Image Credit: Aurore Simonnet/LIGO-Caltech-MIT-Sonoma State

Black hole astrophysics with gravitational-wave catalogs

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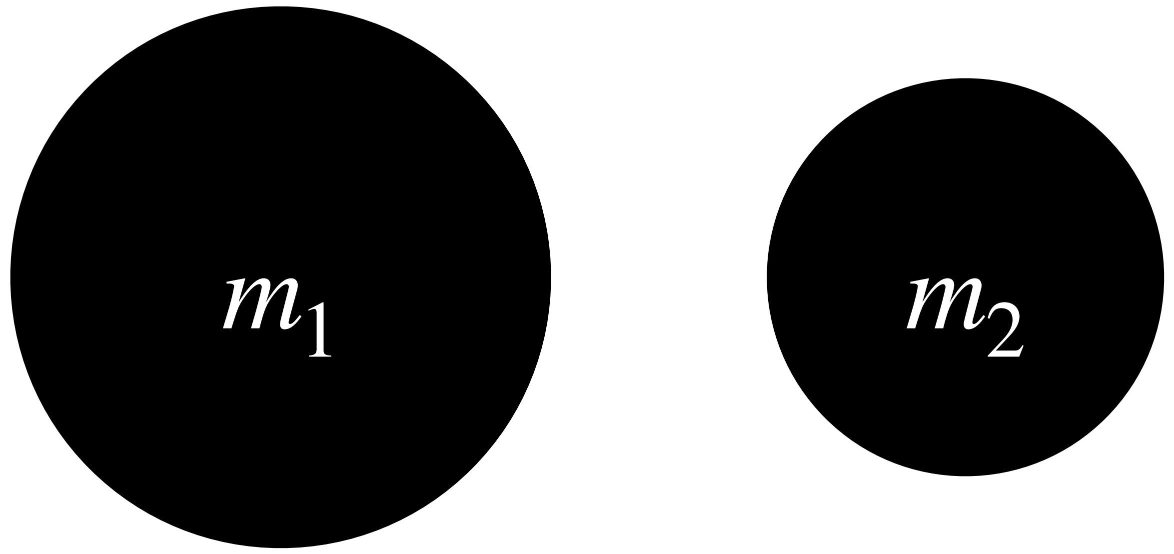
CIERA
Center for Interdisciplinary Exploration and Research in Astrophysics

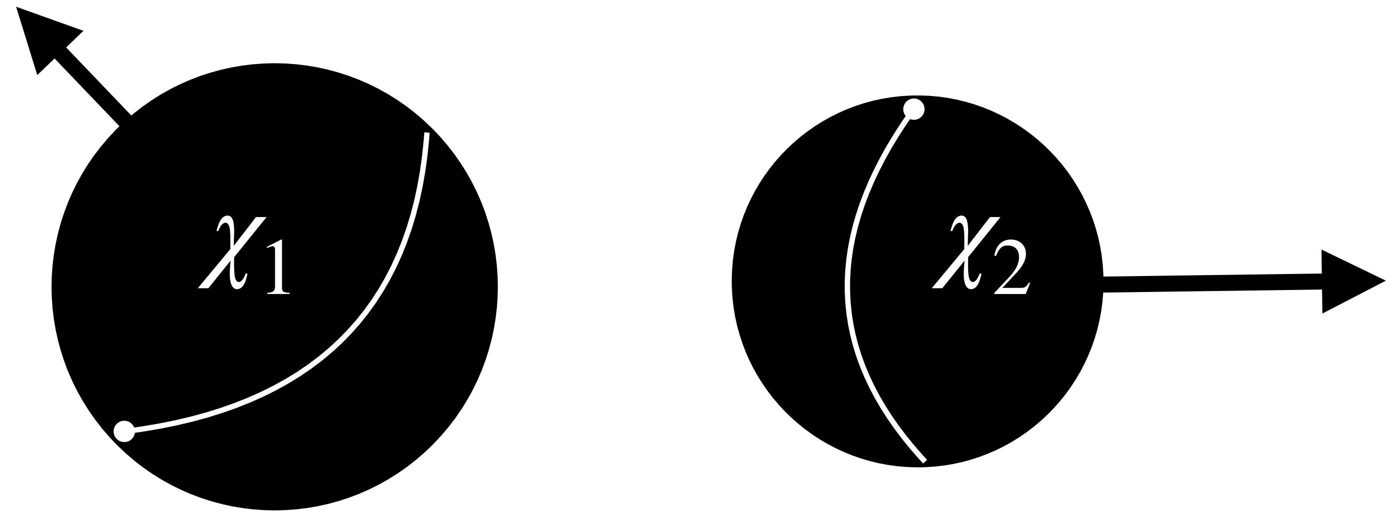
NHFP
NASA Hubble Fellowship Program

Observing Binary Black Holes

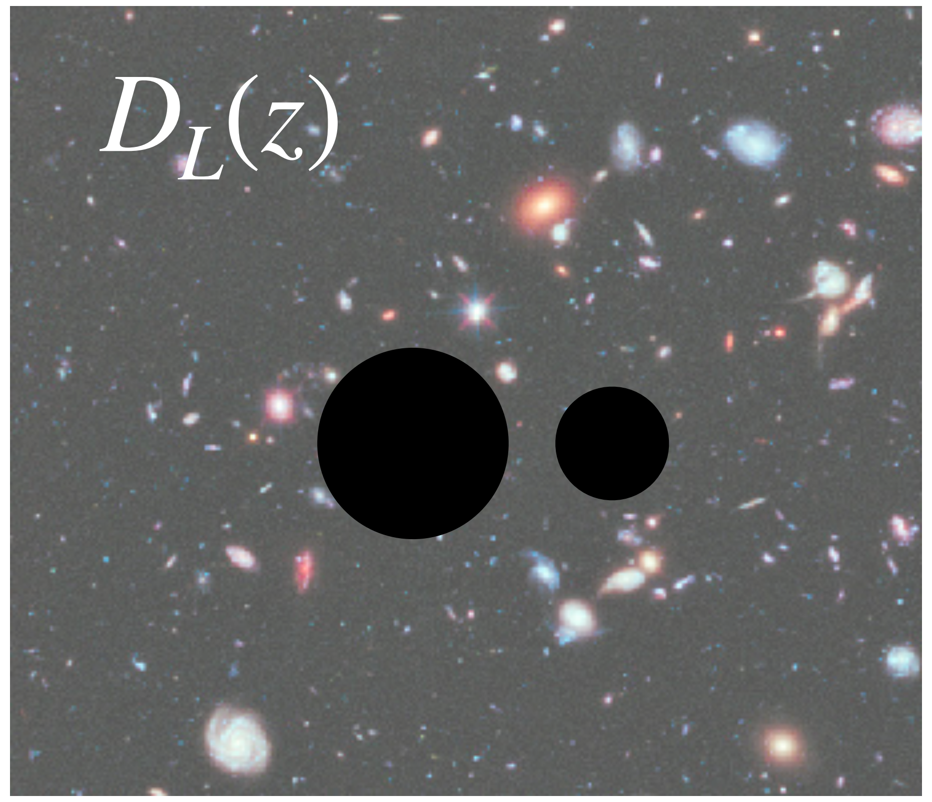
How *big* is each black hole?



How fast are they *spinning*? Where are the spin axes pointing?



Where and *when* did they merge?



How are black holes made?

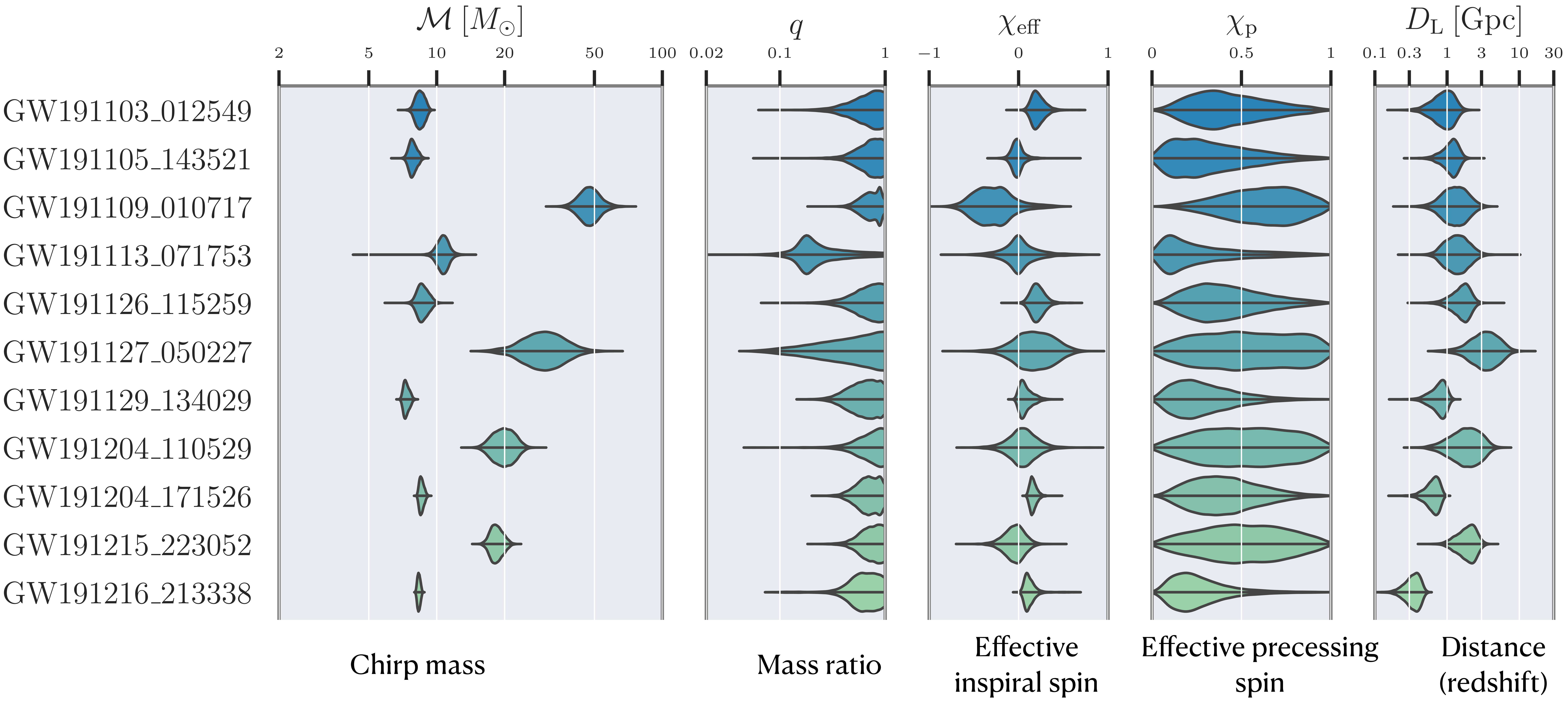
The origins of LIGO-Virgo's black holes

- What were their progenitors? (Likely massive stars)
- Where and when did these massive stars live?
- How did these stars die?
- How did the stellar deaths affect their environments?
- How did these stellar remnants pair up into merger partners?

All of these pieces affect the masses, spins, and merger rates of gravitational-wave events

See Michela's talk

Measuring the parameters of gravitational-wave events



LVK 2021, arXiv:2111.03606

Parameter estimation

Likelihood

$\boxed{\text{Prior}}$

Posterior

$$p(\theta_i \mid \mathrm{data}_i) = \frac{p(\mathrm{data}_i \mid \theta_i)p_0(\theta_i \mid \mathcal{H})}{p(\mathrm{data}_i \mid \mathcal{H})}$$

θ_i: event i's parameters, like masses, spins

Evidence

Parameter estimation assumes a default prior.
Population inference finds the "best" prior, common to all systems

From Single Events to a Population

- Introduce a population model $\mathscr{H}$ with a set of population **hyper-parameters** that describe the **distributions** of masses, spins, redshifts across multiple events
- Example: Fit a power law to black hole masses. Hyper-parameters: power-law slope, minimum black hole mass, maximum black hole mass.
- Take into account **measurement uncertainty** and **selection effects**

Population analysis

Find the "best" prior to use for individual events

$$p_{\mathrm{pop}}(\theta \mid \lambda, \mathscr{H})$$

Parameter estimation likelihood for event *i*

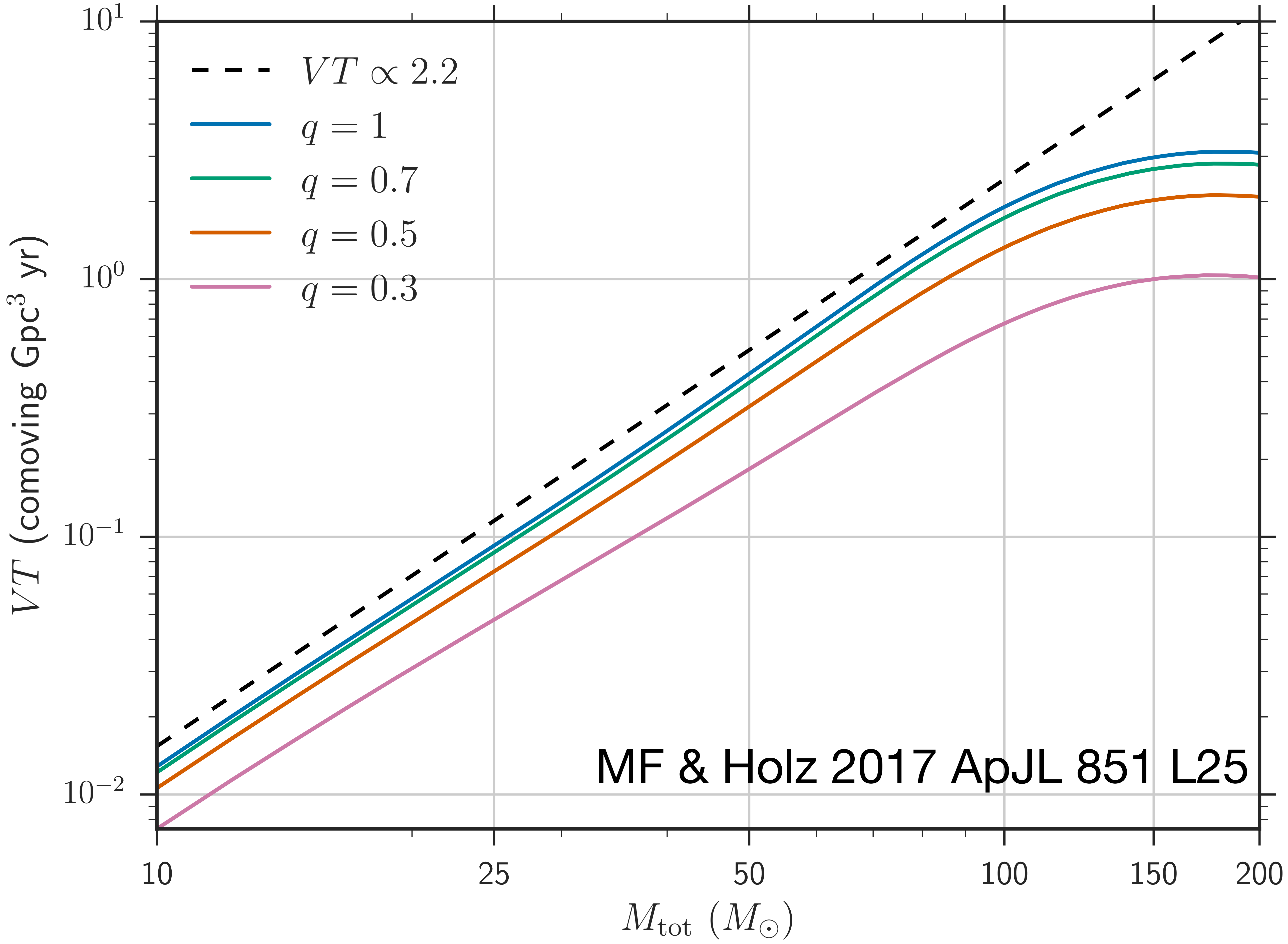
$$p(\mathrm{data} \mid \lambda, \mathscr{H}) = \prod_i \frac{\int p(\mathrm{data}_i \mid \theta_i) \boxed{p_{\mathrm{pop}}(\theta_i \mid \lambda, \mathscr{H})} \mathrm{d}\theta}{\beta(\lambda, \mathscr{H})}$$

Likelihood given population model and hyperparameters

Selection effects: fraction of detectable systems in the population

Example of selection effects:

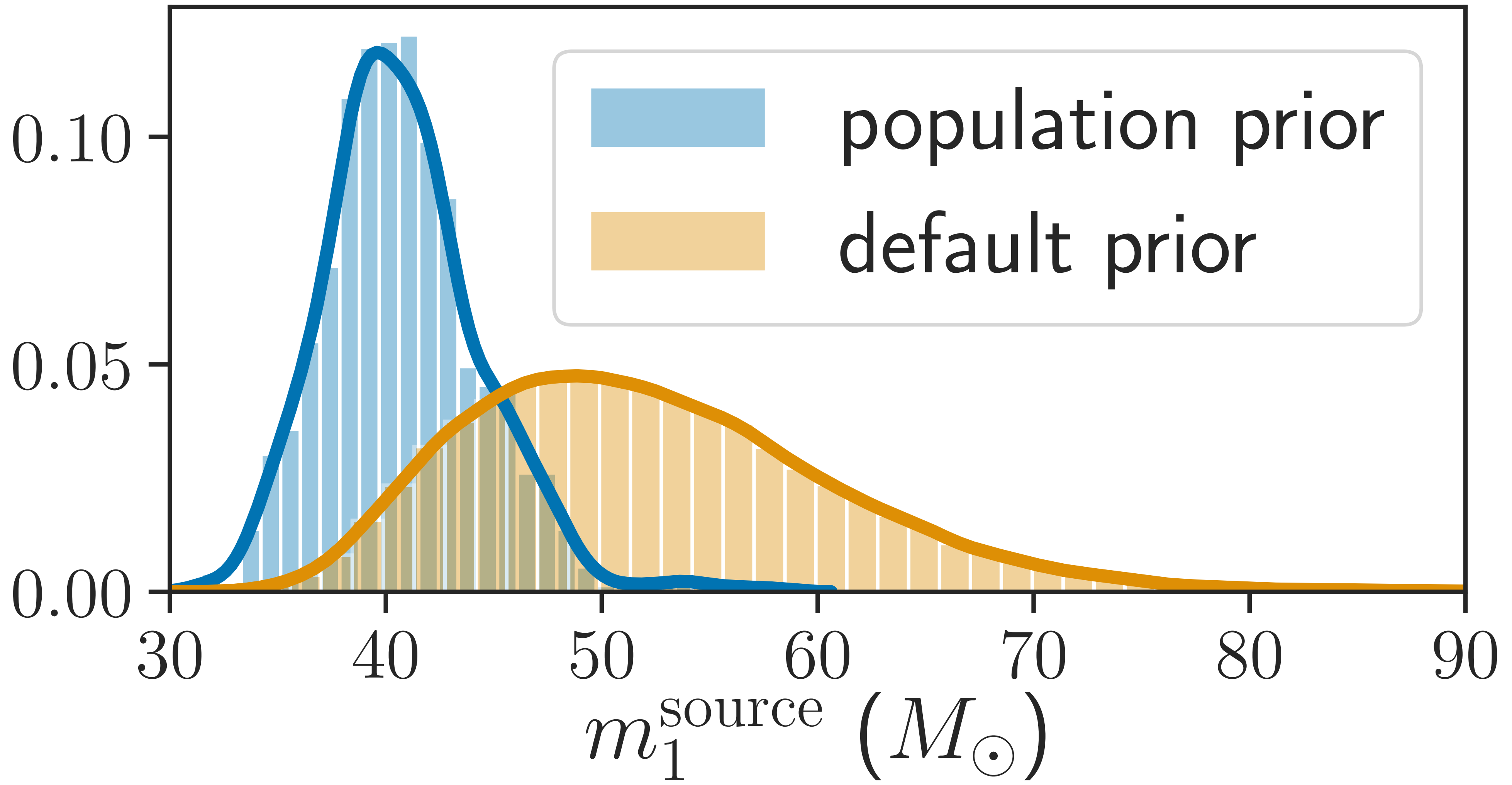
Big black holes are louder than small black holes

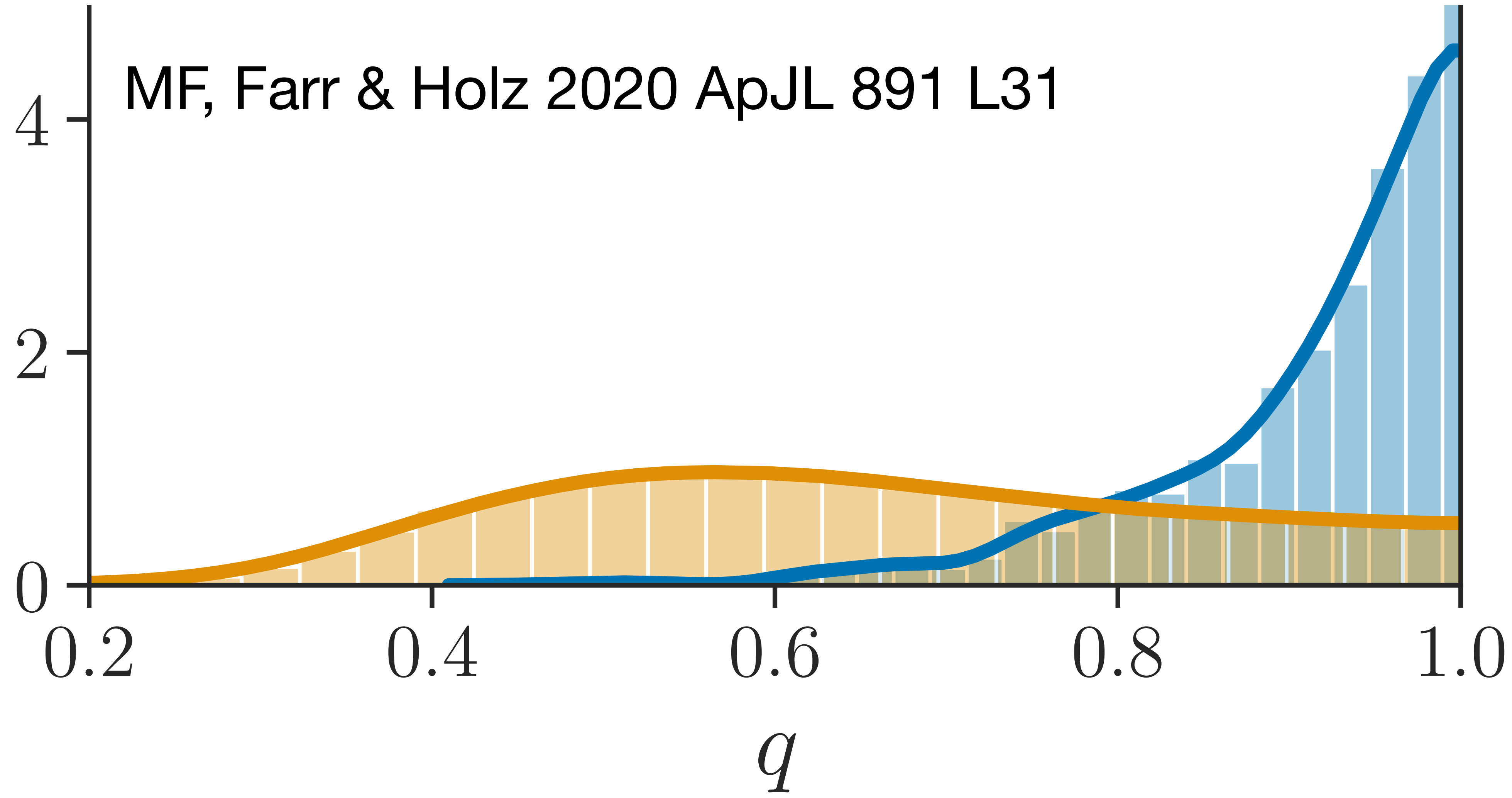


Sensitive volume

Total mass

Population inference also updates our knowledge of the parameters of individual events





Default parameter estimation prior is arbitrary! The likelihood is the same, but different priors ➡ different posteriors.

Default priors are flat in (detector-frame) component masses.

Population prior is the result of fitting the *population model* to all events

Three Astrophysical Lessons

1. **Mass distribution**
2. **Spin distribution**
3. **Evolution with redshift**

Three Astrophysical Lessons

1. **Mass distribution:** transition between neutron stars and black holes
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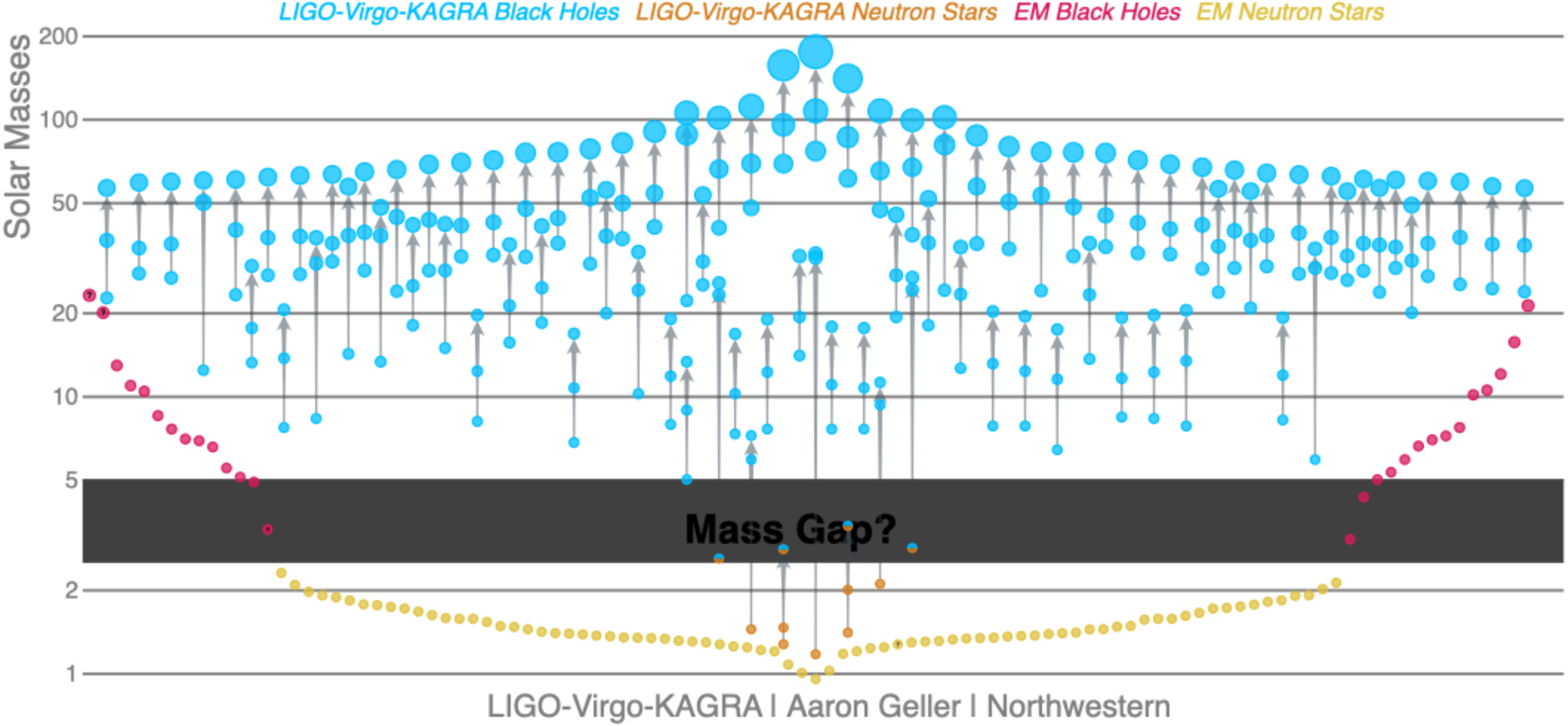
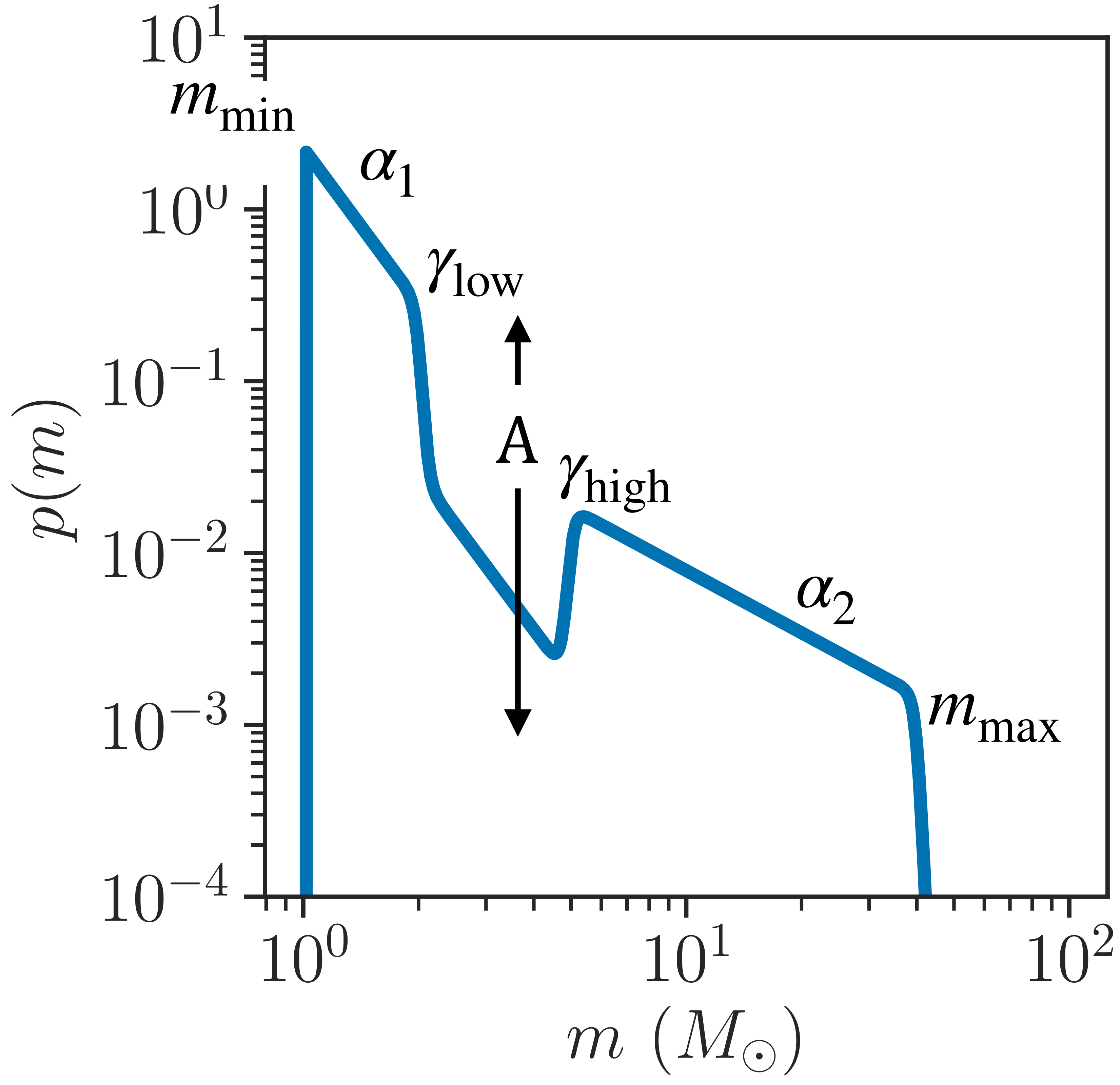
Masses in the Stellar Graveyard



Can we distinguish neutron stars and black holes based on mass?

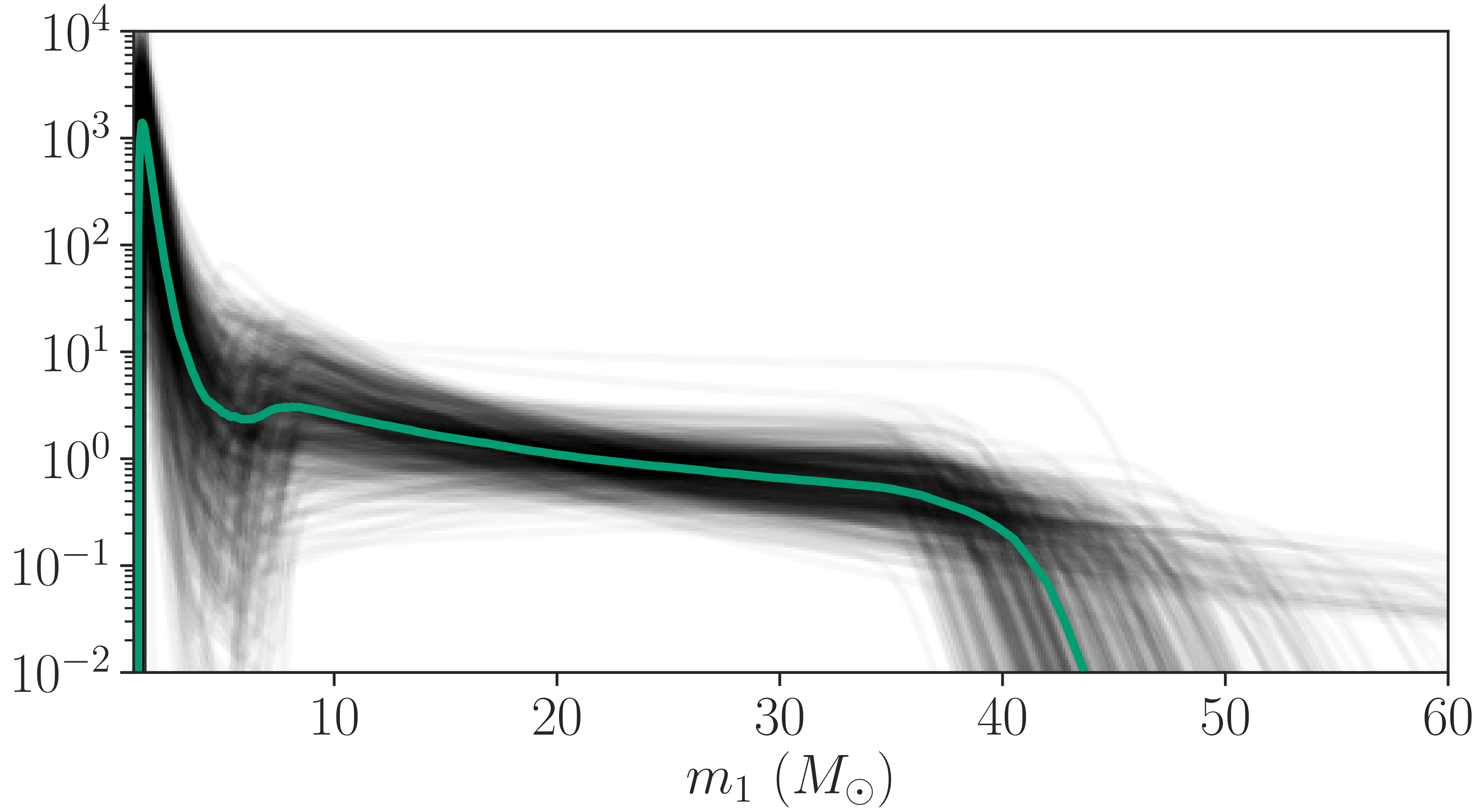
GWTC-1 suggested YES



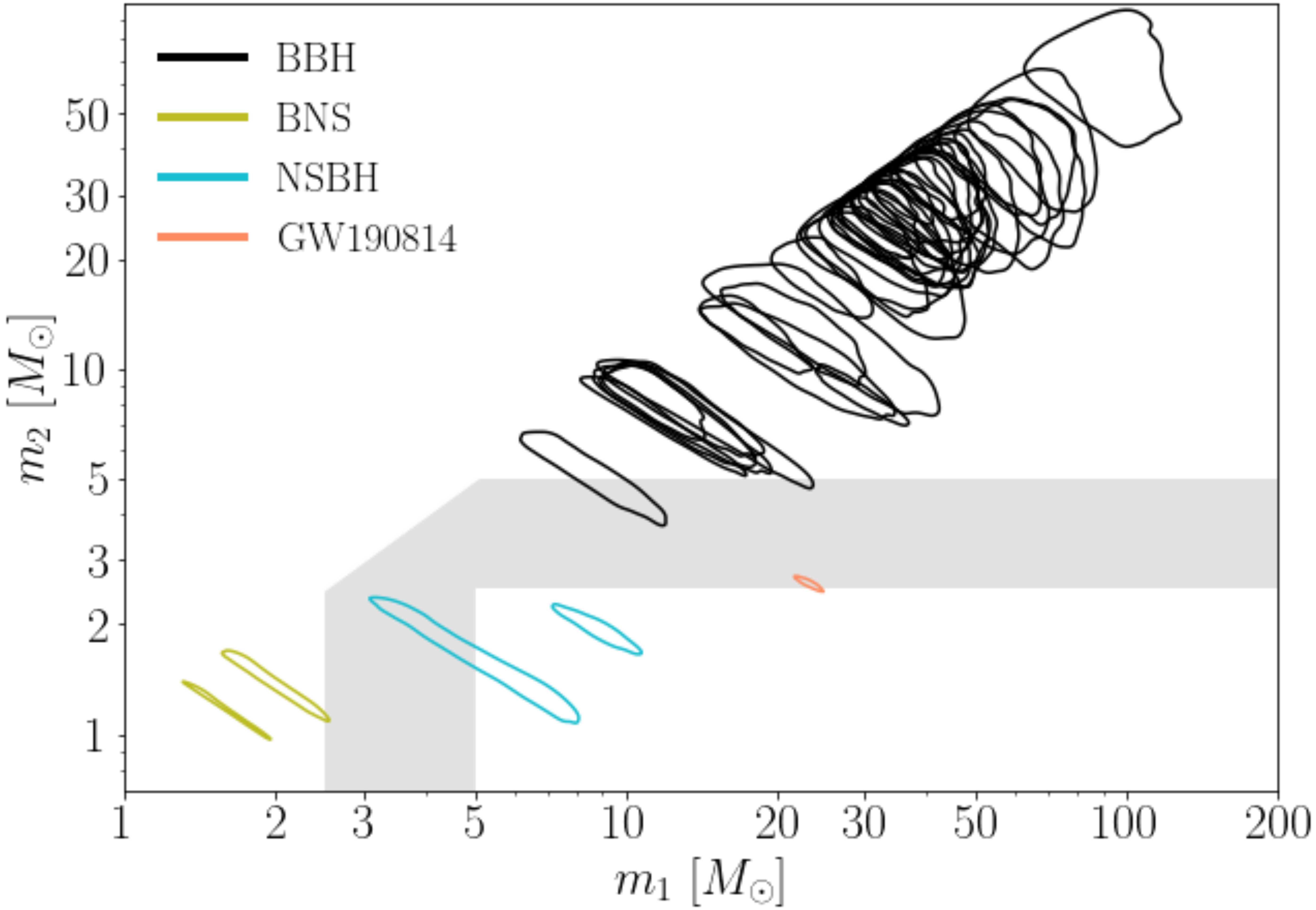
Fit this parametric model

11 events of GWTC-1

Dip between ~2 and ~7 solar masses (around 5 times more likely than no gap)

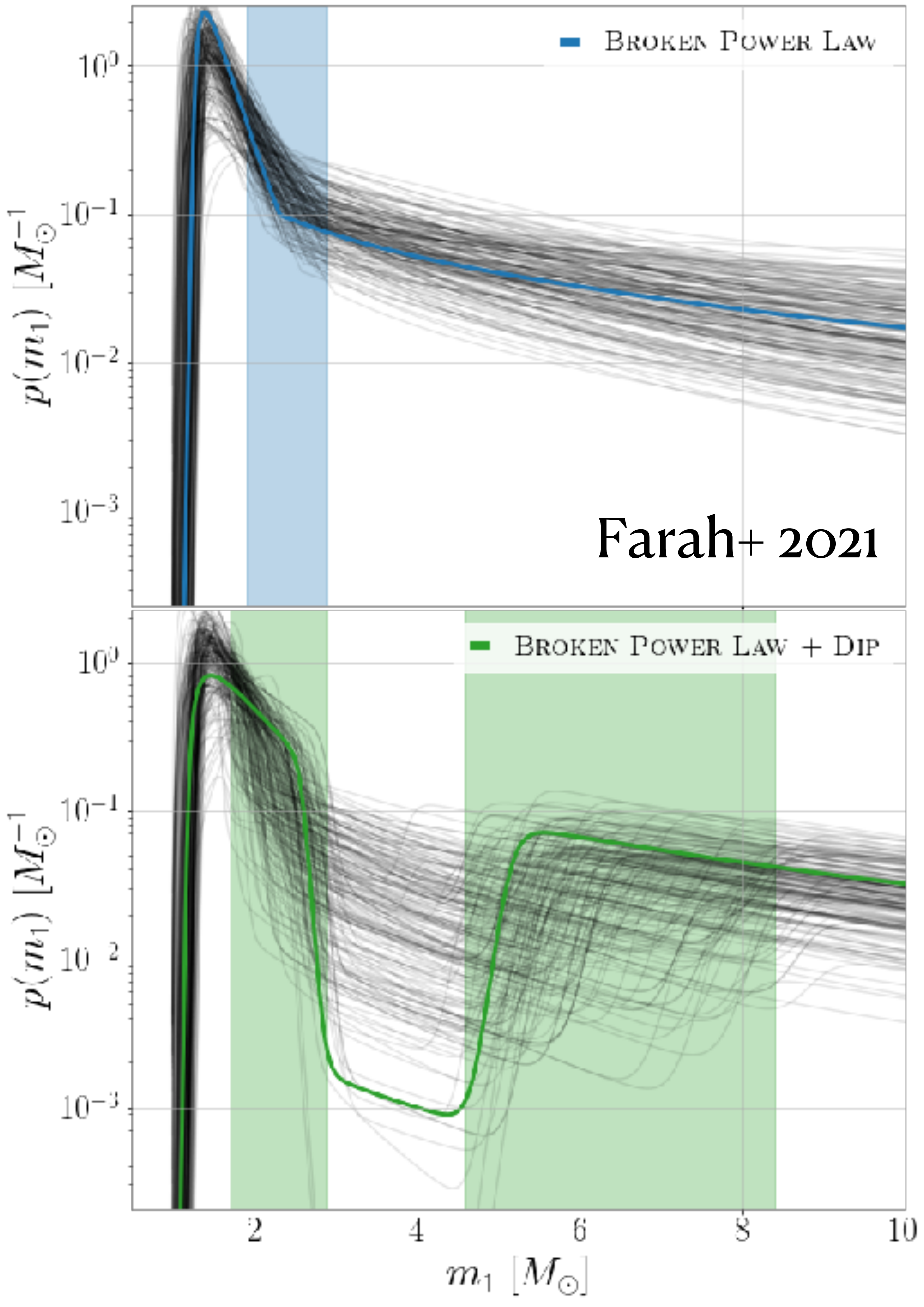


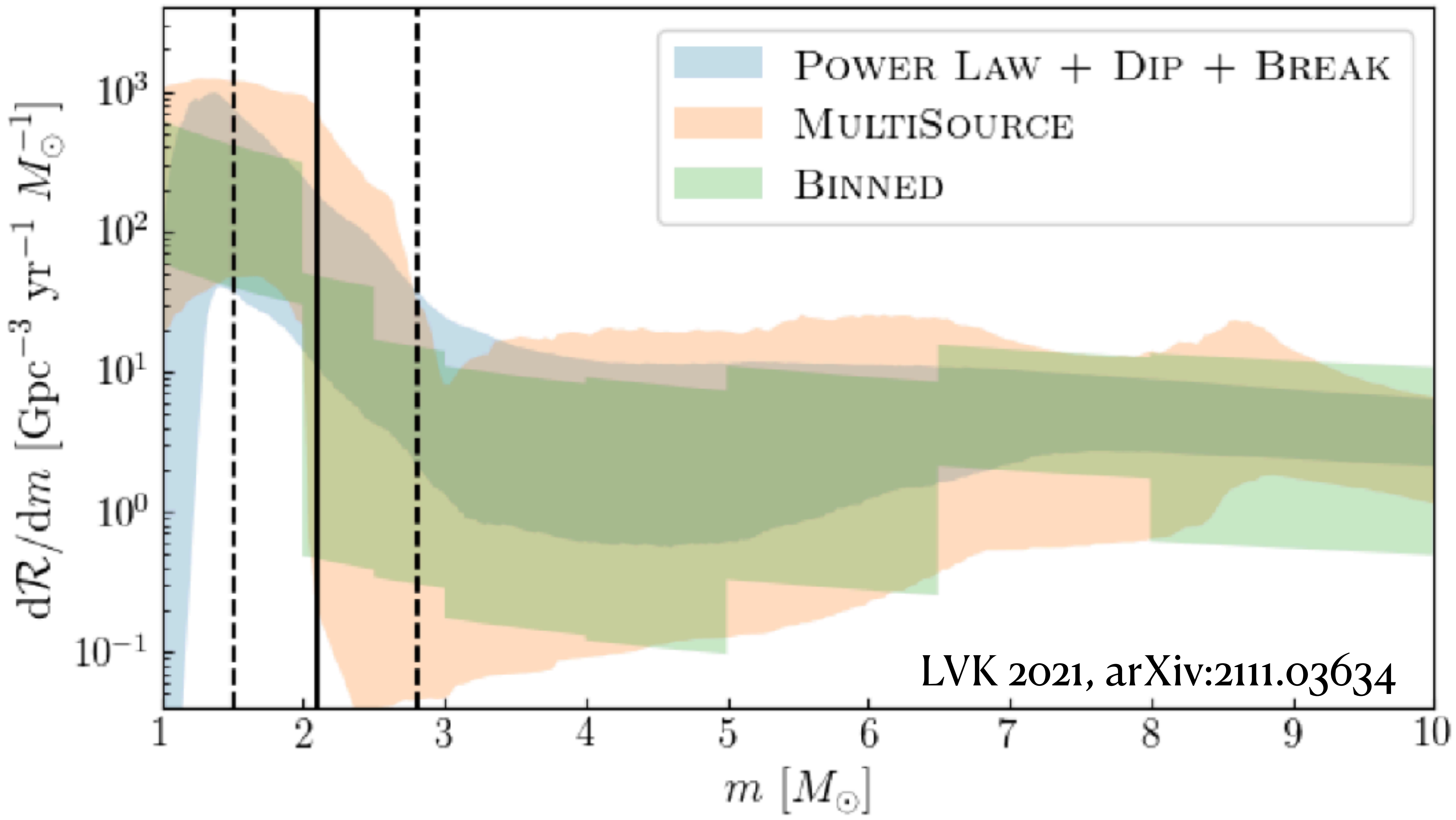
New O3 events are not really filling in the "gap"...



There is a clear deviation from a power law at ~2.4 solar masses

Coincides with neutron star maximum mass?





Three Astrophysical Lessons

1. **Mass distribution**
2. **Spin distribution:** slowly spinning black holes and large spin tilts
3. **Evolution with redshift**

Effective spin parameters

- The gravitational-wave signal can be parameterized by two "effective" spins:
  - The effective inspiral spin measures the total spin along the orbital angular momentum axis,

$$\chi_{\rm eff} = \frac{m_1\chi_1 cos\theta_1 + m_2\chi_2 \cos\theta_2}{m1 + m2}$$

  - The effective precessing spin measures the spin in the orbital plane, perpendicular to orbital angular momentum axis $\chi_{\rm p} \propto \chi_1 \sin\theta_1$

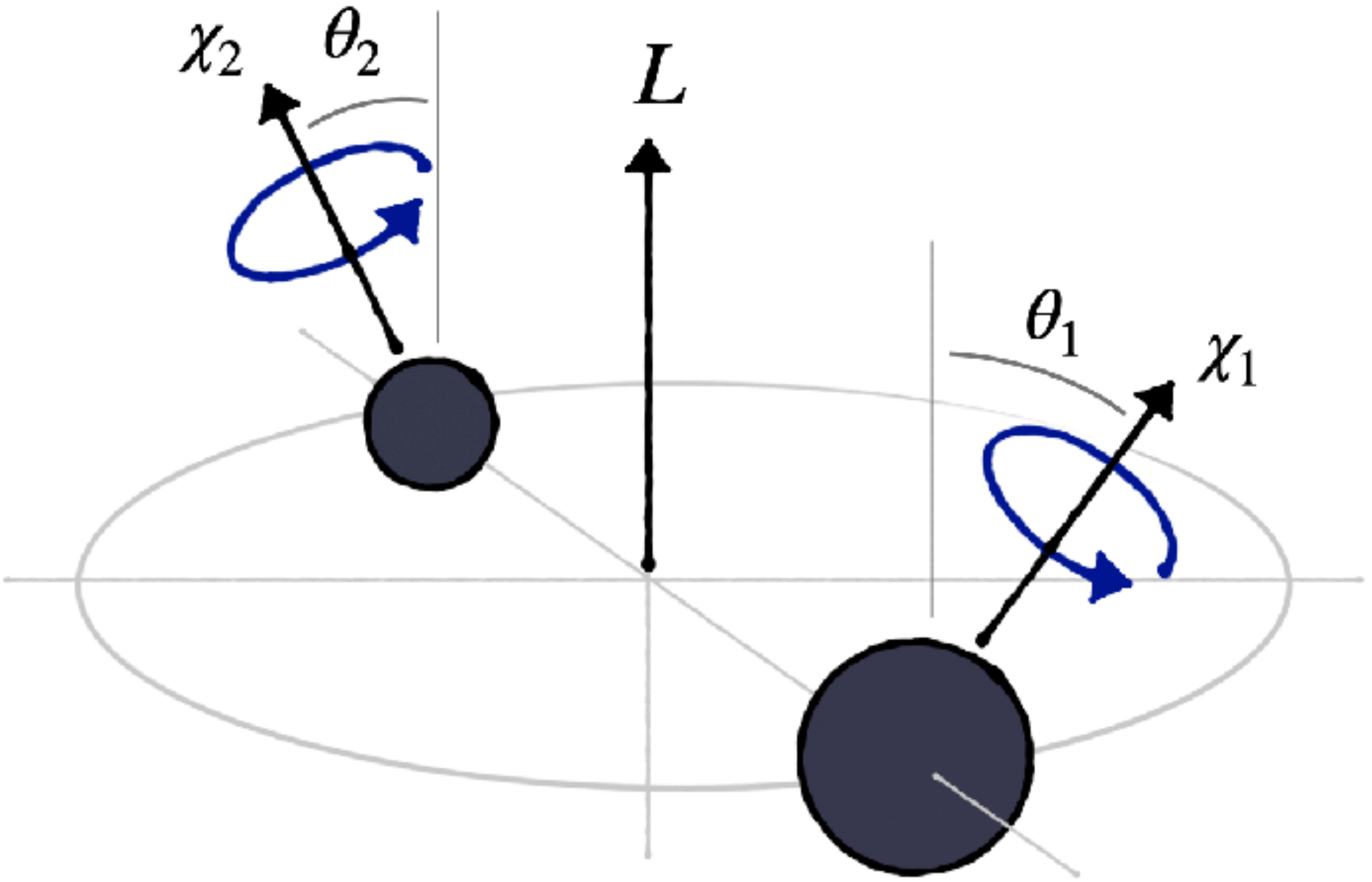
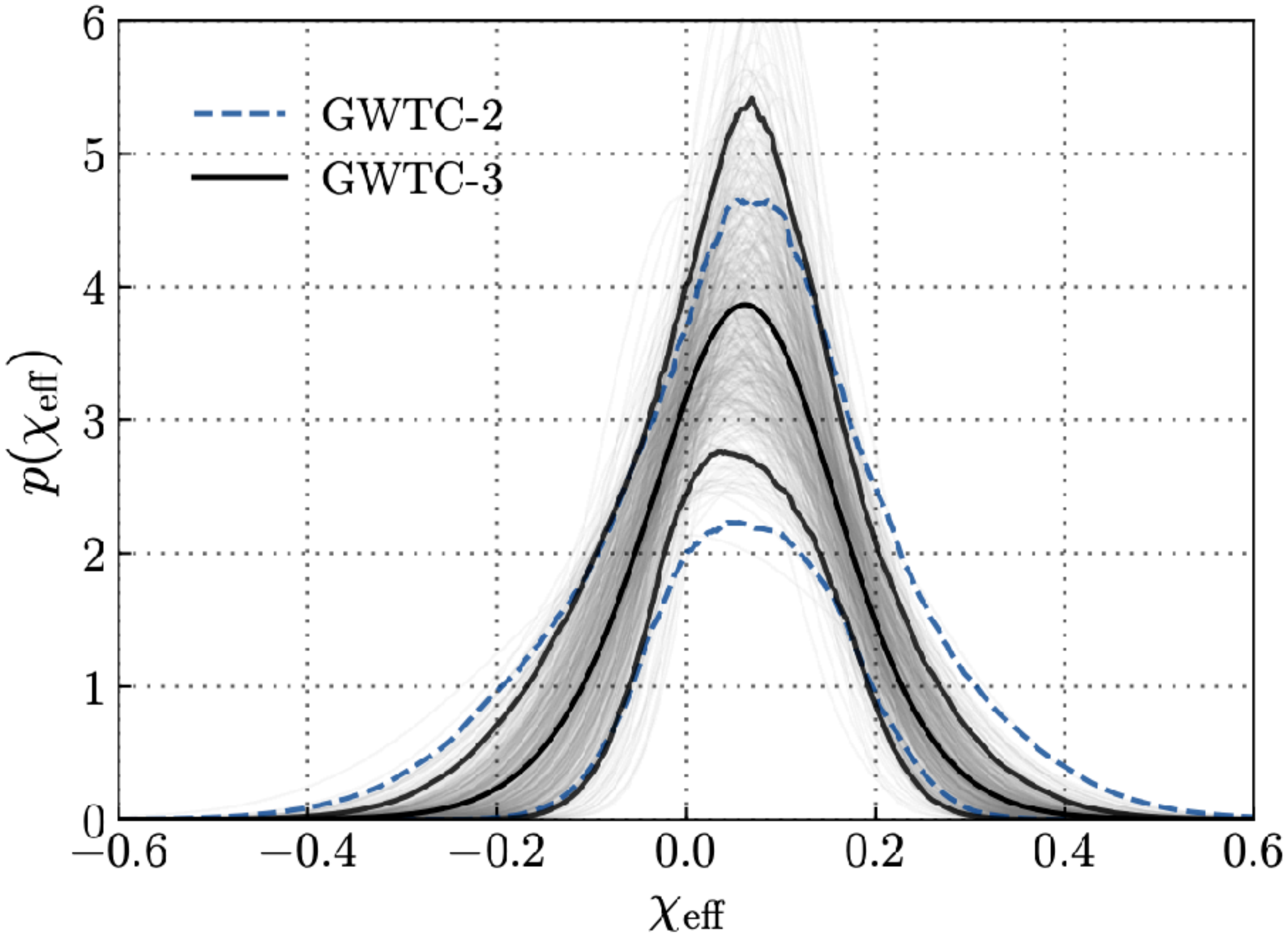


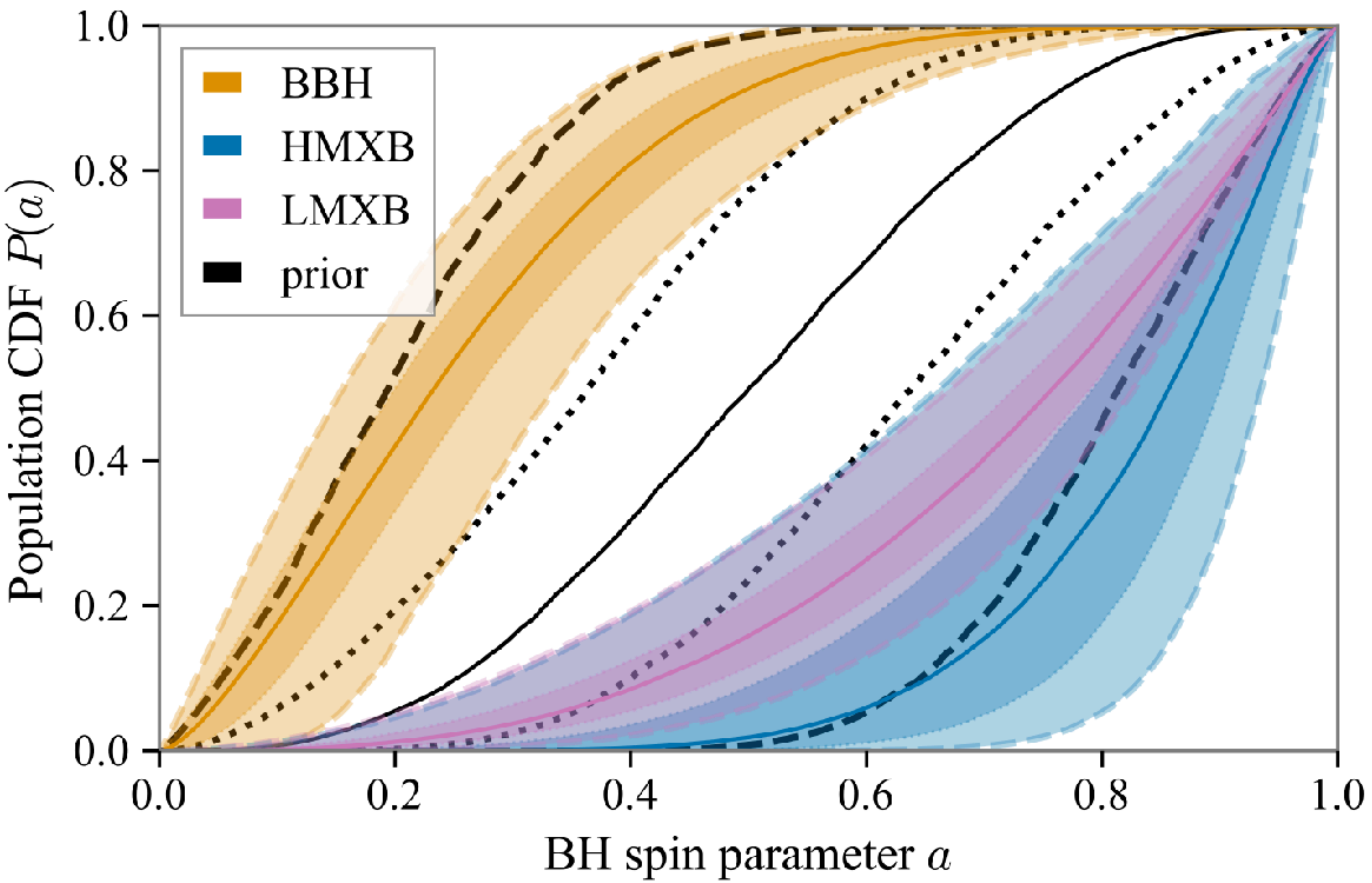
Figure credit: Thomas Callister

Effective inspiral spin population distribution

- $\chi_{\rm eff} < 0$ implies there are spin tilts > 90 degrees
- Current distribution consistent with symmetric $\chi_{\rm eff}$ centered at zero, implies the distribution of spin tilts may be isotropic
- Favors dynamical origin or mixture between dynamical and isolated(?)



Black holes in gravitational-wave systems spin slower than those in X-ray binaries

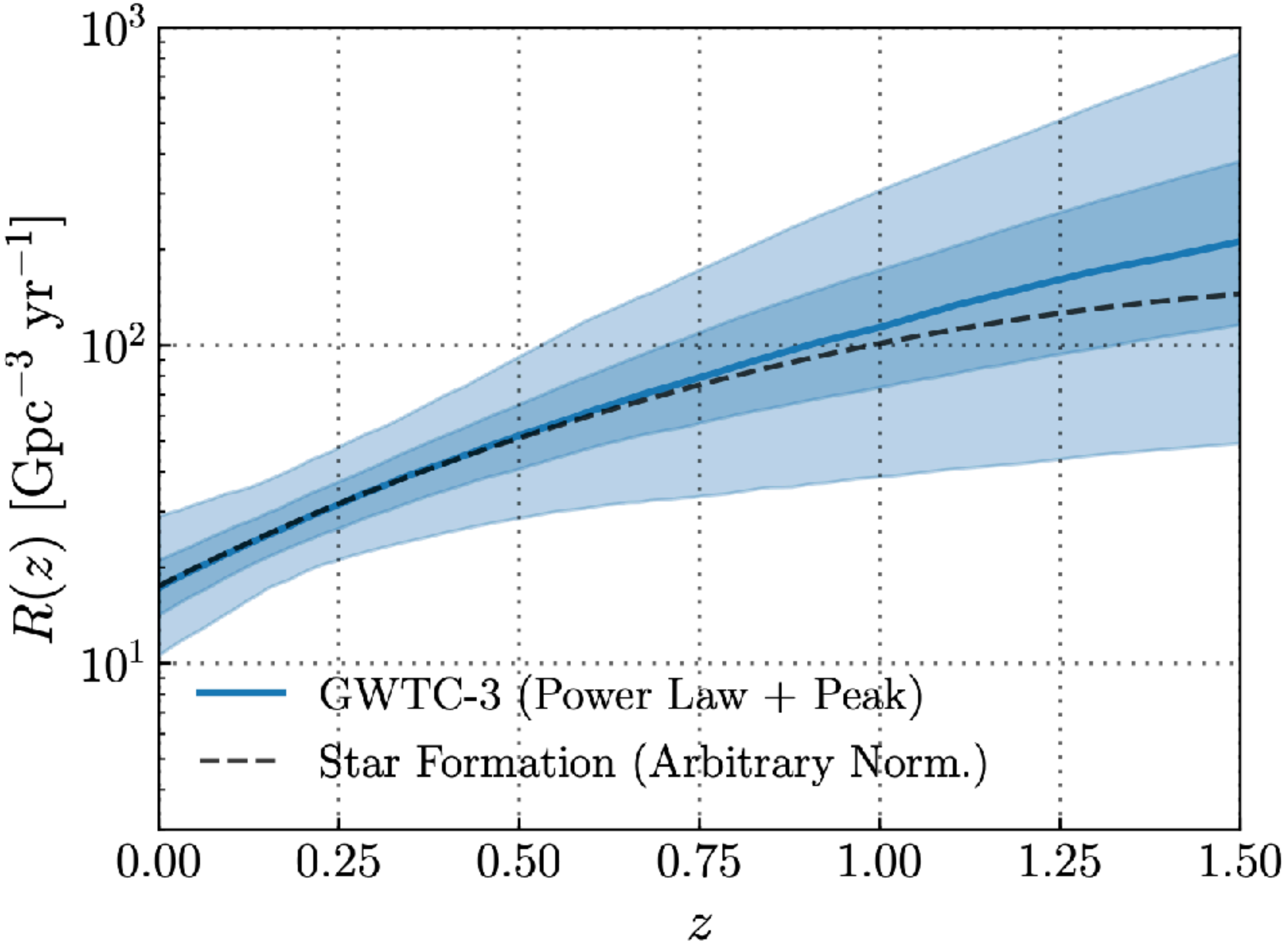


Fishbach & Kalogera 2021, arXiv:2111.02935

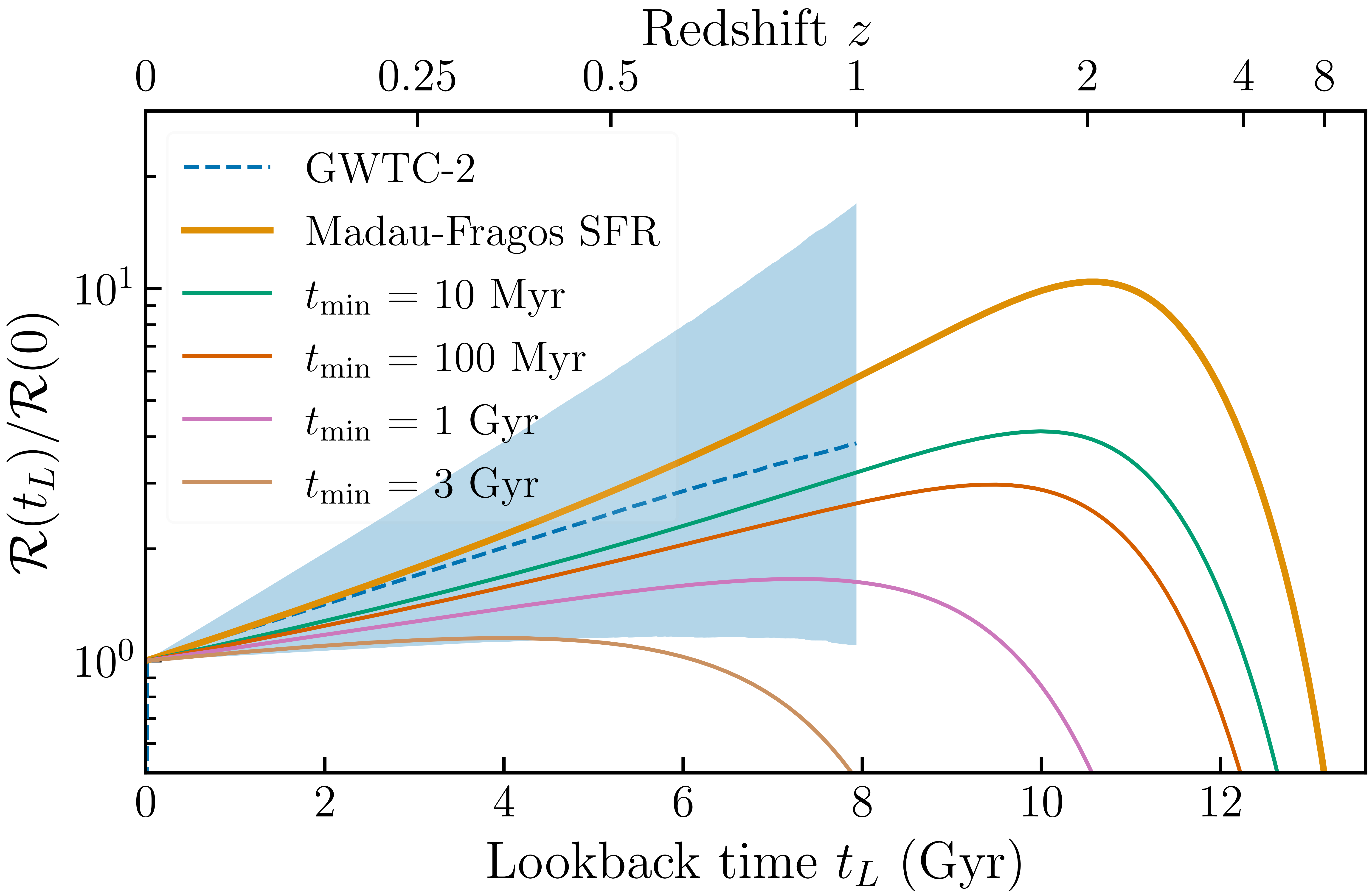
Three Astrophysical Lessons

1. **Mass distribution**
2. **Spin distribution**
3. **Evolution with redshift**: Merger rate evolution matches star formation rate

Binary black hole merger rate density across cosmic time

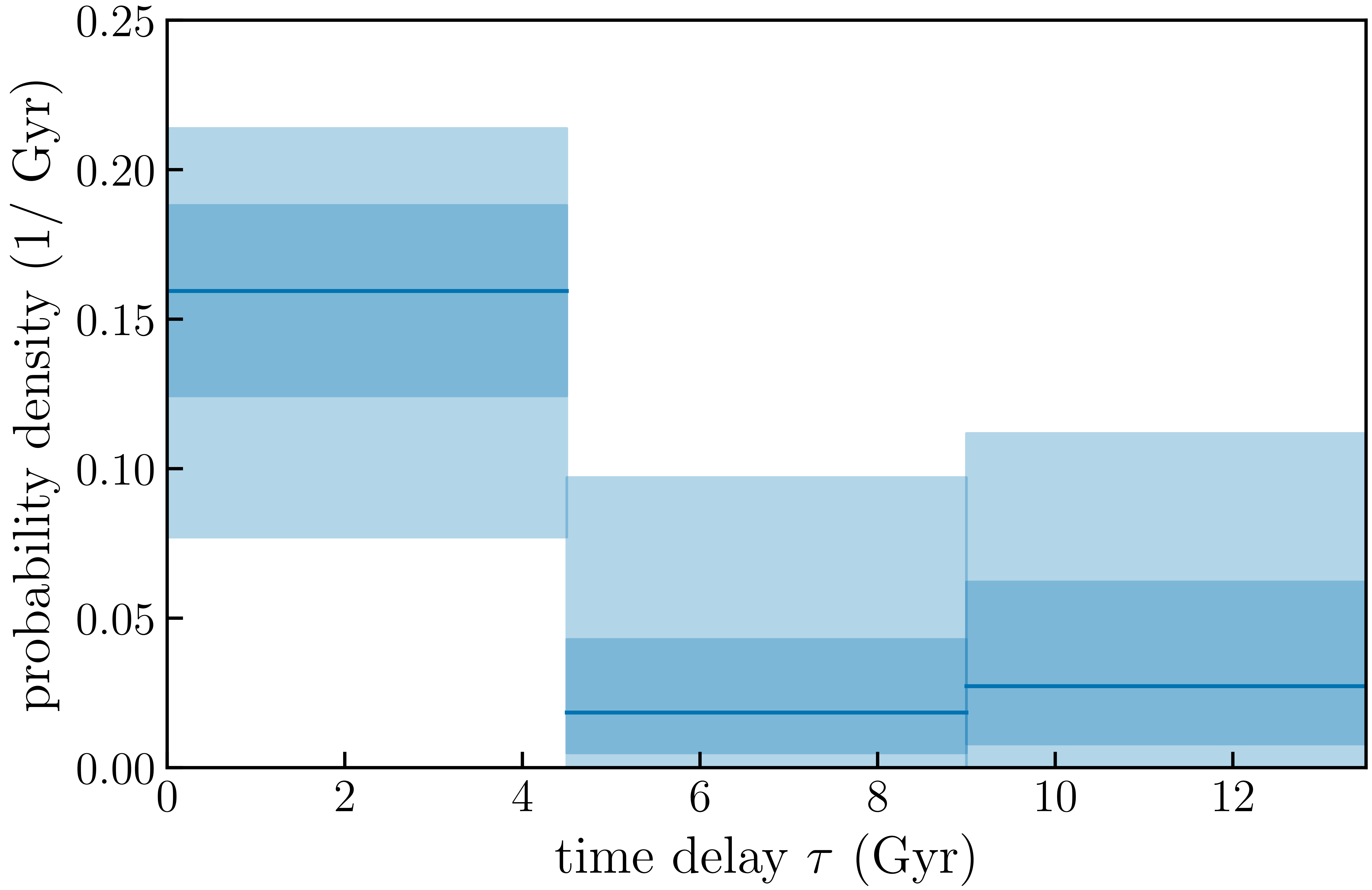


Comparing merger rate evolution to star formation + time delays



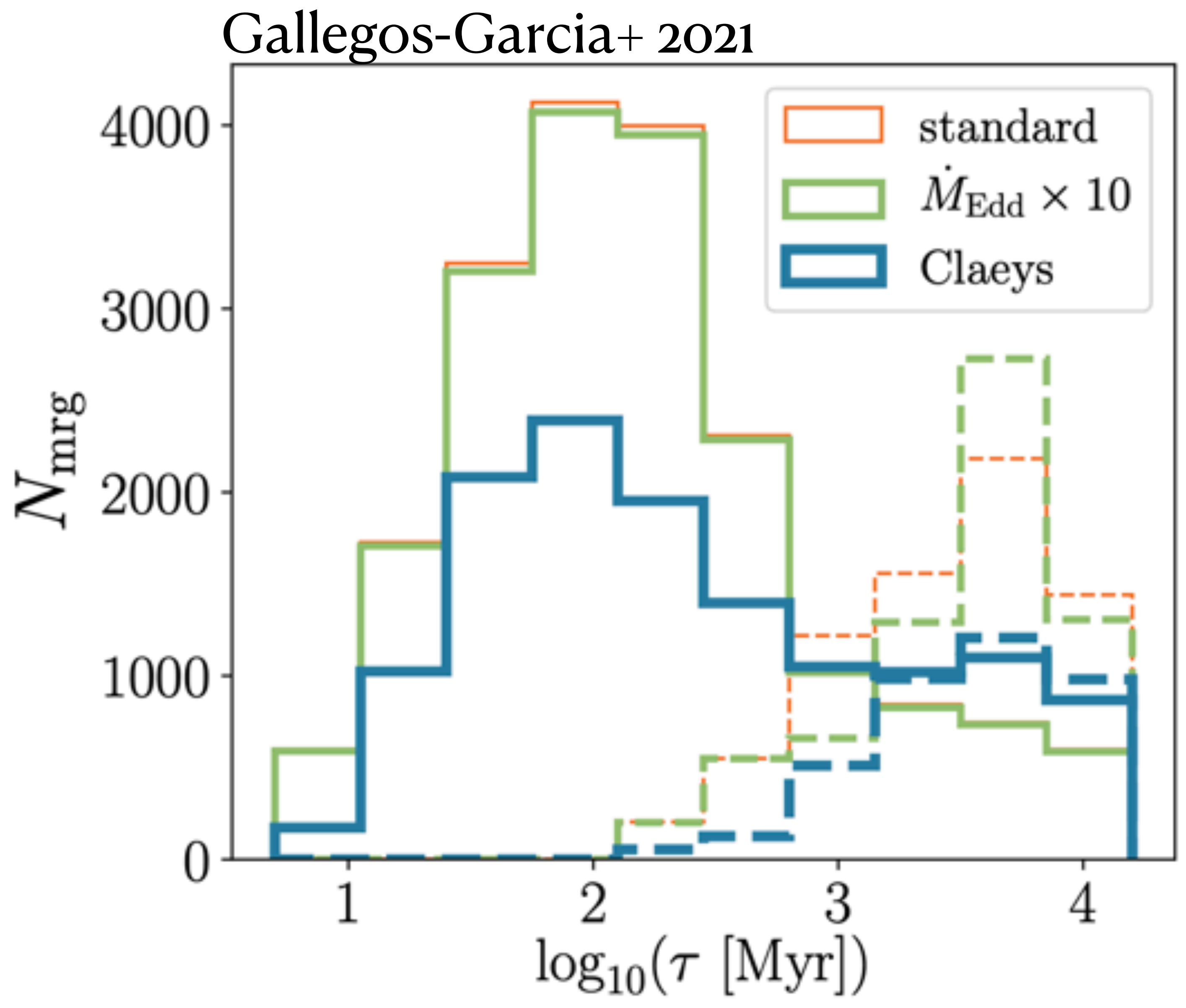
MF & Kalogera 2021, ApJL 914 L30

Data favor short time delays between formation and merger

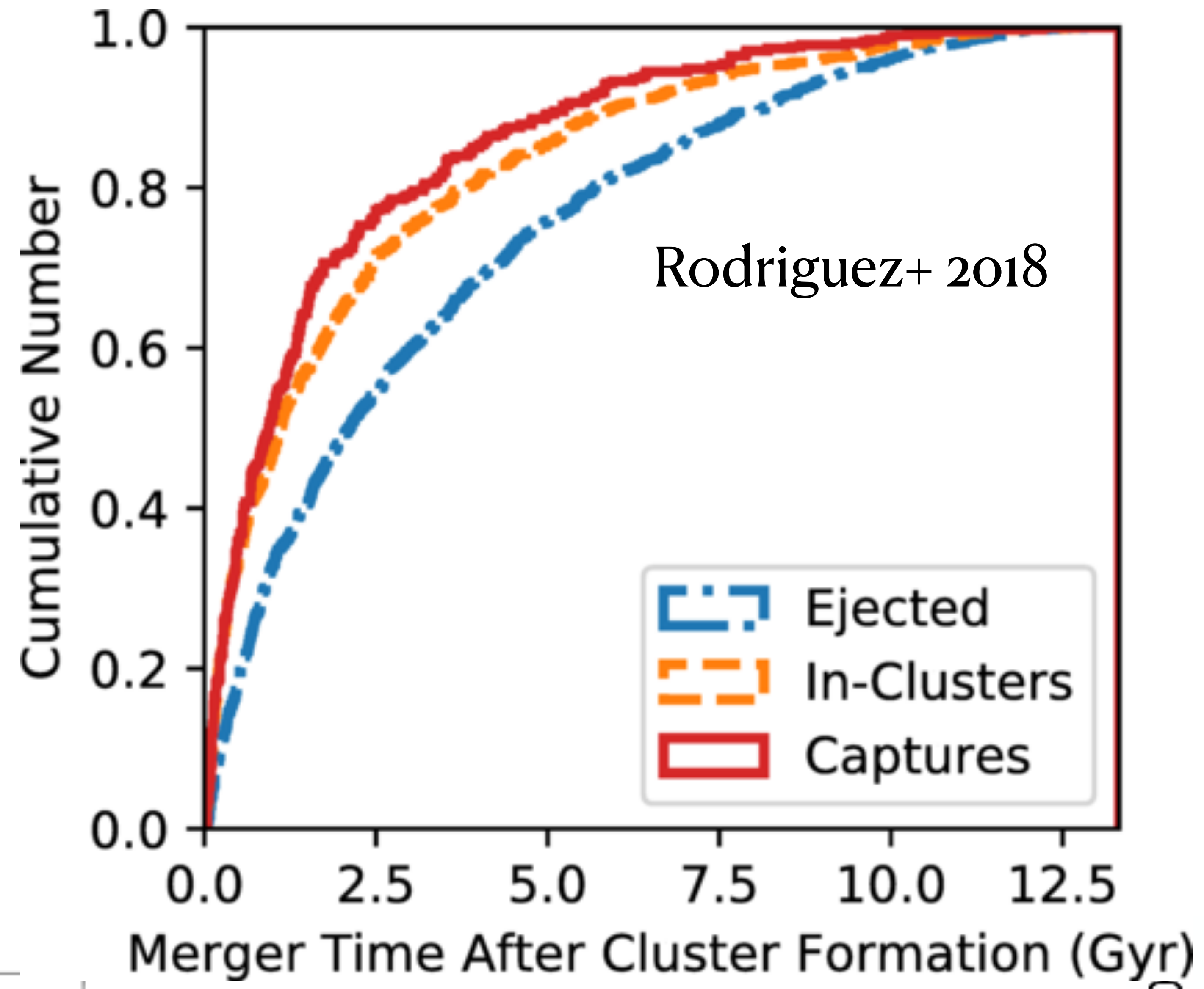


MF & Kalogera 2021, ApJL 914 L30

Predicted time delay distributions

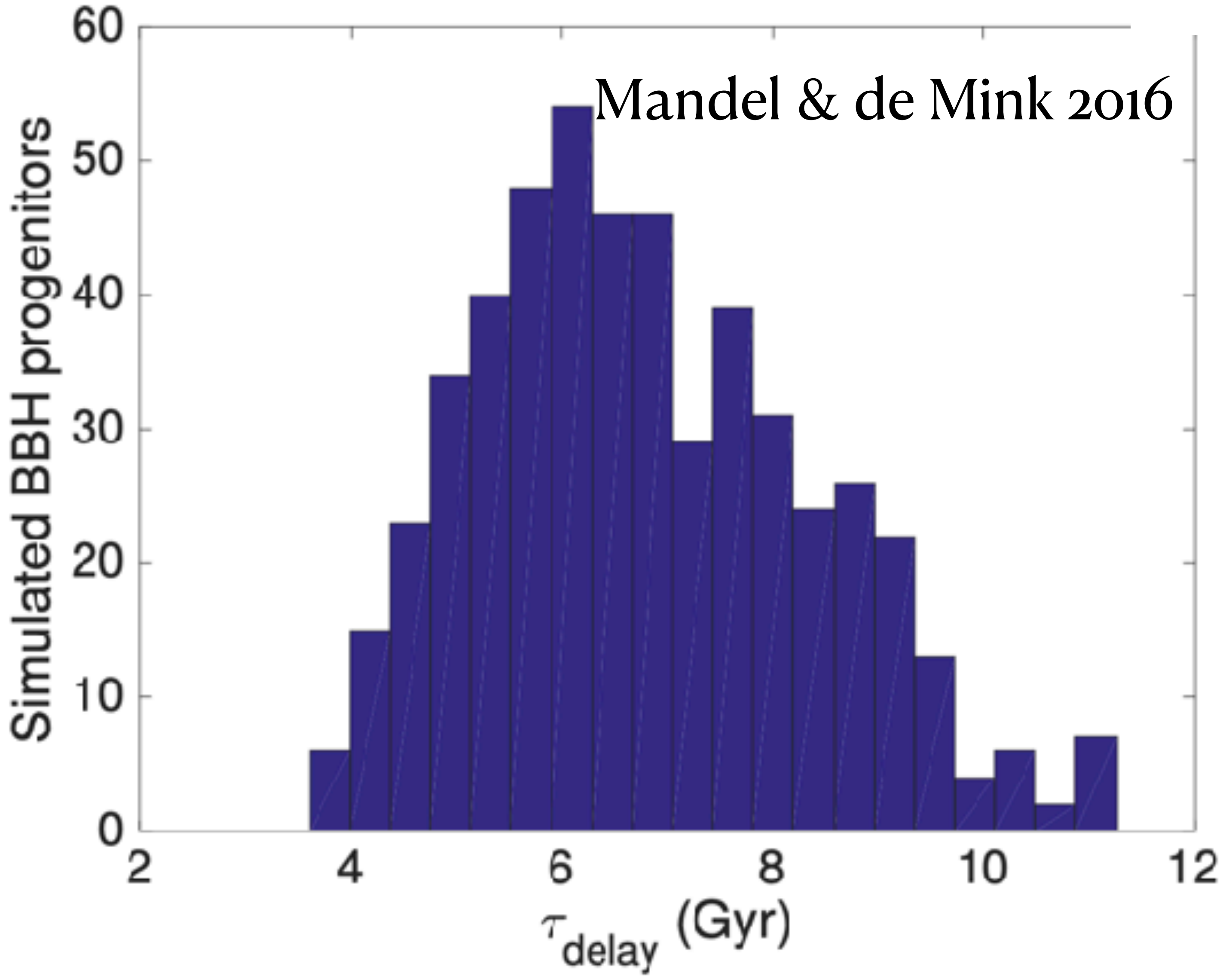


Left: Field binary evolution: common-envelope and stable mass transfer

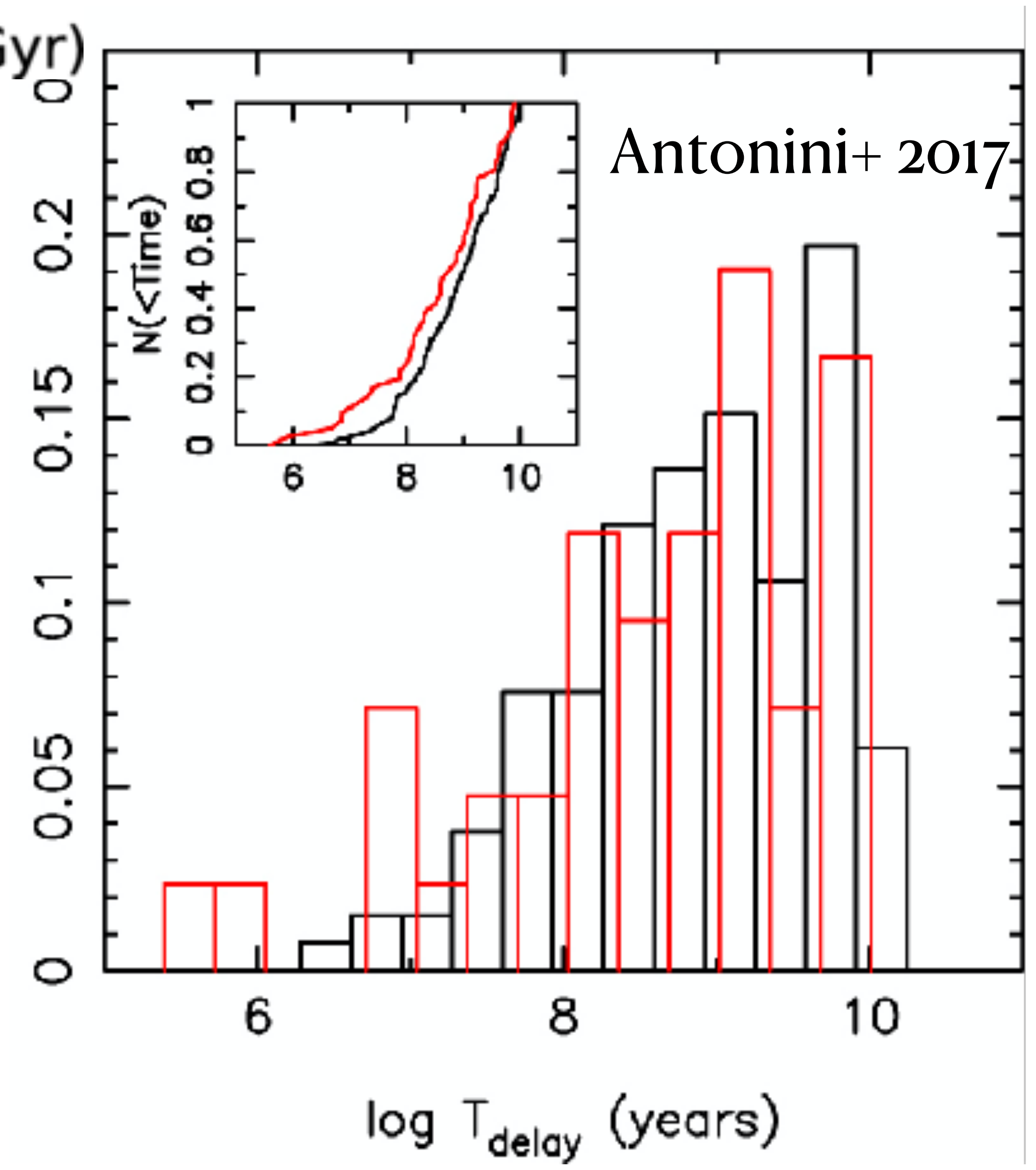


Left: Dynamical assembly in globular clusters

Right: Chemically homogeneous evolution



Right: Stellar triples



Ongoing and future population inference with larger catalogs

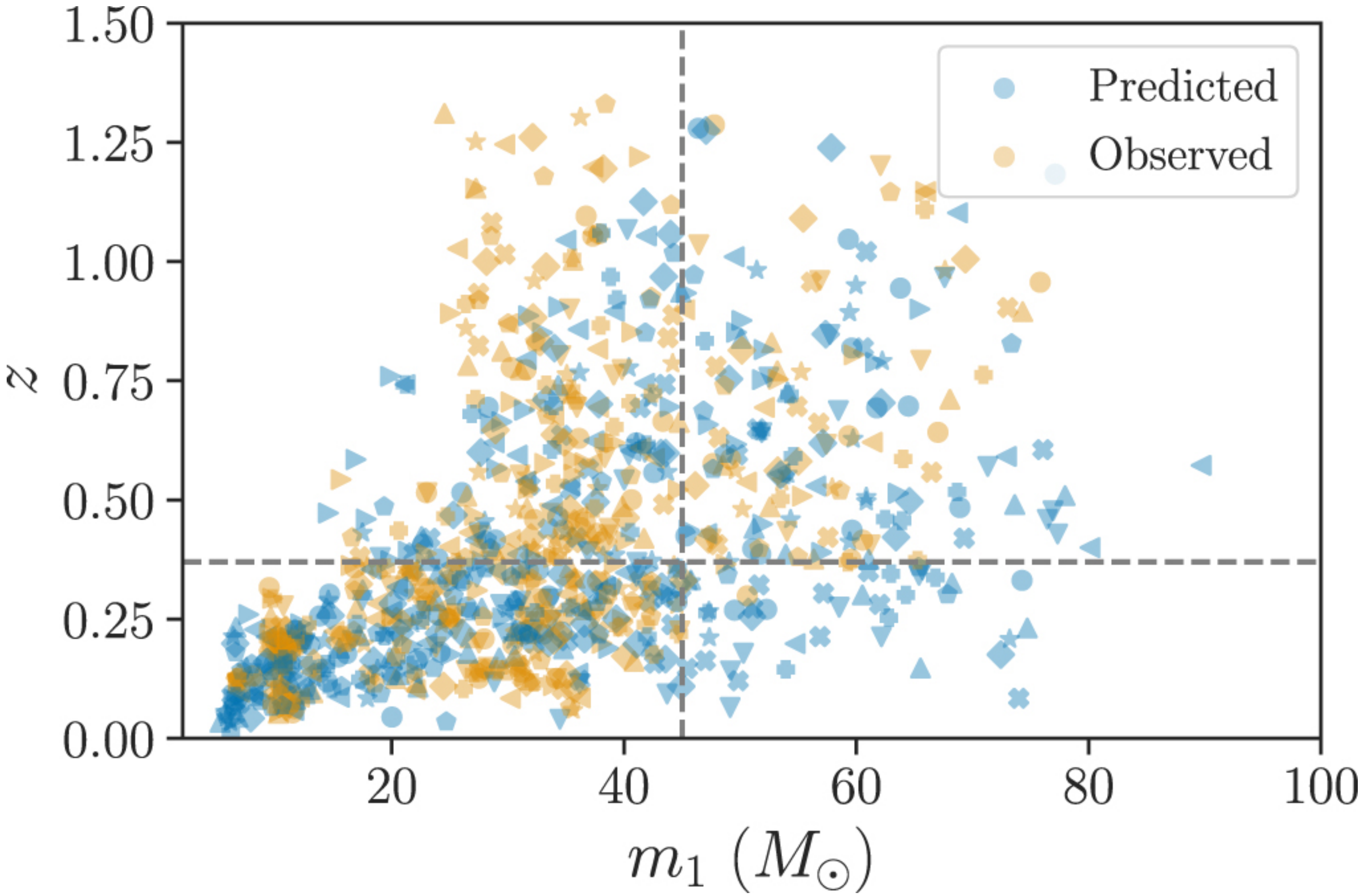
- Model checking and outlier tests
- Towards nonparametric population models
- Population-level correlations between parameters (e.g. mass evolution with redshift, spin correlation with masses)

Ongoing and future population inference with larger catalogs

- **Model checking and outlier tests**: when do we have to upgrade our simple models?
- Towards nonparametric population models
- Population-level correlations between parameters (e.g. mass evolution with redshift, spin correlation with masses)

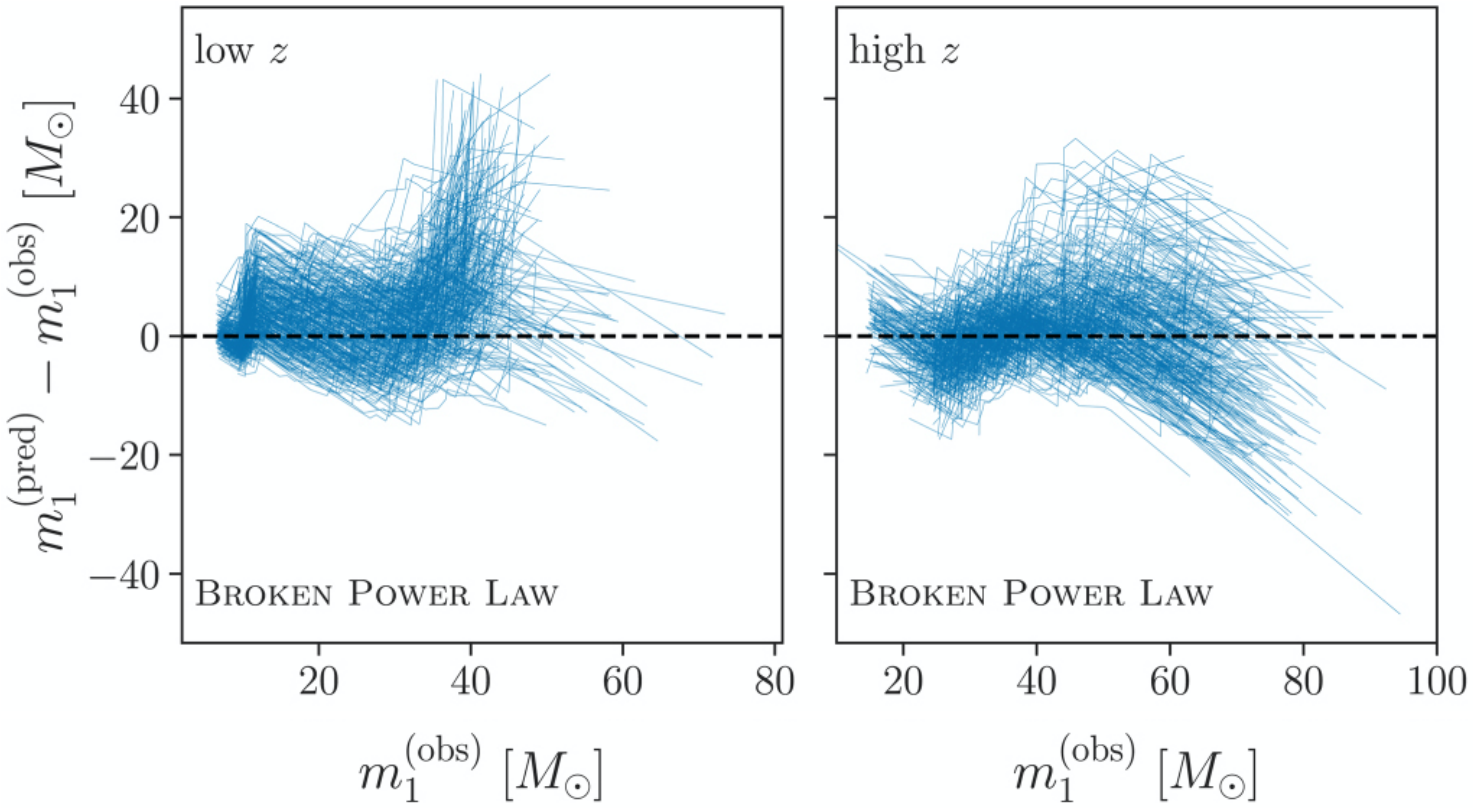
Posterior Predictive Checks

"Goodness of fit:" Unlike with model selection, we don't have to specify an alternative model. But there is also not a single statistic we can use to reject a model.



MF, Doctor, Callister, Edelman, Ye, Essick, Farr, Farr & Holz 2021 ApJ 912 98

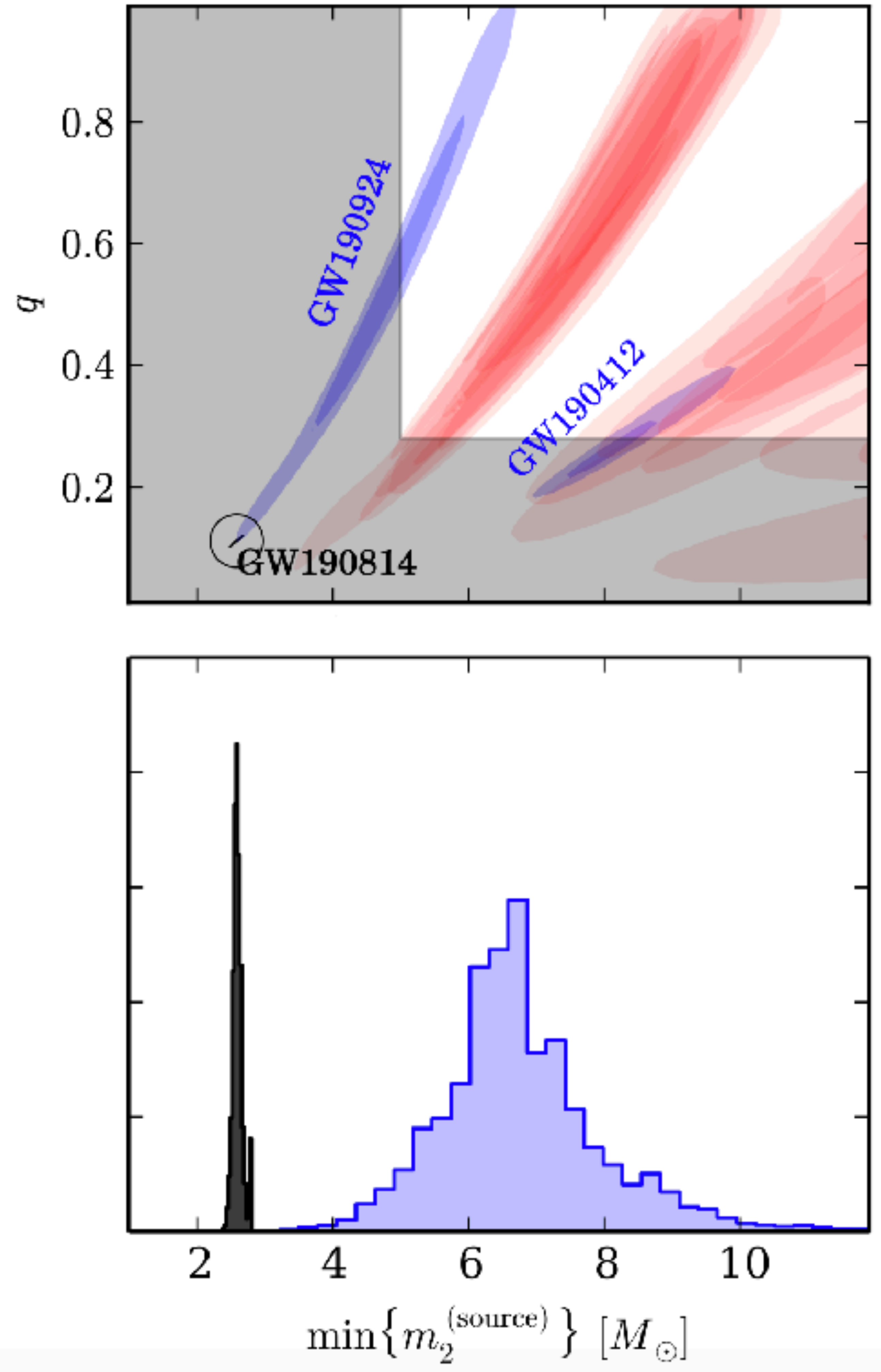
Quantify deviations between predicted and observed events



MF, Doctor, Callister, Edelman, Ye, Essick, Farr, Farr & Holz 2021 ApJ 912 98

Outlier tests with coarse-grained likelihoods

- Typical outlier tests in gravitational-wave population inference do a "leave-one-out" analysis
- This usually involves selecting a potential "outlier" event in advance, which leads to bias
- One solution is to repeat "leave-one-out" for every event in the catalog —> computationally expensive
- Another solution: define a "coarse-grained" likelihood
- Also see Roulet+ Phys. Rev. D 104, 083010 (2021)

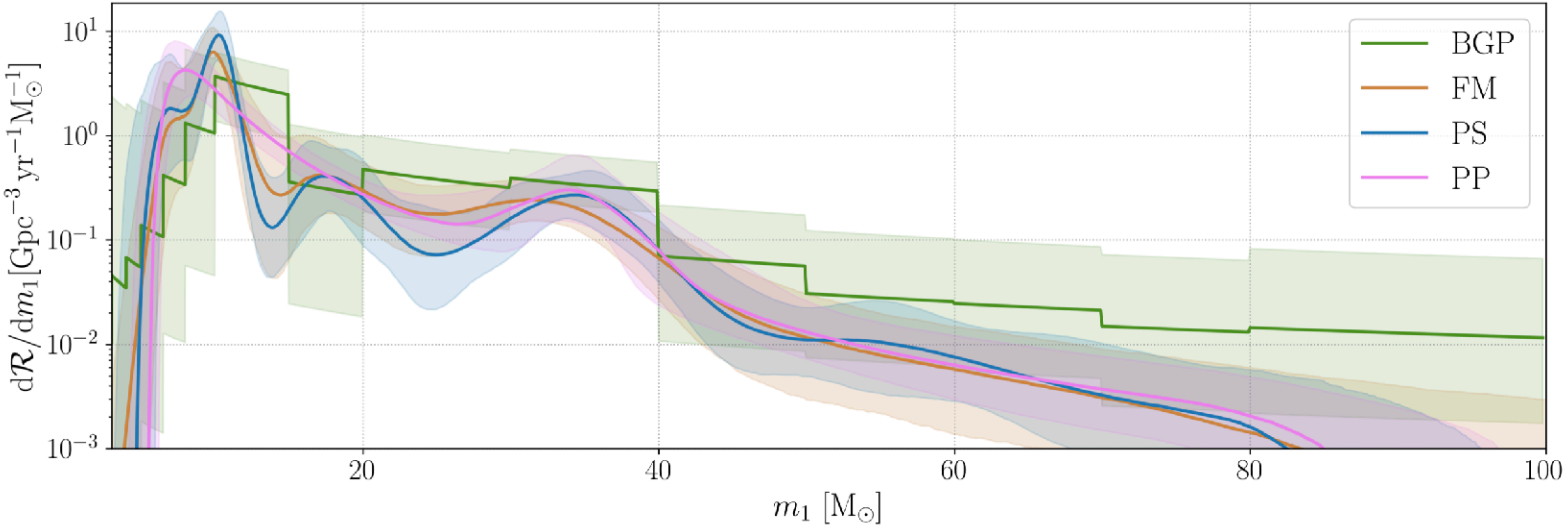


Essick, Farah, Galaudage, Talbot, MF, Thrane & Holz 2021, ApJ in press

Ongoing and future population inference with larger catalogs

- Model checking and outlier tests
- **Towards nonparametric population models**
- Population-level correlations between parameters (e.g. mass evolution with redshift, spin correlation with masses)

Semiparametric population fits in GWTC-3



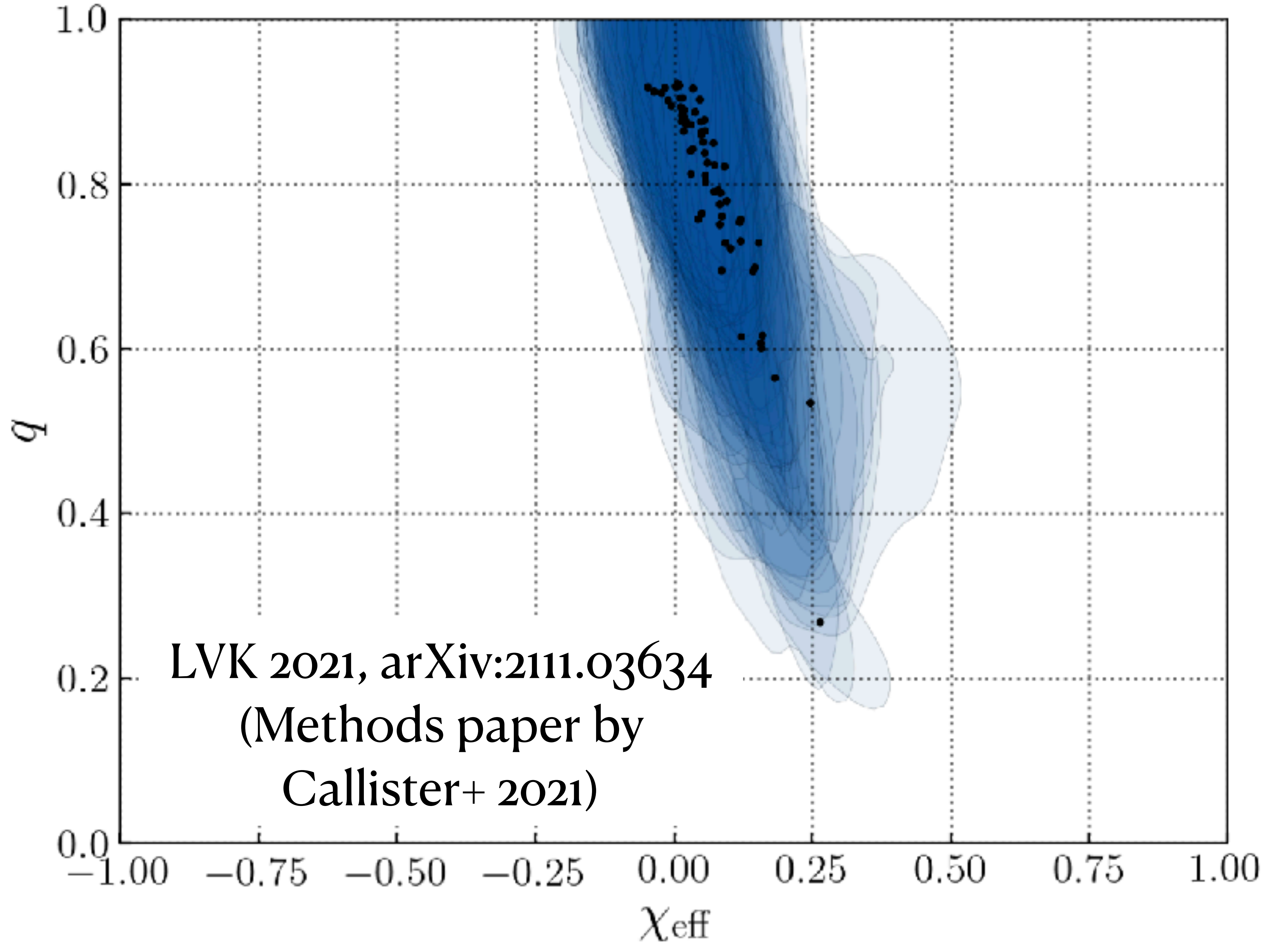
BPG: Binned Gaussian Process on 2D mass plane, FM: flexible mixture model, PS: model deviations from parameterized model as spline

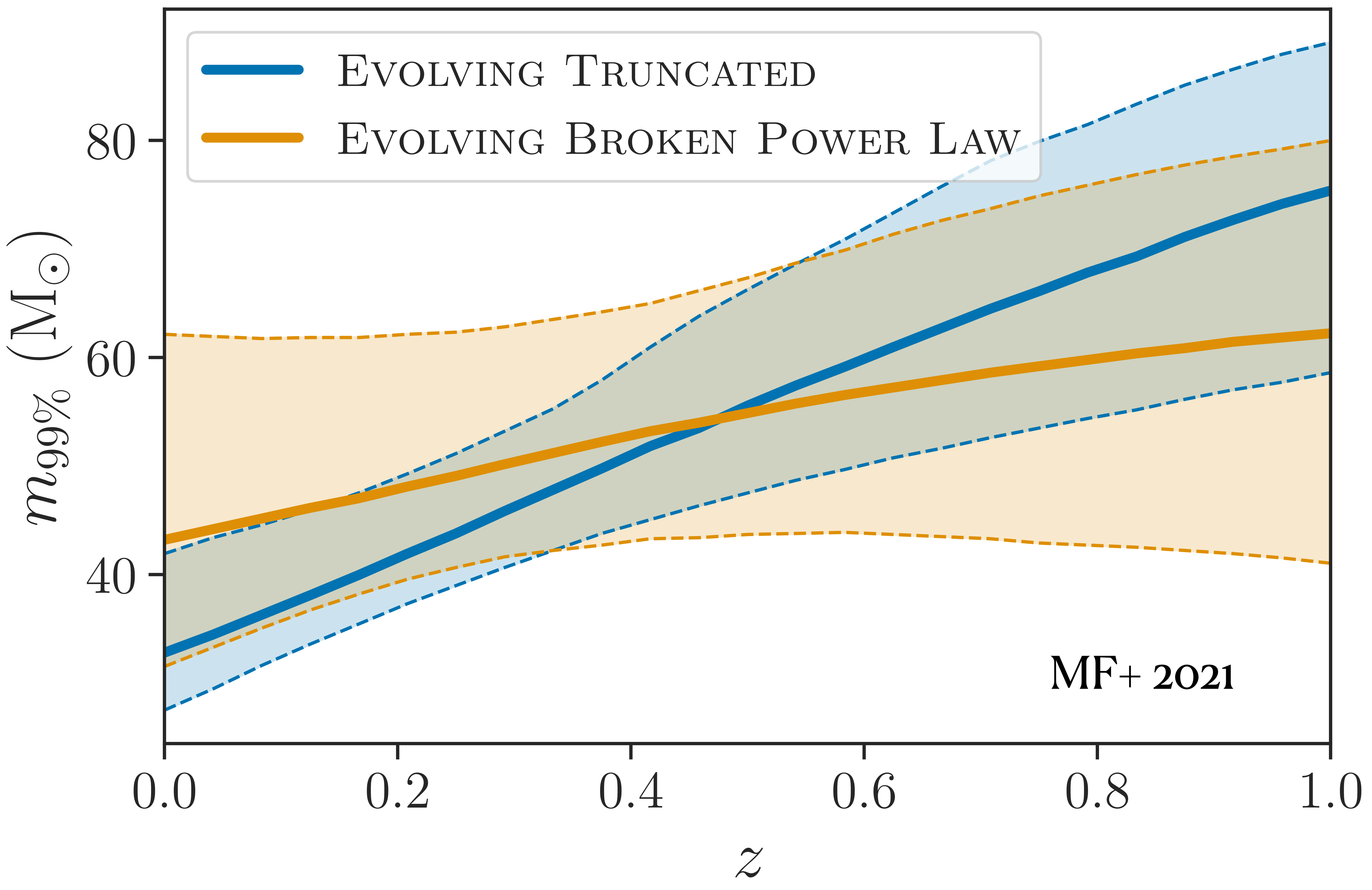
See Bruce's poster

Ongoing and future population inference with larger catalogs

- Model checking and outlier tests
- Towards nonparametric population models
- **Population-level correlations between parameters** (e.g. mass evolution with redshift, spin correlation with masses)

Examples of population-level correlations





Summary

- Astrophysical Lessons
  - Mass distribution: Transition between neutron stars and black holes
  - Spin distribution: Slowly spinning black holes and large spin tilts
  - Evolution with redshift: Merger rate evolution and small time delays
- Future prospects and challenges
  - Model checking and outlier tests: posterior predictive checks, coarse-grained likelihoods
  - Nonparametric population models: Gaussian mixtures, Gaussian process regularized histograms, splines
  - Look for correlations between population distributions

Future prospects and challenges, continued

- Simultaneously model signal and noise population
- How do we interpret the population fits in the context of theoretical models? (See Michela's talk)
- Include cosmology, theory of gravity, neutron star equation of state in population model (see Rachel's talk)
- Leverage information from stochastic background (see Arianna's talk)
- Turn systematic uncertainties into statistical uncertainties (e.g. waveform models, calibration)
- Smarter parameter estimation, sensitivity estimation