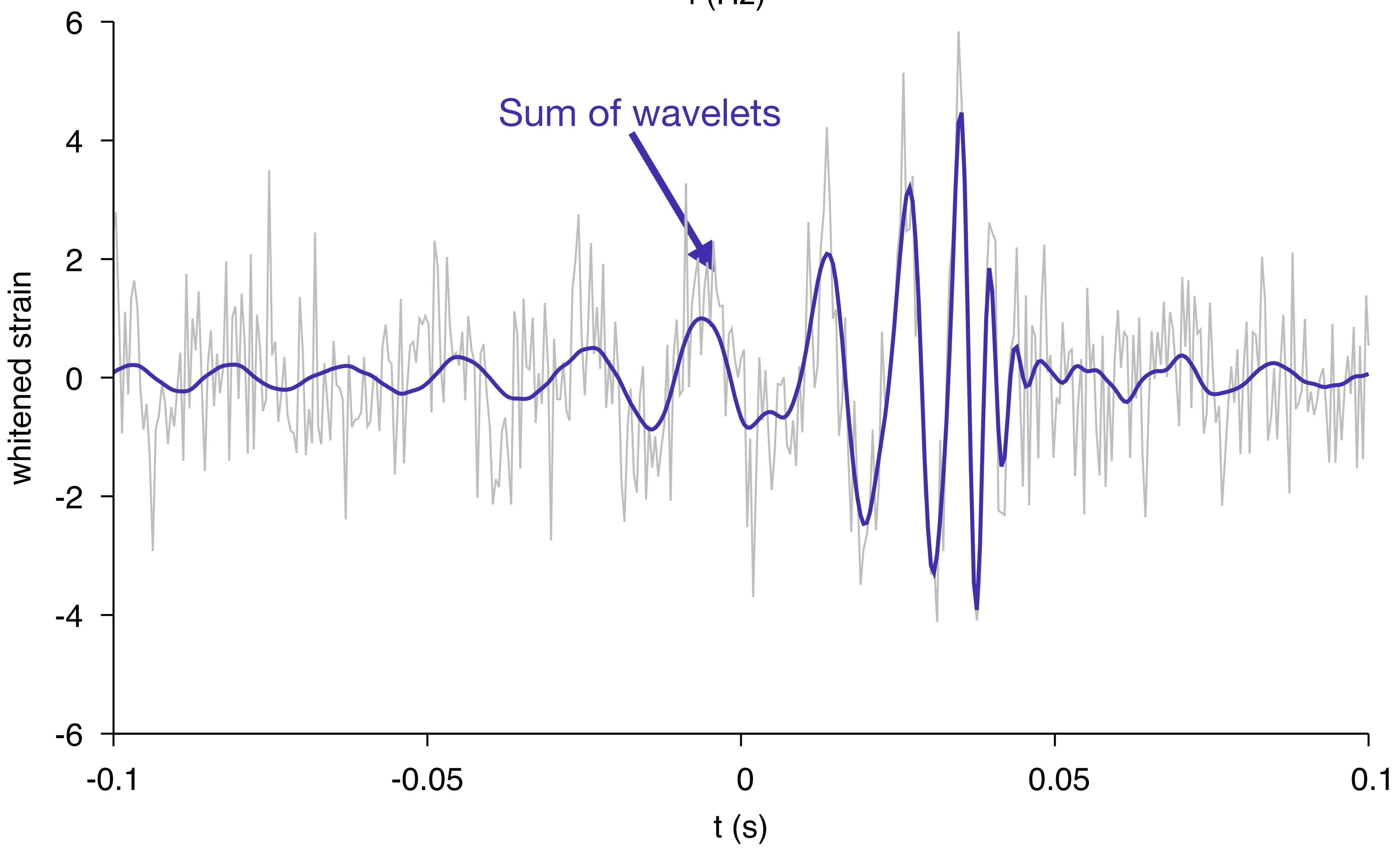
$N_{\mathrm{S}} \times \{f_0, t_0, A, Q, \phi_0\} \cup \{\alpha, \delta, \psi, \epsilon\}$: Generic Signal Model

and/or

$\{m_1, m_2, \mathbf{S}_1, \mathbf{S}_2, \mathbf{L}, \alpha, \delta, D_L, t_0\}$: CBC Model

$N_{\mathrm{cal}} \times \{\delta A_I, \delta\phi_{IFO}\}$: Calibration Model

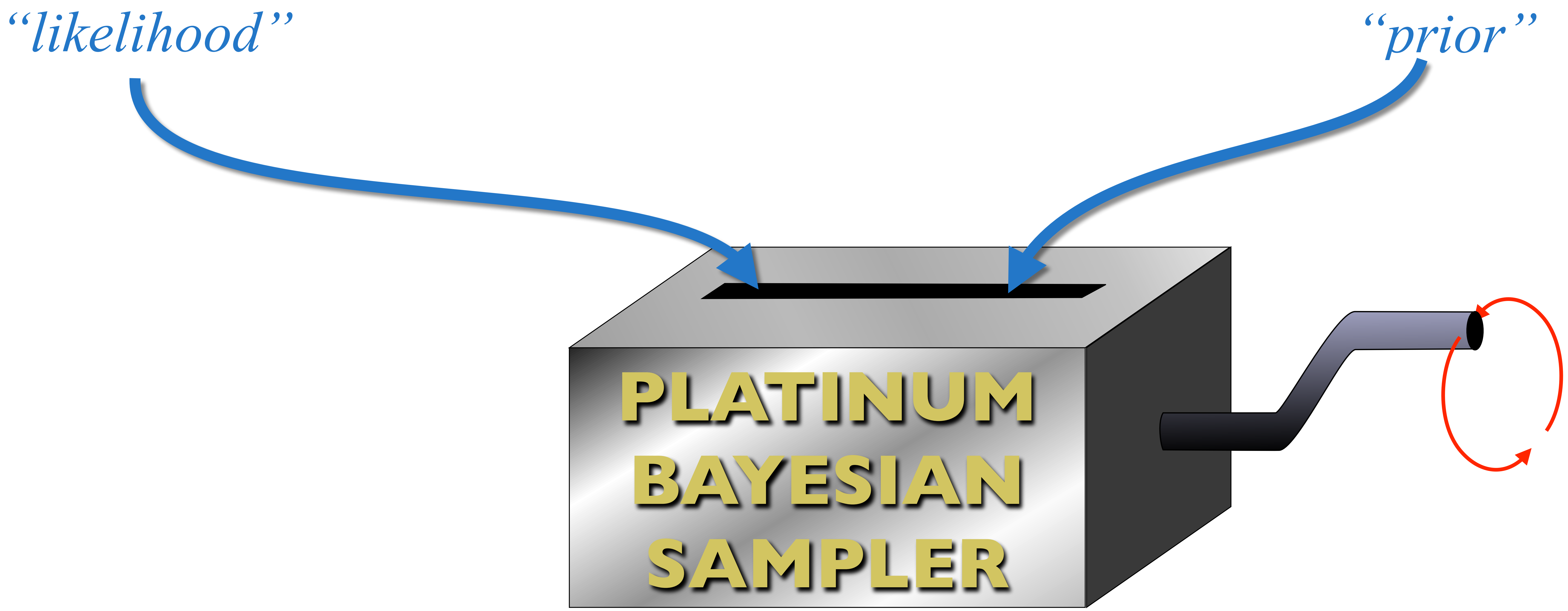




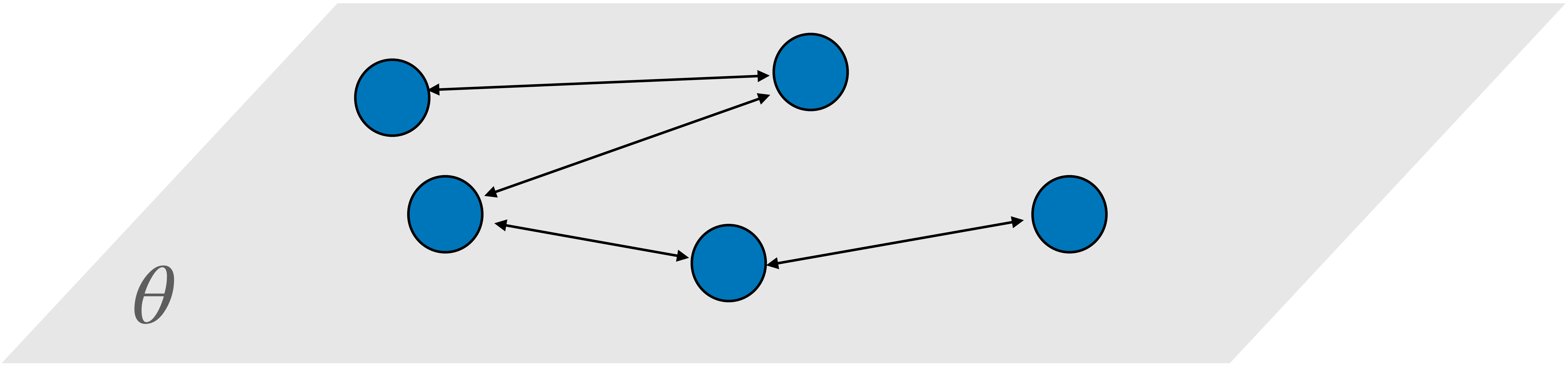
Model everything… likelihood $= p(\mathbf{d} \,|\, \text{signal, noise, glitch})$

Marginalize the stuff you don't care about…

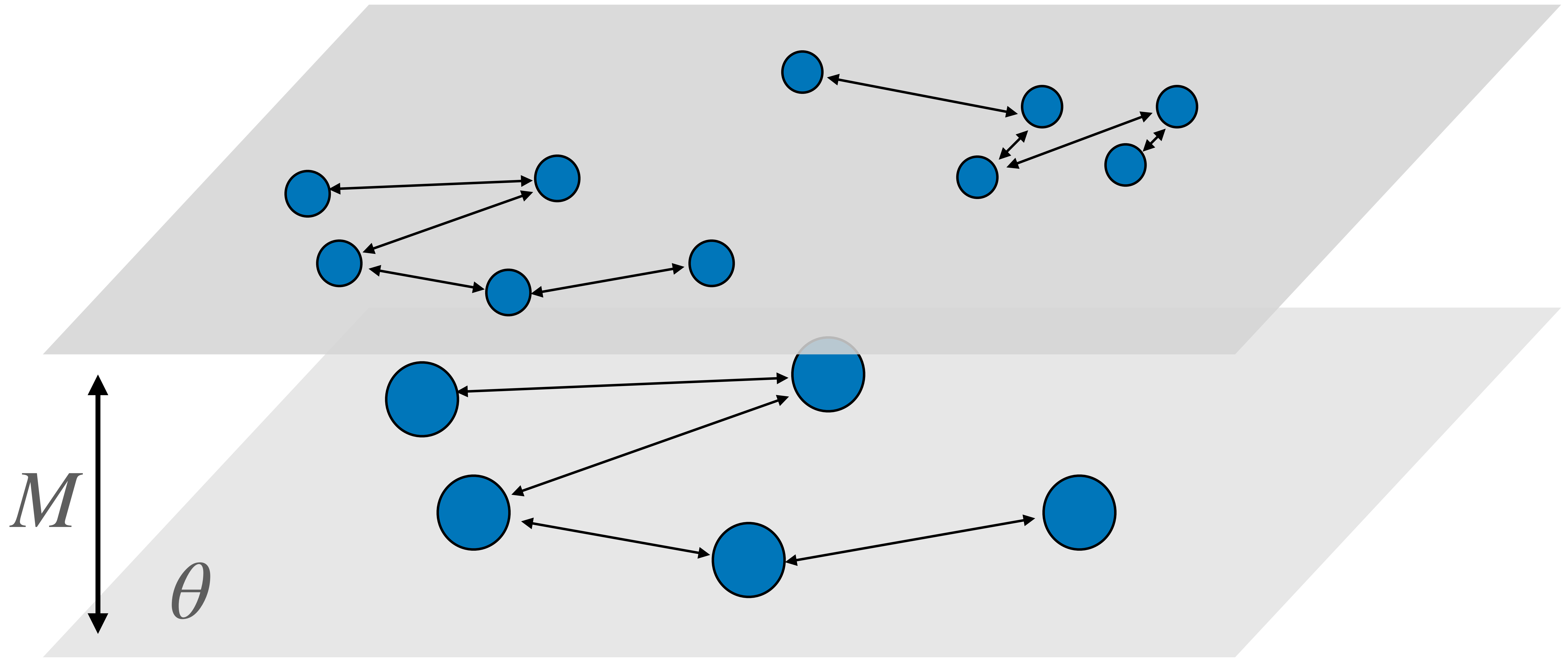
$$p(\text{signal} \,|\, \mathbf{d}) = \int_{\text{glitch,noise}} p(\mathbf{d} \,|\, \text{signal, glitch, noise})$$



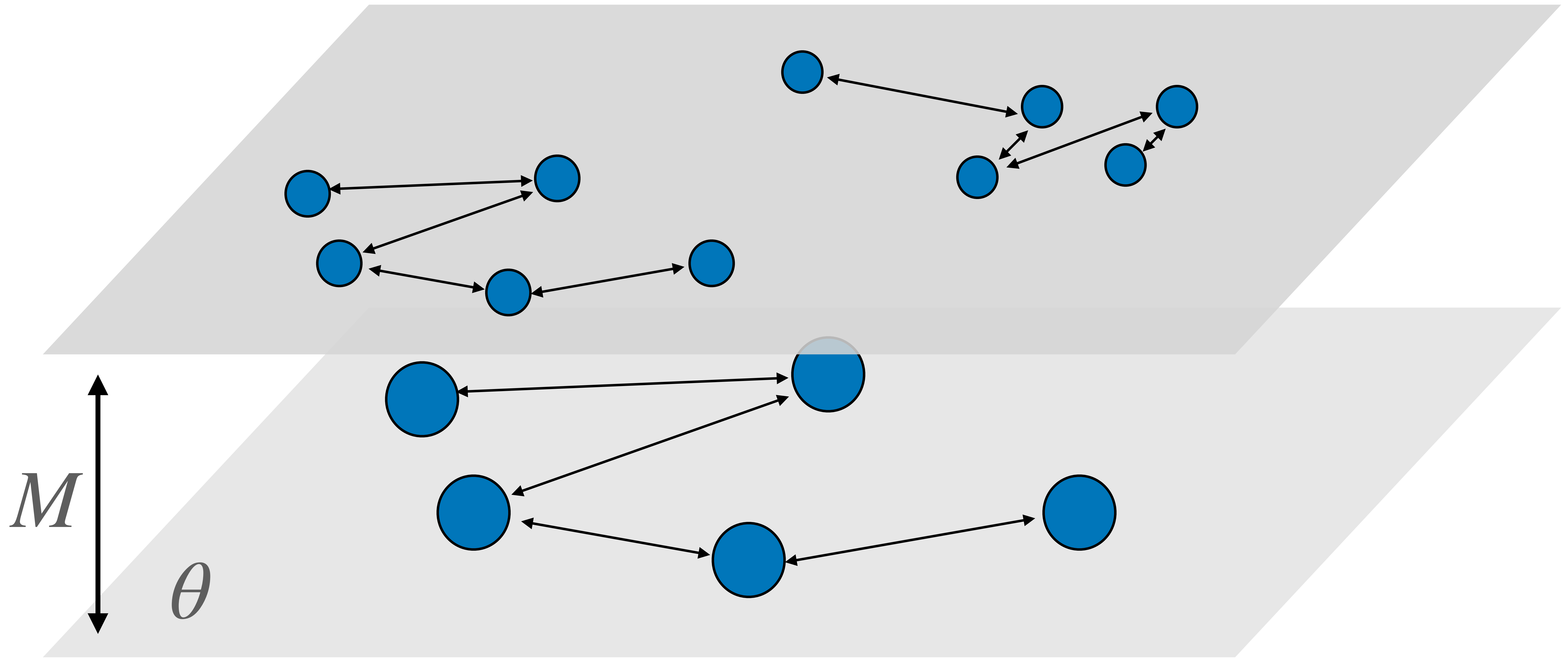
Transdimensional (Reversible Jump) MCMC



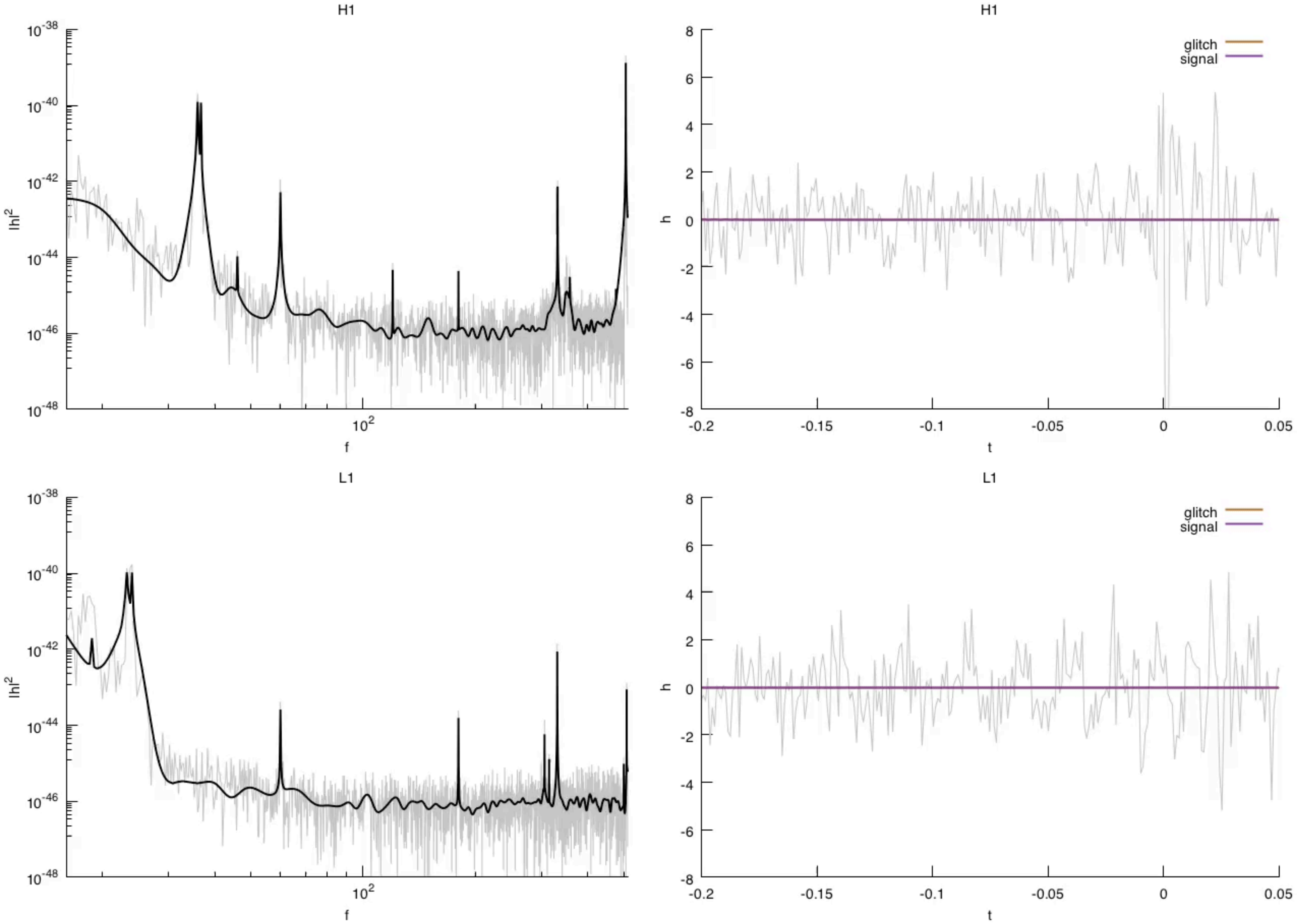
Transdimensional (Reversible Jump) MCMC

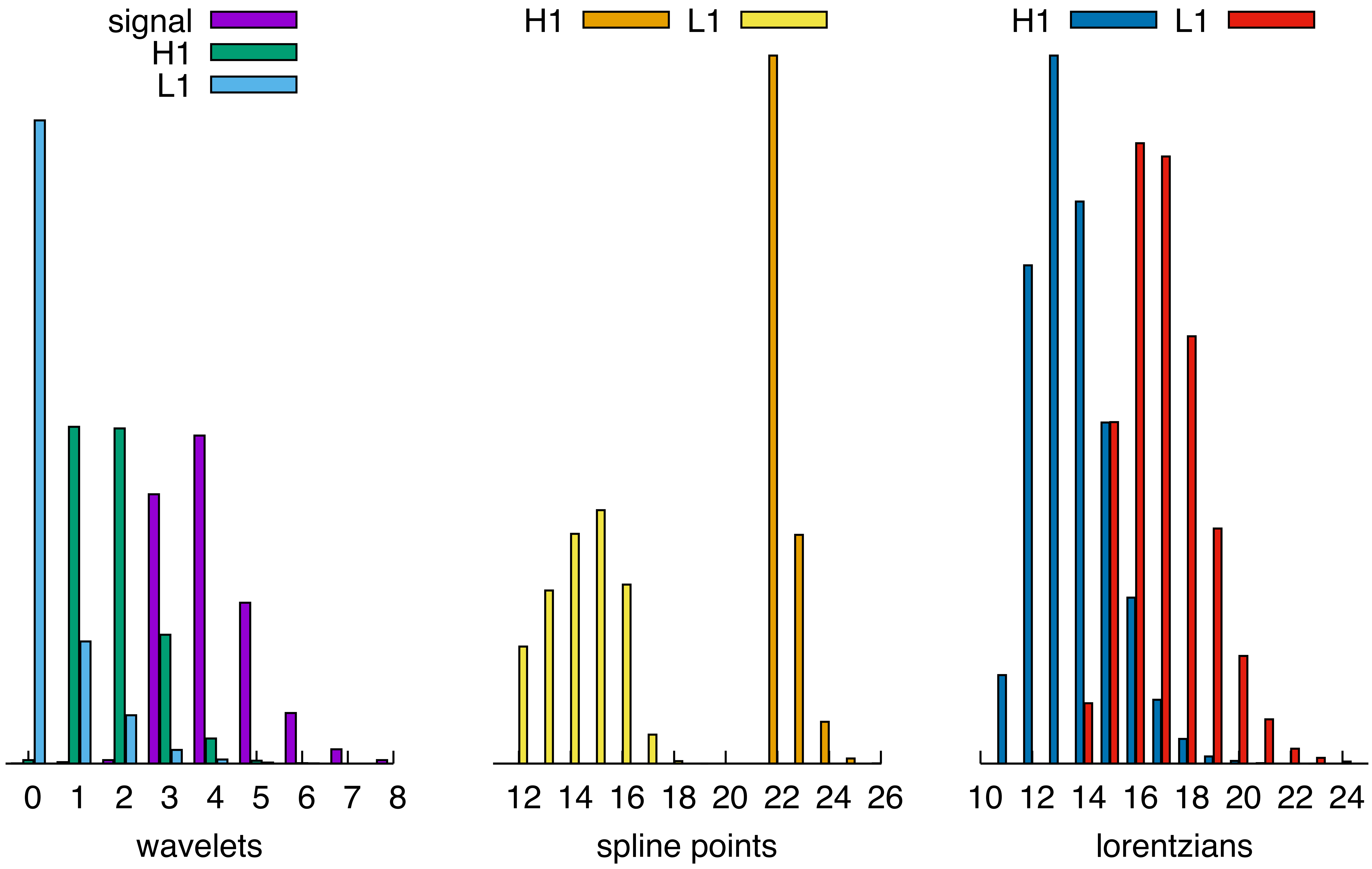


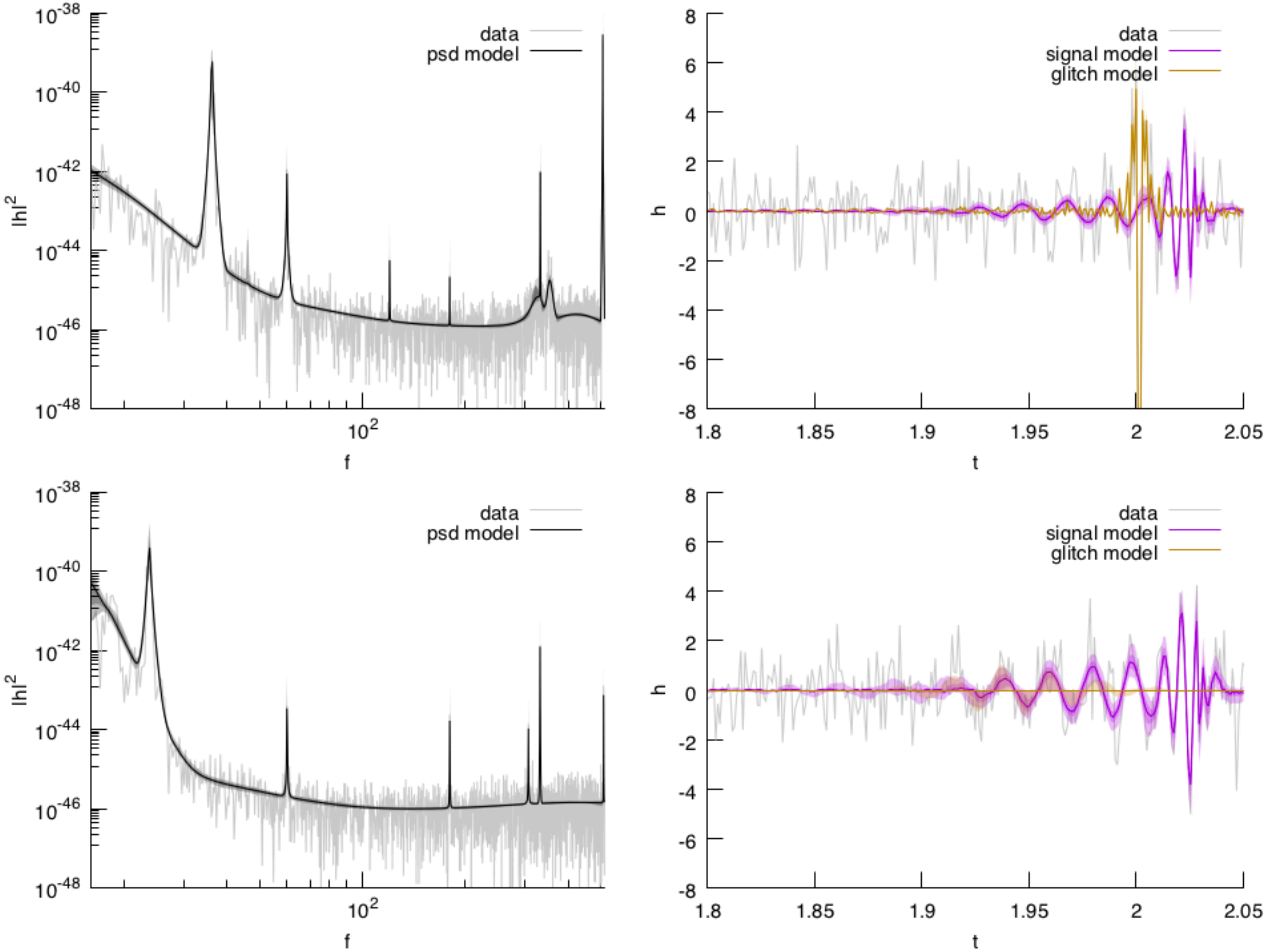
Transdimensional (Reversible Jump) MCMC



$$\frac{\text{counts in model A}}{\text{counts in model B}} \equiv \mathcal{O}_{A,B}$$


H1
|h|^2
f
10^2
H1
glitch
signal
h
t
L1
|h|^2
f
10^2
L1
glitch
signal
h
t


signal
H1
L1
0 1 2 3 4 5 6 7 8
wavelets
H1
L1
12 14 16 18 20 22 24 26
spline points
H1
L1
10 12 14 16 18 20 22 24
lorentzians

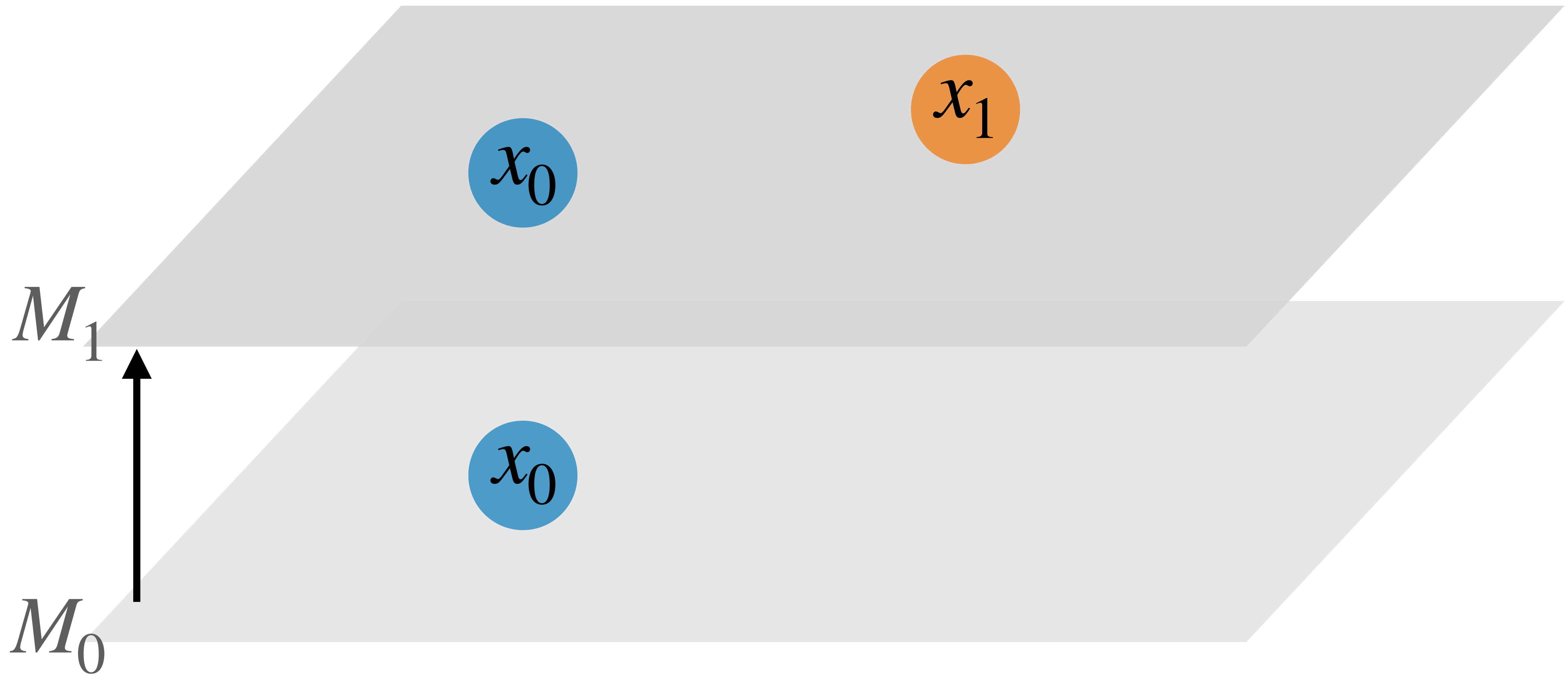


data
psd model
|h|²
f
data
signal model
glitch model
h
t

Transdimensional (Reversible Jump) MCMC
…are notoriously tricky to get mixing…

propose adding parameter set $x : \{x_0\} \rightarrow \{x_0, x_1\}$

$$\alpha_{M_0 \rightarrow M_1} = \min \left[1, \frac{p(\mathbf{d} \mid x_0, x_1)}{p(\mathbf{d} \mid x_0)} \frac{p(x_0)p(x_1)}{p(x_0)} \frac{1}{q(x_1)} \right]$$



Transdimensional (Reversible Jump) MCMC
…are notoriously tricky to get mixing…

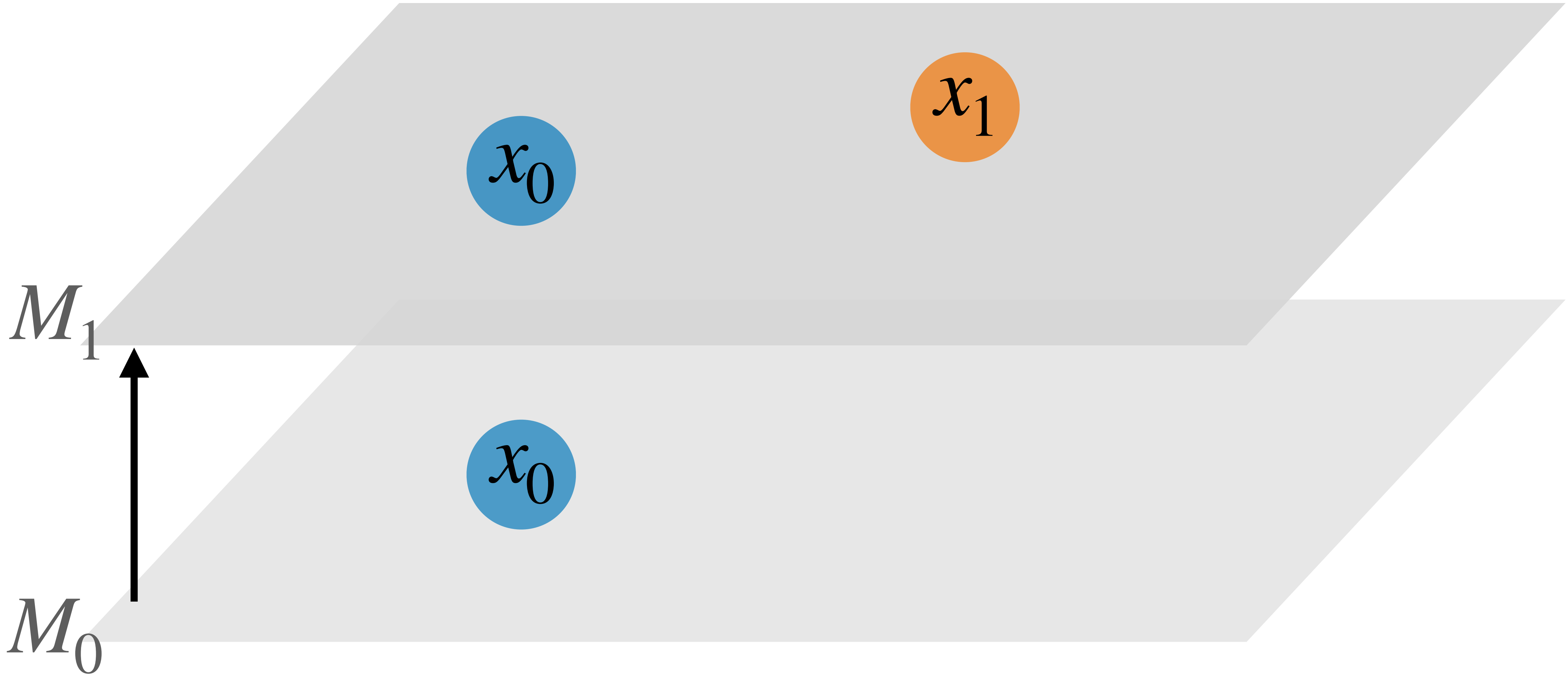
propose adding parameter set $x : \{x_0\} \rightarrow \{x_0, x_1\}$

$$\alpha_{M_0 \rightarrow M_1} = \min \left[1, \frac{p(\mathbf{d} \,|\, x_0, x_1)}{p(\mathbf{d} \,|\, x_0)} \frac{p(x_0)\, p(x_1)}{p(x_0)} \frac{1}{q(x_1)} \right]$$

<<1 unless close to bulk of posterior

penalty for adding prior volume

penalizes "heavy handed" proposal



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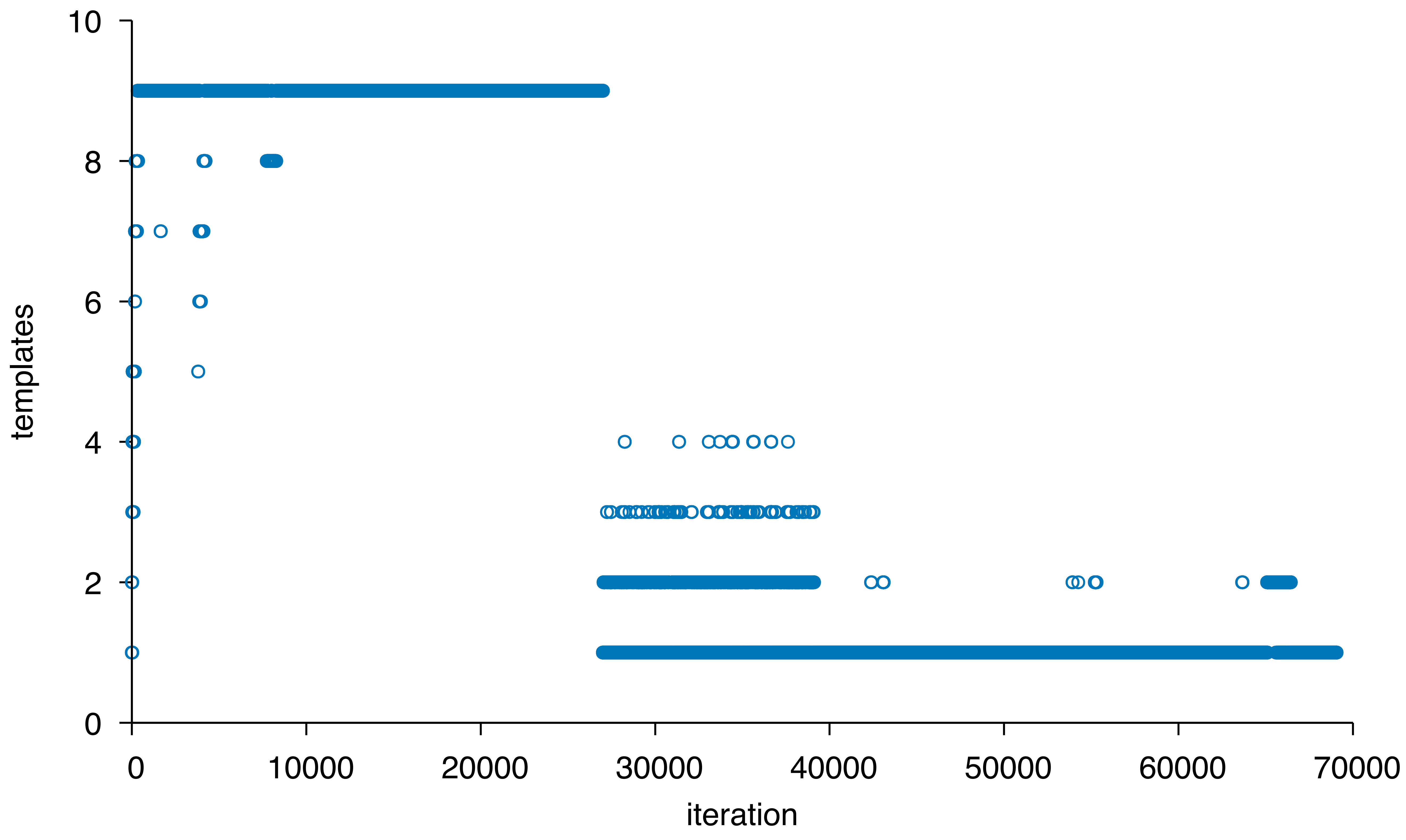

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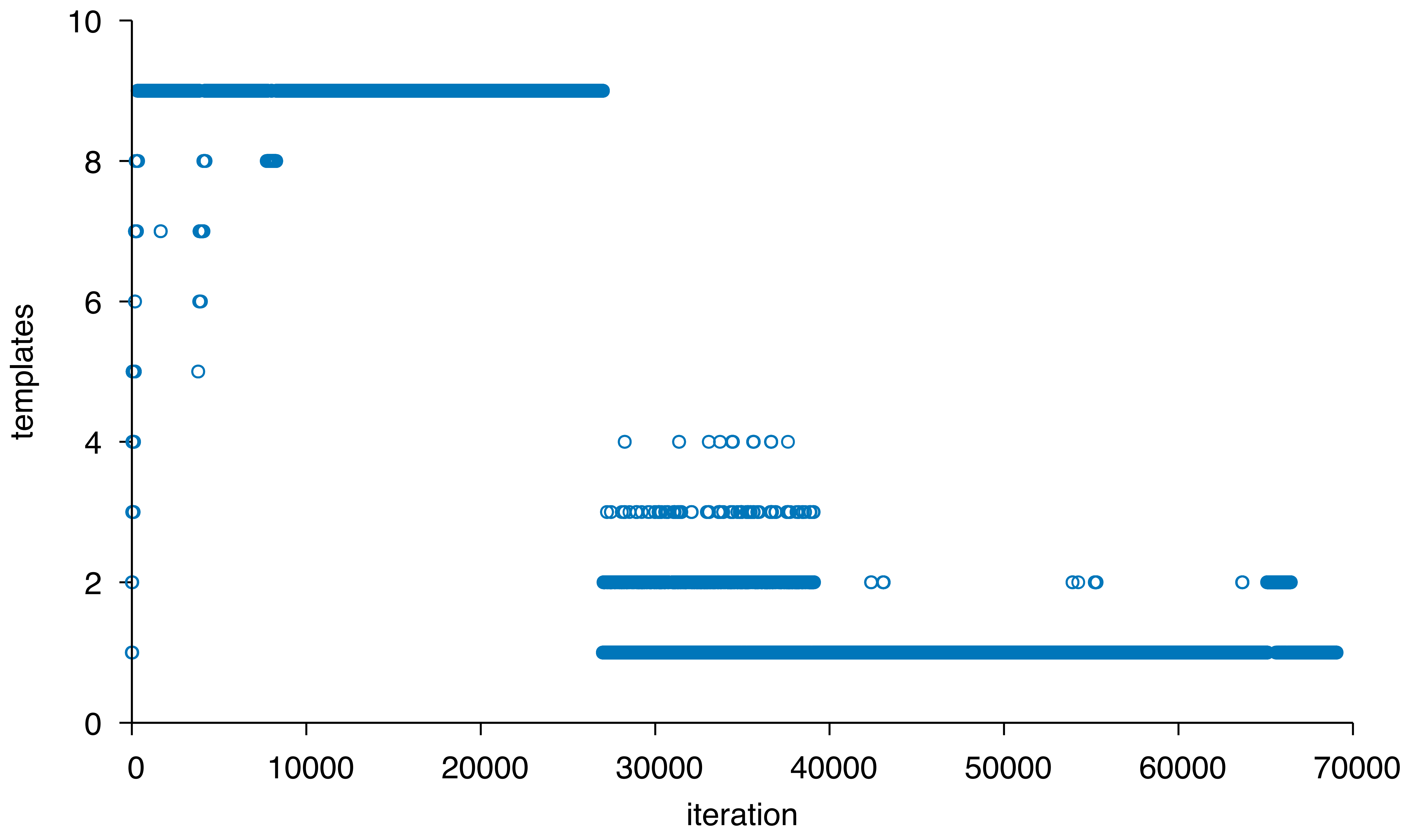
(iii) Having (ii) should make you nervous about the robustness of the sampler



$N_{\text{spline}} \times \{f_0, S_n\}$: Spline Model

$N_{\text{lines}} \times \{f_0, A, Q\}$: Line Model

$N_{\text{G,I}} \times \{f_0, t_0, A, Q, \phi_0\}$: Glitch Model

$N_{\text{S}} \times \{f_0, t_0, A, Q, \phi_0\} \cup \{\alpha, \delta, \psi, \epsilon\}$: Generic Signal Model

and/or

$\{m_1, m_2, \mathbf{S}_1, \mathbf{S}_2, \mathbf{L}, \alpha, \delta, D_L, t_0\}$: CBC Model

$N_{\text{cal}} \times \{\delta A_I, \delta\phi_{IFO}\}$: Calibration Model

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Point estimate of PSD for LIGO-Virgo CBC PE

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Similar algorithms under development for LISA

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