The Computational Challenge of Modeling Core Collapse Supernovae and their Gravitational Wave Emission

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Computational Challenges in Multi-Messenger Astrophysics
GWAWS1
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Outline

- Requirements for constructing a realistic core collapse supernova model.
- Where current three-dimensional models stand.
- Gaps and how they must be addressed (how we intend to address ours).
- Results from our current production runs, including GW predictions.
- Preliminary results from the development of our next-generation CCSN simulation capability.
- Possible need to extend classical neutrino kinetics to quantum neutrino kinetics.
- Takeaways

The efficacy of the neutrino shock reheating/delayed shock mechanism has now been demonstrated by all leading groups across progenitor characteristics (mass, rotation, and metallicity). Nonetheless, significant challenges remain. For recent reviews, see:

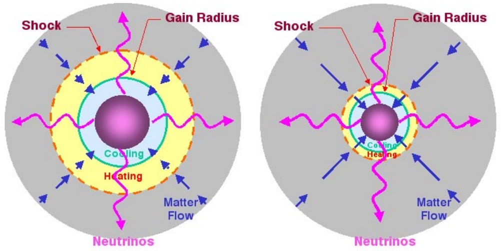
- Mueller, PASA **33** e048 (2016)
- Janka, Melson, and Summa, *Ann. Rev. Nucl. Part. Sci.* **66** 341 (2016)
- Mezzacappa, Endeve, Messer, and Bruenn, *Liv. Rev. Comp. Astr.* **6** 4 (2020)

Necessary Model Components

Three-Dimensional General Relativistic Gravity

Newtonian General Relativistic

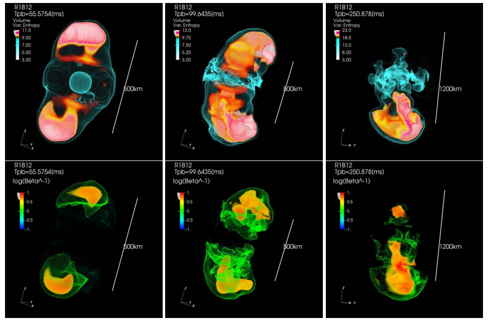
25 M
Model



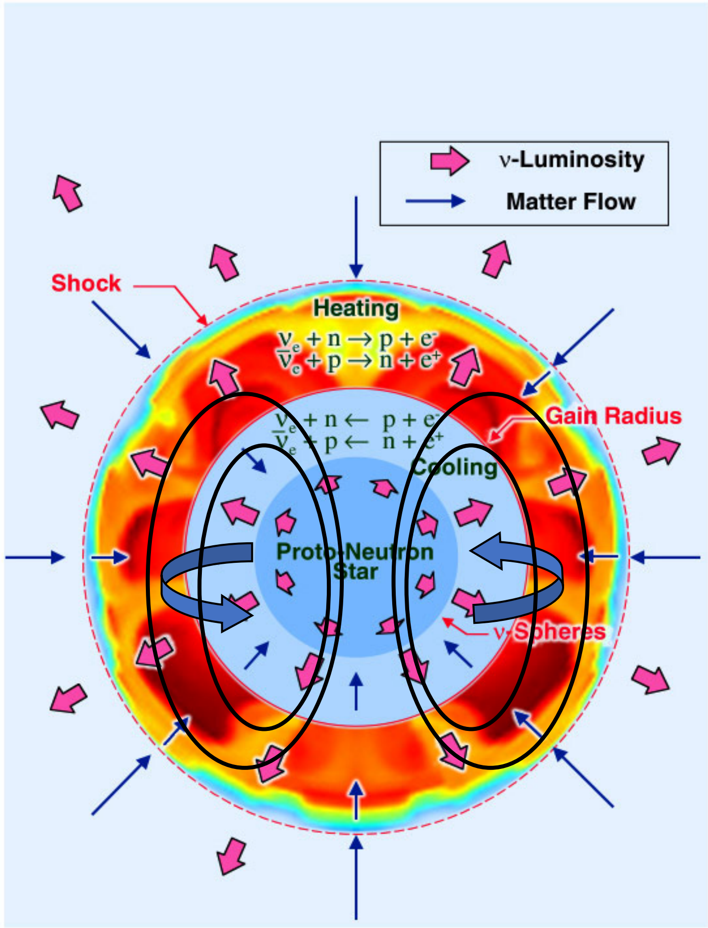
Bruenn, DeNisco, and Mezzacappa, *Ap.J.* **560**, 326 (2001)

Three-Dimensional General Relativistic Magnetohydrodynamics

- Turbulent Convection
- Standing Accretion Shock Instability
- Slow to Rapid Progenitor Rotation
- Magnetic Isotropic Pressure
and other MHD Stresses



Kuroda, Arcones, Takiwaki, and Kotake *Ap.J.* **896**, 102 (2020)



Microphysics

Extensive Set of Weak Interactions
State-of-the-Art Implementations of Them

Suitably Constrained Nuclear Equation of State

Three-Dimensional General Relativistic Neutrino Kinetics

- Neutrino heating depends on the neutrino luminosities, spectra, and angular distributions.

$$\dot{\epsilon} = \frac{X_n}{\lambda_0^a}\frac{L_{\nu_e}}{4\pi r^2}\langle E^2_{\nu_e}\rangle\langle\frac{1}{\mathcal{F}}\rangle + \frac{X_p}{\lambda_0^a}\frac{L_{\bar{\nu}_e}}{4\pi r^2}\langle E^2_{\bar{\nu}_e}\rangle\langle\frac{1}{\bar{\mathcal{F}}}\rangle$$

$$f(t, r, \theta, \varphi, \varepsilon, \theta_p, \varphi_p)$$ Required

$$\{I, \boldsymbol{H}\}(t, \boldsymbol{z}) = \int f(t, \boldsymbol{z}, \omega)\{1, \ell\}d\omega$$ Feasible

Requires a closure prescription.

$$\boldsymbol{K}(\boldsymbol{z}, t) = \int f(t, \boldsymbol{z}, \omega)\ell \otimes \ell d\omega$$

$$\boldsymbol{k} = \frac{1}{2}[(1-\chi)\mathrm{I} + (3\chi - 1)\boldsymbol{h} \otimes \boldsymbol{h}]$$

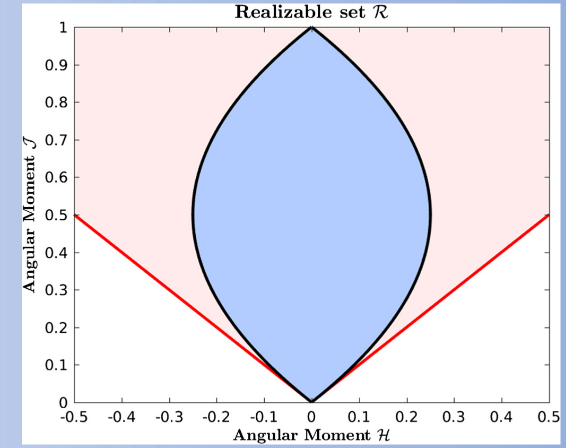
Eddington Factor

$$\boldsymbol{k} = \frac{\boldsymbol{K}}{I}$$

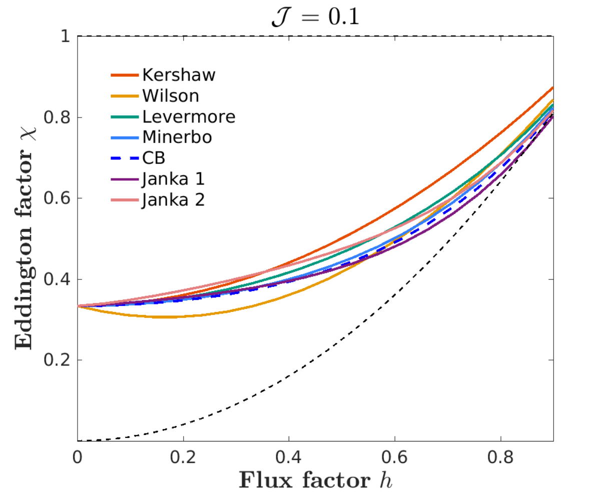
$$\boldsymbol{h} = \boldsymbol{\mathcal{H}}/|\boldsymbol{\mathcal{H}}|$$

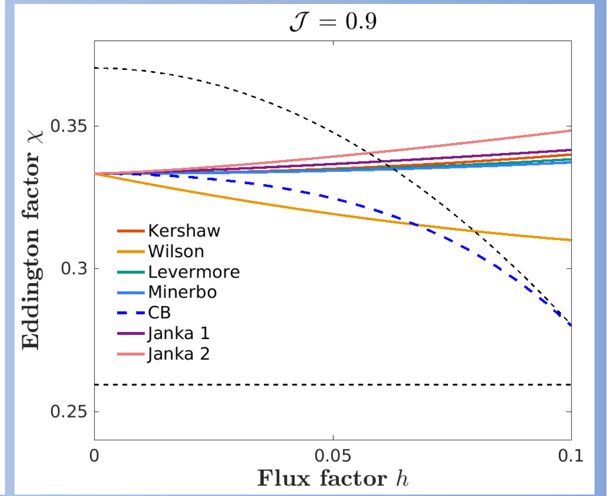
Neutrino Kinetics: A Deeper Look

- Realizability
  - Fermi-Dirac statistics must be obeyed.
    - The distribution function must be bounded.
    - The moments of the distribution functions (e.g., the energy and momentum densities) must be bounded.
- For moments-based approaches, we require a closure.
  - The closure must be realizable – i.e., the Eddington factor is bounded.
- Achieving this in numerical simulations is a significant challenge.
  - Proving that the numerical method is realizable is difficult.
    - At $O(1)$, this has been accomplished [Chu et al. JCP, **389**, 62 (2019)], and in turn leads to restrictions on the time step, which can be used to *guarantee* realizability.
    - In the relativistic case, such proofs have not yet been constructed, implying that the conditions that must be maintained to guarantee realizability in this case are not yet known.



Chu, Endeve, Hauck, and Mezzacappa JCP, **389**, 62 (2019)





Chu, Endeve, Hauck, Mezzacappa, and Messer *Journ. Phys. Conf. Ser.* ***1225***, 012013 (2019)

Neutrino Kinetics: A Deeper Look

- Simulations must endeavor to conserve *both* lepton number and energy.
  - Significant technical challenge – specifically, to develop discretizations of the underlying integro-partial differential equations of neutrino radiation hydrodynamics that conserve both quantities.
- Begin with a reformulation of general relativistic kinetic theory that is *manifestly* conservative for lepton number.
  - Cardall and Mezzacappa, PRD **68**, 023006 (2003)

$$\mathbb{N}[f] = \mathbb{C}[f]$$

$$\mathbb{N}[f] \equiv \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^{\mu}} \left(\sqrt{-g}\, \mathcal{L}^{\mu}{}_{\hat{\mu}} p^{\hat{\mu}} f \right) + E(\mathbf{p}) \left| \det\left[\frac{\partial \mathbf{p}}{\partial \mathbf{u}}\right] \right|^{-1} \frac{\partial}{\partial u^{\hat{i}}} \left(\frac{1}{E(\mathbf{p})} \left| \det\left[\frac{\partial \mathbf{p}}{\partial \mathbf{u}}\right] \right| \left(e F^{\hat{j}}{}_{\hat{\nu}} p^{\hat{\nu}} - \Gamma^{\hat{j}}{}_{\hat{\nu}\hat{\rho}} p^{\hat{\nu}} p^{\hat{\rho}} \right) \frac{\partial u^{\hat{i}}}{\partial p^{\hat{j}}} f \right)$$

  - 3+1 Expressions of the Equations
    - Cardall, Endeve, and Mezzacappa, PRD **88**, 023011 (2013) (Boltzmann)
    - Cardall, Endeve, and Mezzacappa, PRD **87**, 103004 (2013) (Moments)
    - See also Shibata et al. *Prog. Theor. Phys.* **125**, 1255 (2011) and Shibata et al. PRD **89**, 084073 (2014)

For a review, see Mezzacappa, Endeve, Messer, and Bruenn, *Living Reviews in Computational Astrophysics* **6**, 4 (2020)

3D State of the Art

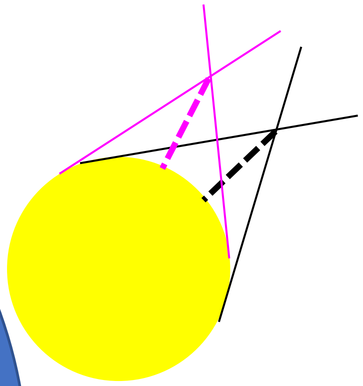
UT-ORNL

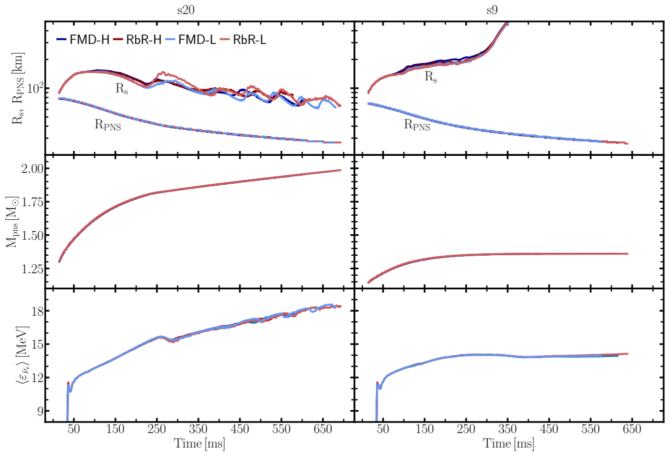
Newtonian
Ray-by-Ray Transport
Partial Weak Physics

Effective Potential
Ray-by-Ray Transport
Extensive Weak Physics

General Relativistic
Three-Dimensional Transport
Partial Weak Physics

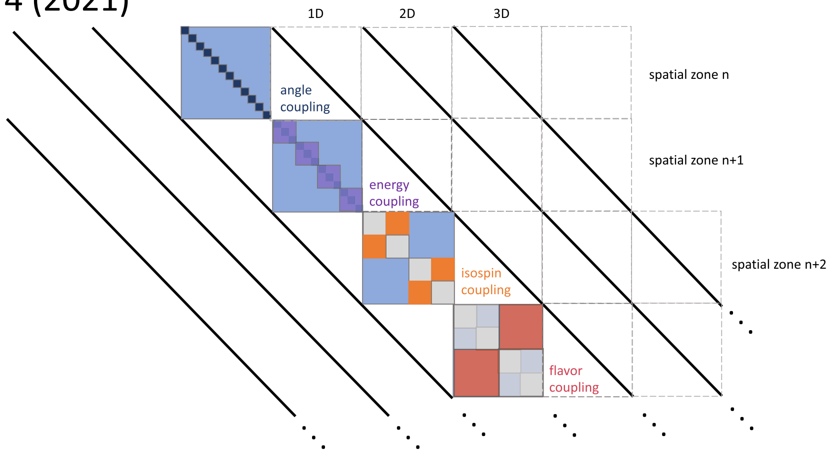
Effective Potential
Three-Dimensional Transport
Extensive Weak Physics





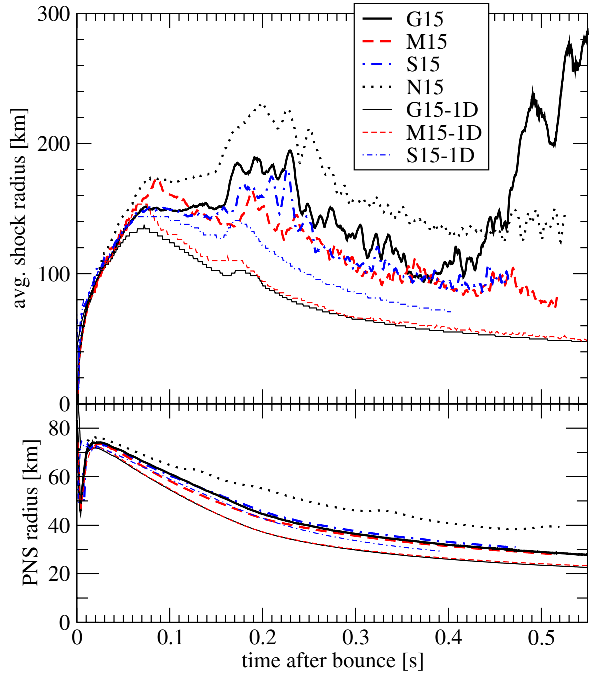
Andresen, Glas, and Janka MNRAS **503**, 3552 (2021)

Mezzacappa, Endeve, Messer, and Bruenn, *Living Reviews in Computational Astrophysics* **6**, 4 (2021)



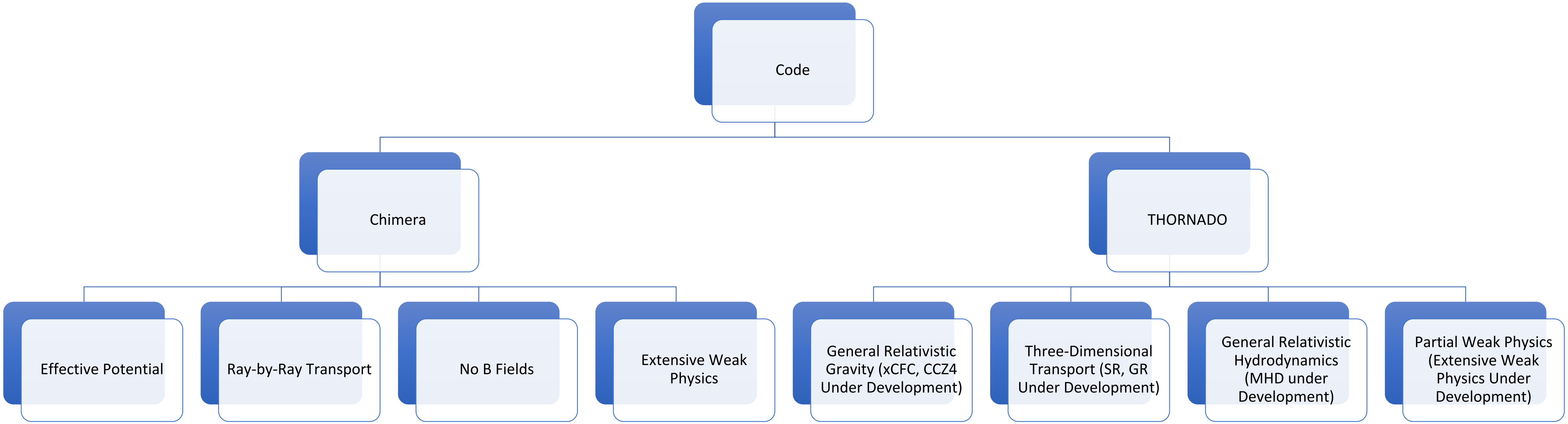
$$\Phi^{\rm GR}(r) = G\int_{\infty}^{r} {\rm d}r'\, \frac{1}{r'^2}\left(m + \frac{4\pi r'^3 (p+P)}{c^2}\right)\frac{1}{\Gamma^2}\left(\frac{\rho_{\rm tot}c^2 + p}{\rho c^2}\right)$$

Rampp and Janka 2002 *A&A* **396**, 361



Mueller, Janka, and Marek *Ap.J.* **756**, 84 (2012)

UT–ORNL Supernova Code Lineage



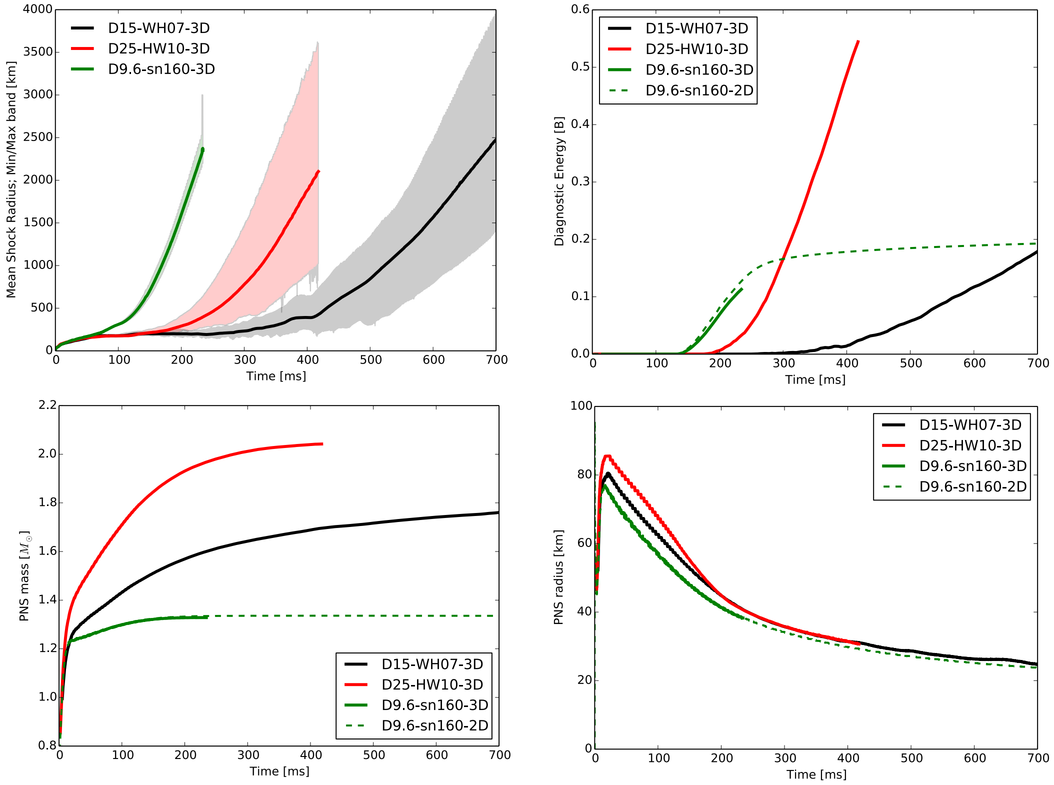
- Based on DG discretizations in phase space (space and neutrino energy).
    - Well suited to capturing *physical* diffusion.
    - Well suited to *simultaneous* conservation of lepton number and energy.
- IMEX time integration.
    - Use implicit methods only where necessary (*local* neutrino–matter interactions).
    - Avoid *global* implicit solves.
- Being developed to satisfy realizability, correct asymptotic behavior, and simultaneous number and energy conservation.

Chimera Models

First Chimera Model: Lentz et al. *Ap.J Lett.* **807** L31 (2015)

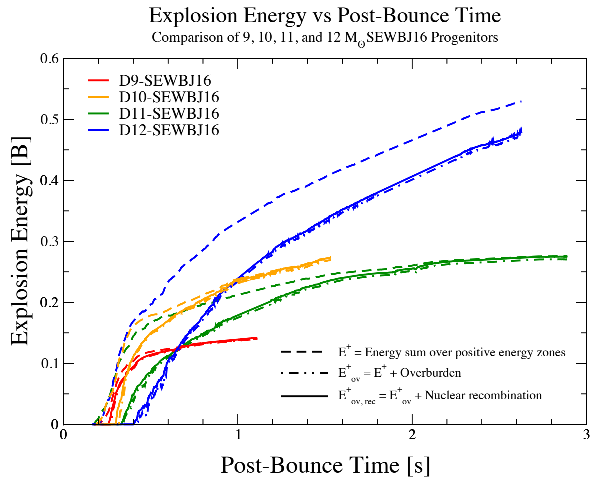
Progenitor Mass (Solar Masses)	Rotating/Perturbed	Progenitor Family/Metallicity/High-Density EOS	Explosion/ Shock Radius (km)	Post-bounce Time (ms)/ Explosion Energy (B)
9.6	N/N	Woosley and Heger (2015)/Zero Metallicity/LS220	Y/9467	467/0.167
15	N/N	Woosley and Heger (2007)/Solar Metallicity/LS220	Y/ 1600	685/?
25	N/N	Heger and Woosley (2010)/Zero Metallicity/LS220	Y/ 2200	405/?

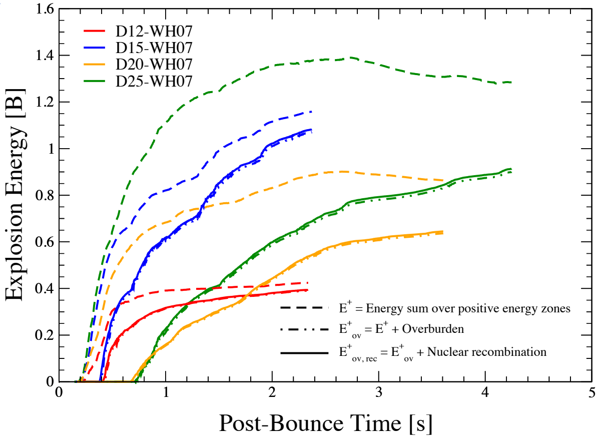
To afford high angular resolution, Chimera underwent significant development to deploy a Yin-Yang grid. The runs reported here had effective angular resolutions of 1 degree in both angular dimensions.



In 3D, we face a significant challenge to determine the most basic observable (explosion energy) given the time scales over which it is determined.

From 2D:





E. J. Lentz, W. R. Hix, J. A. Harris, S. W. Bruenn, O. E. B. Messer, A. Mezzacappa, J. M. Blondin, C. Mauney, and M. Owen, *Ap. J.* in preparation (2021).

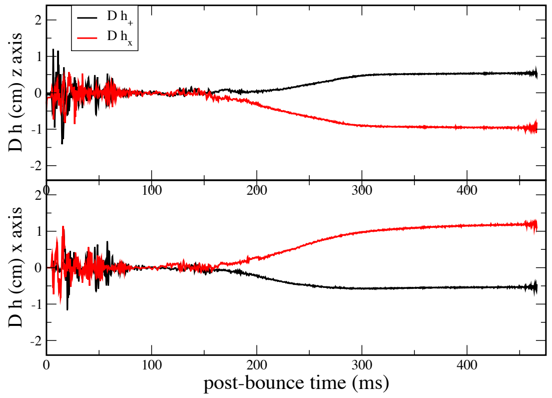
Gravitational Wave Analysis

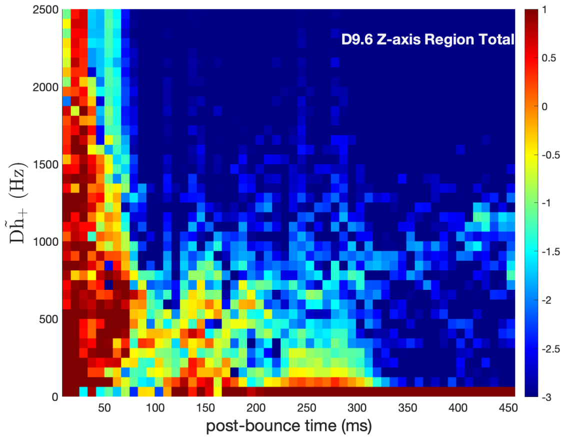
3 Progenitors
3 Disparate Masses
2 Metallicities

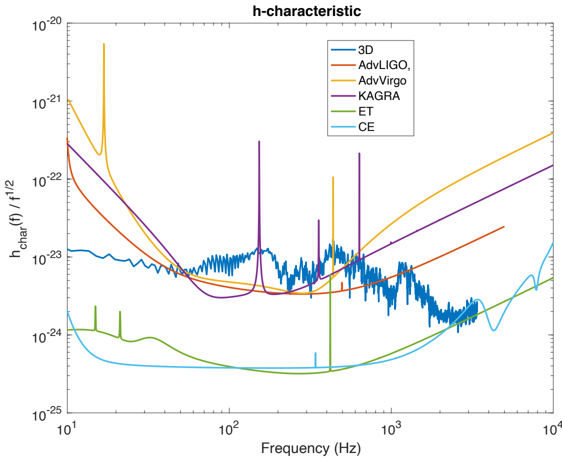
"Complementary" Strain Evolution
"Complementary" Heat Maps

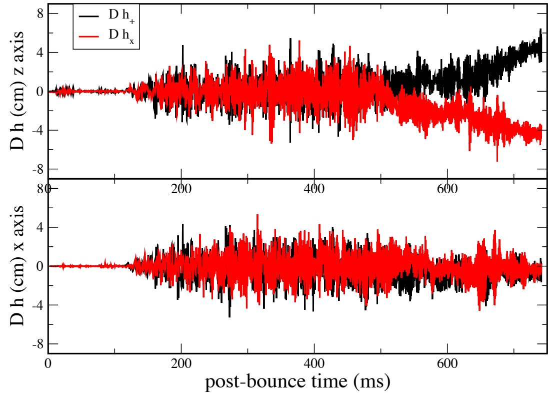
The low-mass progenitor's emission is confined largely to the first 100 ms post bounce.

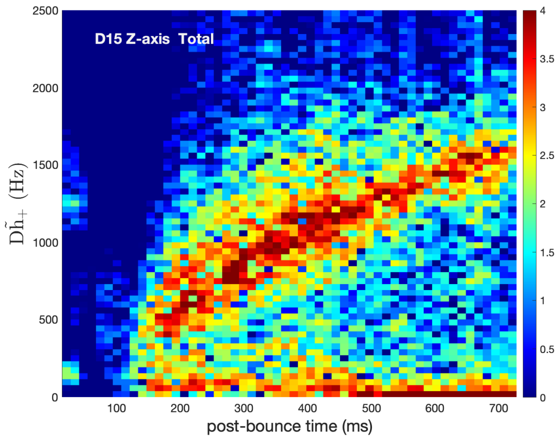
The more massive progenitors exhibit "canonical" emission after 100 ms post bounce.

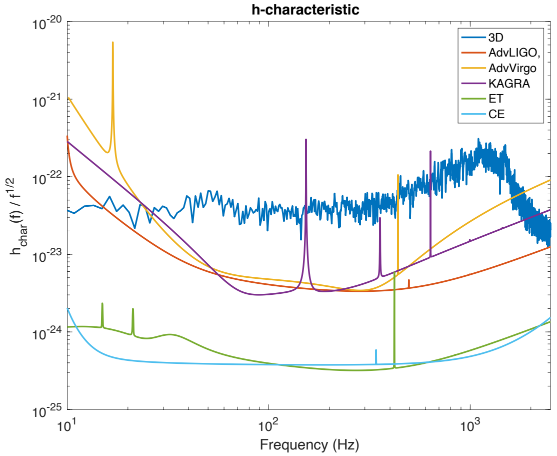


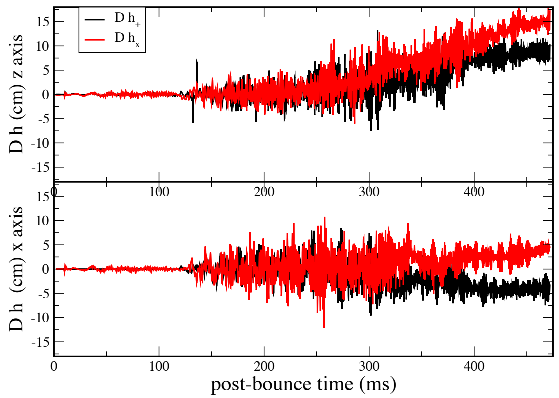


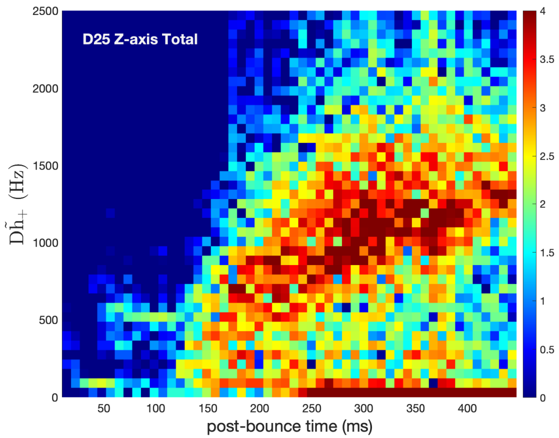


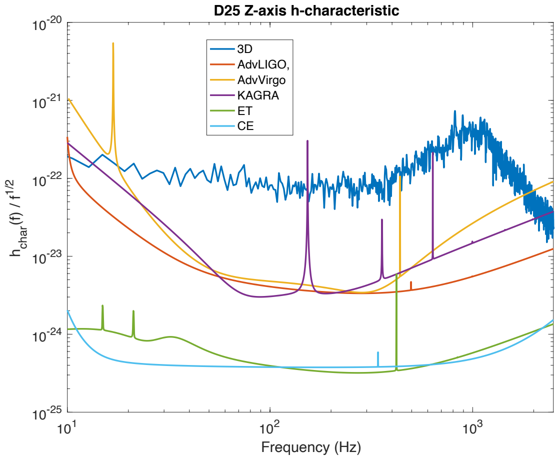






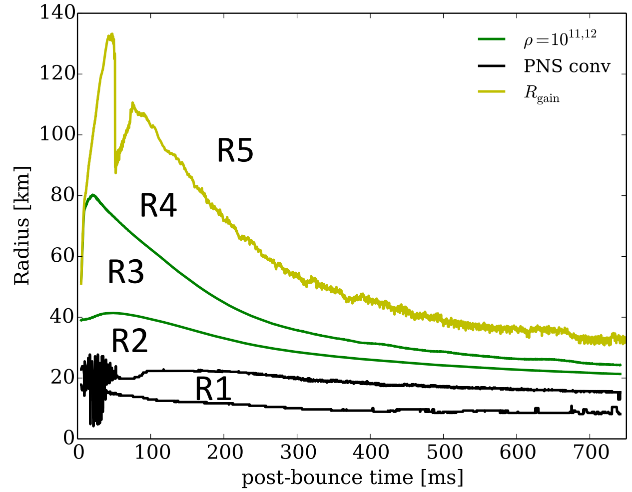


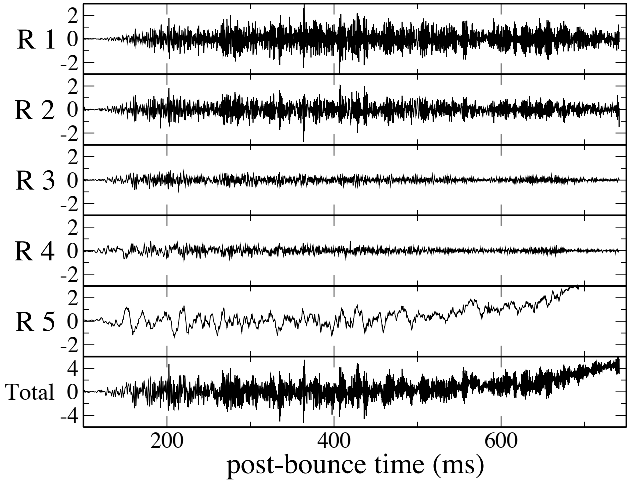


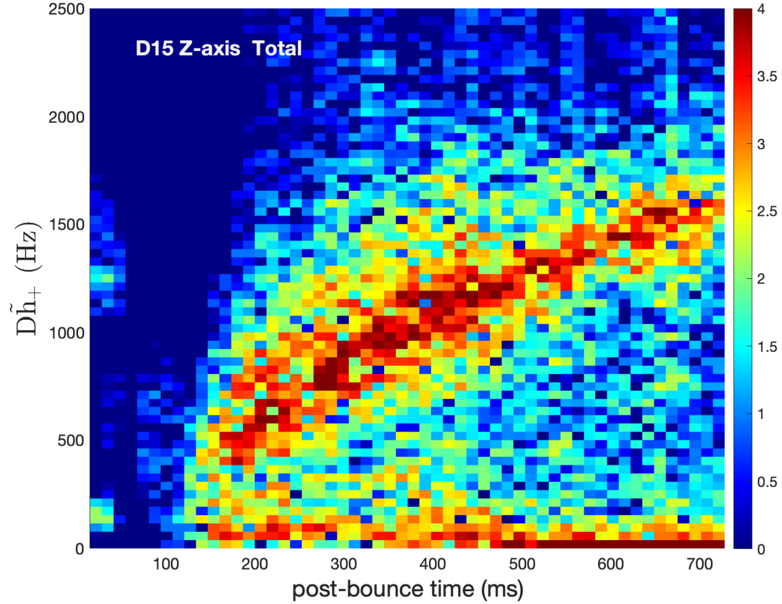


Mezzacappa, Marronetti, Landfield, Lentz, et al. 2021, in preparation

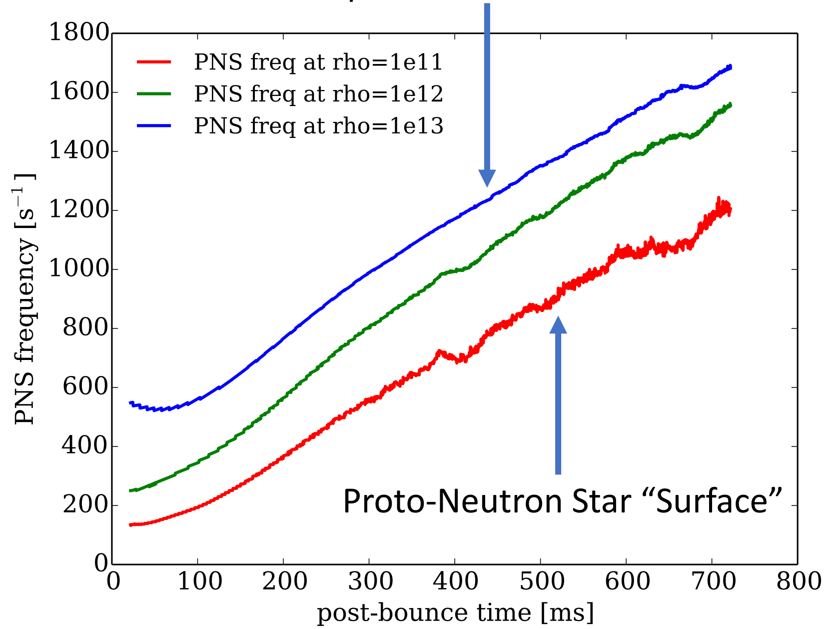
Culling Information about the Proto-Neutron Star







Best "fit" is not at the density we would use to define the proto-neutron star surface.



Ledoux unstable (R1) and stable (R2) regions *both* contribute to (and dominate) high-frequency gravitational wave emission.

$$f_p = \frac{1}{2\pi}\frac{GM}{R^2}\frac{1}{c_s}\sqrt{\Gamma-1}(1-\frac{GM}{Rc^2})^{3/2}$$

Mueller, Janka, and Marek, *Ap.J.* **766** 43 (2013)

Dominant emission mechanism is *within* the proto-neutron star.

3D has broadened the picture! → See also Andresent et al. MNRAS **468** 2032 (2017).

What is the nature of the PNS instabilities?

Ledoux Convection

Doubly Diffusive Instabilities

- Neutron Fingers
- Lepto-Entropy Fingers
- Lepto-Entropy Semi-Convection

Bruenn, Mezzacappa, and Dineva (1995) *Phys. Rep.* **256**, 69
Bruenn and Dineva (1996) *Ap.J. Lett.* **458**, 71
Bruenn, Raley, and Mezzacappa (2004) astro-ph/0404099

Neutron Fingers

Assumption:

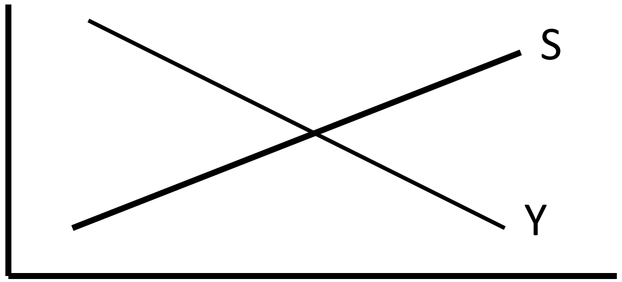
Will occur in crossed entropy and lepton fraction gradients – i.e., in Ledoux stable regions.

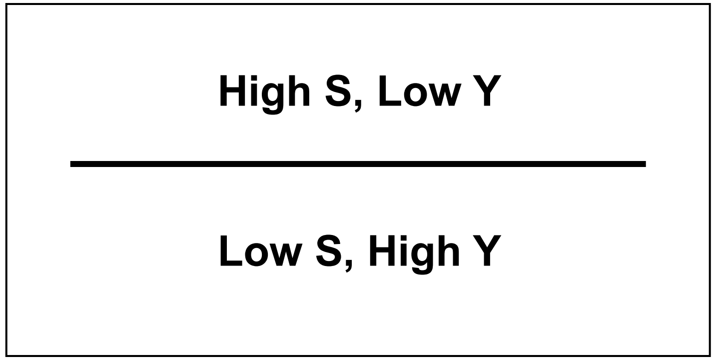
Basis for the assumption:

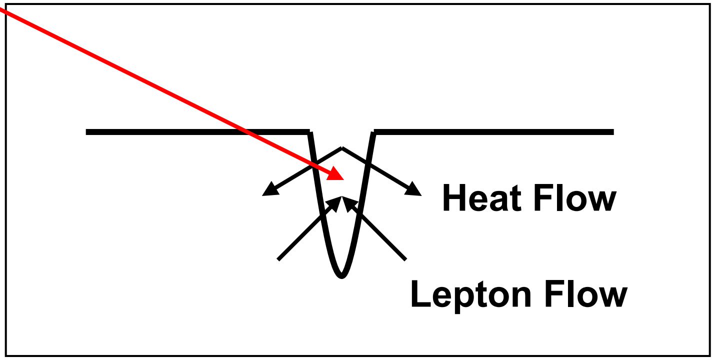
Three flavors mediate energy exchange.
One flavor mediates lepton exchange.

→ Energy exchange dominates.

If heat flow dominates lepton flow, the now low S, low Y perturbation will continue to sink.

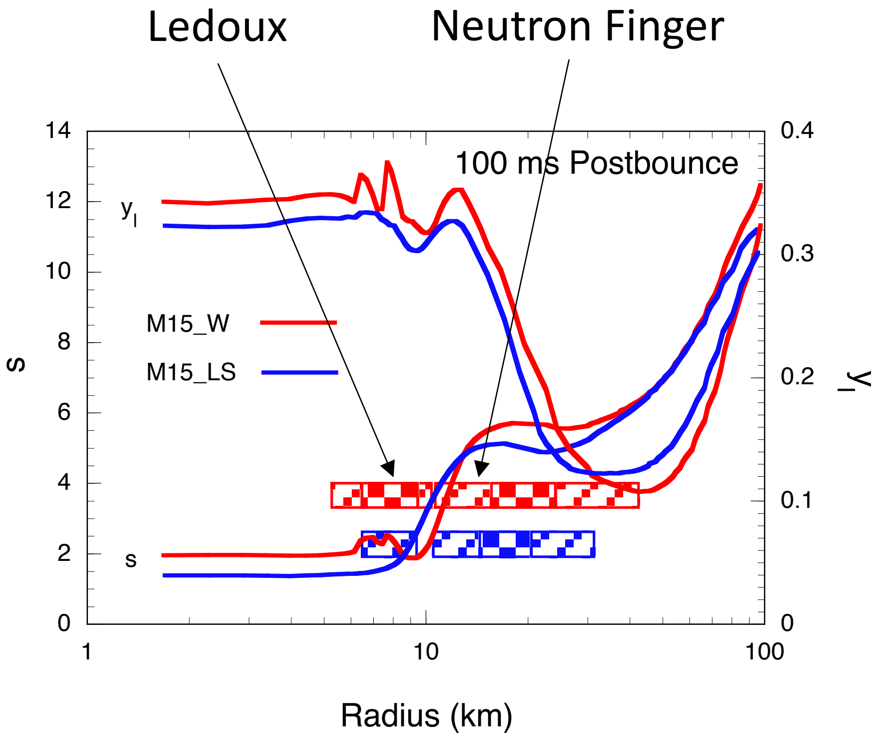






"Doubly Diffusive Instability"

Doubly diffusive instabilities can extend the region of instability beyond the Ledoux unstable regions.



Bruenn, Raley, and Mezzacappa (2004) astro-ph/0404099

Neutron Fingers were invoked by the LLNL group in their explosion models:

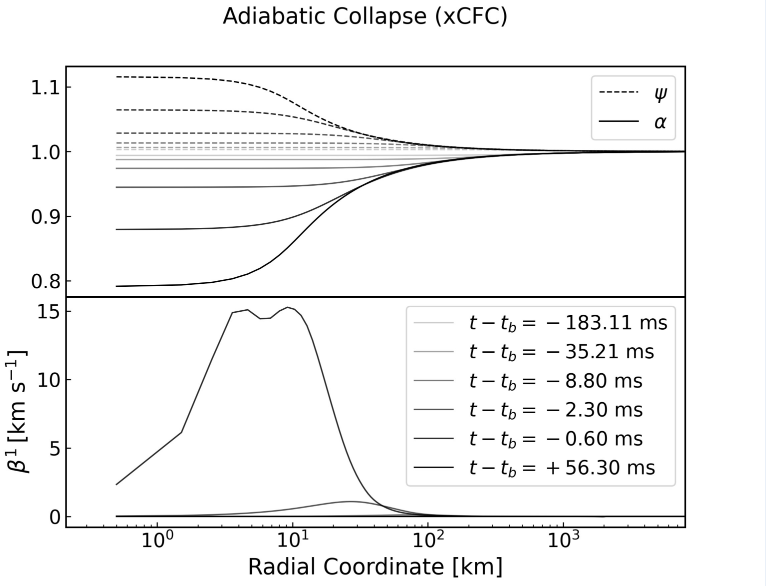
- Smarr, Wilson, Barton, and Bowers, 1981, Ap.J. **246**, 515
- Wilson and Mayle, 1988, Phys. Rep. **163**, 63
- Wilson and Mayle, 1993, Physics Reports, **227**, 97
- Wilson, Mayle, Woosley, and Weaver, 1986, in Annals of the NY Academy of Science, Vol. 470, 12th Texas Symposium on Relativistic Astrophysics, ed. M. Livio and G. Shaviv (Boston: Jones and Bartlett), 267

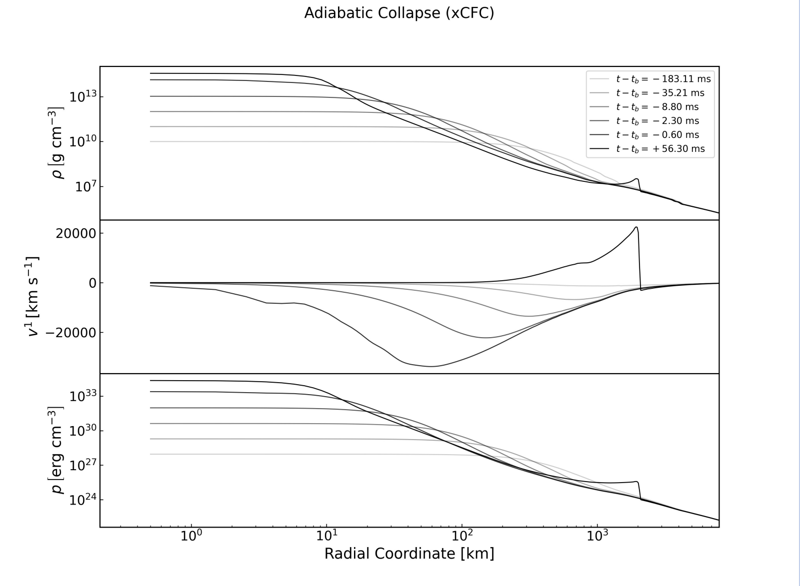
THORNADO Models

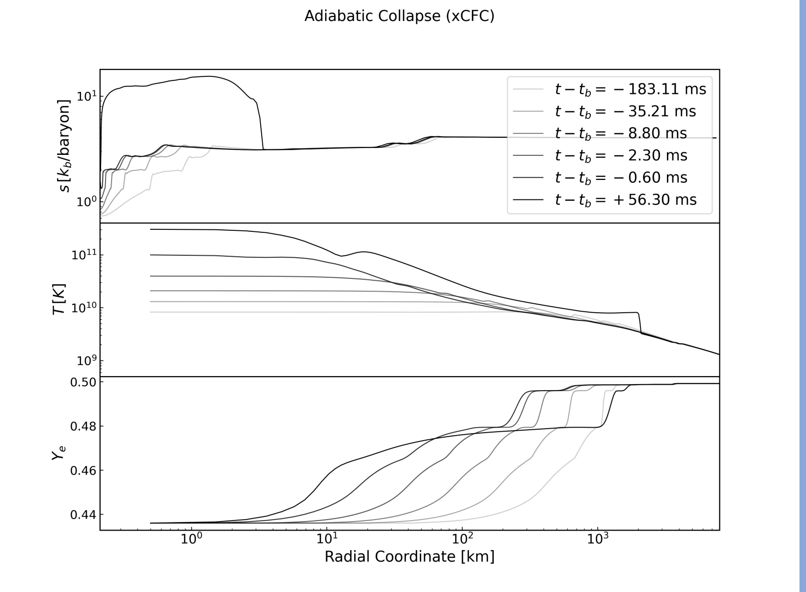
THORNADO

- General Relativistic Gravity
- General Relativistic Hydrodynamics
- General Relativistic Two-Moment Spectral Neutrino Transport

Fully developed and tested through the SR regime.

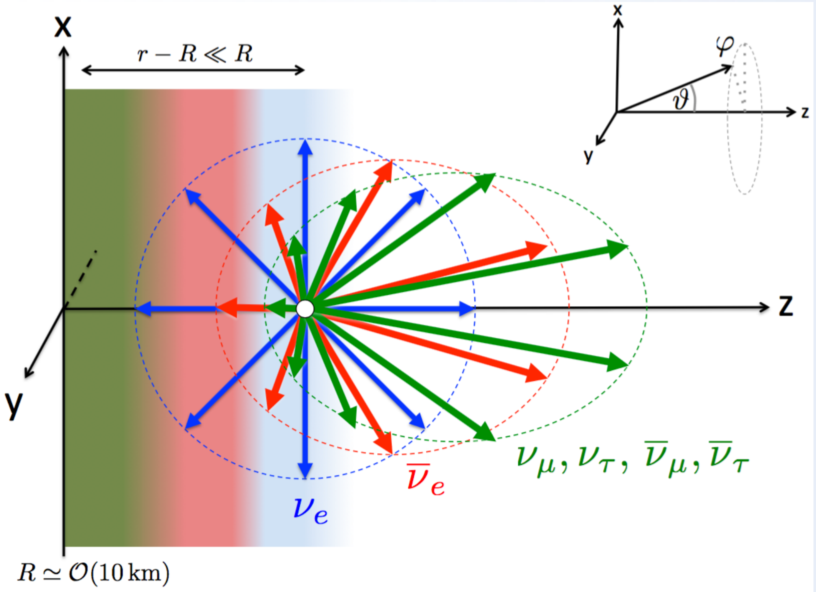






A Common Theme: It's All About the Angular Distributions

- Neutrino shock reheating depends on the neutrino angular distributions.
- Required within the proto-neutron star to accurately capture doubly diffusive instabilities that may occur.
- Classical kinetics with full angular dependence is required to form the foundation for the development of neutrino quantum kinetics.



Neutrino flavor evolution is complicated by neutrino–neutrino interactions, which affect all neutrinos at all energies – i.e., the entire ensemble of neutrinos – collectively.

- Duan, H., Fuller, G. M., & Qian, Y.-Z. 2010, ARNPS **60**, 569

If ν_e and $\bar{\nu}_e$ angular distributions are sufficiently different, "fast flavor instabilities" in the vicinity – i.e., within *O*(m) – of the neutrinospheres may be triggered.

- Sawyer, R. F. 2005, PRD **72**, 045003

Impact on the explosion mechanism?

Takeaways

Recent progress has been great!

Multiple groups have demonstrated the efficacy of the neutrino heating mechanism over a range of progenitor characteristics, in three dimensions.

Current three-dimensional models have allowed us to study associated phenomena such as gravitational wave emission.

There is a great deal of development to be done to arrive at (classical) definitive three-dimensional models.

Full three-dimensionality.
Full general relativity.
Full physics (weak interaction physics, magnetic fields, …).

Quantum kinetics looms large as a potential requirement, the development of which will occupy our community for some time.