MULTI-MESSENGER ASPECTS OF CHARACTERISTIC EVOLUTION

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MULTI-MESSEMGER ORIGINS

COSMIC RAYS

My first contact with astronomy (1958)

GENERAL RELATIVITY -1960's

Main Groups

Syracuse (Peter Bergmann) and Princeton (John Wheeler)

"If you are interested in physics, stay away from relativity. It is the province of mathematicians and no physicist should enter."

— Leonard Schiff to Richard Isaacson

He was right in 1962!

It took a century, but now the field is a highly respectable part of physics and astronomy.

RELATIVITY IN SYRACUSE (1960)

MAIN ACTIVITY

QUANTIZATION (Bergmann, Komar, Arnowitt)

NUMERICAL RELATIVITY DID NOT YET EXIST

IT WAS A GOLDEN ERA

Among long term visitors and postdocs to Syracuse

Ray Sachs: Bondi-Sachs description of gravitational waves

Roy Kerr: the solution for a rotating black hole

Roger Penrose: A new look at everything - asymptotics at infinity, black hole singularity theorems, spinors

My PhD thesis: ASYMPTOTIC BEHAVIOR OF COUPLED GRAVITATIONAL FIELDS

Multi-mesenger Bondi-Sachs theory

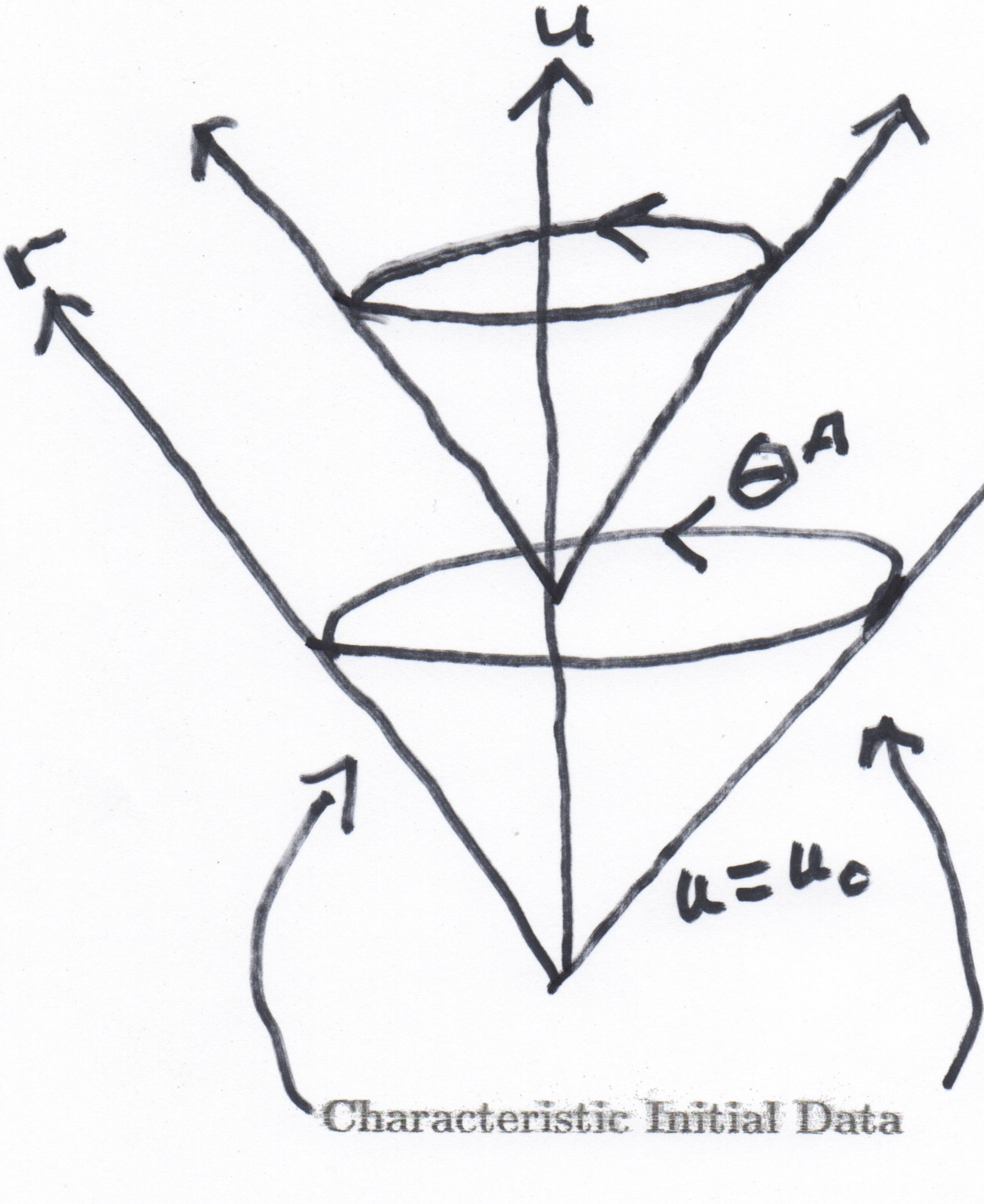
Integration of electromagnetic fields and other matter fields into the Bondi-Sachs formalism

CHARACTERISTIC INITIAL VALUE PROBLEM

Bondi-Sachs coordinates

$$(u, r, \theta^A = (\theta, \phi))$$

u labels family of outgoing null hypersurfaces
r measures areal radius along the outgoing null rays
θ^A anguar coordinates labeling the null rays



Characteristic Initial Data

Curved space Bondi–Sachs coordinates

$$x^\alpha = (u, r, \theta^A)$$

u labels family of outgoing spherical null hypersurfaces

r is areal coordinate along outgoing null rays

$\theta^A = (\theta, \phi)$ label the null rays

Bondi-Sachs metric

$$g_{\alpha\beta}dx^\alpha dx^\beta = -\frac{V}{r}e^{2\beta}du^2 - 2e^{2\beta}dudr + r^2 h_{AB}(d\theta^A - U^A du)(d\theta^B - U^B du)$$

Coordinate Conditions on Metric Components

Null conditions $0 = g_{rr} = g_{rA}$

Areal coordinate r $\Rightarrow \det[g_{AB}] = r^4 \det[q_{AB}] = r^4 q$

Unit sphere metric $q_{AB}d\theta^A d\theta^B = d\theta^2 + \sin^2\theta d\phi^2 \quad q = \sin^2\theta$

$\Rightarrow g_{AB} = r^2 h_{AB} \qquad \det[h_{AB}] = q = \sin^2\theta$

Conformal 2-metric h_{AB} has only two degrees of freedom

6 Metric functions: V, β, U^A, h_{AB}

Coupled Bondi-Sachs Equations

Einstein equations: $E_{\alpha\beta} := G_{\alpha\beta} - 8\pi T_{\alpha\beta} = 0$

Main equations: $E^{\beta}_{r} = 0\,, \quad E_{AB} - \frac{1}{2} g_{AB} g^{CD} E_{CD} = 0$

If main equations are satisfied the Bianchi identities and matter conservation $\nabla_{\beta} E^{\beta}_{\alpha} = 0$ imply

Trivial equation: $g^{AB} E_{AB} = 0$ **and**
$\partial_r(\sqrt{-g} E^r_u) = 0\,, \quad \partial_r(\sqrt{-g} E^r_A) = 0$

Integration $\Rightarrow$ Conservation Conditions $E^r_u = E^r_A = 0$ if satisfied on a worldtube or at a nonsingular worldline $r = 0$ traced out by the vertices of the null cones

At null infinity $\mathcal{I}^+$ the conservation conditions give flux conservation laws for energy-momentum and angular momentum, including the famous Bondi mass loss equation

The main equations form the evolution system for the Bondi-Sachs metric

Evolution of the Bondi-Sachs Metric

Bondi-Sachs metric

$$g_{\alpha\beta}dx^\alpha dx^\beta = -\frac{V}{r}e^{2\beta}du^2 - 2e^{2\beta}dudr + r^2 h_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

Main equations separate into

Hypersurface equations: $E_{r\alpha} = 0$

and

Evolution equations: $E_{AB} - \frac{1}{2}g_{AB}g^{CD}E_{CD} = 0$

Given the data h_{AB} on a null hypersurface N_u the hypersurface Eqs are sequence of radial ODEs for the values of the remaining metric variables (β, U^A, V) on N_u

$$\partial_r \beta = \mathcal{H}_\beta(h_{AC}) + 2\pi r T_{rr}$$

$$\partial_r \left[r^4 e^{-2\beta} h_{AB}(\partial_r U^B) \right] = \mathcal{H}_U(h_{AC}, \beta) + 16\pi r^2 T_{rA}$$

$$2e^{-2\beta}(\partial_r V) = \mathcal{H}_U(h_{AC}, \beta, U^A) + 8\pi[h^{AB}T_{AB} - r^2 T^\alpha_\alpha]$$

where $\mathcal{H}$ represents operators intrinsic to N_u. This also holds for matter fields $T_{\alpha\beta}$ for which N_u are characteristic (e.g. Klein-Gordon and Maxwell fields)

For such coupled fields, given characteristic matter data and gravitational data $h_{AB}|_{N_u}$ on a null hypersurface N_u and the boundary data $(\beta, U^A, \partial_r U^A, V)|_\Gamma$ on an inner worldtube Γ, along with the corresponding matter boundary data, the hypersurface equations determine $(\beta, U^A, \partial_r U^A, V)|_{N_u}$

This lead to a simple evolution algorithm

Evolution Equations

Evolution of characteristic data h_{AB} is simplified by introducing a complex null polarization dyad

$$m^\alpha = (0, 0, m^A) \textbf{ with } h_{AB} m^A m^B = 0$$

tangent to N_u and spanning the angular directions

Evolution equations reduce to complex equation
$E_{AB} - \frac{1}{2} g_{AB} g^{CD} E_{CD} = 0 \Rightarrow m^A m^B E_{AB} = 0$

In terms of metric variables this is a radial ODE for $\partial_u h_{AB}$

$$m^A m^B \partial_r (r \partial_u h_{AB}) = \mathcal{H}(\beta, U^A, V) + 8\pi r^{-1} e^{2\beta} m^A m^B T_{AB}$$

where the righthand side are hypersurface quantities

Given boundary data $\partial_u h_{AB}|_\Gamma$ this leads to a simple update scheme for the gravitational data h_{AB}

Brans-Dicke Scalar-Tensor Gravity

Spherically symmetric oscillation of neutron star would give rise to monopole scalar radiation, unlike the conventional spin-2 tensor component

After a supernova collapse the scalar radiation would damp the spherical oscillation mode in less than a second, producing an intense short burst which could be detectable by a Weber bar antenna

Originally Dicke thought the antenna response to monopole scalar radiation would be spherically symmetric

Instead, a Bondi-Sachs analysis showed the radiative component of the Riemann tensor corresponding to the scalar field is purely transverse to the propagation direction and axially symmetric

An Observable Peculiarity of the Brans-Dicke Radiation Zone

David Robinson and JW (1969) Phys Rev Letters


MATHEMATICAL VS NUMERICAL
Gμν = Tμν
1011010101 1101
010100110101011011

Cauchy-Characteristic Radiation Extraction

Worldtube-Null-Cone Problem

Given null data $h_{AB}|_{N_u}$ and boundary data on inner worldtube Γ

$$(h_{AB}, \beta, V, U^A, \partial_r U^A)|_\Gamma$$

satisfying the supplementary conditions the Main Equations determine a finite difference approximation for $h_{AB}|_{N_{u+\Delta u}}$

Boundary data on Γ is furnished from numerical solution of Einstein's equations carried out by Cauchy evolution of the interior such that it satisfies the supplementary conditions

Given the initial and boundary data the hypersurface equations can be integrated numerically in sequential order and the evolution algorithm can be iterated into the future using a finite difference time-integrator

Implemented as stable, convergent evolution code which propagates the exterior solution to $\mathcal{I}^+$ where radiation strain is computed

PITT NULL CODE (FINITE DIFFERENCE)

SpEC CODE (QUASI-SPECTRAL)

Advantages of Cauchy-Characteristic Extraction

- **Penrose compactification is implemented to conformally map $\mathcal{I}^+$ into a finite grid boundary**
- **The Bondi-Metzner-Sachs (BMS) asymptotic symmetry group at $\mathcal{I}^+$ can be constructed**
- **The BMS group identifies coordinates corresponding to a distant inertial observer**
- **The Bondi news function and radiation strain can be unambiguously computed in inertial coordinates to within finite difference or spectral error.**
- **The Poincare subgroup of the BMS group provides a geometric definition of energy-momentum and angular momentum**
- **The BMS group contains a supertranslation subgroup $u \to u + \alpha(\theta^A)$ comprising a time-independent shift of the slicing of $\mathcal{I}^+$ into spherical cross-sections. This introduces an angular momentum ambiguity associated with a supertranslation of the origin**
- **A supertranslation shift between initial and final retarded times results from the radiation memory effect**

  $\Delta(\textbf{Radiation strain}) = \Delta h_{AB} = h_{AB}(u=\infty,\theta) - h_{AB}(u=-\infty,\theta)$

  which produces net change in relative position $x - y$ of neighboring particles in the radiation zone

  $\Delta(x-y) \sim r^{-1}\Delta(\textbf{Radiation strain})$

The angular momentum and memory effects have only been roughly explored in numerical simulations

The Electromagnetic Analogue

Minkowski metric in outgoing null coordinates (u, r, θ^A)

$$\eta_{\alpha\beta}dx^\alpha dx^\beta = -du^2 - 2drdu + r^2 q_{AB} d\theta^A d\theta^B$$

Assume sources of electromagnetic field are inside a world-tube Γ of radius $r = R$. The outgoing null cones N_u intersect Γ in spheres S_u with angular coordinates θ^A

The Maxwell field $F_{\alpha\beta}$ is represented by vector potential A_α, $F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$ with gauge freedom $A_\alpha \to A_\alpha + \nabla_\alpha \Lambda$

Choice $\Lambda(u, r, \theta^A) = -\int A_r(u, r', \theta^A)dr'$ leads to null gauge $A_r = 0$, which is analogue of Bondi-Sachs coordinate conditions $g_{rr} = g_{rA} = 0$

Remaining gauge freedom $\Lambda(u, x^A)$ used to set $A_u|_{\mathcal{I}^+} = 0$ at future null infinity

Remnant freedom $A_B \to A_B + \partial_B \Lambda(\theta^C)$ is Maxwell analogue of Bondi-Metzner-Sachs supertranslation freedom

Bondi-Sachs Version of Maxwell Equations

Source-free Maxwell equations $\mathcal{E}^\beta := \nabla_\alpha F^{\alpha\beta} = 0$

$$F^{\alpha\beta} = -F^{\beta\alpha} \Rightarrow$$

$$\textbf{IDENTITY} \quad 0 \equiv \nabla_\beta \mathcal{E}^\beta = \nabla_\beta \nabla_\alpha F^{\alpha\beta}$$

$$0 \equiv \nabla_\beta \mathcal{E}^\beta = \frac{1}{\sqrt{-g}} \partial_\beta (\sqrt{-g}\mathcal{E}^\beta) = \partial_u \mathcal{E}^u + \frac{1}{r^2}\partial_r (r^2 \mathcal{E}^r) + \frac{1}{\sqrt{q}} \partial_A (\sqrt{q}\mathcal{E}^A)$$

STRATEGY

Designate $\mathcal{E}^u = 0$ **and** $\mathcal{E}^A = 0$ **as Main Equations**

Designate $\mathcal{E}^r = 0$ **as Conservation Condition**

If Main Equations satisfied then IDENTITY implies

$$0 = \partial_r (r^2 \mathcal{E}^r)$$

so Conservation Condition is satisfied if it is satisfied at any specified value of r**, e.g. on** Γ **or at** $\mathcal{I}^+$

Main Equations form hiearchy

Hypersurface Equation $\mathcal{E}^u = 0$

$$\partial_r (r^2 \partial_r A_u) = \partial_r (\eth_B A^B) = \mathcal{H}_u(A^B)$$

Evolution Equation $\mathcal{E}^A = 0$

$$\partial_r \partial_u A_B = \tfrac{1}{2}\partial_r^2 A_B - \tfrac{1}{2} r^2 \eth^C (\eth_B A_C - \eth_C A_B) + \tfrac{1}{2}\partial_r \eth_B A_u = \mathcal{H}(A^B, A_u)$$

$\eth_A$ **is covariant derivative with respect to** q_{AB}

WorldTube-NullCone Evolution Algorithm

Electromagnetic version of Cauchy-Characteristic Extraction

INITIAL NULL DATA $A_B|_{N_{u_0}}$ **(Dynamical degrees of freedom)**

INITIAL BOUNDARY DATA $\partial_r A_u|_{\Gamma, u=u_0}$ **(Interior sources)**

WORLDTUBE DATA $\partial_u A_B|_\Gamma$ **(Dynamical degrees of freedom)**

GAUGE DATA $A_u|_\Gamma$

SEQUENTIAL RADIAL INTEGRATION SCHEME

Hypersurface Equation $\partial_r(r^2 \partial_r A_u) = \mathcal{H}_u(A_B)$

$$\Rightarrow A_u|_{N_{u_0}}$$

Evolution Eq: $\partial_r \partial_u A_B = \mathcal{H}(A_B, A_u)$

$$\Rightarrow \partial_u A_B|_{N_{u_0}}$$

Finite difference approximation $\Rightarrow A_B|_{N_{u_0+\Delta u}}$

Conservation Condition $r^2 \partial_u \partial_r A_u = \eth^B(\partial_r A_B - \partial_u A_B + \eth_B A_u)$

$$\Rightarrow \partial_u \partial_r A_u|_{\Gamma, u=u_0}$$

Finite difference approximation $\Rightarrow \partial_r A_u|_{\Gamma, u=u_0+\Delta u}$ **and** $A_u|_{N_{u_0+\Delta u}}$

GIVEN $A_B|_{N_{u=u_0+\Delta u}}$ **and** $\partial_r A_u|_{\Gamma, u_0+\Delta u}$ **ITERATION GIVES FINITE DIFFERENCE APPROXIMATION FOR**

$$A_B|_{N_{u_0+n\Delta u}} \qquad \partial_r A_u|_{\Gamma, u=u_0+n\Delta u} \qquad A_u|_{N_{u_0+n\Delta u}}$$

UNFINISHED BUSINESS

IMPLEMENTATION AS EVOLUTION CODE

COUPLED EINSTEIN-MAXWELL CODE

ELECTROMAGNETIC RADIATION MEMORY
Bieri, Garfinkle

Radiation field at null infinity $\mathcal{I}^+$ $\quad E_B(u,\theta^C) = -\partial_u A_B(u,\theta^C)$

Retarded transverse angular components of E-field

NET KICK ON CHARGE q AFTER RADIATION PASSES

Integrate $qE_B = m\frac{dv_B}{du}$

$$\Delta v_B = v_B|_{u=-\infty}^{u=\infty} = (q/m)\int_{u=-\infty}^{u=+\infty} E_B(u,\theta^C)du = -(q/m)\Delta A_B$$

MODE DECOMPOSITION

E-mode $A_B = \partial_B \Phi\,,$ $\quad$ **B-mode** $A_B = \epsilon_{BC}\eth^C\Psi$

Gauge freedom at $\mathcal{I}^+$ $\quad A_B(u,\theta^C) \to A_B(u,\theta^C) + \partial_B\Lambda(\theta^C)$

E-mode component of radiation memory ΔA_B is gauge shift between $u=\pm\infty$ $\quad$ **Analogue of supertranslation shift**

IMPORTANT ASTROPHYSICAL SOURCE

Burst memory - Charge Q ejected to infinity

Dipole radiation $E_B \sim \ddot{d}_B \sim Q\ddot{x}_B \sim Q\dot{V}_B$

$$\Delta A_B = -\int_{-\infty}^{+\infty} E_B(u,x^C)du \sim Q\Delta V_B$$

For charge ejected with escape velocity V in z-direction

$$\Delta A_B|_{\mathcal{I}^+} = \frac{QV}{1-V\cos\theta}\partial_B\cos\theta \quad \textbf{(E-mode)}$$

OUTSTANDING QUESTIONS

CAN THE MEMORY BE DETECTED IN THE RADIO WAVEFORM?

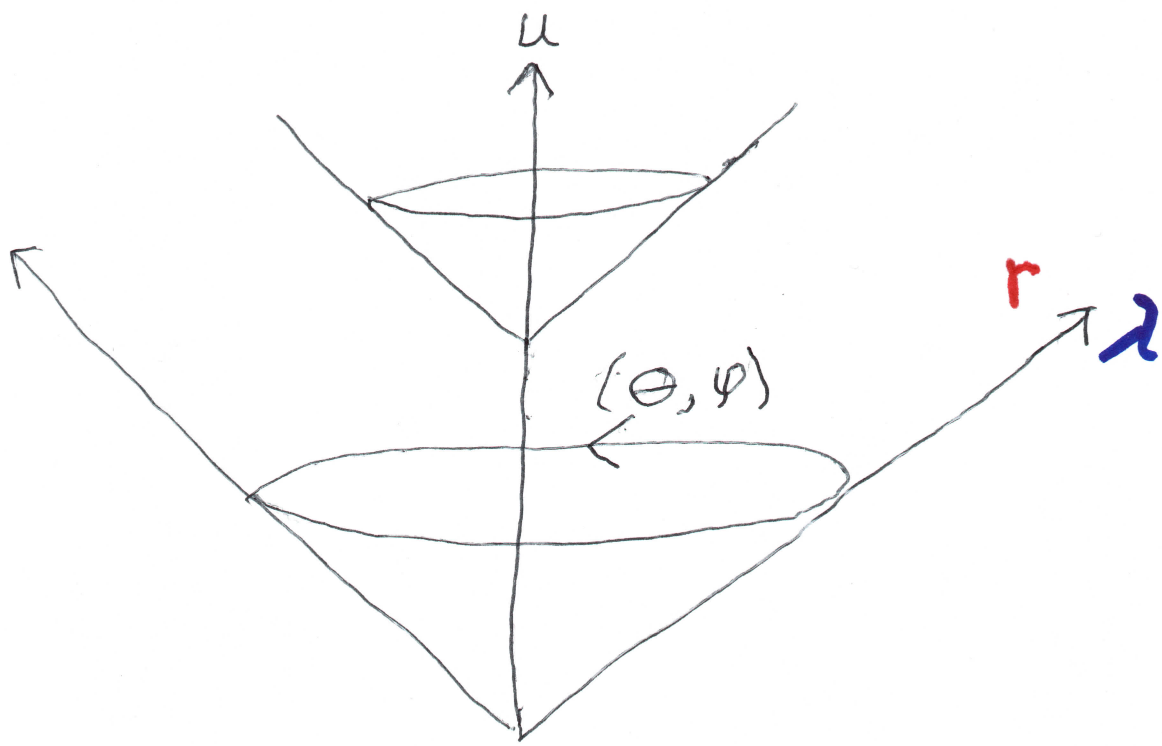
IS THE STRENGTH OF THE KICK STRONG ENOUGH TO AFFECT THE DYNAMICS OF NEARBY PLASMA?

ARE ESCAPING CHARGES FROM A BURST THE ONLY PHYSICALLY SIGNIFICANT SOURCE OF ELECTRO-MAGNETIC RADIATION MEMORY?

IS THERE ANY PHYSICALLY REALISTIC SOURCE OF B-MODE RADIATION MEMORY?

CHARACTERISTIC INITIAL VALUE FORMULATIONS

BONDI-SACHS vs AFFINE-NULL



BONDI-SACHS: areal radius r

AFFINE-NULL: affine parameter λ

APPARENT HORIZON

$$\frac{\partial r}{\partial \lambda} = 0$$

AFFINE-NULL VS BONDI-SACHS
ADVANTAGES - DISADVANTAGES

BOTH TREAT GEOMETRIC QUANTITIES AT $\mathcal{I}^+$

RADIATION STRAIN, ENERGY-MOMENTUM, ANGULAR MOMENTUM, NP CONSTANTS

BONDI-SACHS

Einstein equations → Sequence of ODE's along outgoing null rays → Stable numerical evolution algorithm

BUT areal coordinate r breaks down inside event horizon when null cones begin to refocus ($\frac{\partial r}{\partial \lambda} = 0$) so Bondi-Sachs cannot penetrate the horizon

Except in spherical symmetry, event horizon is reached at different retarded times for different angles so complete $\mathcal{I}^+$ cannot be simulated

AFFINE-NULL

Affine parameter λ only breaks down at caustics where $r = 0$ so Affine-Null can penetrate horizon

BUT Einstein equations do not reduce to sequence of radial ODE's for metric variables,

HOWEVER sequential ODE structure is restored using a non-obvious choice of variables

JW *Phys. Rev. D* **87, 124027 (2014)**

OUTSTANDING PROBLEM

Numerical implementation of Affine-Null evolution