



IPAM Workshop @ UCLA: Source inference and PE in GW Astronomy



# Stochastic Gravitational-Wave Backgrounds: current detection efforts and future prospects

Arianna Renzini

Caltech

November 15th 2021

- **Stochastic Gravitational-Wave Background Signal**
  - **Detection strategies with focus on interferometers**
  - **LVK Stochastic search results**
  - **The future: 3G, LISA**
- 
- See next talk for  
PTAs!*

# Stochastic Gravitational-Wave Backgrounds

incoherent superposition  $\longrightarrow$  unresolved  $\longrightarrow$  stochastic variables

fractional GW  
energy density

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln f} = \frac{32\pi^3}{3H_0^2} f^3 I(f)$$

*discards phase  
information*

GW intensity

$$I(f) = \frac{1}{2} \sum_A |h_A(f)|^2$$

(GW characteristic strain)

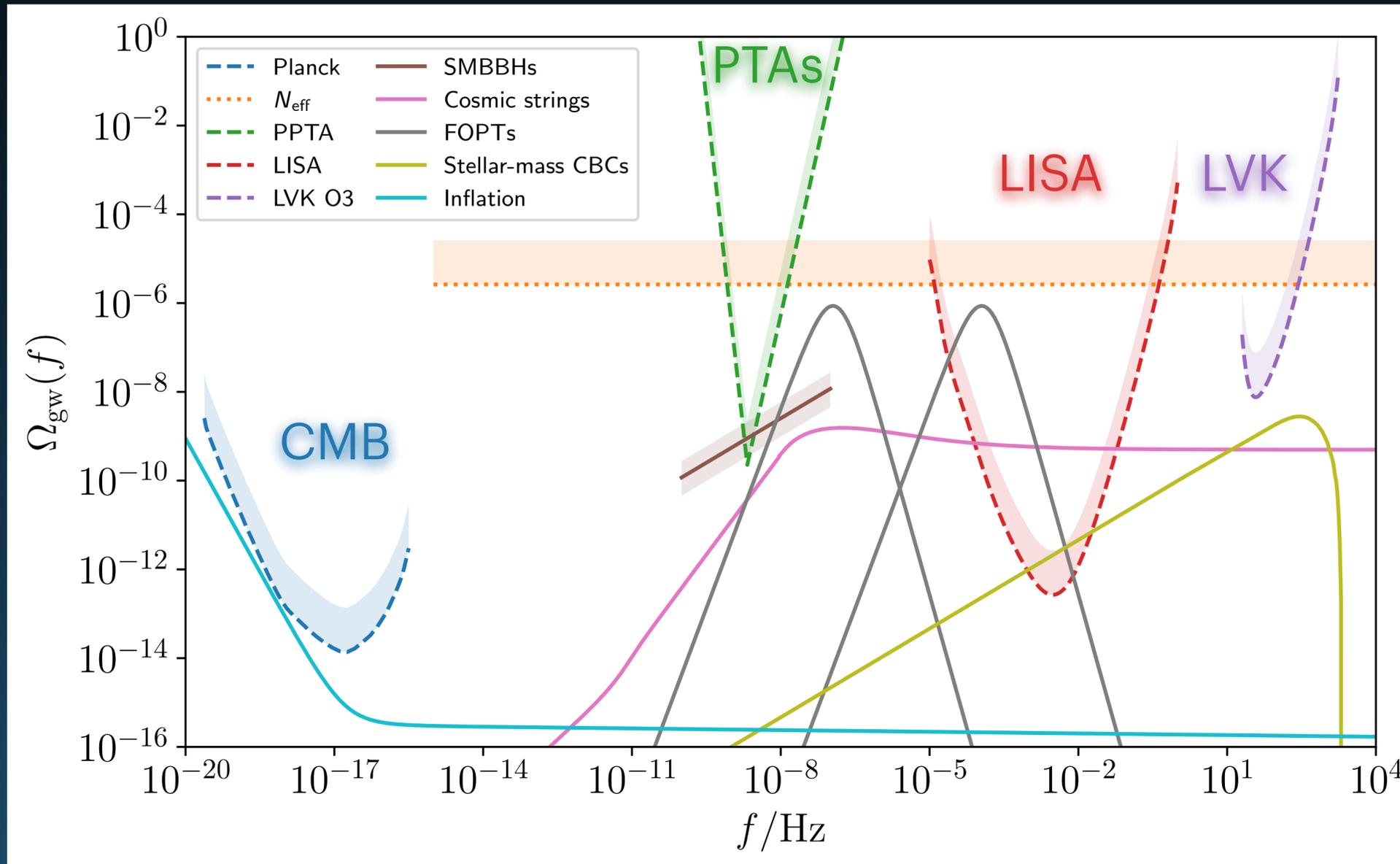
$$\left( \equiv \frac{1}{16\pi} f^{-1} h_c^2 \right)$$

from [Allen & Ottewill '97](#)

# Gravitational-Wave Background Sources

incoherent superposition  $\longrightarrow$  unresolved  $\longrightarrow$  stochastic variables

Primordial



Astrophysical

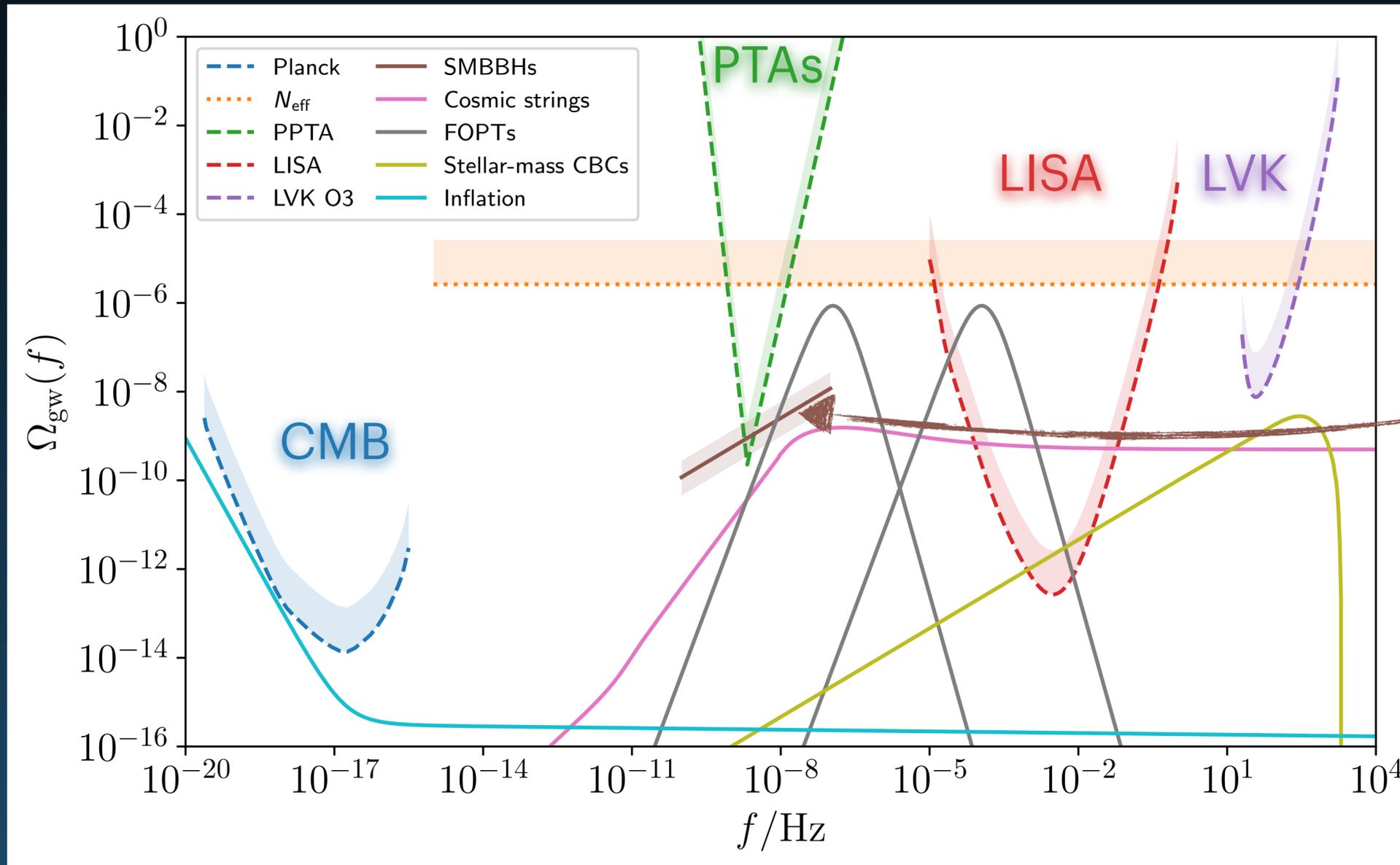
Image Credit: Alex Jenkins  
plot from *AIR et al. in prep*



# Gravitational-Wave Background Sources

incoherent superposition  $\longrightarrow$  unresolved  $\longrightarrow$  stochastic variables

**Primordial**



**Astrophysical**

stellar mass compact binary coalescences

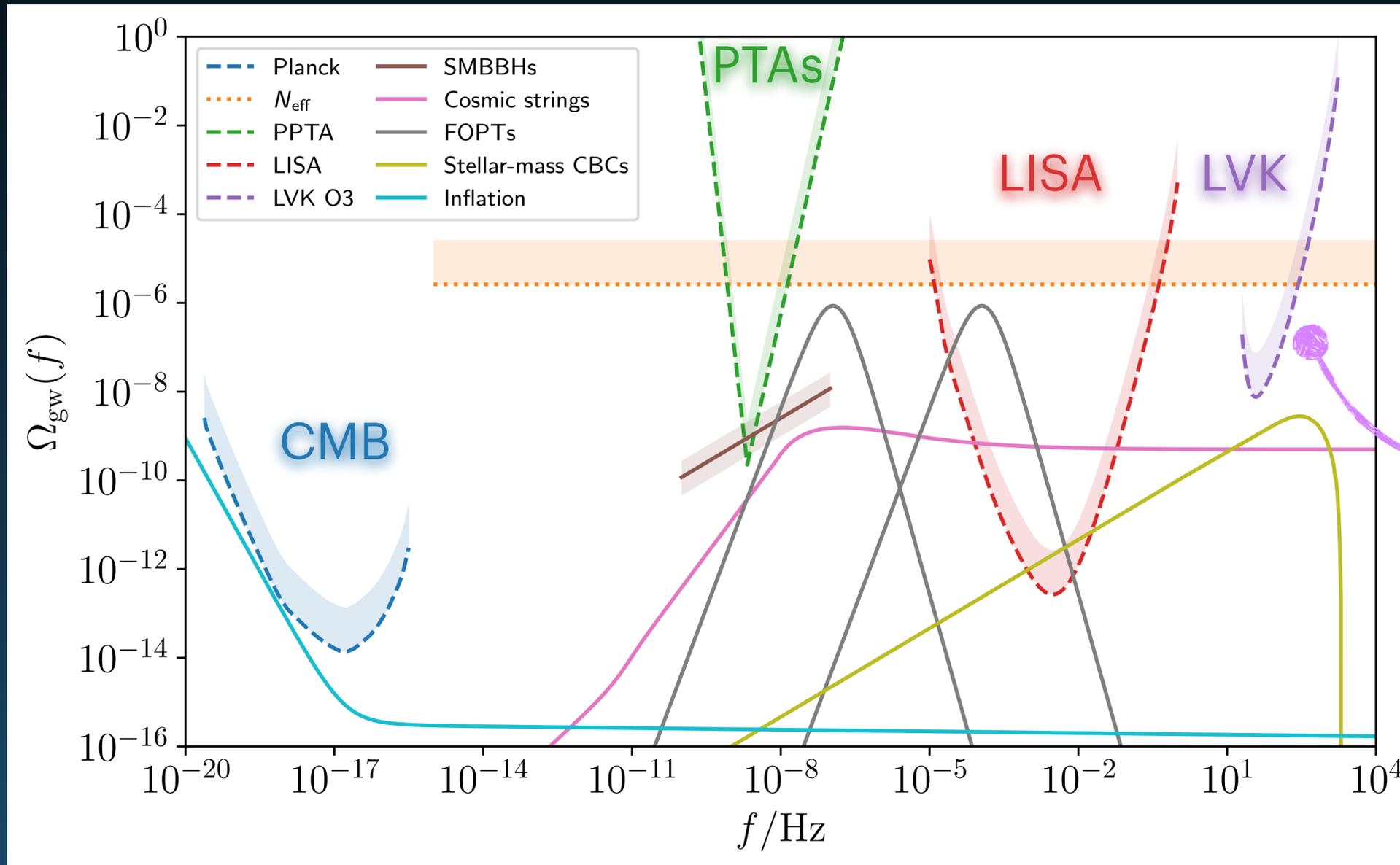
supermassive black hole binary inspirals

Image Credit: Alex Jenkins  
plot from *AIR et al. in prep*

# Gravitational-Wave Background Sources

incoherent superposition  $\longrightarrow$  unresolved  $\longrightarrow$  stochastic variables

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stellar mass black hole and neutron star CBCs

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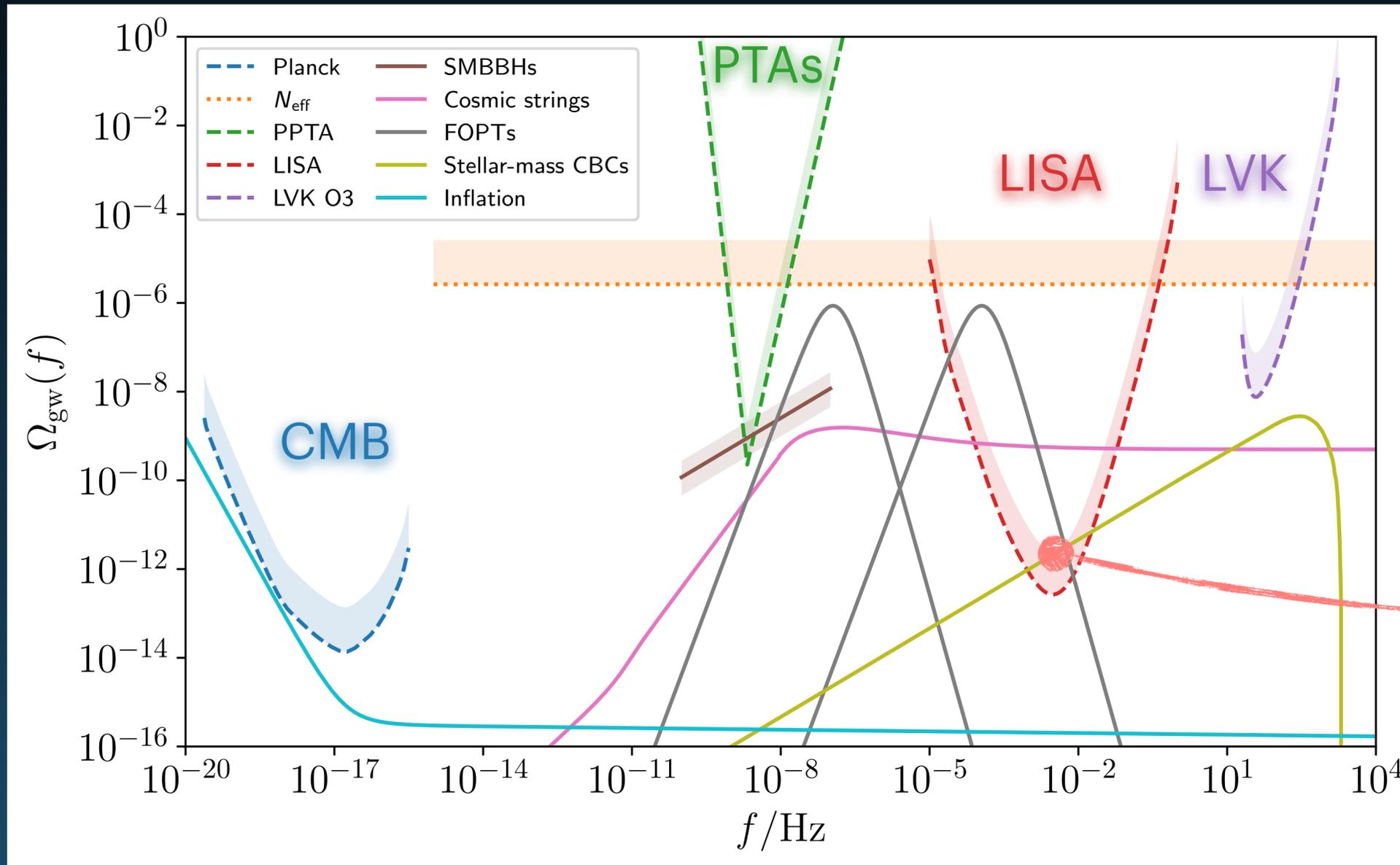
core-collapse supernovas

Image Credit: Alex Jenkins  
plot from *AIR et al. in prep*

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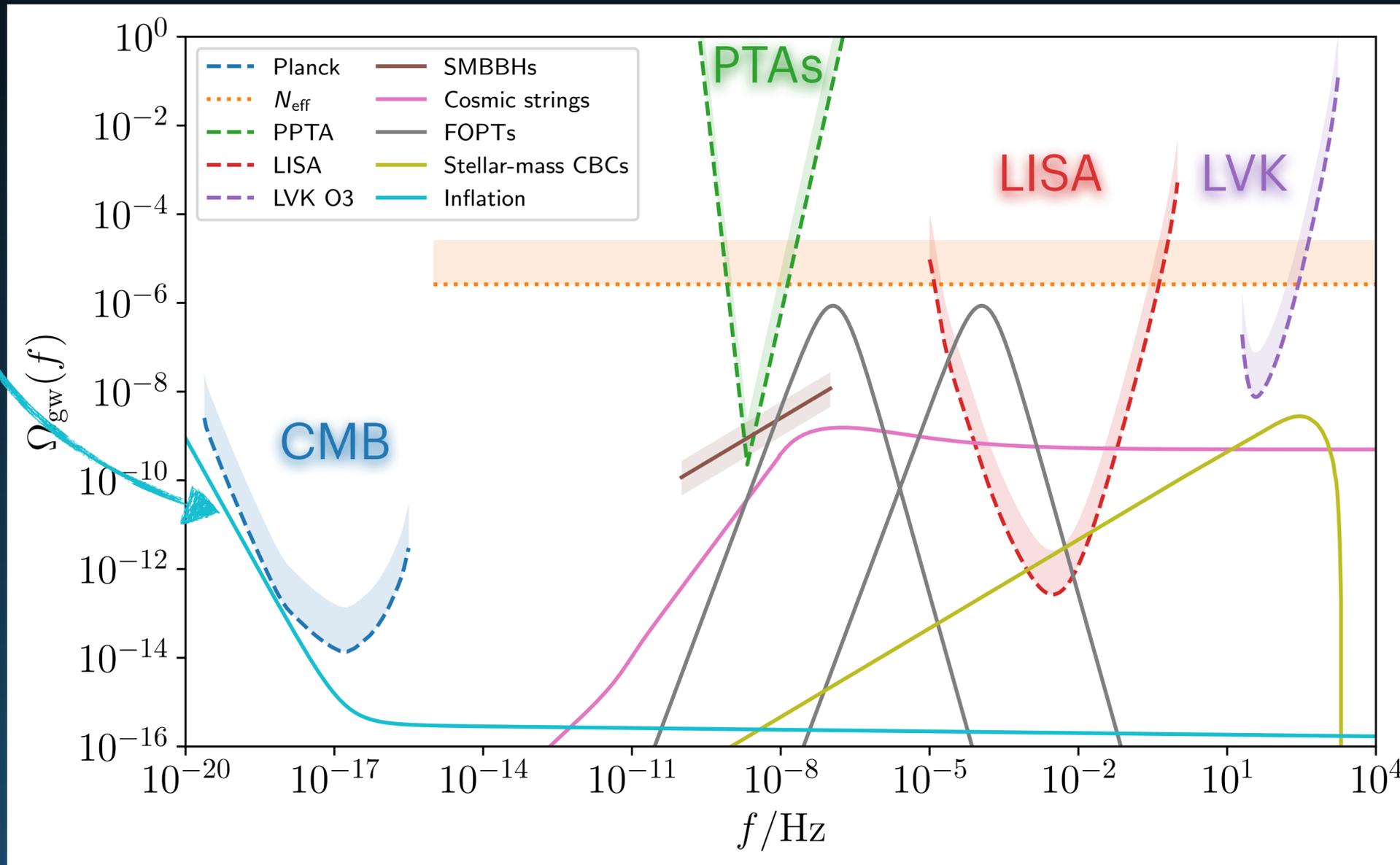
# Gravitational-Wave Background Sources

incoherent superposition  $\longrightarrow$  unresolved  $\longrightarrow$  stochastic variables

**Primordial**

**Astrophysical**

GWs from inflation



stellar mass compact binary coalescences

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Image Credit: Alex Jenkins  
plot from *AIR et al. in prep*

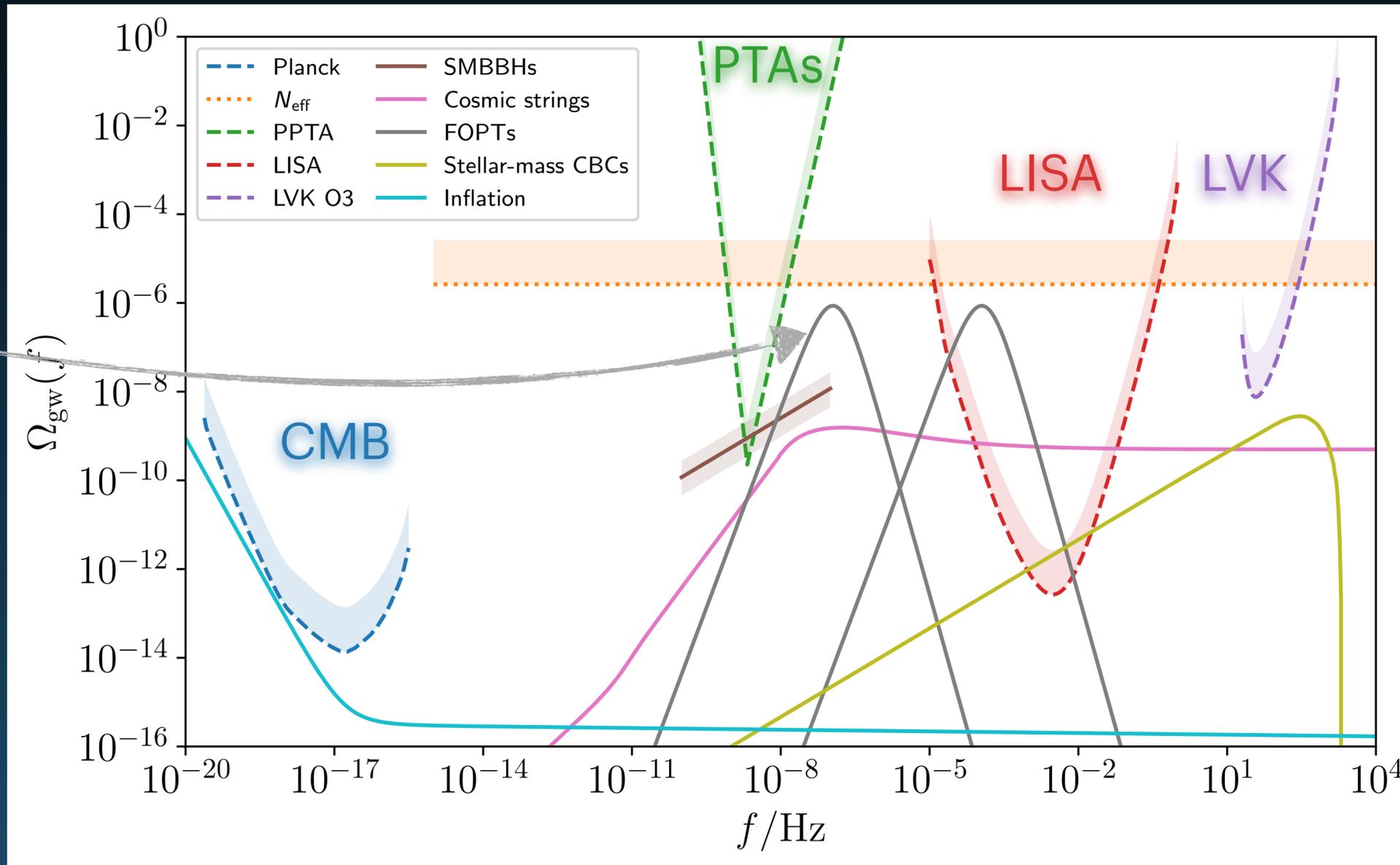
# Gravitational-Wave Background Sources

incoherent superposition  $\longrightarrow$  unresolved  $\longrightarrow$  stochastic variables

## Primordial

GWs from inflation

first order phase transitions



## Astrophysical

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Image Credit: Alex Jenkins  
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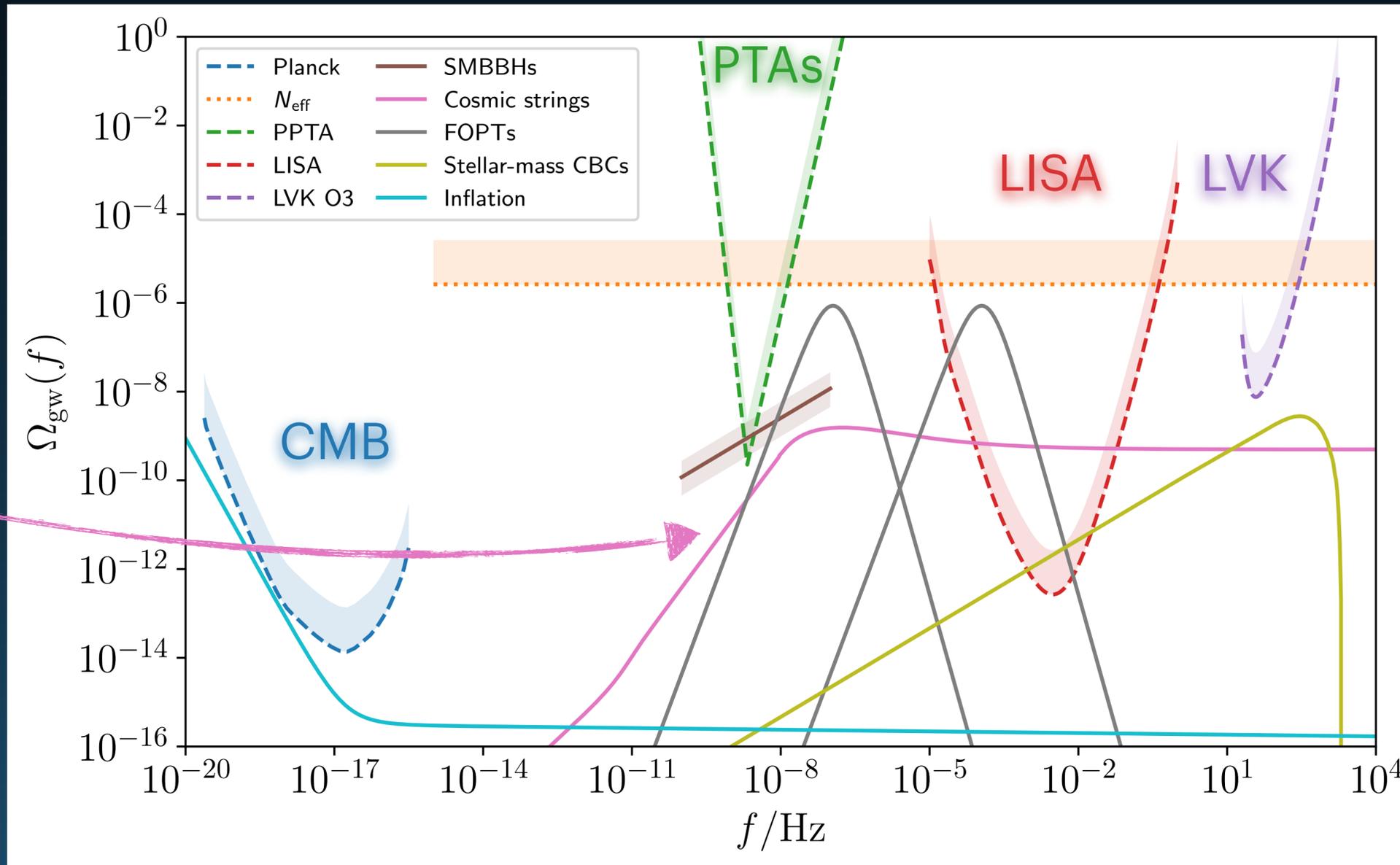
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cosmic strings



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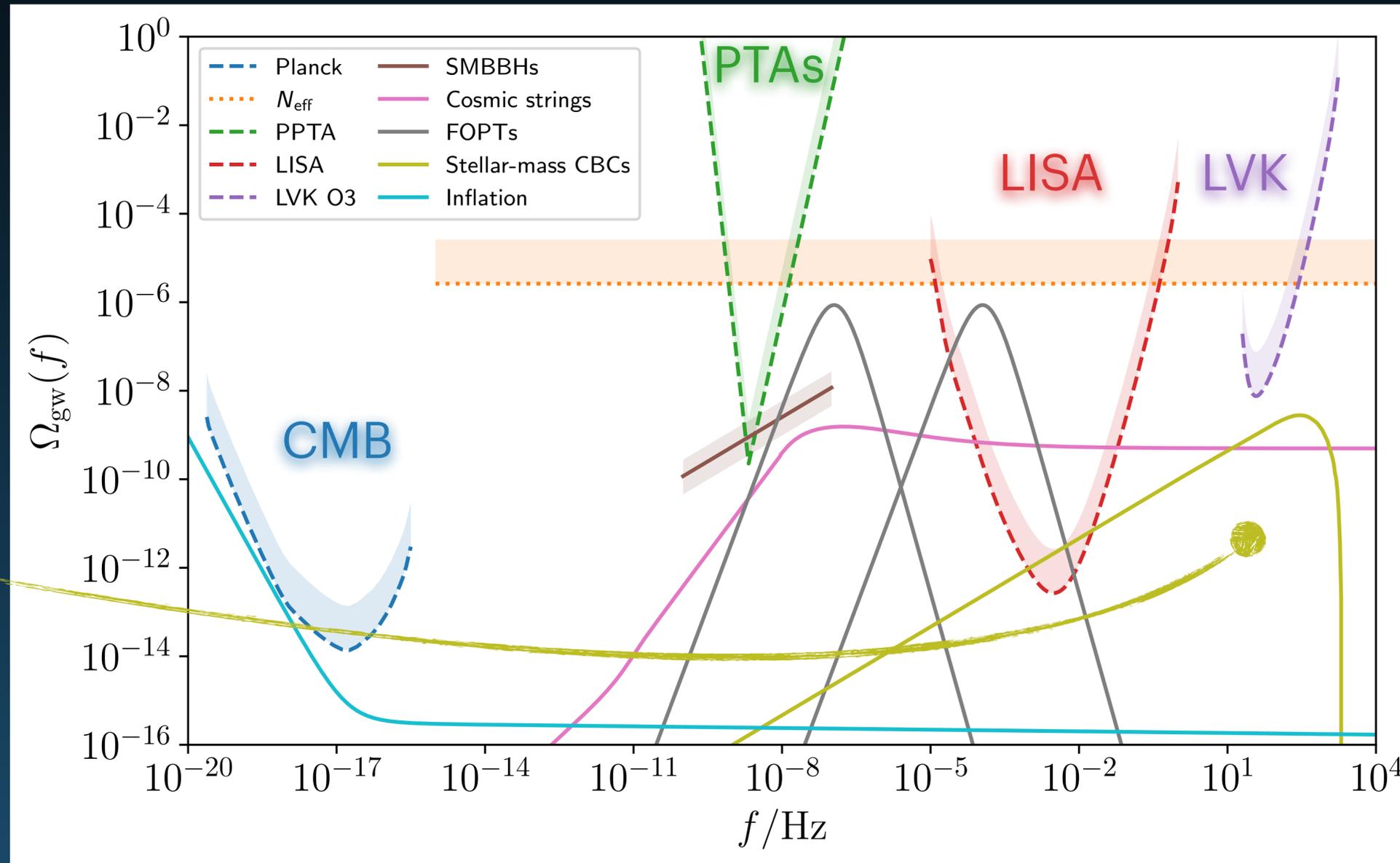
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cosmic strings

primordial black holes



## Astrophysical

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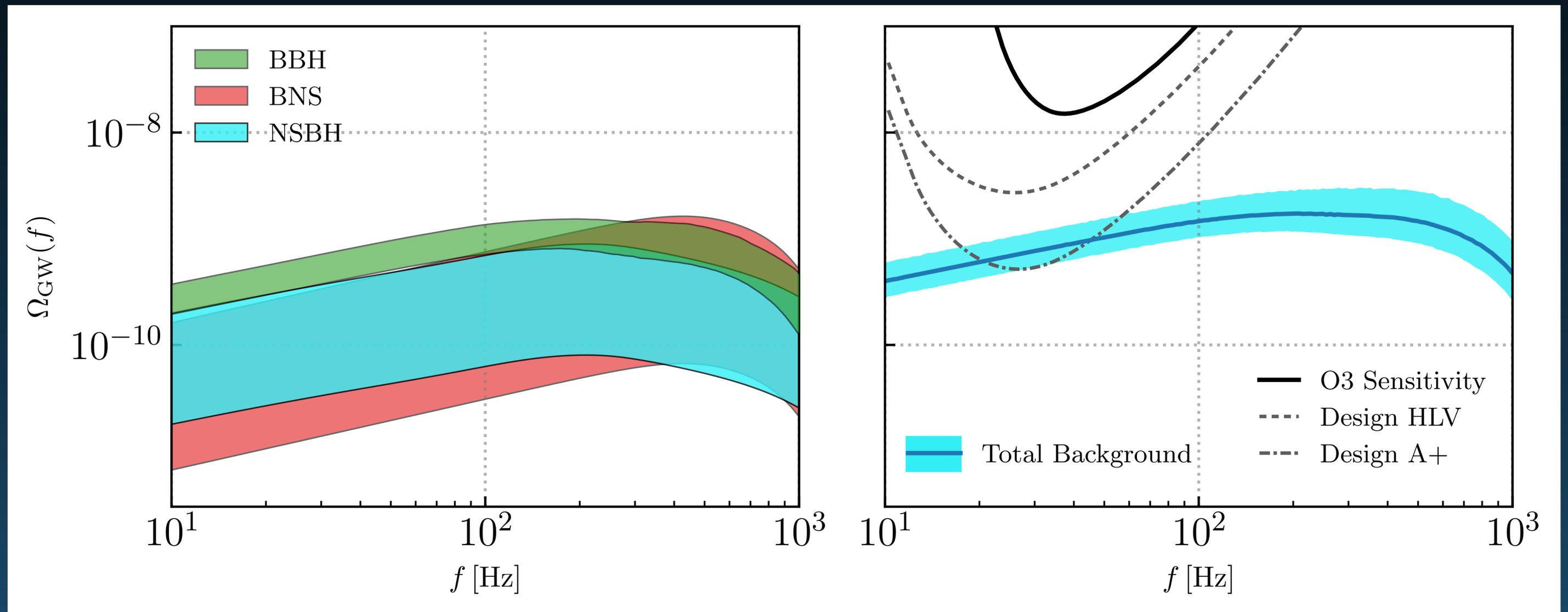
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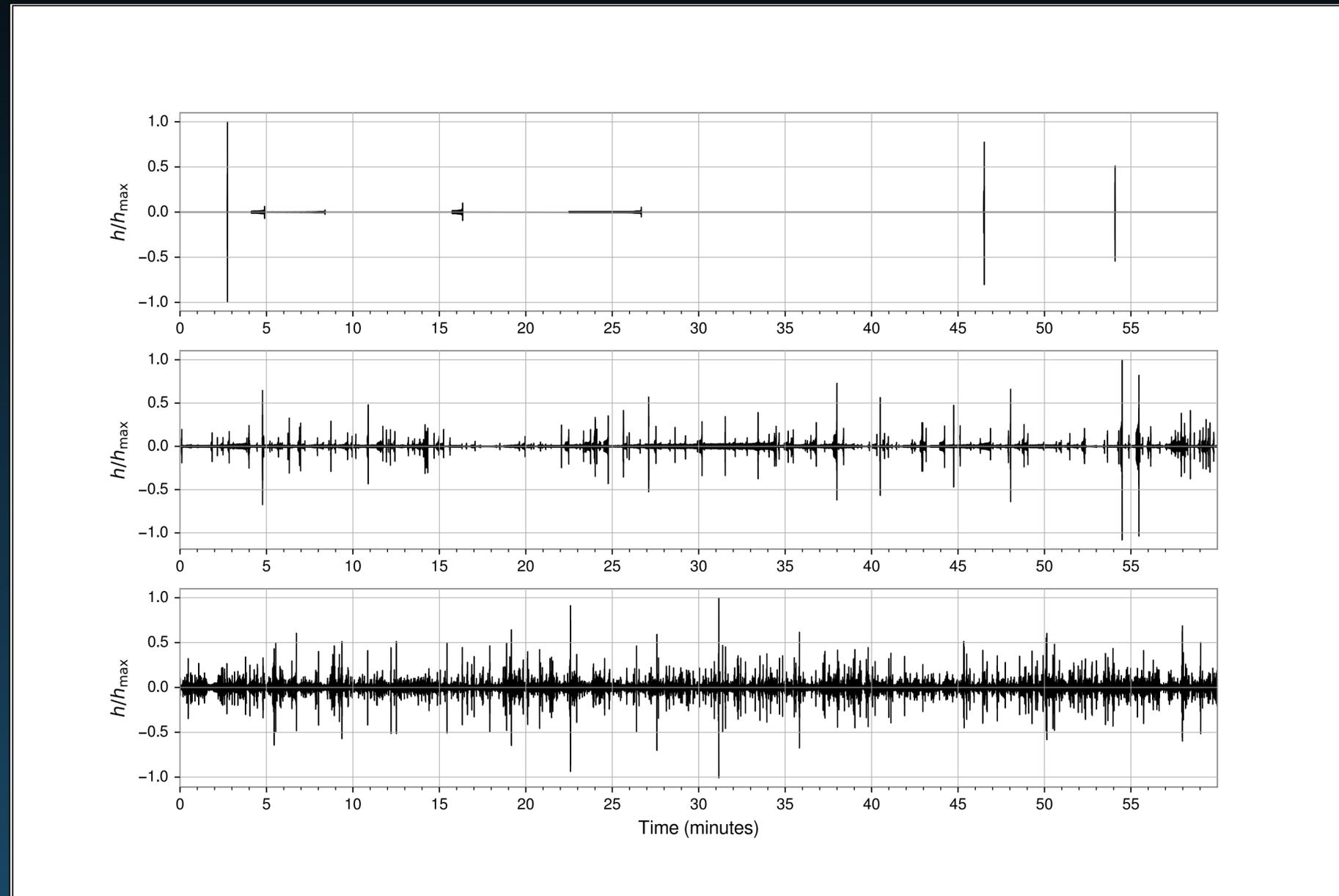
# Binary Black Hole/Neutron Star SGWB estimate from LVK

from [LVK GWTC-3 populations paper](#) out \*last week\*



# Observing SGWBs in the time domain

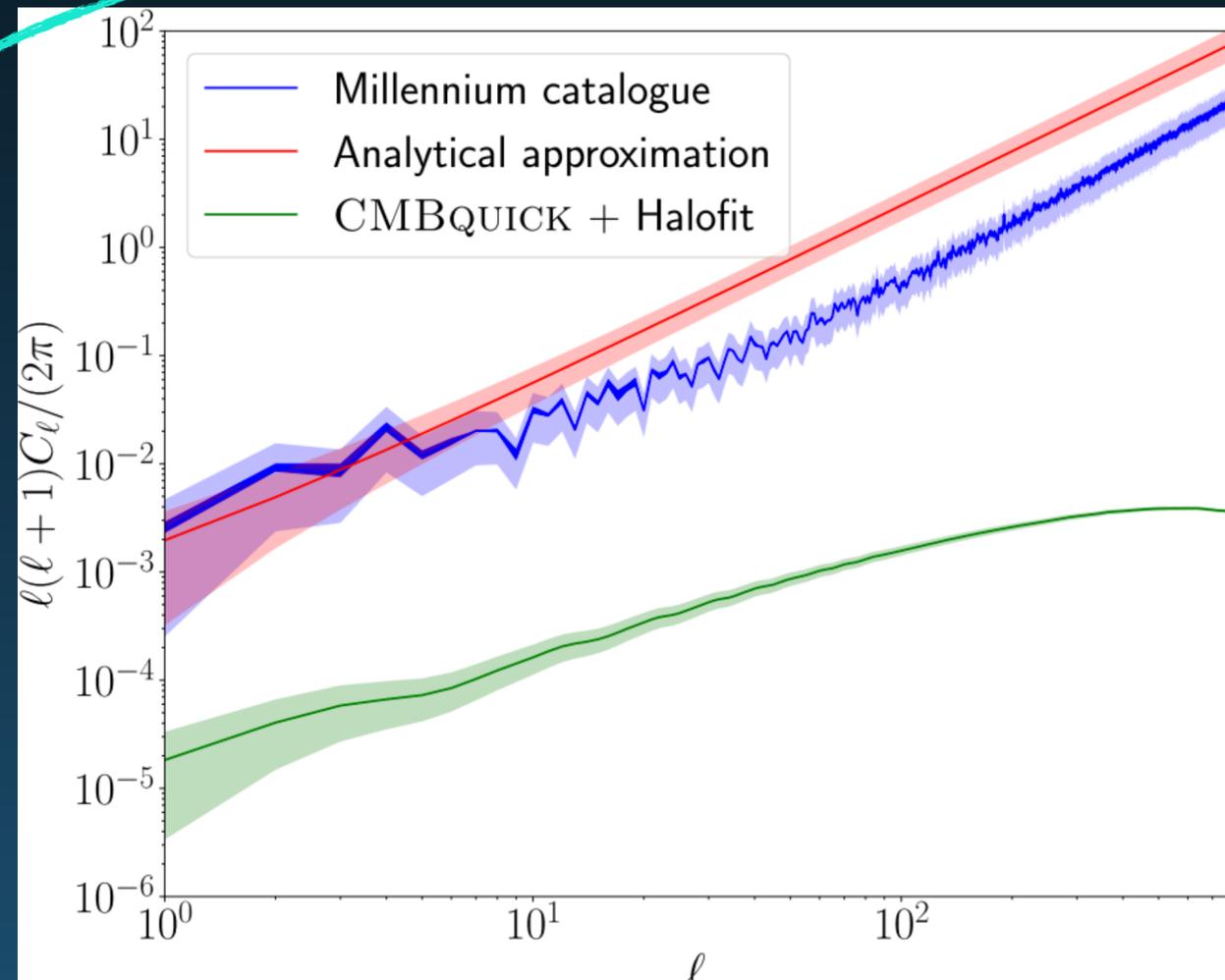
## Gaussianity/non-Gaussianity of continuous/intermittent GWBs



# Anisotropies in Stochastic Backgrounds

**GWBs often assumed statistically isotropic:**

$$\langle \Omega(f, \hat{n}), \Omega(f, \hat{n}') \rangle = \sum_{\ell=0}^{\infty} \frac{2\ell + 1}{4\pi} C_{\ell}^I(f) P_{\ell}(\cos \vartheta)$$



**...like the CMB!**

Expected GWB angular power spectrum from [Cusin et al. '18](#) & [Jenkins et al. '18](#)

# Detection methods: stochastic searches

the cross—correlation statistic

GW detectors collect timestream data which we assume:

$$d(t) = s(t) + n(t)$$

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$$d(t) = s(t) + n(t)$$

**Assuming noise is uncorrelated between detectors, search for GWB with**

**cross correlation:**  $C_{12}(f) = \tilde{d}_1(f) \tilde{d}_2^*(f)$

$$\langle C_{12}(f) \rangle = R_1(f) R_2^*(f) \langle \tilde{h}_1(f) \tilde{h}_2^*(f) \rangle = T_{\text{obs}} \Gamma_{12}(f) I_{\text{GW}}(f)$$

detector responses

overlap reduction  
function

# Detection methods: stochastic searches

the cross—correlation statistic

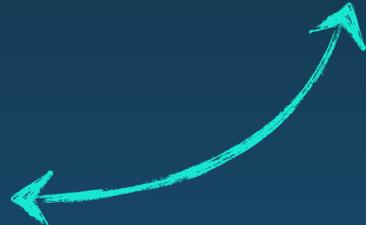
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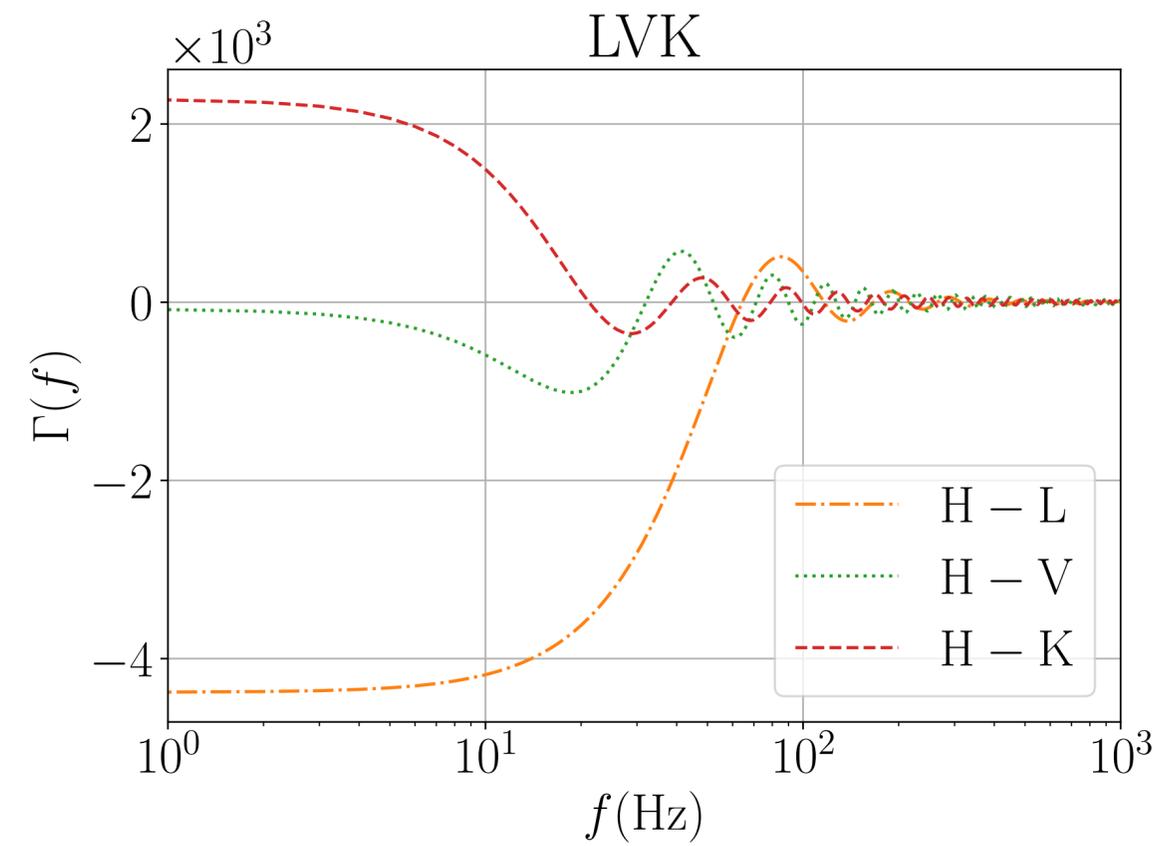
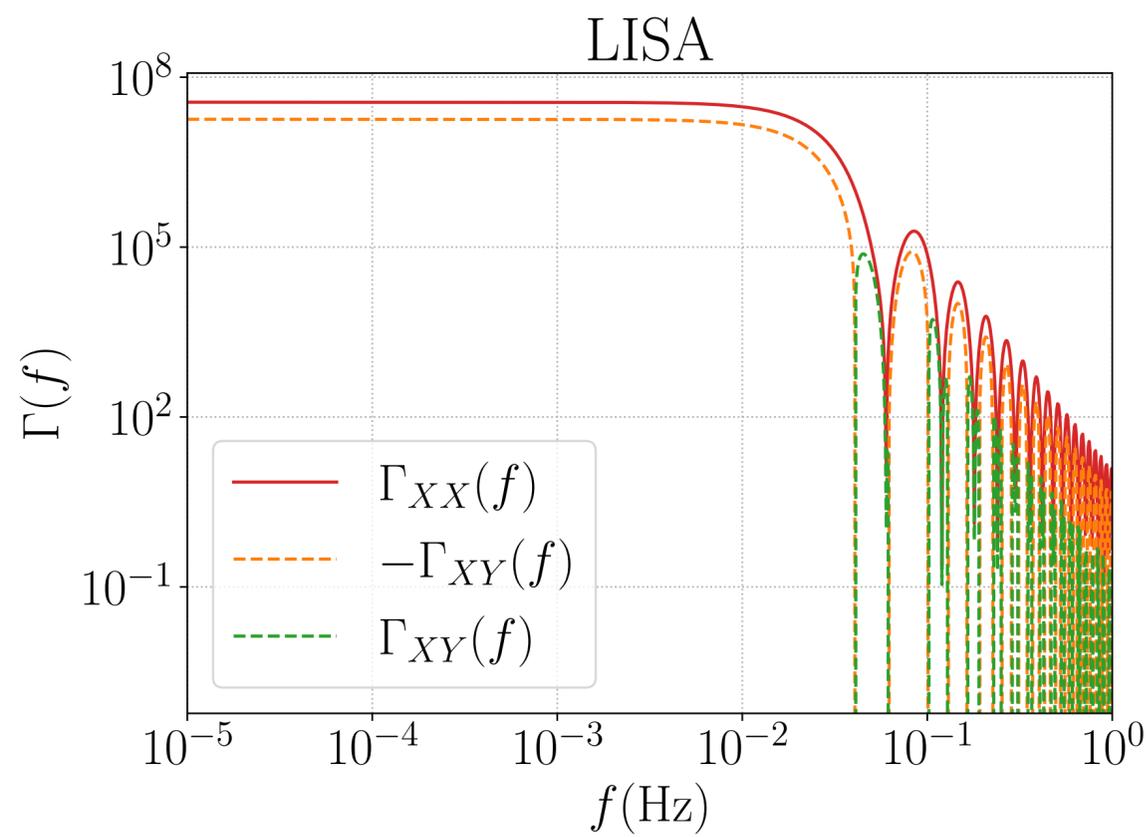
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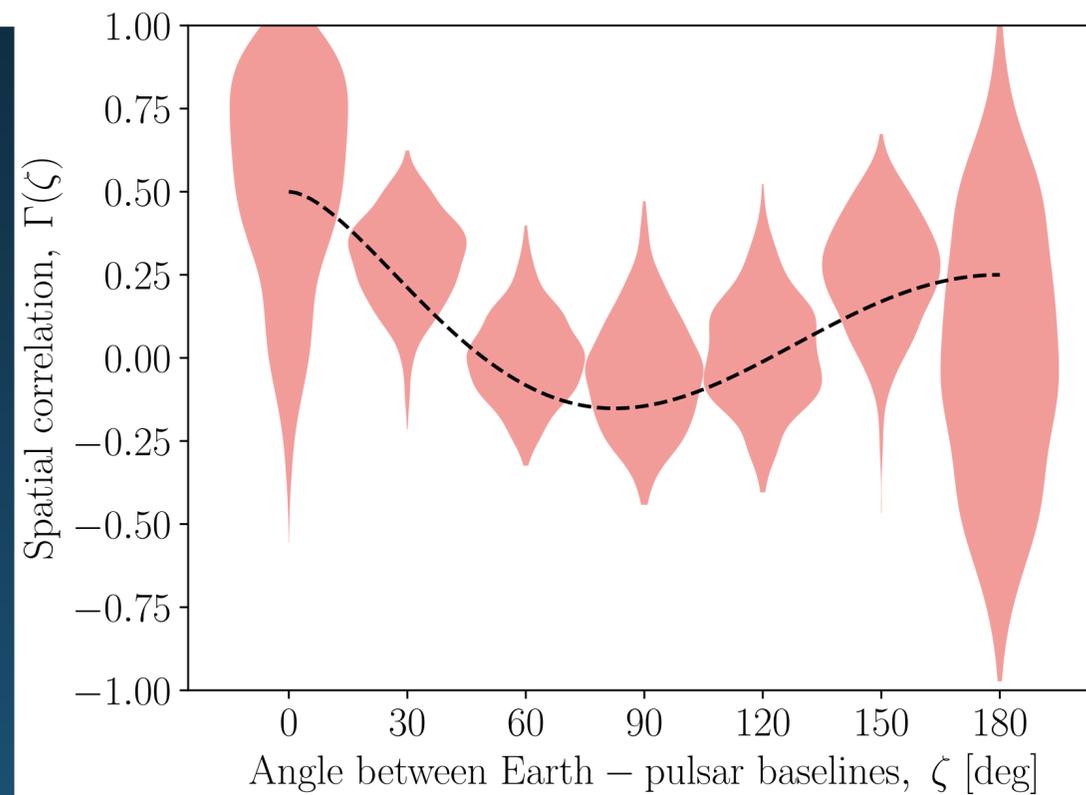
$$\langle C_{12}(f) \rangle = R_1(f) R_2^*(f) \langle \tilde{h}_1(f) \tilde{h}_2^*(f) \rangle = T_{\text{obs}} \Gamma_{12}(f) I_{\text{GW}}(f)$$

$$\langle C_{12}(f) \rangle \propto T_{\text{obs}} \Gamma_{12}(f) f^{-3} \Omega_{\text{GW}}(f)$$


# Overlap functions

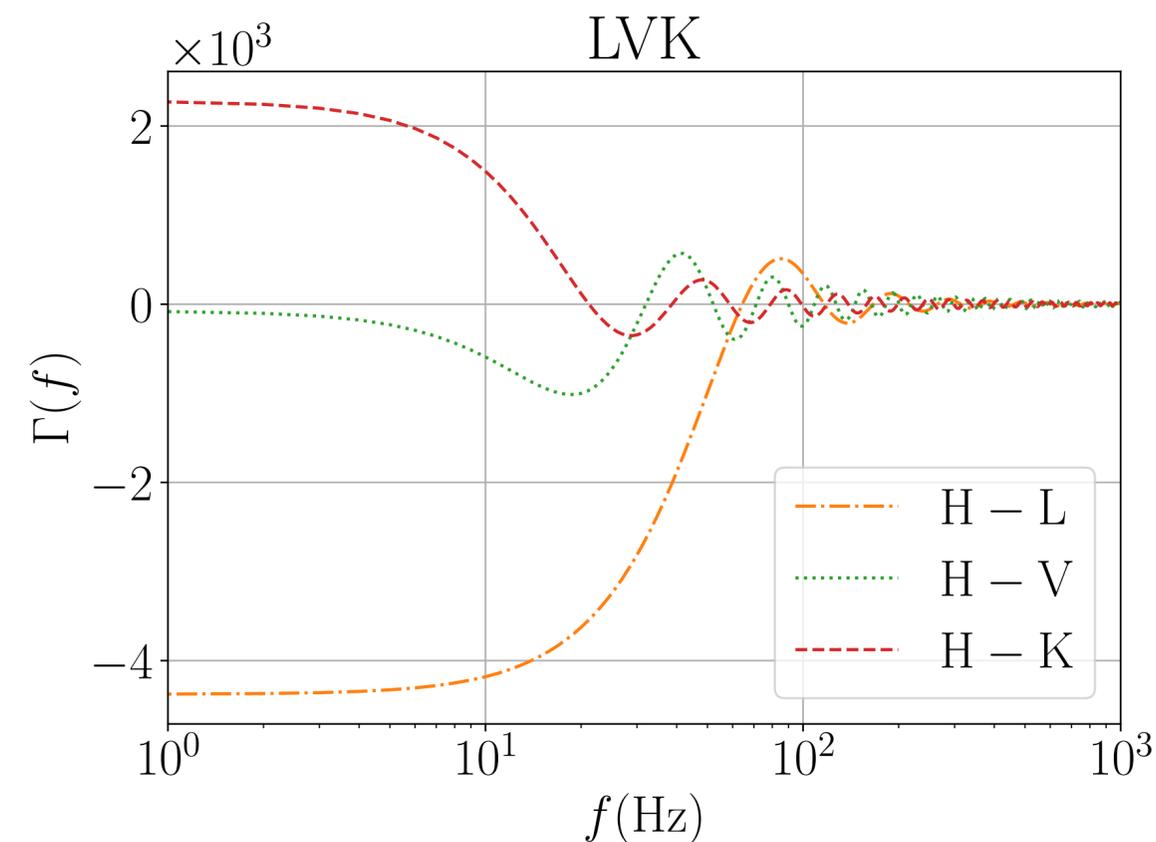
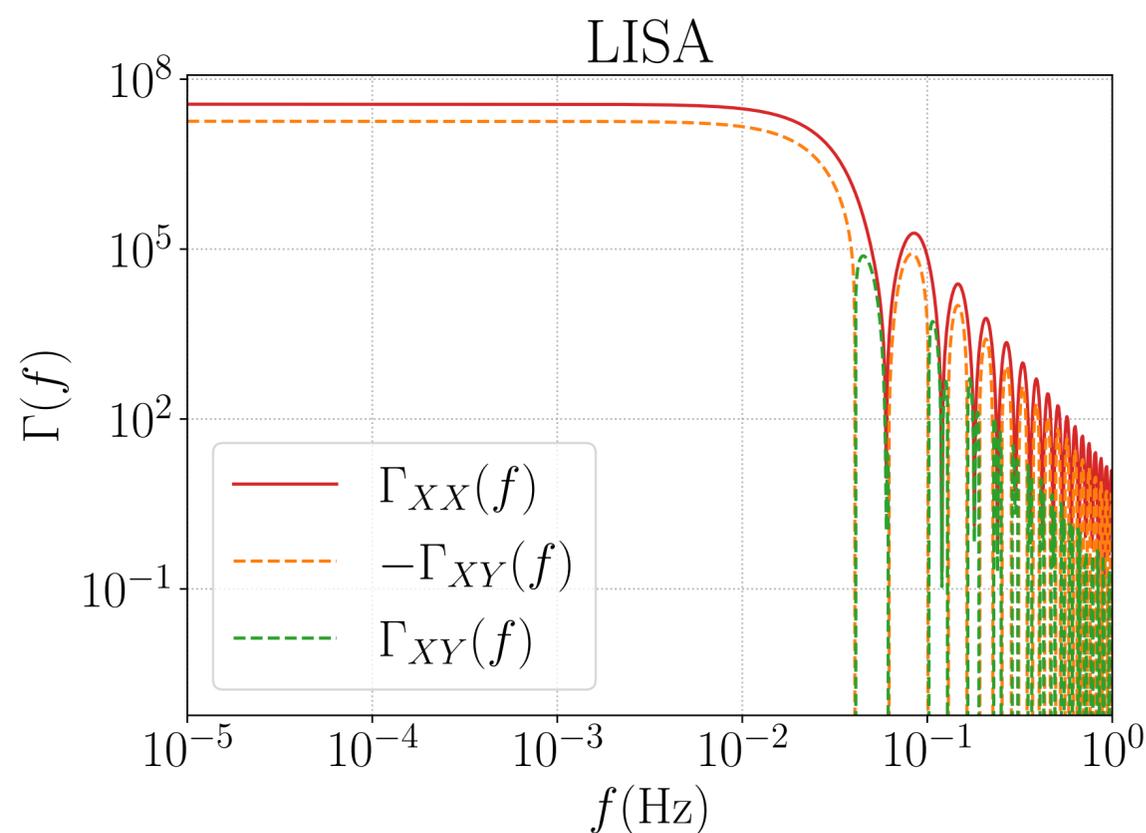


PTAs:



next talk!

# Overlap functions



“large antenna”:

$$L \approx c/f_{\text{GW}}$$

arm transfer function  
is modulated by  $L \cdot f_{\text{GW}}$   
zeros when  $L \equiv f_{\text{GW}}$

“small antenna”:

$$L \ll c/f_{\text{GW}}$$

arm transfer function  
is constant; modulations  
given by **baseline length**

# Stochastic searches: gaussian isotropic signal\*

Gaussian assumption:  $\langle d \rangle = 0$

$$\mathcal{L}(d | I(\hat{n})) \propto \prod_{f, \tau} \frac{1}{|C|^{1/2}} e^{-\frac{1}{2} d^\dagger C^{-1} d}$$

data covariance :

$$C = \Gamma \cdot I + N$$

GWB Intensity

Noise covariance

\*focus on laser interferometers; for PTAs see talk

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*low-signal limit*

[Matas Romano '21](#)



$$\mathcal{L}(C_{ij})_{i \neq j} \propto \prod_{f,\tau} \frac{1}{\sigma^2} e^{\frac{1}{2} (C_{ij} - \langle C_{ij} \rangle) \sigma^{-2} (C_{ij} - \langle C_{ij} \rangle)^*}$$

# Stochastic searches: gaussian isotropic signal

Gaussian assumption:  $\langle d \rangle = 0$

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*low-signal limit*

Matas Romano '21

## MODEL

- signal + uncorr. noise
- s + corr. n + uncorr. n
- corr. n + uncorr. n
- ...

$$\log \mathcal{L}(\hat{\Omega}_{\text{GW}}(f) | \Theta) \propto \frac{1}{2} \sum_{f, \tau} \left( \frac{\hat{\Omega}_{\text{GW}}(f) - \Omega_{\text{M}}(f | \Theta)}{\sigma_{\Omega}^2(f)} \right)^2$$

# Stochastic searches: spectral weighting

- Spectral shape usually fitted to a power law:  $\Omega_{\text{GW}} \propto f^\alpha$
- Either fix  $\alpha$  or keep as parameter in **Bayesian fit**
- binned narrowband frequency fitting

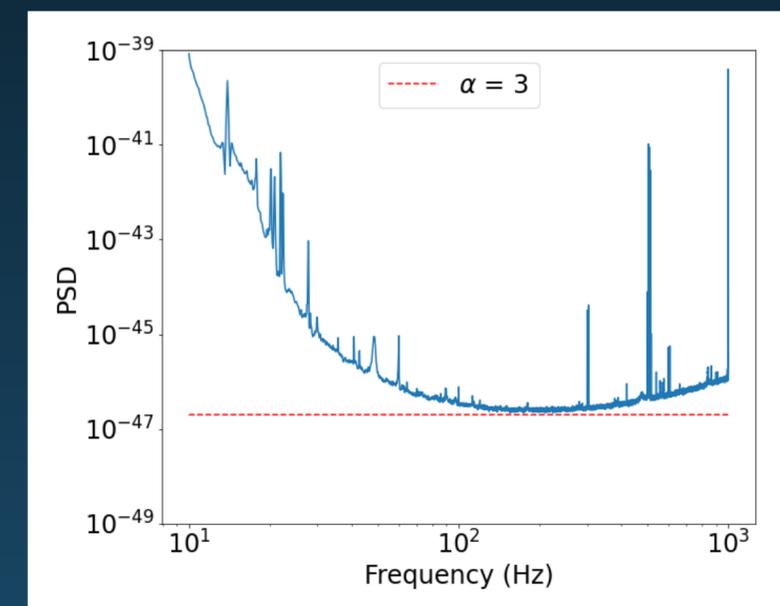
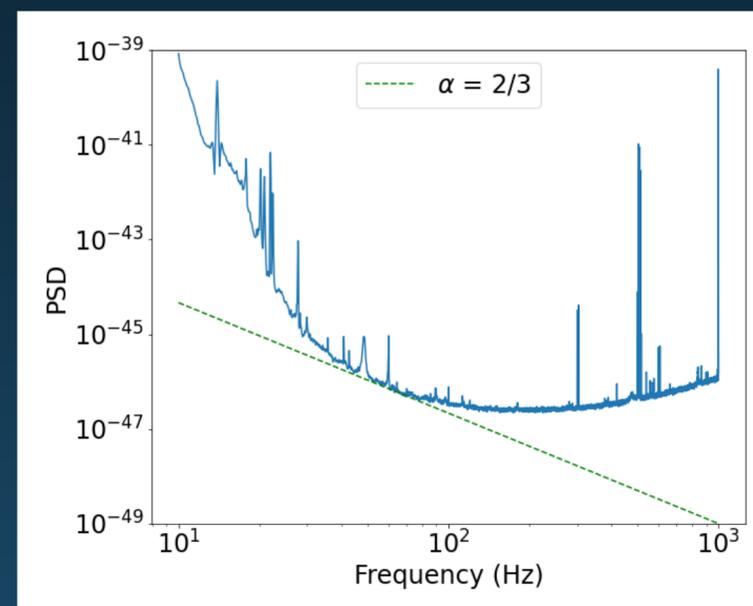
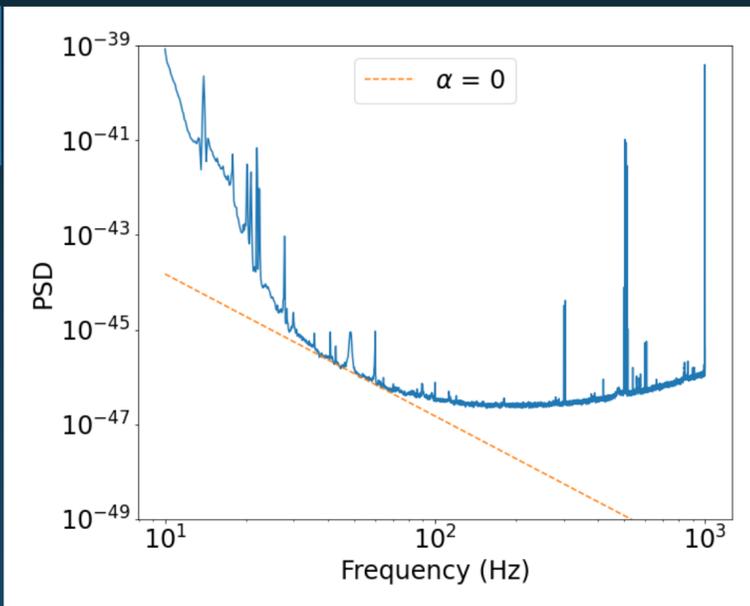
$$\Omega_{\text{GW}} \propto f^\alpha$$

*“cosmological”*

*“inspiral/astro”*

*“best fit”*

LIGO  
NOISE PSD



# Stochastic searches: anisotropic signal

add directional dependence in model:

$$\langle C_{ij}^{\tau}(f) \rangle = \int_{S^2} d\hat{n} \Gamma_{ij}^{\tau}(f, \hat{n}) I(f, \hat{n})$$

adds many parameters to estimate... information quantified by Fisher:

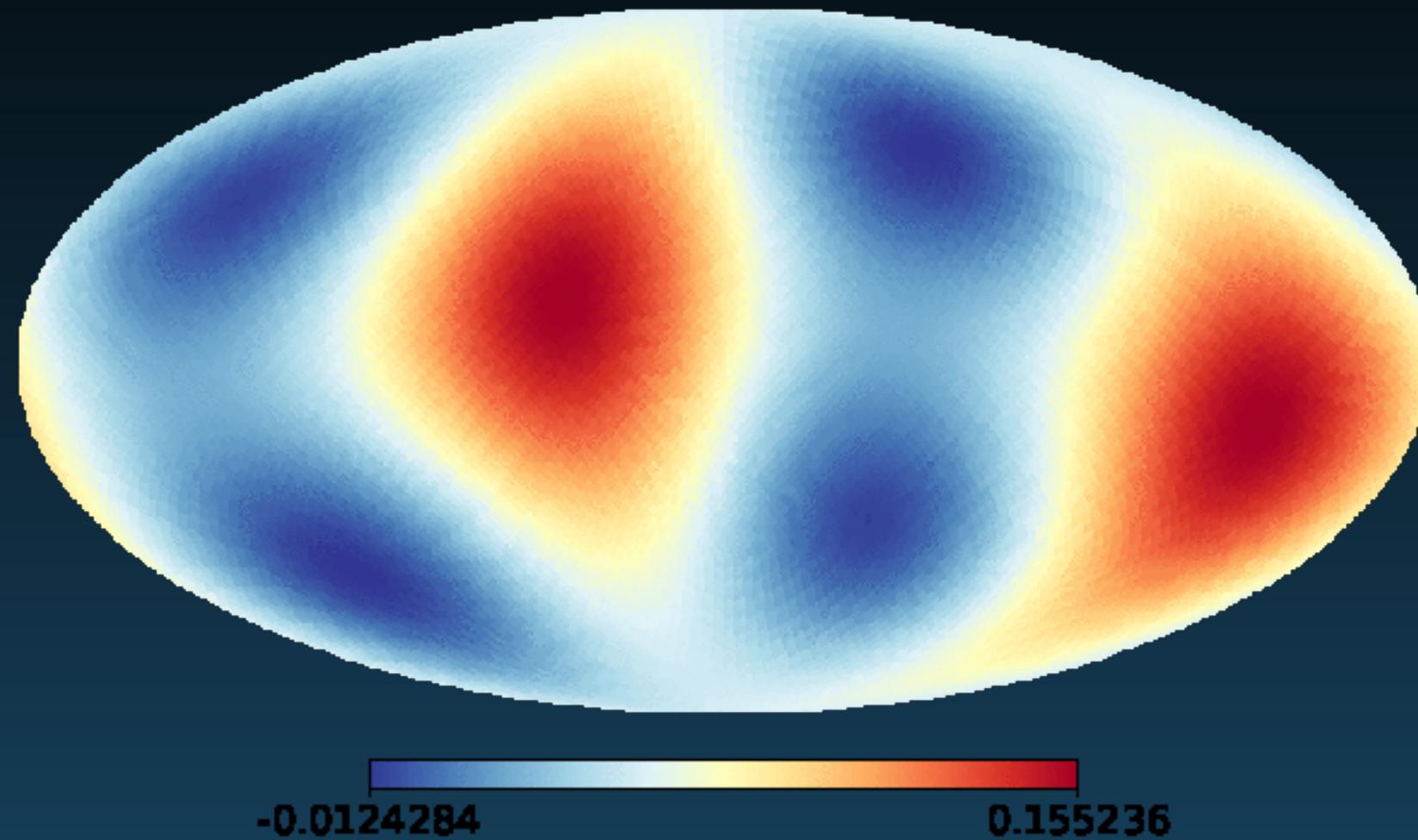
**ML solution**

$$I(\hat{n}) = \mathcal{F}(\hat{n}, \hat{n}')^{-1} z(\hat{n}')$$

Fisher information matrix:  
 $\hat{n} \longleftrightarrow \hat{n}'$  correlations

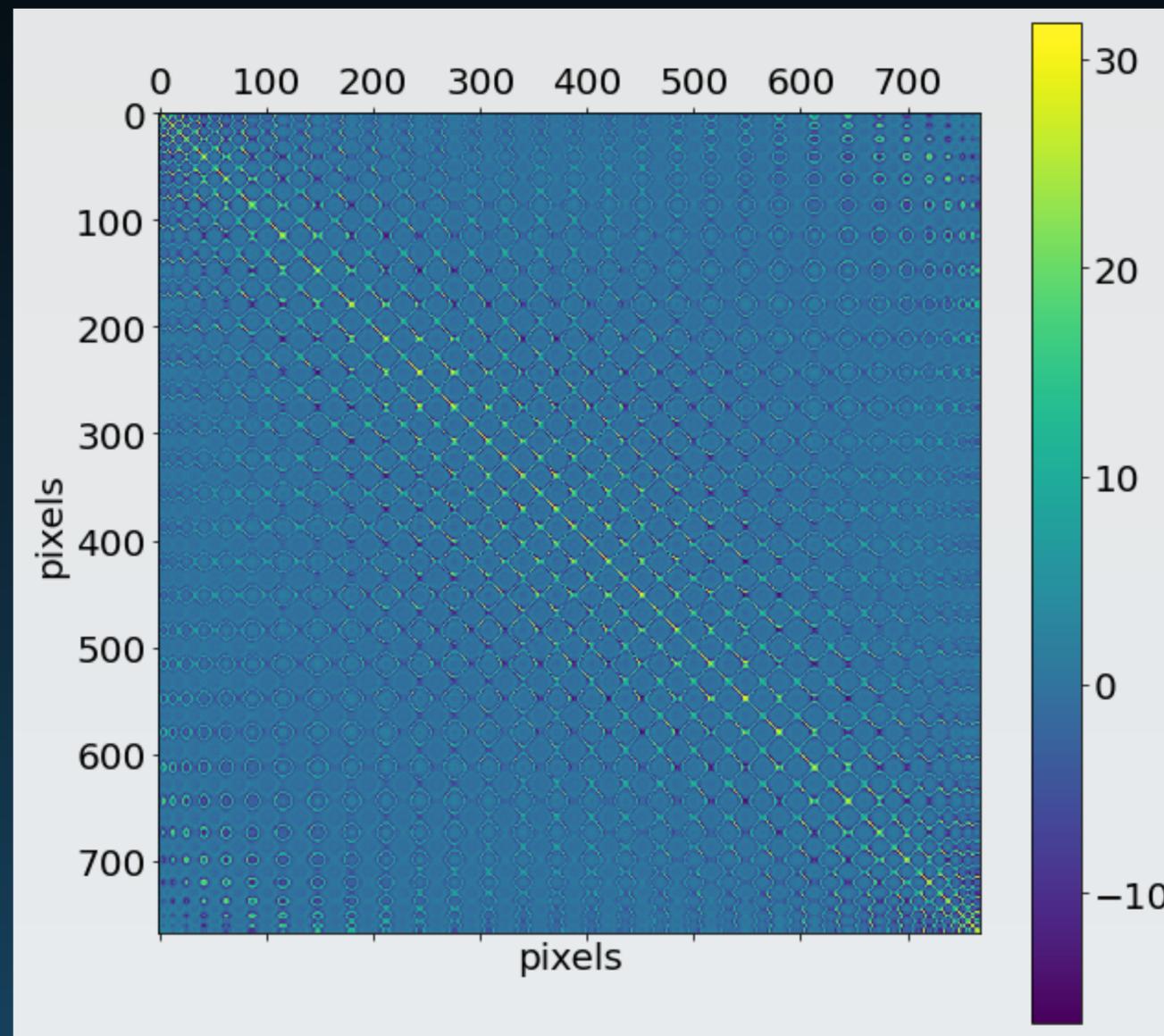
projection map:  
project data on the sky

e.g.: Hanford — Livingston overlap



Scale of the beam  $\leftrightarrow$  detector resolution

# BIG CHALLENGE: inverting $\mathcal{F}$



**improve:**

- ★ Time-dependent sky sampling
- ★ Number of detectors

# Stochastic searches: non-Gaussian signal

intermittent background of CBCs; “deterministic” CBC likelihood:

$$\mathcal{L}(d_i) \propto \prod_{f,\tau} \frac{1}{|C|^{1/2}} e^{\frac{1}{2} (d_i - h_i) C^{-1} (d_i - h_i)^*}$$

gaussian noise + intermittent signal = **Gaussian mixture model:**

$$\mathcal{L}_{\text{full}}(d_i) = \xi \mathcal{L}_s(d_i | h_i) + (1 - \xi) \mathcal{L}_n(d_i | 0)$$

*duty cycle*: probability of there being a CBC signal in the data at any given time

# Stochastic searches: non-Gaussian signal

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“deterministic” =  
use CBC waveforms:  
“The Bayesian Search”  
[Smith & Thrane '18](#)

“deterministic”  
GWB strain model?

~~“deterministic”~~, *stochastic* burst  
search for correlated bursts of  
GW energy, [Drasco & Flanagan, '03](#), Lawrence+(AIR) in prep.

# Stochastic searches with LIGO — Virgo



# LVK results: isotropic search

## Upper Limits on the Isotropic Gravitational-Wave Background from Advanced LIGO's and Advanced Virgo's Third Observing Run

The LIGO Scientific Collaboration, The Virgo Collaboration, and The KAGRA Collaboration\*  
(Dated: January 29, 2021)

We report results of a search for an isotropic gravitational-wave background (GWB) using data from Advanced LIGO's and Advanced Virgo's third observing run (O3) combined with upper limits from the earlier O1 and O2 runs. Unlike in previous observing runs in the advanced detector era, we include Virgo in the search for the GWB. The results of the search are consistent with uncorrelated noise, and therefore we place upper limits on the strength of the GWB. We find that the dimensionless energy density  $\Omega_{\text{GW}} \leq 5.8 \times 10^{-9}$  at the 95% credible level for a flat (frequency-independent) GWB, using a prior which is uniform in the log of the strength of the GWB, with 99% of the sensitivity coming from the band 20-76.6 Hz;  $\Omega_{\text{GW}}(f) \leq 3.4 \times 10^{-9}$  at 25 Hz for a power-law GWB with a spectral index of 2/3 (consistent with expectations for compact binary coalescences), in the band 20-90.6 Hz; and  $\Omega_{\text{GW}}(f) \leq 3.9 \times 10^{-10}$  at 25 Hz for a spectral index of 3, in the band 20-291.6 Hz. These upper limits improve over our previous results by a factor of 6.0 for a flat GWB, 8.8 for a spectral index of 2/3, and 13.1 for a spectral index of 3. We also search for a GWB arising from scalar and vector modes, which are predicted by alternative theories of gravity; we do not find evidence of these, and place upper limits on the strength of GWBs with these polarizations. We demonstrate that there is no evidence of correlated noise of magnetic origin by performing a Bayesian analysis that allows for the presence of both a GWB and an effective magnetic background arising from geophysical Schumann resonances. We compare our upper limits to a fiducial model for the GWB from the merger of compact binaries, updating the model to use the most recent data-driven population inference from the systems detected during O3a. Finally, we combine our results with observations of individual mergers and show that, at design sensitivity, this joint approach may yield stronger constraints on the merger rate of binary black holes at  $z \gtrsim 2$  than can be achieved with individually resolved mergers alone.

O3: first stochastic searches with Virgo!

other models:

- scalar/tensor pol.s
- correlated magnetic noise

CBC backgrounds:

- set upper limits
- combine with resolved searches to get  $\mathcal{R}(z)$

Un-modeled SGWB:

simultaneous fit of spectral index and GWB amplitude

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O3: first stochastic searches with Virgo!

other models:  
• scalar/tensor pol.s  
• correlated  
magnetic noise

$$\Omega_{\text{GW}}^{\text{CBC}}(f = 25 \text{ Hz}) < 3.4 \times 10^{-9}$$

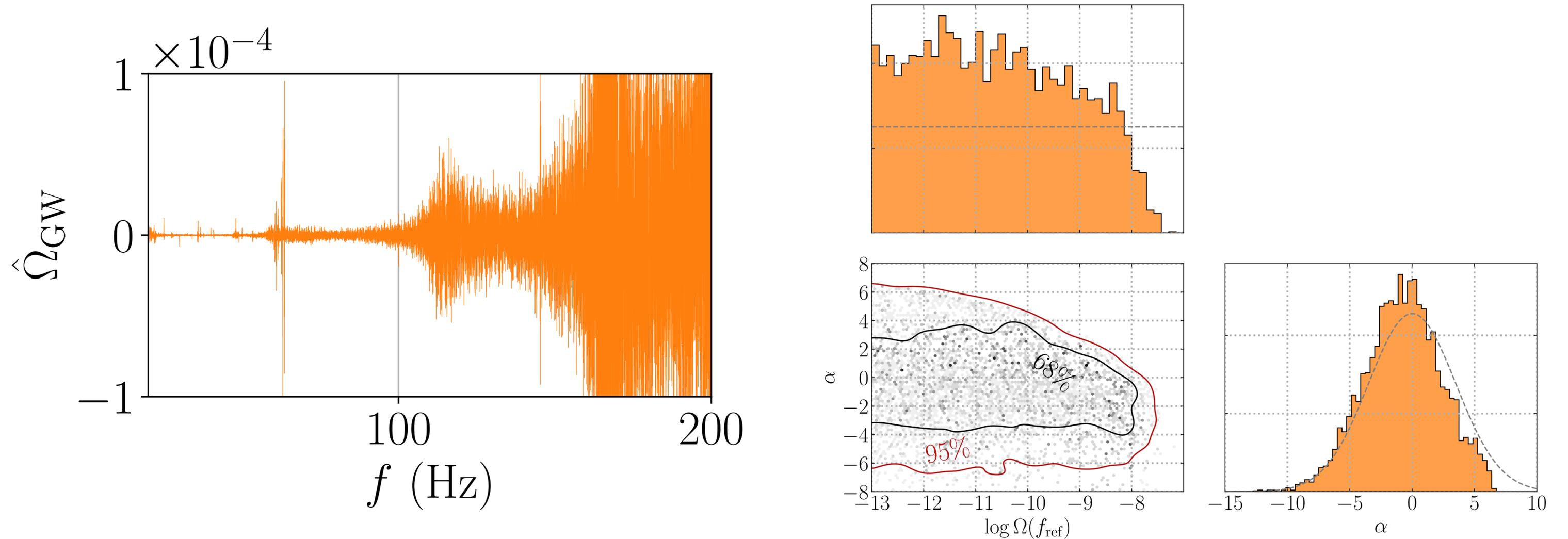
$$\Omega_{\text{GW}}^{\text{CBC, pop}}(f = 25 \text{ Hz}) = 6.9^{+3.9}_{-2.0} \times 10^{-10}$$

CBC backgrounds:  
• set upper limits  
• combine with resolved searches to get  $\mathcal{R}(z)$

Un-modeled SGWB:  
simultaneous fit of spectral index and GWB amplitude

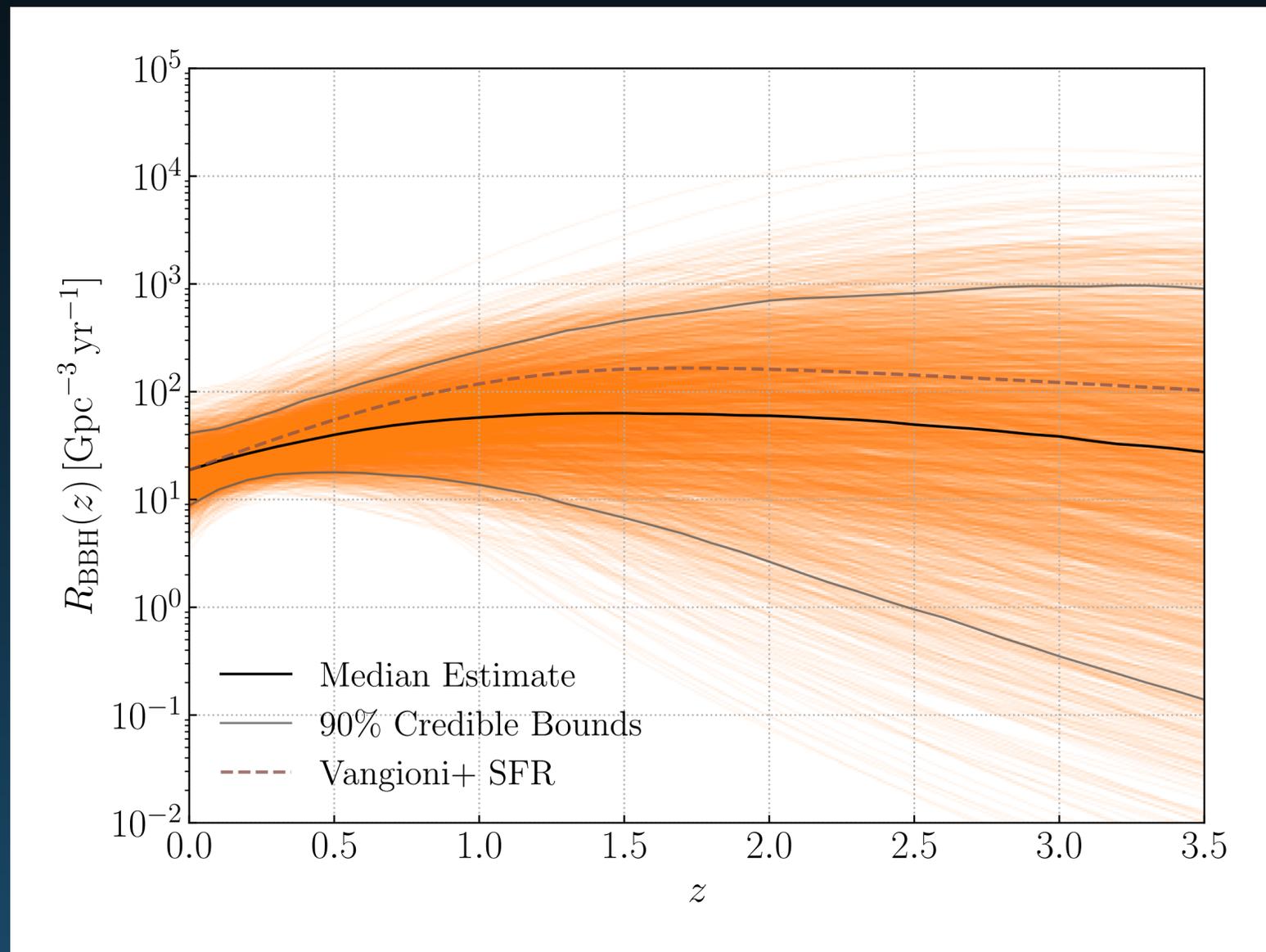
# LVK results: isotropic search

cross-correlation spectrum and posteriors for  $\alpha$  &  $\Omega_{\text{GW}}(f_{\text{ref}})$



# LVK results: isotropic search

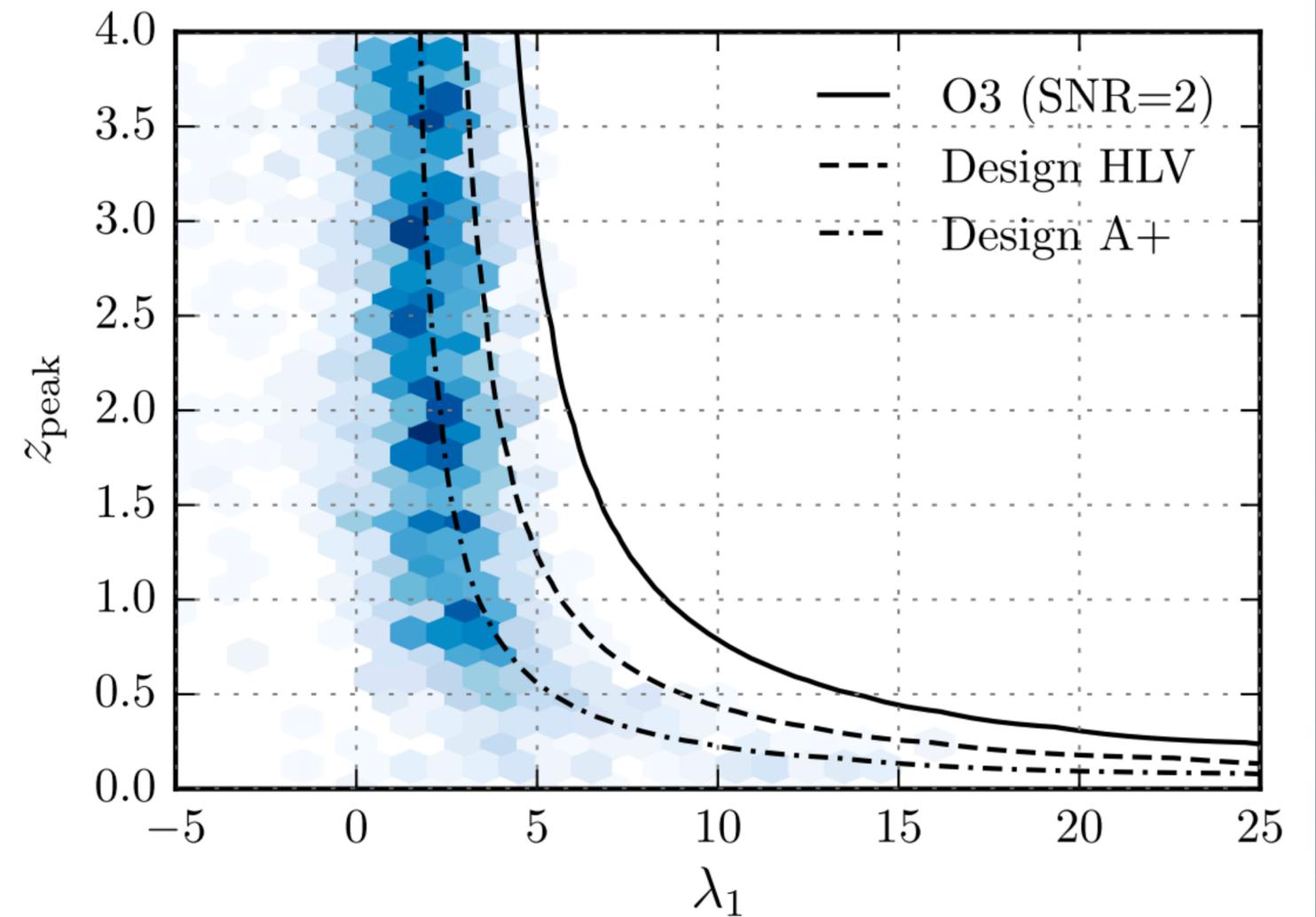
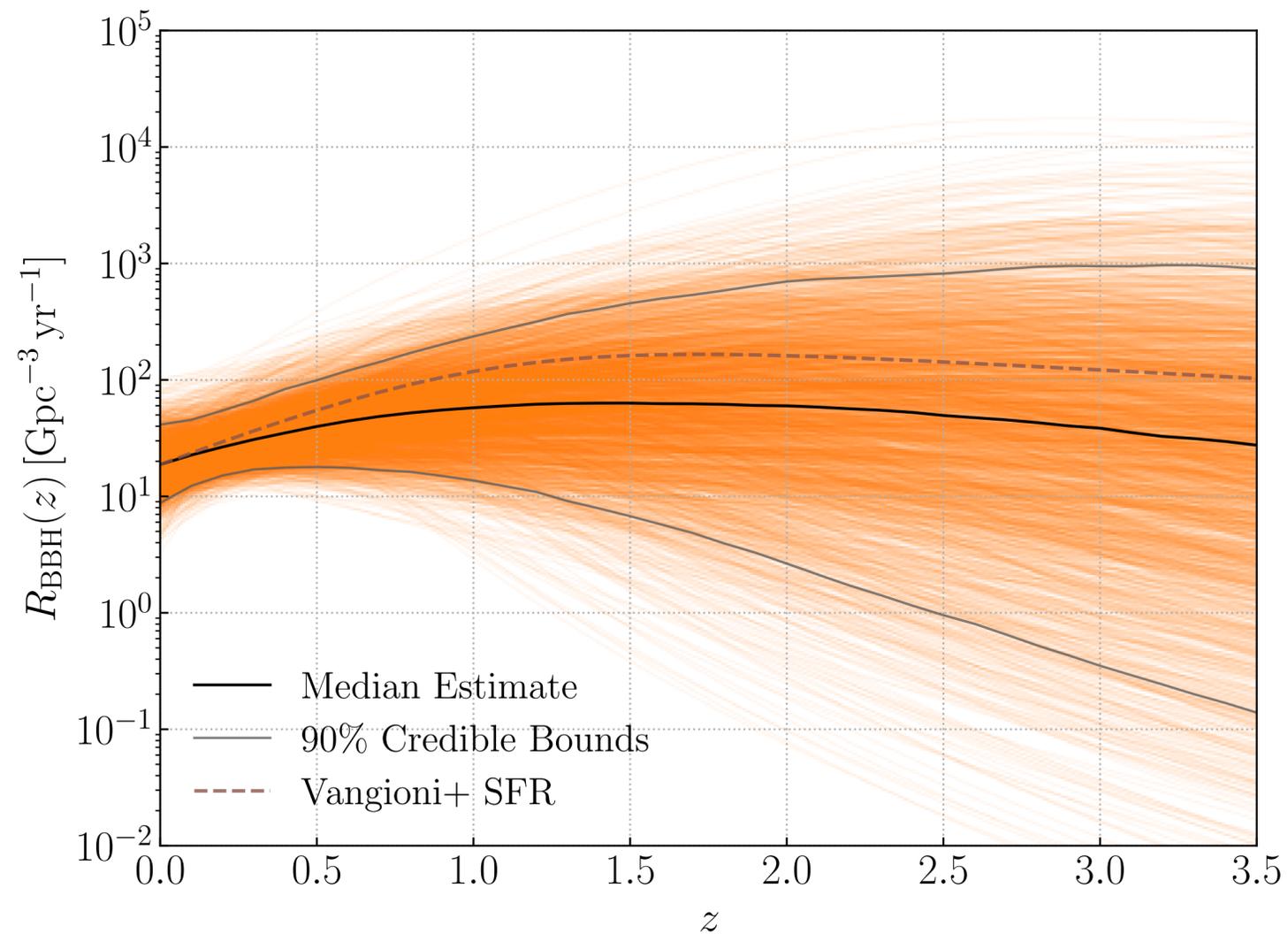
## BBH merger rate constraints



$$R_{\text{BBH}}(z) = \mathcal{C}(\alpha_r, \beta_r, \hat{z}) \frac{R_0(1+z)^{\alpha_r}}{1 + \left(\frac{1+z}{1+\hat{z}}\right)^{\alpha_r+\beta_r}}$$

# LVK results: isotropic search

## BBH merger rate constraints



# LVK results: anisotropic search

## Search for anisotropic gravitational-wave backgrounds using data from Advanced LIGO's and Advanced Virgo's first three observing runs

The LIGO Scientific Collaboration, The Virgo Collaboration, and The KAGRA Collaboration\*  
(Dated: March 15, 2021)

We report results from searches for anisotropic stochastic gravitational-wave backgrounds using data from the first three observing runs of the Advanced LIGO and Advanced Virgo detectors. For the first time, we include Virgo data in our analysis and run our search with a new efficient pipeline called PyStoch on data folded over one sidereal day. We use gravitational-wave radiometry (broad-band and narrow-band) to produce sky maps of stochastic gravitational-wave backgrounds and to search for gravitational waves from point sources. A spherical harmonic decomposition method is employed to look for gravitational-wave emission from spatially-extended sources. Neither technique found evidence of gravitational-wave signals. Hence we derive 95% confidence-level upper limit sky maps on the gravitational-wave energy flux from broadband point sources, ranging from  $F_{\alpha,\Theta} < (0.013 - 7.6) \times 10^{-8} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$ , and on the (normalized) gravitational-wave energy density spectrum from extended sources, ranging from  $\Omega_{\alpha,\Theta} < (0.56 - 9.7) \times 10^{-9} \text{ sr}^{-1}$ , depending on direction ( $\Theta$ ) and spectral index ( $\alpha$ ). These limits improve upon previous limits by factors of 2.8–3.8. We also set 95% confidence level upper limits on the frequency-dependent strain amplitudes of quasi-monochromatic gravitational waves coming from three interesting targets, Scorpius X-1, SN 1987A and the Galactic Center, with best upper limits range from  $h_0 < (1.7 - 2.1) \times 10^{-25}$ , a factor of  $\geq 2.0$  improvement compared to previous searches.

**BBR**

**&**

**NBR:**

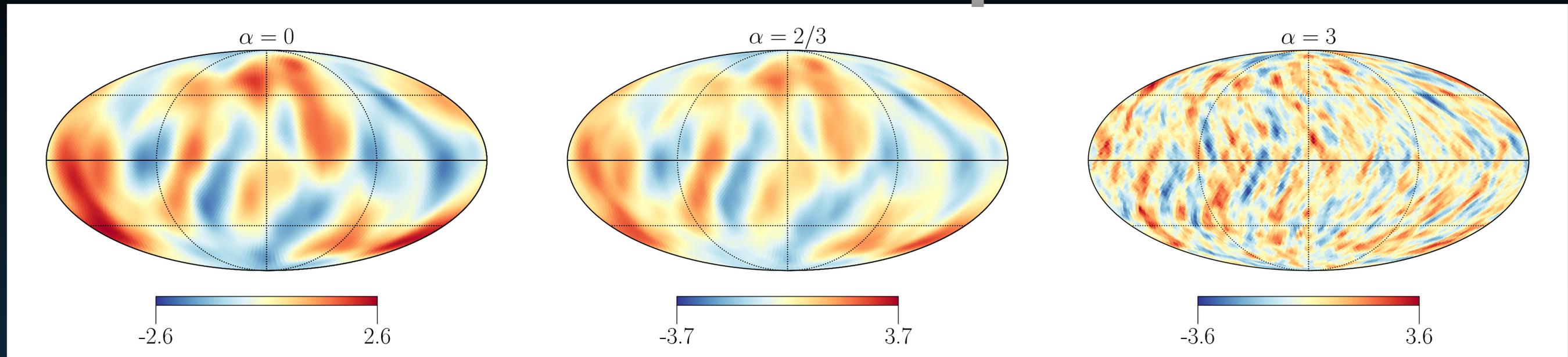
**pixel search for point sources**

**SHD:**

**spherical harmonic search for extended sources**

**Target Search**

# LVK results: anisotropic search



**ASSUME NO CORRELATED  
POWER BETWEEN PIXELS**

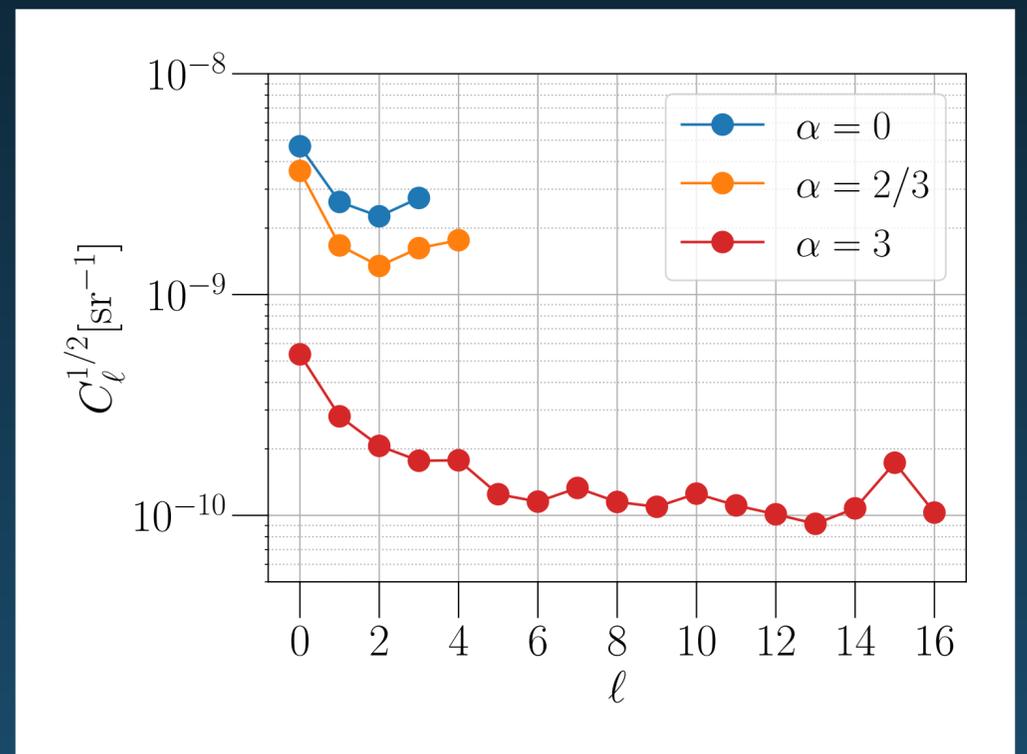
only use diagonal of  $\mathcal{F}$

improvement  $\sim 2.8\text{--}3.8$

**EXTRA CHALLENGE:  
INVERSION OF FULL  $\mathcal{F}$**

use SVD and Virgo

from [LVK collaboration '21](#)



# LVK results: cosmological searches

## ◆ cosmic strings

limit on  $\Omega_{\text{GW}}$   $\longrightarrow$   $\mathcal{G}_\mu < 4 \times 10^{-15}$

→ cosmic string tension

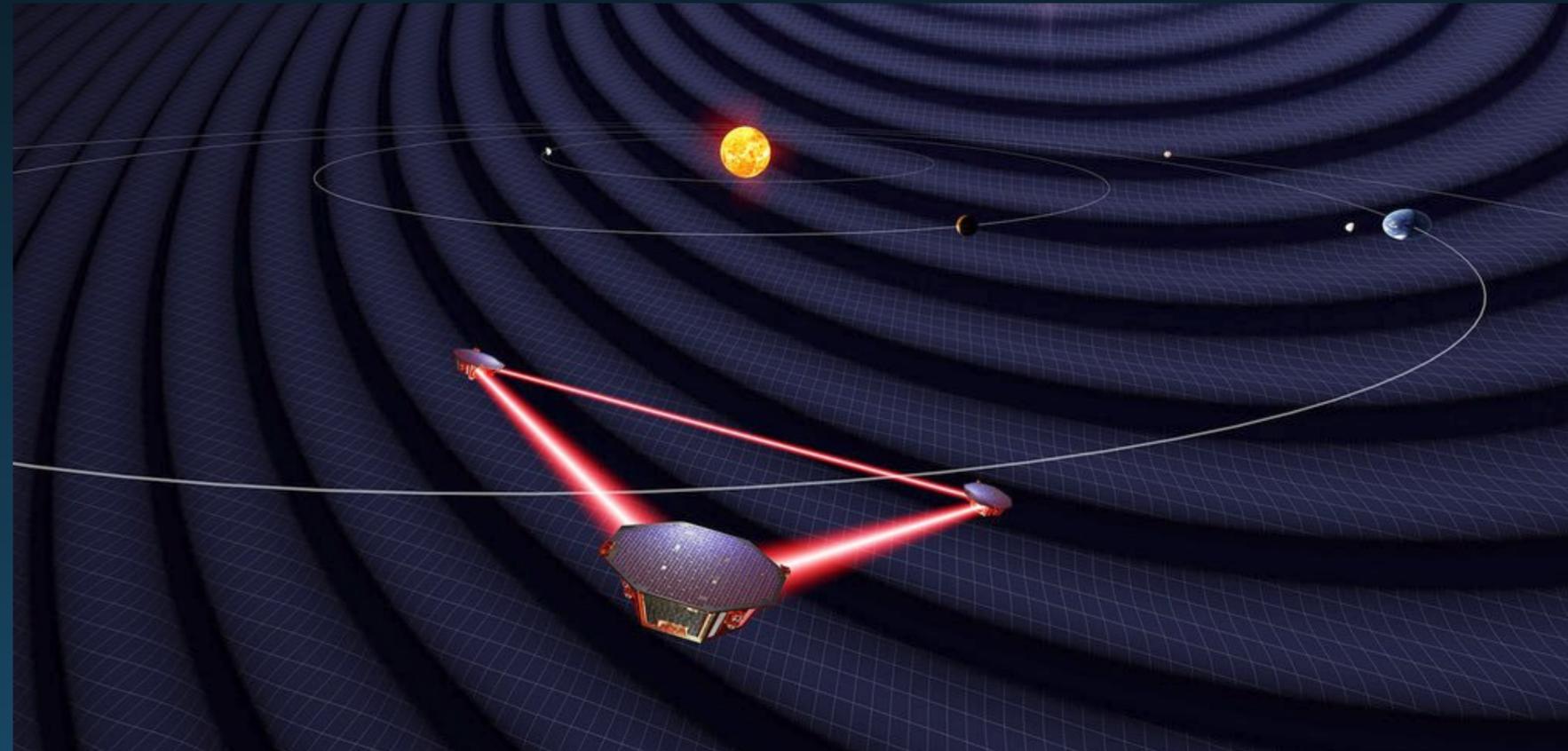
## ◆ primordial black holes

$2M_\odot - 200M_\odot$   $\longleftrightarrow$   $f_{\text{CDM}}^{\text{PBH}} < 1$

fraction of PBHs that make up cold dark matter

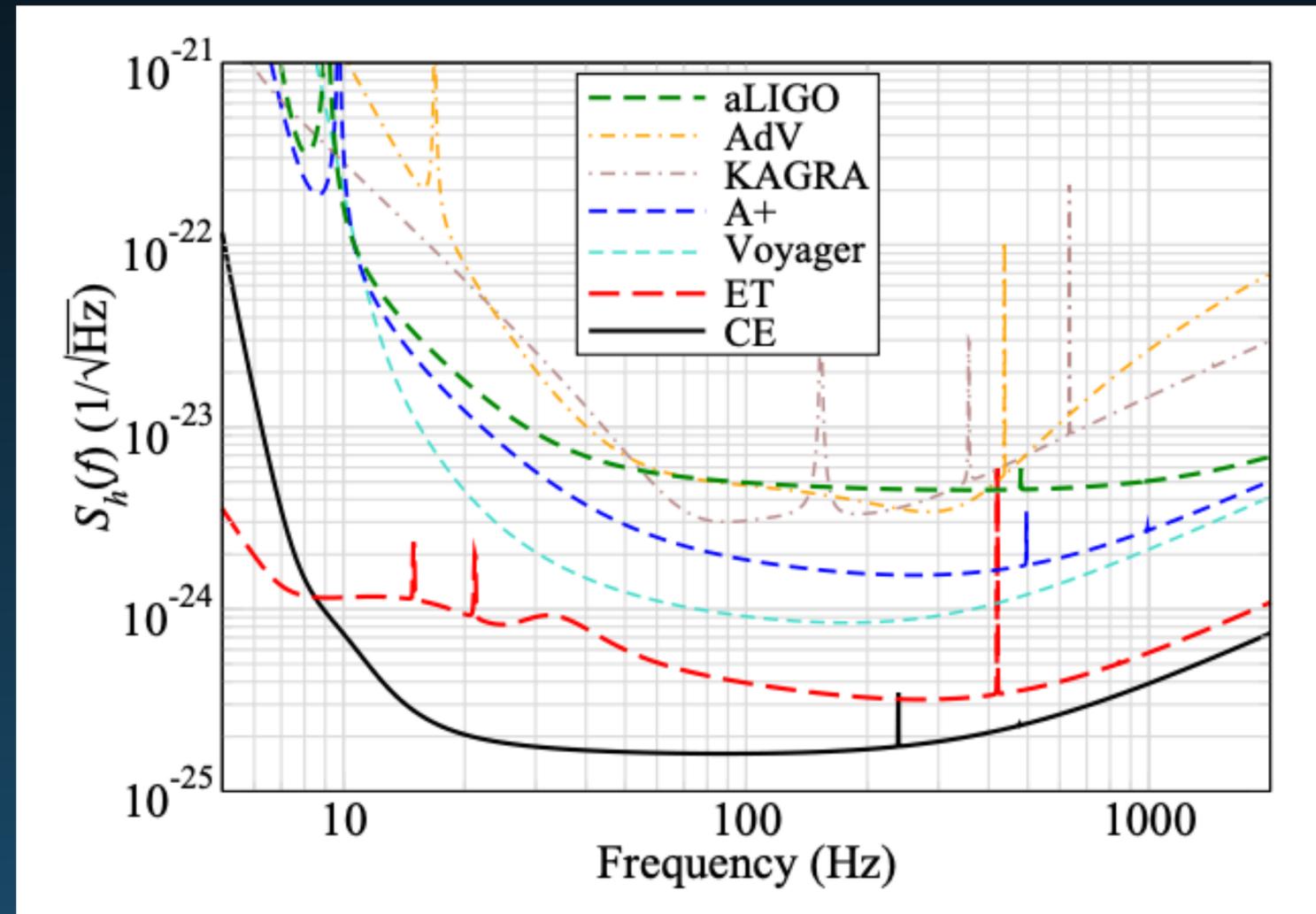
## ◆ first—order phase transitions; superradiant bosonic condensates around BHs; dark photons

# future prospects: 3G, LISA



# Third Generation ground-based Network: ET, CE

Extend the depth of ground surveys up to  $z \approx 20$   $\longrightarrow$  resolve all BBHs!

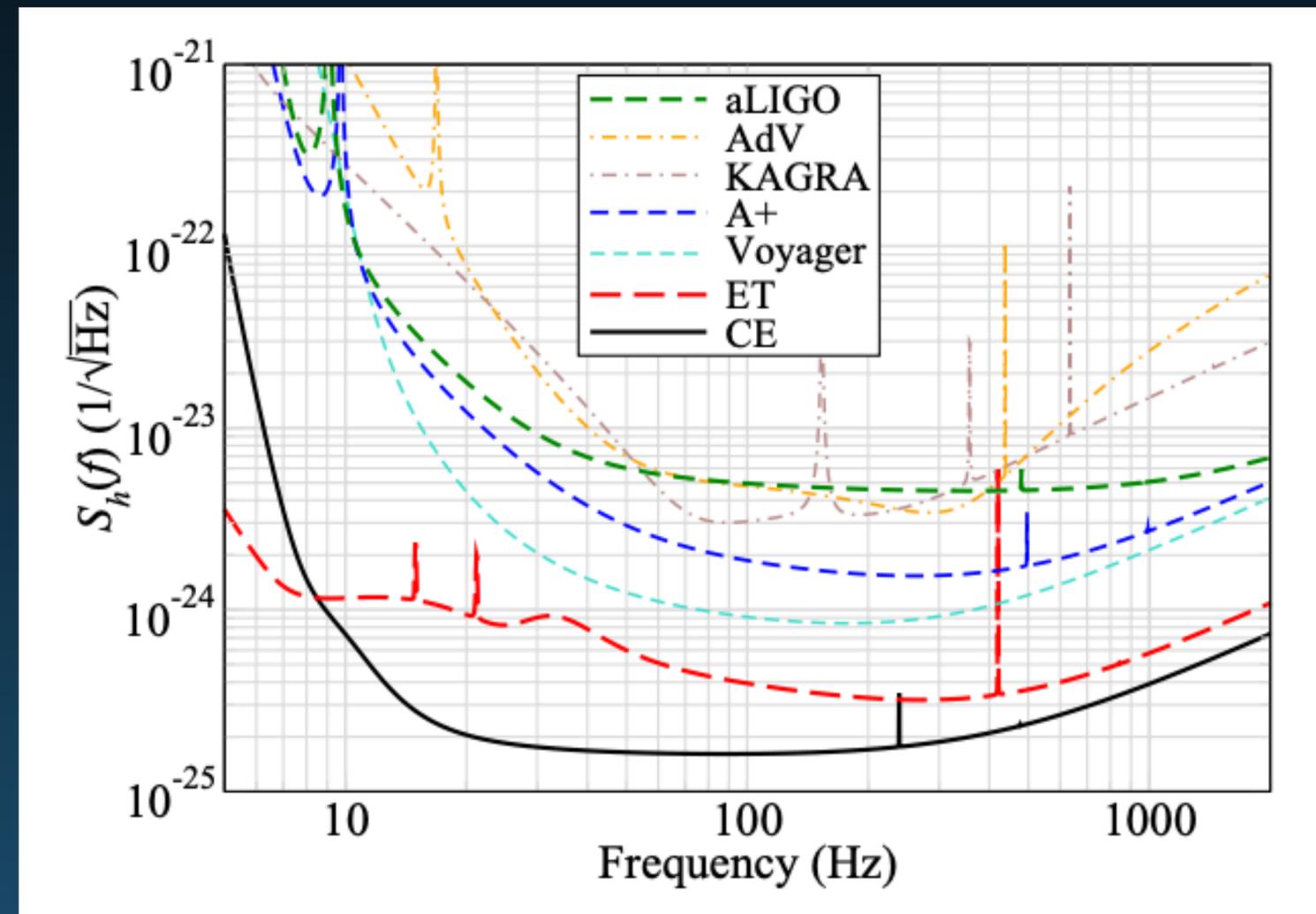


# Third Generation ground-based Network: ET, CE

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“new” stochastic signals within reach:

- BNS/BHNS background
- PBH background
- cosmological backgrounds ...



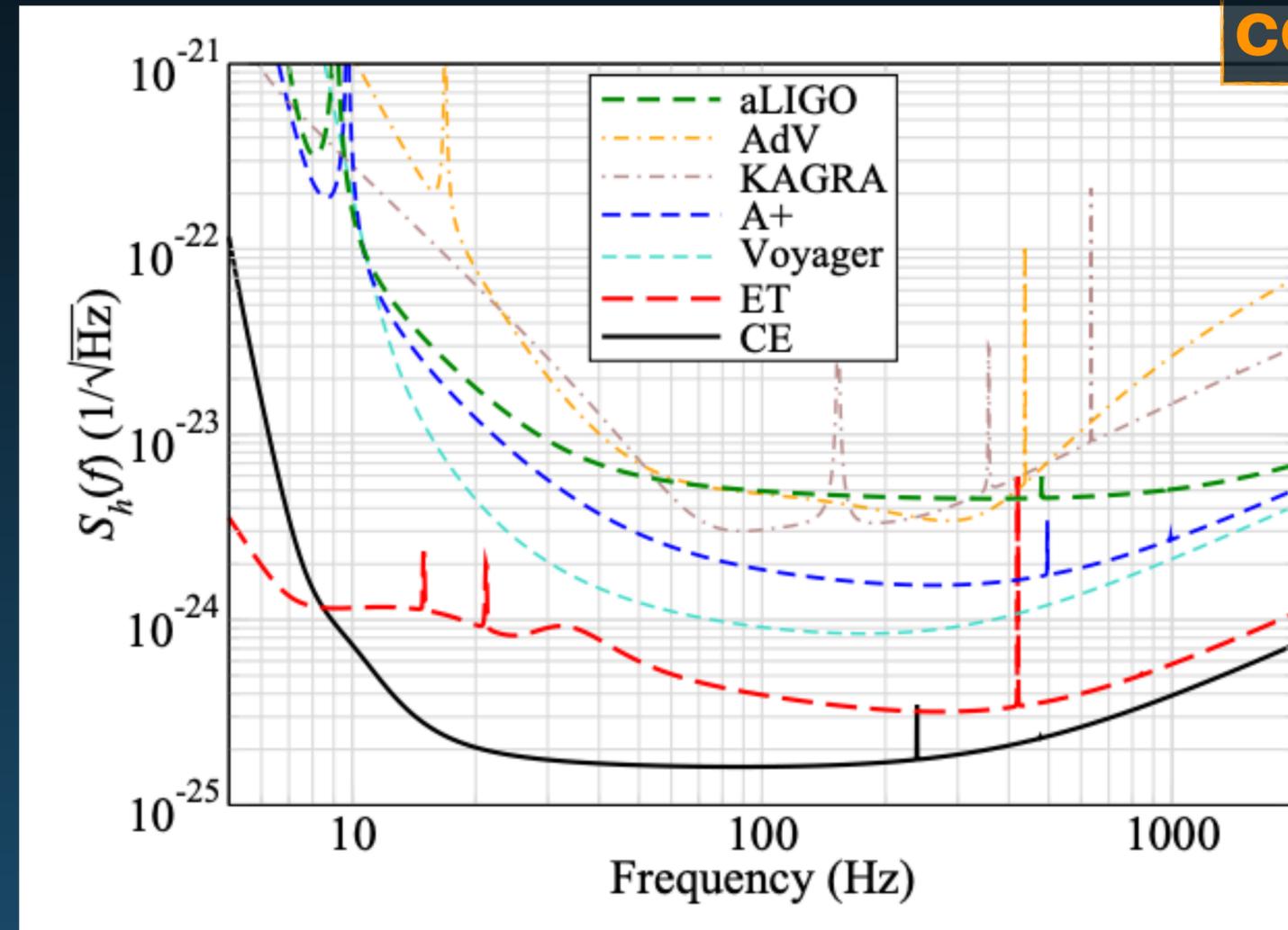
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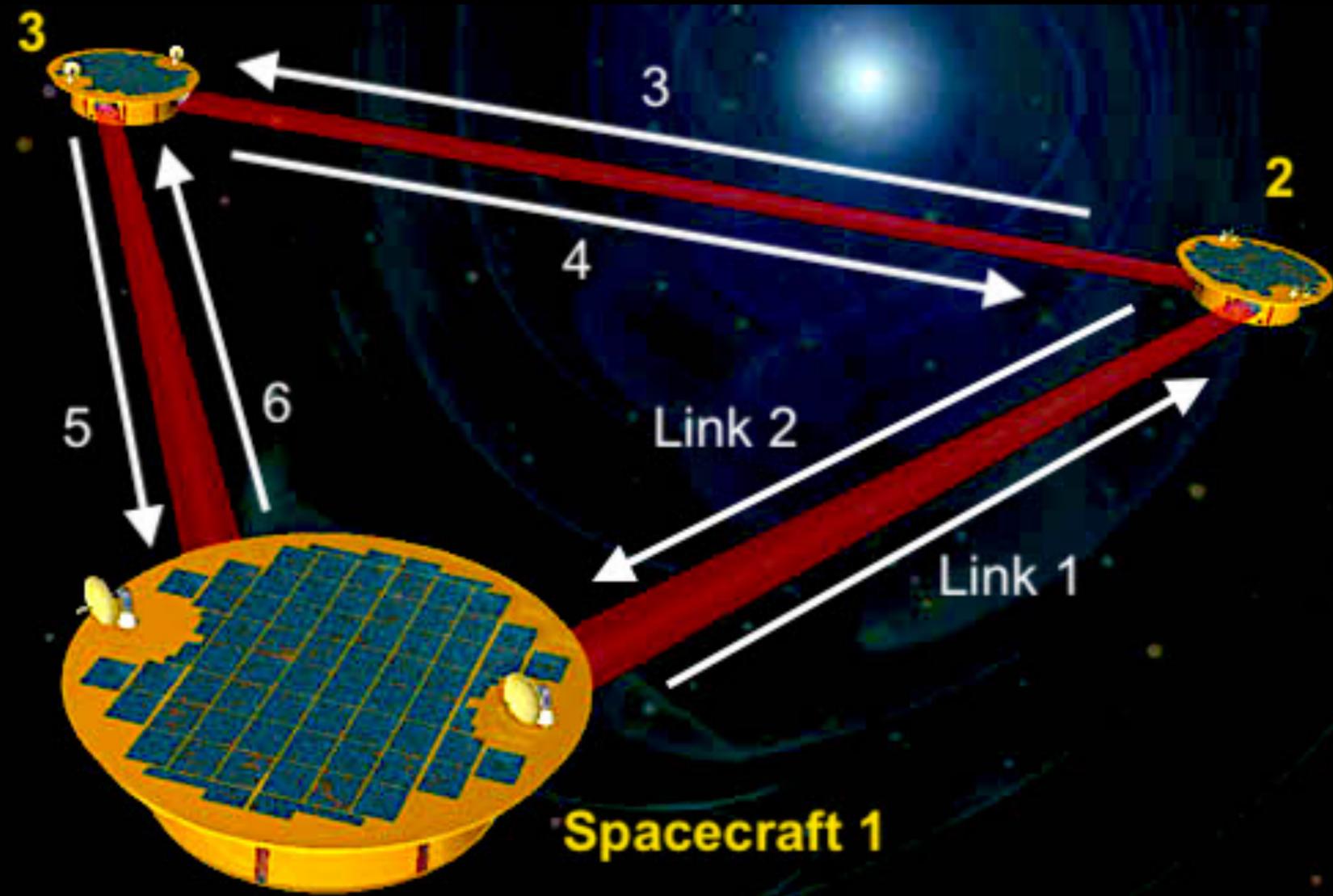
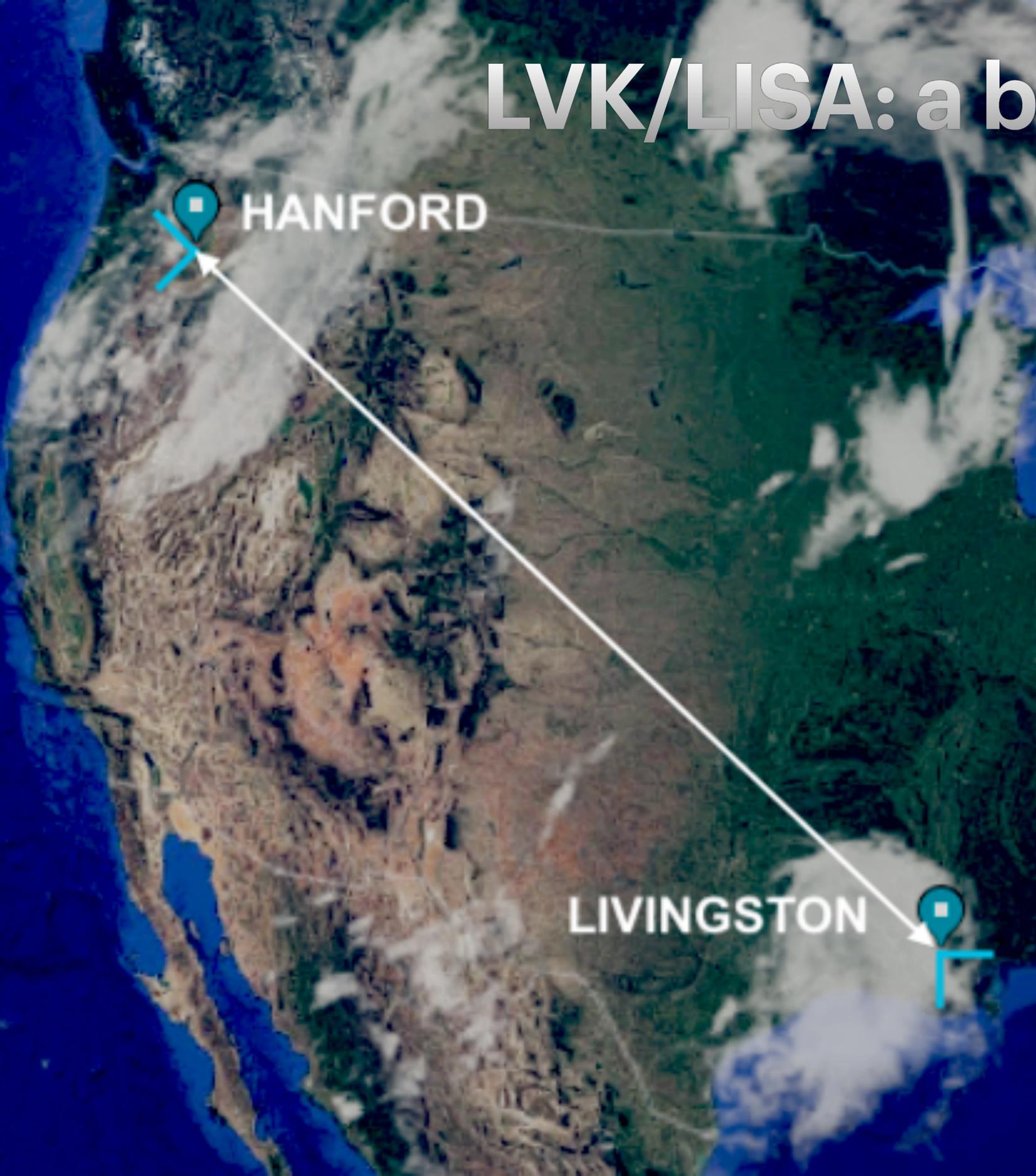
**CHALLENGE:**  
component separation

- BNS/BHNS background
- PBH background
- cosmological backgrounds ...



- subtraction of high SNR signals
  - [Regimbau et al. '17](#)
  - [Cutler & Harms '06](#)
- simultaneous estimation of multiple signals
  - [Biscoveanu et al. '20](#)
  - [Martinovic et al. '20](#)

# LVK/LISA: a big difference?

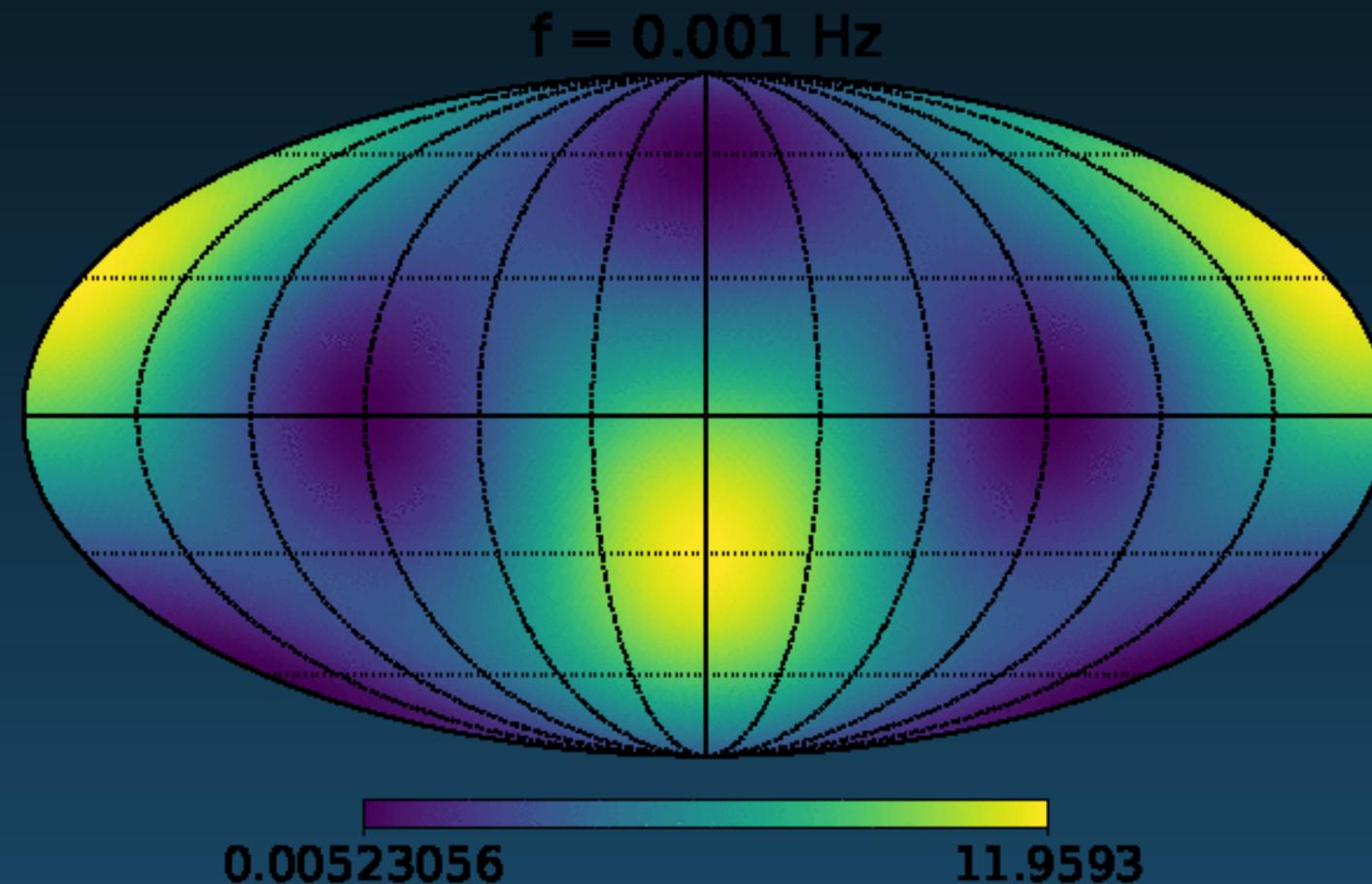


- ▶ not in small antenna limit
- ▶ different treatment of the noise
- ▶ uses time—delayed interferometry (TDI)
- ▶ in space, it's harder to add a detector!

# LISA channel response

Auto-correlated response for TDI = X1 in the Solar System Barycentre frame

**frequency dependence**



strong modulations at higher frequencies!

# Stochastic searches in LISA

with TDI channels

$$X(t) = R_X(t) h(t) + n_X(t)$$

“Michelson—Morley”  
style

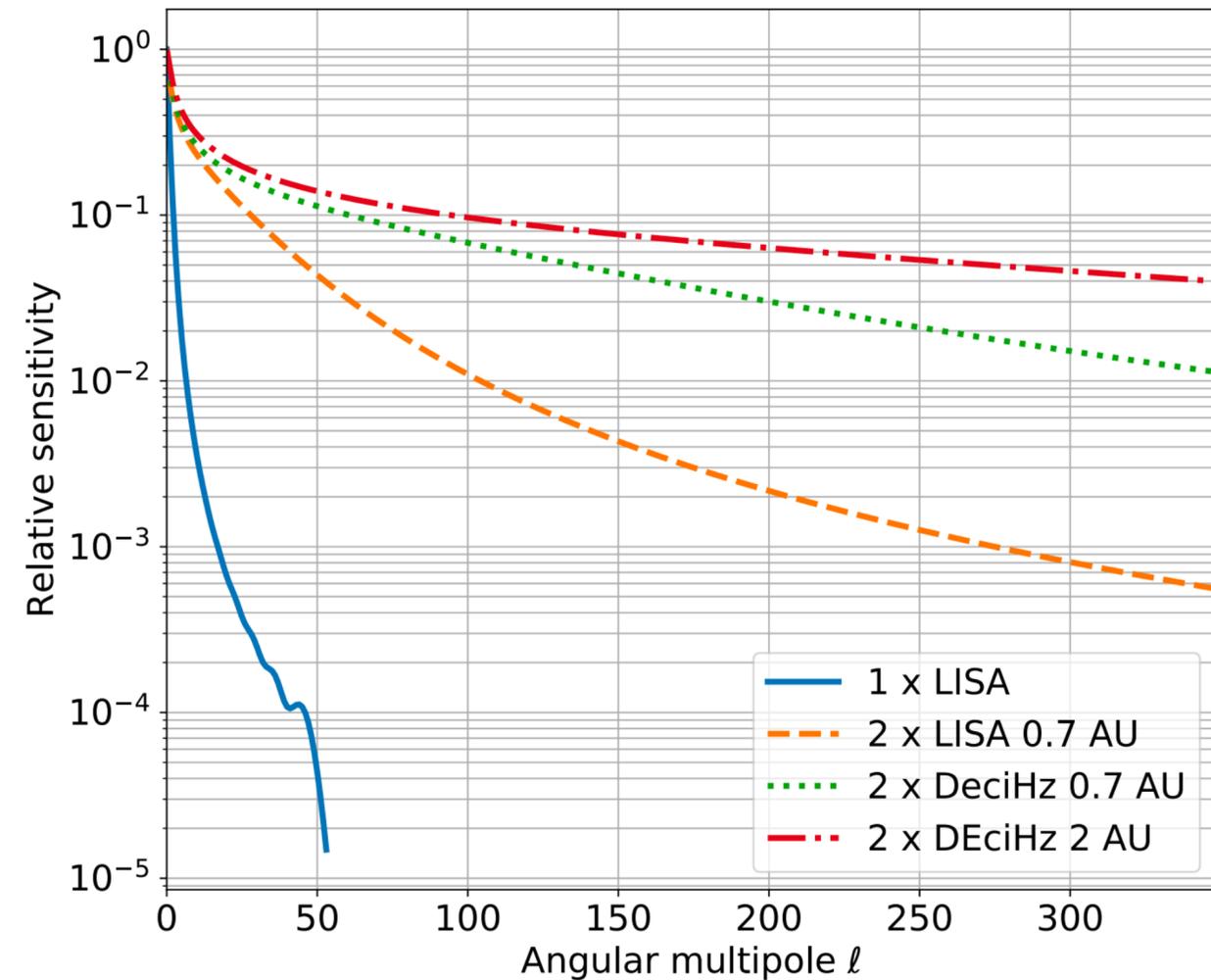
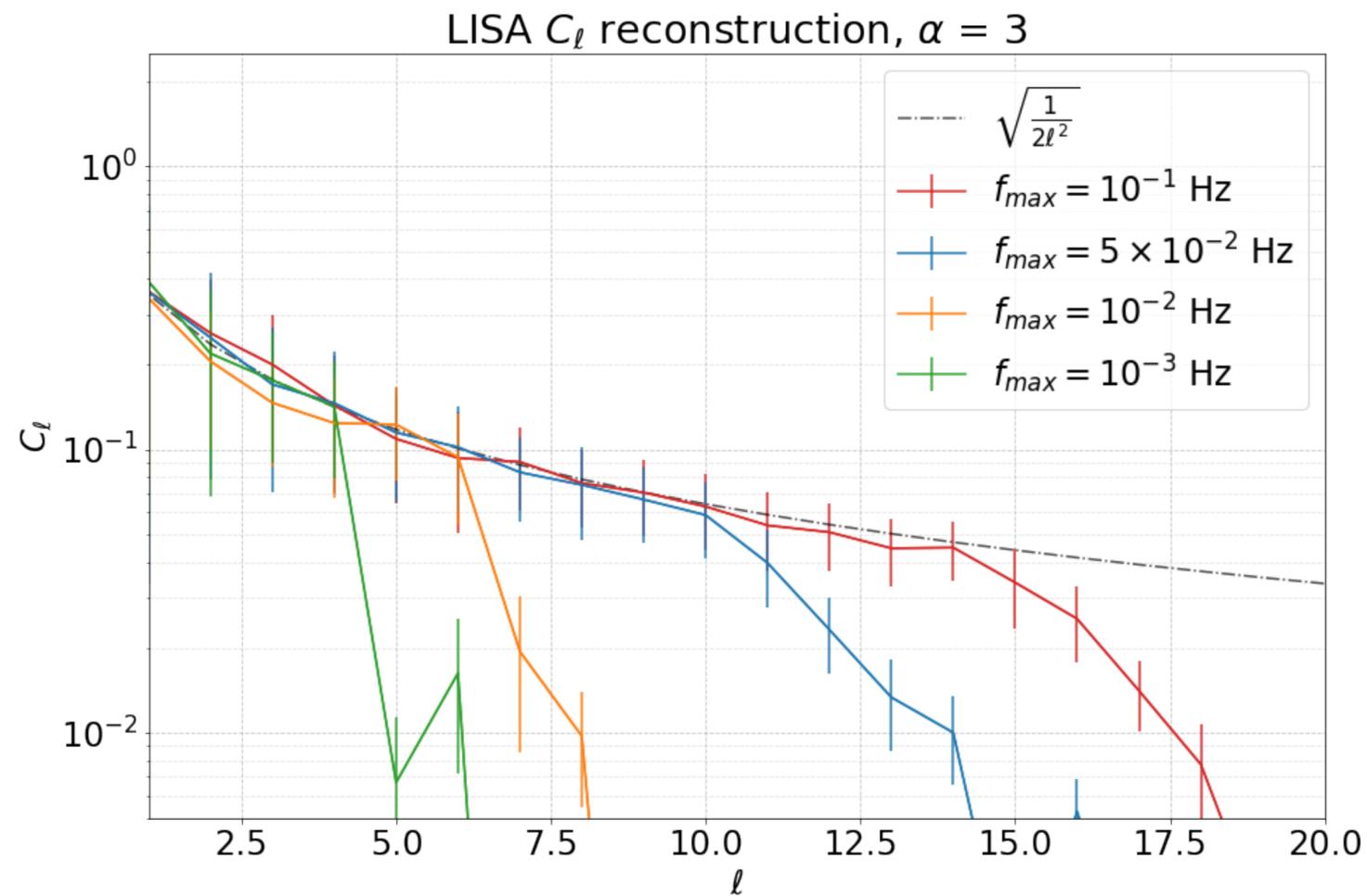
$$X, Y, Z \longleftrightarrow A, E, T$$

diagonal noise  
space

$$\mathcal{L}(d) \propto \prod_{f, \tau} \frac{1}{|C|^{1/2}} e^{\frac{1}{2} T C^{-1} T^*} \quad T = (X, Y, Z)^T$$

**Bayesian framework:** fit both signal and noise params [Adams & Cornish '14](#)

# Angular resolution: LISA and beyond

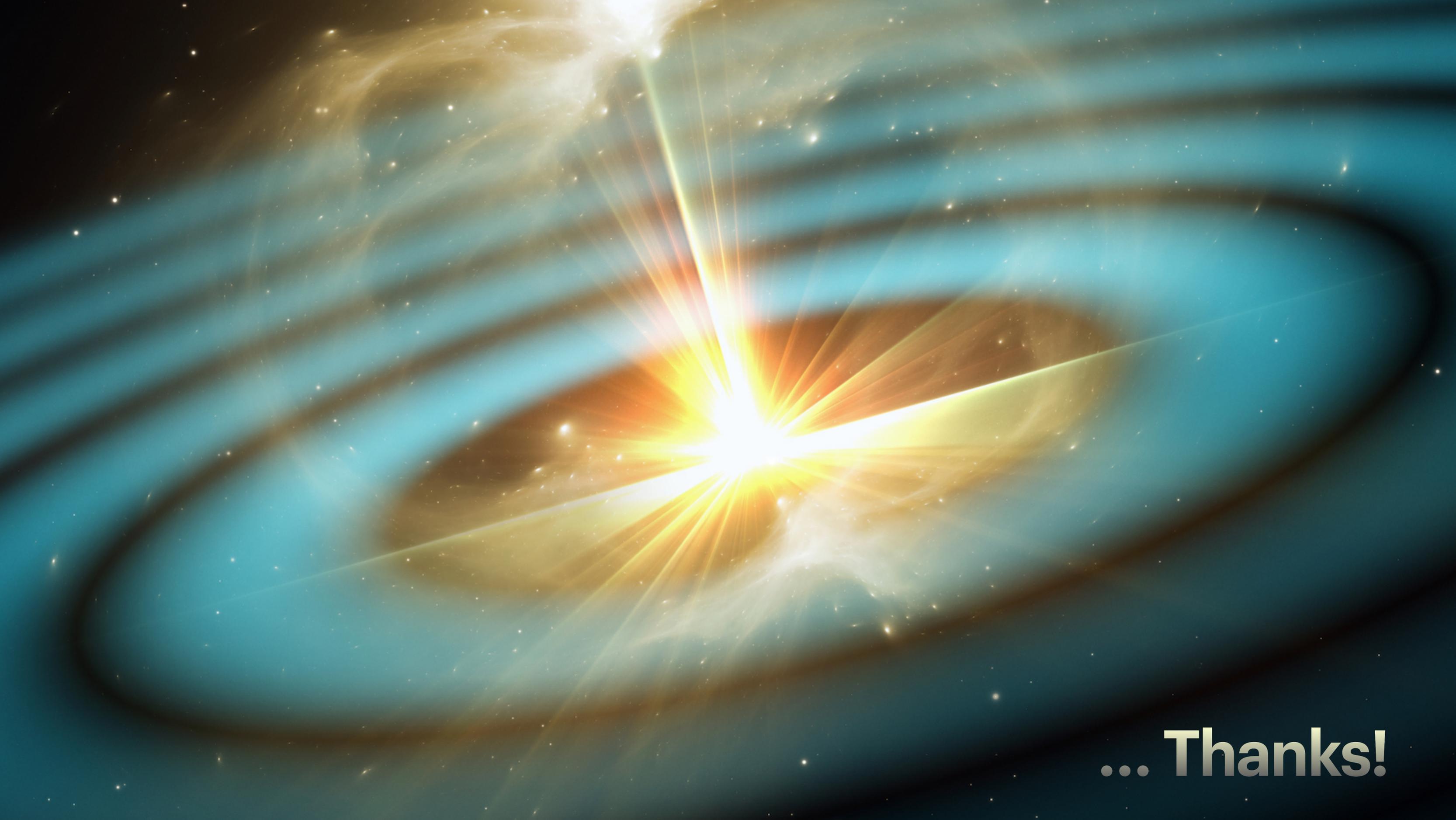


from PRD 102, 043502

**ONLY HOPE: MORE DETECTORS IN SPACE!**

[ESA Voyage 2050](#)

[White Paper](#)



**... Thanks!**