Cascades in networks: a simple theory and applications

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Competitive cascades

Motivation

- Rumors can spread quickly through social networks.
- Examples: political rumors; firms worry about the reputations of their products.
- In the short run, rumors are **irreversible**.
- The **initial seeds** in the social network really matter for what rumor spreads.

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Previous work vs. this talk

- Several recent papers on rumor spread Moreno et al. (2004); Kostka et al. (2008); Trpevski et al. (2010); He et al. (2012); Weng et al. (2012).
- Rumor spread is similar to work on product diffusion: Goyal and Kearns (2012); Bimpikis, Ozdaglar, and Yildiz (2014) (...and Hotelling, 1929)
- Quality and seeding: Fazeli and Jadbabaie (2012a,b,c); Fazeli, Ajorlou, and Jadbabaie (2014).
- Other papers where consumers can switch products many times: Bharathi, Kempe, and Salek (2007); Alon, Feldman, Procaccia, and Tennenholtz (2010); Apt and Markakis (2011); Simon and Apt (2012); Tzoumas, Amanatidis, and Markakis (2012); Borodin, Braverman, Lucier, and Oren (2013); Apt and Markakis (2014); Mei and Bullo (2014).

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- We study the game on the network using **cascade centrality**.
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Model

Preliminaries

- Simple, undirected graph $G(V, E)$.
- A *spreading threshold* for agent i is a random variable Θ_i drawn from a probability distribution with support $[0, 1]$.
- The associated multivariate probability distribution for all the agents in the graph is $f(\boldsymbol{\theta})$.
- Each agent is $i \in V$ assigned a threshold θ_i. Let's define the threshold profile of agents as $\boldsymbol{\theta} := (\theta_i)_{i \in V}$. A **network** G_θ is a graph endowed with a threshold profile.

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Seeding

- Two firms: A spreads rumor a and B spreads rumor b. Rumors are incompatible.
- The state of agent i at time t is denoted $x_i(t) \in \{0, a, b\}$.
- Denote by $S_t^A(G_\theta)$ and $S_t^B(G_\theta)$ the sets of **new** spreaders of rumor a and b in network G_θ at time t resp.
- At time $t = 0$, $x_i(0) = 0$ for all i, and each firm simultaneously chooses **one** agent $S_0^A, S_0^B \in V$ as a seed for their rumor. Overlap in seed sets resolved randomly.

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Linear threshold process dynamics

- Any agent who has not spread any rumor by some period t, **decides to spread a rumor** in time period $t+1$ iff

$$\frac{\#\text{ friends who adopted } a + \#\text{ friends who adopted } b}{\#\text{ friends}} \geq \theta_i$$

i.e. Granovetter's linear threshold model.

- If the threshold is reached, the **probability of spreading rumor** a is

$$\frac{\#\text{ friends who adopted } a \text{ at } t}{\#\text{ friends who adopted } a \text{ at } t + \#\text{ friends who adopted } b \text{ at } t}$$

- Agents use the **latest** spreaders to select the rumor, but **total** spreaders to decide to spread.
- Once an agent spreads rumor a, he remains in state a in all subsequent periods.
- This process converges to a random set: eventual spreaders S^A of rumor a and S^B of rumor b.

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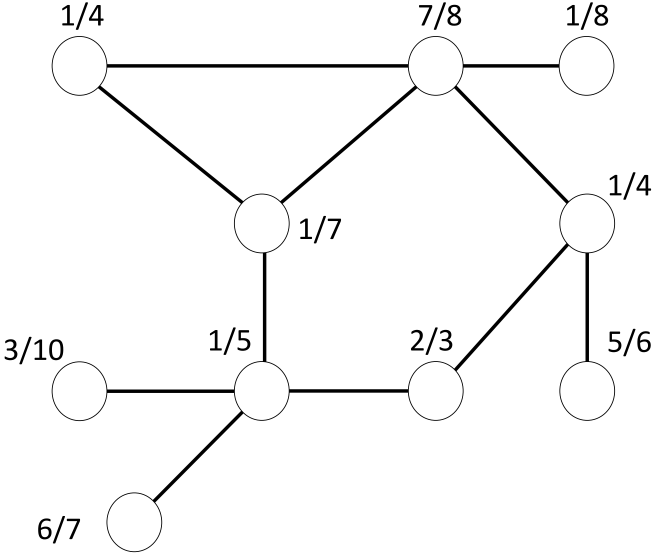
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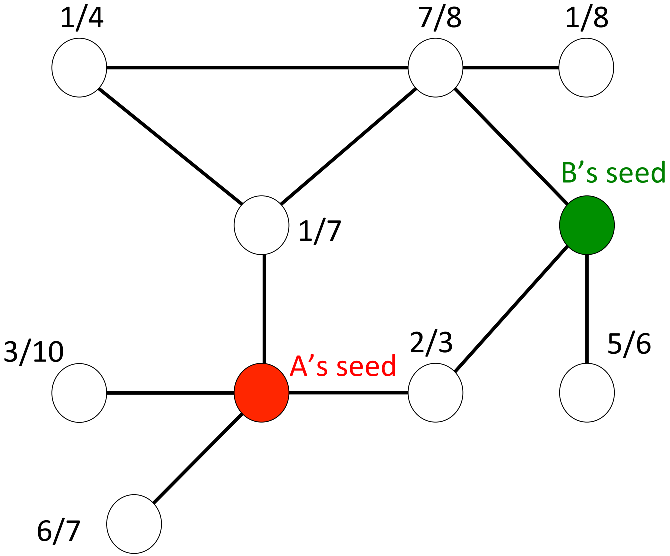
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B's seed
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A's seed
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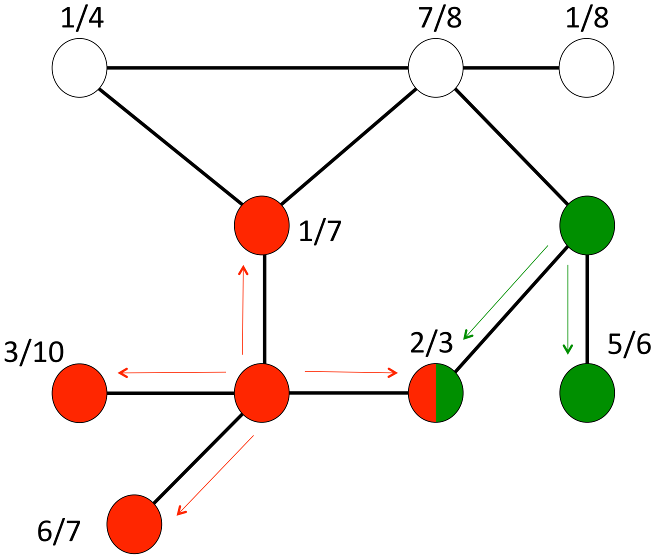
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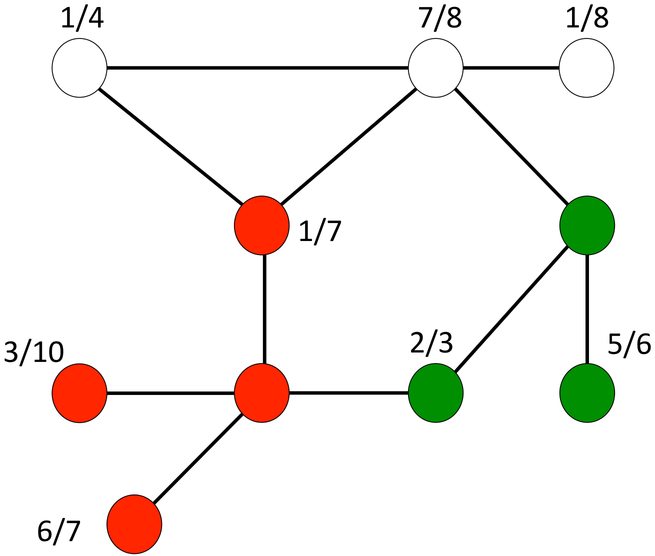
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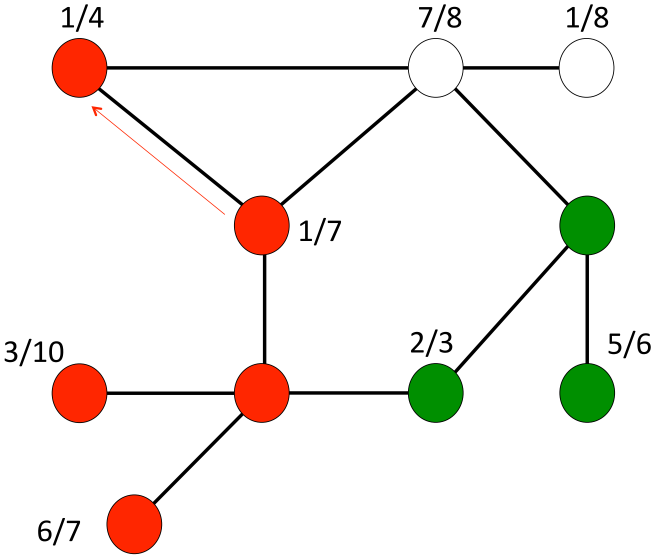
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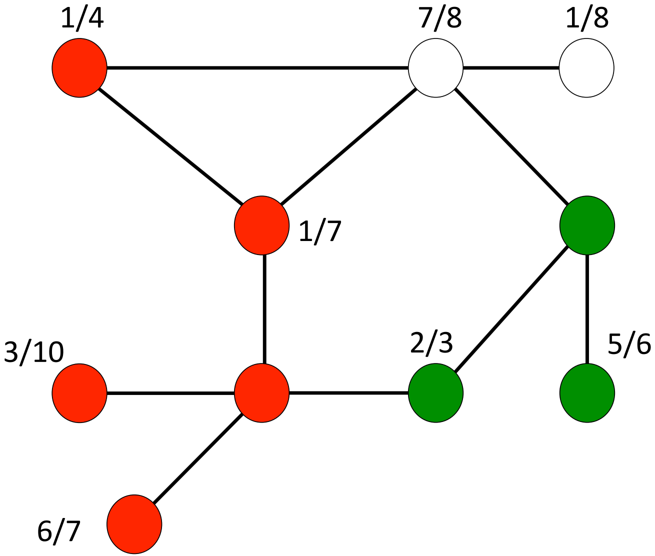
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Expected number of spreaders

- Fixing seeds S_0^A and S_0^B and a graph G, and re-run the process by drawing the agents' thresholds from $f(\theta)$ each time.
- Denoting the probability of any agent spreading rumor a is

$$\mathbb{P}_i^A(G, S_0^A, S_0^B) = \int_{\mathbb{R}^n} |S^A(G_\theta, S_0^A, S_0^B) \cap \{i\}| f(\theta) d\theta$$

- Expected number of spreaders of rumor a is

$$\mathbb{E}[S^A(G, S_0^A, S_0^B)] = \int_{\mathbb{R}^n} |S^A(G_\theta, S_0^A, S_0^B)| f(\theta) d\theta = \sum_{i=1}^{n} \mathbb{P}_i^A(G, S_0^A, S_0^B)$$

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Uniform thresholds

Assumption

For any $G_{\boldsymbol{\theta}}$ and every $i \in V$, $\Theta_i \sim \mathcal{U}(0,1)$ and independent.

Cascade centrality

Definition

Cascade centrality of node i in graph G is the expected number of spreaders of rumor a in that graph given i is the seed and firm A is a monopolist, namely

$$\mathcal{C}_i(G) := \mathbb{E}[S^A(G,\{i\})] = 1 + \sum_{j \in V \setminus \{i\}} \mathbb{P}_j^A(G,\{i\}) = 1 + \sum_{j \in V \setminus \{i\}} \sum_{P \in \mathcal{P}_{ij}} \frac{1}{\chi_P}$$

Cascade centrality

Theorem

The cascade centrality of any node i in G is:

$$\mathcal{C}_i(G) = 1 + d_i - \sum_{j \in V \setminus \{i\}} \sum_{L \in \mathcal{L}_{ij}} \frac{1}{\chi_L}$$

where χ_L is the degree sequence product along a loop.

Game: uniform thresholds

- Action space of firms A and B: $\Sigma := \Sigma_A \times \Sigma_B := V \times V$
- Action profile $\sigma := (\sigma_A, \sigma_B)$ is simply a pair of nodes.
- Payoff profile: $\pi := (\pi_A(\sigma), \pi_B(\sigma))$ is the expected number of spreaders of rumors a and b.

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Game: uniform thresholds

- For $i \neq j$, let us denote $\Xi(i,j)$ as the set of all paths that begin at i and include (but do not necessarily end) at j.

Proposition

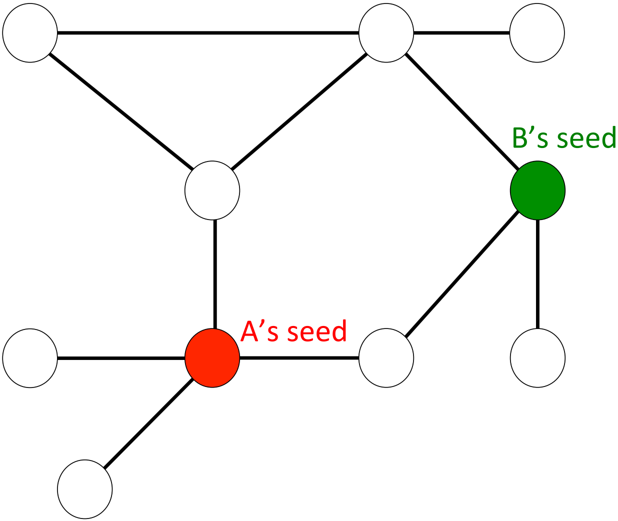
The expected number of spreaders of rumor a (i.e. firm A's payoff) is

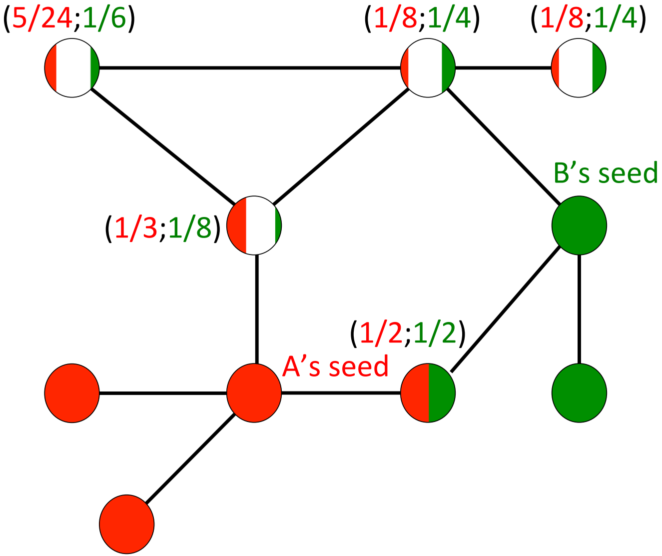
$$\pi_A(\sigma_A, \sigma_B) = \begin{cases} \frac{\mathcal{C}_{\sigma_A}}{2} & \text{if } \sigma_A = \sigma_B \\ \mathcal{C}_{\sigma_A} - \epsilon(\sigma_A, \sigma_B) & \text{if } \sigma_A \neq \sigma_B \end{cases}$$

where

$$\epsilon(i,j) = \sum_{P \in \Xi(i,j)} \frac{1}{\chi_P}$$

It's a constant-sum game.


B’s seed
A’s seed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B's seed
(1/3;1/8)
(1/2;1/2)
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PSNE

- The game is defined as $\Gamma := (\Sigma, \pi)$.

Definition

A profile of actions $\sigma^* := (\sigma_A^*, \sigma_B^*) \in \Sigma$ is a pure-strategy Nash equilibrium if:

- $\pi_A(\sigma_A^*, \sigma_B^*) \geq \pi_A(\sigma_A, \sigma_B^*)$ for all actions $\sigma_A \in \Sigma_A$
- $\pi_B(\sigma_A^*, \sigma_B^*) \geq \pi_B(\sigma_A^*, \sigma_B)$ for all actions $\sigma_B \in \Sigma_B$

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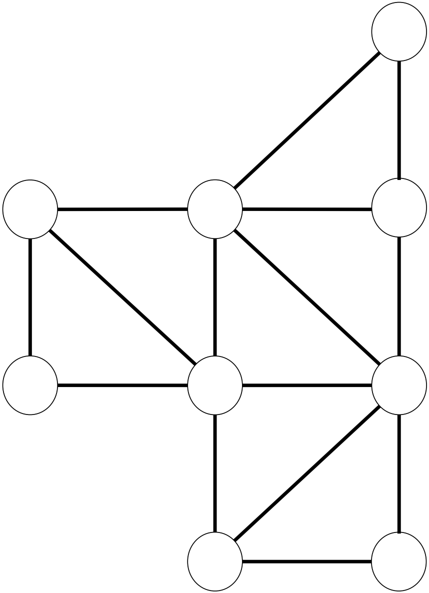
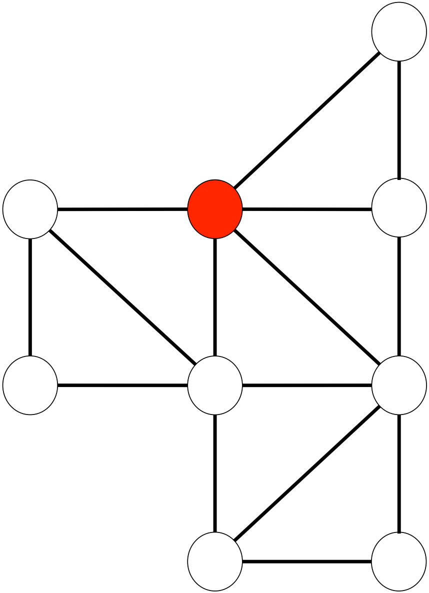
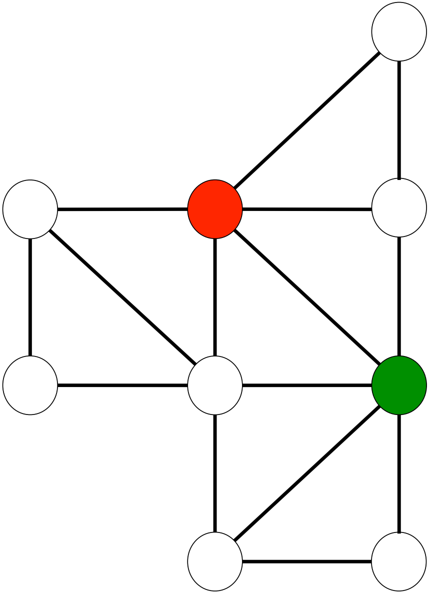
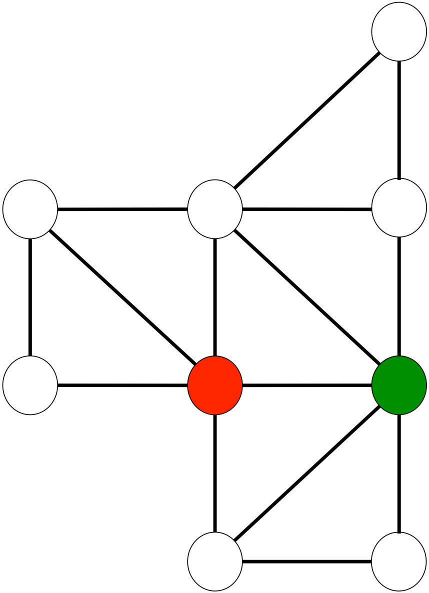
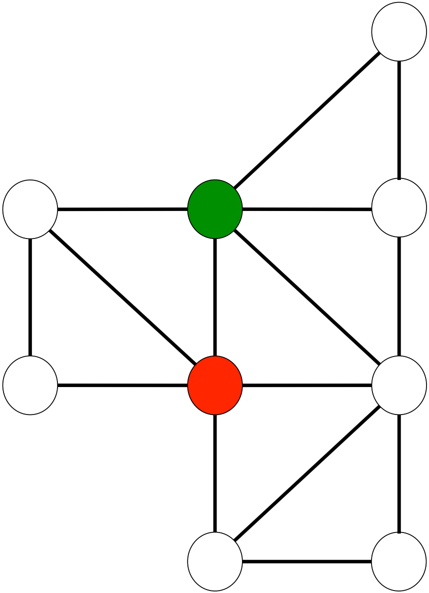
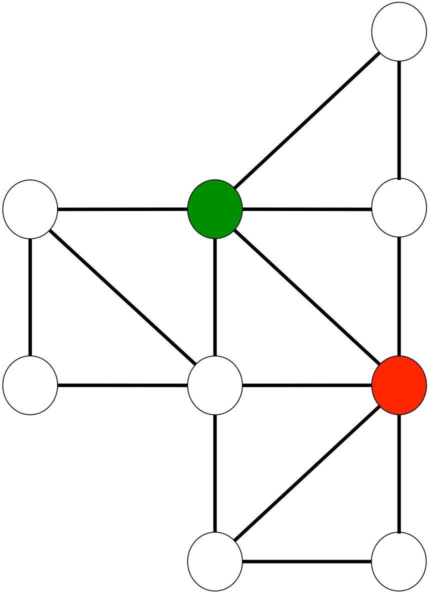












PSNE: existence characterization

Theorem

Consider a duopoly with unit budgets Γ. *Then* Γ *admits at least one PSNE if and only if either:*

1. *There exists* $i \in V$ *such that, for any* $j \in V \setminus \{i\}$*:*

- $$\frac{\mathcal{C}_i}{\mathcal{C}_j} \geq 2 - 2 \cdot \left(\frac{\epsilon(j,i)}{\mathcal{C}_j} \right)$$

then there exists a $\sigma^* = (i, i)$ *PSNE, or...*

PSNE: existence characterization

Theorem

Consider a duopoly with unit budgets Γ. *Then* Γ *admits at least one PSNE if and only if either Condition* **1** *is satisfied or*

2. *There exist* $i, j \in V$ *such that,* $\mathcal{C}_i \geq \mathcal{C}_j$ *and for any* $k \in V \setminus \{i, j\}$*:*

- $\dfrac{\mathcal{C}_i}{\mathcal{C}_k} \geq 1 + \dfrac{\epsilon(i,j) - \epsilon(k,j)}{\mathcal{C}_k}$
- $\dfrac{\mathcal{C}_j}{\mathcal{C}_k} \geq 1 + \dfrac{\epsilon(j,i) - \epsilon(k,i)}{\mathcal{C}_k}$
- $\dfrac{1}{2} + \dfrac{\epsilon(i,j)}{\mathcal{C}_j} \leq \dfrac{\mathcal{C}_i}{\mathcal{C}_j} \leq 2 - 2 \cdot \left(\dfrac{\epsilon(j,i)}{\mathcal{C}_j}\right)$

in which case there exists a $\sigma^* = (i, j)$ *(and* $\sigma^* = (j, i)$ *by symmetry) PSNE.*

Budget multiplier

Definition

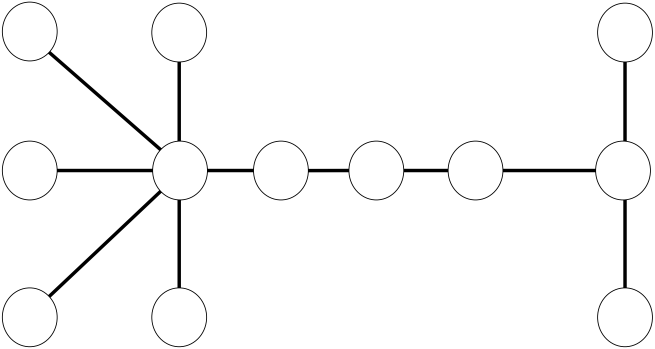
For arbitrary integer budgets $\mathcal{B}_A$ and $\mathcal{B}_B$, the budget multiplier is defined as:

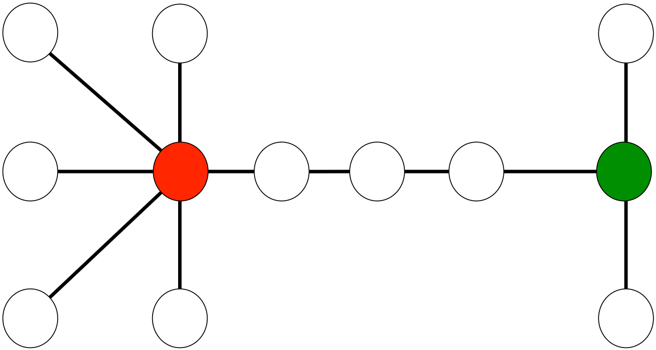
$$\mathrm{BM}\,(\Gamma) := \max_{\sigma \in \Sigma^*} \frac{\pi_A(\sigma)/\pi_B(\sigma)}{\mathcal{B}_A/\mathcal{B}_B}$$

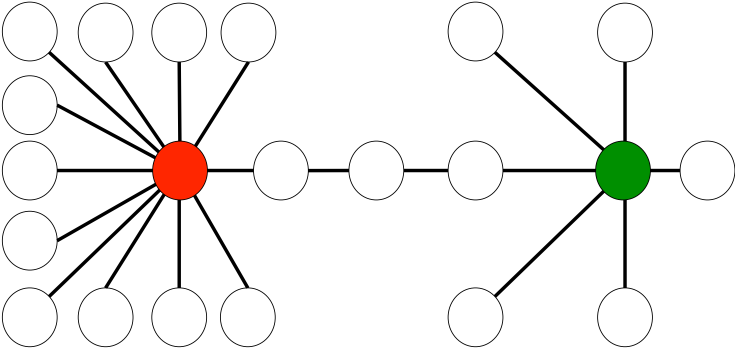
Theorem

For any Γ that admits at least one PSNE,

$$1 \leq BM < 2$$







Price of anarchy

Social planner's objective: $Y(\sigma) := \pi_A(\sigma) + \pi_B(\sigma)$ (i.e. firms' total payoffs).

Definition

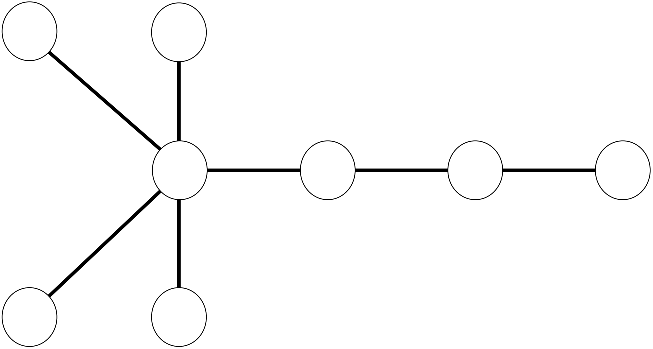
Price of Anarchy is defined as:

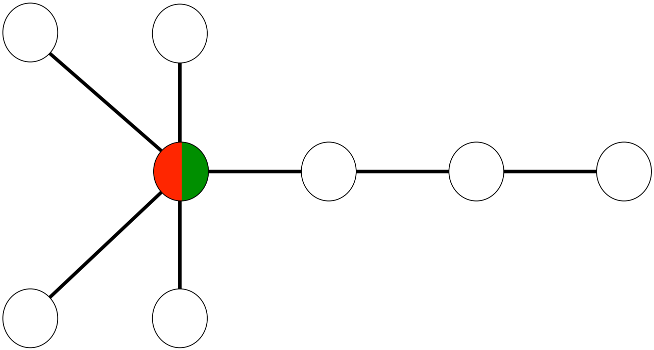
$$\mathrm{PoA}(\Gamma) = \frac{\max_{\sigma \in \Sigma} Y(\sigma)}{\min_{\sigma \in \Sigma^*} Y(\sigma)}$$

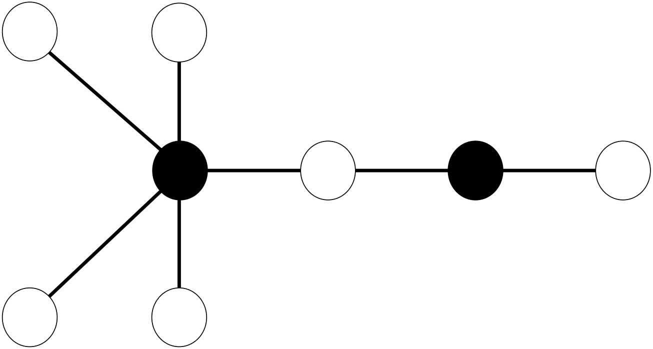
Theorem

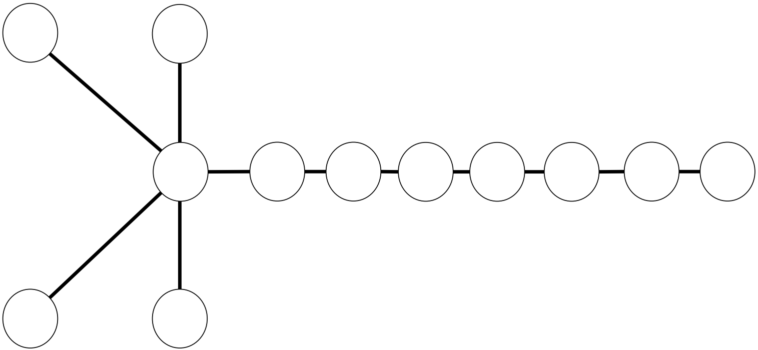
For any Γ that admits at least one PSNE,

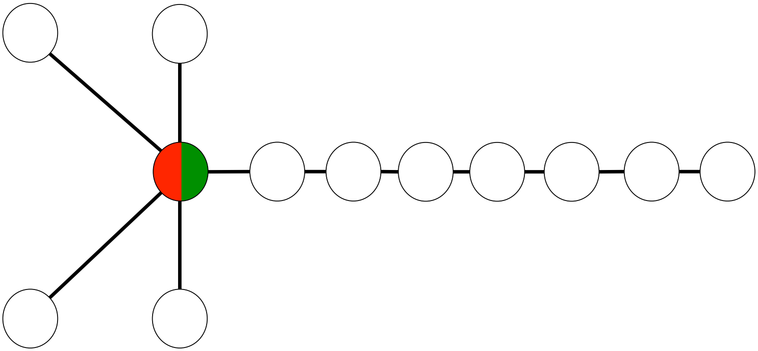
$$1 \leq PoA(\Gamma) < 1.5$$

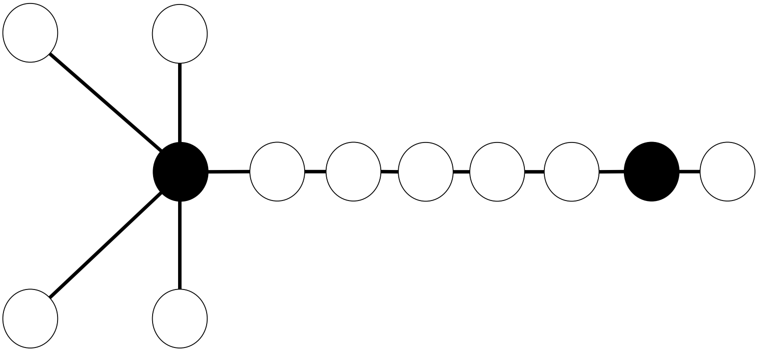












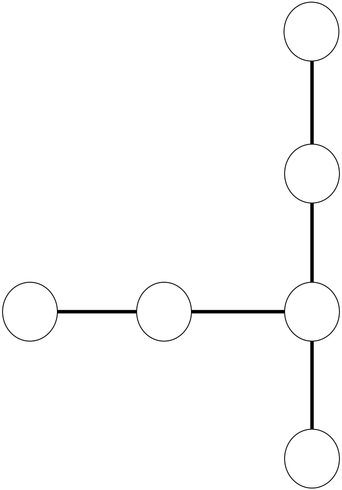
Trees

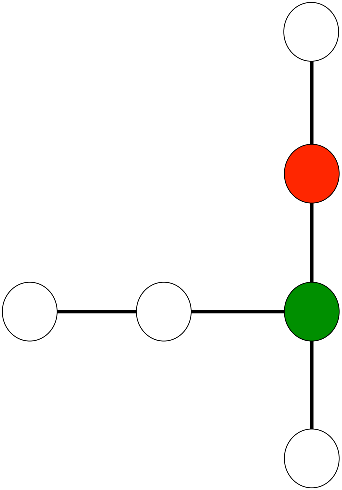
Proposition

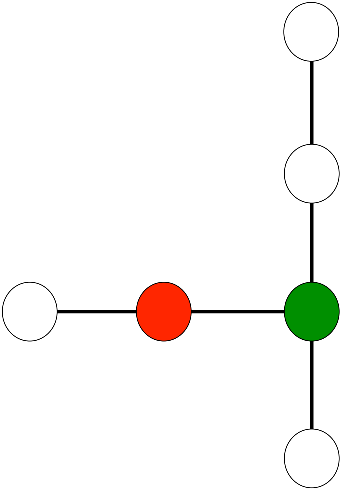
Suppose G is a tree. Then Γ admits at least one PSNE.

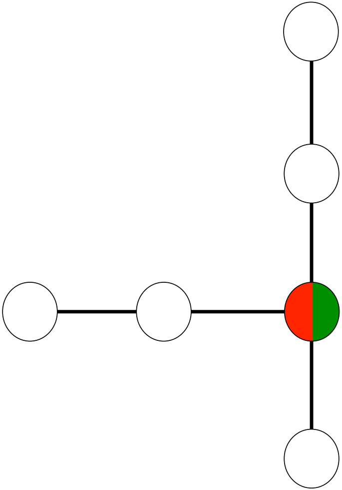
Trees

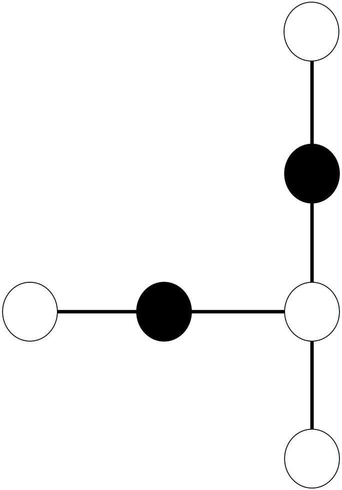
- We provide necessary and sufficient conditions only on the largest and second largest degree of the trees such that:
    - all PSNEs are efficient
    - no PSNEs are efficient
    - at least one PSNE is efficient
    - at least one PSNE is inefficient
    - there is at least one efficient and one inefficient PSNE











Conclusions

- Using a new notion of *cascade centrality*, we analyzed a tractable cascade process on general networks.
- We applied these insights to studying competitive diffusion.
- The competition model can be extended in a few ways (different strength of rumors, sequential entry).

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Future research questions

- Multiple seeds
  - Hard because you need to figure out the optimal seeding problem first.
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