



## Waveform systematics in the gravitational-wave inference of tidal parameters from binary neutron star signals

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#### Outline

Previous talks:

- Q: Why should we care about neutron stars? A: because they are cool (pun intended)
- Q: What can we do with GW data from binary neutron star signals? A: EOS, Cosmology, ...

In the next ~40 minutes:

- Q1: How can we model the GW signal from BNS?
- Q2: How do modelling differences affect GW parameter estimation?

# 1. Modelling of GWs from binary Neutron star systems

 $m_1, m_2, S_1, S_2, \Lambda_1, \Lambda_2, \ldots$ INPUT **OUTPU1 APPROXIMANT**  $h_+, h_ imes$  plus and cross parameters of the source system polarizations

## Phenomenology of a merger



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#### Phenomenology of a merger



Inspiral up to merger: "similar" to a BBH waveform for slowly spinning bodies and q~1 [See Geraint's talk]  $\rightarrow$  We can model this as BBH + corrections! Should be easy right?

#### **Matter effects**

Matter effects are what distinguish NS from point particles (=black holes, BH)

For GW modelling of BNS, the more important ones are:

#### • Tidal effects

- "adiabatic" tidal effects [Damour1983, Flanagan+2007, Damour+2008, Vines+2010,..., Henry+2020]
- o "dynamical" tidal effects [Lai+1994, Hinderer+2016, Steinhoff+2016, Steinhoff+2021]
- Spin induced effects [Poisson1998,Krishnendu+2017]

Additional effects due to resonant modes of neutron stars can be considered (but will not be discussed here)

#### **Adiabatic tides**

- When a NS is subject to the gravitational field of another object, it gets deformed → Tidal effects
- Proportionality constant between external field and quadrupolar deformation  $\rightarrow$  tidal parameter

$$\begin{split} Q_{ij} &= -\lambda E_{ij} \qquad \lambda_i = \frac{\text{Quadrupole deformation of the star}}{\text{External tidal field}} \to \Lambda_i = \frac{\lambda_i}{m_i^5} \\ \rho(p) \\ &+ \underbrace{\frac{1 - g_{tt}}{2} = -\frac{m}{r} - \frac{3Q_{ij}}{2r^3} \left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + O\left(r^{-3}\right)}_{+\frac{1}{2}\mathcal{E}_{ij}x^i x^j + O\left(r^3\right)} \\ &= \underbrace{\lambda_2}_{\text{Tidal deformability}} \end{split}$$

#### **Adiabatic tides**

Generalization to higher multipoles (Otcupole, Hexadecupole, ...)



There exist quasi-universal (=EOS independent) relations between the quadrupolar tidal parameter and the "higher order" ones [Yagi+2016, Carson+2019, Godzieba+2021]



#### **Dynamical tides**

• **Dynamical** Tidal effects (f-mode resonance):

$$\begin{split} L_{\rm DT} &= \frac{1}{4\lambda\omega_f^2} \left[ \dot{Q}^{ij} \dot{Q}^{ij} - \omega_f^2 Q^{ij} Q^{ij} \right] - \frac{1}{2} E_{ij} Q^{ij}, \\ \text{Harmonic Osc.} \end{split}$$

• "Dressing factor" for love numbers:

$$k_{\ell} \mapsto k_{\ell}^{\text{eff}} := \alpha_{\ell m} (\nu, \Omega, \overline{\omega_{f}^{(\ell)}}, X_{\text{A}}) k_{\ell}$$

$$\alpha_{\ell m} = a_{\ell} + b_{\ell} \left\{ \frac{x^{2}}{x^{2} - 1} + \frac{5}{6} \frac{x^{2}}{1 - x^{5/3}} + \frac{x^{2}}{\sqrt{\epsilon}} \left[ \cos\left(\Omega' \hat{t}^{2}\right) \int_{-\infty}^{\hat{t}} \sin\left(\Omega' s^{2}\right) ds - \sin\left(\Omega' \hat{t}^{2}\right) \int_{-\infty}^{\hat{t}} \cos\left(\Omega' s^{2}\right) ds \right] \right\}$$
Resonant part
Fresnel Part



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Resonant part
Fresnel Part

2

 $^{0} = 4$ 

0.01

 $---\ell = 2$  (diss)

0.02

--- f-mode Res.
--- f-mode Res.

--- f-mode Res.

--- Merger

0.03 0.04

Ô

0.05

0.06

0.07

al

#### **Spin-induced effects**

- The spinning motion of companion A creates a distortion in its mass distribution
- In turn, this distorts the gravitational field outside the star
- Clearly, this impacts the orbital motion and the emission of GWs

$$Q_A \simeq -a \chi_A^2 m_A^3$$
  
EOS dependent coefficient, can  
be related to  $\Lambda$ 

#### "State of the art" BNS Waveform models

Current "state of the art" models include most of the effects previously discussed

Three families:

- Post Newtonian approximants (PN) [Krishnendu+2017,Henry+2020,Schmidt+2021]
  - Analytical
  - Fast!
  - **Examples**: TaylorF2, TaylorT4
- Effective One Body approximants (EOB) [Bini+2012, Akcay+2018, Lackey+2018]
  - Semi-analytical, resummed PN + NR
  - Not-as-fast, generally
  - **Examples: TEOBResumS**, SEOBNRv4T (& related surrogate)
- Phenomenological approximants (Phenom) [Dietrich+2017,Kawaguchi+2018,Dietrich+2019]
  - Fits to PN+EOB+NR
  - Fast
  - **Examples**: (any BBH inspiral model) + NRTidal, NRTidalv2, Kawaguchi+ model

#### **PN Waveform models (TaylorF2)**

$$\tilde{h}^{\text{spa}}(f) = \frac{a(t_f)}{\sqrt{\dot{F}(t_f)}} e^{i[\psi_f(t_f) - \pi/4]}, \quad \psi_f(t) \equiv 2\pi f t - 2\phi(t), \qquad t_f = t_{\text{ref}} + M \int_{v_f} \frac{1}{\mathcal{F}(v)} dv, \quad \psi_f(t_f) = 2\pi f t_{\text{ref}} - \phi_{\text{ref}} + 2 \int_{v_f}^{v_{\text{ref}}} (v_f^3 - v^3) \frac{E'(v)}{\mathcal{F}(v)} dv$$

PN phase: orbital + tides + spin +quadrupole-monupole

$$\Psi(f) = \Psi_{\rm O} + \Psi_{\Lambda} + \Psi_S + \Psi_{\rm MQ} \,.$$

#### **Matter contributions**

$$\Delta \Psi(f) = \Psi^{\rm BNS}(f) - \Psi^{\rm BBH}(f) \approx \Psi_{\Lambda} + \Psi_{\rm MQ}$$

$$\Psi_{\rm MQ} = \frac{3}{128\nu} c_{\rm LO}^{\rm MQ} x^{-1/2} (1 + c_1^{\rm MQ, \rm NLO} x + c_{3/2}^{\rm MQ, \rm tail} x^{3/2}) \qquad \Psi_{\Lambda} = c_{\rm LO}^{\Lambda} x^{5/2} (1 + c_1^{\Lambda} x + c_{3/2}^{\Lambda} x^{3/2} + c_2^{\Lambda} x^2 + c_{5/2}^{\Lambda} x^{5/2})$$

#### Quadrupole-Monupole (or spin-spin)

**Tides** 

 $\int^{v_{\rm ref}} E'(v)$ 

#### **EOB Waveform models**

Three ingredients:

• Hamiltonian

$$\begin{split} H_{\rm EOB} &= M \sqrt{1 + 2\nu (\hat{H}_{\rm eff} - 1)}, \\ \hat{H}_{\rm eff} &= \sqrt{p_{r_*}^2 + A(r) \left(1 + \frac{p_{\varphi}^2}{r^2} + 2\nu (4 - 3\nu) \frac{p_{r_*}^4}{r^2}\right)} \end{split}$$

• Waveform

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)} \hat{S}_{\text{eff}}^{(\epsilon)} \hat{h}_{\ell m}^{\text{tail}} f_{\ell m} \hat{h}_{\ell m}^{\text{NQC}}$$

Radiation Reaction

$$\begin{split} \dot{p}_{\varphi} &= \hat{\mathcal{F}}_{\varphi} \ , \\ \dot{p}_{r_*} &= \sqrt{\frac{A}{B}} \left( -\partial_r \hat{H}_{\mathrm{EOB}} + \hat{\mathcal{F}}_r \right) \end{split}$$

For BNS systems on quasi-circular orbits, we may not have the terms in squares (depending on the model)

The Hamiltonian can describe the dynamics along generic orbits



#### **EOB Waveform models**

• In the metric and hamiltonian:

$$\hat{H}_{\text{eff}} = \sqrt{p_r^2 + 4(r)\left(1 + \frac{p_{\varphi}^2}{r^2} + 2\nu(4 - 3\nu)\frac{p_{r_*}^4}{r^2}\right)}, \quad p_{r_*} = \left(\frac{A}{B}\right)^{1/2} p_r \qquad \frac{A = A_0 + A_T}{B = D/A = B_0 + B_T} \qquad \frac{A = A_0 + A_T}{B = D/A = B_0 + B_T}$$
On circular orbits pr = 0  $\rightarrow$  main contribution through A(r)
$$r_c^2(r, \tilde{a}_A, \tilde{a}_B)^{\text{NNLO}} = r^2 + \tilde{a}_Q^2\left(1 + \frac{2}{r}\right) + \frac{\delta a_{\text{NLO}}^2}{r^2} + \frac{\delta a_{\text{NLO}}^2}{r^2}$$

On circular orbits  $pr = 0 \rightarrow main contribution through A(r)$ B(r)  $\rightarrow$  non-circular correction If the system is spinning,  $r \rightarrow rc = centrifugal radius (TEOBResumS)$ 

• In the waveform:

$$h_{\ell m} = h_{\ell m}^{0} + h_{\ell m}^{T} = h_{\ell m}^{\text{Newt}}(\hat{h}_{\ell m}^{0} + \hat{h}_{\ell m}^{T})$$

Currently, corrections available to (2,2), (2,1), (3,3), (3,2), (3,1), (4,4), (4,2)

EOB models which employ "just" PN, adiabatic tides are known to underestimate tidal effects w.r.t NR

→ SEOBNRv4T: dynamical tides [Hinderer+2016, Steinhoff+2016, Steinhoff+2021]

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- $\rightarrow$  TEOBResumS: GSF resummation [Akcay+2018]

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- → SEOBNRv4T: dynamical tides [Hinderer+2016, Steinhoff+2016, Steinhoff+2021]
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For +2,+3,-2 this term is resummed

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→ SEOBNRv4T: dynamical tides [Hinderer+2016, Steinhoff+2016, Steinhoff+2021]





#### **Phenom Waveform models (NRTidalv2)**

$$\begin{split} \frac{\text{Dietrich+2019}}{\psi_T(x) &= -\kappa_{\text{eff}}^T \frac{39}{16\nu} x^{5/2} \tilde{P}_{\text{NRTidalv2}}(x) \qquad \psi_{\text{SS}} = \frac{3x^{-5/2}}{128\nu} \left( \hat{\psi}_{\text{SS, 2PN}}^{(A)} x^2 + \hat{\psi}_{\text{SS, 3PN}}^{(A)} x^3 + \hat{\psi}_{\text{SS, 3.5PN}}^{(A)} x^{7/2} \right) \\ &+ [A \leftrightarrow B] \end{split}$$

#### Amplitude



#### Phenom Waveform models (NRTidalv2)



Dietrich+2019

Much like EOB, while the inspiral is modelled well, the merger description is still not entirely satisfactory for some configurations

#### **Summary Table**

	TaylorF2	TEOBResumS	SEOBNRv4T	PhenomDNRT	PhenomPNRTv2	
Adiabatic tides	2.5 PN	2.5 PN in2.5 PN in1PNHamiltonianHamiltonian		1PN	2.5PN	
Dynamic tides	yes	no*	yes	no	no	
spin-spin	NNLO	NNLO (resummed)	NNLO (resummed)	no	NNLO (PN)	
Additional notes		GSF-resummation, ell=2,,8 electric contributions; ell=2 magnetic contributions; Higher modes in wf	BBH NQC corrections; ell=2,3 electric contributions;	NR fits for phase and amplitude, padé resummed	NR fits for phase and amplitude, padé resummed	

\*yes as of last week :)

# 2. Waveform systematics and effect on the NS radius



#### The problem of PE

In modelled analyses waveform templates are necessary to extract the signal

Different waveform models may recover different source parameters  $\rightarrow$  waveform systematics

- How large is the effect of waveform systematics on tidal parameters/R
- How will this affect future detectors?



#### How should we study systematics?

Many waveform models, very different between each other  $\rightarrow$  systematics are expected!

To study them, we should:

- Compare the approximants in a meaningful way and understand the general behavior of the models w.r.t. one another
- Test our understanding via injection-recovery studies

Note: **both** are needed! We are not really understanding systematics if we don't understand the structural differences between the models.

#### **Measurability of Tidal parameters**

**Fisher matrix**: estimate of the statistical error on a parameter "i" in the high signal to noise ratio regime

$$\sigma_i^2 = (F^{-1})_{ii}$$
$$F_{ij} = (\partial_i h | \partial_j h) \simeq 4 \Re \int \frac{\tilde{A}_h^2}{S_n} (\partial_i \Psi_h \partial_j \Psi_h) df,$$

The integrand indicates at which frequencies most of the information on a certain parameter is located





[Damour+2012.Harry+2018]

#### [Gamba+2020]

## **Comparison of approximants**

Direct comparisons of the GW phase are tricky due to alignment issues

To overcome the problem, one can use a "gauge invariant" quantity:

$$Q_{\hat{\omega}} = \frac{\hat{\omega}^2}{\dot{\hat{\omega}}} = \frac{\mathrm{d}\phi(t)}{\mathrm{d}\ln\hat{\omega}}$$

And compute

$$\Delta Q_{\hat{\omega}} = Q_{\hat{\omega}}^Y - Q_{\hat{\omega}}^X$$

Then, for a fixed value of omega, if the difference is positive:

$$\begin{array}{ll} Q^Y_{\hat{\omega}} &> Q^X_{\hat{\omega}} \\ 1/\dot{\hat{\omega}}^Y &> 1/\dot{\hat{\omega}}^X \\ \dot{\hat{\omega}}^X &> \dot{\hat{\omega}}^Y \end{array}$$

X is more attractive (faster omega evolution) than Y at that frequency



#### **Comparison of approximants**

We observe that:

 Phenom's tidal effects are more attractive (stronger) than TEOB's and its point mass description is close to TEOB's → smaller Λ than TEOB

 TF2's tidal effects are more repulsive (weaker) than TEOB's and the difference between TF2's point mass and TEOB's is large and positive. It partially compensates for the negative → larger Λ than TEOB

#### **Injection study**

Injection-recoveries with LALInference:

- 15 TEOBResumS waveforms with varying Λ and masses;
- GW170817's sky location;
- Advanced LIGO and Virgo design PSD;
- Zero-noise configuration;
- Two cutoffs: 1024 Hz and 2048 Hz
- Recovery with IMRPhenomPv2NRTidal (Phenom) and TaylorF2 (TF2)
- SNR > 80

EOS	M <sub>inj</sub>	$q_{\mathrm{inj}}$	$\tilde{\Lambda}_{inj}$
2B	2.70	1.00	127
SLy	3.00	1.00	191
LS220	3.20	1.00	202
SFHo	2.92	1.00	252
DD2	3.18	1.00	332
SFHo	2.80	1.00	334
ALF2	3.00	1.00	382
SLy	2.68	1.00	401
SLy	2.69	0.88	401
SFHo	2.72	0.88	412
SFHo	2.71	1.00	413
LS220	2.69	0.86	714
LS220	2.68	1.00	715
DD2	2.71	1.00	840
DD2	2.48	1.00	1366

#### **Injections: early inspiral parameters**



## **Injections: Ã** recovery

## Our qualitative expectations are roughly confirmed!



"Cumulative" difference between recovered and injected Lambdas

[Gamba+2020]

and injected Lambda

recovered

Relative difference between

## **Injections: Ã** recovery



[Gamba+2020]

#### **Injections: R**



#### **Injections: importance of spin-induced effects**

Quadrupole-monupole terms too can bias the inference of tidal parameters for highly spinning NS



#### Analysis setup:

- Parallel bilby;
- Very smilar priors/config as bilby catalog, but...

Real data: GW170817

- 1 kHz frequency cutoff
- Small aligned spins (< 0.05)
- Waveform systematics are smaller than  $_{0.0}$  statistical error, but  $\tilde{\Lambda}^{\mathrm{TF2,TEOB}} > \tilde{\Lambda}^{\mathrm{Phenom}^{0}}$
- We find  $R = 12.5^{+1.1}_{-1.8}$  km



Approximant	$\ln p(d \text{Approx.})$
TaylorF2	$523.078 \pm 0.102$
TEOBResumS	$522.585 \pm 0.102$
IMRPhenomPv2NRTidal	$522.261 \pm 0.103$

## Real data: GW170817 (again!)

To account for uncertainties in the approximants, one can:

1. combine samples...

2. ...and/or reweight based on the model evidence Along the second line, new paper yesterday! [Dietrich+2021]

Main idea: sample not only binary parameters but also waveform models

 $p(\boldsymbol{\theta}|\mathbf{d},\Omega) \propto \mathcal{L}(\mathbf{d}|\boldsymbol{\theta},\Omega)\pi(\boldsymbol{\theta}|\Omega),$  $\Omega = \{\Omega_0,\Omega_1,\ldots\Omega_{n-1}\}$ 

Bayesian odds between two models

$$\mathcal{O}_{A/B} = \frac{p_A}{p_B}$$

Way to "marginalize" over approximants uncertainty



#### Future data: 3G, high SNR events

- Estimate of the error on  $\tilde{\Lambda}$  by fitting the width of the injections of slide 34
- Comparison between expected difference and error for state of the art approximants





#### Future data: 3G, high SNR events

- Higher order effects will become measurable, and give different estimates based on their inclusion (or not)
   [Pratten+2021]
- Proof of principle:

#### **TaylorF2+adiabatic tides+dynamical tides**

vs TaylorF2+adiabatic tides

Comments:	6 pages, 3 figures, comments and feedback welcome!
Subjects:	High Energy Astrophysical Phenomena (astro-ph.HE)



#### Future data: 3G, high SNR events

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 [Pratten+2021]



My **personal** opinion: I don't believe in dynamical tides more than I believe in GSF resummation (=would not do PE on dyn. tides parameters), but it is an interesting study showing that even high order PN uncertainty can bias our inference for 3G detectors

#### **Numerical Relativity**

- Need of better NR simulations to understand the strong field behavior
- However, NR too is flawed!
- We consider a handful of BAM simulations taken from the CoRe database (<u>GW database</u>)
- Faithfulness threshold:

$$\mathcal{F} > 1 - rac{\epsilon^2}{2 \rho^2}$$
  $\mathcal{F} = \max_{t_c, \phi_c} rac{(h|k)}{\sqrt{(h|h)(k|k)}}$ 

- For SNR > 80 no simulation exceeds the theoretical faithfulness threshold
- Current NR simulations might not be faithful enough to inform waveform models

TABLE V. Faithfulness values  $\mathcal{F}$  computed considering frequencies from  $f_{\rm low}$  to  $f_{\rm mrg}$  between simulations with the same intrinsic parameters and two different resolutions, extracted at r/M = 1000. The source is situated in the same sky location as GW170817, and the waveform polarizations  $h_+$  and  $h_{\times}$  are computed and projected on the Livingston detector. We employ the aLIGODesignSensitivityPl200087 [23] PSD from pycbc [115] to compute the matches and compare the values obtained to the thresholds  $\mathcal{F}_{\rm thr}$  calculated with Eq. (19) with  $e^2 = 1$  or  $e^2 = 6$ . A tick  $\checkmark$  indicates that  $\mathcal{F} > \mathcal{F}_{\rm thr}$ . Conversely, a cross  $\varkappa$  indicates that  $\mathcal{F} < \mathcal{F}_{\rm thr}$ .

Sim	n <sup>a</sup>	${\cal F}$	SNR					
			14		30		80	
			6	1	6	1	6	1
BAM:0011	[96, 64]	0.991298	1	X	X	X	×	X
BAM:0017	[96, 64]	0.985917	1	×	X	X	×	×
BAM:0021	[96, 64]	0.957098	×	×	×	×	×	×
BAM:0037	[216, 144]	0.998790	1	1	1	×	×	×
BAM:0048	[108, 72]	0.983724	X	X	X	X	×	×
BAM:0058	[64, 64]	0.999127	1	1	1	×	×	×
BAM:0064	[240, 160]	0.997427	1	X	1	X	×	X
BAM:0091	[144, 108]	0.997810	1	1	1	×	×	×
BAM:0094	[144, 108]	0.996804	1	1	1	×	×	×
BAM:0095	[256, 192]	0.999550	1	1	1	1	1	×
BAM:0107	[128, 96]	0.995219	1	×	X	×	×	×
BAM:0127	[128, 96]	0.999011	1	1	1	×	×	×

<sup>a</sup>Number of grid point (linear resolution) of the finest grid refinement, roughly covering the diameter of one NS.

#### **Conclusions**

- To **understand** waveform systematics, it necessary to compare the models and not "just" rely on injection-recovery studies
- Waveform systematics relevant already at design sensitivity...
- ...And dominant for 3G detectors!
- Waveform models **must** improve
- Numerical relativity simulations, too, must improve (higher resolutions, more physics)