Economics for CPS researchers

from Arrow & Nash to non-Coasian worldview (continued)

Galina Schwartz[1]

[1]UC Berkeley

From tastes differences to non-cooperative bargaining
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Team for Research in Ubiquitous Secure Technology

Bargaining: Cooperative and Non-Cooperative

Intro to Non-Cooperative Bargaining Theory

- Property rights and bargaining: how to allocate the rights
- Farrell (1987): examples of allocating the rights
- Rubinstein (1982) [extends & generalizes Stahl (1972)]
- Yildiz (2011) [Nash & Rubinstein approaches reconciled]
- Applications of bargaining

Materials

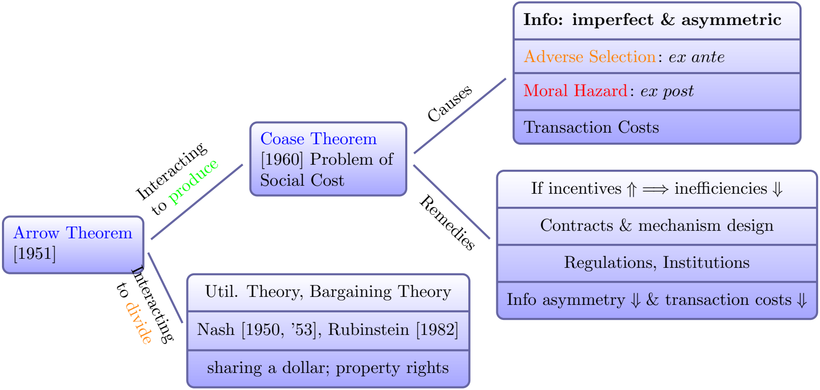
Literature

- J. Farrell, 1987. Information and the Coase Theorem, The Journal of Economic Perspectives, 1 - 2, 113-129. http://www.jstor.org/stable/1942984
- A. Rubinstein, 1982. Perfect Equilibrium in a Bargaining Model. Econometrica 50-1, 97 - 109. http://www.jstor.org/stable/1912531
- M. Yildiz, 2011, Nash meets Rubinstein in final-offer arbitration, Economics Letters,110- 3, 226-230. http://dx.doi.org/10.1016/j.econlet.2010.10.020

Further readings

- T. Palfrey, H. Rosenthal, 1989. Underestimated probabilities that others free ride: An experimental test. Mimeo, Caltech and CMU; T. Palfrey, H. Rosenthal, 1990. Testing game-theoretic models of free riding : new evidence on probability bias and learning, Working paper #549, MIT. http://hdl.handle.net/1721.1/64219
- T. Palfrey, H. Rosenthal, 1991. Testing for effects of cheap talk in a public goods game with private information, Games and Economic Behavior, 3, 183-220, http://dx.doi.org/10.1016/0899-8256(91)90022-7
- T. Palfrey, H. Rosenthal, N. Roy. 2015. How Cheap Talk Enhances Efficiency in Public Goods Games. Social Sciences Working Paper 1400: Caltech. http://people.hss.caltech.edu/~trp/SSWP%201400.pdf
- J. Sutton, 1986. Non-Cooperative Bargaining Theory: An Introduction, Review of Economic Studies 53-5, 709-724. http://www.jstor.org/stable/2297715
- C. Stevens, 1966. Is Compulsory Arbitration Compatible With Bargaining?, Industrial Relations, 5, 38-52.
- Anat Admati and Motty Perry, 1991. Joint Projects without Commitment, The Review of Economic Studies 58-2, 259-276. http://www.jstor.org/stable/2297967

Arrow impossibility theorem & its progenies

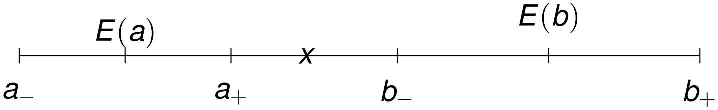


Arrow, Coase, Nash Bargaining [+ the tree of knowledge]

Public Goods when tastes differ (Farrell 1987)

Room temperature (conflict?): office mates [Players A & B]

x	public outcome [temperature]
$u(x, a)$	player A dis-utility
$v(x, b)$	player B dis-utility
$a \in [a_-, a_+]$	player A taste [private info]
$b \in [b_-, b_+]$	player B taste [private info]
$U = u + v$	aggregate societal loss
0	zero costs [free electricity]
uniform	independently dist. tastes on resp. intervals



Preferred temperature(s) intervals for player A & B

Farrell1987: effects of private information

Question: How to allocate property rights in the most socially beneficial way (in the presence of hidden information)?

Each player minimizes:

$$u(x, b) = -\beta(x - b)^2 \text{ [Player B dis-utility]}$$

$$u(x, a) = -\alpha(x - a)^2 \text{ [Player A dis-utility],}$$

Private knowledge of a and b; for others a and b are uniformly (independently) distributed on intervals $[a_-, a_+]$ and $[b_-,\ b_+]$
Constants α and β – known parameters

Possibilities for choosing x ?

Possible games [procedures of finding x]

Possible allocations of property rights (for x)

- Building manager dictates x to minimize social dis-utility [Conditional on his limited information]
- Building manager (aka Social Planner) designs a mechanism to find optimal x [uncovers tastes via standard revelation procedure]
- Player A dictates x; player B compensates A [Player B offers menu of contracts]
- Player B dictates x; player A compensates B [Player A offers menu of contracts]]

We will start with a benchmark of perfect information

Dixit & Olson (2000): public good provision with volunteer participation [draw on Palfrey & Rosenthal (1984)]

N	identical players
V	per person benefit
$V \times N$	societal benefit
C	cost of public good
IN/OUT	participate or not in financing
n	number of players IN

'Everyone here? Good. Meeting topic: Setting world record for shortest meeting. All in favor say aye. Ayes have it. Meeting over."

Dixit & Olson (2000): an illustration

	IN	OUT
		
Payoffs	$V - \frac{C}{n}$	V [free riders]

Comparison of P&R1989 and D&O2000

Dixit & Olson (2000): No private information

	Player 2	
	IN (contribute)	*OUT* (don't)
IN	$1 - C/2, 1 - C/2$	$1 - C, 1$
OUT	$1, \quad 1 - C$	$0, \quad 0$

Palfrey & Rosenthal (1989): Private information

	Player 2	
	IN (contribute)	*OUT* (don't)
IN	$1 - c_1, 1 - c_2$	$1 - c_1, 1$
OUT	$1, \quad 1 - c_2$	$0, \quad 0$

Each player knows own cost, and a dist. function $P(\cdot)$ from which the other player cost is drawn.

Public goods provision with private information

2	identical players
$V = 1$	per player benefit
1×2	societal benefit
c_i	player i cost if IN (contributes)
$P(\cdot)$	distribution of costs
IN/OUT	contribute or not

OUT,OUT

IN, OUT

IN,IN

P&R1989

	IN	OUT
		
Payoffs	$1 - c_i$	1 [free rider]

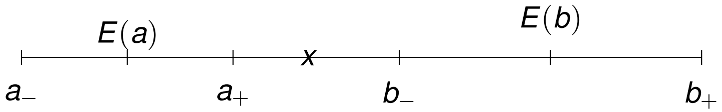
Proposition

Any symmetric equilibrium of P&R1989 game is mixed.

Comparing F1987 and P&R1989

Farrell (1987)

How to determine x? [Games differ by how x is chosen and by whom]



Palfrey and Rosenthal (1989)

	Player 2	
	IN (contribute)	*OUT* (don't)
IN	$1 - c_1, 1 - c_2$	$1 - c_1, 1$
OUT	$1, \quad 1 - c_2$	$0, \quad 0$

Both papers have private info (tastes/costs) Players know own taste/cost, and a dist. function from which the other player taste/cost is drawn.

Farrell 1987 setup summary Player A dictates x (has property rights for x)

$x(a)$		outcome
$p(a)$		payment for outcome $x(a)$
$p(\cdot)$, $x(\cdot)$		menu choice of B [for a given a]
a		preferred point of A, wlg $a \leq b$
b		preferred point of B
α	$\alpha + \beta = 1$	known taste' weight of A
β		known taste's weight of B
$E(a)$	$\frac{a_- + a_+}{2}$	
$E(b)$	$\frac{b_- + b_+}{2}$	
r	$var(a) = \frac{(a_- + a_+)^2}{12}$	
s	$var(b) = \frac{(b_- + b_+)^2}{12}$	
a	is uniform on $[a_-, a_+]$	
b	is uniform on $[b_-, b_+]$	

Benchmark of perfect information

The average conflict C between B and A is $C = E(b) - E(a)$

perfect information case: For Pareto efficiency

$$W(a, b) = -\min\left\{\beta(x - b)^2 + \alpha(x - a)^2\right\}.$$

Thus, benchmark of perfect information gives socially optimal x^*:

$$x^* = \frac{\alpha a + \beta b.}{\alpha + \beta} = \alpha a + \beta b, \text{ when } \alpha + \beta = 1.$$

$E(W^*)$ – aggregate welfare in social optimum (on average, at first best [benchmark of perfect information]

$$E(W^*) = -\int_{a_-}^{a_+}\int_{b_-}^{b_+}\{W(a, b)\} f(a)f(b)dadb$$

Social Optimum Benchmark [the lowest dis-utility]

Benchmark of perfect information

This lowest dis-utility is achievable when the values (a, b) are given is $W^*(a, b)$. This dis-utility is unequal to the average dis-utility:

$$W^*(a, b) \lesseqgtr E(W^*)$$

Could mechanism design could help to illicit hidden information, and (in repeated case) achieve "almost balance the budget"?

Building Manager dictates property rights "on average"

Building manager dictates x [knows only prob. distributions of a & b; he does not know the realizations]

$\longrightarrow$ His choice of x should depend on public information only:

$$x^{SP} = \alpha E(a) + \beta E(b).$$

compare with benchmark of perfect information

$$x^* = \alpha a + \beta b$$

Welfare loss (relative to perfect information)

$$W^{SP} < W^* < 0.$$

$$W^* - W^{SP} = \beta^2 r + \alpha^2 s.$$

Social Optimum via Mechanism Design By Building Manager

Building manager designs a mechanism

The goal is to elicit truthful realizations of a and b. Players must pay to Building Manager (the amount of externality of A on B - net effect of a' on player B payoff)

$$p_A^{MD} = \beta\left(\alpha a' + \beta b - b\right)^2 \text{ and } p_B^{MD} = \alpha\left(\beta b' + \alpha a - a\right)^2.$$

Then: a' and b' truthfully reported, and $x^{MD} = \beta a' + \alpha b'$ is first-best. If no lump-sum transfer back, the cost of revelation is

$$p_A^{MD} + p_B^{MD} = \beta\left(\alpha a' + \beta b - b\right)^2 + \alpha\left(\beta b' + \alpha a - a\right)^2.$$

If the players can refuse to participate in mech design, and instead rely on “on average” allocation, will they do that?

Player Welfare with Mechanism Design

$$W_A^{MD} = -\beta(x-b)^2 - \alpha\,(x-a)^2 = W^*(a,b) = W_B^{MD}$$

$$\begin{aligned} W^{MD} &= -\beta(x-b')^2 - \alpha(x-a')^2 - p_A^{MD} - p_B^{MD} \\ &= \left[-\beta\left(\beta a' + \alpha b' - b'\right)^2 - \alpha\left(\beta a' + \alpha b' - a'\right)^2\right] \times 2 \\ &= 2W^*(a,b) < W^*(a,b). \end{aligned}$$

Mechanism design achieves truthful revelation, BUT

Aggregate welfare (of player A & B) is lower than with complete info.

If Building Manager is also a player, optimum is achieved [problematic]

Social Optimum via Mechanism Design:

Could player payments to Building Manager return to them? I

How return player payments (to Building Manager) back to them?
[This will reduce player dis-utility.]
Only possible to use lump-sum (a constant transfer)
Let building manager return lump-sum $E(W^*)$ to the occupants, i.e., on average (in expectation) incentive payments are returned to players

$$W_A^{MD} + W_B^{MD} = 2W^*(a,b) + E(W^*),$$

and

$$W_A^{MD} = W_B^{MD} = W^*(a,b) - \frac{E(W^*)}{2} \lesseqgtr \frac{E(W^*)}{2}$$

If the players could refuse to participate (vote?), will they choose so? For what parameters & outside option(s)? For what values of tastes?

Player A has initial property rights

Optimal allocation (with a possibility of contracting)

Can Player B improve the allocation (relative to the allocation optimal for player A)? How?
Intuition: Player B can offer player A a payment in exchange for more favorable (for player B) choice(s) of x.

Incentive Problem: how player B chooses $x(a)$ and $p(a)$

Optimization problem of player B: to design a menu $(x(a), p(a))$ to maximize [the expected value]

$$\tilde{u}(x, b) = v(x(a), b) - p(a) = -p(a) - \beta(x(a) - b)^2,$$

where $x(a)$ is an outcome induced by the payment of $p(a)$ to player A.

Player A has initial property rights Player B offers a menu of contracts

Player A reveals a truthfully if

$p(a) - \alpha(x(a) - a)^2 \geq p(a') - \alpha(x(a') - a')^2$. [IC=incentive compatibility]

FOCs are:

$$\frac{dp(a)}{da} = 2\alpha(x(a) - a)\frac{dx(a)}{da}$$

Incentive compatability for A (to accept the offer from B)

$$-p(a) - \alpha(x(a) - a)^2 \geq 0. \quad \text{[IR=individual rationality (participation)]}$$

[IC] is automatic if B sets $x(a) = a$ and $p(a) = 0$. Generically, only one constraint [IC or IR] binds strictly. Let z denote such a that player A is indifferent between taking payment $p(z)$ or not:

$$p(z) - \alpha(x(z) - z)^2 = 0$$

$$p(z) = \alpha(x(z) - z)^2$$

Player A (or B) has initial property rights Side payments between the players are allowed

If A sets $x(a)$ (for a compensation $p(a)$ from B), his best choice is

$$x(a) = x^* - \alpha(a_+ - a) \le x^* = \beta a + \alpha b,$$

Inefficiency in expectation is $\alpha^2 (a_+ - a)^2$ or $4\alpha^2 r$

$$W^* - W^A = 4\alpha^2 r$$

If B sets x (for a compensation $p(x)$ from A) he will set

$$x(b) = x^* + \beta(b_+ - b) \ge x^*.$$

$$W^* - W^B = 4\beta^2 s$$

Comparison of games [differ by procedure that sets x]

Benchmark of perfect information vs

Possible allocations of property rights (for x)

- Building manager dictates x; preferred when $\alpha^2 \times r \approx \beta^2 \times s$ [inefficiency is $\alpha^2 \times r + \beta^2 \times s$]
- Building manager designs a mechanism to set x [possibly improves on Building manager dictate]
- Player A dictates x; is preferred when α and r are relatively low [inefficiency is $4\alpha^2 \times r$]
- Player B dictates x; is preferred when β and s are relatively low [inefficiency is $4\beta^2 \times s$]

Comparison of possible allocations: Conclusion

First best is unattainable

With private info, efficiency via mechanism design is unattainable.

Second best depends on parameters

The "second best" property rights allocation could occur at different games (see previous slide).

Non-cooperative vs Cooperative Bargaining

Why bargaining is important for CPS

1. Conflicts of individual and group preferences
2. No truthful preference revelation [in general, even with 2 parties]
3. Need to allocate ownership rights

Why cooperative bargaining is not enough

- ? – obvious?
- Two pillars of bargaining theory
  - Axiomatic Nash Bargaining Solution
  - Rubinstain Solution of Alternating Bargaining Game
- Stahl-Rubinstein B. [differences from Nash B.]
  - strategic
  - protocol details matter (bargaining procedure)
  - versatile – can account for delay, risk, costs...

Foundations of non-cooperative bargaining theory

An asset (surplus)

Rubinstein's alternating bargaining model

X	set of agreements (x_1, x_2)
x_i	player i share, $x_i \geq 0$
D	Disagreement [worse than any agreement]
t	time, $t = 1, 2, \ldots$

Why alternating offers?

Time preferences drive the model

From Nash to Rubinstein: Dividing a fixed pie

Cooperative: Nash

- Perfect information
- Known player utilities
- Axioms N1- N5 [à la Arrow (1951)]

Noncooperative: Rubinstein

- from automata to humans
- allocation depends on
  - player preferences (objectives)
  - environment features (asset properties)
  - bargaining protocol

From Nash to Rubinstein: Examples

Rubinstein and Nash coincide for discount rates close to 1.

Cooperative: N

- Perfect info
- Axioms N1- N5
- Unique solution

Examples

?

Nash solution: impractical
important methodologically
widely used in the literature

Noncooperative: R

- Alternating offers
- Impatient players
- Axioms R1 - R6
- Unique solution

Examples

- Legal: private (divorce)
- Legal: corporate (patent litigation)
- personal: hm... dividing a pie

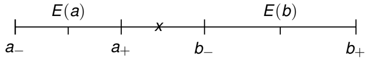
Farrell'87 vs Rubinstein'82

Farrell: allocation of rights matters!

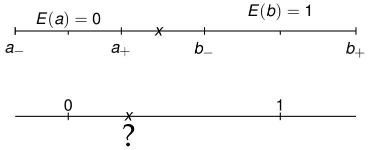
- Applied [simplistic, but realistic]
- How to evaluate inefficiencies
- Thoughts provoking

R: HOW the rights are allocated

- Theoretical [axiomatic]
- Focus on bargaining protocol(s)
- Connects with Nash bargaining
- Opens a new field (cited 5000+ times)



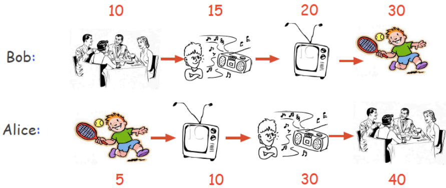
Farrell: temperature intervals preferred by players A & B



Rubinstein: Players 1 & 2 [A & B]

Dividing an asset via bargaining

An asset (surplus)



Surplus sharing problem: bargaining approach

v	Buyer's valuation
c	Seller's cost
S	Surplus, $S = v - c$
p	Price

Trade can happen at any $p \in [c, v]$.

Nash bargaining solution

Unique p exists (under very stylized axioms).

Rubinstein alternating offers model

Examples of non-cooperative bargaining

- Dividing a dollar
- Negotiating a sale price of a good (car)
- Negotiating wages [with given profit]

Formal Rubinstein's Model

$$X = \left\{ (x_1, x_2) \in \mathbb{R}_+^2 | x_1 + x_2 = 1 \text{ \& } x_i \geq 0 \text{ for } i = 1, 2 \right\}.$$

A bargaining game of alternating offers is am extensive form game where players have *complete transitive reflective* ($x \sim x$) preference ordering $\succeq_i$ over the set of outcomes $(X \times T) \cup \{D\}$.
Solution concept: Subgame Perfect Nash Equilibrium (SPE)

Rubinstein's Axioms

(R1) [Disagreement D is the worst outcome] For every $(x, t) \in X \times T$ we have $(x, t) \succeq_i D$

(R2) [Pie is desirable] $\forall t \in T, x \in X$,and $y \in X$ we have $(x, t) \succeq_i (y, t)$ iff $x_i > y_i$

(R3) [Time is valuable] $\forall t \in T, s \in T$,and $x \in X$ we have $(x, t) \succeq_i (x, s)$ if $t < s$ and strict preference if $x_i > 0$

(R4) [Continuity] Let $\{(x_n, t)\}_{n=1}^{\infty}$ & $\{(y_n, s)\}_{n=1}^{\infty}$ be sequences of points in $X \times T$ s.t. $\lim_{n \longrightarrow \infty} x_n = x$ & $\lim_{n \longrightarrow \infty} y_n = y$. Then $(x, t) \succeq_i (y, s)$ if $\forall n \ (x_n, t) \succeq_i (y_n, s)$.

(R5) [Stationarity] $\forall t \in T, x \in X$,and $y \in X$ we have $(x, t) \succeq_i (y, t + 1)$ iff $(x, 0) \succeq_i (y, 1)$

(R6) [Increasing loss to delay] The difference $x_i - v_i(x_i, 1)$ increases in x_i

Rubinstein's Axioms: from preferences to utilities

Proposition (From preferences to utilities)

Preference ordering $\succeq_i$ satisfies R2 - R4 iff i's preferences over $X \times T$ can be represented by continuous utility function $U_i : [0, 1] \times T \to \mathbb{R}$ increasing in its first (i's share), and decreasing in its second (the period of agreement) argument, if share is positive.

From R5 and R2 - R4, $\forall \delta \in (0, 1)$ we have $U_i : [0, 1] \times T \to \mathbb{R}$ and $U_i(x_i, t) = \delta^t u_i(x_i)$

Note: no concavity assumption on preferences $u_i(x_i)$

Rubinstein's Axiom R6: net present value

Discussion of Axiom R6

Define $v_i : [0,1] \times T \to [0,1]$

$$v_i(x_i, t) = \begin{cases} y_i & \text{if} \quad (y,0) \sim_i (x,t) \\ 0 & \text{if} \quad (y,0) \succ_i (x,t) \quad \forall y \in X \end{cases}$$

$v_i(x_i, t)$ is net present value of (x, t) for player i even if $v_i(x_i, t) = 0$. [a slight abuse of term]. If $v_i(x_i, t) > 0$, player i is indifferent between $v_i(x_i, t)$ and x_i at $t = 0$.
Axiom (R6) is weaker than concavity of u_i.

Examples that comply with Axioms R1 - R6

Constant discount rates

$U_i(x_i, t) = \delta_i^t x_i$ and $U_i(D) = 0$.
[With constant discount rates $v_i(x_i, t) = \delta_i^t x_i$]

Constant costs of bargaining

$U_i(x_i, t) = x_i - c_i t$ (if $x_i \geq c_i t$ and $U_i(D) = -\infty$.)
[Constant per period costs c_i of bargaining for each player i]
$v_i(x_i, t) = x_i - c_i t$ if $x_i \geq c_i t$
and
$v_i(x_i, t) = 0$ otherwise.

Rubinstein's Model: the root of uniqueness

Lemma (In search of uniqueness)

If preference ordering $\succeq_i$ of each player satisfies R2 - R6 there exists a unique pair $(x^*, y^*) \in X \times X$ s.t. $y_1^* = v_1(x_1, t)$ and $x_2^* = v_2(y_2, t)$

Player 1	proposes	$\bar{x}$
	accepts	$x_1 \geq \bar{x}_1$
Player 2	proposes	$\bar{x}$
	accepts	$x_1 \leq \bar{x}_1$

To find SPE – have to specify strategies for all possible histories (including off-equilibrium)

Rubinstein's Model: Equilibrium

Let (x^*, y^*) be SPE. Then:

$$y_1^* = v_1(x_1^*, 1) \text{ and } x_2^* = v_2(y_2^*, 1)$$

$$y_1^* = \delta_1 x_1^* \text{ and } x_2^* = \delta_2 y_2^*$$

$$x^* = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right) \text{ and } y^* = \left(\frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}, \frac{1-\delta_1}{1-\delta_1\delta_2}\right)$$

With $\delta_1 = \delta_2 = \delta$

$$x^* = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$$

Reminder: Alice and Bob engage in (Nash) Bargaining

Nash bargaining solution

v	Buyer valuation
c	Seller cost
$d = 0$	if no trade

$$U_S(p-c) = (p-c)^\alpha$$

$$U_B(v-p) = (v-p)^\beta.$$

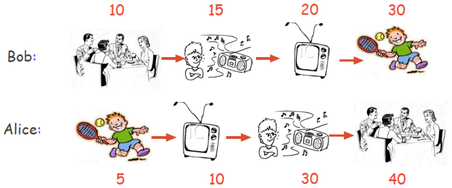
Nash product

$$(p-c)^\alpha (v-p)^\beta, \text{ with } p \in [c, v].$$

The price p^* of exchange:

$$p^* = v\frac{\alpha}{\alpha+\beta} + c\frac{\beta}{\alpha+\beta}.$$

Ceteris paribus, smaller α(or β) moves p closer to c(or v).



Applying Nash product (dinner)

40	Alice valuation
10	Bob's valuation
S	Surplus $S = 40 + 10$

$$\max_p (40-p)(10+p), \text{ with } \alpha = \beta = 1$$

$$p^* = 15 \text{ \& } U_{Alice} = U_{Bob} = 25.$$

Surplus divided equally: $(1/2, /1/2)$

Reconciling of Nash and Rubinstein

Yildiz 2011 Model of final-offer arbitration (Based on Stevens 1966)

- Perfect information
- Discount factor δ
- Three periods, $t \in (0, 1, 2)$

If no agreement before the (pre-defined deadline), arbitrator chooses between the existing offers of the parties (sequential moves).

Solution concept: Subgame Perfect Nash Equilibrium (SPE)

Main results

Let Arbitrator maximize welfare using Nash social welfare function. Then, there exists a unique SPE. It coincides with SPE of Rubinstein's model. This result extends to games allowing pre-arbitration negotiations.

Model of Arbitration

1 and 2	players (negotiators)
A	player arbitrator
3	periods $t \in (0, 1, 2)$
δ	discount factor of the negotiators
(x_0, y_0)	Player 1 offer at $t = 0$
(x_1, y_1)	Player 2 offer at $t = 1$
(x_2, y_2)	Player A offer at $t = 2$
$(0, 0) \in X$	Disagreement

$(x_2, y_2) \in \{(x_0, y_0), (x_1, y_1)\}$
$(x, y) \in X$, X convex and compact set, $X \subset \mathbb{R}^2_+$
No discount factor is needed (defined) for Arbitrator.
Negotiators are allowed to accept offers before $t = 2$

Payoffs

Arbitrator's utility

$$u_A(x, y) = xy.$$

Player payoffs (if the game ends in period t)

$$(x\delta^t, y\delta^t).$$

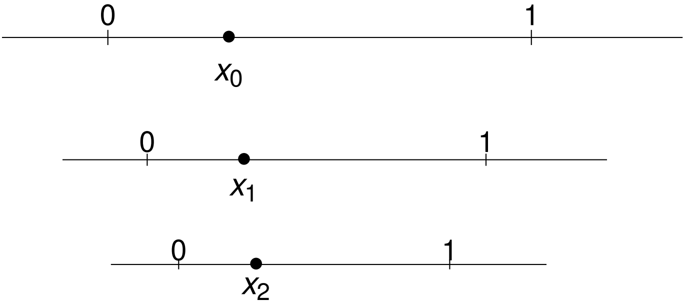
Let the function f

$$f : x \mapsto \max y | (x, y) \in X$$

the function f is continuous, concave and strictly decreasing $f(0) > 0$, and there exists $\bar{x}$ s.t. $f(\bar{x}) = 0$. [standard]

Illustration

(x_0, y_0) Player 1 offer at $t = 0$
(x_1, y_1) Player 2 offer at $t = 1$
(x_2, y_2) Player A offer at $t = 2$, where $(x_2, y_2) \in \{(x_0, y_0), (x_1, y_1)\}$



[for $\delta = 0.8$] $(x\delta^t, y\delta^t)$ Player payoffs with agreement at t

Equilibrium

Proposition

The unique SPE of sequential final-offer arbitration model is identical to a unique R1982 SPE in infinite-horizon alternating-offer bargaining model. Player 1 offers

$$(x_0, y_0) = (x^R, f(x^R)),$$

and Player 2 accepts.

Proof: by backward induction.

Intuition of the proof

Consider a transferable utility

$$X = \{(x, y) | x + y = 1\}.$$

Nash solution is $(1/2, 1/2)$. $\longrightarrow$
Arbitrator will choose an offer closer to $(1/2, 1/2)$. Fix (x_0, y_0). Then:
If Player 2 offers x_1 that is closer to $1/2$ than x_0, Player 1 accepts x_1.
(because Player 1 payoff from arbitration is $\delta x_1 < x_1$.
If Player 2 offers $x_1 > \delta x_0$, even with x_0 closer to 1/2 than x_1, Player 1 accepts x_1. (because his payoff from arbitration is $\delta x_0 < x_1$).
Altogether: If Player 2 offers

$$x_1^*(x_0) = \min\{\delta x_0, 1 - x_0\},$$

Player 1 accepts.

Intuition of the proof (cont.)

Player 2 max payoff $x_1^*(x_0)$ at $t = 1$ is reached at $x_0 = \frac{1}{1+\delta}$. (i.e., $= x^R$)
Thus, at $t = 0$, Player 2 clearly should accept any offer in which $x_0 \leq x^R$ (because he cannot improve on such offers).
Next, let $x_0 > x^R = \frac{1}{1+\delta}$. Then:

$$1 - x_0 \leq \delta x_0 \iff x_0 \geq \frac{1}{1+\delta}$$

$$y_0 = 1 - x_0 < \frac{\delta}{1+\delta} < \delta x_0$$

and

$$y_1 = 1 - \min\{\delta x_0, (1 - x_0)\} = x_0 \geq \frac{1}{1+\delta}.$$

Thus, if $x_0 > x^R = \frac{1}{1+\delta}$, player 2 rejects, and offers $(1 - x_0)$. Then, his payoff is $\delta y_1 = \delta x_0 > y_0$. To sum: Any offer $x_0 < \frac{1}{1+\delta}$ is accepted, and $x_0 \geq \frac{1}{1+\delta}$ rejected. In equilibrium: $x_0^* = \frac{1}{1+\delta} = x_0^R$

Endogenous Final-Offer Arbitration Model

The game with $t \in (0, 1, 2)$ can be extended to $t \in (0, 1, 2, \ldots)$. Player 1 offers on even dates, and player 2 on odd dates. The other player decides to:

- accept
- reject and file for arbitration
- reject and make a counter-offer

Proposition

The unique SPE of the endogenous final-offer arbitration model is identical to a unique R1982 SPE in infinite-horizon alternating-offers bargaining model. Player 1 offers

$$(x_0, y_0) = (x^R, f(x^R)),$$

and Player 2 accepts.

Rubinstein and Nash: the axioms

R1	[Disagreement D is the worst] $\forall (x,t) \in X \times T$ have $(x,t) \succeq_i D$	N1	$\mathcal{B} = (U,d)$: utility possibility set U and the disagreement d
R2	[Pie is desirable] $\forall t \in T, x \in X$, and $y \in X$ we have $(x,t) \succeq_i (y,t)$ iff $x_i > y_i$	N2	PAR Cannot improve a player utility without negative effect on the opponent
R3	[Time is valuable] $\forall t \in T, s \in T$,and $x \in X$ we have $(x,t) \succeq_i (x,s)$ if $t < s$ and strict preference if $x_i > 0$	N3	SYM Solution is symmetric for symmetric players (identical utilities and $d_1 = d_2$).
R4	[Continuity] Let sequences $\{(x_n,t)\}_{n=1}^{\infty}$ & $\{(y_n,s)\}_{n=1}^{\infty}$ s.t. $\lim_{n\longrightarrow\infty} x_n = x$ & $\lim_{n\longrightarrow\infty} y_n = y$. Then $(x,t) \succeq_i (y,s)$ if $\forall n$ $(x_n,t) \succeq_i (y_n,s)$.	N4	LIN Invariant to an affine transformation of utility function ($f(\cdot)$ is independent of origin & units)
R5	[Stationarity] $\forall t \in T, x \in X, y \in X$. Then, $(x,t) \succeq_i (y,t+1)$ iff $(x,0) \succeq_i (y,1)$	N5	IIA A new game defined on a subset containing the original disagreement point and solution, has the same solution as original game.
R6	[Increasing loss to delay] The difference $x_i - v_i(x_i,1)$ increases in x_i		

Why should the outcomes of cooperative and non-cooperative bargaining to coincide?

Repord Cards from ASCE: poor grades

2013 REPORT CARD FOR AMERICA'S INFRASTRUCTURE ASCE

NAVIGATION MENU

CATEGORY	1988*	1998	2001	2005	2009	2013
AVIATION	B-	C-	D	D+	D	D
BRIDGES	-	C-	C	C	C	C+
DAMS	-	D	D	D+	D	D
DRINKING WATER	B-	D	D	D-	D-	D
ENERGY	-	-	D+	D	D+	D+
HAZARDOUS WASTE	D	D-	D+	D	D	D
INLAND WATERWAYS	B-	-	D+	D-	D-	D-
LEVEES	-	-	-	-	D-	D-
PUBLIC PARKS AND RECREATION	-	-	-	C-	C-	C-
RAIL	-	-	-	C-	C-	C+
ROADS	C+	D-	D+	D	D-	D
SCHOOLS	D	F	D-	D	D	D
SOLID WASTE	C-	C-	C+	C+	C+	B-
TRANSIT	C-	C-	C-	D+	D	D
WASTEWATER	C	D+	D	D-	D-	D
PORTS	-	-	-	-	-	C
AMERICA'S INFRASTRUCTURE GPA	C	D	D+	D	D	D+
COST TO IMPROVE	C	-	$1.3 TRILLION	$1.6 TRILLION	$2.2 TRILLION	$3.6 TRILLION

Infrastructure CPS: many public goods features

From individual to ~~societal~~ group preferences

1. Conflicts of individual and group preferences
2. No truthful preference revelation [in general, even with 2 parties]
3. Arrow, Nash & Coase: a connection of information and incentives

Public goods vs club (free to club members) goods

- infrastructures related
  - free internet [Starbucks]
  - free roads, highways, bridges
- free coffee; free internet [at work]
- free shipping [Amazon prime members]

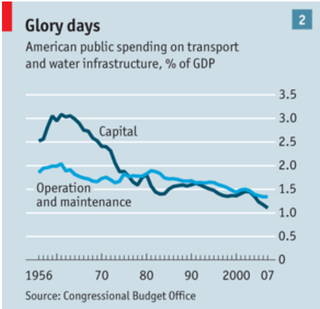
US infrastructures grade: D+ : not a coincidence

American society of civil engineers http://www.infrastructurereportcard.org/grades

Behind D+: Where do the funds come from (US)?

- Long Term[LT] construction
- Medium & Short Term [MT & ST] operations and maintenance

Aver. daily commute	LT (Congress) MT (Congress & Taxes)	Taxes



Courtesy of: http://www.economist.com/node/18620944

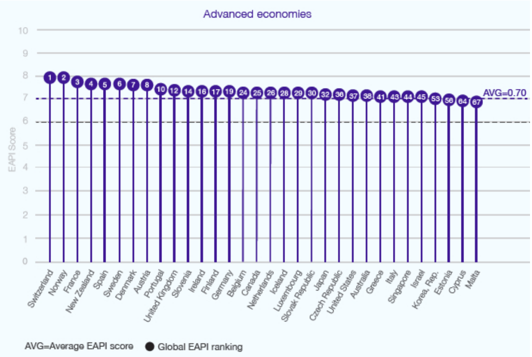
Other report cards for infrastructures: US vs others

US infrastructure lags behind

ICT | Energy architecture performance

Pillar 6: Availability and use of ICTs

Rank	Country/Economy	Score (1–7)
1	Sweden	6.5
2	United Kingdom	6.4
3	Finland	6.4
4	Netherlands	6.4
5	Denmark	6.4
6	Korea, Rep.	6.4
7	Norway	6.4
8	Singapore	6.2
9	Luxembourg	6.1
10	Japan	6.0
11	Hong Kong SAR	6.0
12	Estonia	6.0
13	United States	5.9



Sources: Government stats.; World Economic Forum Reports (WEF) reports, Courtesy of:

http://www.weforum.org/reports/global-energy-architecture-performance-index-report-2015

Bargaining theory for next gen CPS?

Importance of property rights allocation in Non-Coasian World

Coase [1960] : decentralized conditions for allocative efficiency
wrong conditions [& unlikely to hold]

Efficiency depends on allocation of rights

The means of allocating the rights

- dictatorial [centralized]
- via pricing (money) [mixed; as if de-centralized]
- via voting [de-centralized] (possibly unsolvable (Arrow))
- via bargaining [sort of de-centralized]

Resilient CPS: how to assign rights? who should assign the rights?

Assigning ownership & control rights for data. Balance of interests:
•data collector [utility] •data analytics [MDM] •customer