# *Tracking a continuous gravitational-wave signal with a hidden Markov model*

***Ling Sun (OzGrav ANU), Nov 2021, IPAM Workshop***

*** This material is based upon work supported by NSF's LIGO Laboratory which is a major facility fully funded by the National Science Foundation.*

# Outline

## Part A

- *An overview of continuous/long-transient gravitational waves*
- *Major challenges in searches for these waves*

## Part B

- *Hidden Markov model tracking technique*
- *Applications in gravitational-wave data analysis*

# Outline

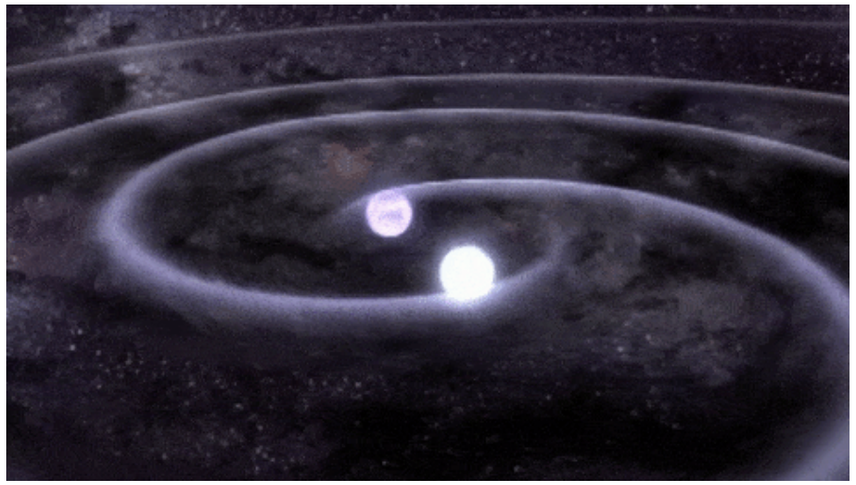
## Part A

- *An overview of continuous/long-transient gravitational waves*
- *Major challenges in searches for these waves*

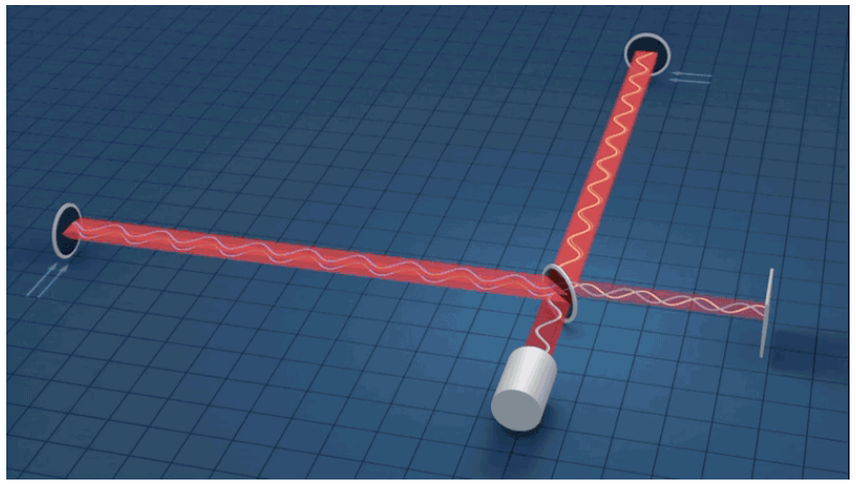
## Part B

- *Hidden Markov model tracking technique*
- *Applications in gravitational-wave data analysis*

# Gravitational waves are directly observed



*Credit: NASA/Dana Berry, Sky Works Digital*



*Credit: LIGO/T. Pyle*

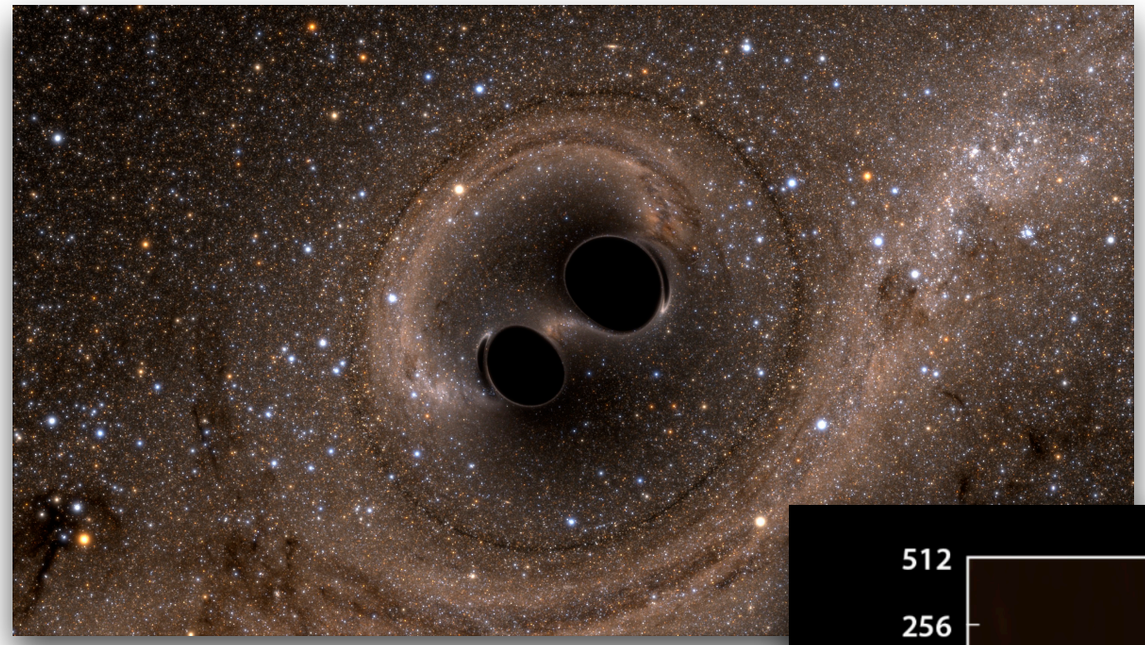
$$h_0 = \frac{\Delta L}{L}$$

**~$10^{-19}$ m**
**< 1/10,000 of a proton!**

**4 km (LIGO)**
**3 km (Virgo/KAGRA)**

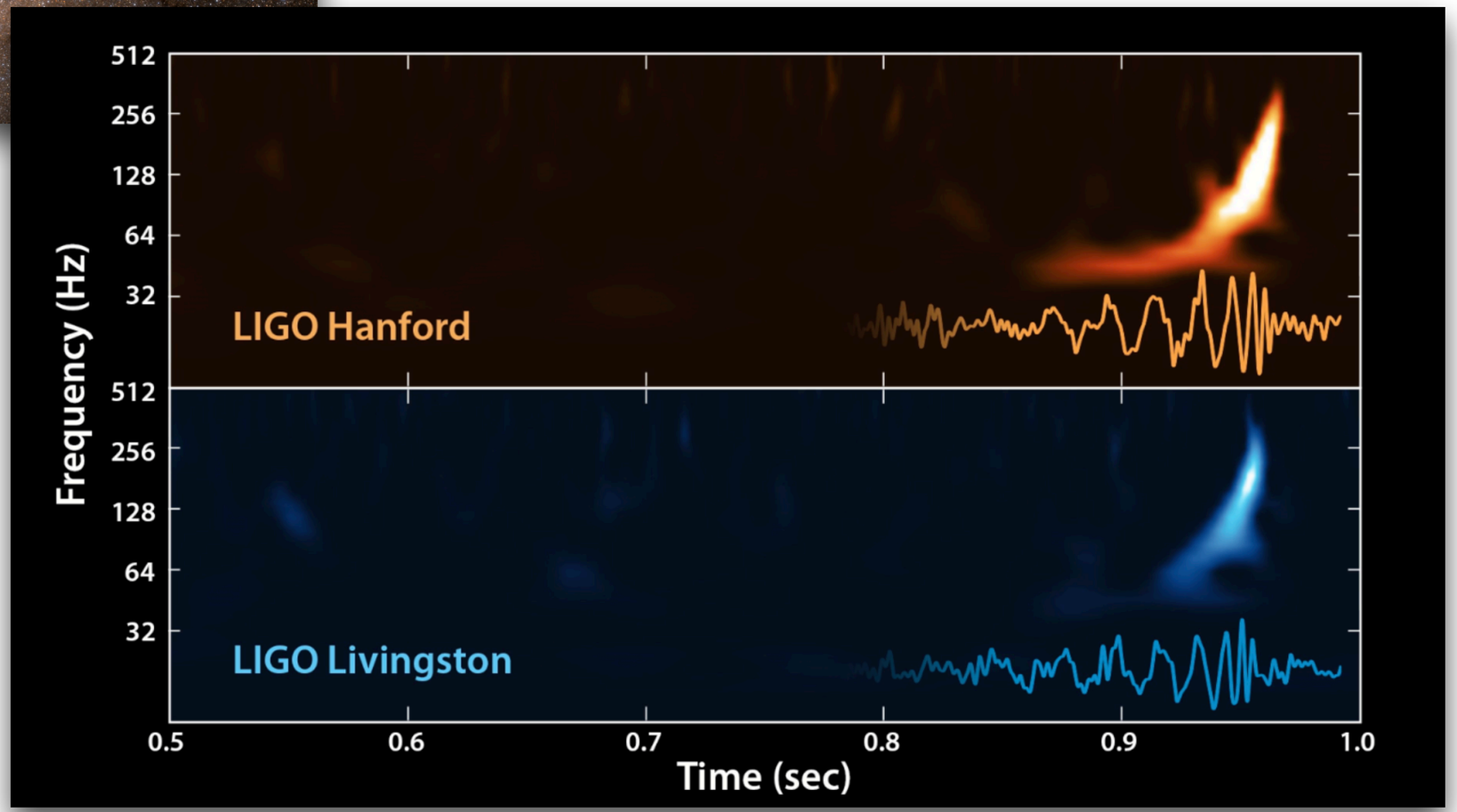

*Credit: ICRR, Univ. of Tokyo/LIGO Lab/Caltech/MIT/Virgo Collaboration*

# *The first detection of a binary black hole merger –– GW150914*

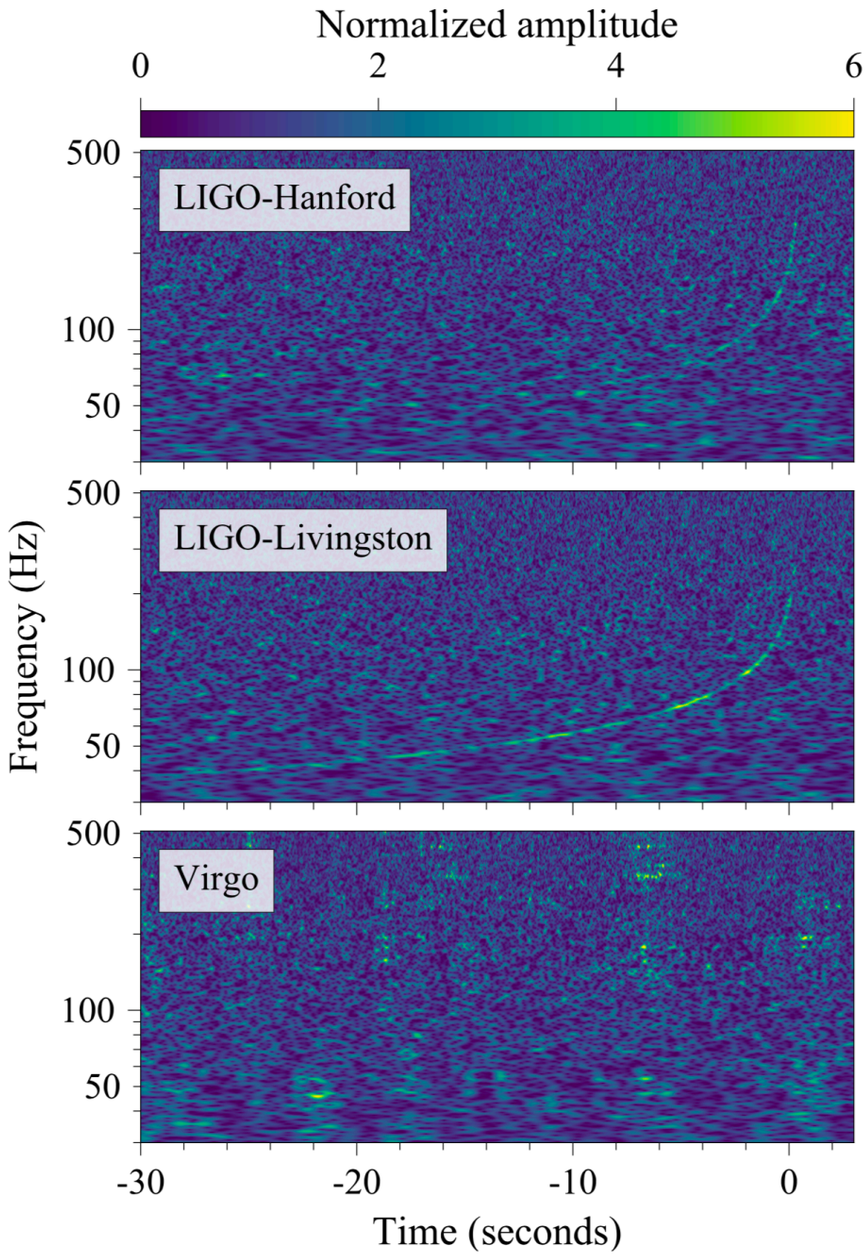


*Credit: SXS*

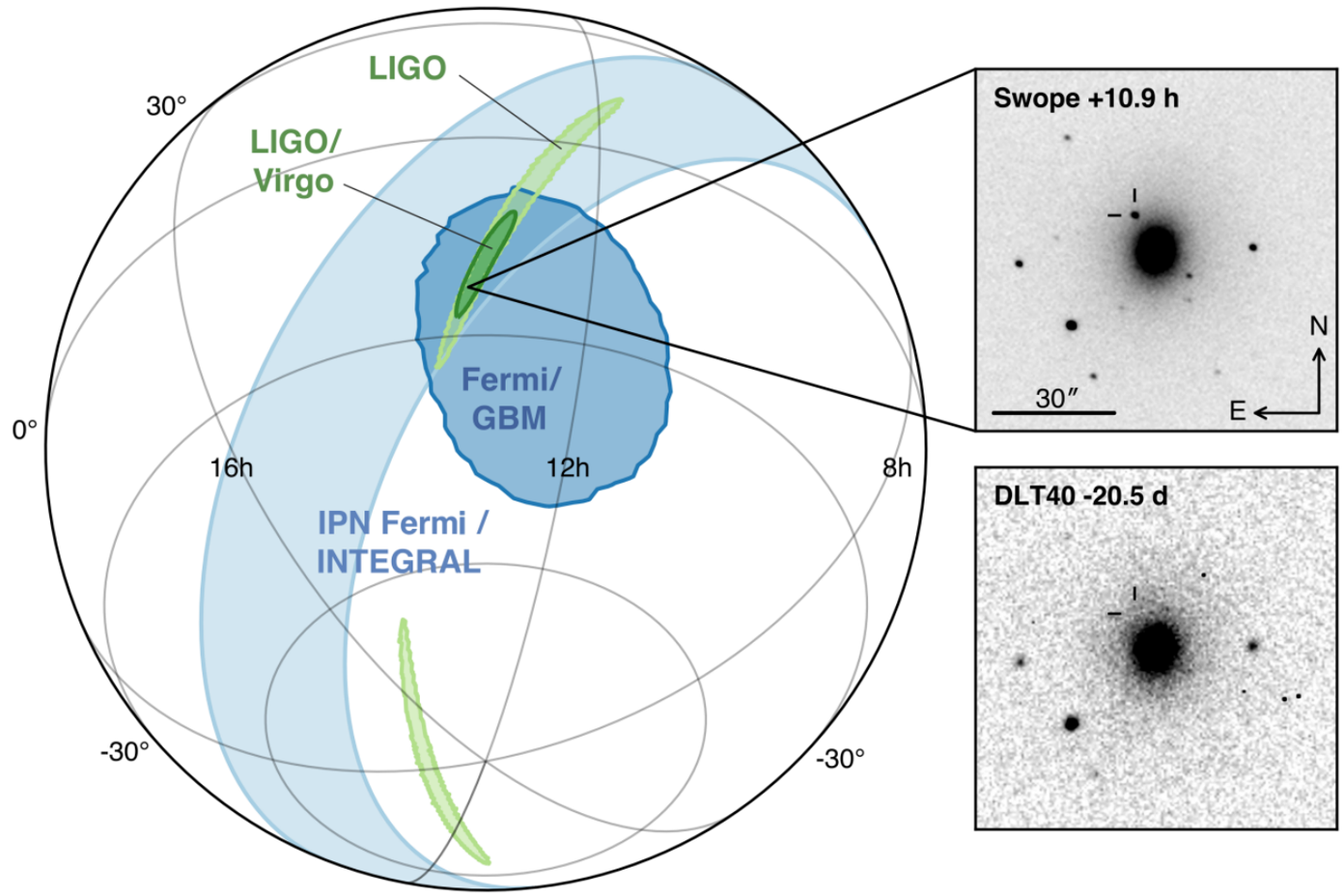
*Credit: Caltech/MIT/LIGO Lab*



# *The first multi-messenger event*
# *–– binary neutron star merger GW170817*

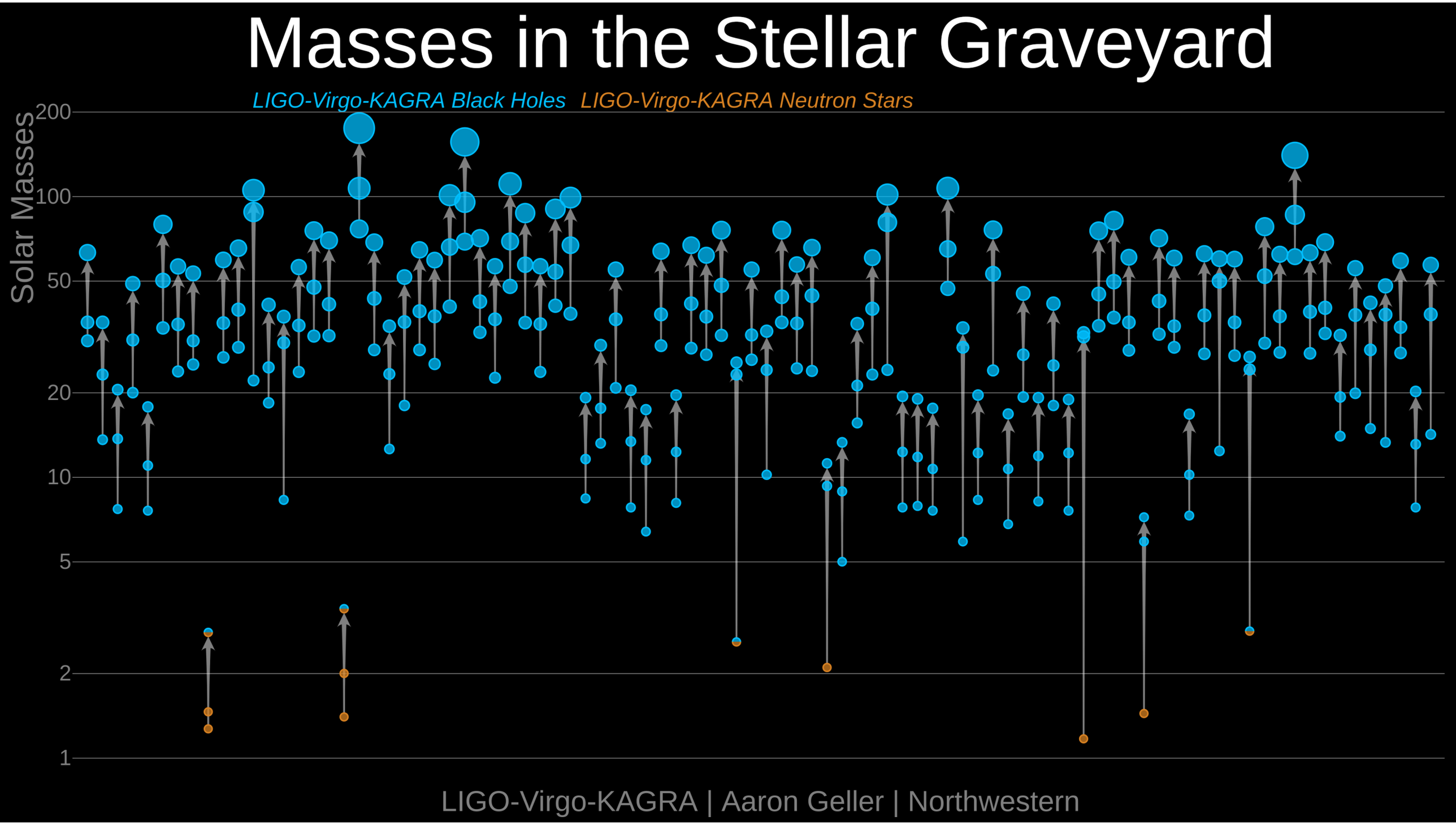



*Abbott+, Phys. Rev. Lett. 119, 161101 (2017)*

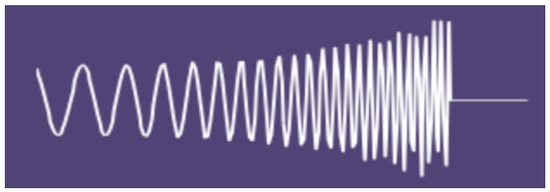


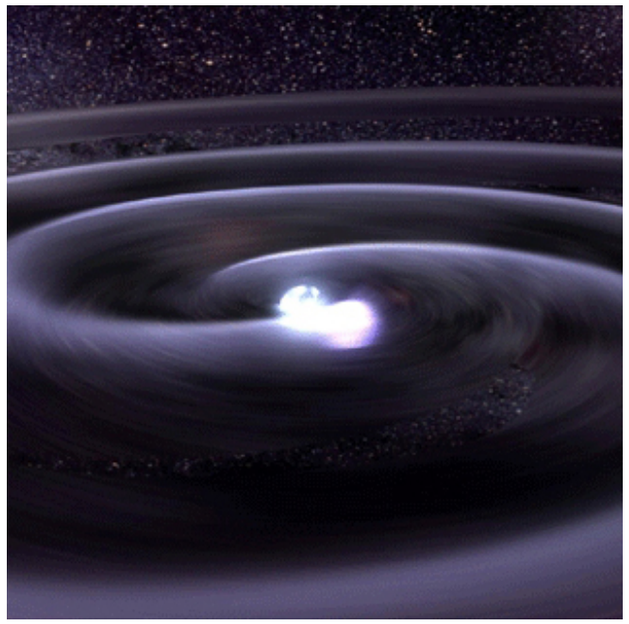

*Abbott+, Astrophys. J. Lett. 848, L12 (2017)*

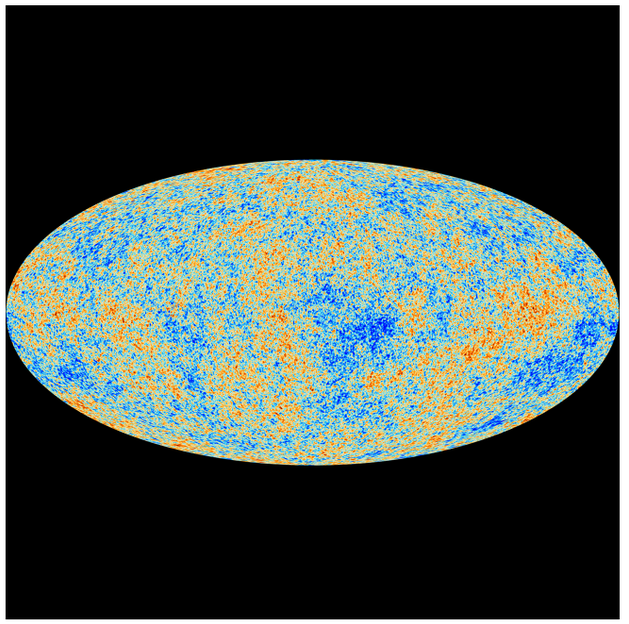
## *All the compact binary mergers detected so far*

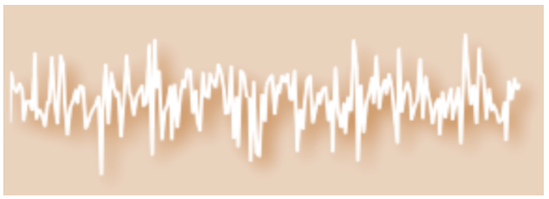


# *Other sources of gravitational waves*

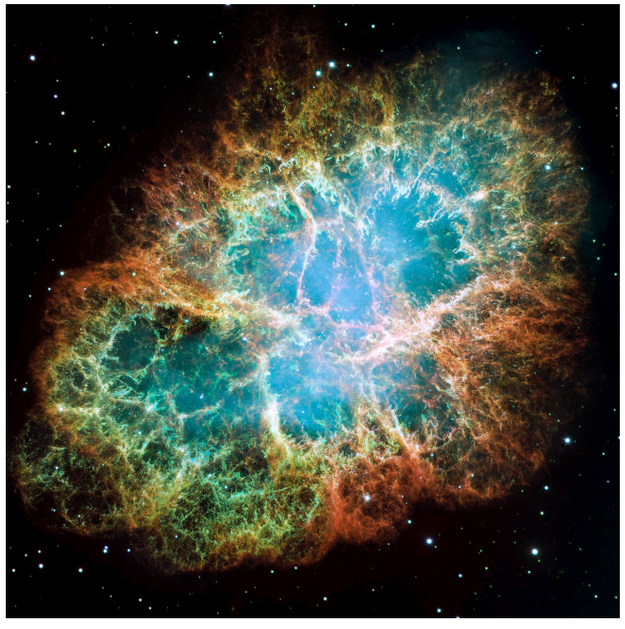


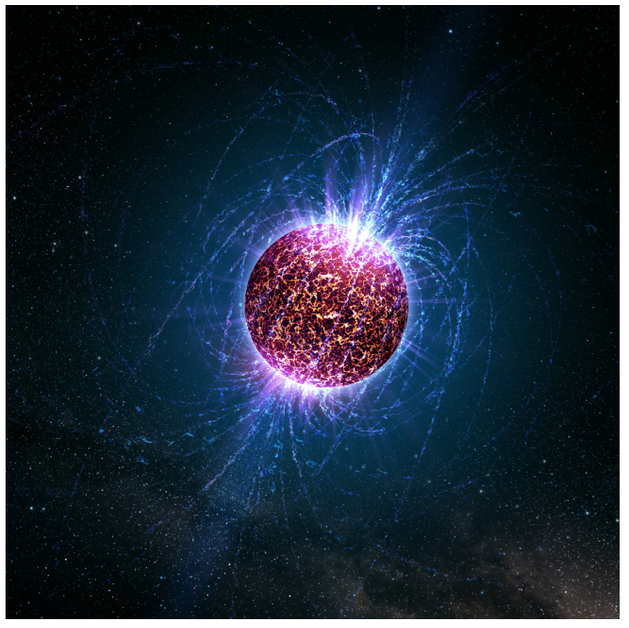






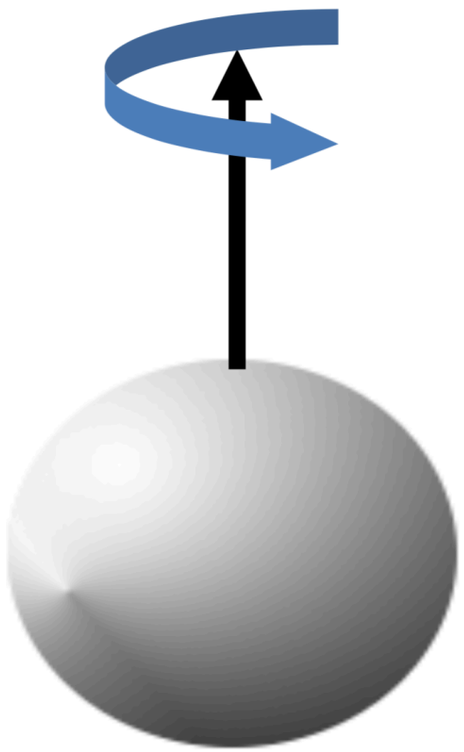
*seconds* *years*





# Continuous waves from neutron stars

- *Non-axisymmetric deformation due to elastic stresses or magnetic field*
- *Signal is weak but persistent*



$$f_{\mathrm{GW}} = 2 f_{\mathrm{rotation}}$$

$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz} f_{\mathrm{GW}}^2}{D} \epsilon$$

$$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

***ellipticity***

$$h_0 \approx 10^{-25} \left( \frac{\epsilon}{10^{-5}} \right) \left( \frac{I_{zz}}{10^{45}\,\mathrm{g\,cm^2}} \right) \left( \frac{f_{\mathrm{GW}}}{100\,\mathrm{Hz}} \right)^2 \left( \frac{1\,\mathrm{kpc}}{D} \right)$$

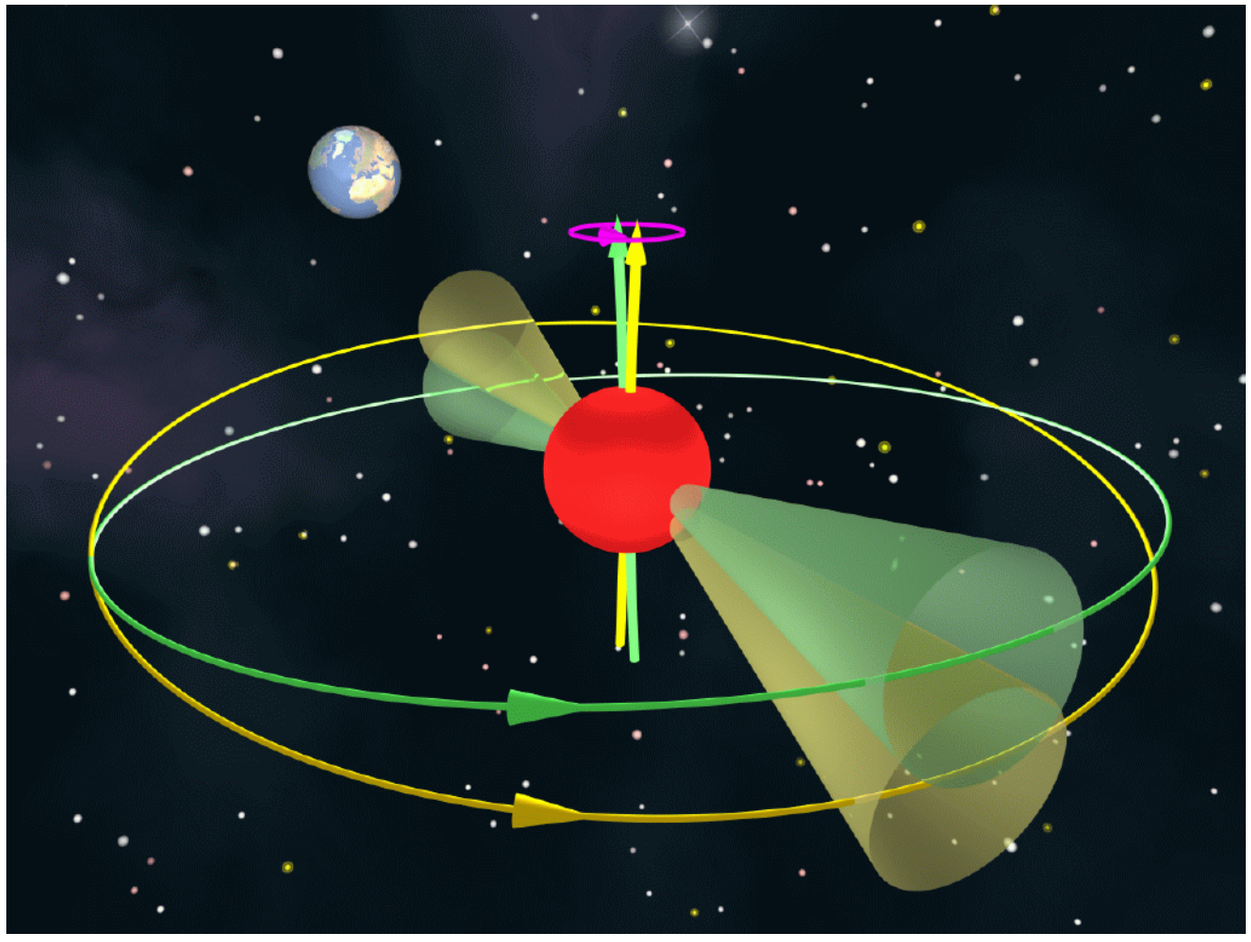
*cf. GW150914 peak* ***$h_0 \sim 10^{-21}$***

# *Continuous waves from neutron stars*

- *Free precession around the rotation axis*

$$f_{\mathrm{GW}} \sim f_{\mathrm{rotation}} + f_{\mathrm{precession}}$$

$$f_{\mathrm{GW}} \sim 2f_{\mathrm{rotation}} + 2f_{\mathrm{precession}}$$

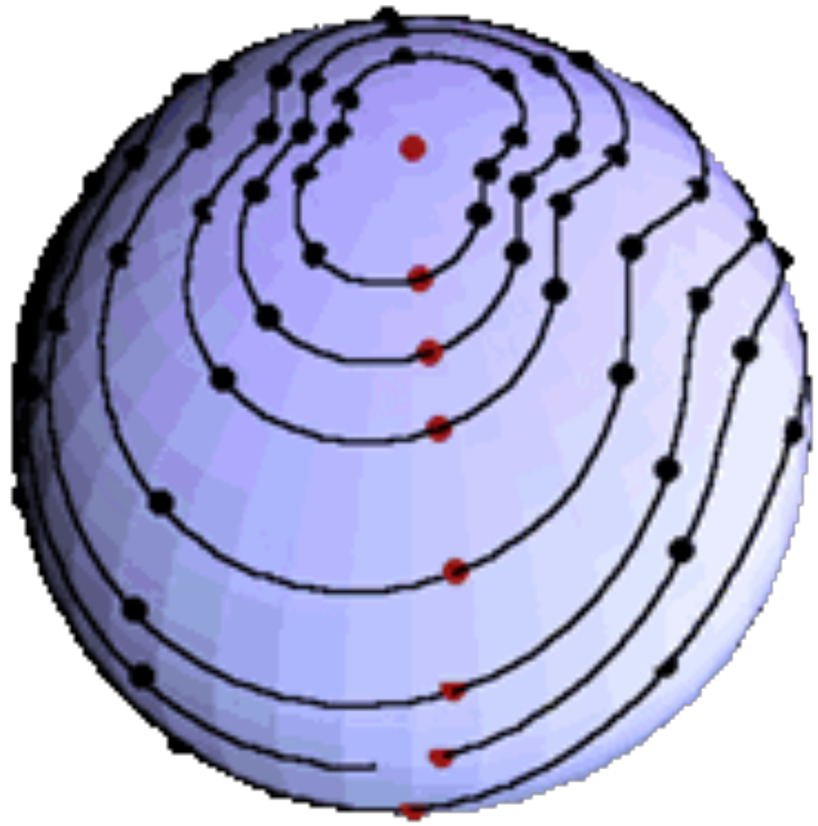


*Credit: M. Kramer*

# *Continuous waves from neutron stars*

- *r-modes, long-lasting oscillations in the fluid that makes up most of the star —– a fluid wave travelling around the star and driven by the Coriolis force due to rotation (see Rossby waves)*

$$f_{\rm GW} \sim 4 f_{\rm rotation}/3$$



*Credit: C. Hanna and B. Owen*

***r-mode amplitude***

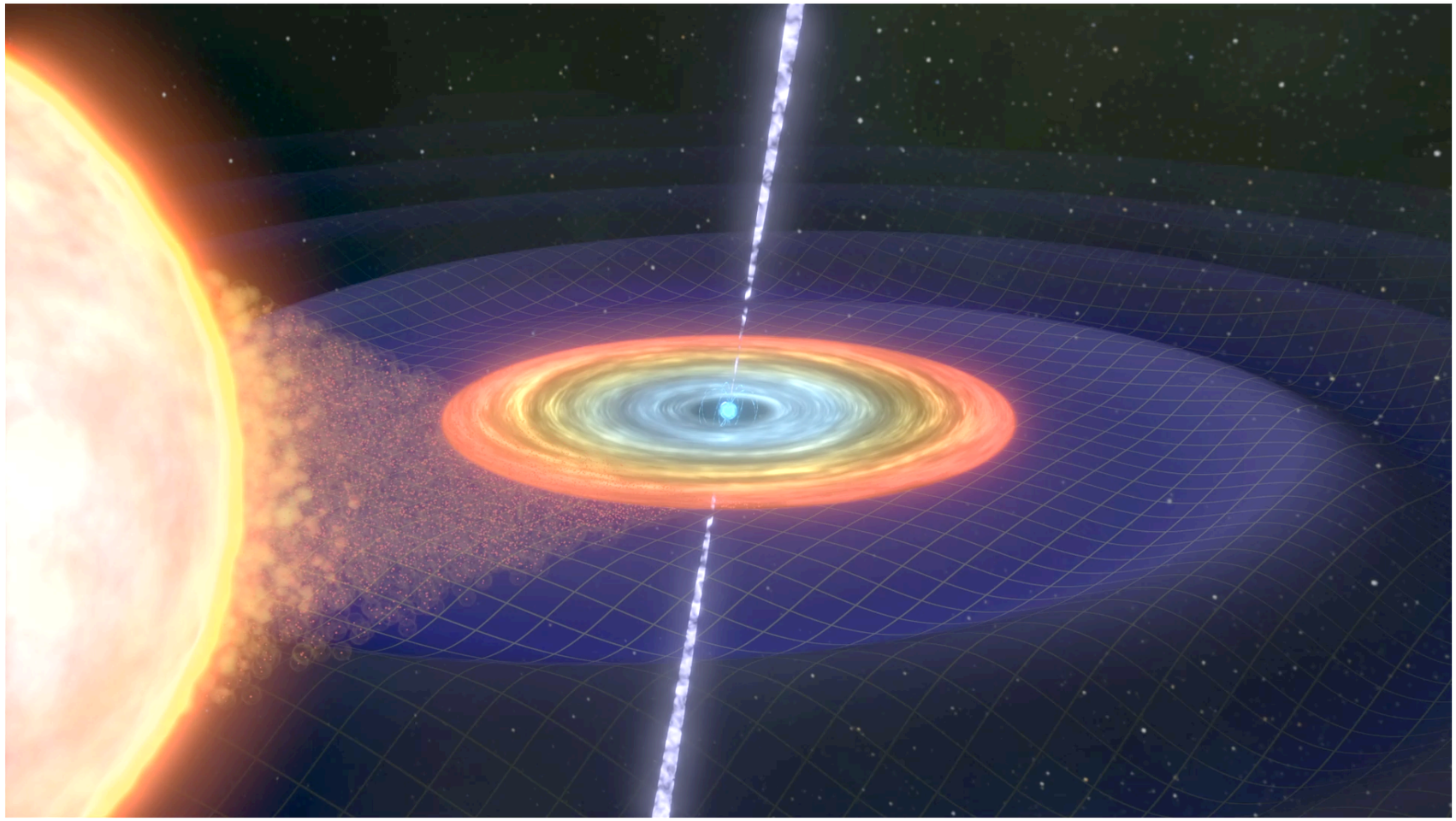
$$h_0 \simeq 3.6 \times 10^{-26} \left(\frac{\alpha}{10^{-3}}\right) \left(\frac{f_{\rm GW}}{100\,{\rm Hz}}\right)^3 \left(\frac{1\,{\rm kpc}}{D}\right)$$

## *Continuous waves from neutron stars*

- *Deformation due to matter accretion in the binary system*

$$f_{\mathrm{GW}} \sim 2 f_{\mathrm{rotation}}$$

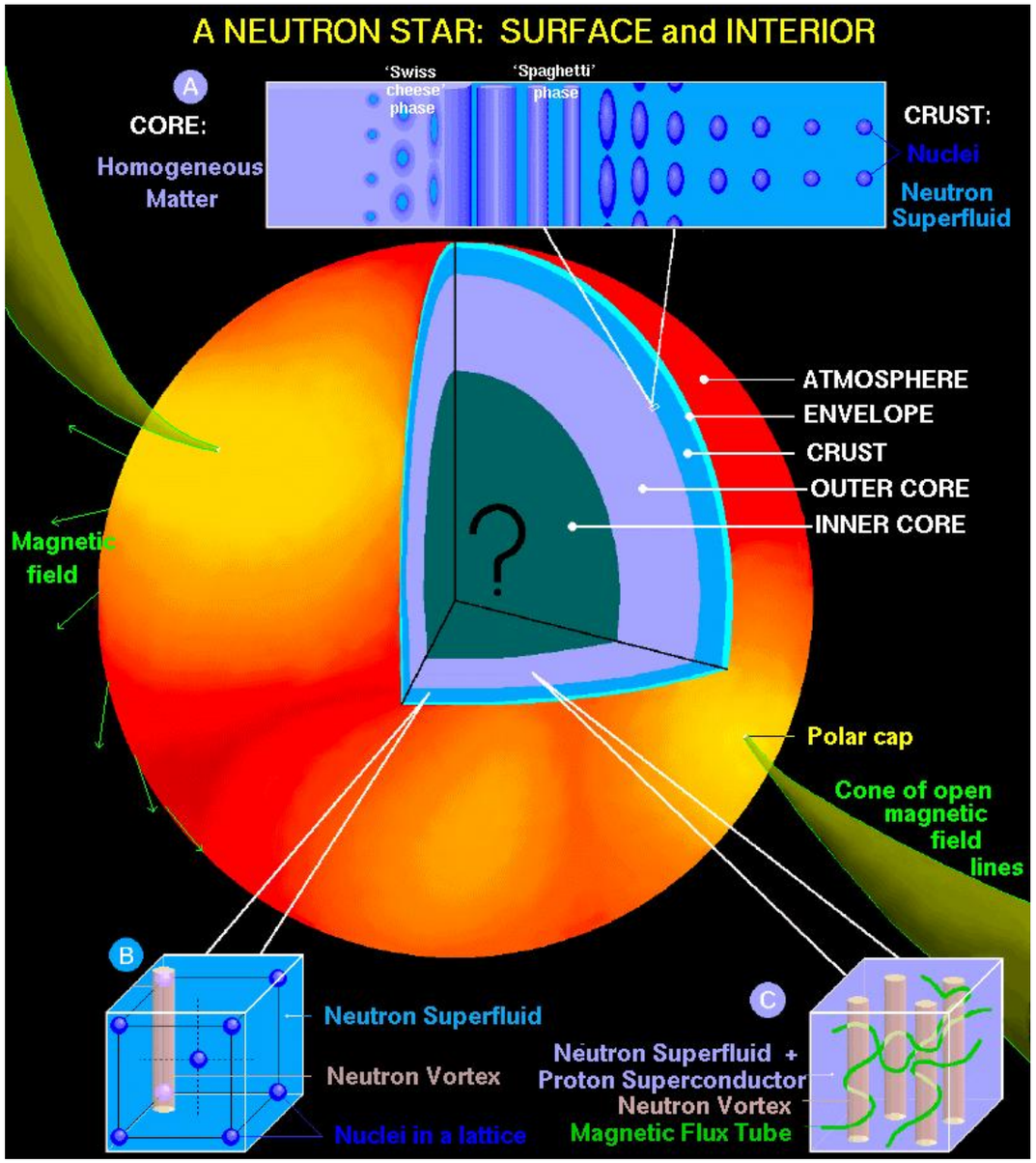
- *Accretion is a natural method of powering GW emission*
- *Torque-balance theory —– accretion spins the star up; GW emission slows it down*
- *Signal frequency might be wandering slightly due to accretion*



*Credit: Mark Myers, OzGrav-Swinburne*

# *What can we learn from continuous waves?*

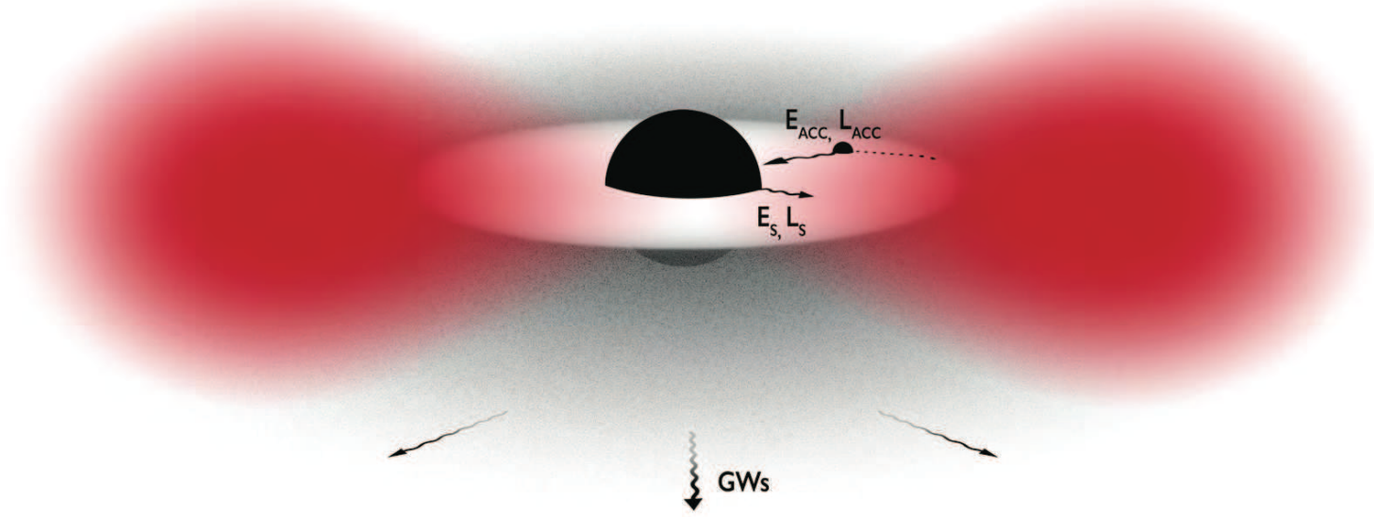
- *Interior structure of neutron star*
- *Neutron star properties, e.g., mass, spin, ellipticity*
- *Nuclear equation of state*
- *May discover exotic states of matter*
- *Multi-messenger studies, e.g., mass and magnetic field structure inferred from GW/EM relative phase*
- *Testing General Relativity*
- *… and so on*




*http://www.astro.umd.edu/~miller/nstar.html*

# Continuous waves from boson clouds

- *Beyond Standard Model theories predict the existence of* ***new ultralight boson particles*** *(e.g., QCD axion, string axion) —— dark matter candidates*
- *Ultralight boson fields around Kerr BHs can extract the BH's rotational energy through* ***Superradiance***
- *Superradiant* ***instability*** *—— when Compton wavelength ~ BH size, a macroscopic cloud could form*



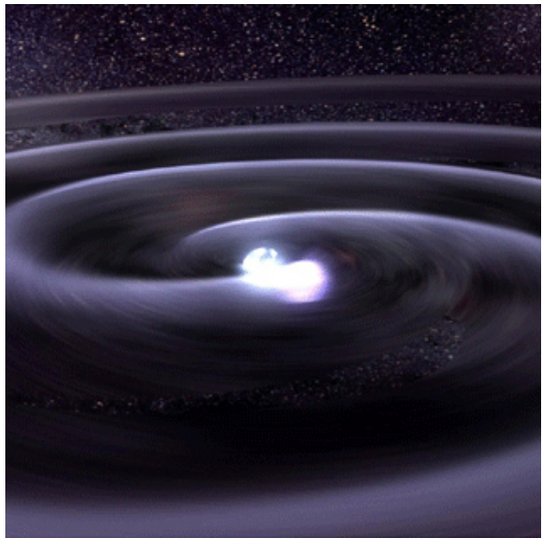

*Brito, Cardoso, Pani (2015)*

$$\omega_{\mathrm{GW}} = 2\omega_{\bar{n}}$$

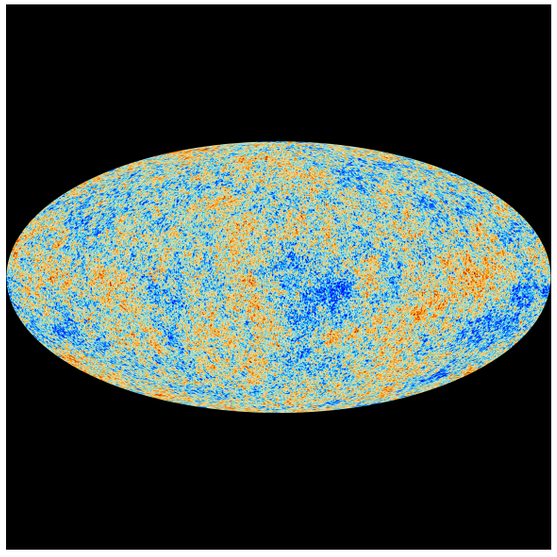
*Continuous quasi-monochromatic signals*

- ***What can we learn?*** *—— Probe a parameter space that is inaccessible by conventional particle physics experiments; may detect new particles*

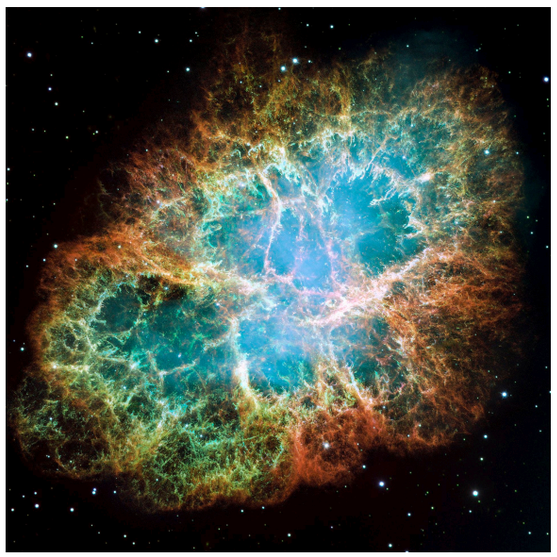
# *Long-transient gravitational waves*



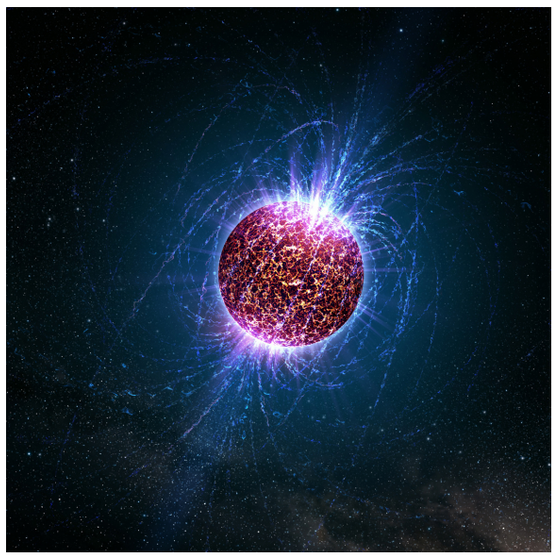
***Time-scale could be in between the transient and long traditional search regimes (~seconds to days).***



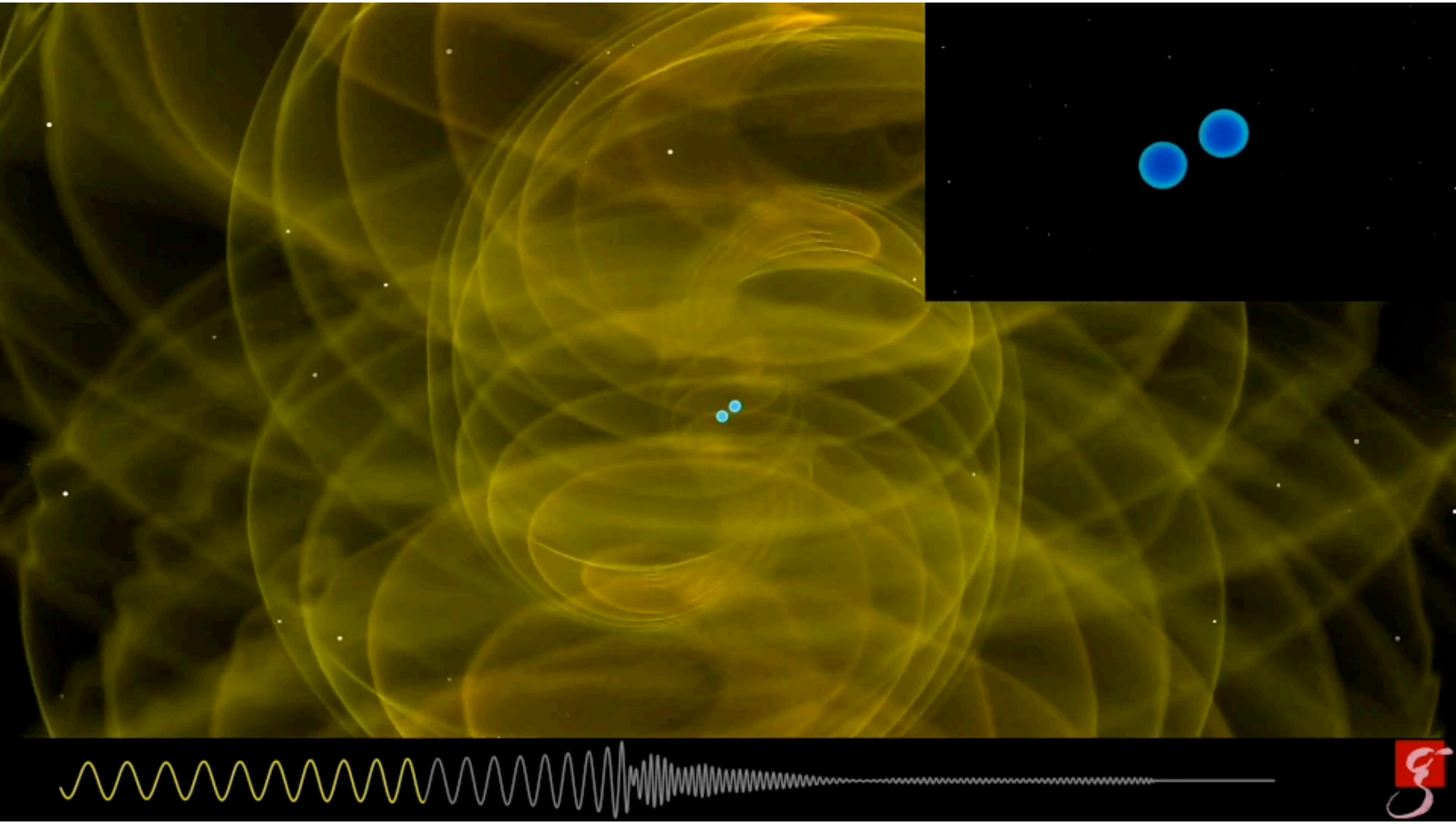
***seconds*** — **long-transient GW** — ***years***



- ***BNS postmerger remnants***
- ***Newly born supernova remnants***
- ***Glitching pulsars***
- ***Accreting objects***
- ***… etc.***



# *Post-merger signal from a binary neutron star merger*



# *Different types of searches*

| | | | |
|---|---|---|---|
| ***Targeted*** | *known sky position, ephemeris available* | *known pulsars* | *least expensive* |
| ***Directed*** | *known sky position, unknown frequency* | *NSs in supernova remnants, and low-mass x-ray binaries; boson clouds around known BHs* | *intermediate* |
| ***All-sky*** | *unknown sky position, unknown frequency* | *any possible signal from any position and any kind of sources* | *most expensive* |

## *Difficulties in the searches for these waves*

- ***Long-duration integration*** *—— computationally expensive*

- ***Large parameter space***
  - *Unknown frequency range*
  - *Large frequency evolution rate*

- ***Unknown spin-wandering***
  - *Spin-wandering has been seen in electromagnetic observations*
  - *Fluctuation of magnetospheric or superfluid torques*
  - *Fluctuation of accretion torque*
  - *Cannot integrate over a long duration coherently*

- ***Uncertainty in signal models***
  - *Uncertainty in the theories*
  - *Minor features not taken into account in simplified signal models*

# Outline

## Part A

- *An overview of continuous/long-transient gravitational waves*
- *Major challenges in searches for these waves*

## Part B

- *Hidden Markov model tracking technique*
- *Applications in gravitational-wave data analysis*

## *Bayes' theorem*

$$\mathbf{X} = \{x_0, x_1, \cdots, x_N\}$$

*Observational data sequence*

$$\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_M)$$

*Vector parameter, e.g.,* $f, \dot{f}, \cdots$

*Posterior* *Likelihood* *Prior*

$$p(\boldsymbol{\theta}|\mathbf{X}) = \frac{p(\mathbf{X}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\mathbf{X})}$$

*Posterior probability distribution & Bayes's theorem*

$$p(\mathbf{X}) = \int d\boldsymbol{\theta} p(\mathbf{X}|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

*The evidence —– a normalisation factor*

## *Maximum a posteriori (MAP) estimate*

$$\mathbf{X} = \{x_0, x_1, \cdots, x_N\}$$

*The **maximum a posteriori (MAP)** estimator maximises the posterior probability*

$$\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_M)$$

$$\hat{\boldsymbol{\theta}}_{\text{map}} = (\hat{\theta}_{1_{\text{map}}}, \hat{\theta}_{2_{\text{map}}}, \cdots, \hat{\theta}_{M_{\text{map}}})$$

$$p(\boldsymbol{\theta}|\mathbf{X}) = \frac{p(\mathbf{X}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\mathbf{X})}$$

$$\hat{\theta}_{i_{\text{map}}} = \arg\max_{\theta_i} p(\boldsymbol{\theta}|\mathbf{X})$$

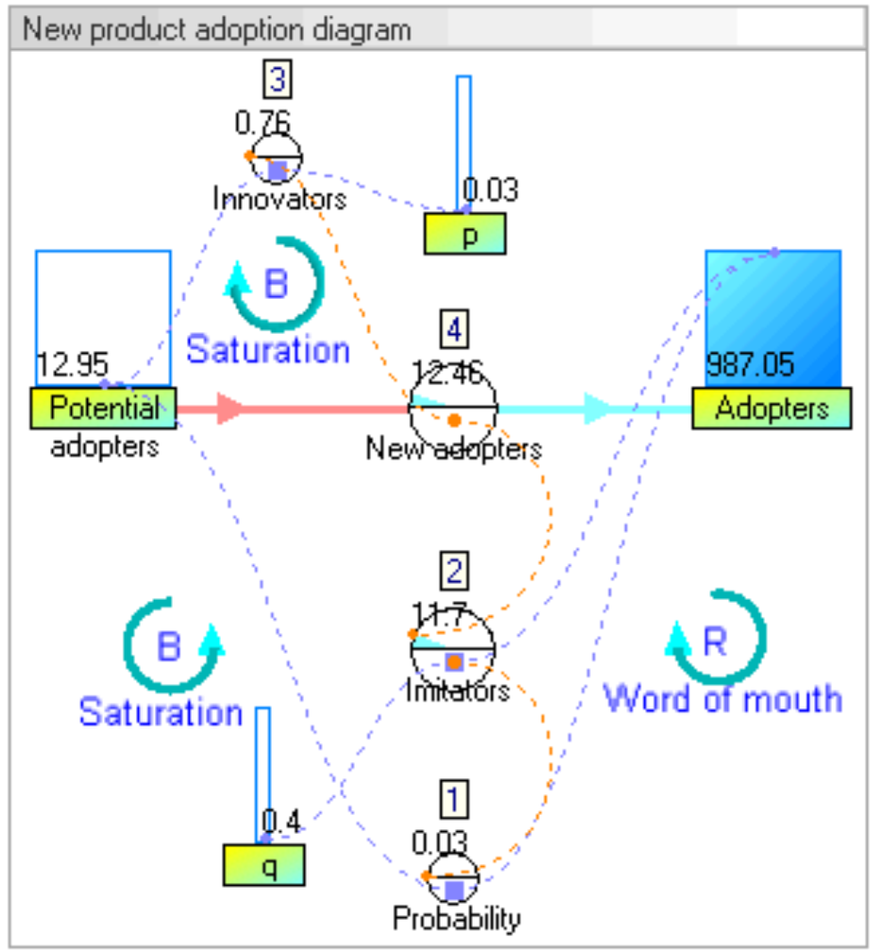
$$p(\mathbf{X}) = \int d\boldsymbol{\theta} p(\mathbf{X}|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

- *For detection, we also need some statistics to evaluate the significance, e.g., setting a threshold on the posterior probability.*

# *System, state, tracking*

- *The **system** we consider here:*
  - ➡ *It evolves; we know something about the dynamics*
  - ➡ *We can measure it at intervals over time*
- ***State** –– Possible configuration of the system*
- *We **track** the state of a system*

- *System –– e.g, a spinning neutron star*
- *State –– GW frequency at a given time*
- *Track –– GW frequency over time*




*From wikipedia: Dynamic stock and flow diagram of model New product adoption (model from article by John Sterman 2001)*

## ***The uncertainties***

- *We cannot know the exact state of a system in most situations*

- *Uncertainties may come from:*
  - ➡ *Dynamic evolution*
  - ➡ *Measurement at each time interval*

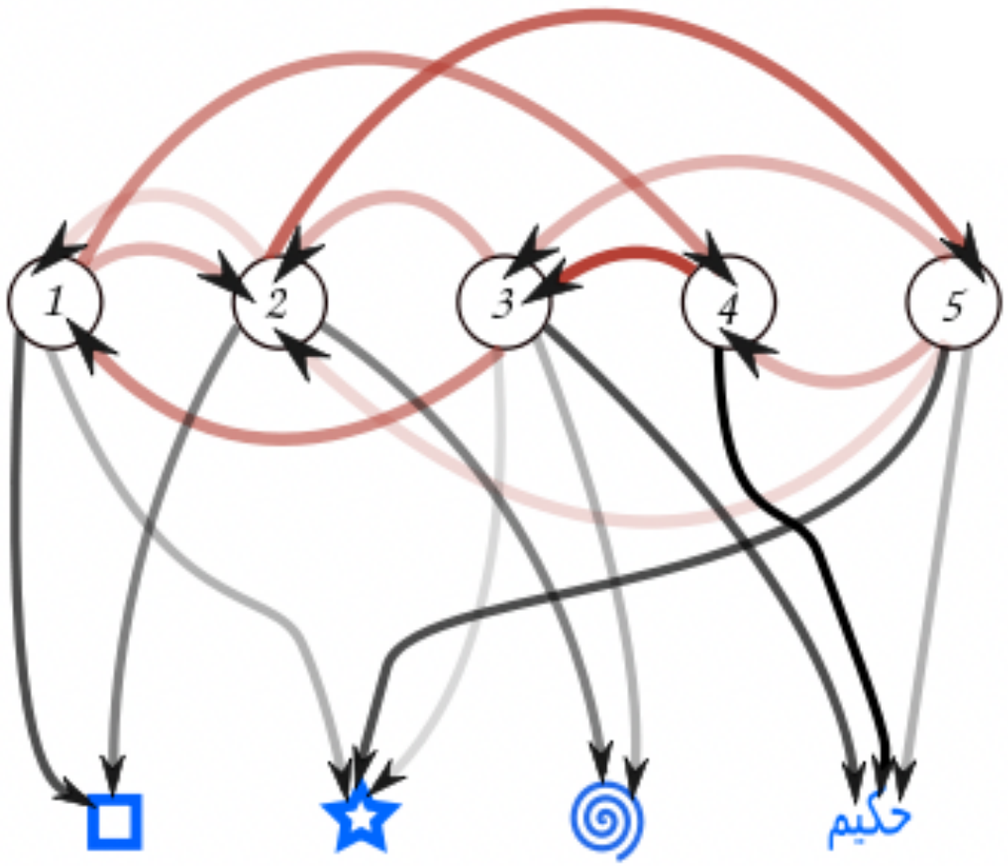
- *Dynamic evolution —– unknown spin wandering, varying spin-down/spin-up rate*
- *Measurement —– noisy data and weak signal*

- *Calculate the posterior probability and capture the uncertainty in the tracking*

# Hidden Markov Model

**Emission Probability** **Transition Probability**

1.0 1.0




**Hidden States**

*GW signal frequency*

**Observable States**

*noisy data*


**Observation Sequence**

*Q: hidden path*

*O: observation sequence*

**Hidden states: $q_1$, $q_2$, $q_3$ …**

**Observable states: $o_1$, $o_2$, $o_3$ …**

$$P(Q|O) = L_{o(t_{N_T})q(t_{N_T})} A_{q(t_{N_T})q(t_{N_T-1})} \cdots \times L_{o(t_1)q(t_1)} A_{q(t_1)q(t_0)} \Pi_{q(t_0)},$$

**L: Emission Probability**

**A: Transition Probability**

- *Problem: to find the maximum a posteriori (MAP) hidden path given an observation sequence*

## ***Applications***

- *Speech recognition/synthesis*
- *Part-of-speech tagging*
- *Machine translation*
- *Handwriting recognition*
- *Music signal processing*
- *Cryptanalysis*
- *Financial modelling*
- *Target tracking models*
- *radar and sonar analysis*
- *mobile telephony*
- *Protein folding*
- *Single-molecule kinetic analysis*
- *Metamorphic virus detection*
- *Bioinformatics*

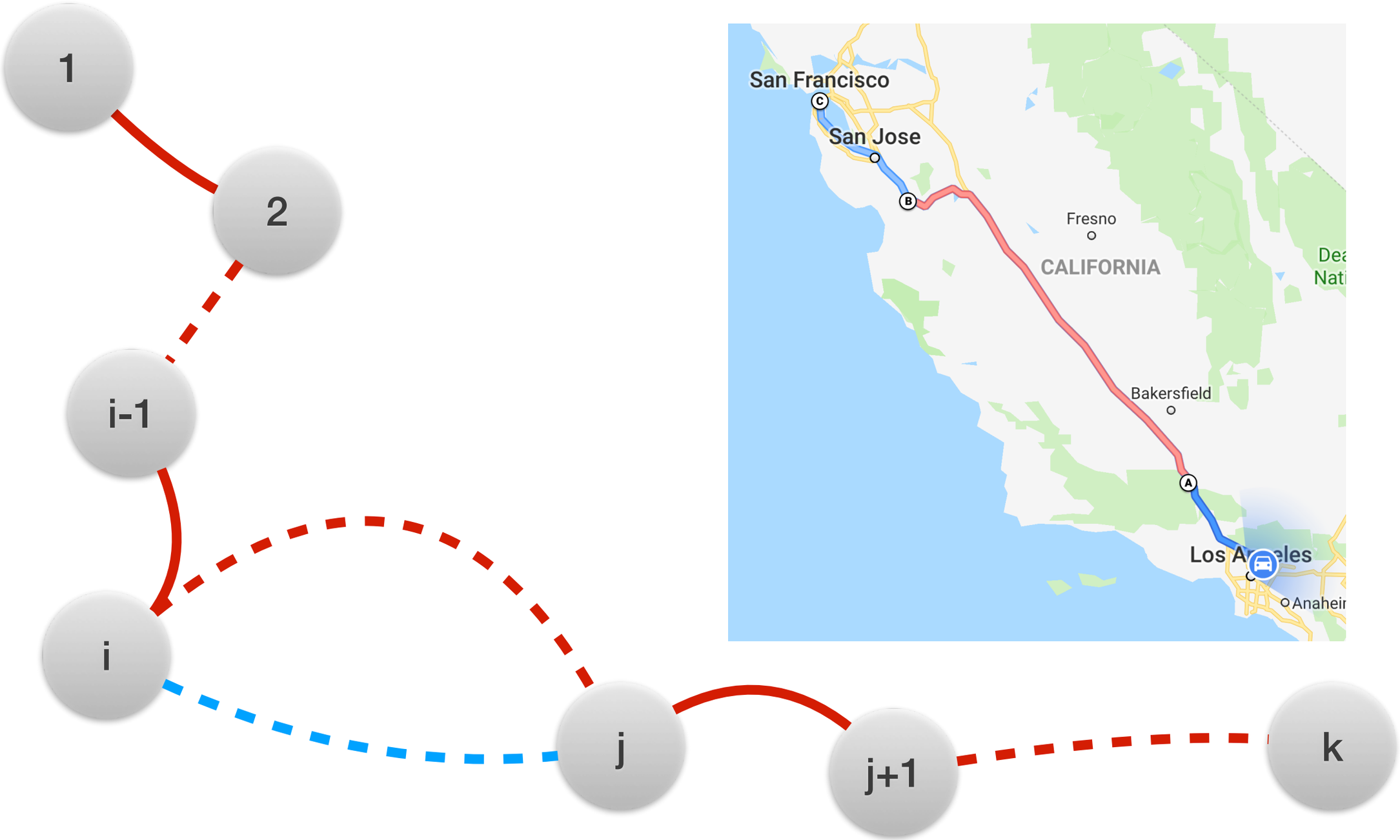
## *How to efficiently solve the HMM problem*

- ***Viterbi algorithm*** *[Viterbi (1967)] —– a dynamic programming algorithm to efficiently find the maximum a posteriori (MAP) path given an observation sequence*

- ***Bellman's principle of optimality*** *[Bellman (1957)] —– given the optimal path of states from the first step to the last, then any subpath is also optimal. It is the key idea of dynamic programming to recursively solve a complex optimization problem in stages.*

# Bellman's principle of optimality

- *Given the optimal path of states [q*(1),⋯ ,q*(i),⋯ ,q*(j),⋯ ,q*(k)], any subpath [q*(i), · · · , q*(j)] must also be optimal.*

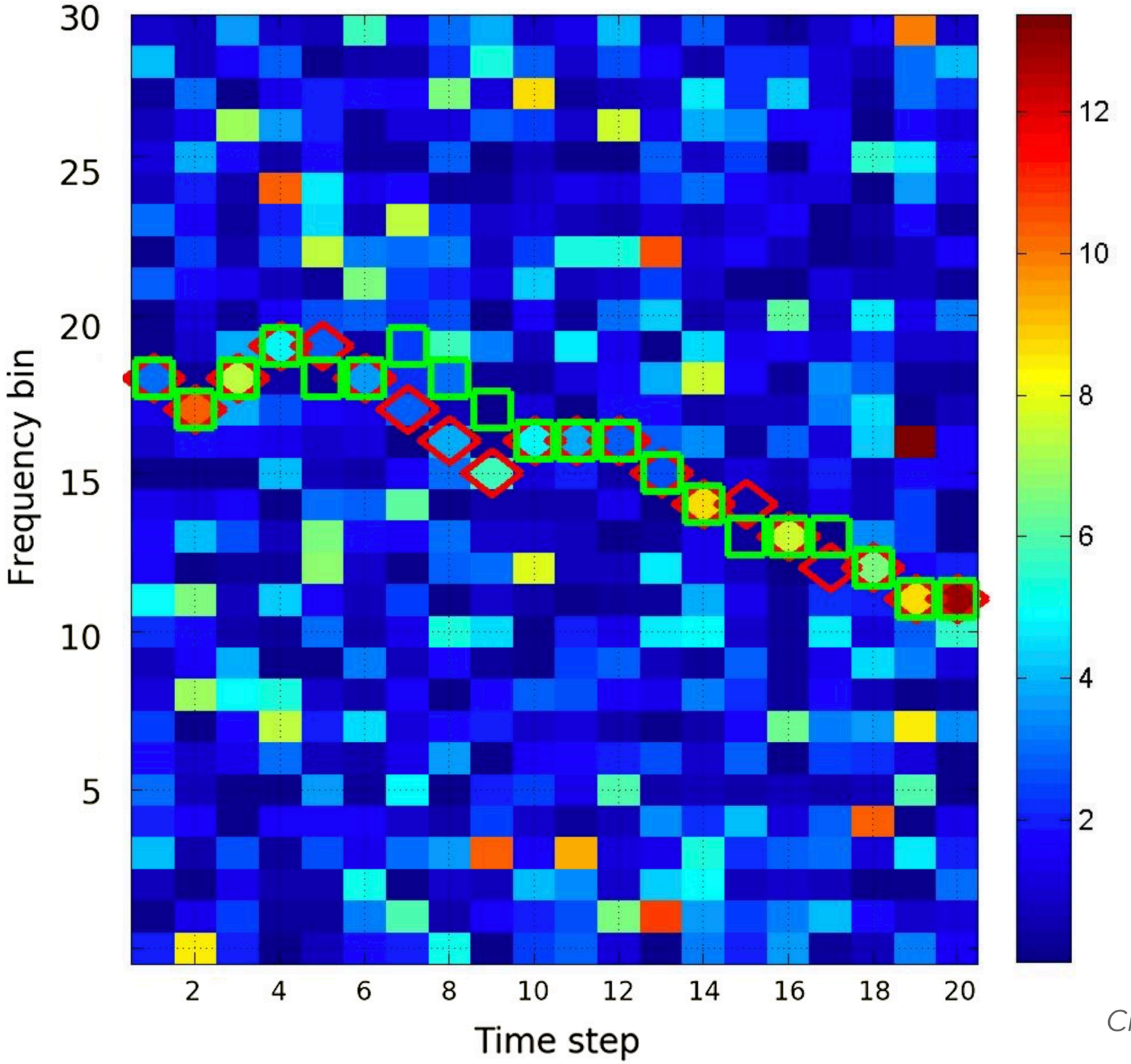


## *The Viterbi algorithm*

- *At **every forward step** t > 1 in recursion, Viterbi eliminates all but $\mathbf{N_q}$ possible state paths (the MAP path leading to each end state at each step; in total $\mathbf{N_q}$ hidden states).*

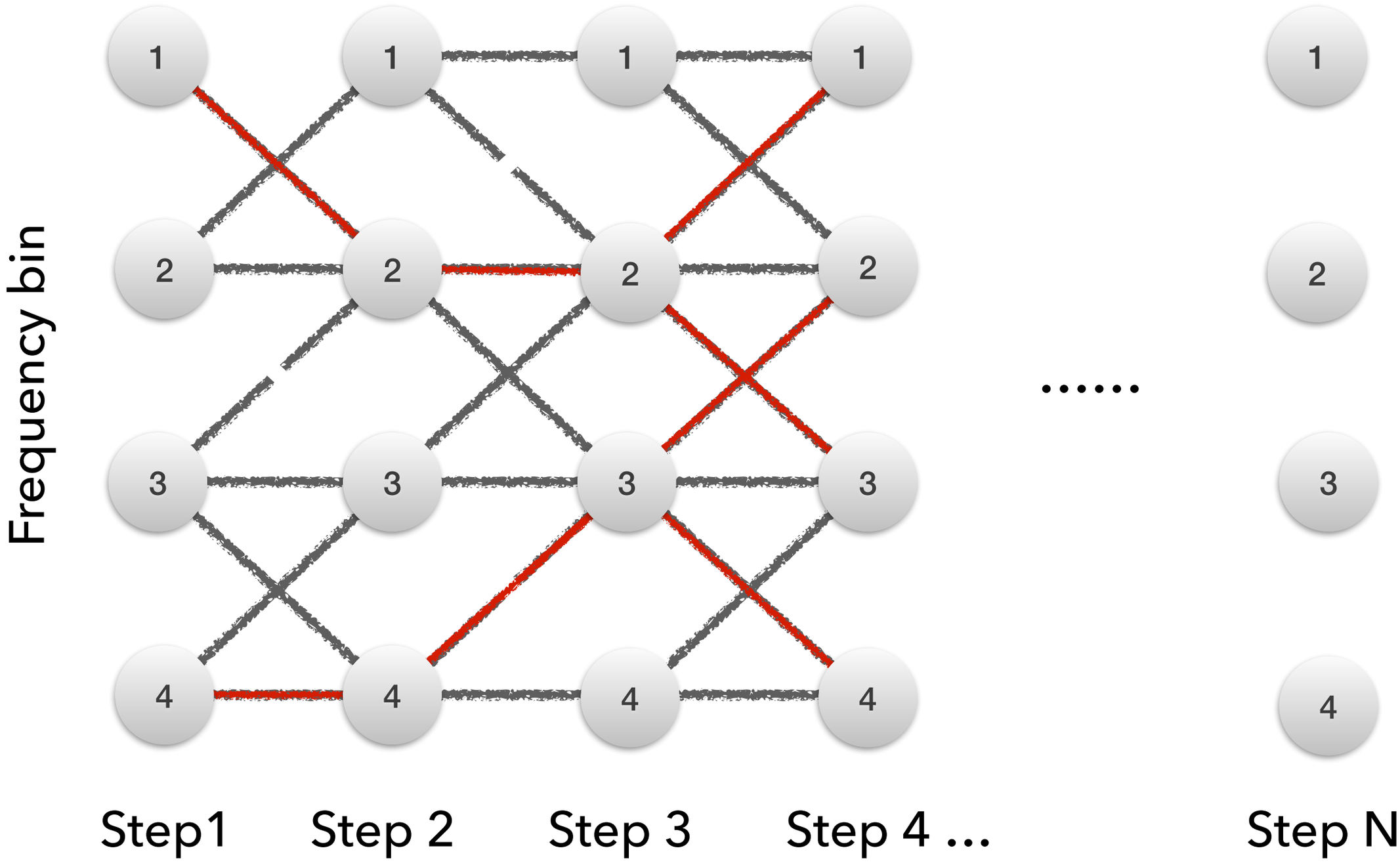
- *At the termination, find the optimal path among the $\mathbf{N_q}$ MAP paths. Then **backtrack** the optimal path from the end.*

# *Viterbi algorithm and the optimal path*



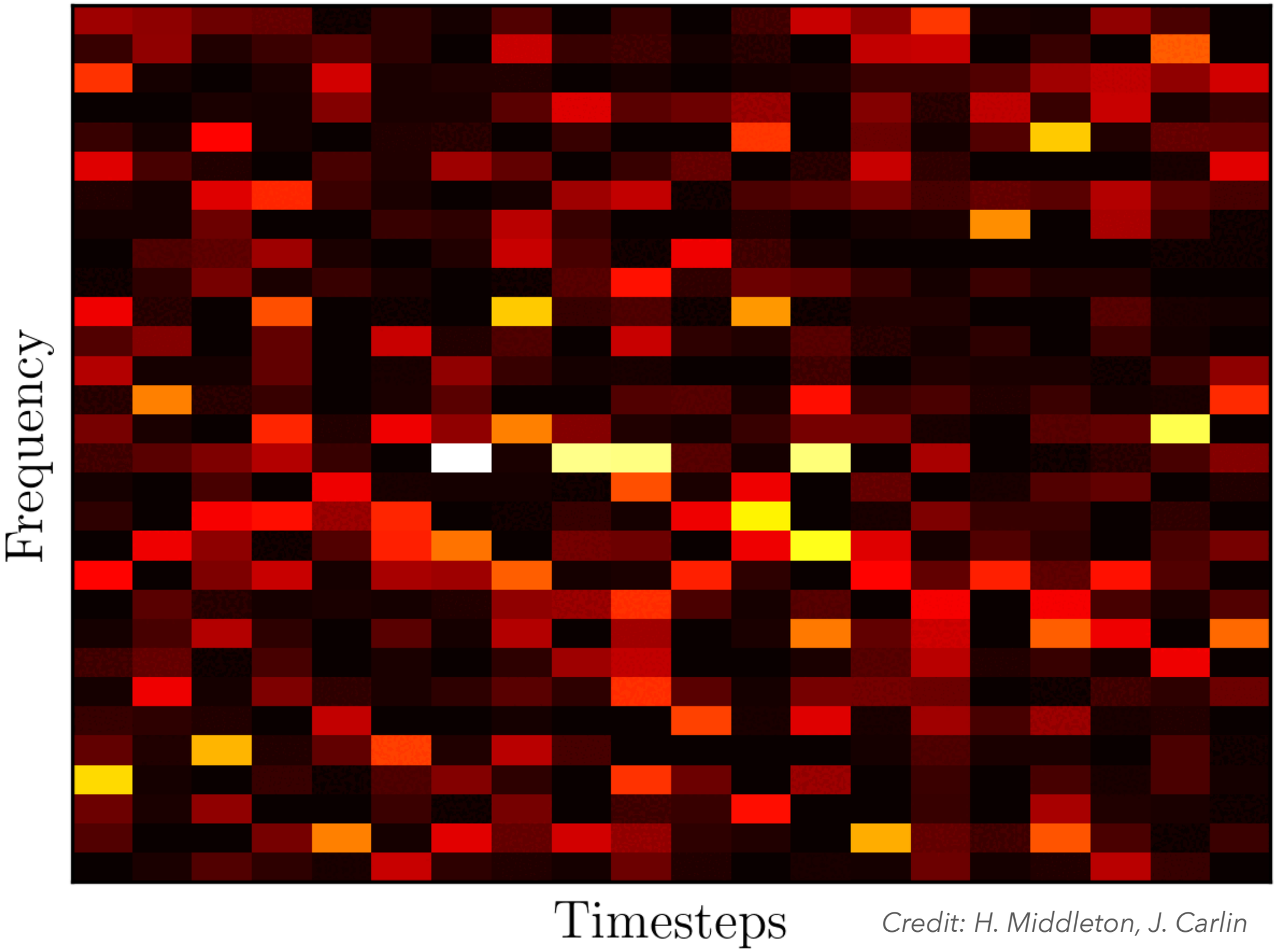

*Credit: S. Suvorova*

# How does Viterbi work –– Dynamic programming




- *Number of path comparison:* $N_q \times 3^{N_{\text{step}}}$ → $N_q \times 3 \times N_{\text{step}}$

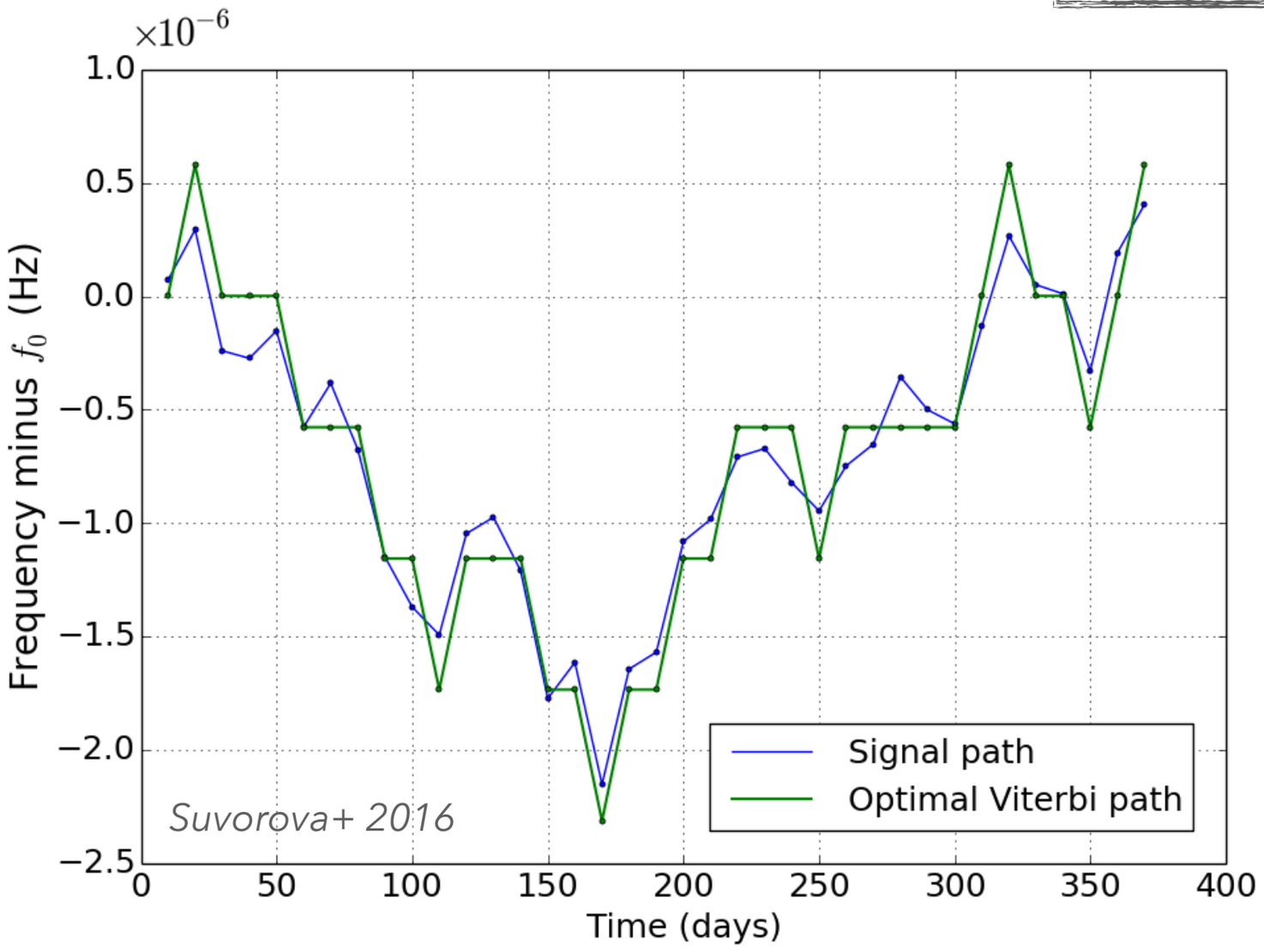
# *Tracking example*




*Credit: H. Middleton, J. Carlin*

# *Applications in GW analysis*
# *—— look for continuous waves from neutron stars*

*Fixed frequency with spin wandering*

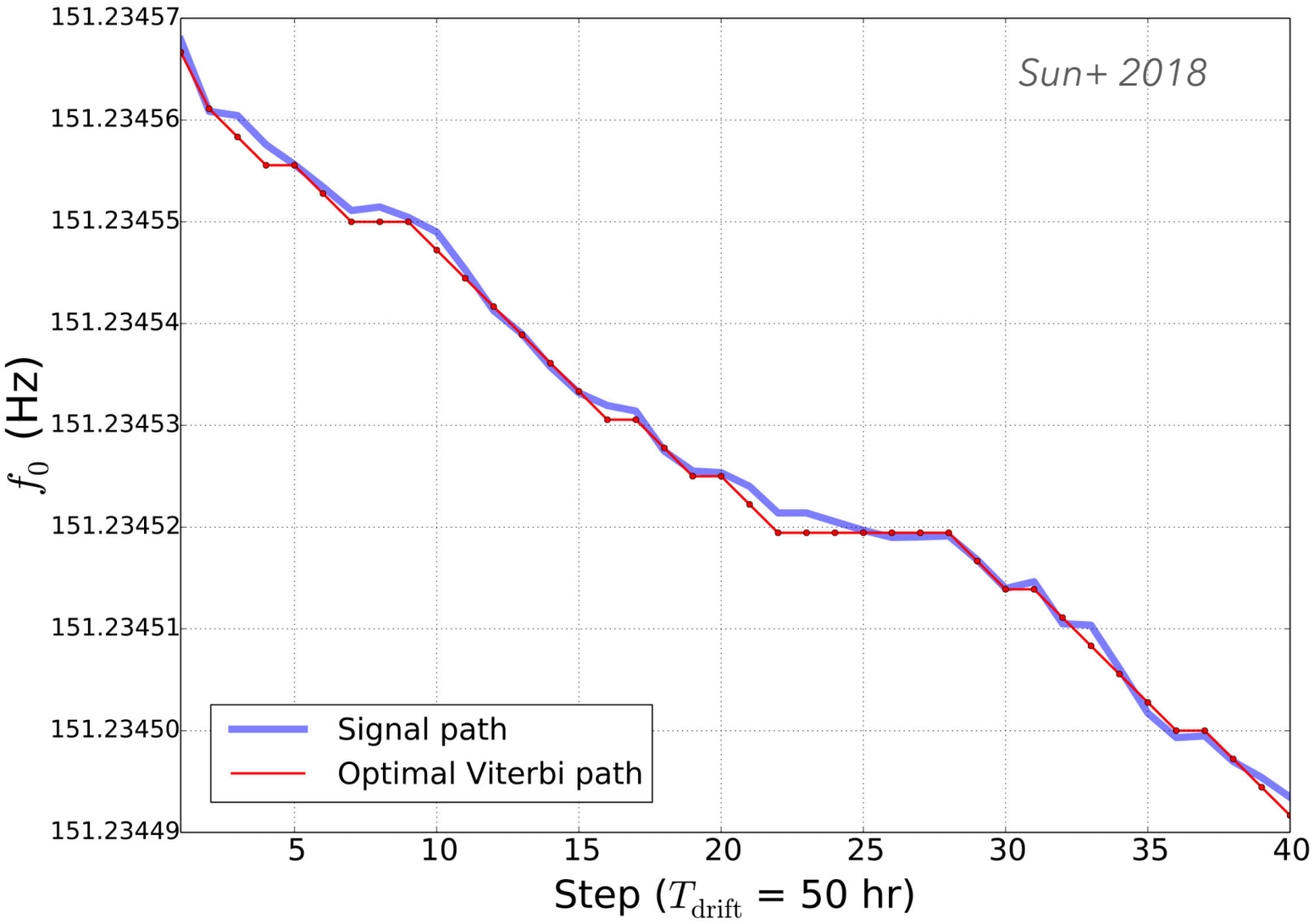



- *A neutron star in accreting binary system is spinning at a stabilised frequency.*
- *There is spin-wandering effect due to the fluctuation in the accretion torque.*

*** These are synthetic signals injected into simulated noise.*

# *Applications in GW analysis*
# *–– look for continuous waves from neutron stars*

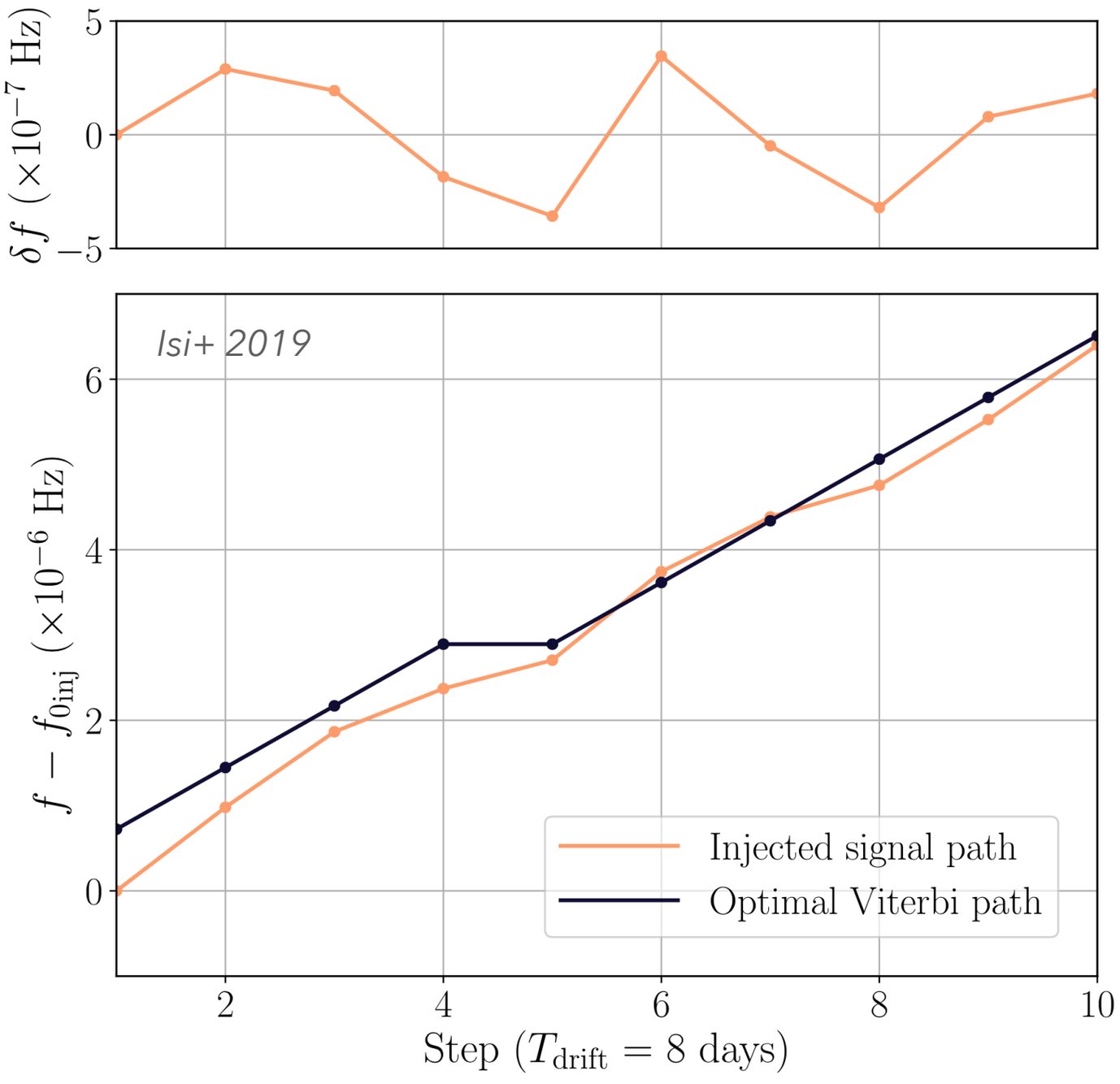
*Spin-down signal with spin wandering*




- *An isolated neutron star is spinning down due to the loss of energy.*
- *There is also small random fluctuation on top of the secular spin down.*

*** These are synthetic signals injected into simulated noise.*

# *Applications in GW analysis*
# *–– look for continuous waves from ultralight boson clouds*

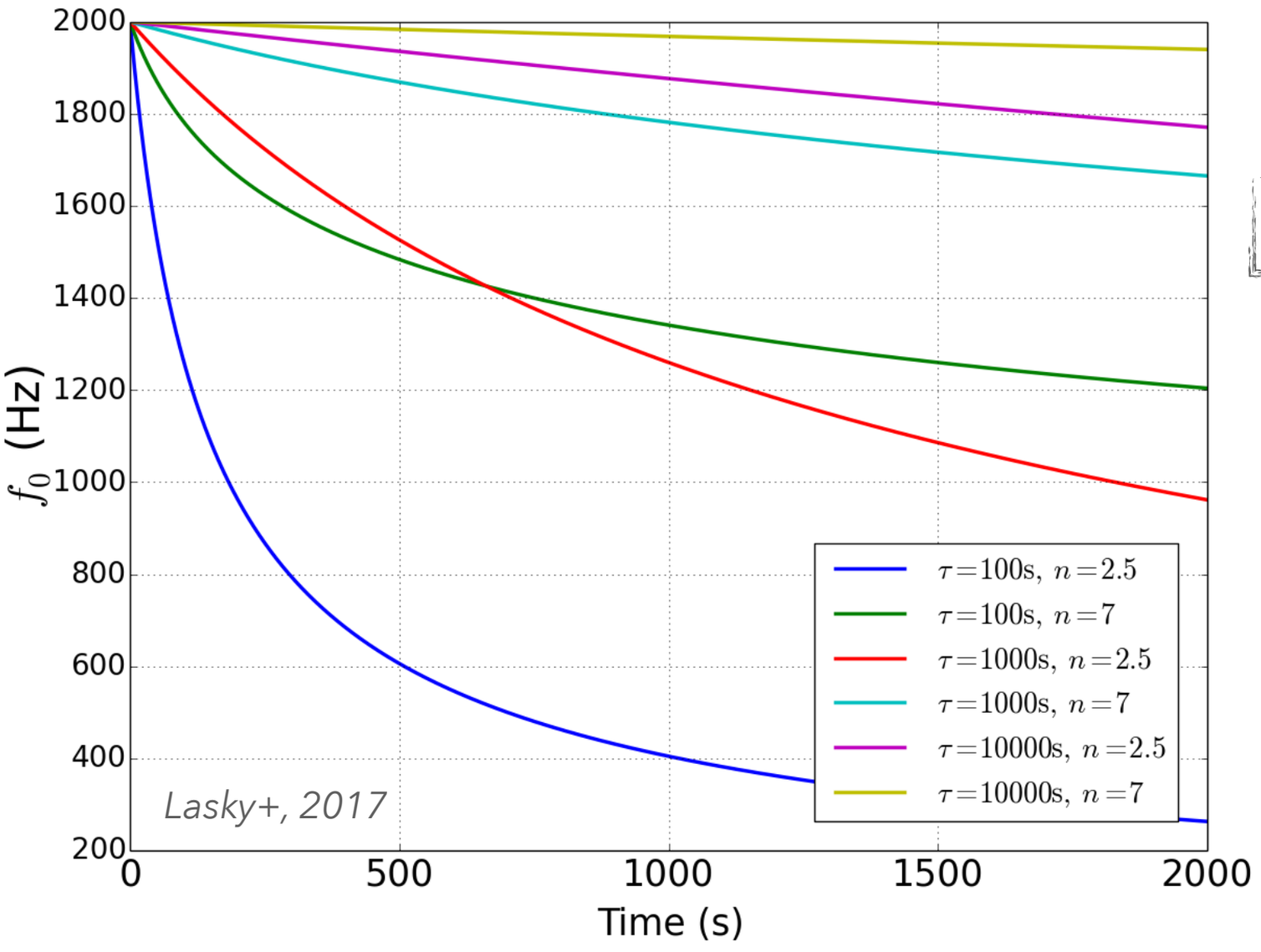



*Spin-up signal with spin wandering*

- *Theories predict signals with increasing frequencies.*
- *The signal might not strictly follow a power law due to, e.g., particle self-interaction, unpredicted features, etc.*

*** These are synthetic signals injected into simulated noise.*

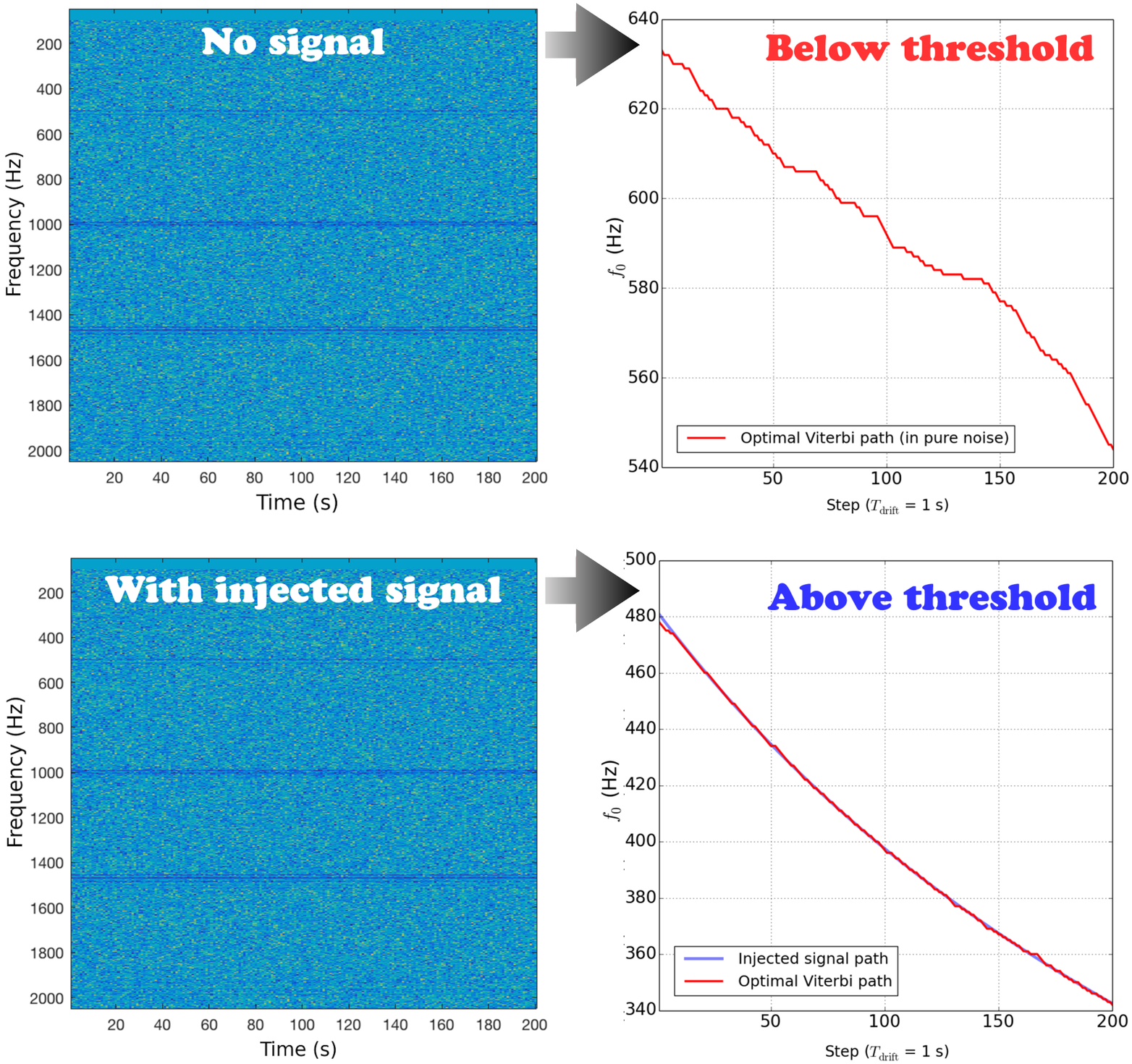
# *Applications in GW analysis*
# *–– look for long-transient waves from BNS merger remnants*




*A rapidly spinning-down signal*

- *For example, if a millisecond magnetar is formed after the merger, it spins down very quickly.*
- *There are many possibilities of the frequency evolution.*

# *Applications in GW analysis*
# *–– look for long-transient waves from BNS merger remnants*




- *Can efficiently find the most likely signal path (max power along a physical path)*
- *Also robust against transient noise*

*This method has been widely applied to many long-duration/ continuous wave searches, including the analyses in the latest observing run.*

*Abbott+ 2017; Sun & Melatos 2018*

# ***Summary***

- *There are many other smart algorithms used in the continuous wave searches.*

- *No detections yet for continuous/long-duration gravitational waves, but start to probe interesting physical regions*

- *With advanced signal processing and data analysis techniques and further improved detectors, new discovery is at the door!*

*Image: virtulearn*