Waveform systematics in the gravitational-wave inference of tidal parameters from binary neutron star signals

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Outline

Previous talks:

- Q: Why should we care about neutron stars? A: because they are cool (pun intended)
- Q: What can we do with GW data from binary neutron star signals? A: EOS, Cosmology, ...

In the next ~40 minutes:

- Q1: How can we model the GW signal from BNS?
- Q2: How do modelling differences affect GW parameter estimation?
1. Modelling of GWs from binary Neutron star systems

\(m_1, m_2, S_1, S_2, \Lambda_1, \Lambda_2, \ldots\)

**INPUT**

parameters of the source system

**APPROXIMANT**

**OUTPUT**

\(h_+, h_\times\) plus and cross polarizations
Phenomenology of a merger

- **Inspiral** (≥ mins)
- **Merger** (ms)
- **Early postmerger** (~ 10 ms)
- **Viscous postmerger** (≥ 1s)
- **Secular postmerger** (≥ days)

- **Matter ejection**
- **Spin-down**
- **Neutron star**
- **Outflows**
- **Jet**
- **ISM**

Image Credits: Matteo Breschi

**Gravitational radiation**

**Electromagnetic radiation**
Phenomenology of a merger

THIS TALK

NOT DISCUSSED
Phenomenology of a merger

Inspiral up to merger: “similar” to a BBH waveform for slowly spinning bodies and $q \sim 1$ [See Geraint’s talk] → We can model this as BBH + corrections! Should be easy right?

From: Interpreting binary neutron star mergers: describing the binary neutron star dynamics, modelling gravitational waveforms, and analyzing detections [Dietrich+2021]
Matter effects

*Matter effects* are what distinguish NS from point particles (=black holes, BH)

For GW modelling of BNS, the more important ones are:

- **Tidal effects**

- **Spin induced effects** [Poisson1998, Krishnendu+2017]

Additional effects due to resonant modes of neutron stars can be considered (but will not be discussed here)
Adiabatic tides

- When a NS is subject to the gravitational field of another object, it gets deformed → Tidal effects
- Proportionality constant between external field and quadrupolar deformation → tidal parameter

\[
Q_{ij} = -\lambda E_{ij}
\]

\[
\lambda_i = \frac{\text{Quadrupole deformation of the star}}{\text{External tidal field}} \rightarrow \Lambda_i = \frac{\lambda_i}{m_i^5}
\]

\[
\rho(p) + \frac{1-g_{tt}}{2} = -\frac{m}{r} - \frac{3Q_{ij}}{2r^3} \left( n^i n^j - \frac{1}{3} \delta^{ij} \right) + O \left( r^{-3} \right) + \frac{1}{2} \varepsilon_{ij} x^i x^j + O \left( r^3 \right)
\]

Perturbed static spherically symm. metric

\[
\lambda_2
\]

Tidal deformability
Adiabatic tides

Generalization to higher multipoles (Otcupole, Hexadecupole, …)

There exist quasi-universal (=EOS independent) relations between the quadrupolar tidal parameter and the “higher order” ones \( [\text{Yagi+2016, Carson+2019, Godzieba+2021}] \)
Dynamical tides

- **Dynamical Tidal effects (f-mode resonance):**

\[
L_{DT} = \frac{1}{4\lambda\omega_f^2} \left[ \dot{Q}^{ij} Q^{ij} - \omega_f^2 Q^{ij} Q^{ij} \right] - \frac{1}{2} E_{ij} Q^{ij},
\]

Harmonic Osc.

- **“Dressing factor” for love numbers:**

\[
k_{\ell} \rightarrow k_{\ell}^{\text{eff}} := \alpha_{\ell m}(\nu, \Omega, \bar{\omega}_f^{(\ell)}, X_A) k_{\ell}
\]

\[
\alpha_{\ell m} = a_\ell + b_\ell \left\{ \frac{x^2}{x^2 - 1} + \frac{5}{6} \frac{x^2}{1 - x^{5/3}} + \frac{x^2}{\sqrt{\epsilon}} \left[ \cos \left( \Omega' \tilde{t}^2 \right) \int_{-\infty}^{\tilde{t}} \sin \left( \Omega' s^2 \right) ds - \sin \left( \Omega' \tilde{t}^2 \right) \int_{-\infty}^{\tilde{t}} \cos \left( \Omega' s^2 \right) ds \right] \right\}
\]

Resonant part

Fresnel Part
Dynamical tides

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\]

Resonant part \quad \text{Fresnel Part}
Spin-induced effects

- The spinning motion of companion A creates a distortion in its mass distribution
- In turn, this distorts the gravitational field outside the star
- Clearly, this impacts the orbital motion and the emission of GWs

\[ Q_A \approx -a \chi A^2 m_A^3 \]

EOS dependent coefficient, can be related to \( \Lambda \)
“State of the art” BNS Waveform models

Current “state of the art” models include most of the effects previously discussed.

Three families:

- **Post Newtonian approximants (PN)** \([Krishnendu+2017, Henry+2020, Schmidt+2021]\)
  - Analytical
  - Fast!
  - **Examples**: TaylorF2, TaylorT4

- **Effective One Body approximants (EOB)** \([Bini+2012, Akcay+2018, Lackey+2018]\)
  - Semi-analytical, resummed PN + NR
  - Not-as-fast, generally
  - **Examples**: TEOBResumS, SEOBNRv4T (& related surrogate)

- **Phenomenological approximants (Phenom)** \([Dietrich+2017, Kawaguchi+2018, Dietrich+2019]\)
  - Fits to PN+EOB+NR
  - Fast
  - **Examples**: (any BBH inspiral model) + NRTidal, NRTidalv2, Kawaguchi+ model
PN Waveform models (TaylorF2)

\[ \tilde{h}^{\text{spa}}(f) = \frac{a(t_f)}{\sqrt{\dot{F}(t_f)}} e^{i[\psi_f(t_f) - \pi/4]}, \quad \psi_f(t) \equiv 2\pi ft - 2\phi(t) \]

PN phase: orbital + tides + spin + quadrupole-monopole

\[ \Psi(f) = \Psi_O + \Psi_\Lambda + \Psi_S + \Psi_{MQ}. \]

Matter contributions

\[ \Delta\Psi(f) = \Psi_{BNS}(f) - \Psi_{BBH}(f) \approx \Psi_\Lambda + \Psi_{MQ} \]

\[ \Psi_{MQ} = \frac{3}{128\nu} c_{\nu}^{\text{MQ}} x^{-1/2} \left( 1 + c_1^{\text{MQ,NLO}} x + c_3^{\text{MQ,tail}} x^{3/2} \right) \]

\[ \Psi_\Lambda = c_{\nu}^{\Lambda} x^{5/2} \left( 1 + c_1^{\Lambda} x + c_3^{\Lambda} x^{3/2} + c_2^{\Lambda} x^2 + c_5^{\Lambda} x^{5/2} \right) \]

Quadrupole-Monopole (or spin-spin)

Tides
EOB Waveform models

Three ingredients:

- **Hamiltonian**

\[ H_{EOB} = M \sqrt{1 + 2\nu(\hat{H}_{\text{eff}} - 1)}, \]

\[ \hat{H}_{\text{eff}} = \sqrt{p_{r*}^2 + A(r) \left( 1 + \frac{p_{r*}^2}{r^2} + 2\nu(4 - 3\nu)\frac{p_{r*}^4}{r^2} \right)} \]

- **Waveform**

\[ h_{\ell m} = h_{\ell m}^{(N_{\text{e}})} S_{\text{eff}} \hat{h}_{\ell m}^\text{tail} f_{\ell m} \hat{h}_{\ell m}^\text{NQC} \]

- **Radiation Reaction**

\[ \dot{p}_\varphi = \hat{F}_\varphi, \]

\[ \dot{p}_{r*} = \sqrt{\frac{A}{B}} \left( -\partial_r \hat{H}_{\text{EOB}} + \hat{F}_r \right) \]

The Hamiltonian can describe the dynamics along generic orbits.

For BNS systems on quasi-circular orbits, we may not have the terms in squares (depending on the model).
EOB Waveform models

- In the metric and hamiltonian:

\[
\hat{H}_{\text{eff}} = \sqrt{\frac{p_r^2}{r} + A(r) \left( 1 + \frac{p_r^2}{r^2} + 2\nu(4-3\nu) \frac{p_r^4}{r^2} \right)} , \quad p_r = \left( \frac{A}{B} \right)^{1/2} p_r
\]

On circular orbits \( p_r = 0 \rightarrow \) main contribution through \( A(r) \)

\( B(r) \rightarrow \) non-circular correction

If the system is spinning, \( r \rightarrow r_c = \) centrifugal radius (TEOBResumS)

- In the waveform:

\[
h_{\ell m} = h_{\ell m}^0 + h_{\ell m}^T = h_{\ell m}^{\text{Newt}} (\hat{h}_{\ell m}^0 + \hat{h}_{\ell m}^T)
\]

Currently, corrections available to (2,2), (2,1), (3,3), (3,2), (3,1), (4,4), (4,2)
EOB: Enhancing tidal effects close to merger

EOB models which employ “just” PN, adiabatic tides are known to underestimate tidal effects w.r.t NR → SEOBNRv4T: dynamical tides [Hinderer+2016, Steinhoff+2016, Steinhoff+2021]
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→ TEOBResumS: GSF resummation [Akcay+2018]
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\[
A_T(u) = \sum_{l \geq 2} A^{(\ell+)}_{A}^{LO}(u) \hat{A}_{A}^{(\ell+)}(u) + A^{(\ell-)}_{A}^{LO}(u) \hat{A}_{A}^{(\ell-)}(u) + (A \leftrightarrow B)
\]

\[
\hat{A}_{A}^{(\ell+)}(u) = 1 + \alpha_{1A}^{(\ell+)} u + \alpha_{2A}^{(\ell+)} u^2.
\]
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\]

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\hat{A}_A^{(\ell+)}(u) = 1 + \alpha_{1A}^{(\ell+)} u + \alpha_{2A}^{(\ell+)} u^2.
\]

PN

GSF

For +2,+3,-2 this term is resummed
EOB: Enhancing tidal effects close to merger

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→ SEOBNRv4T: dynamical tides [Hinderer+2016, Steinhoff+2016, Steinhoff+2021]

→ TEOBResumS: GSF resummation [Akcay+2018]
Phenom Waveform models (NRTidalv2)

Phase

\[ \psi_T(x) = -\kappa_{\text{eff}} \frac{39}{16\nu} \frac{x^{5/2}}{x^{5/2}} \tilde{P}_{\text{NRTidalv2}}(x) \]

\[ \psi_{SS} = \frac{3x^{-5/2}}{128\nu} \left( \hat{\psi}_{SS, 2PN} x^2 + \hat{\psi}_{SS, 3PN} x^3 + \hat{\psi}_{SS, 3.5PN} x^{7/2} \right) + [A \leftrightarrow B] \]

\[ \tilde{P}_{\text{NRTidalv2}}(x) = \frac{1 + \tilde{n}_1 x + \tilde{n}_{3/2} x^{3/2} + \tilde{n}_2 x^2 + \left( \tilde{n}_{5/2} x^{5/2} + \tilde{n}_3 x^3 \right)}{1 + \left( \tilde{d}_1 x + \tilde{d}_{3/2} x^{3/2} + \tilde{d}_2 x^2 \right)} \]

Amplitude

\[ \tilde{A}_{\text{NRTidalv2}} = -\sqrt{\frac{5\pi \nu}{24}} \frac{9M^2}{D_L} \frac{\kappa_{\text{eff}} x^{13/4}}{x^{13/4}} \frac{1 + \frac{449}{108} x + \frac{22672}{9} x^{2.89}}{1 + d x^4} \]

Dashed circles: coefficients fit to NR simulations + hybrids

Dietrich+2019
Phenom Waveform models (NRTidalv2)

Much like EOB, while the inspiral is modelled well, the merger description is still not entirely satisfactory for some configurations.
## Summary Table

<table>
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<th>TaylorF2</th>
<th>TEOBResumS</th>
<th>SEOBNRv4T</th>
<th>PhenomDNRT</th>
<th>PhenomPNRTv2</th>
</tr>
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<tr>
<td><strong>Adiabatic tides</strong></td>
<td>2.5 PN</td>
<td>2.5 PN in Hamiltonian</td>
<td>2.5 PN in Hamiltonian</td>
<td>1PN</td>
<td>2.5PN</td>
</tr>
<tr>
<td><strong>Dynamic tides</strong></td>
<td>yes</td>
<td>no*</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td><strong>spin-spin</strong></td>
<td>NNLO</td>
<td>NNLO (resummed)</td>
<td>NNLO (resummed)</td>
<td>no</td>
<td>NNLO (PN)</td>
</tr>
<tr>
<td><strong>Additional notes</strong></td>
<td>----</td>
<td>GSF-resummation, ell=2,..,8 electric contributions; ell=2 magnetic contributions; Higher modes in wf</td>
<td>BBH NQC corrections; ell=2,3 electric contributions;</td>
<td>NR fits for phase and amplitude, padé resummed</td>
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</tbody>
</table>

*yes as of last week :)

GSF-resummation, ell=2,..,8 electric contributions; ell=2 magnetic contributions; Higher modes in wf
BBH NQC corrections; ell=2,3 electric contributions; NR fits for phase and amplitude, padé resummed

NR fits for phase and amplitude, padé resummed
2. Waveform systematics and effect on the NS radius
The problem of PE

In modelled analyses waveform templates are necessary to extract the signal.

Different waveform models may recover different source parameters → waveform systematics.

- How large is the effect of waveform systematics on tidal parameters/R?
- How will this affect future detectors?
How should we study systematics?

Many waveform models, very different between each other → systematics are expected!

To study them, we should:

- Compare the approximants in a meaningful way and understand the general behavior of the models w.r.t. one another
- Test our understanding via injection-recovery studies

Note: both are needed! We are not really understanding systematics if we don’t understand the structural differences between the models.
Measurability of Tidal parameters

**Fisher matrix**: estimate of the statistical error on a parameter “i” in the high signal to noise ratio regime

\[
\sigma_{i}^{2} = (F^{-1})_{ii}
\]

\[
F_{ij} = (\partial_{i} h | \partial_{j} h) \simeq 4 \Re \int \frac{\tilde{A}_{h}^{2}}{S_{n}} (\partial_{i} \Psi_{h} \partial_{j} \Psi_{h}) \, df,
\]

The integrand indicates at which frequencies most of the information on a certain parameter is located

**Tidal effects** are measured at high (> 100 Hz) frequencies, where the noise of the detector is large
Comparison of approximants

Direct comparisons of the GW phase are tricky due to alignment issues.

To overcome the problem, one can use a “gauge invariant” quantity:

\[ Q_\omega = \frac{\dot{\omega}^2}{\dot{\omega}} = \frac{d\phi(t)}{d \ln \omega} \]

And compute

\[ \Delta Q_\omega = Q_\omega^Y - Q_\omega^X \]

Then, for a fixed value of omega, if the difference is positive:

\[ Q_\omega^Y > Q_\omega^X \]
\[ \frac{1}{\dot{\omega}^Y} > \frac{1}{\dot{\omega}^X} \]
\[ \dot{\omega}^X > \dot{\omega}^Y \]

X is more attractive (faster omega evolution) than Y at that frequency.
Comparison of approximants

We observe that:

- **Phenom**'s tidal effects are **more attractive** (stronger) than TEOB's and its point mass description is close to TEOB's
  → **smaller \( \Lambda \) than TEOB**

- **TF2**'s tidal effects are **more repulsive** (weaker) than TEOB's and the difference between TF2's point mass and TEOB's is large and positive. It partially compensates for the negative
  → **larger \( \Lambda \) than TEOB**
Injection study

Injection-recoveries with LALInference:

- 15 TEOBResumS waveforms with varying $\tilde{\Lambda}$ and masses;
- GW170817's sky location;
- Advanced LIGO and Virgo design PSD;
- Zero-noise configuration;
- Two cutoffs: 1024 Hz and 2048 Hz
- Recovery with IMRPhenomPv2NRTidal (Phenom) and TaylorF2 (TF2)
- SNR > 80
Injections: early inspiral parameters

Total mass and mass ratio are (usually) recovered quite well!

\[ Gamba+2020 \]
Injections: $\tilde{\Lambda}$ recovery

Our qualitative expectations are roughly confirmed!

Relative difference between recovered and injected Lambda

"Cumulative" difference between recovered and injected Lambdas

-10%

+5%

-10%

[Gamba+2020]
Injections: Ā recovery

In the worst (single event) case, ± 5%

[Gamba+2020]
Injections: R

~ 5% error on the radius, the qualitative picture we observed is the same

[Rossella Gamba - IPAM, 19.11.21]
Injections: importance of spin-induced effects

Quadrupole-monopole terms too can bias the inference of tidal parameters for highly spinning NS

Samajdar+2019
Real data: GW170817

Analysis setup:

- Parallel bilby;
- Very similar priors/config as bilby catalog, but...
  - **1 kHz frequency cutoff**
  - Small aligned spins (< 0.05)
- Waveform systematics are smaller than statistical error, but $\tilde{\Lambda}^{TF2, TEOB} > \tilde{\Lambda}^{Phenom}$
- We find $R = 12.5_{-1.8}^{+1.1}$ km

| Approximant                | $\ln p(d|\text{Approx.})$    |
|----------------------------|-------------------------------|
| TaylorF2                   | $523.078 \pm 0.102$           |
| TEOBResumS                 | $522.585 \pm 0.102$           |
| IMRPhenomPv2NRTidal        | $522.261 \pm 0.103$           |
Real data: GW170817 (again!)

To account for uncertainties in the approximants, one can:

1. combine samples...
2. ...and/or reweight based on the model evidence

Along the second line, new paper yesterday! [Dietrich+2021]

Main idea: sample not only binary parameters but also waveform models

\[
p(\theta | d, \Omega) \propto \mathcal{L}(d|\theta, \Omega)\pi(\theta|\Omega),
\]
\[
\Omega = \{\Omega_0, \Omega_1, \ldots \Omega_{n-1}\}
\]

Bayesian odds between two models

\[
\mathcal{O}_{A/B} = \frac{p_A}{p_B}
\]

Way to “marginalize” over approximants uncertainty
Future data: 3G, high SNR events

- Estimate of the error on $\tilde{\Lambda}$ by fitting the width of the injections of slide 34
- Comparison between expected difference and error for state of the art approximants
Future data: 3G, high SNR events

- Higher order effects will become measurable, and give different estimates based on their inclusion (or not)

- Proof of principle:
  - $\text{TaylorF2 + adiabatic tides + dynamical tides}$
  - $\text{TaylorF2 + adiabatic tides}$

[Pratten+2021]

Comments: 6 pages, 3 figures, comments and feedback welcome!
Future data: 3G, high SNR events

- Higher order effects will become measurable, and give different estimates based on their inclusion (or not)

- Proof of principle:
  
  \textbf{TaylorF2+adiabatic tides+dynamical tides} vs \textbf{TaylorF2+adiabatic tides}

**My personal** opinion: I don’t believe in dynamical tides more than I believe in GSF resummation (=would not do PE on dyn. tides parameters), but it is an interesting study showing that even high order PN uncertainty can bias our inference for 3G detectors

[Pratten+2021]
Numerical Relativity

- Need of better NR simulations to understand the strong field behavior

- However, NR too is flawed!

- We consider a handful of BAM simulations taken from the CoRe database (GW database)

- Faithfulness threshold:

\[ \mathcal{F} > 1 - \frac{e^2}{2\rho^2} \]

\[ \mathcal{F} = \max_{\epsilon, \Phi} \frac{(h|k)}{\sqrt{(h|h)(k|k)}} \]

- For SNR > 80 no simulation exceeds the theoretical faithfulness threshold

- Current NR simulations might not be faithful enough to inform waveform models

<table>
<thead>
<tr>
<th>Sim</th>
<th>SNR</th>
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<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>[96, 64]</td>
<td>14</td>
<td>[96, 64]</td>
<td>30</td>
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<tr>
<td>[96, 64]</td>
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<td>[96, 64]</td>
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<tr>
<td>[108, 72]</td>
<td>14</td>
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<tr>
<td>[64, 64]</td>
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<td>[240, 160]</td>
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<tr>
<td>[128, 96]</td>
<td>30</td>
<td>[128, 96]</td>
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</tr>
</tbody>
</table>

\( ^a \)Number of grid point (linear resolution) of the finest grid refinement, roughly covering the diameter of one NS.
Conclusions

- To **understand** waveform systematics, it necessary to compare the models and not “just” rely on injection-recovery studies
- Waveform systematics relevant already at design sensitivity…
- ...And dominant for 3G detectors!
- Waveform models **must** improve
- Numerical relativity simulations, too, must improve (higher resolutions, more physics)