Economics for CPS researchers: from Arrow & Nash to non-Coasian worldview

Galina Schwartz[1]

[1]UC Berkeley

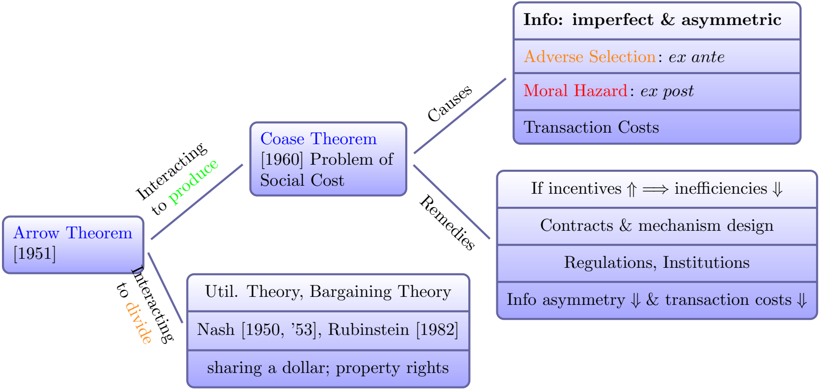
Talk 1& 2 at IPAM summer school
July 8, 2015

From Arrow, Coase, & Nash to CPS

Arrow theorem Coase theorem & Nash bargaining

- Preferences
- Arrow Impossibility Theorem
- From preferences to utility functions
- Cooperative bargaining theory [Nash bargaining solution]
- Coase Theorem
- Welcome to a non-Coasian World
- Non-cooperative bargaining theory [Stahl-Rubenstein]

Arrow impossibility theorem & its progenies



Arrow, Coase, Nash Bargaining [+ the tree of knowledge]

Preferences: Individual and Social

Literature

- K. Arrow, 1951. Social Choice and Individual Values. Wiley, NewYork
- J., Geanakoplos, 2005. Three brief proofs of Arrow's Impossibility Theorem. Econ Theory 26(1), 211-215. http://link.springer.com/article/10.1007%2Fs00199-004-0556-7

Further readings

- A. Gibbard, 1973. Manipulation of voting schemes: a general result, Econometrica 41 (4): 587-601.
- M. A. Satterthwaite, 1975. Strategy-proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions, Econ Theory 10: 187 - 217.
- E. Kalai, E. Muller, M. A. Satterthwaite, 1979. Social welfare functions when preferences are convex, strictly monotonic, and continuous, Public Choice 34-1, 87-97 http://link.springer.com/article/10.1007/BF00125755
- P. Ray, 1973. Independence of Irrelevant Alternatives, Econometrica. 41-5, 987-991. http://www.jstor.org/stable/1913820

The roots and their connections

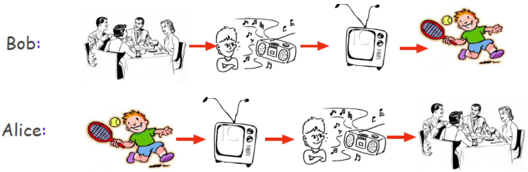
Literature

- R.A. Coase, 1960. The problem of social cost. Journal of Law and Economics, 1-44.
- J. Nash Jr, 1950. The Bargaining Problem, Econometrica, 18- 2, 155-162.
  http://www.jstor.org/stable/1907266
- J. Nash Jr, 1953. Two-Person Cooperative Games, Econometrica, 21-1, 128-140.
  http://www.jstor.org/stable/1906951
- A. Dixit and M. Olson, 2000. Does voluntary participation undermine the Coase Theorem?, Journal of Public Economics, 76-3, 309-335. http://www.sciencedirect.com/science/article/pii/S0047272799000894

Further readings

- A. Rubinstein, 1982. Perfect Equilibrium in a Bargaining Model. Econometrica 50-1, 97 - 109.
  http://www.jstor.org/stable/1912531
- M. Yildiz, 2011, Nash meets Rubinstein in final-offer arbitration, Economics Letters, 110- 3, 226-230,
  http://dx.doi.org/10.1016/j.econlet.2010.10.020
- J. Farrell, 1987. Information and the Coase Theorem, The Journal of Economic Perspectives, 1 - 2, 113-129.
  http://www.jstor.org/stable/1942984
-  P. Samuelson, 1995. Some uneasiness with the Coase Theorem, Japan and the World Economy, 7-1, 1-7.
  http://dx.doi.org/10.1016/0922-1425(94)00040-Z
- T. Palfrey & H. Rosenthal, 1984. Participation and the provision of discrete public goods. Journal of Public Economics, 24-2, 171-193. http://www.sciencedirect.com/science/article/pii/0047272784900239

Preferences of individuals



Preferences of an individual over alternatives

Preferences P_i (or $\succeq_i$) of an individual i over alternatives (x,y,...)

- I1 Preferences $\succeq$ of each individual are *complete* i.e., defined for all pairs of alternatives
- I2 *Strict* preference: if $x \succ y$ then not $y \succ x$
- I3 Ties are allowed, i.e., could be indifferent $x \sim y$
- I4 *Transitive*: if $x \succeq y$ & $y \succeq z$ then $x \succeq z$

Finite set of alternatives; finite set of individuals.

Preferences of the Society

How to aggregate individual preferences?

Definition (Social Preferences=Constitution)

Let a constitution be a function that associates N-tuple (or profile) of transitive individual preferences with another transitive preference. Constitution defines social preference ordering.

Definition (Dictator)

An individual is a dictator if for every strict preference that he has, society strictly prefers the same ordering.

Arrow Theorem: descriptive assumptions

(A1) The number of individuals is finite.

(A2) There are at least three social alternatives.

(A3) All combinations of individual preference orderings are admissible; for each combination a social preference ordering must exist.

(A4) If everyone prefers *a* to *b*, then *a* is socially preferred to *b*.

(A5) Let an individual have the same preference ordering for *a* and *b* in two combinations. Let the same be true for every individual. Then, the social preference between *a* and *b* shall be the same in the two cases.

(A6) There is no individual such that when he prefers one alternative to another, then the first is socially preferred to the second.

Arrow Theorem: Formal Assumptions

(A1) Society is a set of N individuals; $|N| \geq 1$

(A2) (Alternatives) $|A| \geq 3$

(A3) (Social Preference) societal ranking of all alternatives: societal preference profile π

(A4) (Unanimity) If $\forall n\ a \succeq_n b$ then $a \pi b$

(A5) (Independence of Irrelevant Alternatives) Societal ranking of a & b depends only on how individuals rank a & b

(A6) (No Dictator)

(A1) The number of individuals is finite.

(A2) There are at least three social alternatives.

(A3) All combinations of individual preference orderings are admissible; for each combination a social preference ordering must exist.

(A4) If everyone prefers a to b, then a is socially preferred to b.

(A5) Let an individual have the same preference ordering for a and b in two combinations. Let the same be true for every individual. Then, the social preference between a and b shall be the same in the two cases.

(A6) There is no individual such that when he prefers one alternative to another, then the first is socially preferred to the second.

Arrow (1951): Impossibility Theorem

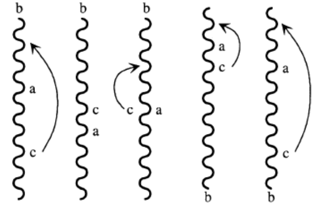
Theorem (Arrow: Version 1)

(A1) - (A6) cannot hold simultaneously.

Theorem (Arrow: Version 2)

Any constitution that respects transitivity, independence of irrelevant alternatives and unanimity is a dictatorship.

Arrow Theorem, the proof

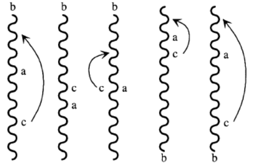


- (Extremal lemma) For an arbitrary alternative *b*, consider a profile in which all individuals put it at the very top or very bottom of his ranking (some on top, and some – on the bottom). Then, society must rank *b* this way too (top or bottom).

  Proof Assume the reverse: $a \succ_\pi b$ and $b \succ_\pi c$. Then, move *c* above *a* (without touching *b*). Then, the order of *ab* and *cb* is not affected (because *b* is in extreme position). From transitivity $a \succ c$, but unanimity now gives $c \succ_\pi a$, which is a contradiction.

Definition Individual $n^*(b)$ is pivotal (locally) if social pref. profile switches the ranking of *b* when n^* switches his (top to bottom).

Arrow Theorem, the proof (cont.)



- II Let n^* be pivotal (decisive). By unanimity, n^* exists. Let π_1 & π_2 be social profiles with b at the bottom & top. (exist from Lemma 1).
- III Then, n^* is a dictator for any pair a and c. Proof:
  Leave b untouched in the bottom. Assume being in π_2 (b ranked top). Let n^* change his prefs. s.t. $a \succ_{n^*} b \succ_{n^*} c$. Then, in new social pref. π_3, from IIR, we must have $b \succ_{\pi_3} c$ (remains as in π_2). Also, in π_3, we have $a \succ_{\pi_3} b$ ([as in π_1], since n^* is pivotal). Then, by transitivity, in π_3 we have $a \succ_{\pi_3} c$, i.e., n^* is a dictator.
- IV Last, show that n^* will be a dictator for all alternatives. Do the same (top-bottom construction) with c as with b. Then, there exists some (other?) dictator n^{**} for c, i.e., $n^{**}(c)$. From III, he is a dictator for ab. But from II, n^* can affect b. Thus, $n^* = n^{**}$, and Theorem is proven.

Other important results

Condorcet' Paradox

Even if individual preferences are transitive, social preferences may fail to be. (there might be cycles).

Gibbard–Satterthwaite Theorem

Consider preference reporting in a restrictive settings with only 2 players with interdependent preferences. Then, truthfulness is gone!

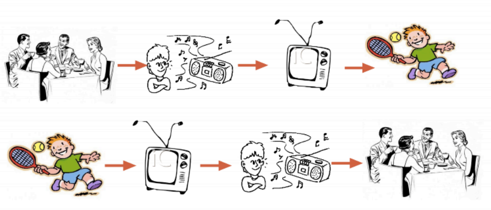
Kalai et. al.

The paper extends Arrow Impossibility results to convex and continuous preferences.

Arrow's results are robust.

Arrow Impossibility result & ramifications

Bob



Alice

From individual to societal preferences

1. Conflicts of individual and societal preferences
2. No easy & efficient aggregation of preferences
3. No truthful preference revelation [in general]
4. Arrow Theorem allows to connect information and incentives

Applying Arrow Impossibility to CPS

Modern CPS are complex systems with interdependencies

- more than 1 agent
- with more than 2 alternatives

In large scale CPS we expect

- conflicts of preferences cannot be fully remedied
- from preferences to objective functions [how? – standard result(s)]
- no truthful preference reporting
- no fully efficient mechanisms
- $\Rightarrow$ Efficiency quest: how to approach?

From Arrow to Nash

Nash: efficient allocation between two parties

Assumptions

- Perfect information
- Known player utilities
- Axioms N1- N5 [similar to Arrow (1951)]

Comments

- Nash constructs a unique solution
- His results generalize to N players
- Non-cooperative counterpart: Stahl-Rubinstein bargaining
- M. Yildiz (2011) – Nash & Rubinstein approaches reconciled

From preferences to utilities

Preferences of an individual

Let X denote a set of alternatives. A *preference relation P* is complete and transitive preferences on X.

- **complete** P ($\succ, \sim, \prec$)is defined for all pairs of alternatives
- **transitive** if xPy & yPz then xPz

A preference relation can be represented by a utility function $u : X \to \mathbb{R}$ in the following sense: $x \succeq y \iff u(x) \geq u(y) \forall x, y \in X$

Utility of an individual

Theorem (From ordinal preferences to ordinal utilities)

Let X be finite (or countable). A relation can be presented by a utility function if and only if it is complete and transitive.

Preferences to utilities: ordinal

Finite (or countable) case

Let $U : X \to \mathbb{R}$ represent some preference relation. Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly increasing function. Then, $f \cdot U$ also represent this preferences.

Continuous case

Theorem (Continuous ordinal utility representation)

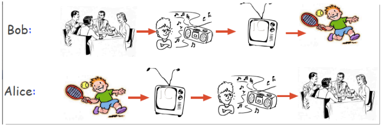
Assume the set of alternatives X is a compact, convex subset of a separable metric space. A preference relation has a continuous ordinal representation if and only if it is continuous.

Definition Pref. relation P is continuous, if for any $a \succ b$ there exists balls B_a and B_b, s.t. for all $x \in B_a$ and $y \in B_b$, pref. order remains the same: $x \succ y$

Remark Let P be continuous; $x' \succ x \succ x''$. Then for any continuous $u : [0,1] \to X$ with $u(1) = x'$ & $u(0) = x''$, there exists $t \in [0,1]$ s.t. $u(t) = x$.

An example: Alice's utility function

Ordinal



Possible utilities for Alice: Ordinal vs Cardinal

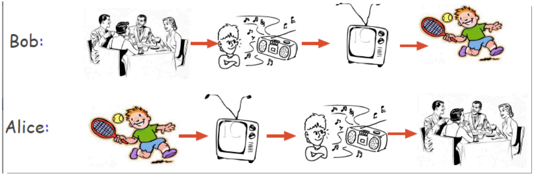
Cardinal

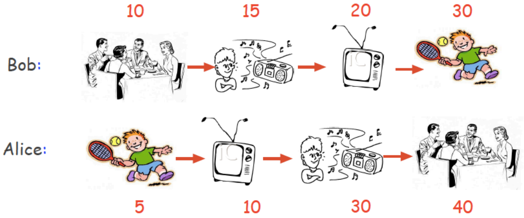
5	10	30	40
50	100	300	400
5	6	7	8

Preferences = ranking of alternatives (higher, lower, indifferent)
Transitive prefs. = mapping to ordinal utilities exists
Cardinal prefs. = values assigned: "prefer by how much" [numeraire (money)]

An example of utility functions for Alice and Bob

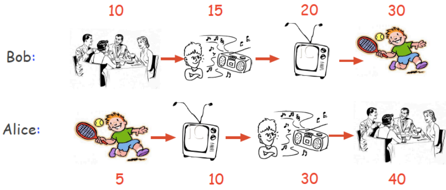
How to introduce exchanges between Alice and Bob?





A canonical setup of bargaining problem

An asset (surplus)



Surplus sharing problem: bargaining approach

v	Buyer's valuation
c	Seller's cost
S	Surplus, $S = v - c$
p	Price

Trade can happen at any $p \in [c, v]$.

Nash bargaining solution*** *No relation with Nash equilibrium*

Unique p exists (under very stylized axioms).

Nash Bargaining Program: Axioms

(N1) Bargaining Problem $\mathcal{B} = (U, d)$ between 2 players defined on a compact convex payoff space U, with d being disagreement point.

(N2) Pareto [PAR] Bargaining solution $f(\cdot)$ satisfies Pareto property

(N3) Symmetry [SYM] Symmetric players = symmetric solution

(N4) Linear invariance [LIN]: Independence of Utility Origin [IUO] and units [IUU]

(N5) Independence of Irrelevant Alternatives [IIA] Consider $\mathcal{B}$ with payoff space U, disaggrement point d, and solution $\bar{u}$. Obtain $\tilde{\mathcal{B}}$ from $\mathcal{B}$ by restricting U to $\tilde{U} \subset U$, s.t. $d \in \tilde{U}$ and $\bar{u} \in \tilde{U}$. Then, the solution $\bar{u}$ is also the solution of $\tilde{\mathcal{B}}$.

(N1) $\mathcal{B} = (U, d)$: utility possibility set U and the status-quo (threat point / disagreement)d

(N2) PAR Cannot improve one player utility without negative effect on the opposing player

(N3) SYM Solution is symmetric if players are symmetric: identical utilities and $d_1 = d_2$.

(N4) LIN Invariant to an affine transformation of utility function: $f(\cdot)$ is independent of utility origin and units

(N5) The solution of an original bargaining game remains the solution of a new game defined on a subset of the original game, if the subset contains the original disagreement point and the original solution.

Nash Bargaining Program: main result

Proposition (Nash bargaining solution)

Assume that the solution function $f(\cdot)$ satisfies Axioms N2 - N5 (i.e., PAR, SYM, LIN and IIA). Then, Nash bargaining solution is the only solution of the problem $\mathcal{B}$

Nash product.

Nash shows that solving the problem is equivalent to solving:

$$\max_{u_1, u_2} = (u_1 - d_1)(u_2 - d_2), \text{ s.t., } u_1, u_2 \in U.$$

Let (u_1^*, u_2^*) be max. Then,

$$f_1(U, d) = u_1^* \text{ and } f_1(U, d) = u_2^*.$$

Alice and Bob engage in Nash Bargaining

Nash bargaining solution

v	Buyer valuation
c	Seller cost
$d = 0$	if no trade

$$U_S(p-c) = (p-c)^{\alpha}$$

$$U_B(v-p) = (v-p)^{\beta}.$$

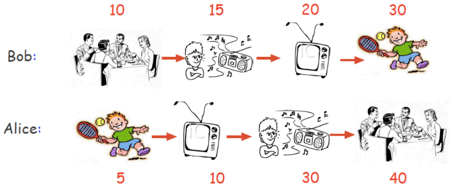
Nash product

$$(p-c)^{\alpha}(v-p)^{\beta}, \text{ with } p \in [c, v].$$

The price p^* of exchange:

$$p^* = v\frac{\alpha}{\alpha+\beta} + c\frac{\beta}{\alpha+\beta}.$$

Ceteris paribus, smaller α(or β) moves p closer to c(or v).



Applying Nash product (dinner)

40	Alice valuation
10	Bob's valuation
S	Surplus $S = 40 + 10$

$$\max_p (40-p)(10+p), \text{ with } \alpha = \beta = 1$$

$$p^* = 15 \ \& \ U_{Alice} = U_{Bob} = 25.$$

Arrow Theorem with formal assumptions: a reminder

Theorem (Arrow Impossibility)

Any constitution (social preference ordering) that respects transitivity, independence of irrelevant alternatives (IIA) and unanimity is a dictatorship.

- (A1) Society = a finite set $\mathfrak{N}$ of individuals$\{1, ..., n, ..., N\} = \mathfrak{N}$; with $N \in \mathbb{N}$ and $N \geq 1$.
- (A2) A set of alternatives $\mathfrak{A}$ with $|\mathfrak{A}| > 2$; Individual preferences aP_nb defined $\forall a, b \in \mathfrak{A}$.
- (A3) Societal preference profile π ranks all alternatives
- (A4) Unanimity: If $\forall n \in \mathfrak{N}$, we have $aP_nb \Rightarrow a\pi b$
- (A5) Independence of Irrelevant Alternatives: [IIA] $\leftarrow$
  $a\pi b$ depends only on aP_nb
- (A6) No Dictator

(N5) [IIA]: The solution of original bargaining game is the solution of a new game defined on a subset of the original game, if the subset contains the original disagreement point and the original solution.

Independence of irrelevant alternatives (IIA) Arrow & Nash: different meanings

Deceptive similarity: IIA Arrow $\neq$ IIA Nash

(A5) IIA Arrow: Societal ranking of *a* & *b* depends only on how individuals rank *a* & *b* (not on how they rank other alternatives).

(N5) IIA Nash: The solution of original game remains the solution of a new bargaining game defined on a subset of original game, if this subset contains the original disagreement point and solution.

The difference of Arrow & Nash IIA

Arrow IIA: same set of alternatives; the rankings may change.

Nash IIA: the set of alternatives changes, the utility function does not.

P. Ray, 1973. Independence of Irrelevant Alternatives, Econometrica. 41-5, 987-991.

Coase Theorem: a formulation

Theorem (Coase)

With (C1) - (C2), property rights allocation and legal liability assignment are irrelevant.

- C1 Information is perfect
- C2 Transaction costs are zero

Critical thoughts about Coase Theorem

- Is Coase Theorem sound (mathematically)?

How Arrow & Coase theorems relate?

Question: How to achieve Coasian efficiency?

How to re-assigns the rights? Nash Bargaining solves the re-assignment (efficiently). Could that help to refute Coase theorem?

A short answer

If Nash could bring mathematical validity to Coase, he would destroy Arrow Th. (for 2 players under C1-C2). But C1-C2 have no effect on the Arrow's proof. If Arrow Th. is correct, Coase Th. must be wrong.

- Indeed ... Coase Th. was shown to fail [by many papers].
- Dixit & Olson (2000) ⟵ discuss now

Arrow theorem retains its robustness. And...despite being wrong, Coase theorem retains its importance.

Dixit & Olson (2000): public good provision with volunteer participation [draw on Palfrey & Rosenthal (1984)]

N	identical players
V	per person benefit
$V \times N$	societal benefit
C	cost of public good
IN/OUT	participate or not in financing
n	number of players IN

"Everyone here? Good. Meeting topic: Setting world record for shortest meeting. All in favor say aye. Ayes have it. Meeting over."

Dixit & Olson (2000): on route to an equilibrium

	IN	OUT
		
Payoffs	$V - \frac{C}{n}$	V [free riders]

$$(M-1)V < C < MV, M \text{ is min \# of IN players to produce} : n \geq M$$

Proposition

If $M < N$, only mixed equilibrium exists in which the public good is produced.

Dixit & Olson (2000): Herb's optimization

Let P be the prob. that Herb chooses IN (participates in bargaining).

$$\text{If IN} \sum_{n=M}^{N} P^{n-1}(1-P)^{N-n} \frac{(N-1)!}{(n-1)!(N-n)!} \left[V - \frac{C}{n} \right],$$

$$\text{If OUT} \sum_{n=M}^{N-1} P^{n}(1-P)^{N-1-n} \frac{(N-1)!}{n!(N-n-1)!} V$$

Dixit & Olson (2000): Computing equilibrium

To find the equilibrium probability P

$$\sum_{n=M}^{N} P^{n-1}(1-P)^{N-n}\frac{(N-1)!}{(n-1)!(N-n)!}\left[V-\frac{C}{n}\right]$$
$$=\sum_{n=M}^{N-1} P^{n}(1-P)^{N-1-n}\frac{(N-1)!}{n!(N-n-1)!}V$$

$$\frac{b(N,M,P)}{\sum_{n=M}^{N} b(N,M,P)}=\frac{C}{MV},$$

numerator – exactly M successes, denominator – M or more successes:

$$b(N,M,P)=P^{M}(1-P)^{N-M}\frac{N!}{M!(N-M)!}.$$

Dixit & Olson (2000): numerical results

Equilibrium probabilities Q of successful financing the public good

Table 2
Individual probability, P, of choosing IN, and cumulative probability, Q, of success with $M = 2$

	$C = 1.1$		$C = 1.5$		$C = 1.9$	
N	P	Q	P	Q	P	Q
3	0.71	0.80	0.50	0.50	0.13	0.51×10^{-1}
6	0.31	0.59	0.18	0.28	0.37×10^{-1}	0.19×10^{-1}
15	0.11	0.51	0.59×10^{-1}	0.22	0.12×10^{-1}	0.13×10^{-1}
60	0.27×10^{-1}	0.48	0.14×10^{-1}	0.20	0.26×10^{-2}	0.11×10^{-1}

Table 3
Individual probability, P, of choosing IN, and cumulative probability, Q, of success with $M = 50$

	$C = 49.1$		$C = 49.5$		$C = 49.9$	
N	P	Q	P	Q	P	Q
60	0.84×10^{-1}	0.60×10^{-43}	0.49×10^{-1}	0.97×10^{-55}	0.10×10^{-1}	0.11×10^{-88}
100	0.18×10^{-1}	0.27×10^{-58}	0.10×10^{-1}	0.10×10^{-70}	0.20×10^{-2}	0.26×10^{-105}
150	0.91×10^{-2}	0.74×10^{-62}	0.51×10^{-2}	0.23×10^{-74}	0.10×10^{-2}	0.48×10^{-109}
250	0.46×10^{-2}	0.56×10^{-64}	0.25×10^{-2}	0.16×10^{-76}	0.51×10^{-3}	0.28×10^{-111}

Dixit & Olson (2000): repeated versions of the game

Finitely repeated game

If more than 1 meeting is allowed: even lower chances of financing (finite number of repetitions). Efficiency fails strikingly.

Infinitely repeated game

Lastly, assume the game is repeated infinitely many times (and $\delta \to 1$). Then, still no luck.

Dixit & Olson [D&O2000]: Nash barg. meets Herb

Public goods provision: not even close to Coasian efficiency

- D&O2000: Financing public goods in Coasian setting.

  To account for incentives, D&O2000 allow free participation.

- Public good provision eludes Coasian efficiency.
- Remedies: bureaucratic restrictions (to improve participation)

  This interferes with (unrelated player) choices. [anti-Coasian in spirit]

Arrow theorem holds under perfect information; Coase – does not.

A mere possibility of non-participation ==> enormous inefficiencies.

Dark forces of Arrow impossibility theorem overpower Nash cooperation

Mandatory impositions are needed in the domain of public goods.

Coase Theorem???

Coase Theorem is methodologically invaluable

Definition

Coasian world is a hypothetical environment where Coase Theorem holds (miraculously).

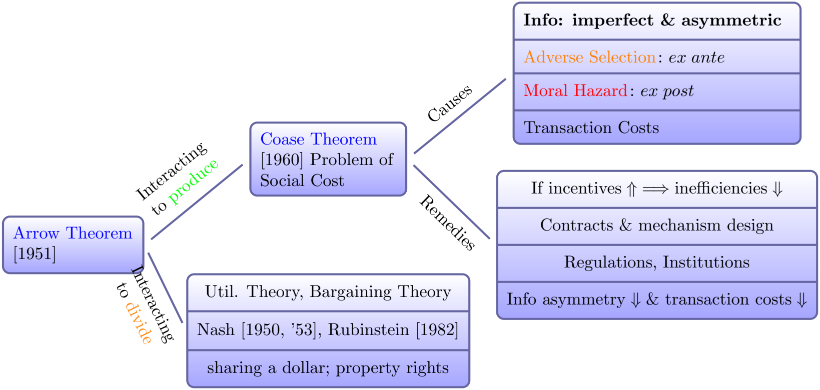
Coasian world: Benchmark

Coasian world is a useful social welfare benchmark.
Conditions C1 - C2 almost never hold. The further the departure from C1-C2, the higher the welfare gap with Coasian benchmark.
Improve efficiency $\Leftrightarrow$ reduce welfare gap

- improve information
- reduce transaction costs

Caveats apply [Counterexamples to Coase theorem]

Arrow impossibility theorem & its progenies



After-math of Coase Theorem: the branches

How these past century news matter NOW?

From Arrow, Nash and Coase to .... ?

- Arrow Theorem [1951]: robust impossibility of constructing well-behaved social preferences for well-behaved individual ones
- Efficient bargaining solution
  cooperative: Nash [1950, '53] ; strategic: Stahl[1971]-Rubinstein [1982]
- Coase [1960] : decentralized conditions for allocative efficiency
  wrong conditions [& unlikely to hold]
- Efficiency depends on allocation of rights $\longleftarrow$ Tomorrow
  Agency / contract theory: studies such environments

Summary

Ubiquitous inefficiencies in large scale (public) systems

HOW TO [eat the cake and keep it?]

- Trade-offs between inefficiencies: no global efficiency
- Public goods provision $\Leftrightarrow$ mandatory impositions

Samuelson on public goods

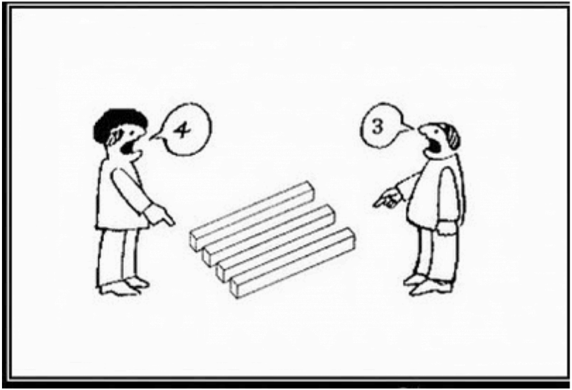
My point is that the same thing applies to legal assignment of property rights. We have paradoxes in public-utility economics. In the Tragedy of the Open Road we encounter the Braess Paradox, in which supplying a new road segment leads under laissez-faire to a slower ride for everybody. We can contrive, in this instance, a variety of alternative property rights that would make for Pareto-Optimality. (One could be: give the State a right to impose

6 *P. Samuelson / Japan and the World Economy 7 (1995) 1–7*

tolls; but constrain in the property-grant the right of the bureaucrats to deviate from the just-right pattern of tolls. Or contrive an equivalent Dutch Auction to pick a private monopolist with effectively the same property rights.) But in the time it takes me to write this paragraph, exogenous change and dynamic endogenous change will cause the data of the problem to change in such a way as to entail a different set of property rights. If the Social Contract makes all earlier property rights capable of being abrogated to adjust to new Pareto-Optimal needs, we are in the old quagmire of moral hazard, inverse externalities, and so forth. No doubt wise Twenty-First-Century Hayeks or Lerners will come forward to give persuasive counsel on how a Mixed Economy will want to compromise among divergent instrumentalities so as to optimize various definitions of social and individual welfares.

Engineers and Economists: the similarities

Engineers and Economists



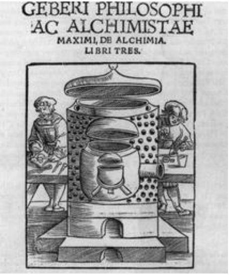
are almost identical
both professions are quantitative

Not so recent history: the alchemists

Science is ~~not~~ magic: Jabir (Geber) ibn Hayyan

He who performs not practical work nor makes experiments will never attain to the least degree of mastery.

Jabir (Geber) ibn Hayyan, B. 821AD



The theory:
Base metals consist of
- sulfur & • mercury

The elixir = turns (base) metals into gold

Elixir = catalist

- ★ If red, it turns base metals into gold
- ★ If white, it turns them into silver

History of the philosophers' stone

The philosophers' stone

There exists in Nature a certain pure substance, which when discovered and brought by art to its perfect state, will convert to perfection all imperfect bodies that it touches.

Arnold of Villanova, Spanish alchemist (14th century)

- If red, it turns base metals into gold
- If white, it turns them into silver

Like unicorn, the philosophers' stone has all manner of striking qualities – except existence. ... Without it, chemistry would not be what it is today.
... But then, chemistry was essentially a murky business.

Strathern, Paul. 2001. Mendeleyev's Dream: The Quest for the Elements. New York: St. Martins Press, p. 58-59.

A search for perfection continues

Human aspiration: a search of perfection

- The elixir of life (China [600AD], Greece [300AD]) $\Longrightarrow$
- (Al-íksir) elixir on Arabic [8 century "Islamic golden age"] $\Longrightarrow$
- The philosopher's stone [11 century] $\Longrightarrow$
- The quest of Coasian world [20 century] $\Longrightarrow$
- Efficiency [mantra of 21 century?]

★ today's gold = efficiency

★ today's silver = perfect information & zero transaction costs.

Like unicorn, the Coasian world has all manner of striking qualities – except existence. ... Without it, economic science would not be what it is today.
... But ... getting to Coasian world is a murky business.