Multi-oscillating Boson Stars

Mathematical and Numerical Aspects of Gravitation

IPAM, UCLA
October 26, 2021

Matt Choptuik
Dept. of Physics and Astronomy
UBC

Outline

- Introduction / overview of boson stars
- Multi-oscillating stars in AdS
- Multi-oscillating stars in asymptotically flat spacetime
- Conclusions

Joint work with
Ramon Masachs, Jorge Santos and
Benson Way (2018,2019)


100% Rigor-free

Overview of boson stars

- In asymptotically flat spacetime: stationary configurations of general relativistically self-gravitating, massive, complex scalar field, possibly with nonlinear self-interaction

- Long history: First constructed by Kaup (1968) and shortly thereafter and independently by Ruffini and Bonazzola (1969)

- Astrophysical/cosmological significance remains unclear, but at various times have been proposed as, e.g:

  - Candidates for gravitationally compact objects in lieu of black holes

  - Candidates for dark matter halos of galaxies, and dark matter in general

- Computationally: less challenging to simulate in many respects than fluid matter (no shocks); basis for good model problems

Overview of boson stars

- Lagrangian (with quartic self-interaction)

$$L = -\frac{1}{2} g^{\mu\nu} \phi^*_{;\mu} \phi_{;\nu} - \frac{1}{2} m^2 |\phi|^2 - \frac{1}{4} \lambda |\phi|^4$$

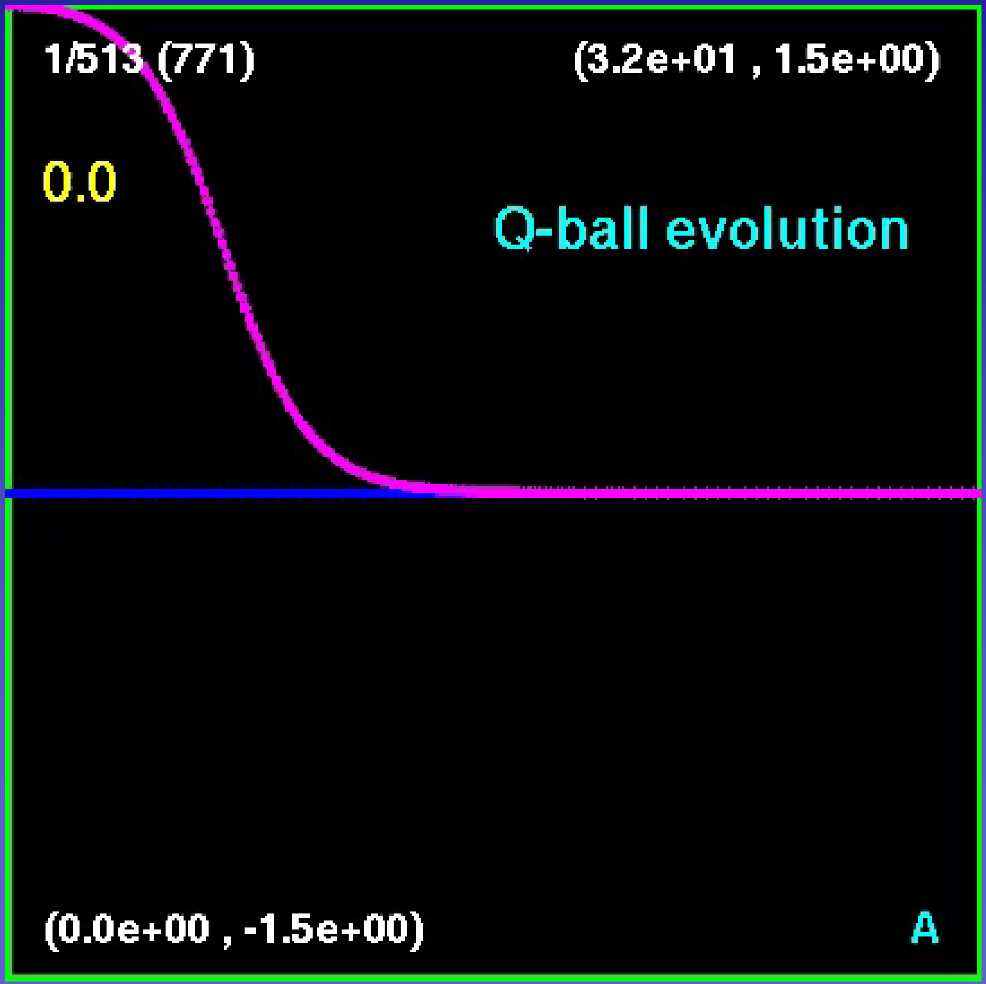
- Impose spherical symmetry, and time-harmonic ansatz for scalar field

$$ds^2 = -B(r) dt^2 + A(r) dr^2 + r^2 d\Omega^2$$

$$\phi(t,r) = \psi(r) e^{i\omega t}$$

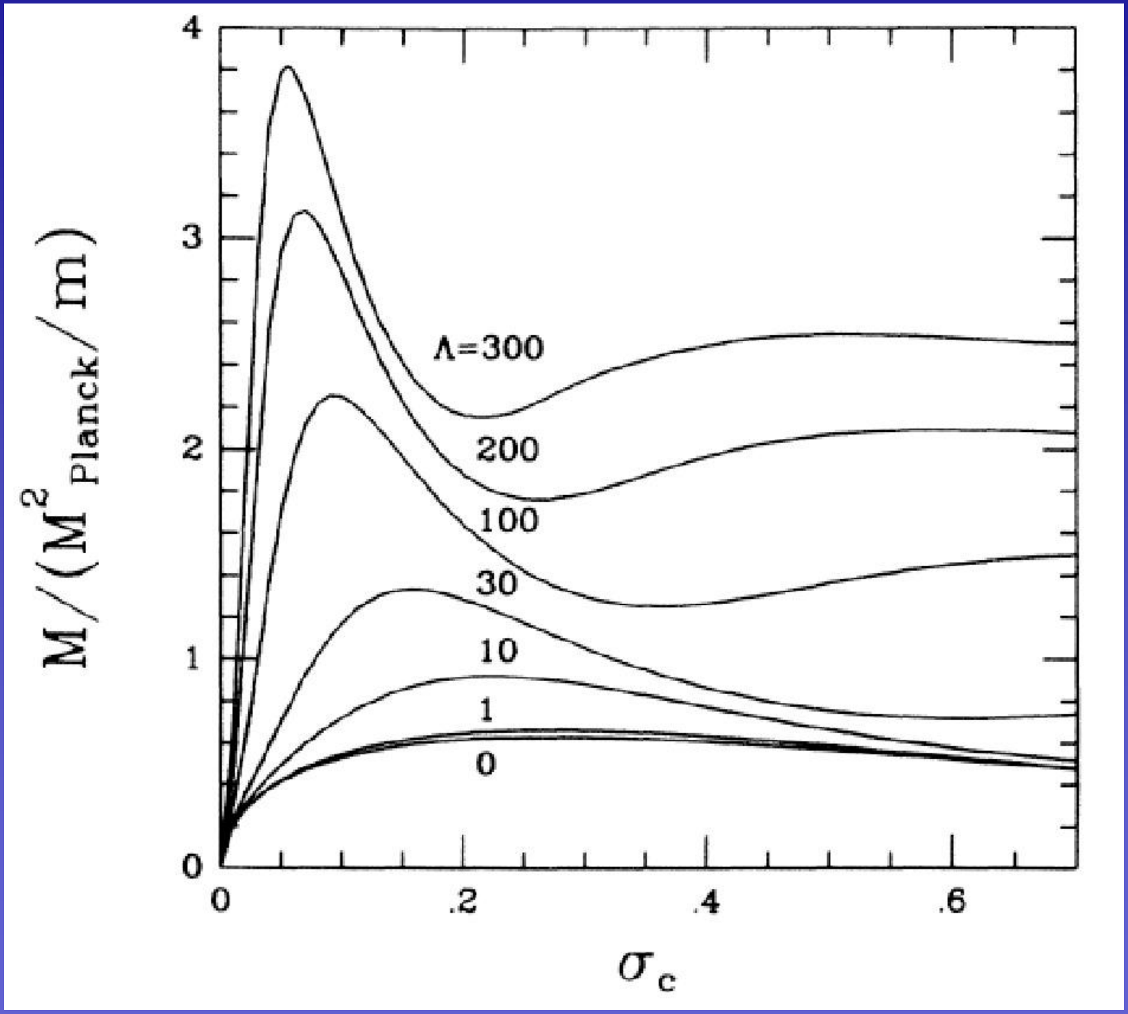
- Get eigenvalue problem (set of ODEs) for any $\psi(0)$ finite energy solution exists only for specific values of ω (or vice versa; in general have countable infinity of solutions)

Boson star evolution



Families of boson stars

(Colpi, Wassermann & Shapiro (1986))



Multi-oscillating Boson Stars in AdS

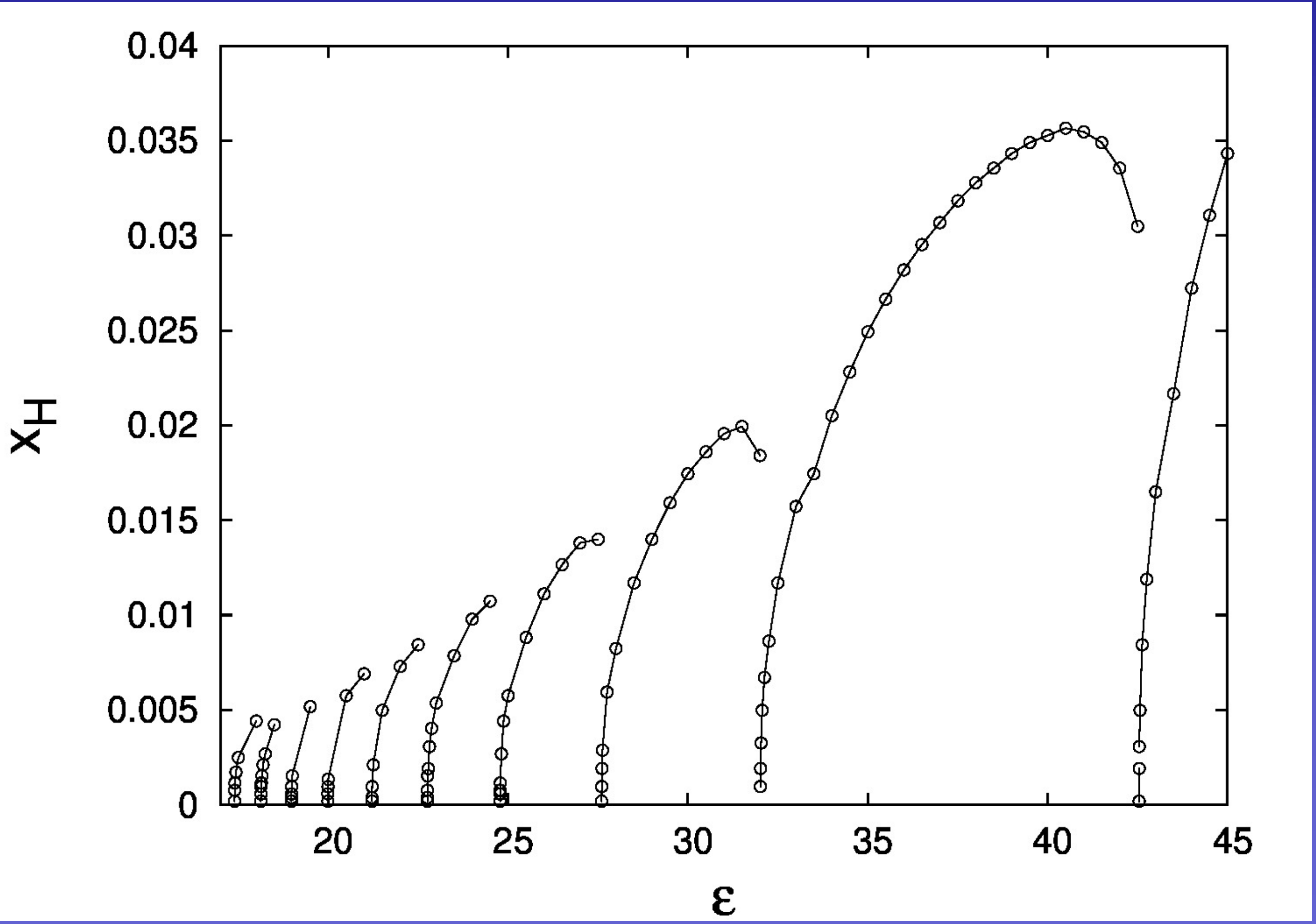
(MWC, Santos, Way, (2018))

Motivation

- Asymptotically de Sitter and asymptotically flat spacetime known to be stable (Friedrich (1986), Christodoulou & Klainerman (1993))
- What about AdS (with reflecting boundary)?
- Heuristic considerations suggest arbitrarily small excitations can form black holes (Dafermos & Holzegel (2006), Dafermos (2006))
- Convincing numerical evidence presented by Bizon and Rostworowski (2011) based on collapse of massless scalar field
- As we heard yesterday, work by Moschidis makes heuristic ideas from numerical results and elsewhere rigorous

Scalar collapse in AdS: Black hole mass versus initial field amplitude

(from Bizon & Rostworowski (2011))



Motivation

- Numerical evidence relatively quickly suggested that not all scalar field configurations would be unstable
- Example: Buchel, Liebling and Lehner (2013)—constructed stationary configurations of a massless, complex field
- Numerical calculations indicated that at least some of these AdS boson stars might be nonlinearly stable
- Current view is that the noncollapsing data constitute “islands of stability”; extent of the islands remains an open problem
- Has implications for interpretation within the field theory dual provided by the AdS/CFT correspondence

Prelude

- Consider perturbation theory about AdS, with massless scalar field, ϕ
- Take perturbative expansion

$$\phi(t,x) = \sum_{p=1}^{\infty} \epsilon^p \phi^{(p)}(t,x) \qquad g_{\mu\nu}(t,x) = \sum_{p=0}^{\infty} \epsilon^p g_{\mu\nu}^{(p)}(t,x)$$

- Linear order: perturbative soln is linear combination of normal modes

$$\phi^{(1)}(t,x) = \sum_{n=0}^{\infty} \left(a_n^+ e^{+i\omega_n^{(0)} t} + a_n^- e^{-i\omega_n^{(0)} t} \right) P_n(x)$$

If $a_n^\pm \neq 0$ for at least two distinct n, secular term proportional to $\epsilon^3 t$ appears, leading to breakdown in pert. theory at timescale $1/\epsilon^2$; evidence suggests this is timescale at which collapse occurs

Prelude

- Now consider $a_n^{\pm} \neq 0$ for only one value of n. Then secular terms can be removed and can construct scalar field configuration that oscillates at a single frequency
- This is precisely a boson star in AdS
- Other oscillating configurations
  - Real scalar field: oscillon
  - Pure gravitational field (non spherically symmetric): geon
- Are all stable (or quasi-stable) configurations close to oscillators?
- What other types of oscillators are there?

Construction of an oscillator

- Linearly perturb AdS to find normal mode with frequency $\omega_n^{(0)}$
- Correct mode with higher orders in perturbation theory (frequency also picks up corrections)
- Nonperturbatively, oscillators have spectral expansion

$$\phi(t,x) = \sum_{k=-\infty}^{\infty} \sum_{l=0}^{\infty} A_{k,l} e^{ik\omega_1 t} P_l(x)$$

where ω_1 can be used as a parameter, with AdS recovered for $\omega_1 = \omega_n^{(0)}$

- Solutions can be computed by treating t as a periodic variable and solving resulting boundary value problem

Construction of a double oscillator

- Iterate process: perturb (single) oscillator

$$\phi(t,x) = \left(\sum_{k,l} A_{k,l} e^{ik\omega_l} P_l(x) \right) + \epsilon e^{-i\omega_2 t} \delta\phi(t,x)$$

where $\delta(t,x)$ is periodic in time with frequency ω_1

- Note: $\phi(t,x)$ is not periodic since ω_1 and ω_2 are not necessarily commensurate

- However, at linear order in ϵ, ω_2 will appear as an eigenvalue in the EOM; all perturbation functions will remain periodic in time with frequency ω_1

- Can thus solve system as BVP using standard approach for solving eigenvalue problems

Construction of a double oscillator

- Reflecting boundary of AdS leads to spectrum of normal modes for the frequency ω_2
- Nonpertubatively, solution takes spectral form

$$\phi(t,x) = \sum_{k_1,k_2=-\infty}^{\infty} \sum_{l=0}^{\infty} A_{k_1,k_2,l} e^{ik_1\omega_1 t + ik_2\omega_2 t} P_l(x)$$

which is "periodic" on the two frequencies ω_1 and ω_2

- Now consider alternative spectral expansion

$$\phi(t_1,t_2,x) = \sum_{k_1,k_2,l} A_{k_1,k_2,l} e^{ik_1\omega_1 t_1 + ik_2\omega_2 t_2} P_l(x)$$

which is periodic on t_1 and t_2

Construction of a double oscillator

- Then make the replacement

$$\partial_t \to \partial_{t_1} + \partial_{t_2}$$

- Double oscillator can now be constructed by solving a BVP in x, t_1 and t_2 where t_1 and t_2 are periodic with frequencies ω_1 and ω_2, respectively

- Clearly, can extend this process indefinitely, leading to multi-oscillators that are periodic on more and more frequencies

Details of construction

- Set AdS length scale, Newton constant to unity; spacetime dimension 5

- Metric and massless complex scalar field ansatz:

$$ds^2 = \frac{1}{\cos^2 x}\left(-\alpha\beta^2 dt^2 + \frac{dx^2}{\alpha} + \sin^2 x \, d\Omega_3\right)$$

$$\phi = \cos^4 x e^{i\omega_1 t}\left(\phi_+ + i\phi_-\right)$$

$$\alpha = 1 - \sin^2 x \cos^2 x \, A$$

$$\beta = 1 - \cos^8 x \, \delta$$

Details of construction

- Time dependence in factor $e^{i\omega_1 t}$ cancels out in equation of motion so can treat A, δ and $\phi_\pm$ as periodic in time with frequency ω_2

- Scalar field is doubly periodic with frequencies ω_1 and ω_2

- Introduce first order variables

$$\cos x \partial_x \phi_\pm - 4 \sin x \phi_\pm = \Phi_\pm$$

$$\partial_t \phi_\pm = \alpha\beta \frac{\Pi_\pm}{\cos x} \pm \omega_1 \phi_\pm$$

- Momentum constraint (used as independent check)

$$\partial_t A = 2 \frac{\cos^4 x}{\sin x} \alpha^2 \beta \left(\Phi_+ \Pi_+ + \Phi_- \Pi_- \right)$$

Details of construction

- Slicing condition, Hamiltonian constraint, scalar field evolution equations

$$\cos x \partial_x \delta - \sin x \left(8 + \cos^8 x S\right)\delta = -\sin x S$$

$$\sin x \partial_x A + \cos x \left(4 + \sin^2 \cos^8 x S\right) A = \cos^3 x S$$

$$\partial_t \Phi_\pm = \beta\left(\alpha \partial_x \Pi_\pm - A_\Phi \tan x \Pi_\pm\right) \pm \omega_1 \Phi_\mp$$

$$\partial_t \Pi_\pm = \beta\left(\alpha \partial_x \Phi_\pm + A_\Pi \cot x \Phi_\pm\right) \pm \omega_1 \Pi_\mp$$

$$S = \Phi_+^2 + \Phi_-^2 + \Pi_+^2 + \Pi_-^2$$

$$A_\Phi = 3 - \frac{1}{2}\left[9 - 5\cos(2x)\right]\cos^4 x A$$

$$A_\Pi = 3 - \frac{1}{2}\left[3 - 5\cos(2x)\right]\sin^2 x \cos^2 x A$$

Details of construction

- Stage 1: Find boson stars and their perturbations; set

$$\Phi_+(t,x) = \Phi_0(x) + \epsilon\left[\cos(\omega_2 t)\delta\Phi_+(x)\right]$$
$$\Phi_-(t,x) = \epsilon\left[\sin(\omega_2 t)\delta\Phi_-(x)\right]$$
$$\Pi_+(t,x) = \epsilon\left[\sin(\omega_2 t)(t)\delta\Pi_+(x)\right]$$
$$\Pi_-(t,x) = \Pi_0(x) + \epsilon\left[\cos(\omega_2 t)\delta\Pi_-(x)\right]$$
$$A(t,x) = A_0(x) + \epsilon\left[\cos(\omega_2 t)\delta A(x)\right]$$
$$\delta(t,x) = \delta_0(x) + \epsilon\left[\cos(\omega_2 t)\delta\delta(x)\right]$$

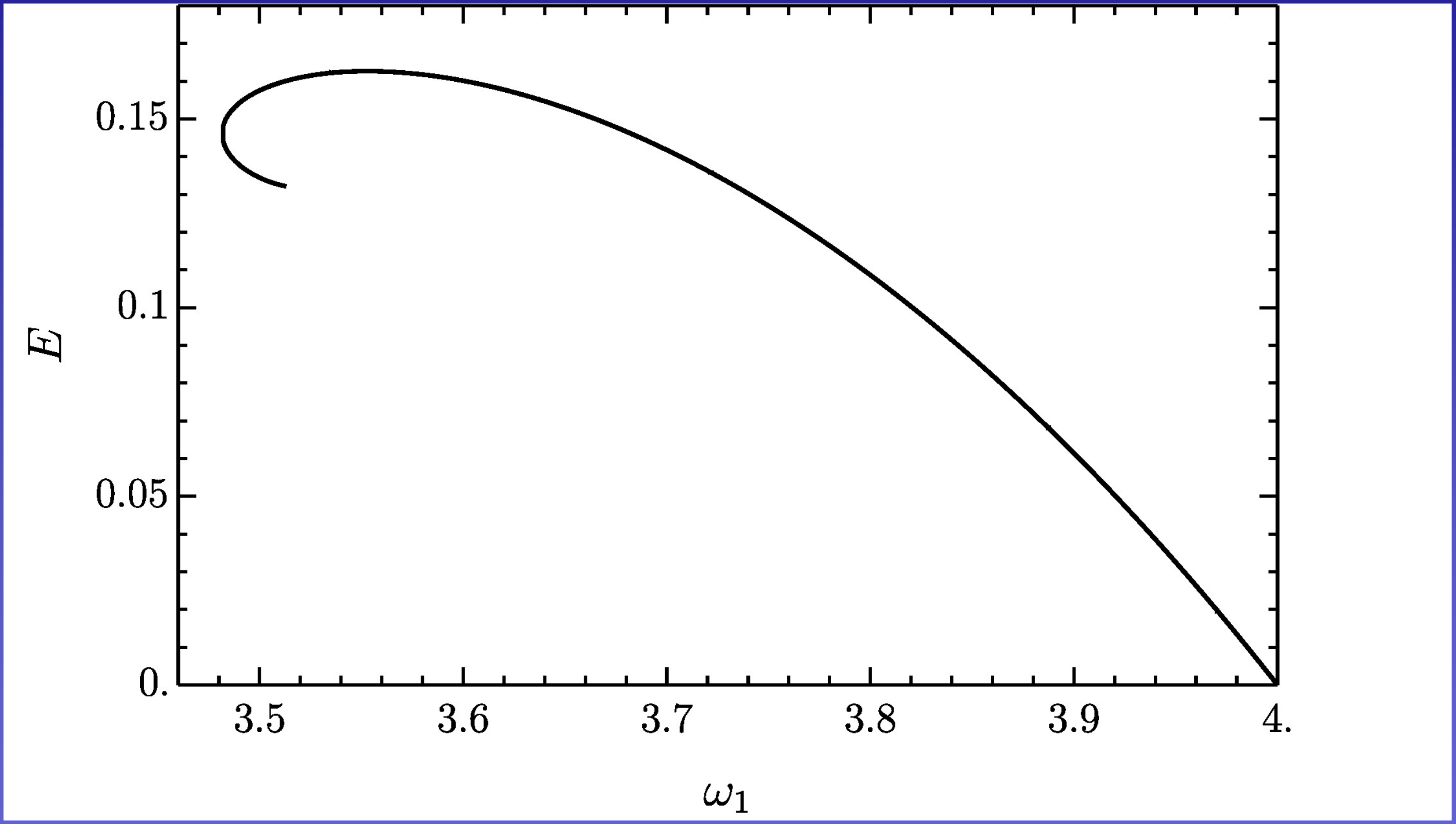
- With $\epsilon = 0$ get a set of ODEs that can be solved to obtain a boson star; then linearly expand in ϵ to get another set of ODEs for perturbation functions as an eigenvalue problem with eigenvalue ω_2

- Solve these ODE systems using Fourier spectral methods

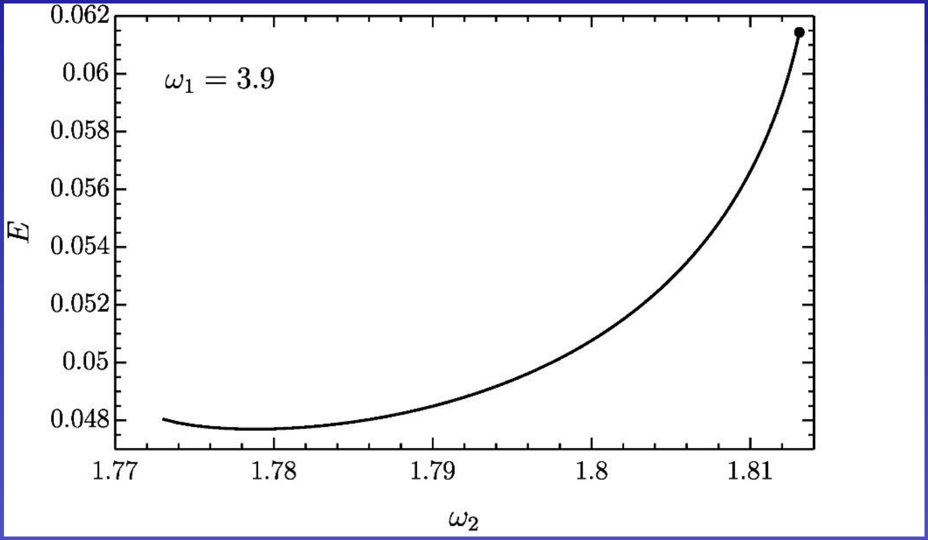
Details of construction

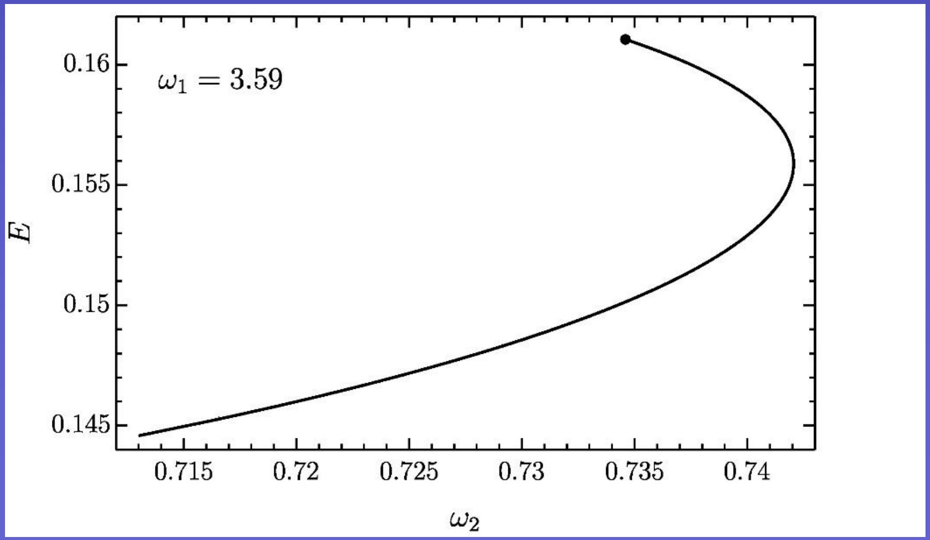
- Stage 2: Use perturbed boson stars as initial conditions for Newton-Raphson solution of full, nonlinear BVP resulting from treating t as a periodic coordinate with frequency ω_2

Boson star energy (total mass) vs ω_1

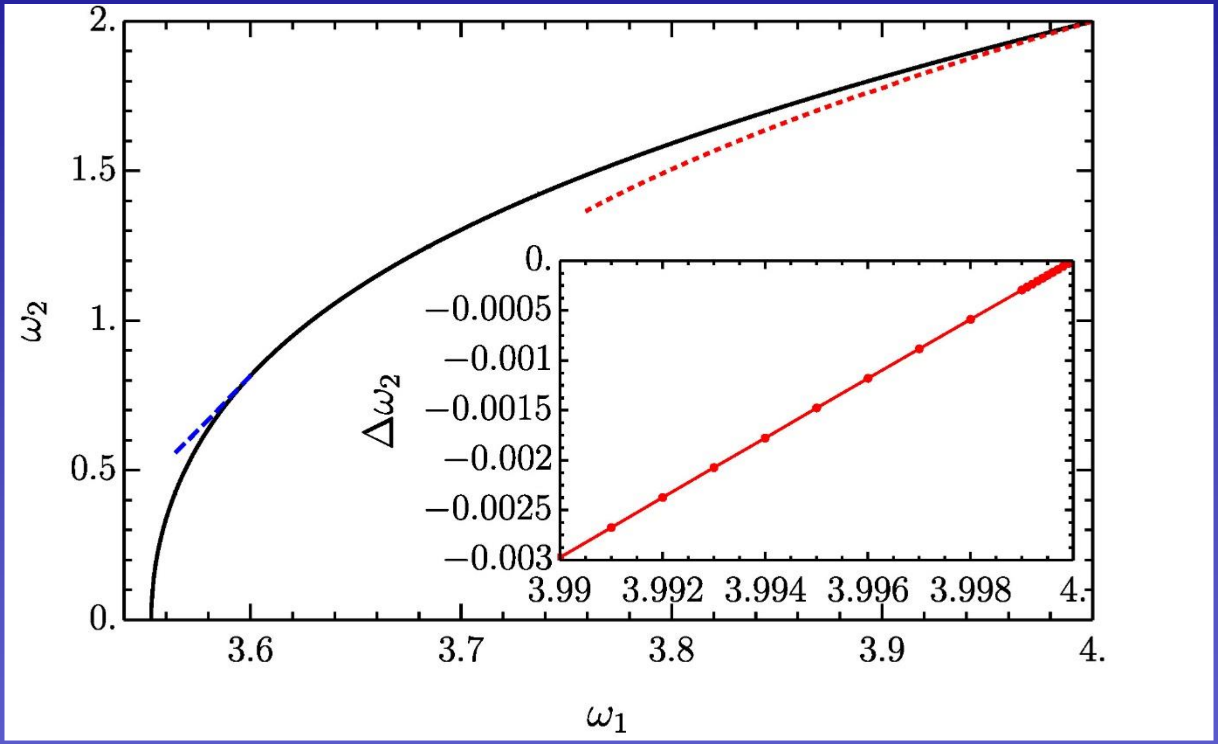


Double oscillators: Energy vs ω_2





Double oscillators: Parameter space



Black: Boson stars
Red: Turning point in energy
Blue: Turning point in frequency

Multi-oscillating Boson Stars in Asymptotically Flat Spacetime

(MWC, Masachs, Way (2019))

Early Work

(Hawley & MWC (2003))

- Seidel and Suen (1991) numerically constructed oscillating, quasi-stationary self-gravitating configurations of a real, massive scalar field

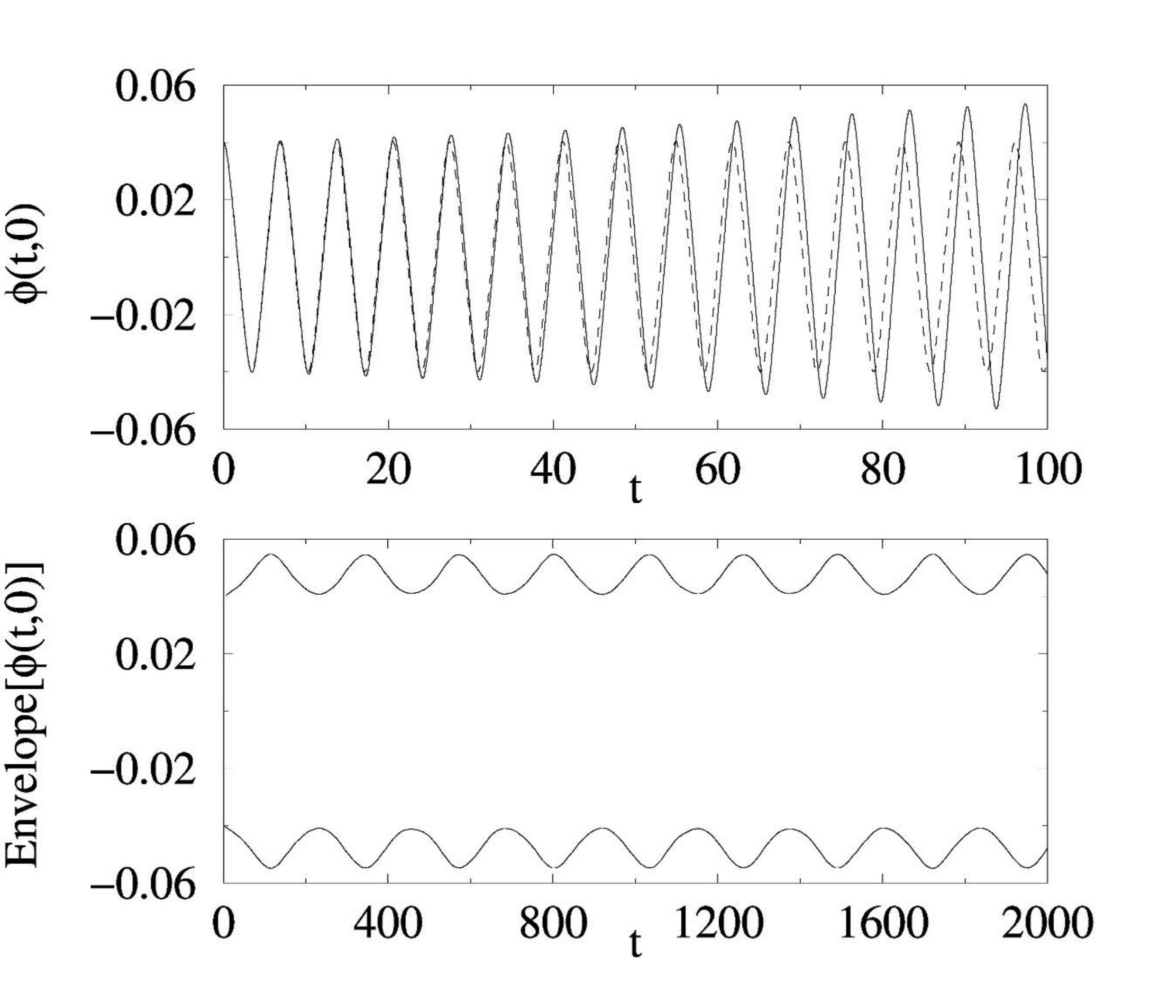
- Consider two components of complex scalar field with boson star ansatz

$$\phi_1(t,r) = \psi(r)\cos(\omega t)$$
$$\phi_2(t,r) = \psi(r)\cos(\omega t + \delta)$$
$$\delta = \pm\pi / 2$$

- Then setting $\delta = 0$ can resolve Einstein equations, find configuration very close to Seidel & Suen solution

- Can then generalize to arbitrary $\delta \neq 0$

"Multi-scalar" star



Provided strong evidence for existence of multi-field, multi-frequency, quasi-stationary solutions

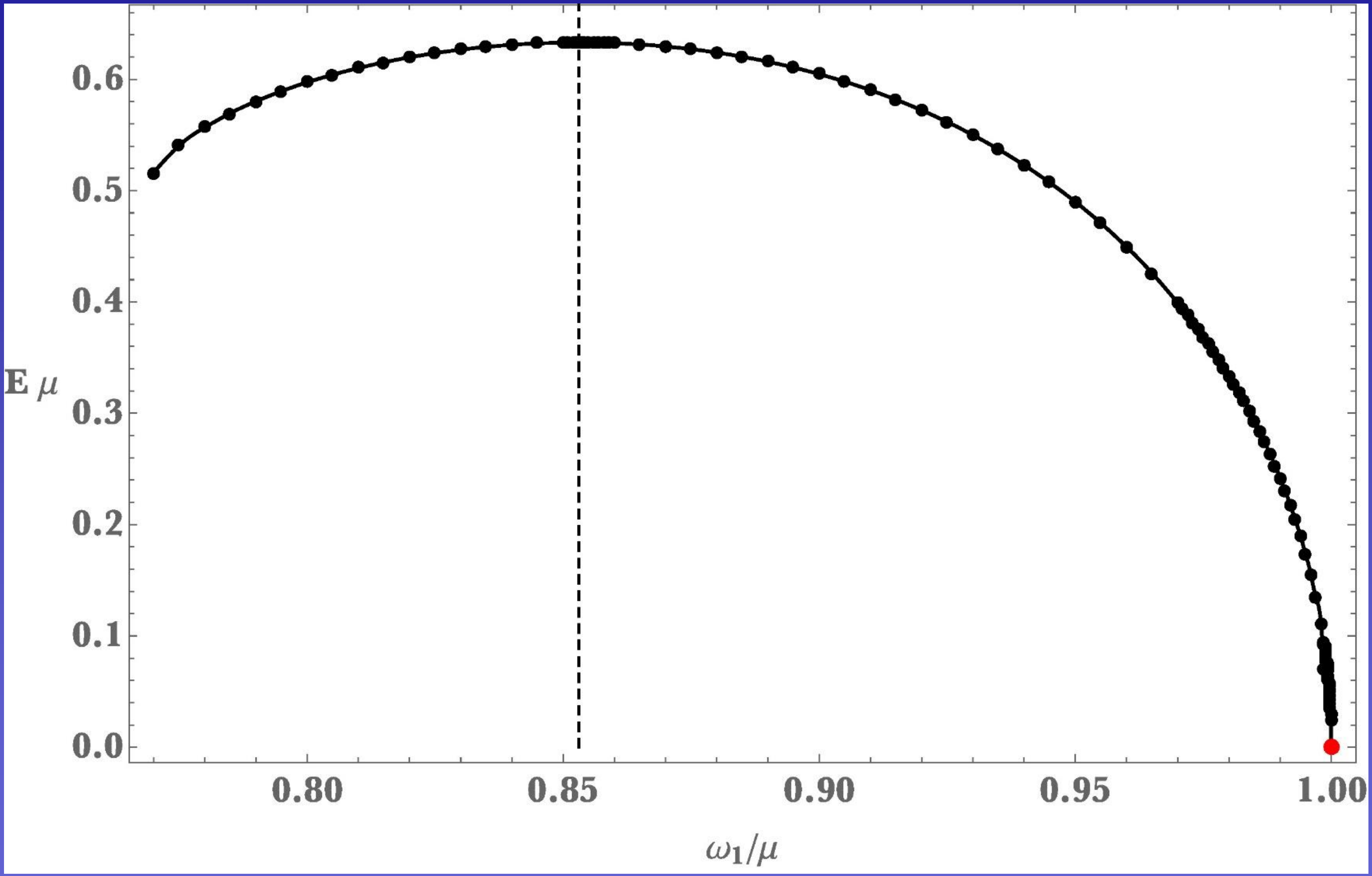
Prelude

- Consider massive (m = 1) complex scalar field
- Boson star ansatz

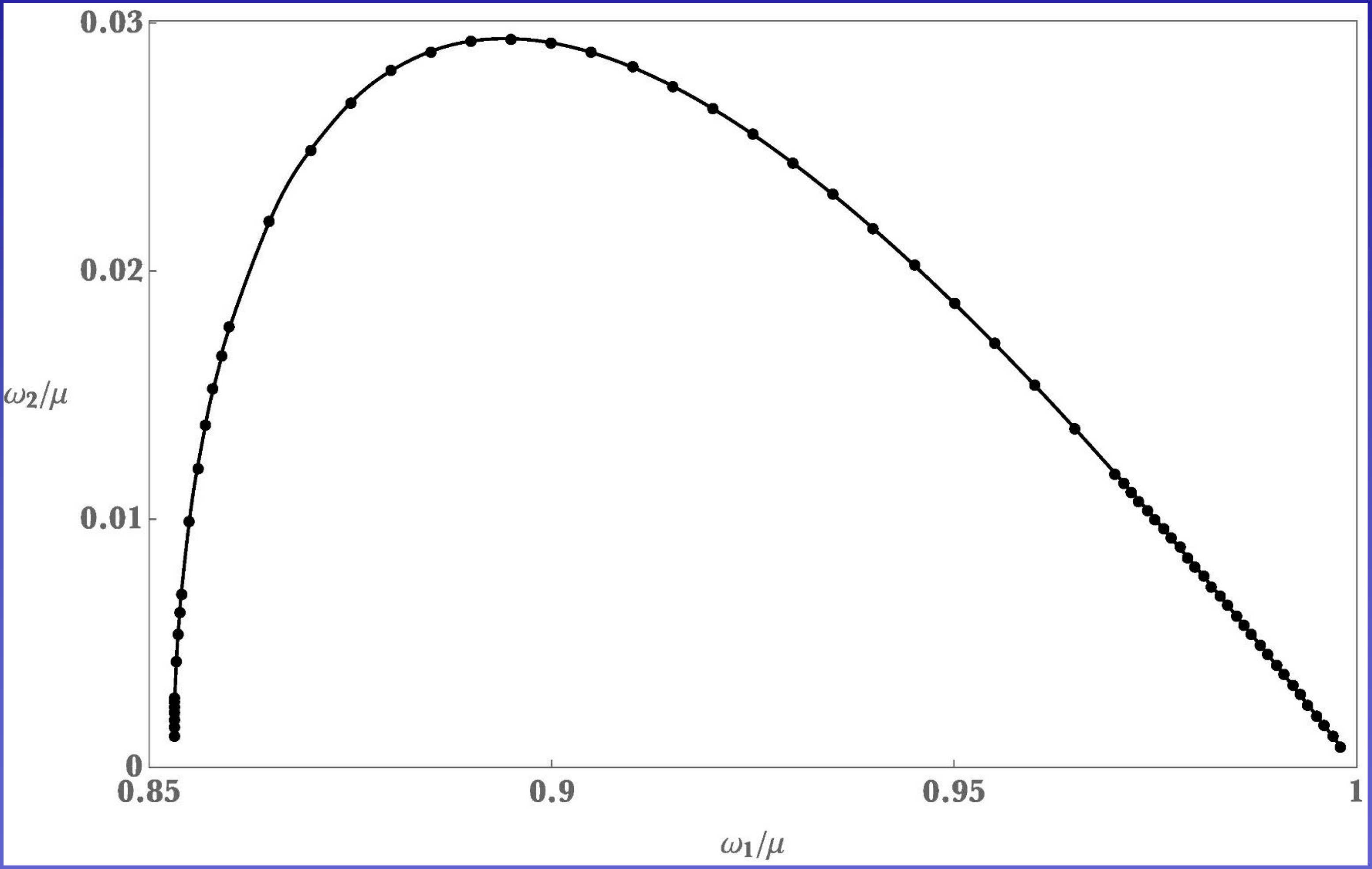
$$\phi(t,r) = e^{i\omega_1 t}\psi(r)$$

- Again, boson stars constitute one-parameter family (family parameter ω_1 or $\psi(0)$ e.g.)
- Can determine lowest perturbative mode; has frequency ω_2
- As with AdS case, perturbative mode can be "promoted" into nonlinear regime, resulting in double oscillator

Boson star energy vs frequency



Perturbative frequency of boson stars



Details of construction

- Metric and field ansatz

$$ds^2 = -\delta f dt^2 + \frac{dr^2}{f} + r^2 d\Omega_2$$

$$\phi = e^{i\omega_1 t}\left(\phi_r + i\phi_i\right)$$

- Introduce compactified spatial coordinate

$$r = r_s \rho \frac{\sqrt{2-\rho^2}}{1-\rho^2}$$

Details of construction

- Redefine basic dependent variables via

$$\delta = 1 - (1-\rho^2) f_1$$
$$f = 1 - \rho^2 (2-\rho^2)(1-\rho^2) f_2$$
$$\phi_r = (1-\rho^2)^2 f_3$$
$$\phi_i = (1-\rho^2)^2 f_4$$

- Now consider quasiperiodic function f on k frequencies

$$f(t,\rho) = \sum_{n_1,\ldots,n_k} A_{n_1,\ldots,n_k}(\rho) e^{in_1\omega_1 t + \ldots + in_k \omega_k t}$$

- Has same spectral information as

$$f(t_1,\ldots,t_k,\rho) = \sum_{n_1,\ldots,n_k} A_{n_1,\ldots,n_k}(\rho) e^{in_1\omega_1 t_1 + \ldots + in_k \omega_k t_k}$$

Details of construction

- Double oscillators can then be found by setting $\partial_t \to \partial_{t_1} + \partial_{t_2}$ promoting $f_i(t,\rho) \to f_i(t_1,t_2,\rho)$ and demanding that each time coordinate t_i be periodic with frequency ω_i

- In general this process yields a BVP in 3 coordinates but problem is simplified by incorporating ω_1 into an overall phase, which removes the dependence on t_1

- Adopt time Fourier series

$$f_{1,2,3}(t,\rho) = \sum_k \hat{f}^{(k)}_{1,2,3}(\rho)\cos(k\omega_2 t_2)$$

$$f_4(t_2,\rho) = \sum_k \hat{f}^{(k)}_4(\rho)\sin(k\omega_2 t_2)$$

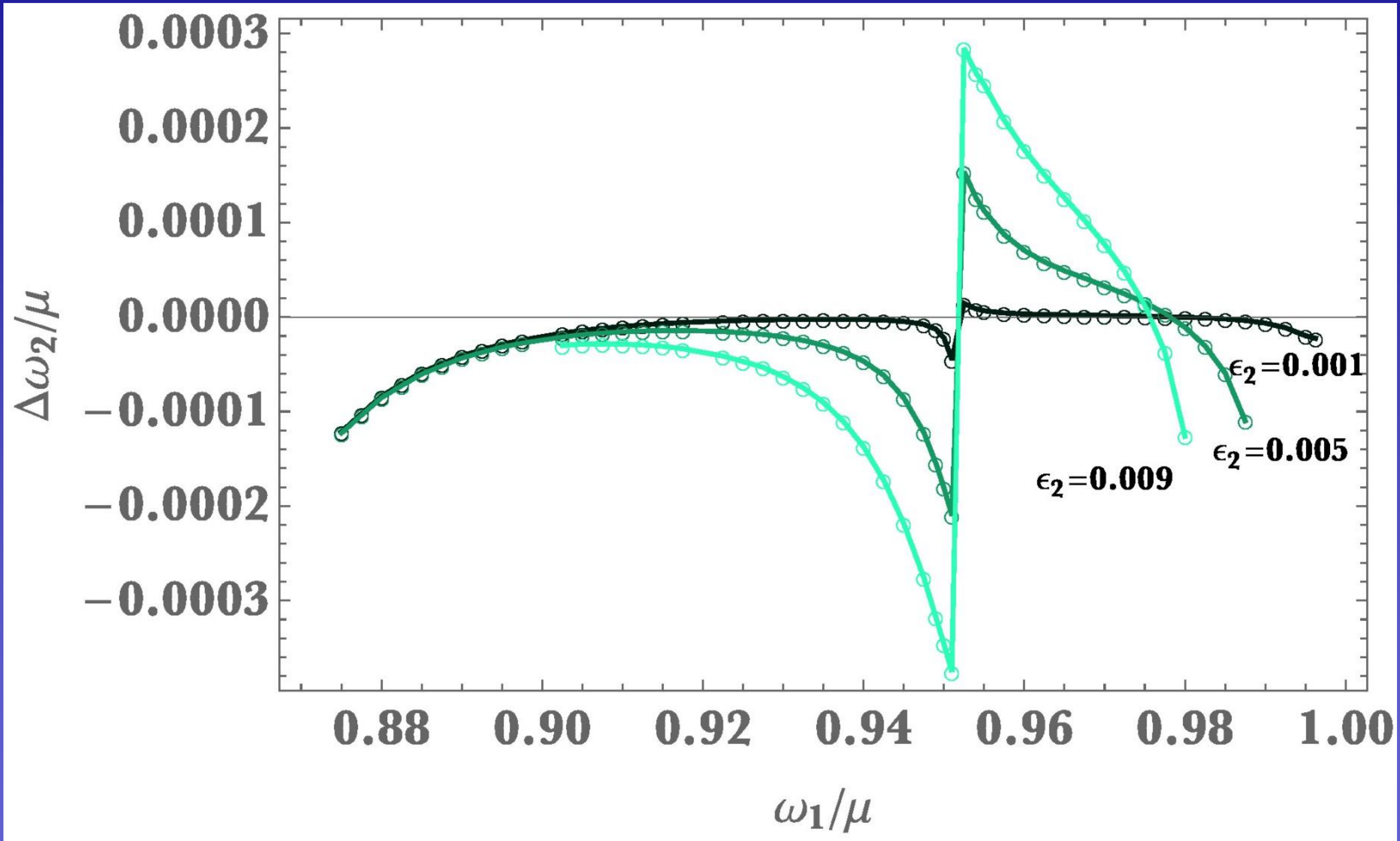
- Diagnostic quantities

$$E = \frac{r_s}{2} f_2(1) \qquad M(\rho) = \frac{r_s \rho^3 (2-\rho^2)^{3/2}}{2} f_2(\rho)$$

Details of construction

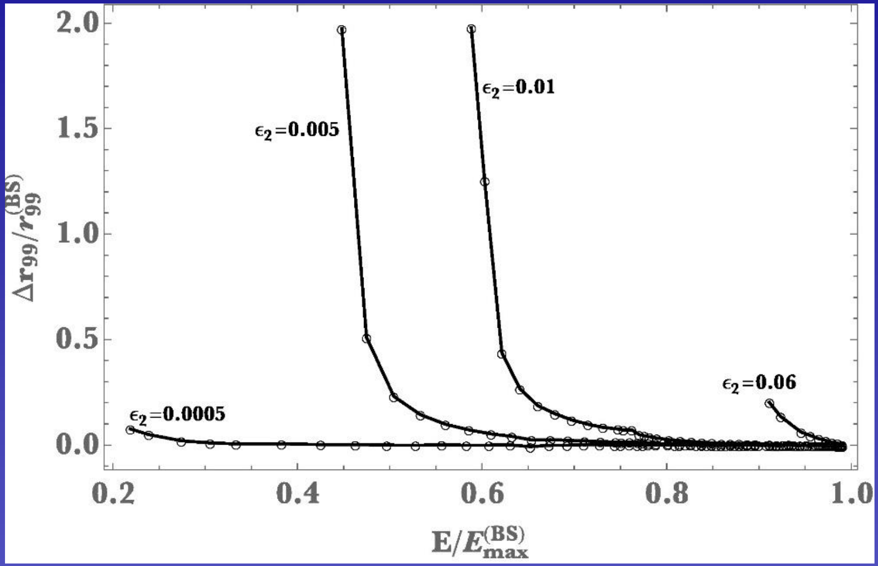
- Solve double-oscillator equations using Fourier spectral methods in t_2 and fourth order finite differences in ρ
- Use an overall Newton-Raphson method with boson star solutions as initial estimates

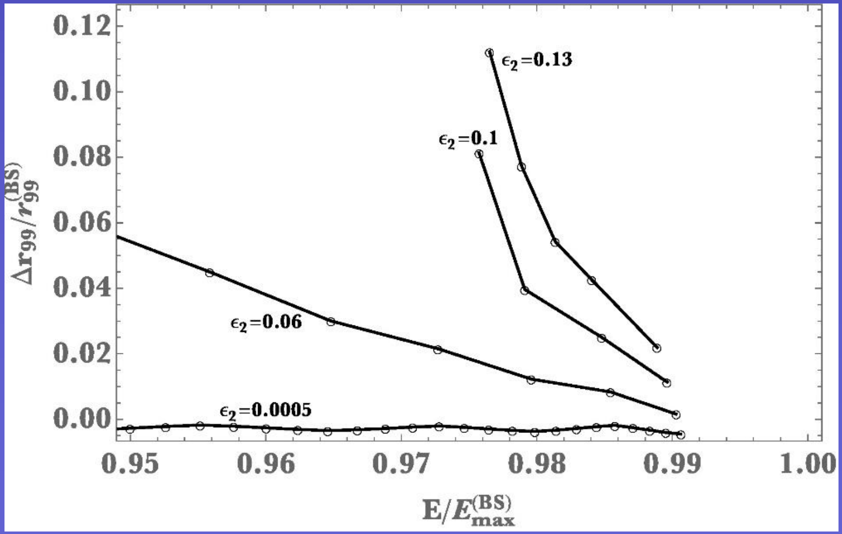
Change in ω_2 as function of ω_1



$\epsilon_2 = \hat{f}_4^{(1)}(0)$ (equivalent to ω_2)

Size of star as function of energy





Conclusions

Conclusions

- Although we have constructed only double oscillators, it seems clear that process can be extended to produce configurations that oscillate on any number of incommensurate frequencies
- For AdS case is tempting to conjecture that islands of stability are precisely mapped out by such configurations
- Stability of these double-oscillators (and multi-oscillators in general) remains an open question, but fact that perturbed single-oscillator stars appear stable for both AdS and AF cases suggest that at least some of them are (quasi)-stable (as well as evidence from previous calculations, e.g. Hawley & MWC (2003))
- Construction should extend to related models, such as Q-balls (stationary configurations of nonlinearly self-coupled scalar fields in flat spacetime)