Black Hole Formation in Randall-Sundrum II Braneworlds

Matt Choptuik

Dept. of Physics & Astronomy, UBC

Computational Challenges in Gravitational Wave Astronomy

IPAM, UCLA

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Based on Daoyan Wang’s PhD Thesis

arxiv:1505.00093

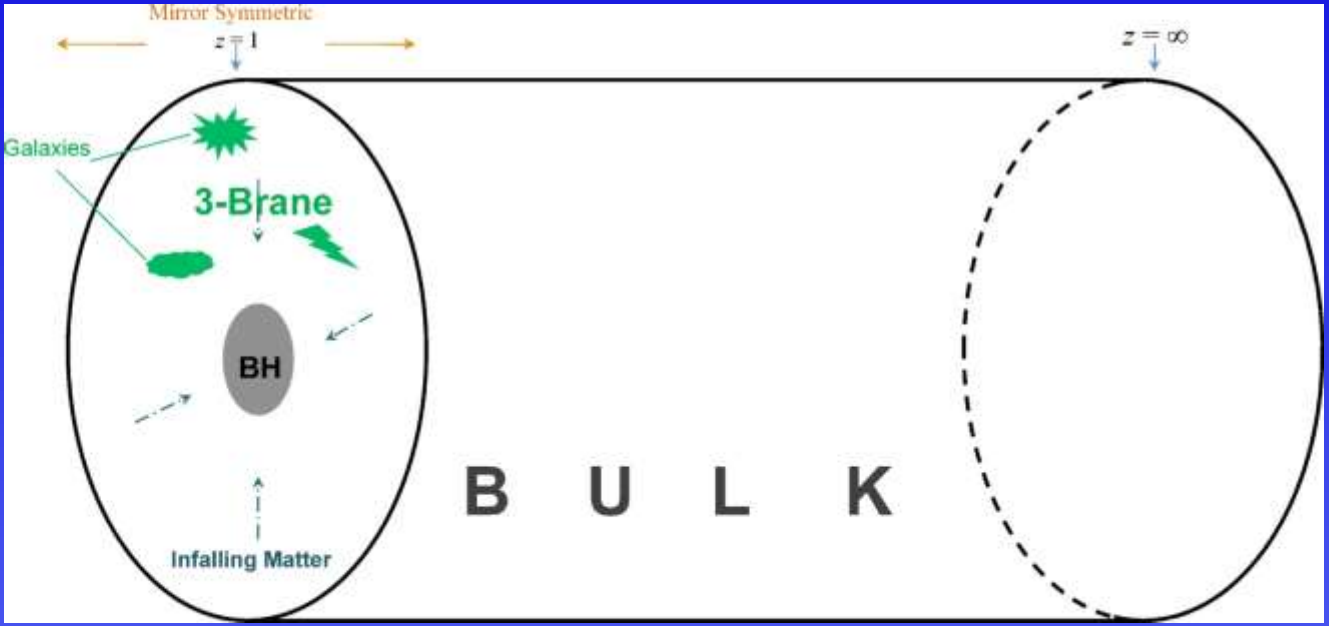
and

arxiv:1604.04832 (DW & MWC, *PRL* 2016)

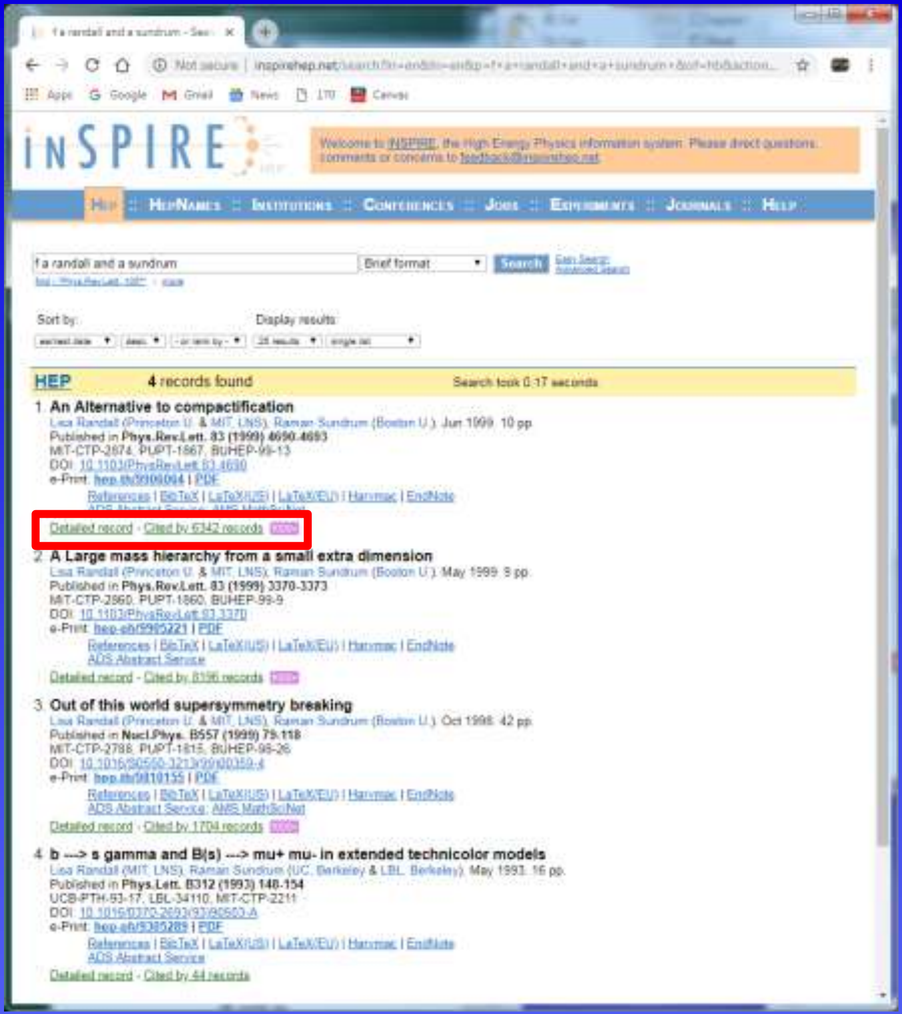
Outline

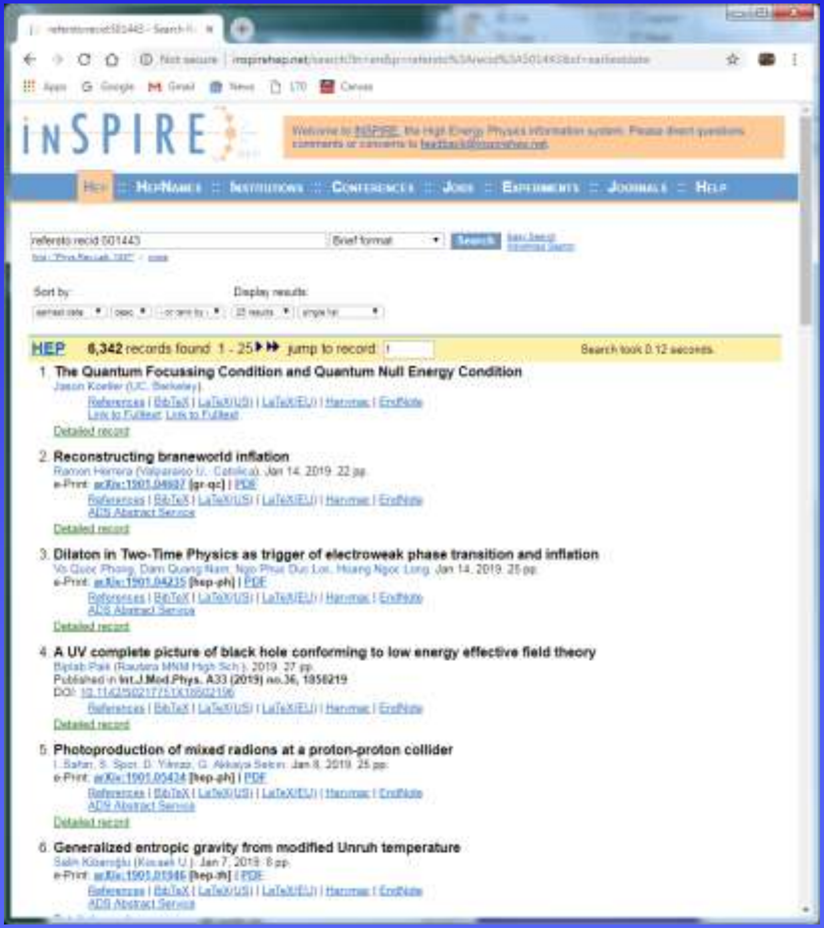
- Motivation & overview of Randall-Sundrum II braneworld
- Overview of current work
- Challenges & solutions
- Implementation
- Results
- Summary & speculation

Randall-Sundrum II braneworld (RS II)
(Randall & Sundrum 1999)



- Observable universe could be 3+1 dimensional “brane” (membrane) embedded in a higher dimensional “bulk” spacetime
- RS II is a 4+1 dimensional model containing single brane on which all matter is confined; only gravity propagates into bulk
- Bulk has negative cosmological constant; brane has tension, so gravitates; universe has Z_2 symmetry (“mirror symmetry”) about brane





Citations

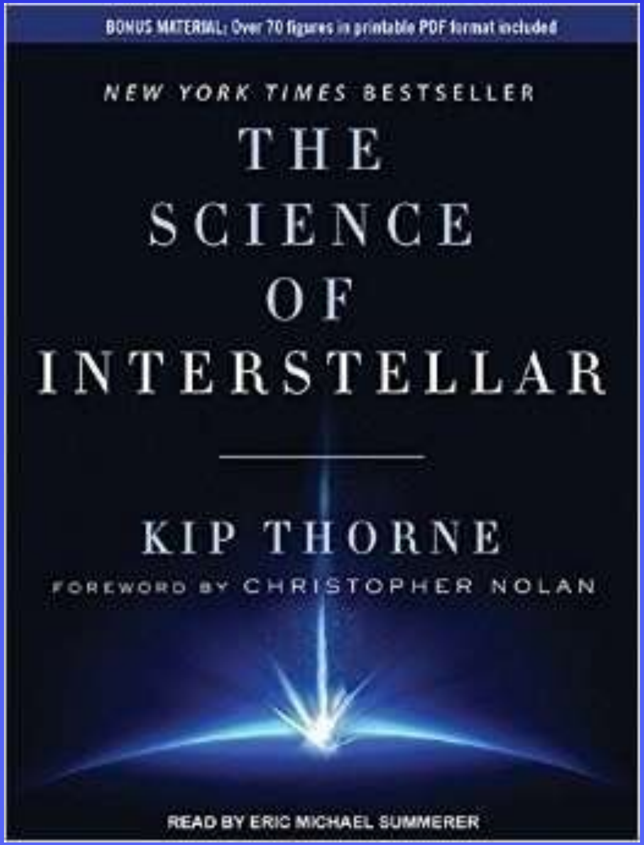
Oct 20 2016: 5778

Jan 23 2019: 6342

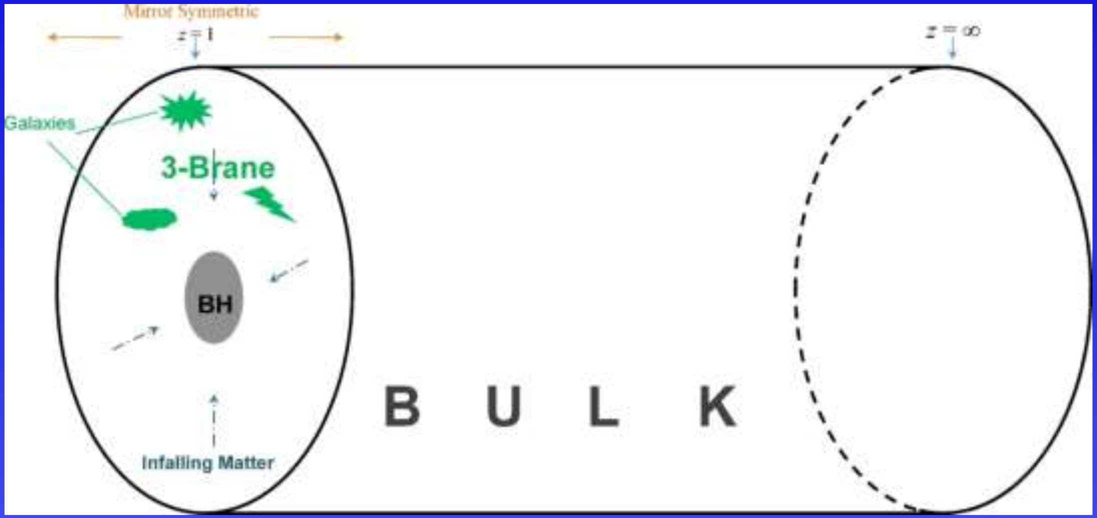
Why the continuing interest in RS II?

- Extra spatial dimensions are a fact of life in string theories (e.g.) and perhaps in our universe
- Brane models such as RS II provide alternative to compactification (Kaluza-Klein) for hiding extra dimensions from our normal view
- RS II particularly notable in that it recovers GR on the brane at low energies even though the extra dimension is infinite
- Also provides a framework for the celebrated AdS/CFT correspondence
- Much more, including …

Hollywood buy-in



Randall-Sundrum braneworld II (RS II)



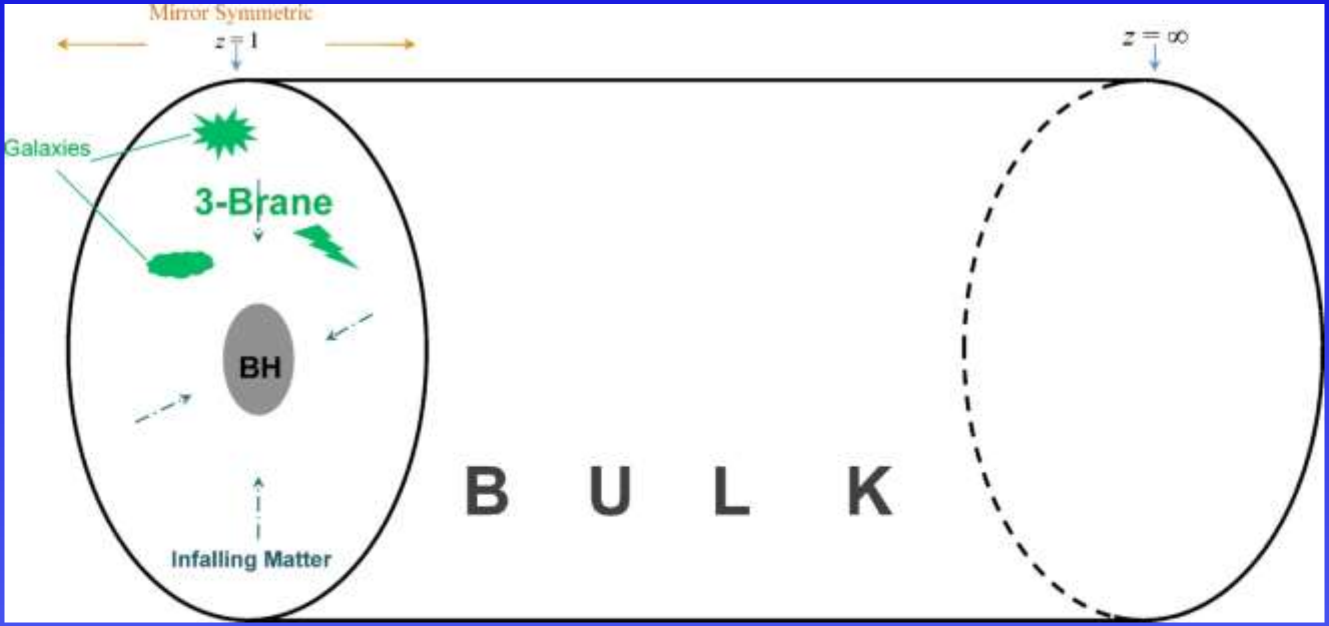
- Governing equations in bulk: Vacuum 5D Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = k_d T_{\mu\nu} = 0$$

- Governing equations on brane: field equations for matter, same as usual 4D case

- Matter & tension on brane couple to bulk through Israel junction conditions

Randall-Sundrum braneworld II (RS II)

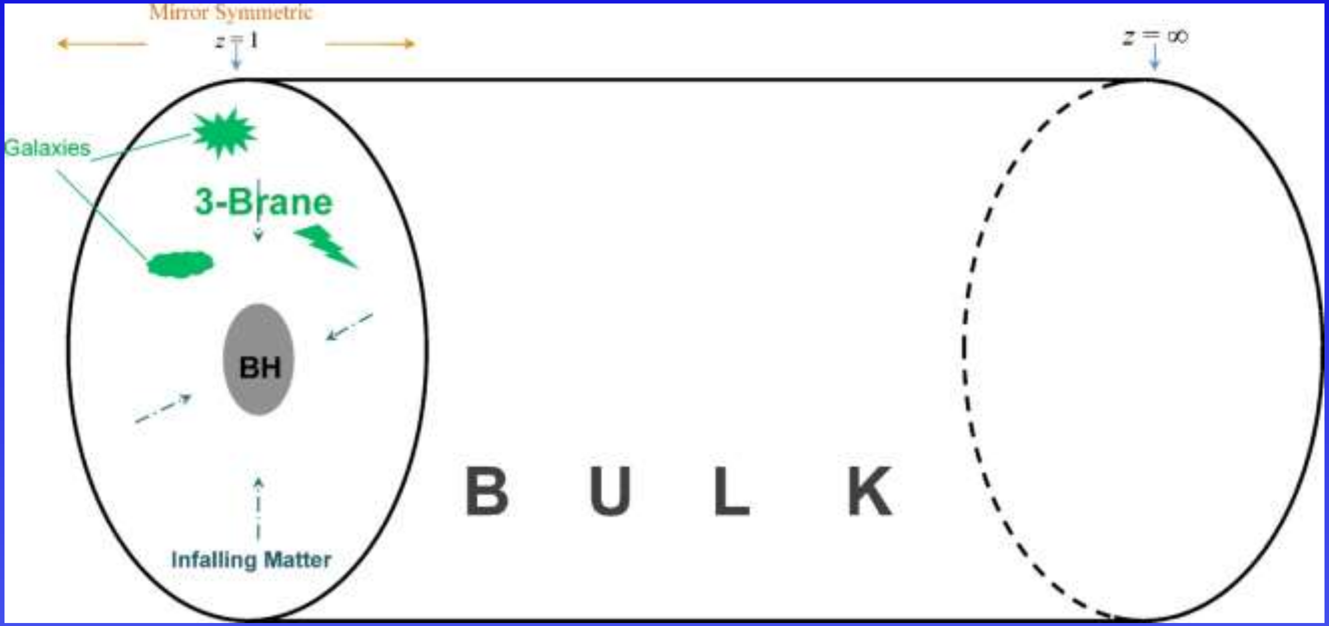


- At low energies can show that we recover Einstein equations on the brane
- At high energies (strong field), situation is not clear

Key questions

- What is the nature of black objects in the scenario?
  - If model is to be consistent with reality as we currently understand it, there must be solutions that describe stationary black holes on the brane

- What is the end point of gravitational collapse of matter on the brane?



- One vacuum solution:

$$ds^2 = \frac{\ell^2}{z^2}\left(h_{ab}dx^a dx^b + dz^2\right) \quad \text{where} \quad z \geq \ell$$

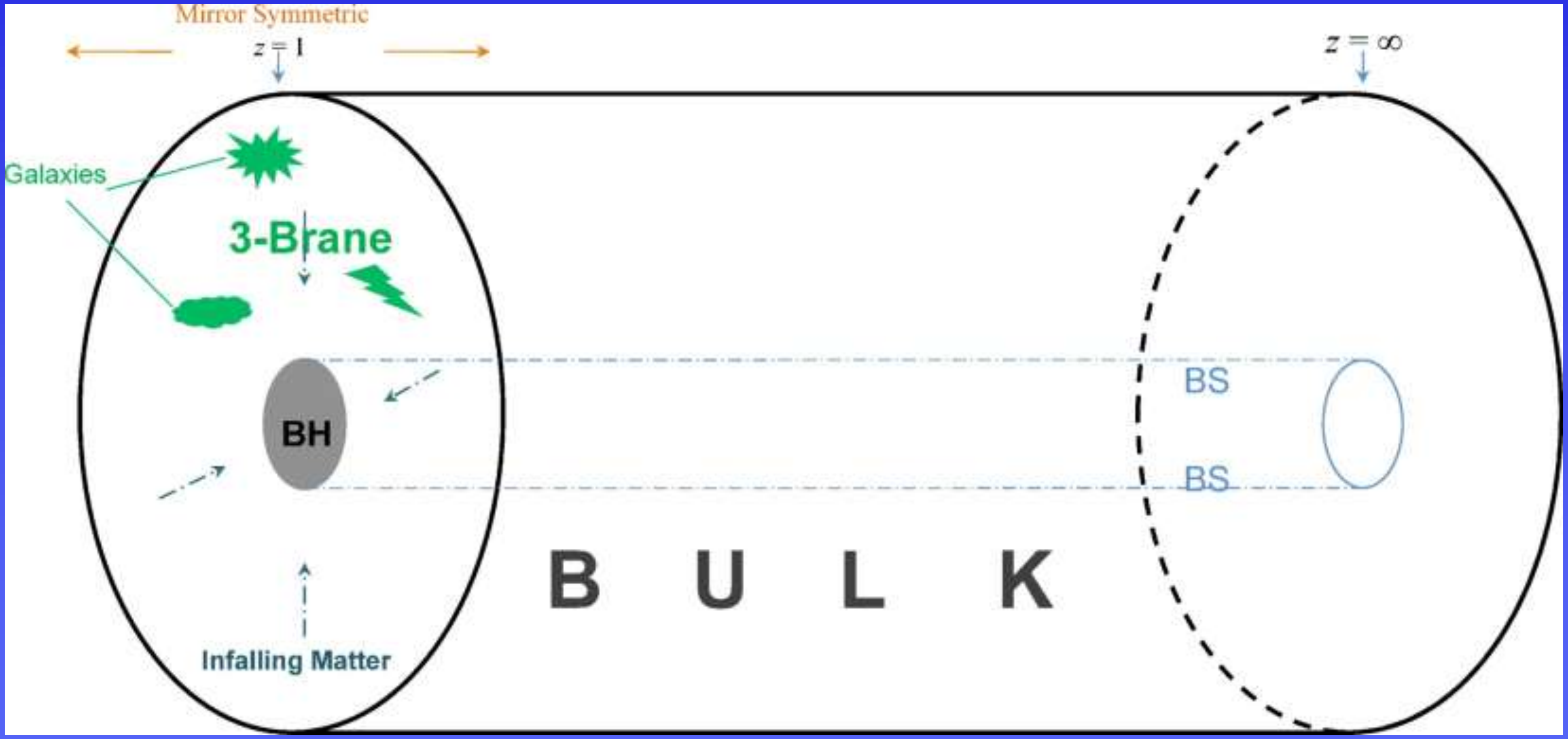
h_{ab} is the metric of a Ricci-flat soln of Einstein's eqns in 4D, x^a , a=0,1,2,3 are the brane coordinates, z is the coordinate of the extra dimension and l is the AdS length scale

Vacuum solutions

- When h_{ab} is the Minkowski metric, the above spacetime is a part of a Poincare patch of an anti-de Sitter (AdS) spacetime with AdS length, l

- When h_{ab} is a 4-d black hole metric, the spacetime describes a black string

Black string



Vacuum solutions

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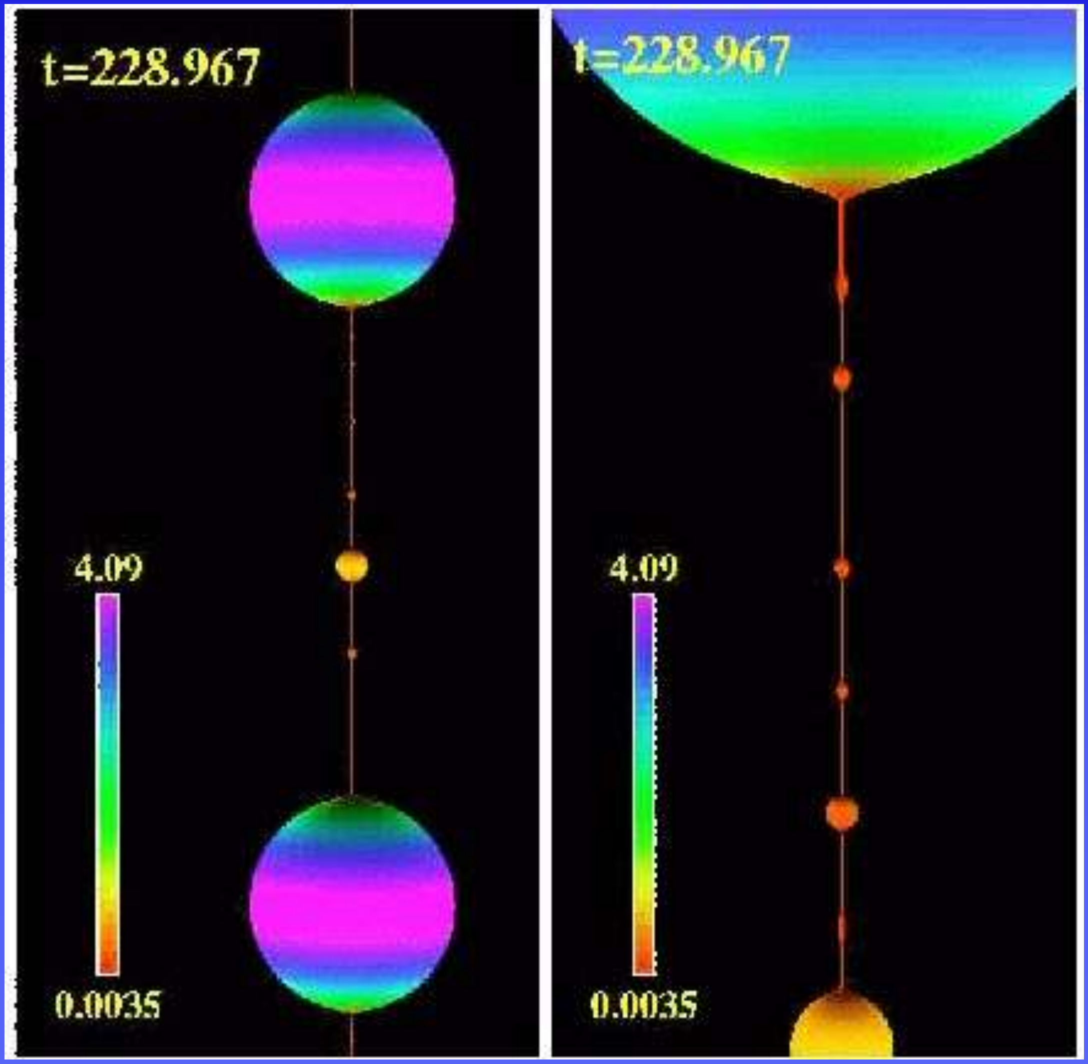
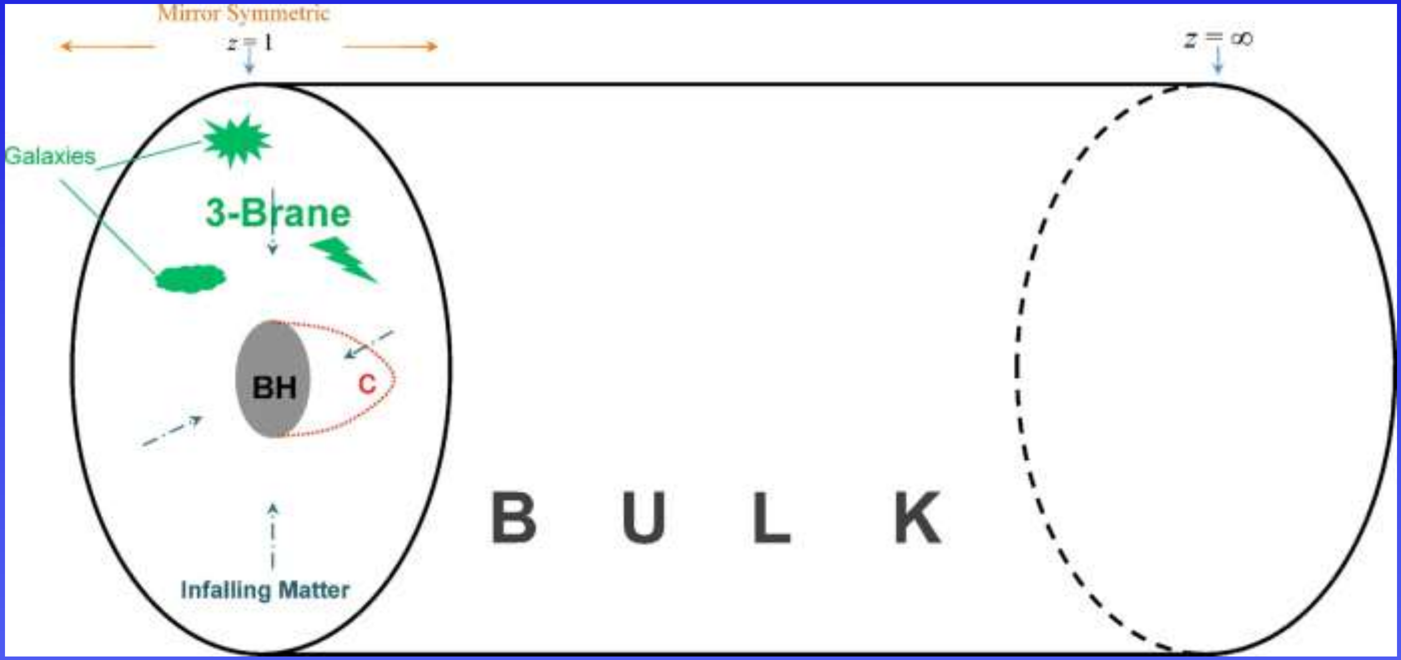
- When h_{ab} is a 4-d black hole metric, the spacetime describes a black string
  - However, these solutions are unstable via the Gregory-Laflamme instability (Gregory 2000)
  - Don’t expect to form from collapse processes

Black string instability (4D)
(Lehner & Pretorius 2010)



Proposal
(Chamblin, Hawking & Reall 2000)



- End point of gravitational collapse is "black cigar" (i.e. type of black hole in 5D)
  - conjectured that such solutions exist, are stable and are unique

- There followed a long period of conflicting studies, both numerical (N) and closed form/analytical (A)
    - N: Time symmetric initial data with apparent horizons (AH) (Shiromizu-Shibata, 2000)
    - A: Large holes don't exist (Tanaka 2002; Emparan et al 2002)
        - Argued via AdS/CFT that any large black hole formed on the brane would quickly radiate into the extra dimension
    - N: Small black holes exist (Kudoh et al 2003)
    - N: In 6D black holes can be large (Kudoh 2004)
    - A: Arguments for existence based on fact that problem is strongly coupled (Fitzpatrick et al 2006, Gregory et al 2008)
    - N: Time symmetric initial data with large AH (Tanahashi & Tanaka, 2008)
    - N: No solutions, large or small (Yoshino 2008, Kleihaus et al 2011)

Breakthrough

- A few years ago, two different groups (Figueras and Wiseman (FW) 2011, Abdolrahimi et al 2012) numerically constructed black hole solutions with a range of sizes; all extend into the bulk; essentially realizes proposal of Chamblin et al

- Construction involved solution of elliptic PDEs resulting from static ansatz, and perturbation of an AdS_5-CFT_4 black hole solution that itself was found numerically

- (Non-linear) stability of solutions not immediately apparent

- Relation of solutions to gravitational collapse also not clear

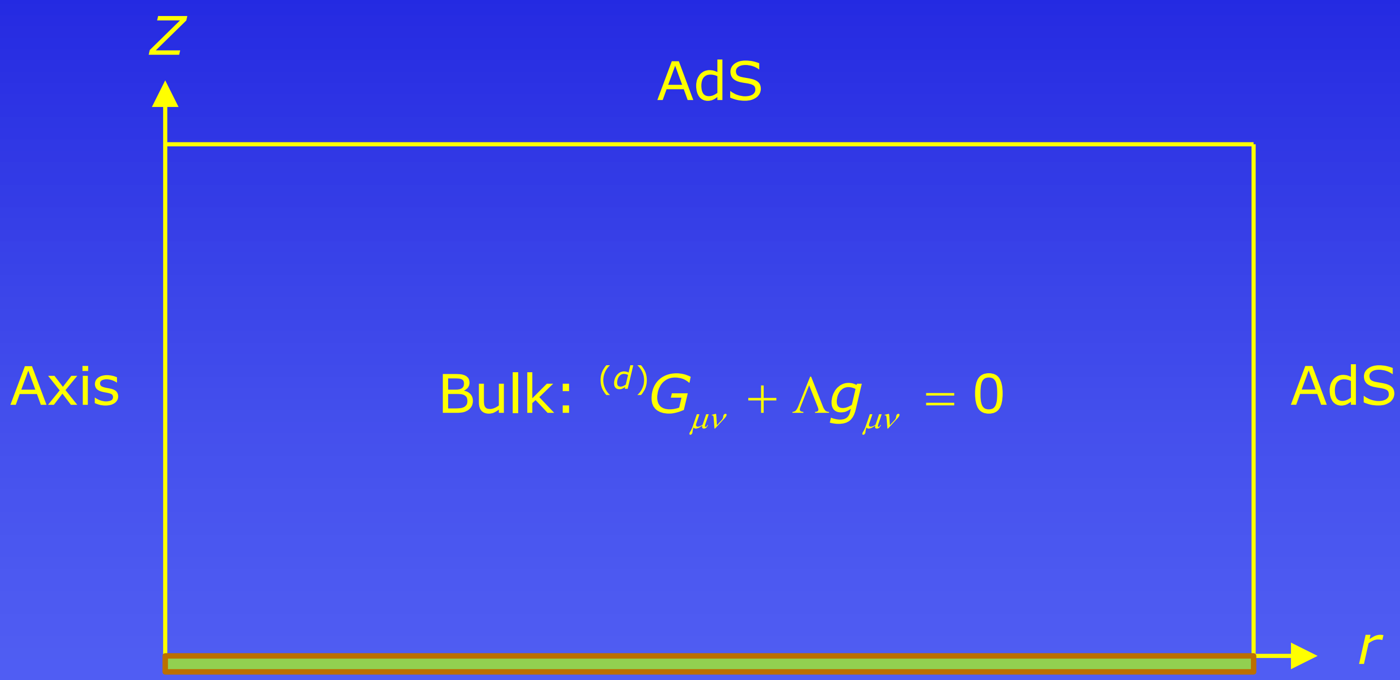
Current work

- Direct assault on the problem using numerical relativity
- Study collapse of simple matter content on the brane (massless scalar field) to see what happens in strong field regime
- To keep problem computationally tractable, impose spherical symmetry on the brane; problem then becomes axisymmetric about z axis (extra dimension)
- Adopt cylindrical coordinates: computational problem is solution of PDEs in (t,r,z)
- First study of fully coupled, noninear dynamics in a braneworld scenario

Challenges & solutions (partial list)

- Stable evolution
  - Generalized harmonic approach to Einstein's equations
  - Need modifcation for use with cylindrical coordinates
- Regularity
  - Possible regularity problems in vicinity of axis
  - Treated using general approach that focuses on regularity of functions, rather than equations
- Brane provides delta-function (distributional) source
  - Both brane tension and matter content (scalar field) contribute
  - Use Israel junction conditions: provide certain boundary conditions on metric components
  - Use coordinate freedom to supply some others

Schematic of setup



Brane $(z = \ell)$: Israel's junction condition

$$\nabla^a \nabla_a \Phi = 0$$

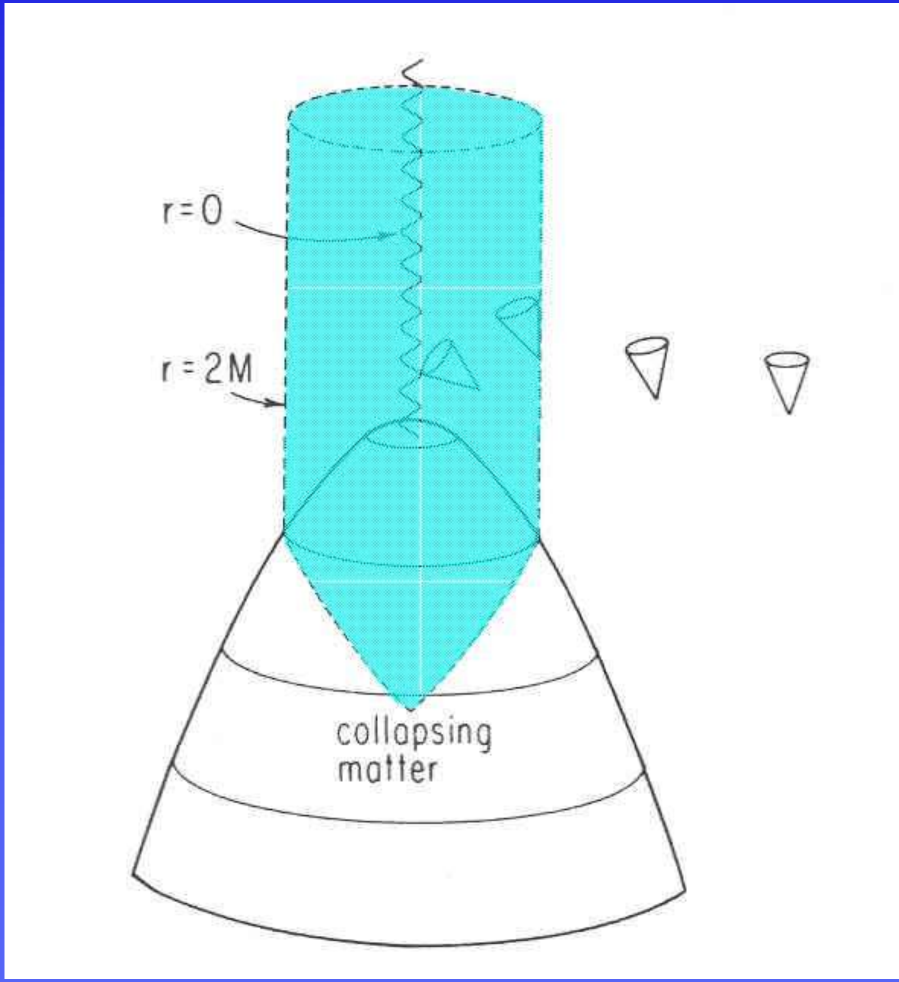
Challenges & solutions (cont.)

- AdS boundary conditions with generalized harmonic
  - Perform conformal transformation to flat background; can then use experience gleaned from asymptotically flat 3+1 studies (black hole collisions, Pretorius 2005 e.g.)
- Constraint damping
  - Need to control constraints associated with gauge functions
  - Regular damping works in interior
  - Problems arise near brane; treat by imposing constraints exactly on brane

Challenges & solutions (cont.)

- Black hole formation
  - Want to evolve as long as possible after horizon formation in attempt to identify static end states (assuming they exist)
  - Use apparent horizon (AH) as surrogate for event horizon and implement black hole excision once AH has been detected
  - Note: AH on brane does not necessarily coincide with intersection of AH in bulk with brane

Black hole excision



Implementation

- Spherical symmetry on brane, problem axisymmetric about z axis
- Use cylindrical coordinates: (t,r,z)
- Use generalized harmonic approach, modified for use with curvilinear coordinates, constraint damping
- Apply conformal transformation, yielding flat background metric
- Choose regularized variables for both metric and gauge source functions

Implementation

- Discretize using second order finite differences (equations kept in second order form), use Kreiss-Oliger dissipation, Newton-Gauss-Seidel iteration for update

- Use compactified spatial coordinates so that spatial infinities are mapped to finite coordinate locations and exact (Dirichlet) conditions can be imposed

- Gauge choice
  - Depending on type of evolution (size of black hole that forms), use
    - damped wave gauge
    - lapse driver (Pretorius 2006)
    - harmonic ($h_t = h_r = h_z = 0$)
  - Always blend to lapse driver for long time evolution once apparent horizon is detected

Implementation

- Apparent horizon detection and excision
  - Choose coordinates so that AH is smooth across brane
  - Axisymmetry: locate AH by solving certain ODE with shooting technique
  - Numerical irregularity serious issue at and near excision surface; best treatment uses one-sided dissipation operators developed by Calabrese et al 2004
  - Approach to AH location effectively precludes parallelization and adaptive mesh refinement: unigrid, uniprocessor operation

- Apparent horizon analysis
  - Study time development of AH in attempt to identify stationary states
  - Minimize coordinate effects by embedding geometry in flat background

Initial data

- Specify scalar field as time-symmetric Gaussian pulse

$$\Phi_0 \equiv \Phi(0, r) = A \exp((r-1)^2 / .01)$$

use A as control parameter

- Then solve Hamiltonian constraint using special two-function approach to fix metric variables

Diagnostic quantities (animations)

- Kretschmann scalar

$$K(t,r,z) \equiv R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}$$

- Lapse function

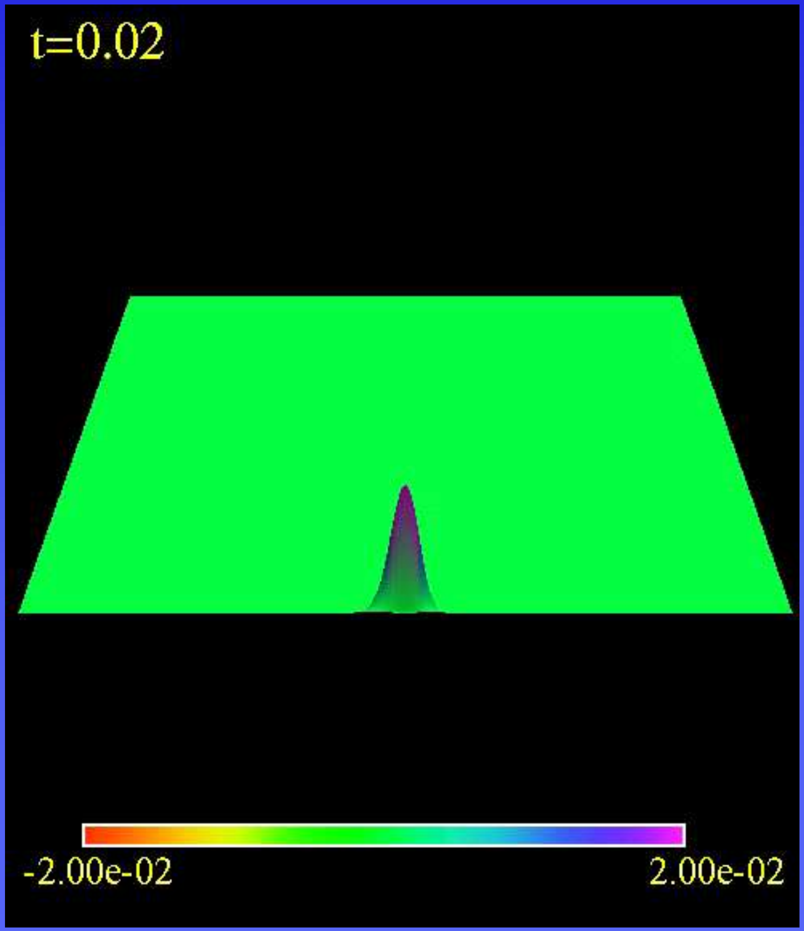
$$\alpha(t,r,z)$$

- Scalar field

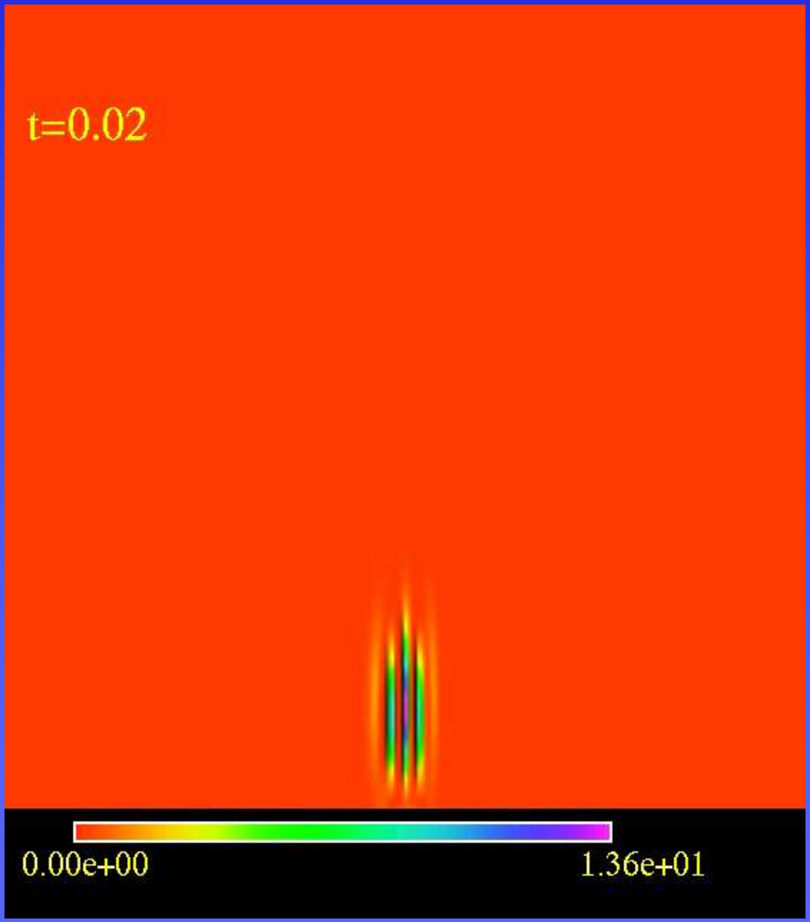
$$\Phi(t,r)$$

Weak field evolution $A = 0.03$

$\Phi(t, r)$



$K(t, r, z)$



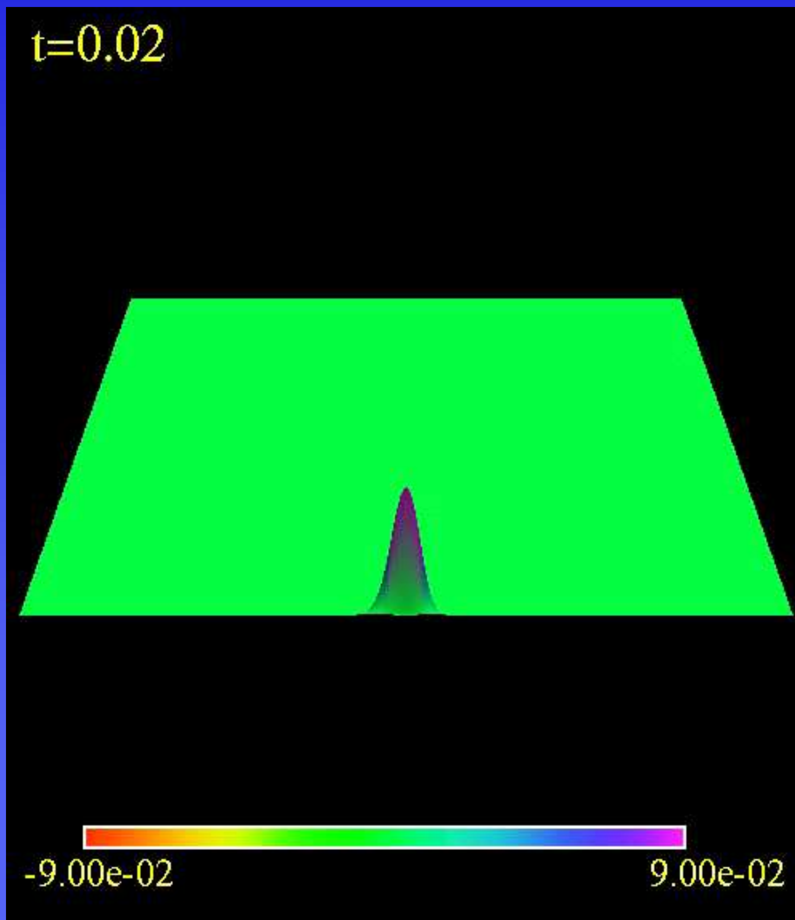
z z

r r

Strong field evolution $A = 0.16$

$\Phi(t,r)$

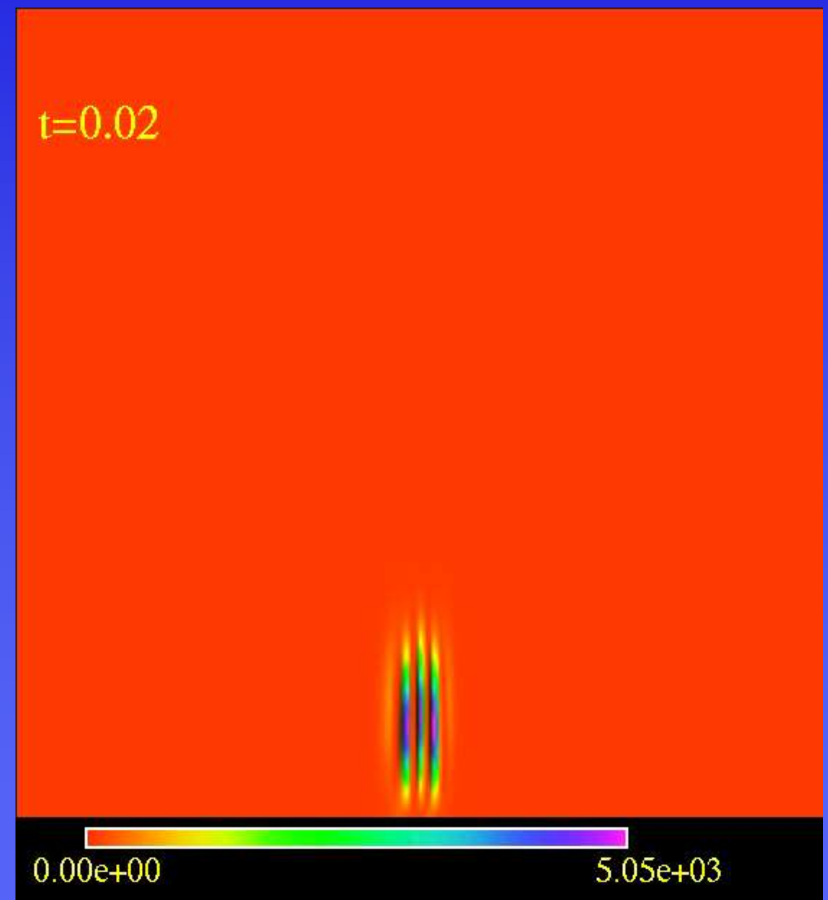
z



r

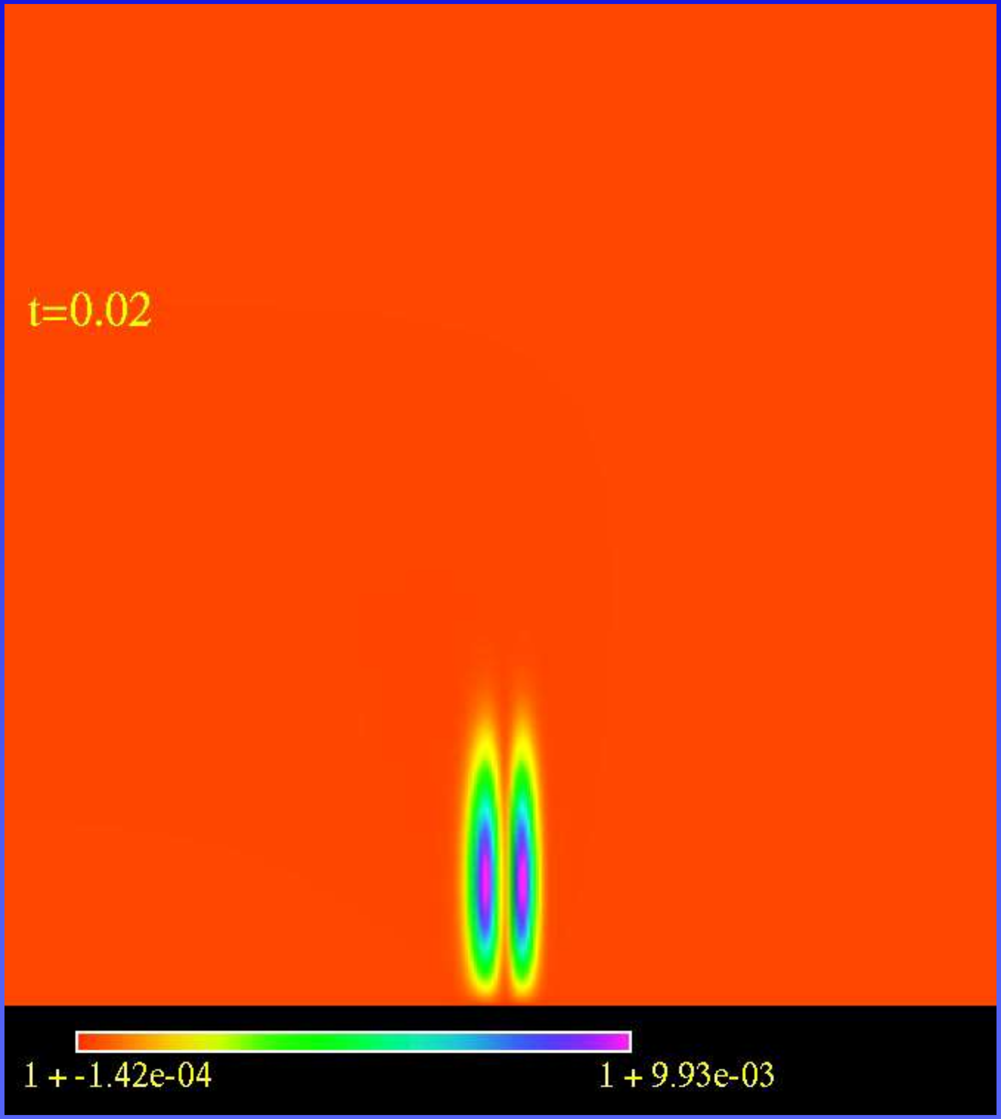
$K(t,r,z)$

z



r

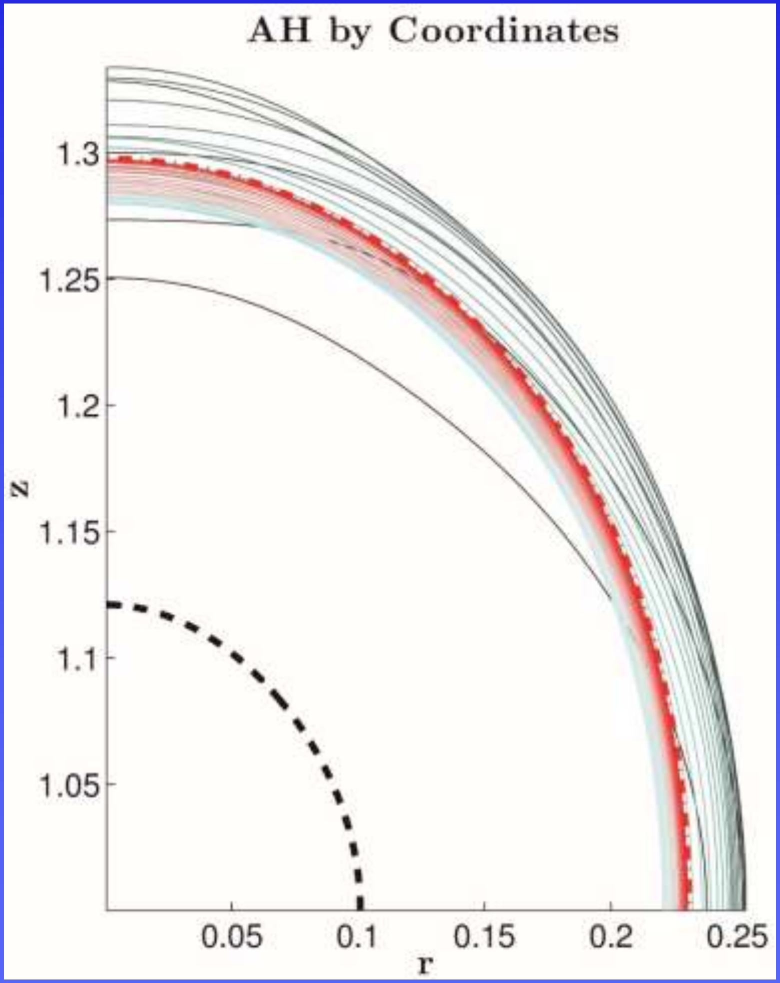
Strong field evolution: $\alpha(t, r, z)$



z

r

Evolution of apparent horizon
(non-compactified computational coordinates)



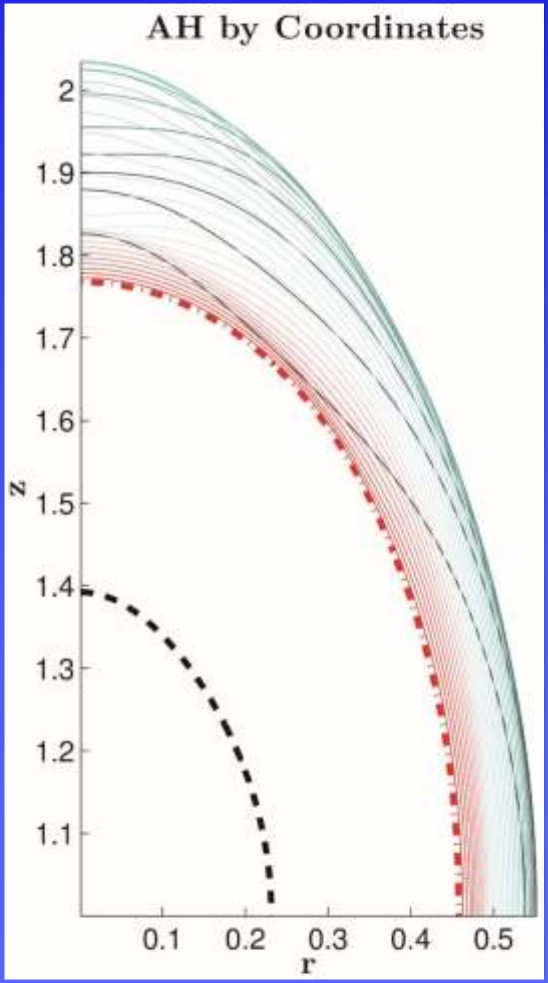
Evolution of AH

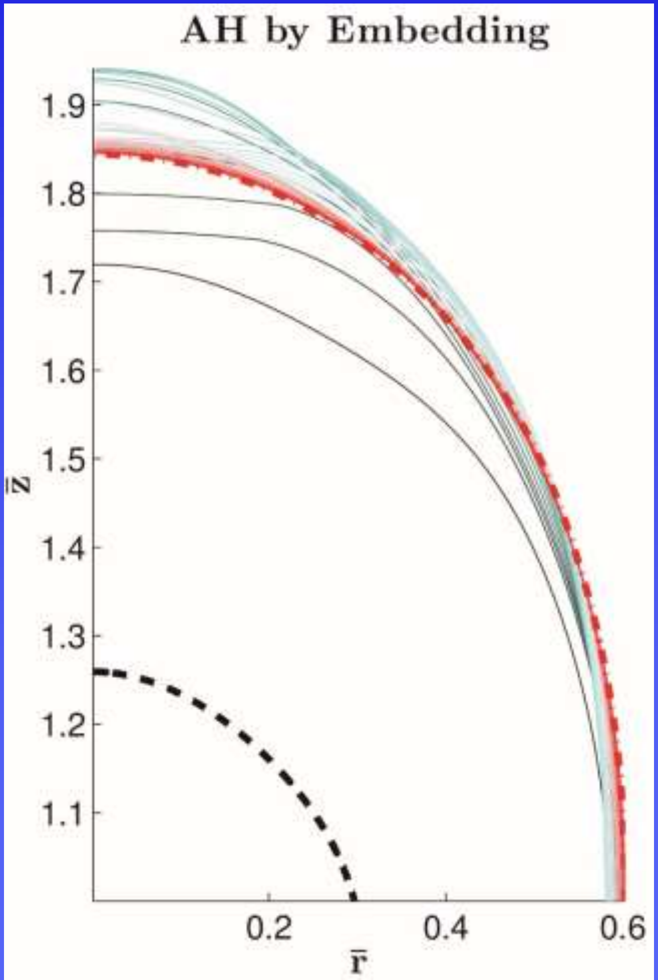
- Some evidence that configurations are settling into a stationary state, at least for some amount of coordinate time

- Attempt to mimimize coordinate effects (time dependence of spatial coordinates) by isometrically embedding AH curves into flat background:

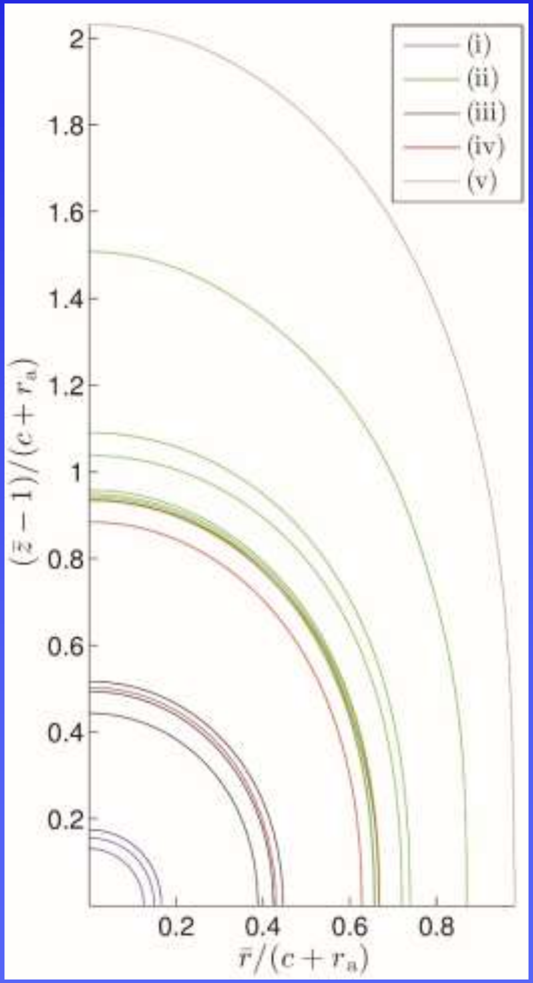
$$ds^2 = \frac{\ell^2}{\bar{z}^2}\left(-dt^2 + d\bar{r}^2 + \bar{r}^2 d\bar{\Omega}^2 + d\bar{z}^2\right)$$

AH evolution in computational and flat background coordinates





Indications of uniqueness of final states



- Call near-stationary late time configurations "apparently stationary states"
- Shown are such states from selected calculations using 5 distinct familes of initial data
- Overlap among families suggests uniqueness of sequence of solutions (no hair)

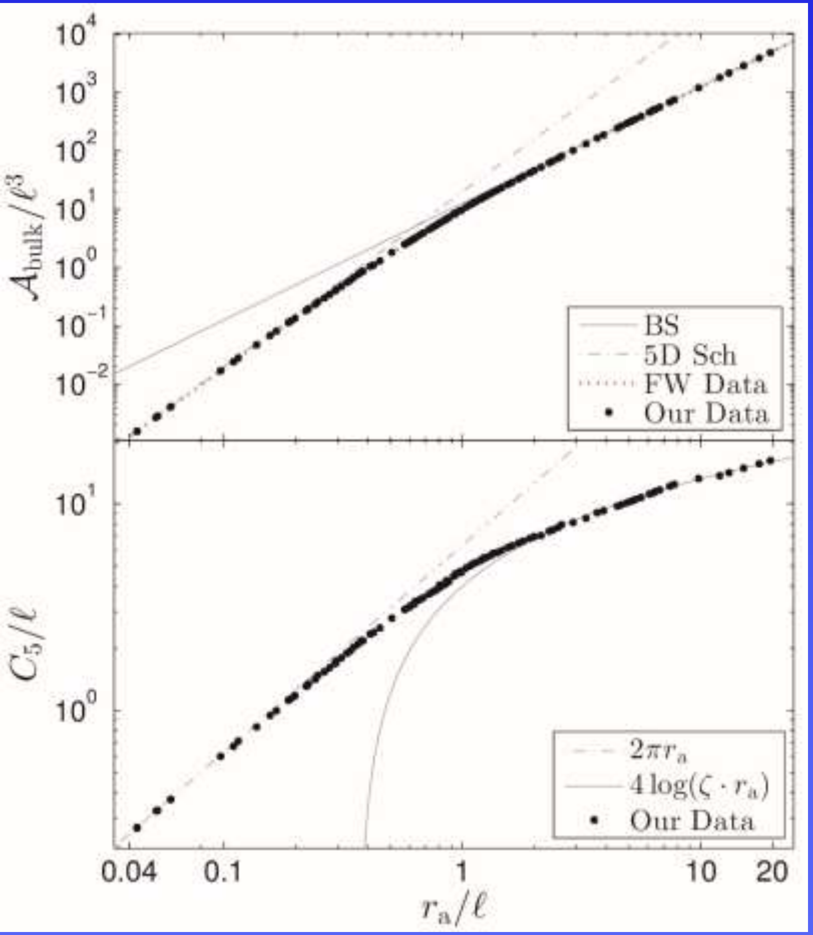
AH geometry as function of size

- Define size of black hole

$$r_a = \sqrt{A_{brane} / 4\pi}$$

  and compute 5D proper circumference C_5

- Comparison with Figueras-Wiseman data (computed from static ansatz) is very good



Here: $\zeta \approx 2.71$ Figueras & Wiseman: $\zeta \approx 2.8$

Summary

- Constructed code to solve the coupled Einstein massless-scalar equations in the context of the Randall-Sundrum II braneworld
- Successful implementation required solution of several novel problems arising from the peculiarities of the model
- Find evidence for a single-parameter family of stable, static black hole solutions (finite extension into the bulk) conveniently characterized by size on the brane
- Good agreement between solutions found here, and those computed previously from a static ansatz

Speculation

- Is Randall-Sundrum II still viable?
  - Pardo et al, (2018) claim that model is *not* ruled out by GW170817
    - Leakage of graviton energy from brane to bulk is very inefficient (Randall & Sundrum (1999))
  - Would be very interesting to, for example, simulate binary black hole (or neutron star) inspiral and coalescence and compare to 4D asymptotically flat case
  - Would also be extremely challenging; 5D and several of the strategies adopted here won't generalize easily (or at all) to the symmetry-free case
  - Are there shortcuts for such testing?