Cosmology with dark sirens: Attempting to solve the Hubble Constant tension

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Queen Mary University of London

LSC LIGO Scientific Collaboration

Outline

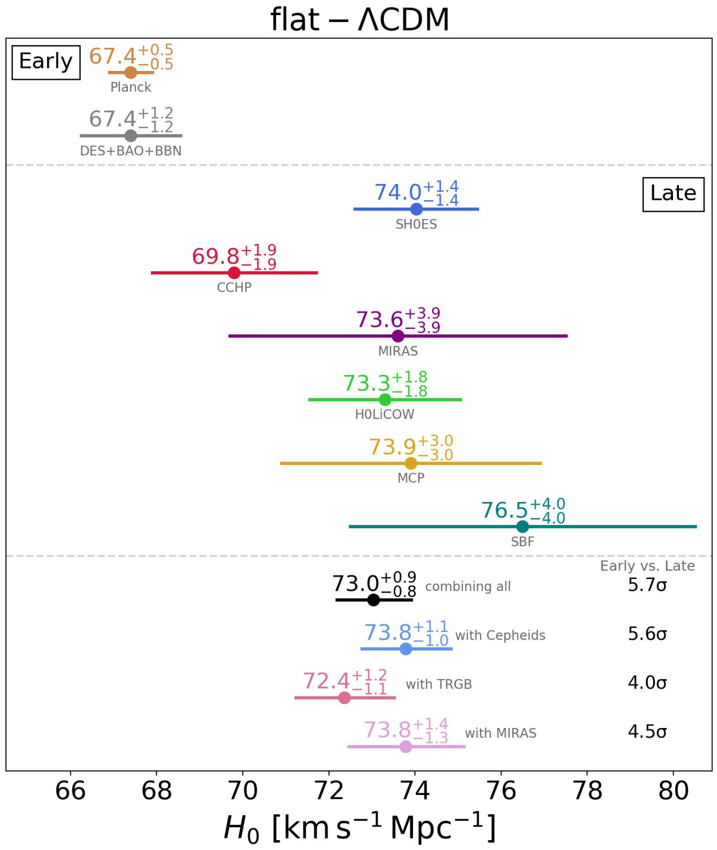
1. Why use gravitational waves for cosmology?
2. A bayesian methodology
3. Dealing with galaxy catalogue incompleteness
4. Dealing with real GW data and galaxy catalogues
5. How population and cosmological parameters intertwine
6. The current state of affairs, and where do we go from here?

Why use gravitational waves for cosmology?

Luminosity distance: directly measurable from the amplitude of the gravitational wave signal.

$$cz \approx H_0 d_L$$ (for low redshift)

Redshift: degenerate with mass in the GW signal. Must be found through other methods



Verde, L., Treu, T. & Riess, A.G. Tensions between the early and late Universe. *Nat Astron* **3,** 891–895 (2019). https://doi.org/10.1038/s41550-019-0902-0

*a simplified view

Cosmology with standard sirens*

Detect GW merger

Provides sky localisation and luminosity distance

Detect EM counterpart?

Yes

Associate with host galaxy and use galaxy's redshift[2]

No

Use galaxy catalogues to get possible host redshifts[1]

Measure H_0

[1] Schutz, B. *Nature* **323,** 310–311 (1986)

[2] The LIGO Scientific Collaboration and The Virgo Collaboration., Abbott, B.,et al. *Nature* **551**, 85–88 (2017)

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E.g. **GW170817 and NGC4993**

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Compensate for galaxy catalogue incompleteness

Measure H_0

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Use galaxy catalogues to get possible host redshifts[1]

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Correct for GW selection effects

Measure H_0

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A Bayesian approach

The posterior on H_0 for N_{det} GW events:

$$p(H_0|\{x_{\mathrm{GW}}\}, \{D_{\mathrm{GW}}\}) \propto p(H_0)p(N_{\mathrm{det}}|H_0) \prod_i^{N_{\mathrm{det}}} p(x_{\mathrm{GW}\,i}|D_{\mathrm{GW}\,i}, H_0)$$

A Bayesian approach

The posterior on H_0 for N_{det} GW events:

The prior on H_0

$$p(H_0|\{x_{\mathrm{GW}}\},\{D_{\mathrm{GW}}\}) \propto \boxed{p(H_0)}\, p(N_{\mathrm{det}}|H_0) \prod_{i}^{N_{\mathrm{det}}} p(x_{\mathrm{GW}i}|D_{\mathrm{GW}i}, H_0)$$

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Rates term
(independent of H0 for a 1/R prior [1])

[1] M. Fishbach et al 2018 *ApJL* **863** L41

A Bayesian approach

The posterior on H_0 for N_{det} GW events:

Contribution from a single GW event

The prior on H_0

$$p(H_0|\{x_{\mathrm{GW}}\},\{D_{\mathrm{GW}}\}) \propto \boxed{p(H_0)}\,\boxed{p(N_{\mathrm{det}}|H_0)}\prod_i^{N_{\mathrm{det}}}\boxed{p(x_{\mathrm{GW}i}|D_{\mathrm{GW}i},H_0)}$$

Rates term
(independent of H0 for a 1/R prior [1])

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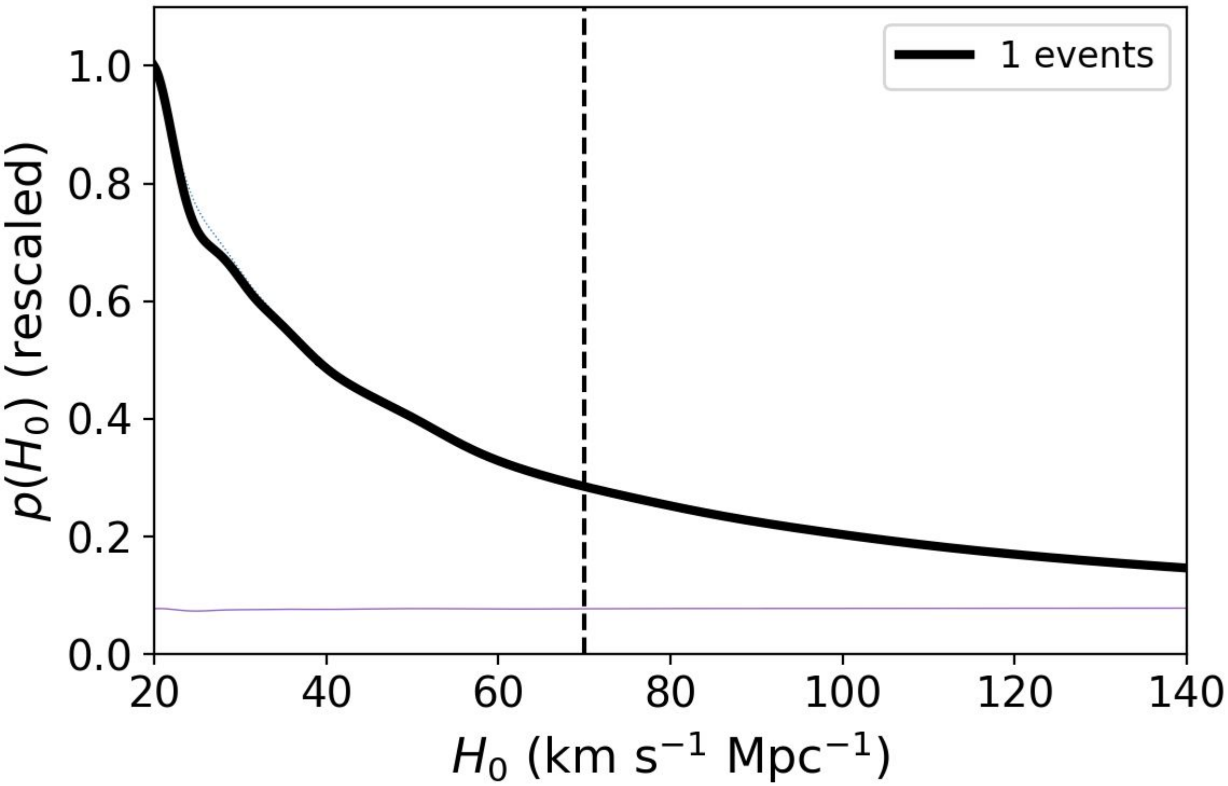
Combining multiple events to measure H_0



A Bayesian approach

Contribution from one event:

$$p(x_{\mathrm{GW}}|D_{\mathrm{GW}}, H_0) = \frac{p(D_{\mathrm{GW}}|x_{\mathrm{GW}}, H_0)p(x_{\mathrm{GW}}|H_0)}{p(D_{\mathrm{GW}}|H_0)}$$

A Bayesian approach

Contribution from one event:

= 1 (by definition)

$$p(x_{\rm GW}|D_{\rm GW}, H_0) = \frac{\cancel{p(D_{\rm GW}|x_{\rm GW}, H_0)}\,p(x_{\rm GW}|H_0)}{p(D_{\rm GW}|H_0)}$$

A Bayesian approach

Contribution from one event:

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GW selection effects

GW selection effects

Our detectors aren't infinitely sensitive. We are biased towards detecting "louder" events.

$$p(D_{\rm GW}|H_0) = \int_{\rho>\rho_{\rm th}}^{\infty} p(x_{\rm GW}|H_0) dx_{\rm GW}$$

What fraction of events from the GW population pass the threshold for detection?

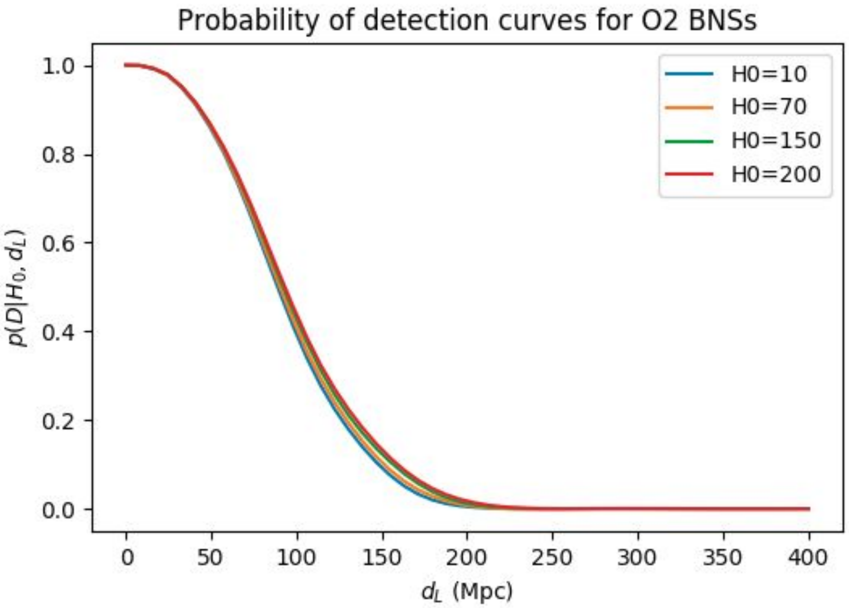


Fig: Probability of detection as a function of luminosity distance for a population of binary neutron stars with O2-like sensitivity.

Using a galaxy catalogue to provide redshift information

$$p(H_0|\{x_{\mathrm{GW}}\},\{D_{\mathrm{GW}}\},I) \propto p(H_0|I)p(N_{\mathrm{det}}|H_0,I)\prod_i^{N_{\mathrm{det}}} \underbrace{p(x_{\mathrm{GW}i}|D_{\mathrm{GW}i},H_0,I)}$$

Contribution from a single event:

$$p(x_{\mathrm{GW}}|G,D_{\mathrm{GW}},s,H_0,I) = \frac{\sum_{i=1}^{N} p(x_{\mathrm{GW}}|z_i,\Omega_i,s,H_0,I)p(s|z_i,I)p(s|M(z_i,m_i,H_0),I)}{\sum_{i=1}^{N} p(D_{\mathrm{GW}}|z_i,\Omega_i,s,H_0,I)p(s|z_i,I)p(s|M(z_i,m_i,H_0),I)}$$

Using a galaxy catalogue to provide redshift information

Sum over N galaxies in the catalogue

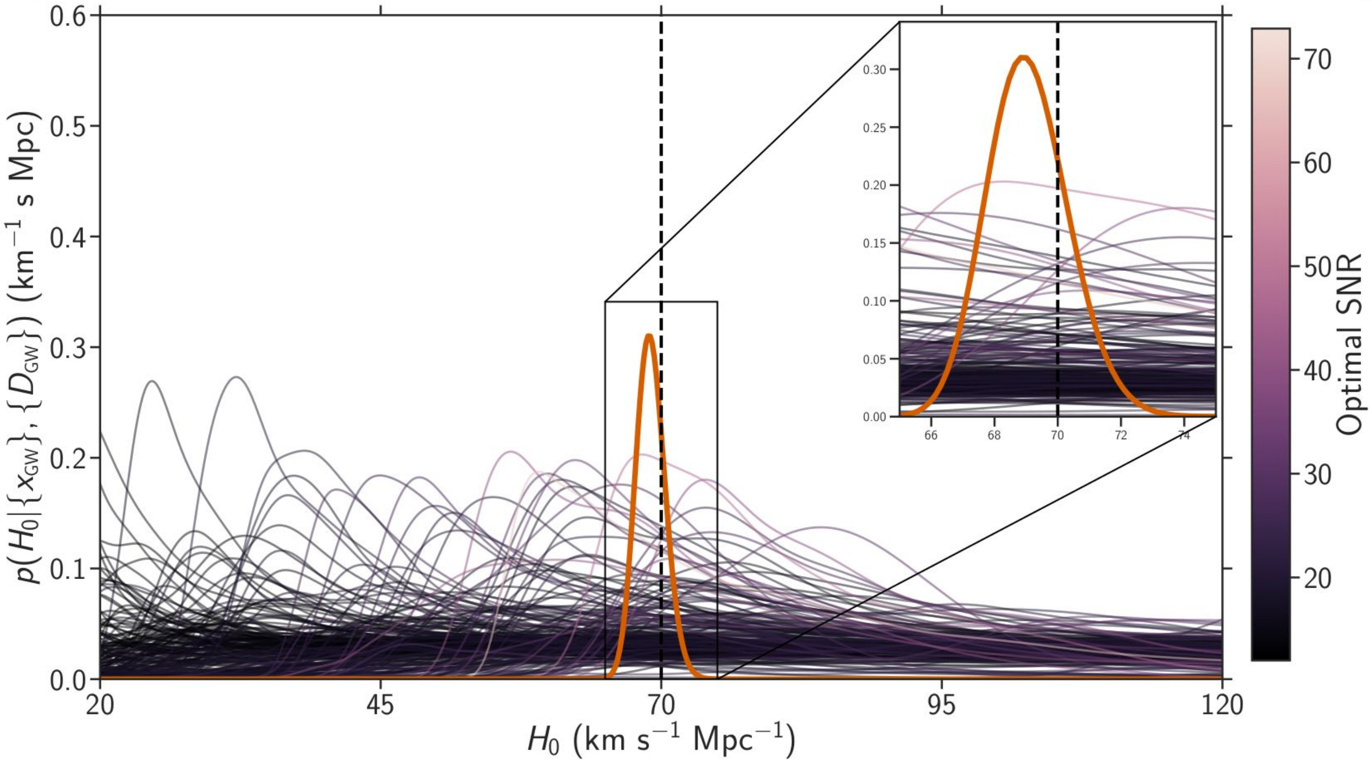
GW likelihood evaluated at redshift and sky location of galaxy *i*

GW merger redshift dependence

Luminosity weighting

$$p(x_{\rm GW}|G, D_{\rm GW}, s, H_0, I) = \frac{\boxed{\sum_{i=1}^{N}} \boxed{p(x_{\rm GW}|z_i, \Omega_i, s, H_0, I)} \boxed{p(s|z_i, I)} \boxed{p(s|M(z_i, m_i, H_0), I)}}{\sum_{i=1}^{N} p(D_{\rm GW}|z_i, \Omega_i, s, H_0, I) p(s|z_i, I) p(s|M(z_i, m_i, H_0), I)}$$

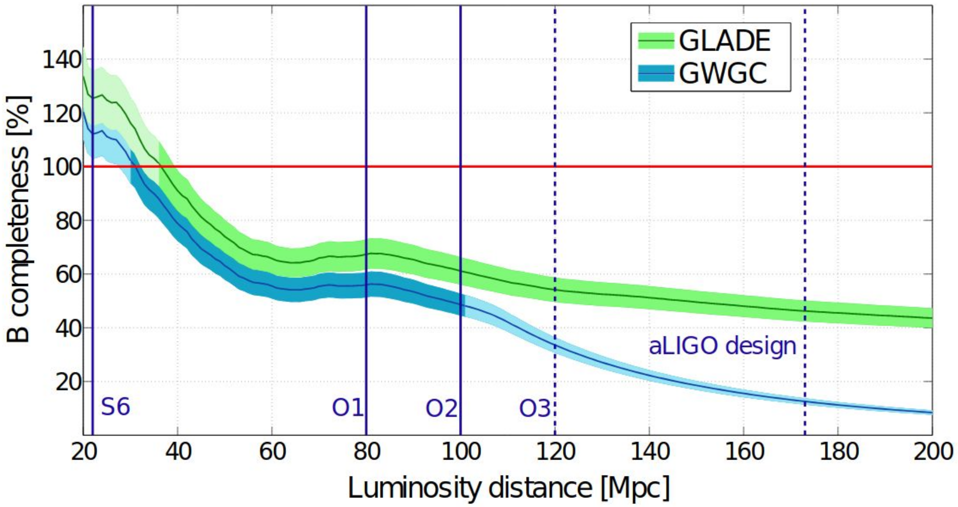
An example with 250 (mock) binary neutron stars



"Cosmological inference using gravitational wave standard sirens: A mock data analysis"

R. Gray *et al.* Phys. Rev. D **101**, 122001 – Published 8 June 2020

What if the host galaxy is not in the catalogue?



Galaxy surveys are flux limited: galaxies with (apparent) magnitudes fainter than some threshold won't be in the catalogue.

How do you compensate for this?

GLADE: A Galaxy Catalogue for Multi-Messenger Searches in the Advanced Gravitational-Wave Detector Era
G. Dálya *et al.* *MNRAS* **479** 2 (2018) 2374–2381

What if the host galaxy is not in the catalogue?

$$p(x_{\mathrm{GW}}|D_{\mathrm{GW}}, H_0) = \sum_{g=G,\bar{G}} p(x_{\mathrm{GW}}|g, D_{\mathrm{GW}}, H_0)p(g|D_{\mathrm{GW}}, H_0)$$

$$= p(x_{\mathrm{GW}}|G, D_{\mathrm{GW}}, H_0)p(G|D_{\mathrm{GW}}, H_0) + p(x_{\mathrm{GW}}|\bar{G}, D_{\mathrm{GW}}, H_0)p(\bar{G}|D_{\mathrm{GW}}, H_0)$$

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In catalogue

Not in catalogue

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$$= \boxed{p(x_{\mathrm{GW}}|G, D_{\mathrm{GW}}, H_0)}p(G|D_{\mathrm{GW}}, H_0) + p(x_{\mathrm{GW}}|\bar{G}, D_{\mathrm{GW}}, H_0)p(\bar{G}|D_{\mathrm{GW}}, H_0)$$

GW likelihood assuming the host galaxy is inside the galaxy catalogue

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$$p(x_{\mathrm{GW}}|D_{\mathrm{GW}}, H_0) = \sum_{g=G,\bar{G}} p(x_{\mathrm{GW}}|g, D_{\mathrm{GW}}, H_0)p(g|D_{\mathrm{GW}}, H_0)$$

$$= p(x_{\mathrm{GW}}|G, D_{\mathrm{GW}}, H_0)\boxed{p(G|D_{\mathrm{GW}}, H_0)} + p(x_{\mathrm{GW}}|\bar{G}, D_{\mathrm{GW}}, H_0)\boxed{p(\bar{G}|D_{\mathrm{GW}}, H_0)}$$

Probability that the host galaxy is (or is not) inside the galaxy catalogue

(Sums to 1)

R. Gray *et al.* Phys. Rev. D **101**, 122001 – Published 8 June 2020

What if the host galaxy is not in the catalogue?

$$p(x_{\mathrm{GW}}|D_{\mathrm{GW}},H_0) = \sum_{g=G,\bar{G}} p(x_{\mathrm{GW}}|g,D_{\mathrm{GW}},H_0)p(g|D_{\mathrm{GW}},H_0)$$

$$= p(x_{\mathrm{GW}}|G,D_{\mathrm{GW}},H_0)p(G|D_{\mathrm{GW}},H_0) + \boxed{p(x_{\mathrm{GW}}|\bar{G},D_{\mathrm{GW}},H_0)}p(\bar{G}|D_{\mathrm{GW}},H_0)$$

GW likelihood assuming the host galaxy is outside the galaxy catalogue

R. Gray *et al.* Phys. Rev. D **101**, 122001 – Published 8 June 2020

What if the host galaxy is not in the catalogue?

$$p(x_{\rm GW}|L \quad\quad , H_0)$$

$$= p(x_{\rm GW}|G, D \quad\quad p(\bar{G}|D_{\rm GW}, H_0)$$

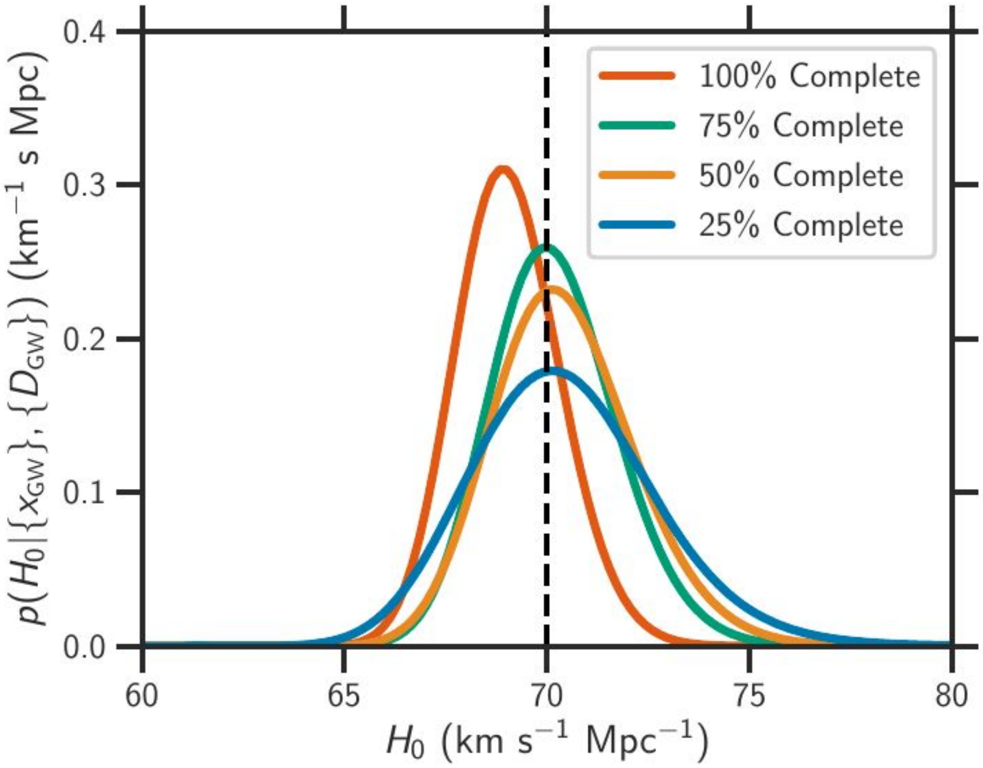
To model incompleteness need to make a few assumptions:

1. **Apparent magnitude threshold of galaxy catalogue**
2. **Assume general distribution of galaxies in the universe (e.g. uniform in comoving volume)**
3. **Assume the luminosity distribution of said galaxies (e.g. Schechter function)**

outside the

R. Gray *et al.* Phys. Rev. D **101**, 122001 – Published 8 June 2020

The impact of incompleteness



"Cosmological inference using gravitational wave standard sirens: A mock data analysis"
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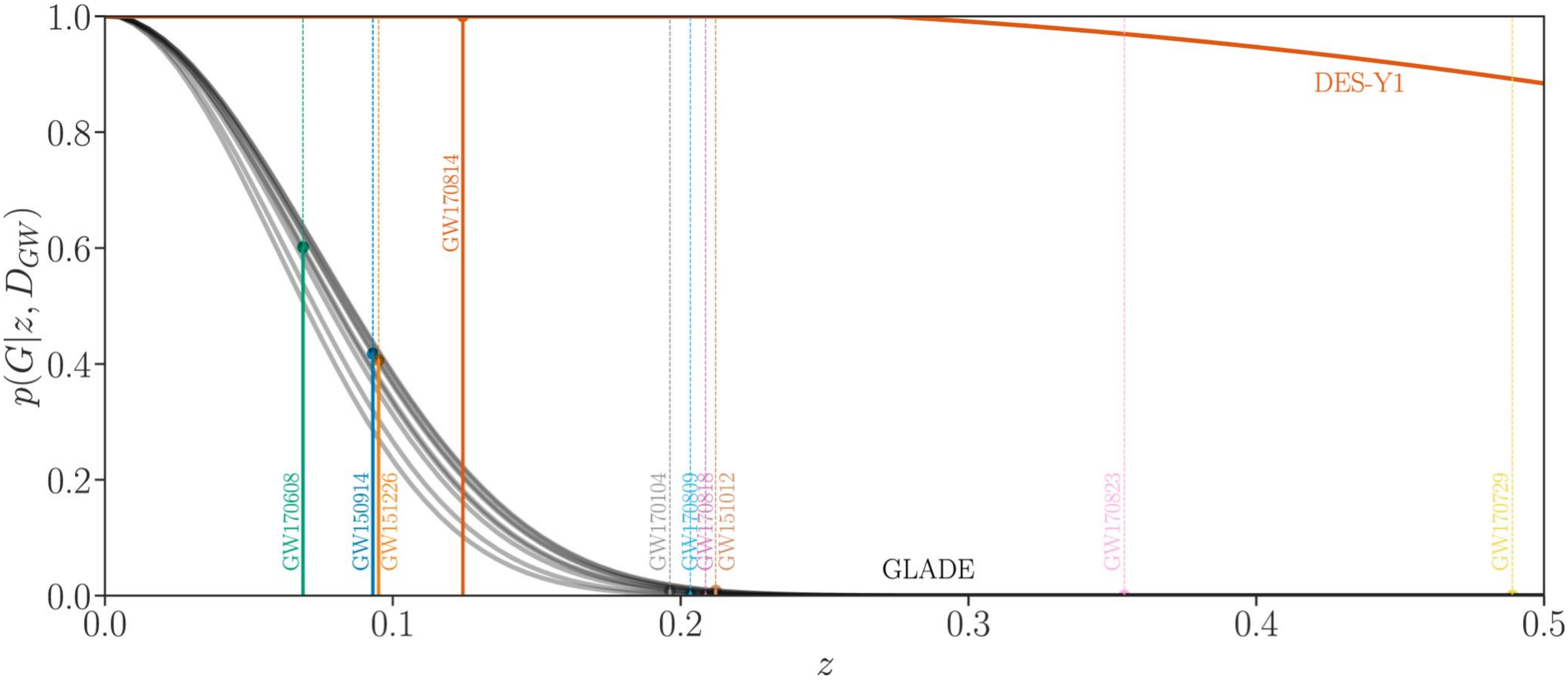
Measuring the Hubble constant with real data

For O1 and O2 there were:

- One binary neutron star detection [1]
    - Plus one electromagnetic counterpart [2]
- Ten binary black hole detections [3]
    - One all-sky galaxy catalogue (GLADE) [4]
    - One highly complete but partial sky catalogue (DES) [5]

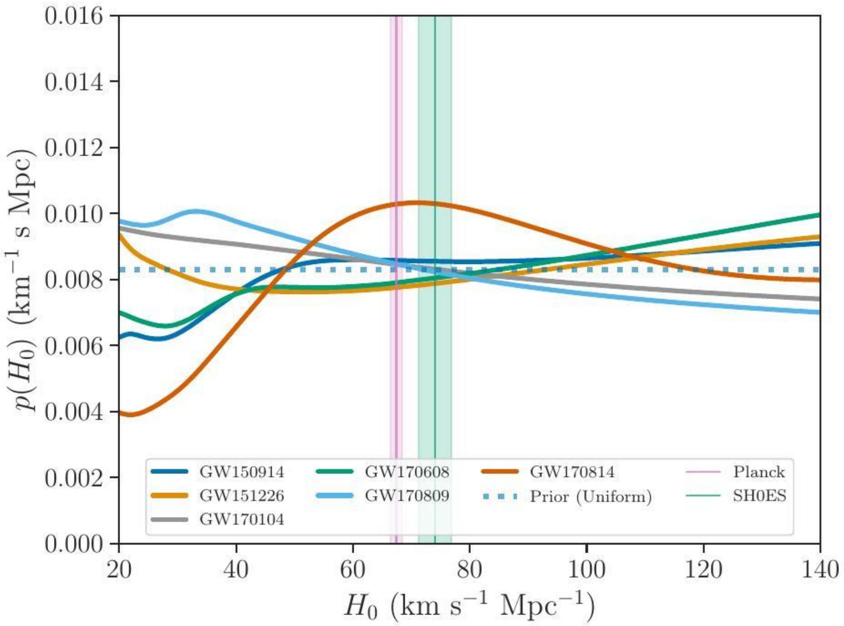
[1] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) *Phys. Rev. Lett.* **119**, 161101 – Published 16 October 2017
[2] B. P. Abbott et al 2017 *ApJL* **848** L12
[3] B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), *Phys. Rev. X* **9**, 031040 – Published 4 September 2019
[4] G. Dálya *et al. Monthly Notices of the Royal Astronomical Society*, Volume 479, Issue 2, September 2018, Pages 2374–2381
[5] A. Drlica-Wagner *et al.* 2018 *ApJS* **235** 33

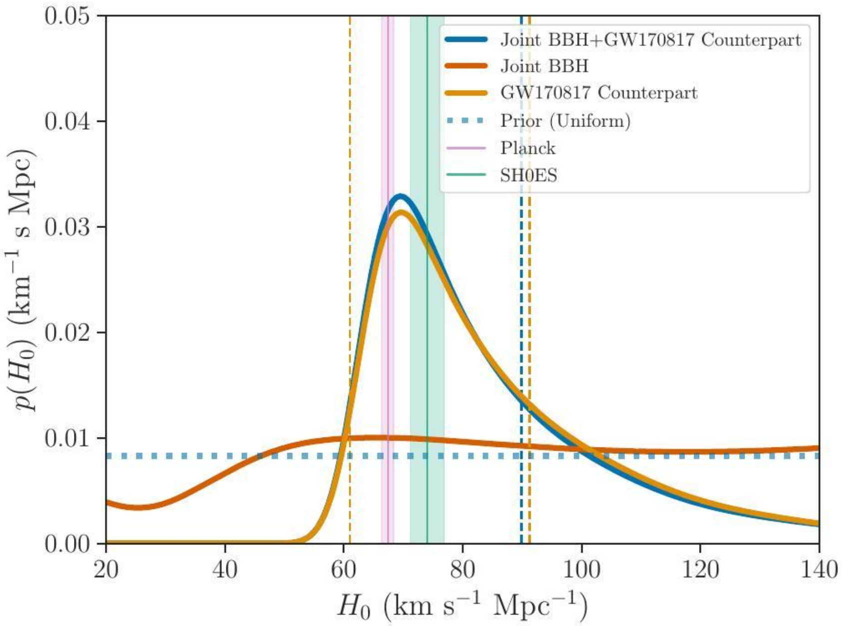
Probability that the host galaxy is in the catalogue



B. P. Abbott *et al.* A Gravitational-wave Measurement of the Hubble Constant Following the Second Observing Run of Advanced LIGO and Virgo. 2021 *ApJ* **909** 218. Figure 1. https://doi.org/10.3847/1538-4357/abdcb7

Constraining H_0 with 10 BBHs, 1 BNS, an EM counterpart and two galaxy catalogues

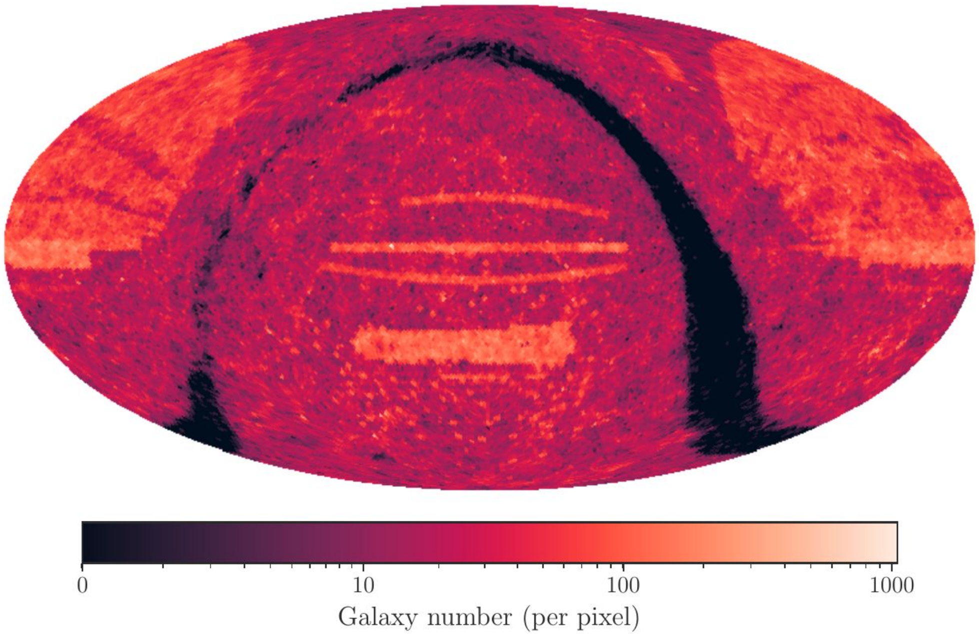




B. P. Abbott *et al.* A Gravitational-wave Measurement of the Hubble Constant Following the Second Observing Run of Advanced LIGO and Virgo. 2021 *ApJ* **909** 218. Figure 3 (left) and figure 4 (right). https://doi.org/10.3847/1538-4357/abdcb7

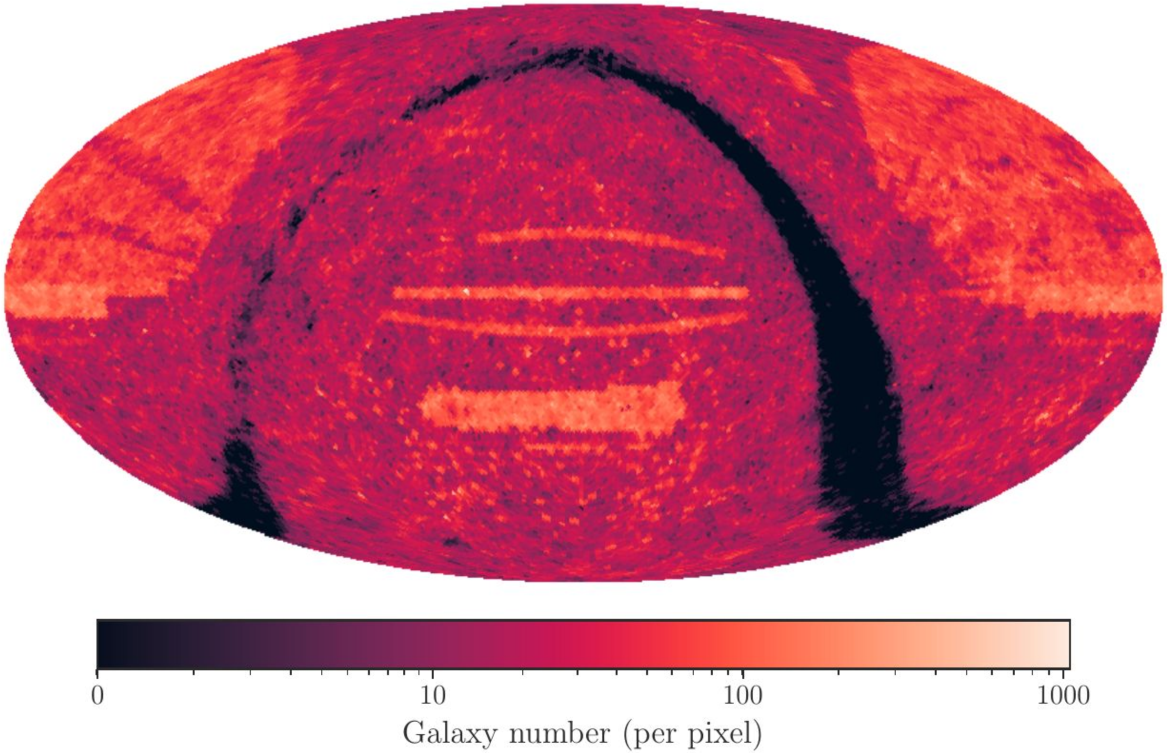
The problem with real galaxy catalogues

- Large redshift uncertainties.
- Unreliable redshift measurements at high redshift.
- Galaxy luminosities are redshifted from source frame.
- Non-uniform galaxy catalogue completeness across the sky.

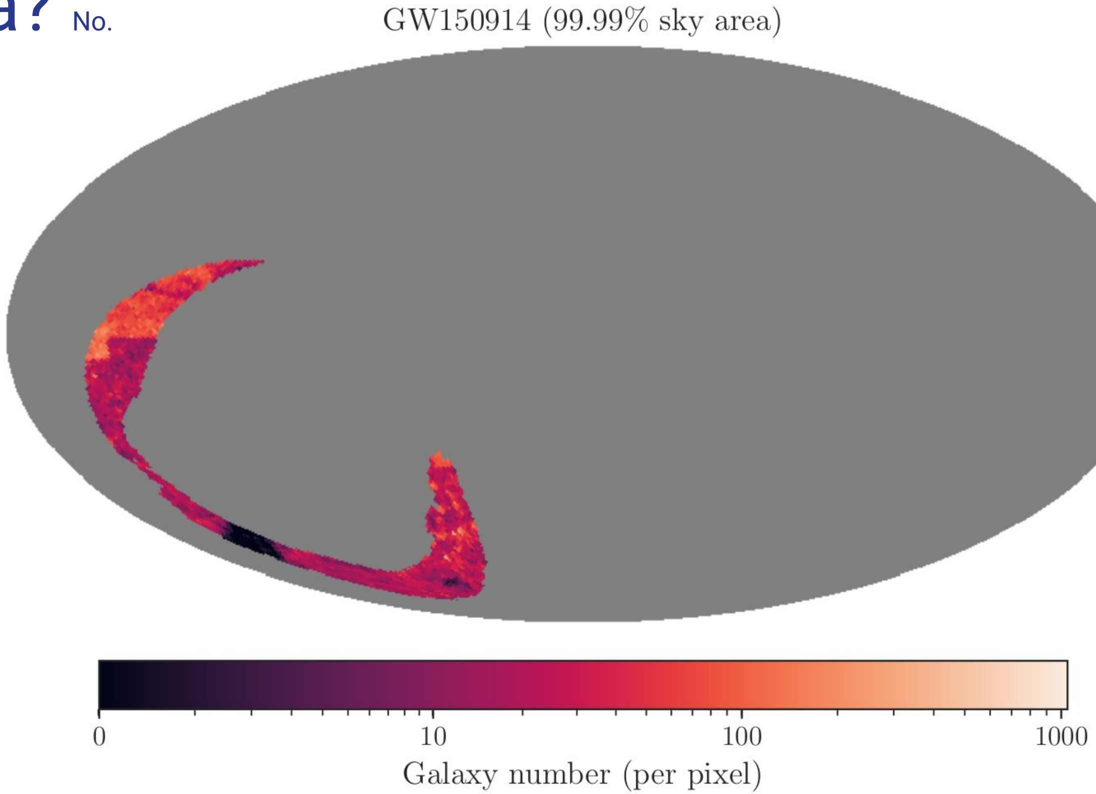


“A Pixelated Approach to Galaxy Catalogue Incompleteness: Improving the Dark Siren Measurement of the Hubble Constant”
R. Gray, C. Messenger, J. Veitch. e-Print: 2111.04629 [astro-ph.CO]

Is assuming uniform m_{th} in the sky area of an event a good idea?



Is assuming uniform m_{th} in the sky area of an event a good idea? No.



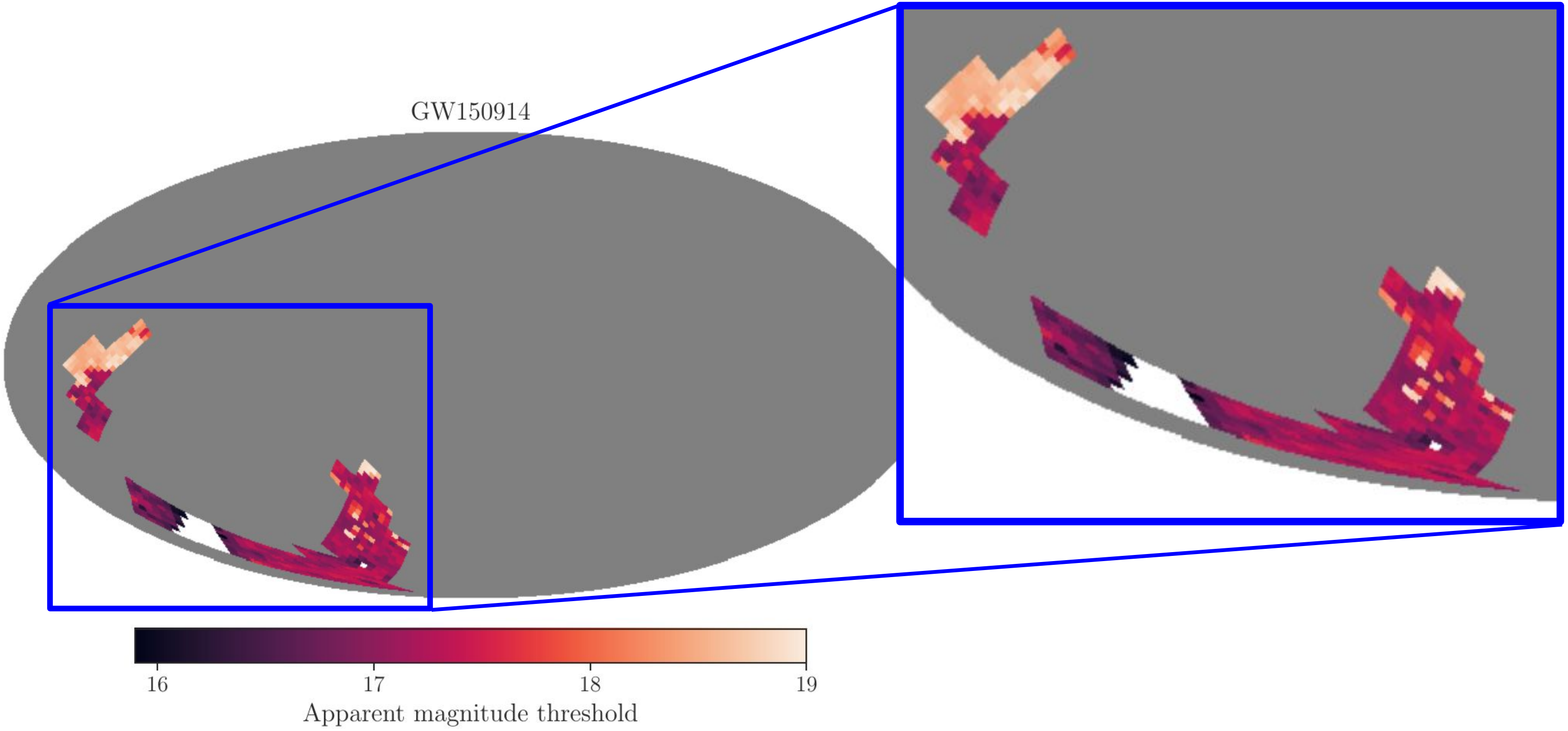
Pixelating the analysis

$$p(x_{\mathrm{GW}}|D_{\mathrm{GW}}, H_0, I) = \sum_i^{N_{\mathrm{pix}}} p(x_{\mathrm{GW}}|\Omega_i, D_{\mathrm{GW}}, H_0, I)\, p(\Omega_i|I)$$

$$= \frac{1}{N_{\mathrm{pix}}} \sum_i^{N_{\mathrm{pix}}} \underbrace{p(x_{\mathrm{GW}}|\Omega_i, D_{\mathrm{GW}}, H_0, I),}$$

Can now estimate the completeness of the catalogue *per pixel*

How does m_{th} vary in the sky area of an event?



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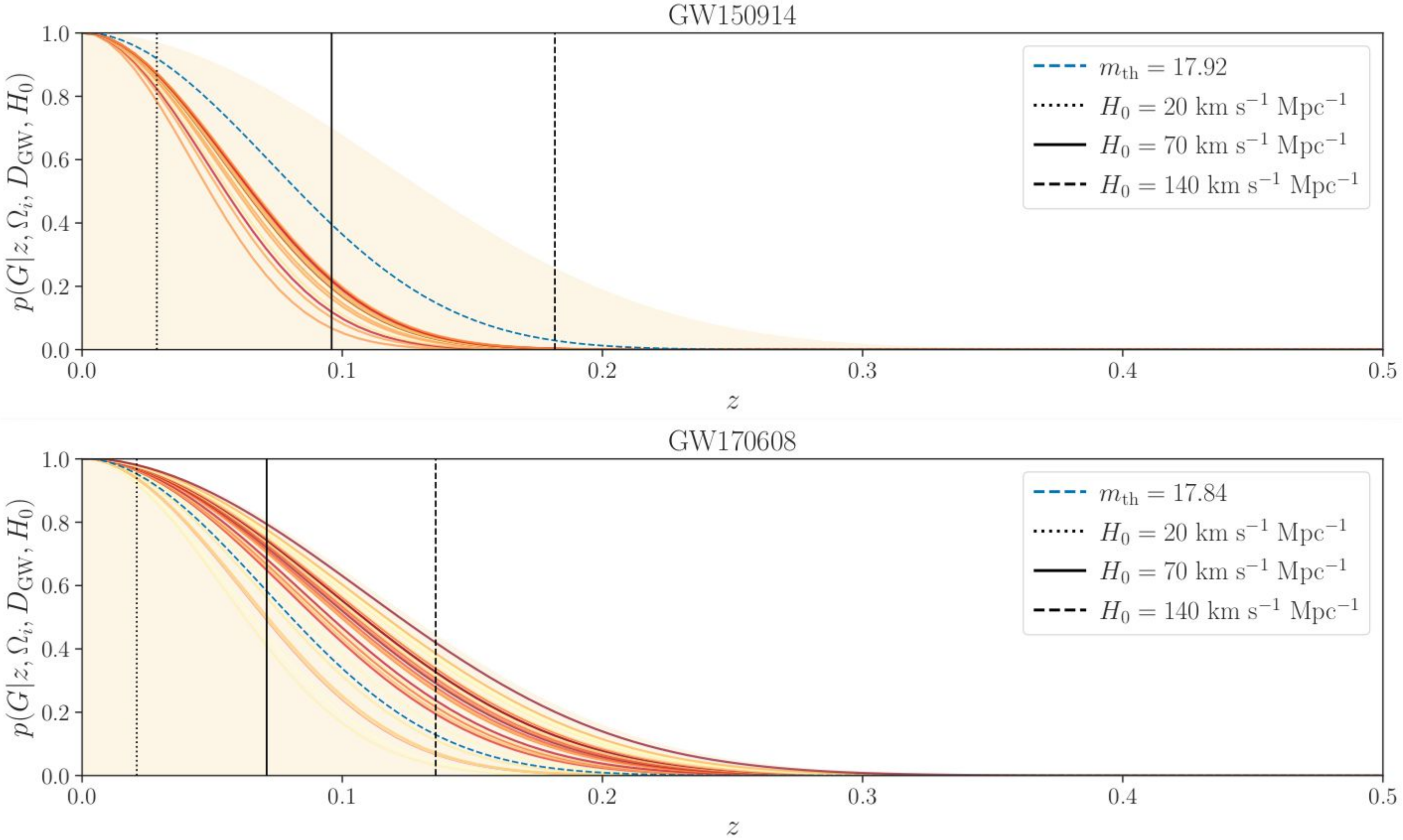

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Impact of pixelation on a single event

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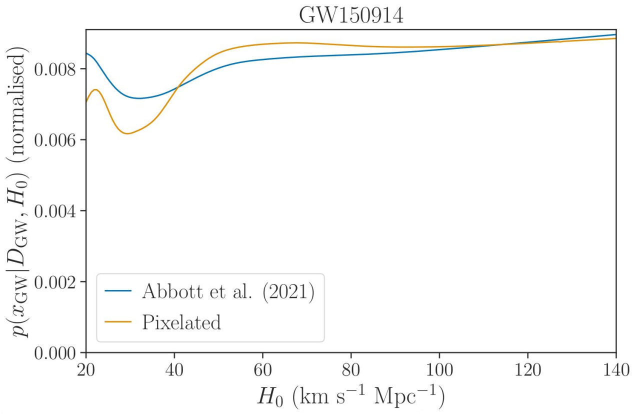
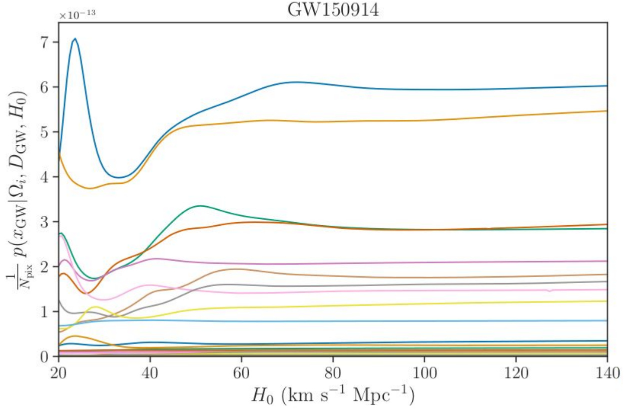
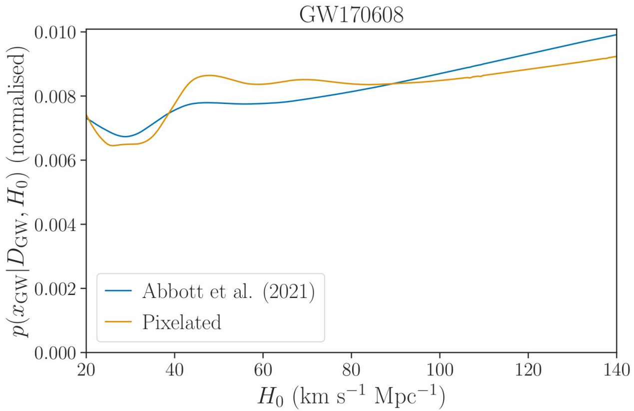
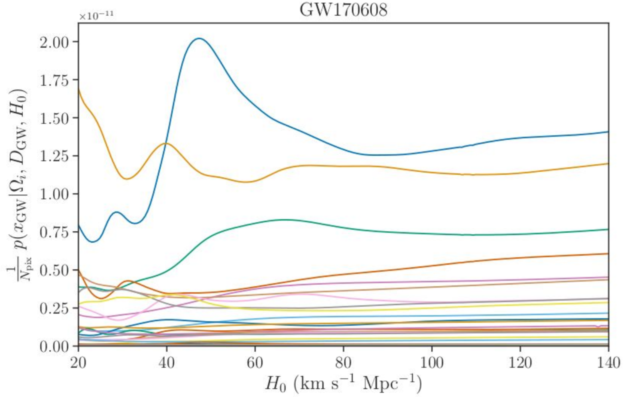
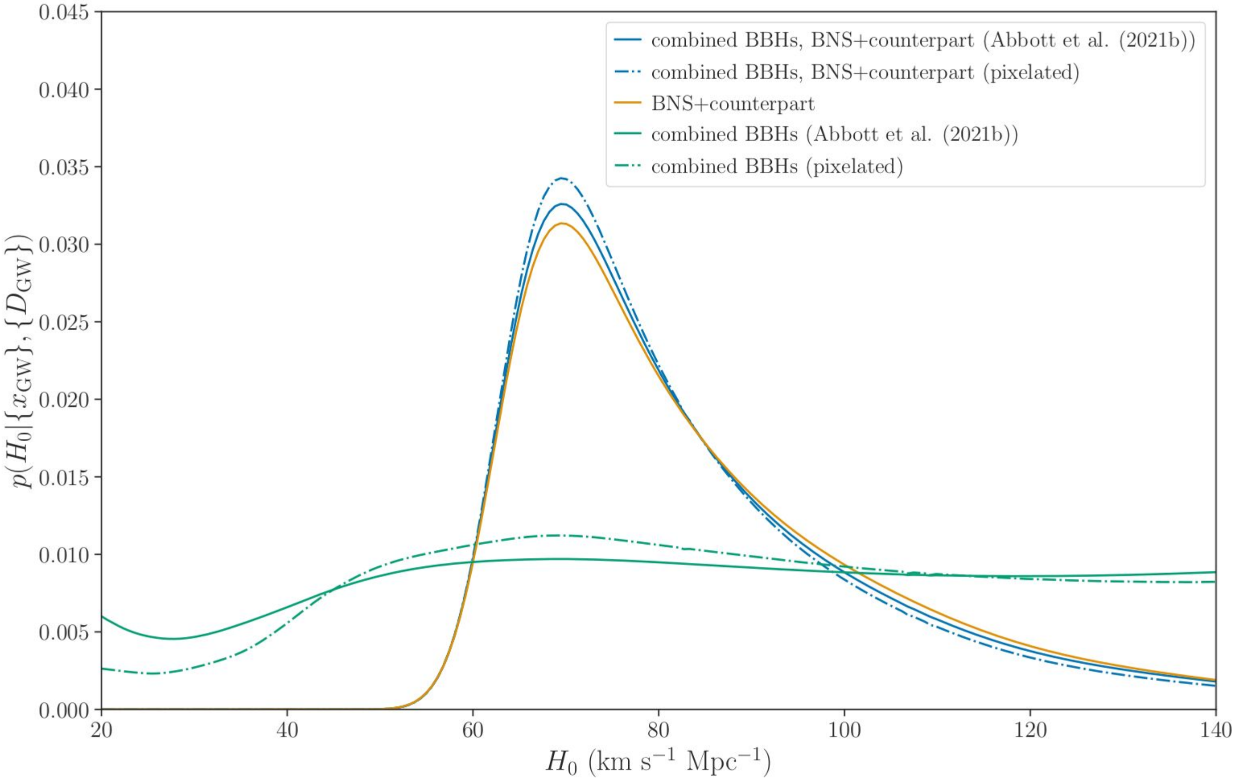








How much does pixelation improve things?



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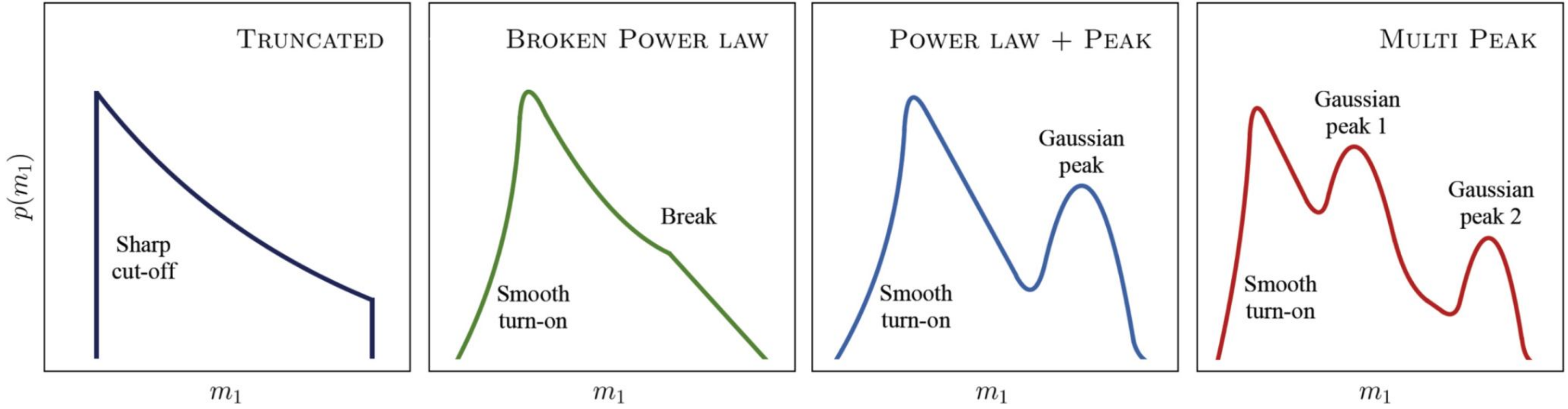
R. Gray, C. Messenger, J. Veitch. e-Print: 2111.04629 [astro-ph.CO]

What is the mass distribution of the BBH population?

We measure detected (redshifted) GW mass, not source-frame mass:

$$\mathcal{M}_z \equiv \mathcal{M}(1+z)$$

Low H_0 → low redshift → higher source frame mass → inconsistency with mass prior?



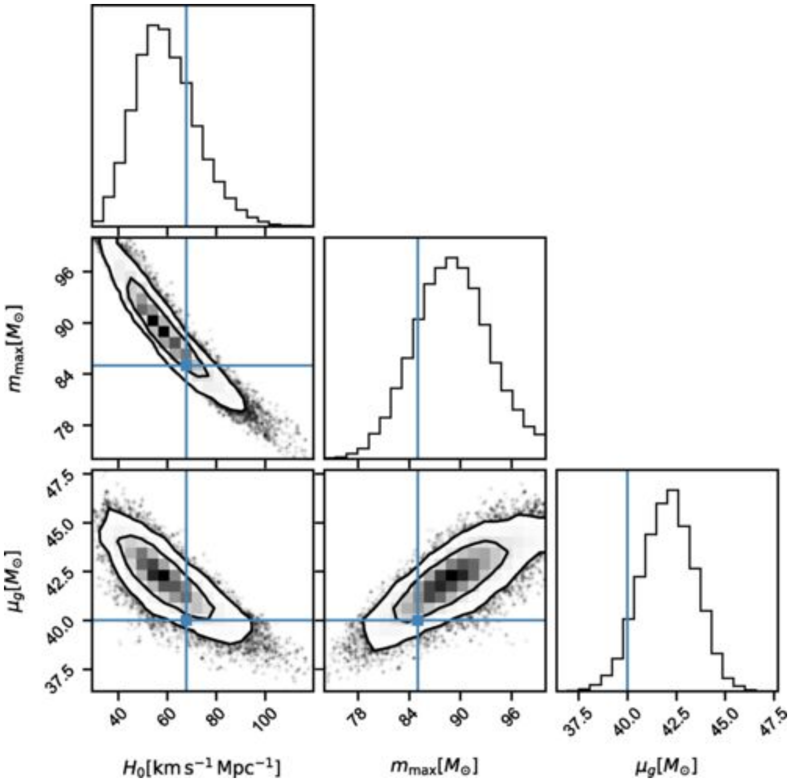
Population Properties of Compact Objects from the Second LIGO–Virgo Gravitational-Wave Transient Catalog
R. Abbott *et al* 2021 *ApJL* **913** L7. https://doi.org/10.3847/2041-8213/abe949

Correlation between population and cosmological parameters

64 (simulated) BBH detections with O2/O3-like sensitivity (assuming power-law + peak mass distribution).

Sharp features in the mass distribution are correlated with H_0: can measure H_0 with BBH detections alone! (See also, *e.g.* [1]).

[1] Will M. Farr *et al* 2019 *ApJL* **883** L42



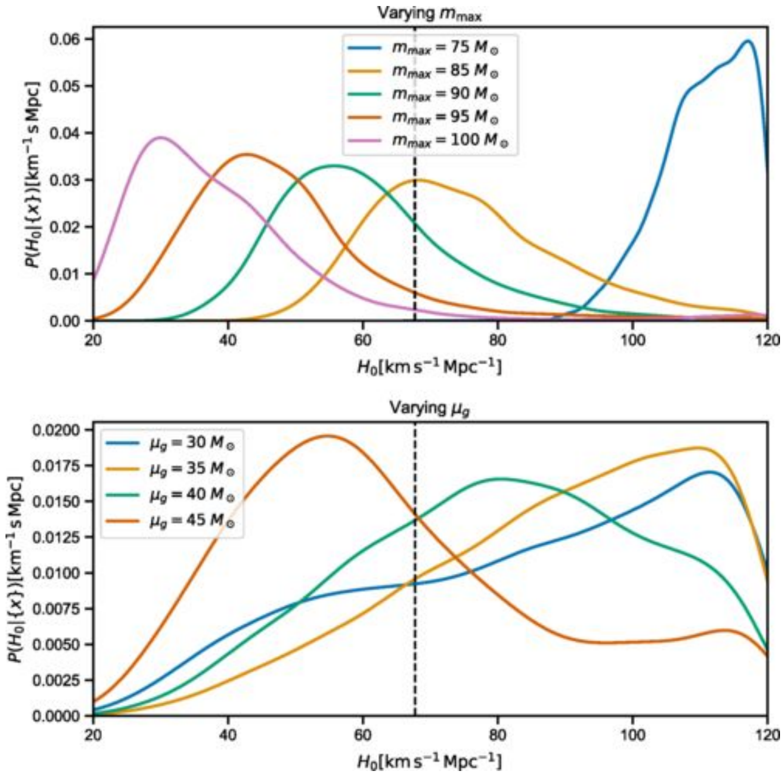
S. Mastrogiovanni et al. *Phys. Rev. D* **104**, 062009, Sept 2021

The impact of wrongly assuming a population prior

The impact of mis-assuming the population mass prior is to bias the estimate of H_0.

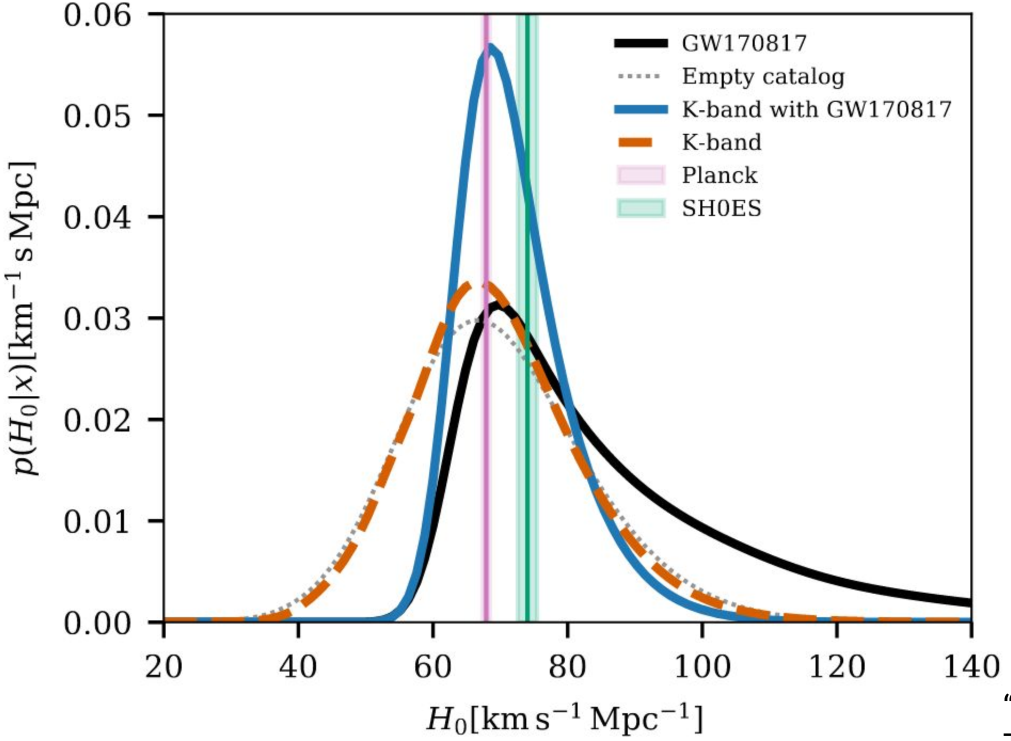
True values: m_{max} = 85 M_{sun}, μ_g = 40 M_{sun}

This is a potential problem for the galaxy catalogue analysis: current analyses assume a fixed mass distribution.



S. Mastrogiovanni et al. *Phys. Rev. D* **104**, 062009, Sept 2021

Cosmology results from GWTC-3



Results from 47 compact binaries (42 BBH, 2 NSBH, 2 BNS, 1 asymmetric mass binary) with SNRs > 11, with the GLADE+ galaxy catalogue. The population mass distributions are fixed.

“Constraints on the cosmic expansion history from GWTC-3”, The LIGO Scientific, Virgo and KAGRA collaborations: arXiv:2111.03604 (public as of last week)

Challenges that remain*

* a non-exhaustive list

Galaxy catalogues:

- K corrections and redshift estimates that are valid at high redshifts
- Finding a suitable luminosity function (also evolves with redshift)
- And more…

Gravitational waves:

- Incorporating the uncertainty in population hyperparameters into cosmological parameter estimation with galaxy catalogues

When will we have an answer?

So when will we measure the Hubble constant (to a high enough precision to comment meaningfully on the Hubble constant tension)?

It depends.

- More GW detectors coming online, and improved detector sensitivity → more detections and better localised events.
- Deeper galaxy surveys become available → more informative contributions from individual events.
- Depending on the quality of events, need $\mathcal{O}(100)$ - $\mathcal{O}(1000)$ detections.

Either way, the next 5 years should be interesting…

Conclusions

- A lot of progress has been made using GW dark sirens for cosmological measurements, in terms of handling galaxy catalogue incompleteness and modelling GW selection effects.
- However a lot of challenges remain, both in our understanding of galaxy catalogues and the GW population.

Further reading on the dark siren/galaxy catalogue method (also non-exhaustive):

- Walter Del Pozzo, *Phys. Rev. D* **86**, 043011 (2012)
- Chen et al., *Nature* volume **562**, pages 545–547 (2018)
- M. Fishbach et al 2019 *ApJL* **871** L13
- M. Soares-Santos et al 2019 *ApJL* **876** L7
- A. Palmese et al 2020 *ApJL* **900** L33
- Vasylyev and Filippenko 2020 *ApJ* **902** 149
- Finke et al JCAP08(2021)026

Thanks to the LIGO Scientific, Virgo and KAGRA collaborations. This material is based upon work supported by NSF's LIGO Laboratory which is a major facility fully funded by the National Science Foundation.