How do networks form? Strategic network formation

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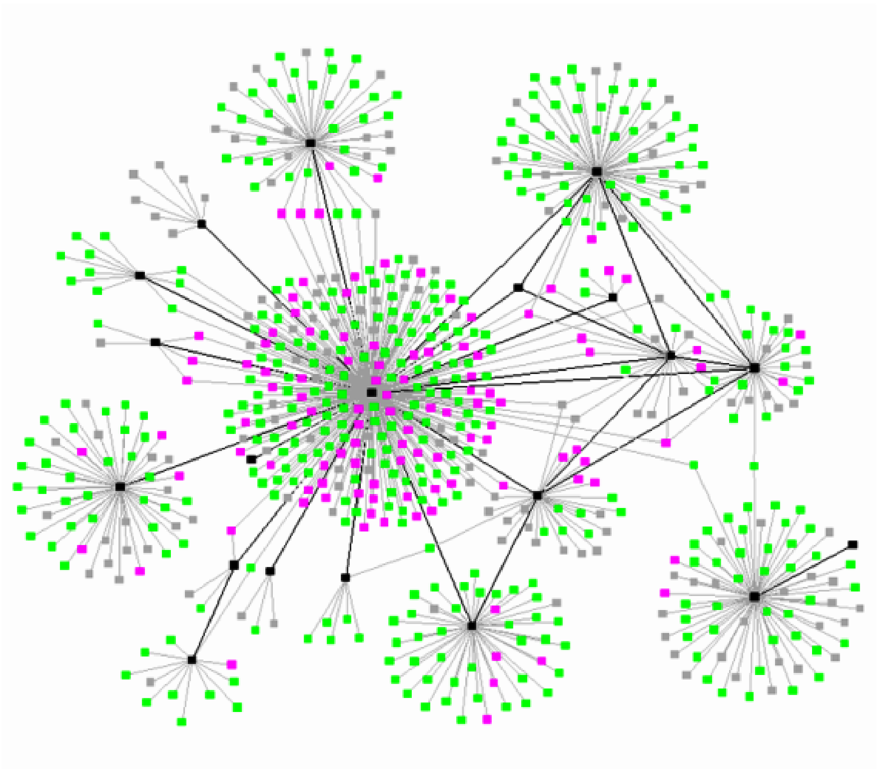
Acknowledgement: ONR

Networks

- Social networks
  - Friendship networks
  - Work networks
  - Scientific networks
  - Expertise networks
- Economic networks

Etc.



Network Science - literature

- Large literature on network analysis from data (infer social ties, communities, etc.) – [Barabasi][Kleinberg] etc.
- Limitations:
  - Cannot explain why and how networks form (analysis is ex-post)
  - Does not explain what we should expect to see
  - Does not allow predictions
  - Cannot assess effect of policies and/or social norms on networks

Our agenda

- Build a model of endogenous network evolution with incomplete information and learning
- Understand how agent learning and network formation co-evolve
- Establish methods for “guiding” network formation

Exogenous vs. Endogenous

Exogenously determined

- ❑ Predetermined by exogenous events

- Analyze ***given*** linking patterns
- How do agents learn about the ***exogenous environment***?
- How should information be ***disseminated***?
- Do agents in the network ***reach consensus?*** Are they ***herding***?

Endogenously evolving

- ❑ Determined by strategic choices of agents

- Analyze ***evolving*** linking patterns
- How do agents learn about the exogenous environment and ***each other***?
- How does information ***shape the network***?
- Do agents in the network ***cooperate? compete***?

Related Works - Network Formation

- Network formation under **complete information**
  - Homogeneous agents: [Jackson&Wolinsky'96], [Bala&Goyal'00], [Watts'01]
  - Heterogeneous agents: [Galeotti&Goyal'10], [Zhang&van der Schaar'12'13]
  - Known parameters, payoffs, everything – nothing to learn

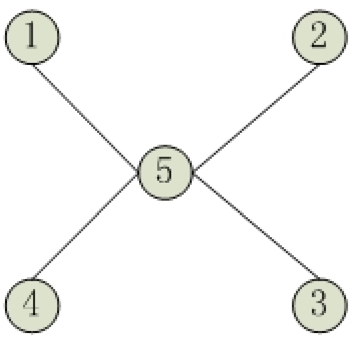
- Network formation under **incomplete information**

  [Song&van der Schaar'14]
  - Incomplete information matters!
  - Model is oversimplified, learning is actually gradual

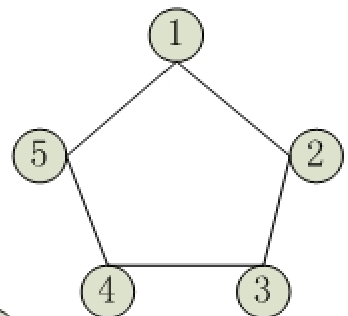
- These models are inadequate
- Unrealistic
- Not useful for prediction or guidance
- Cannot reason about welfare

…As a result, limited prediction power

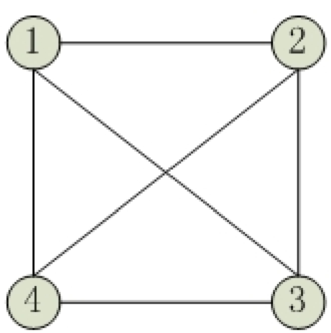
Theoretical predictions:
Simple networks



Star network



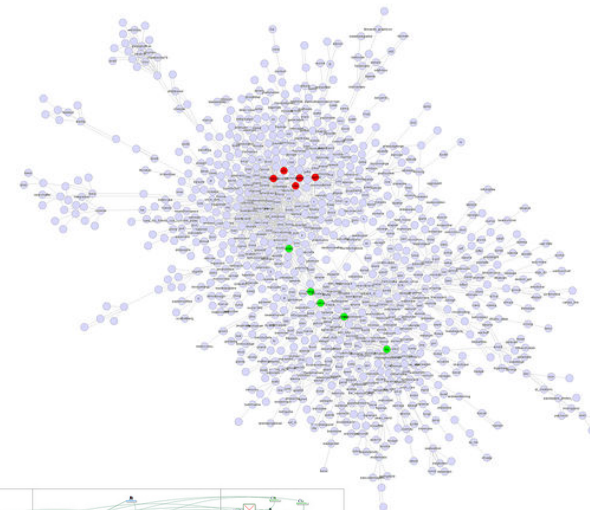
Wheel Network

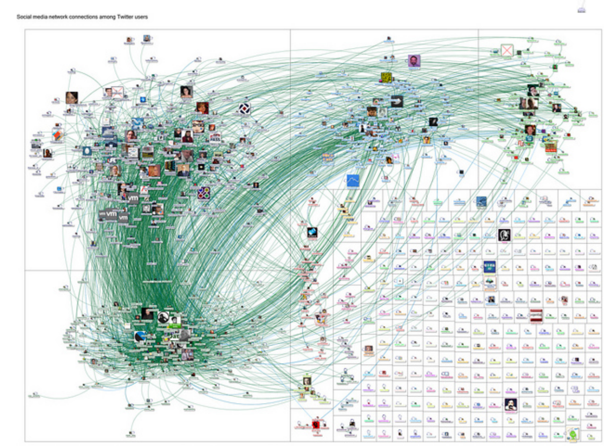


Complete network

VS

Actual networks:
Complicated





New Model Needed

Desideratum: Tractable model for

- analyzing impact of learning,
- analyzing co-evolution of network structures
- computing social welfare,
- guiding network formation to achieve desired goals

We have gone only a few steps in this agenda…

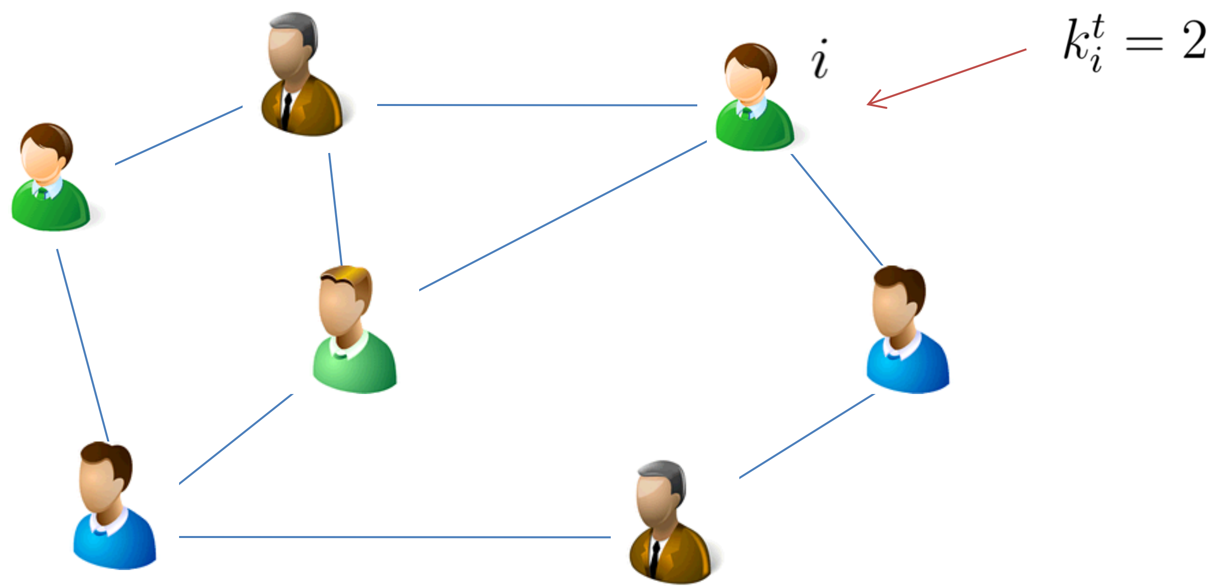
Joint work with Simpson Zhang (Economics, UCLA)

S. Zhang, M. van der Schaar, "Reputational Learning and Network Dynamics"

(http://medianetlab.ee.ucla.edu/papers/Simpson_networks_2015.pdf)

Network Model

- Infinite horizon continuous time
  - Interactions are on-going, not synchronized
- *N* agents, initially linked according to G^0
  - Physical/geographical/communication connection constraints
  - Planned
- Network evolves over time G^t
  - $k_i^t = \sum_j g_{ij}^t$: number of links (neighbors) of agent i at time t

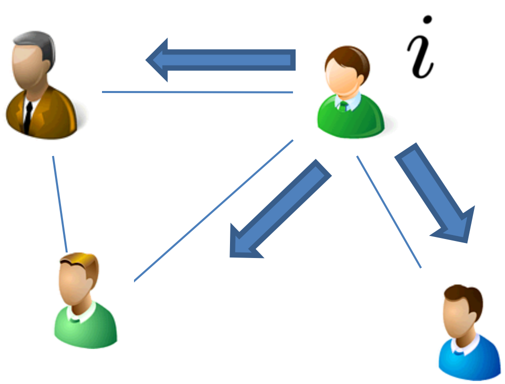


Agent Quality

- Agent *i* has true quality q_i
  - Unknown a priori
  - Prior beliefs: drawn from a distribution – here $\mathcal{N}(\mu_i, \sigma_i^2)$
  - Different agents, different beliefs
    - Good agents, bad agents
- Benefit *i* provides to *j* = noisy

$$db_{ij}(t) = q_i dt + \tau_i^{-1/2} dZ(t)$$

- Assumption: Summary information = Average over links



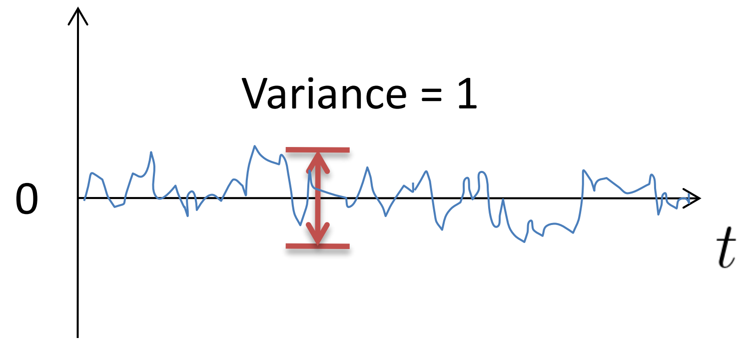
$$dB_i(t) = q_i dt + (k_i^t \tau_i)^{-1/2} dZ(t)$$

Per-capita benefit sent by agent i up to time t

Noisy Benefit Flow

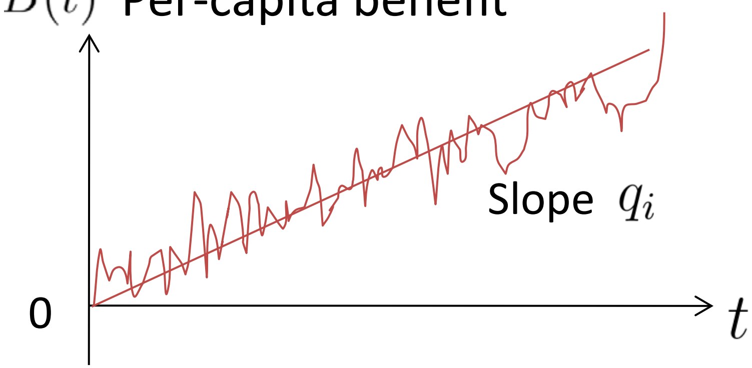
$$dB_i(t) = \underbrace{q_i dt}_{\text{Benefit reflecting the true quality}} + \underbrace{(k_i^t \tau_i)^{-1/2} dZ(t)}_{\text{Noise term}}$$

$Z(t)$ Standard Brownian Motion (SBM)



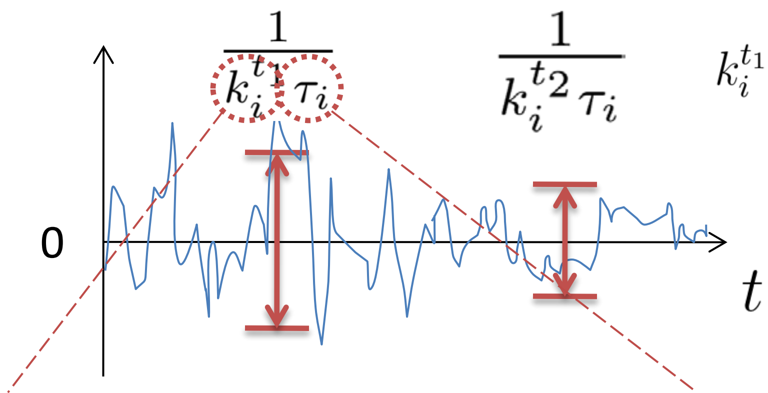
Without noise

$B(t)$ Per-capita benefit



With noise

Noise: "Modulated" SBM



$k_i^{t_1} < k_i^{t_2}$

Number of current neighbors

Base precision of an agent (measure of noisiness of benefit flow)

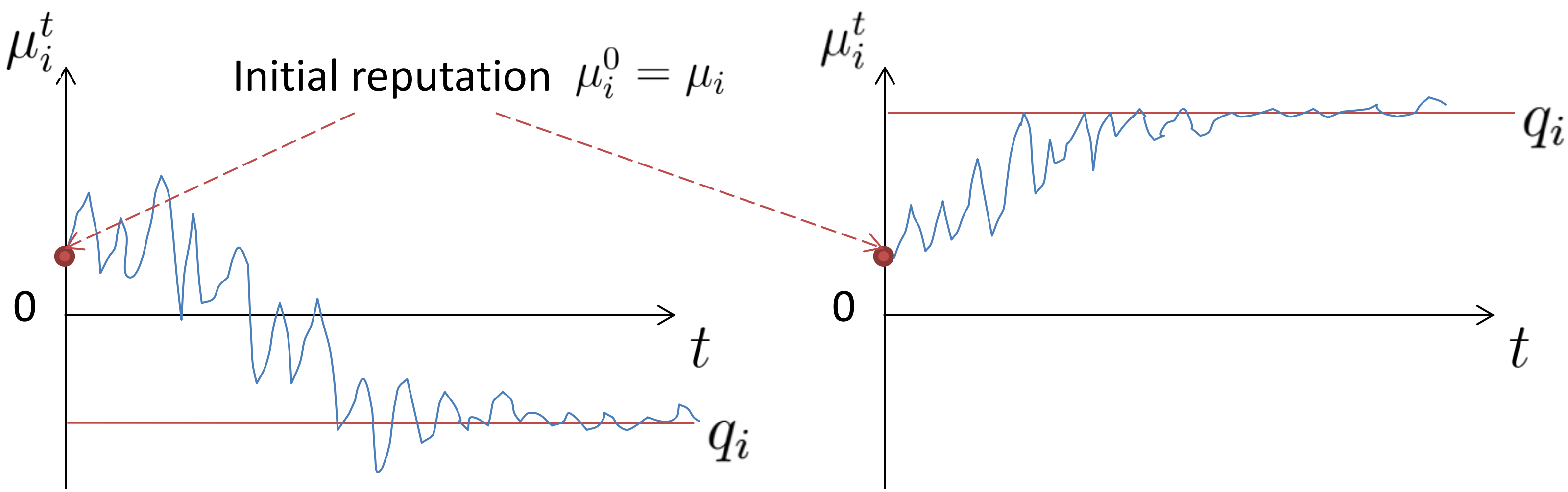
Larger base precision & more neighbors
→ Less noise

Reputation

- Expected quality conditional on observed benefit history

$$\mu_i^t = E[q_i | \{b_i^t\}_{t'=0}^t]$$

  - Updated according to Bayes rule (learning)
  - **Suppose** always connected and generating benefit flow



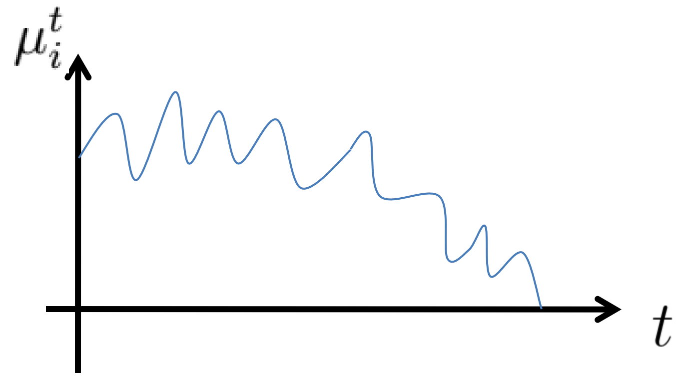
Low quality agent will be learned to be of low quality

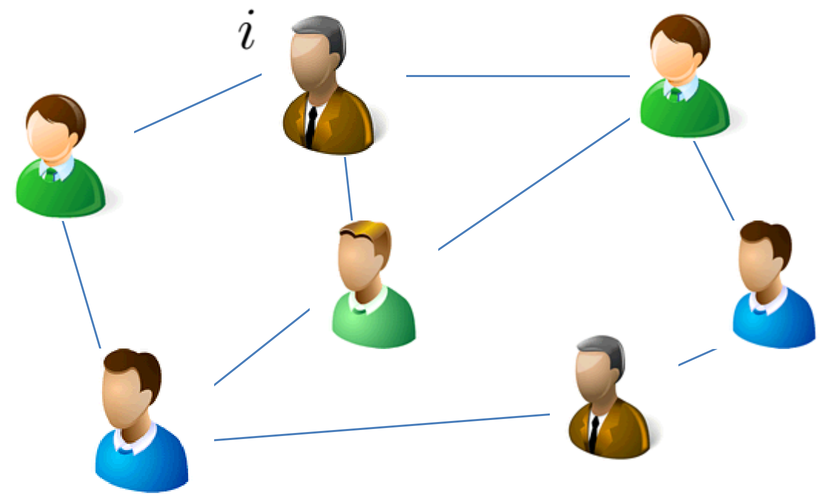
High quality agent will be learned to be of high quality

Network Evolution

Agents are myopic

- Goal: Maximize instantaneous utility
  - Connect $\mu_i^t > 0$
  - Disconnect $\mu_i^t \leq 0$





Agent *i*'s neighbors cut off links with Agent *i*

All Agent *i*'s neighbors have the same information/belief, so all cut/not cut link to Agent *i*

Agent *i* gets ostracized from the network

Learning about Agent *i*'s neighbors slows down (since they have fewer links)

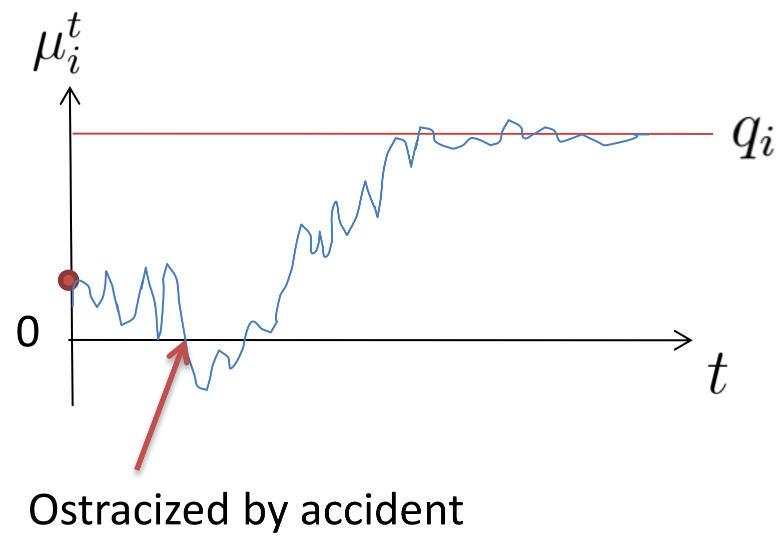
Process continues and more agents may be ostracized

Stability

Stability = Network does not change over time

Theorem. From any initial configuration, convergence to a stable network always occurs in finite time

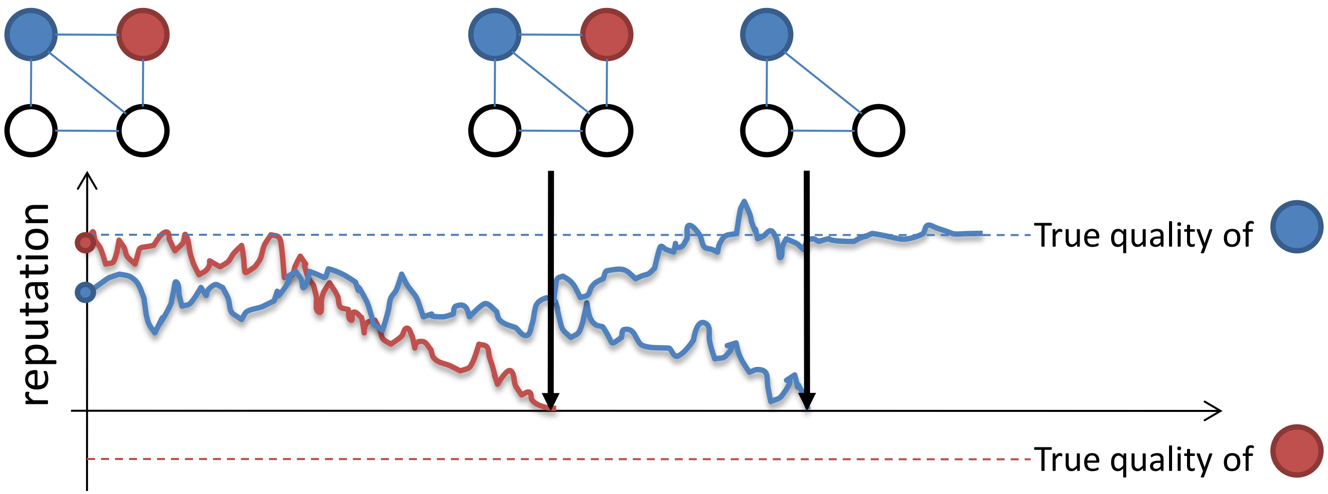
- Low quality agents
  - Always learned to be low quality → will always be ostracized (never in any limiting stable network)
- High quality agents
  - If learned to be high quality → will stay in the network forever
  - If believed to be low quality (by accident) → will be ostracized

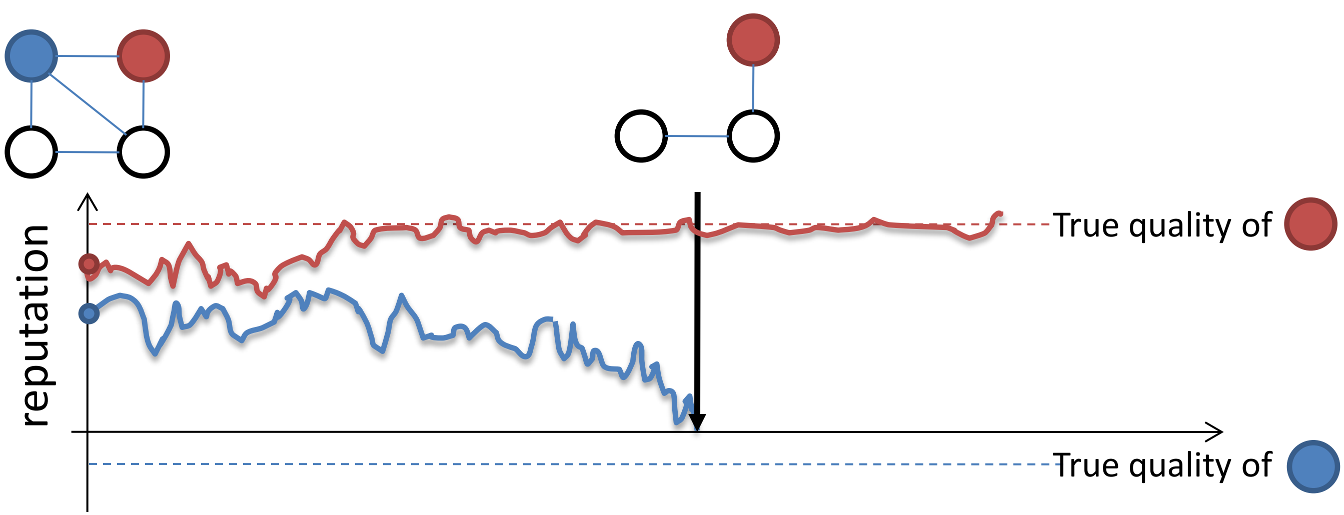


MANY possible stable networks!

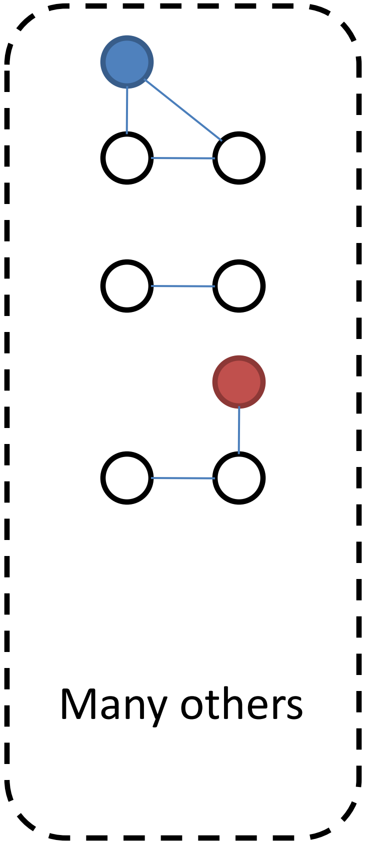
Which one emerges? Random! Different probabilities

Random Evolution

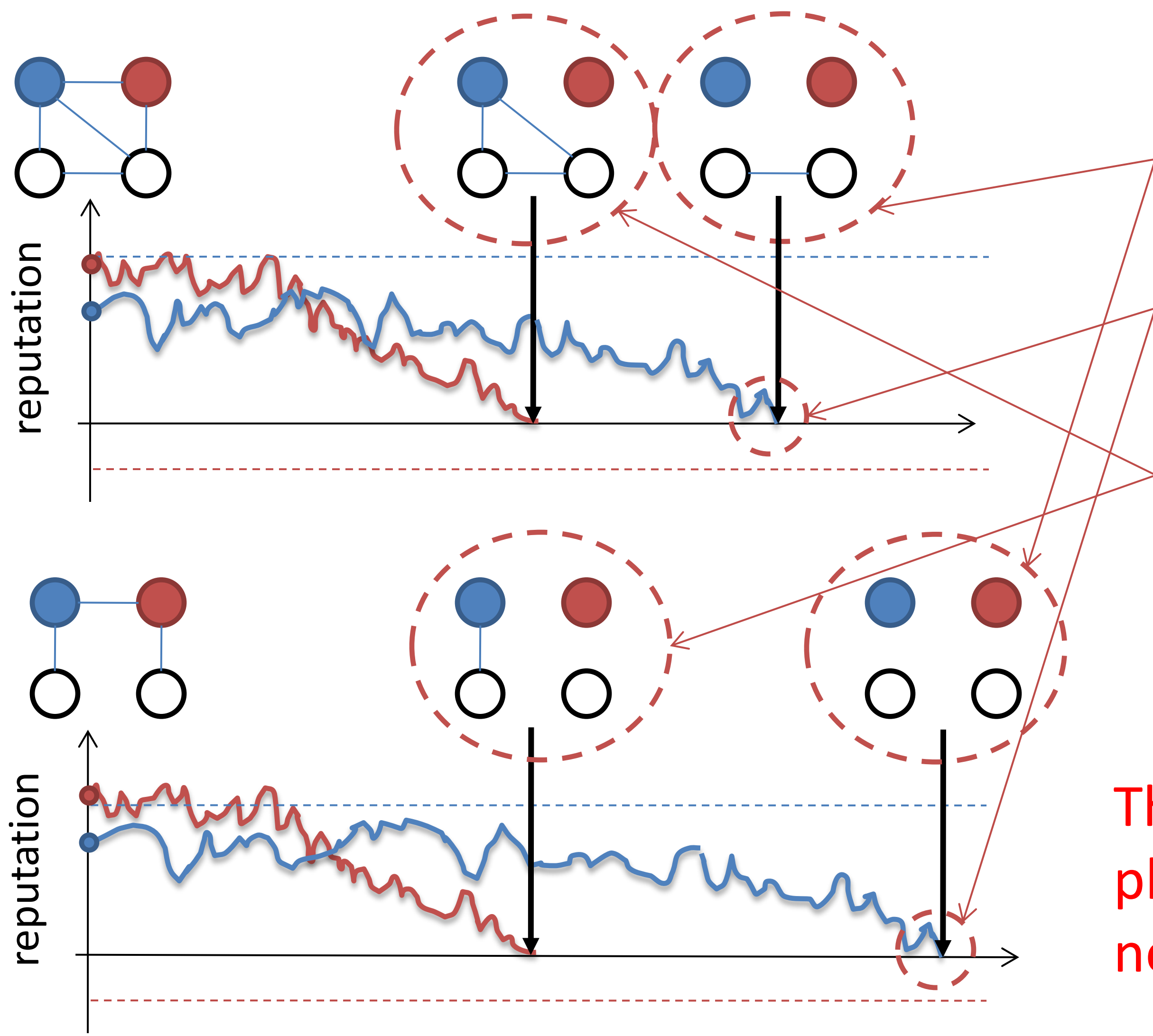




Stable Networks



Initial Network Matters!



Different stable networks

Different time when it becomes stable

Different intermediate networks

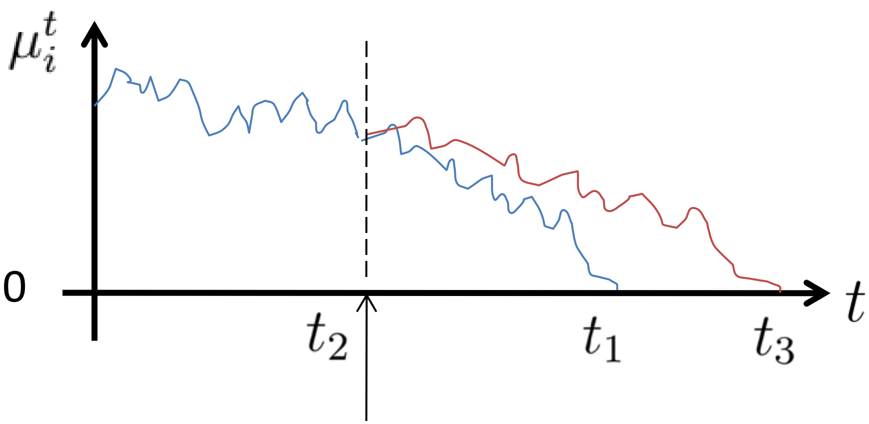
There is scope for planning the initial network!

Ostracism

Proposition. The probability that agent *i* is ostracized in the long run is *independent* of the initial (connected) network.

(The time it takes for agent *i* to be ostracized is *not* independent of the initial network.)

– Scaling effect:



One neighbor is ostracized
→ Fewer links

Changes when the hitting occurs

Does not change whether the hitting occurs

Does not change whether the agent stays in the stable network in this realization

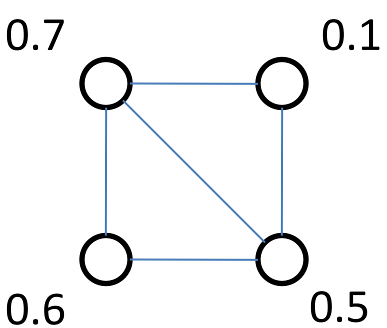
What networks can emerge and be stable?

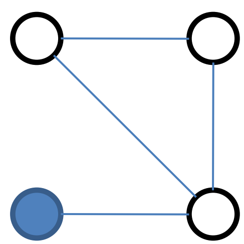
- Ex-ante probability that agent i with initial reputation μ_i is never ostracized

$$\int_0^\infty (1 - \exp(-\frac{2}{\sigma_i^2}\mu_i q_i))\phi\left((q_i - \mu_i)\frac{1}{\sigma_i}\right) dq_i$$

Theorem. Beginning from an initial configuration G^0, a network G can emerge and be stable with positive probability if and only if G can be reached from G^0 by sequentially ostracizing agents (Explicit formula for this probability.)

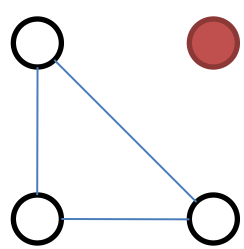
Example:





= 0

Not feasible



= 0.7x0.6x0.5x(1-0.1)

Guiding network formation

- Planner's goal
  - Maximize long-term welfare (discount factor ρ)
- What does the planner know?
  - The initial reputations of agents
  - *Not* the true quality of agents
- What can the planner do?
  - Set an initial connectivity of the network

Social Welfare

- How to define social welfare?
  - Path of network evolution is random
    - **It is not only about the limit stable network, but also about the intermediate networks that matter**
  - The "in expectation" perspective
    - Initial reputation (Prior belief about agents' quality)
    - Initial network topology

Definition: *Ex ante* discounted long-term sum benefit

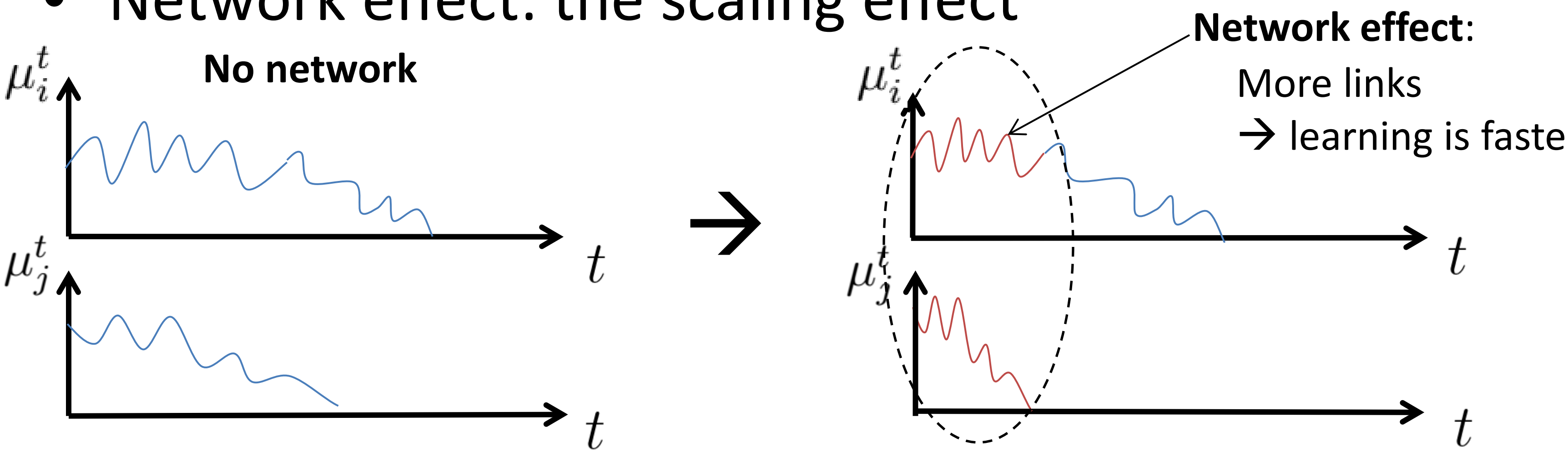
$$W = \boxed{\int_{q_1=-\infty}^{\infty} \cdots \int_{q_N=-\infty}^{\infty}} \sum_{i,j} \int_0^{\infty} \underbrace{e^{-\rho t}}_{\text{Discounting}} q_j \underbrace{P(L_{ij}^t | \boldsymbol{q}, G^0)}_{\text{Survival probabilities of links}} \phi(\frac{q_N - \mu_N}{\sigma_N}) dq_N \ldots \phi(\frac{q_1 - \mu_1}{\sigma_1}) dq_1 dt$$

Expectation using prior belief

Extremely difficult to compute: numerous conditional probabilities

Ex Post → Ex Ante

- Network effect: the scaling effect



Compute distributions

Reconstruct realization
Compute *ex post* welfare

Theorem. The ex ante social welfare can be computed in a closed form as follows

$$W = \boxed{E_{\hat{\varepsilon}}} \sum_i \left(\frac{1 - e^{\boxed{-\rho M_i(\mathbf{t})}}}{\rho} \sum_{j:g_{ij}^0=1, t_j=\infty} \frac{\mu_j}{\boxed{P(S_j)}} \right)$$

Mi(t) – hitting time in the realization

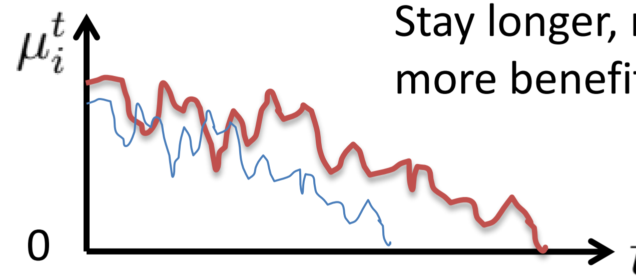
How learning affects individuals' welfare?

$$dB_i(t) = q_i dt + (k_i^t \tau_i)^{-1/2} dZ(t)$$

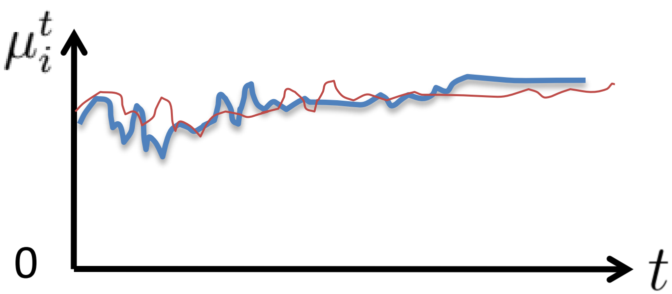
Base precision of an agent: information sending speed

- Low quality agents
  - Want to be learned about more slowly

Stay longer, receive more benefit

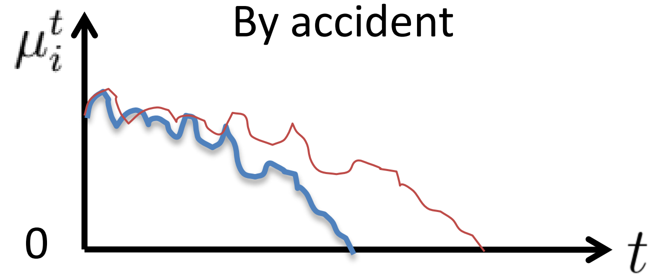


- High quality agents
  - Want to be learned about more quickly?



Not affected in this case

By accident



Worse off in this case

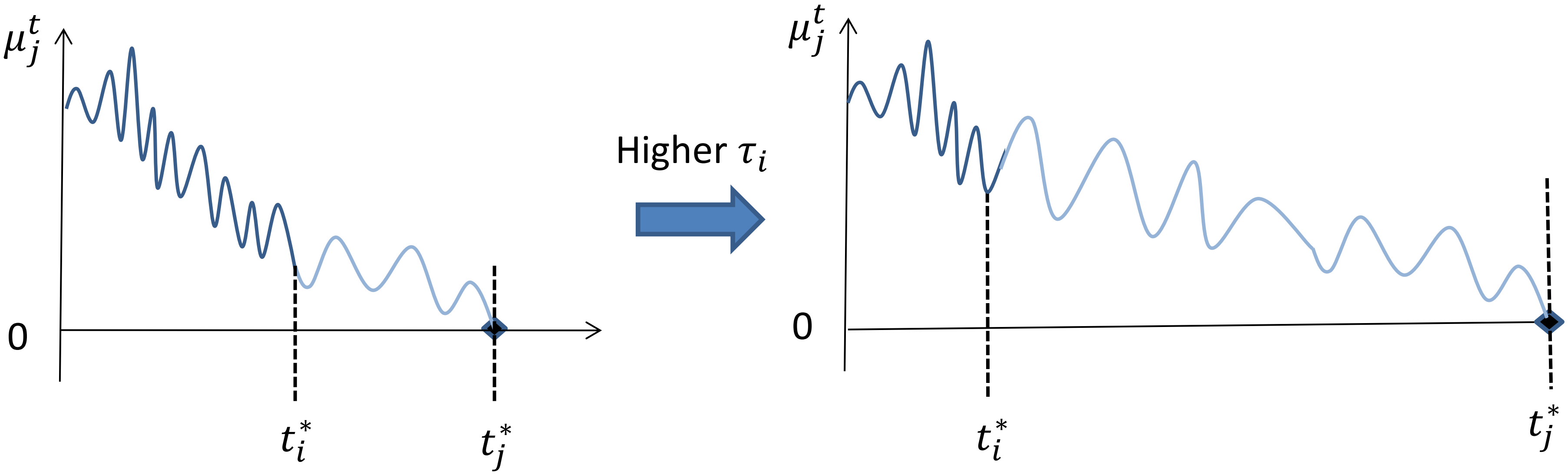
High quality agents also want to be learned about more slowly

Impact of Learning Speed on Welfare

Theorem. For any initial network, each agent i's welfare is ***decreasing*** in its base precision τ_i.
Further, multiplying all agents' base precisions by the same factor $d > 1$ decreases the total *ex ante* social welfare.

Theorem. For any initial network without cycles, increasing any agent i's base precision τ_i ***increases*** the welfare of each of i's neighbors.

Increasing Agent i's Precision helps its Neighbor



Neighbor j's hitting time increases!

Agent j gets more benefits from network!

Optimal Initial Network G^0

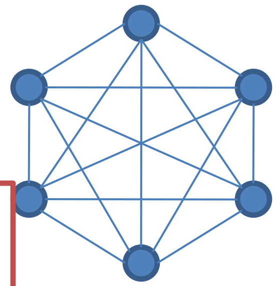
- Depends on planner's patience ρ

- Completely impatient – only the initial network matters
- Completely patient – only the limit stable network matters

- These cases are NOT very interesting
- Intermediate patience $0 < \rho < 1$?

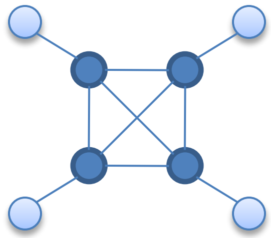
Optimal Initial Network

- Fully connected network



Theorem. A fully connected initial network is optimal if all prior mean qualities are sufficiently high (depending on ρ)

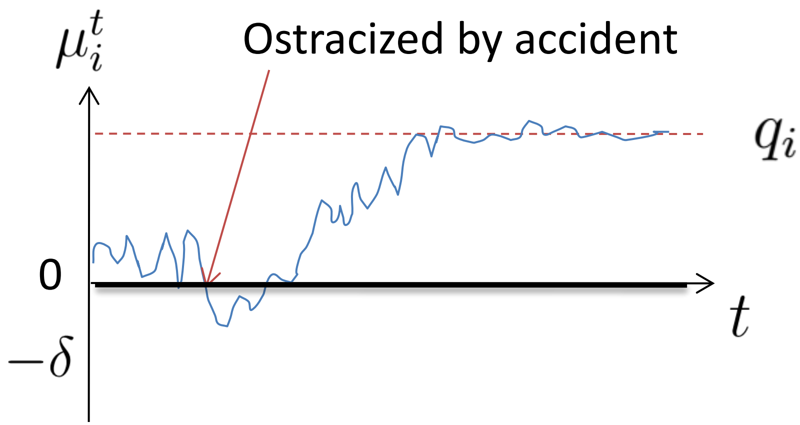
- Core-periphery network
  - Heterogeneous agents: two initial reputations μ_H μ_L

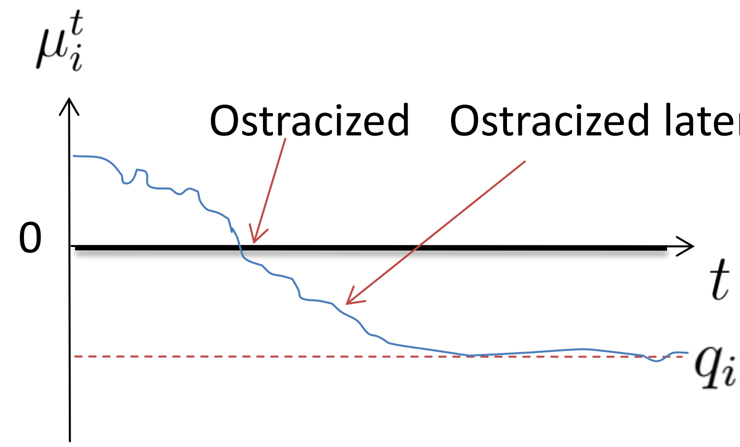


Theorem. A core-periphery initial network is optimal if μ_H is sufficiently higher than μ_L (depending on ρ)

- Why?
  - High quality in the core → learned more quickly
  - Low quality in the periphery → less harm

Encouraging experimentation





Theorem. (1) $\exists\, \underline{\delta}$ s.t. $W(\delta) > W(0)$ for all $\delta > \underline{\delta}$
(2) $\delta^* = \arg\max_\delta W(\delta)$ exists and is finite.

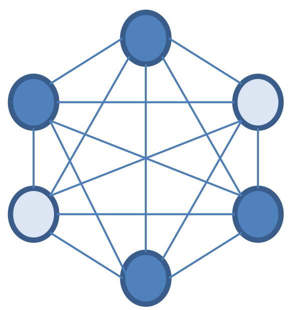
– Experimentation promotes learning, but weakens punishments

– Optimal (computable) amount of experimentation

Incorporating Agent Entry

- Our model can be tractably extended to allow agents to *enter* the network over time
  - E.g. a firm does not hire all workers immediately, but introduces them in a sequential order (designer *not* monitoring the network)

Initial Network



$t = 1$

$t = 2$

Reconstruct realization based on agent entry time

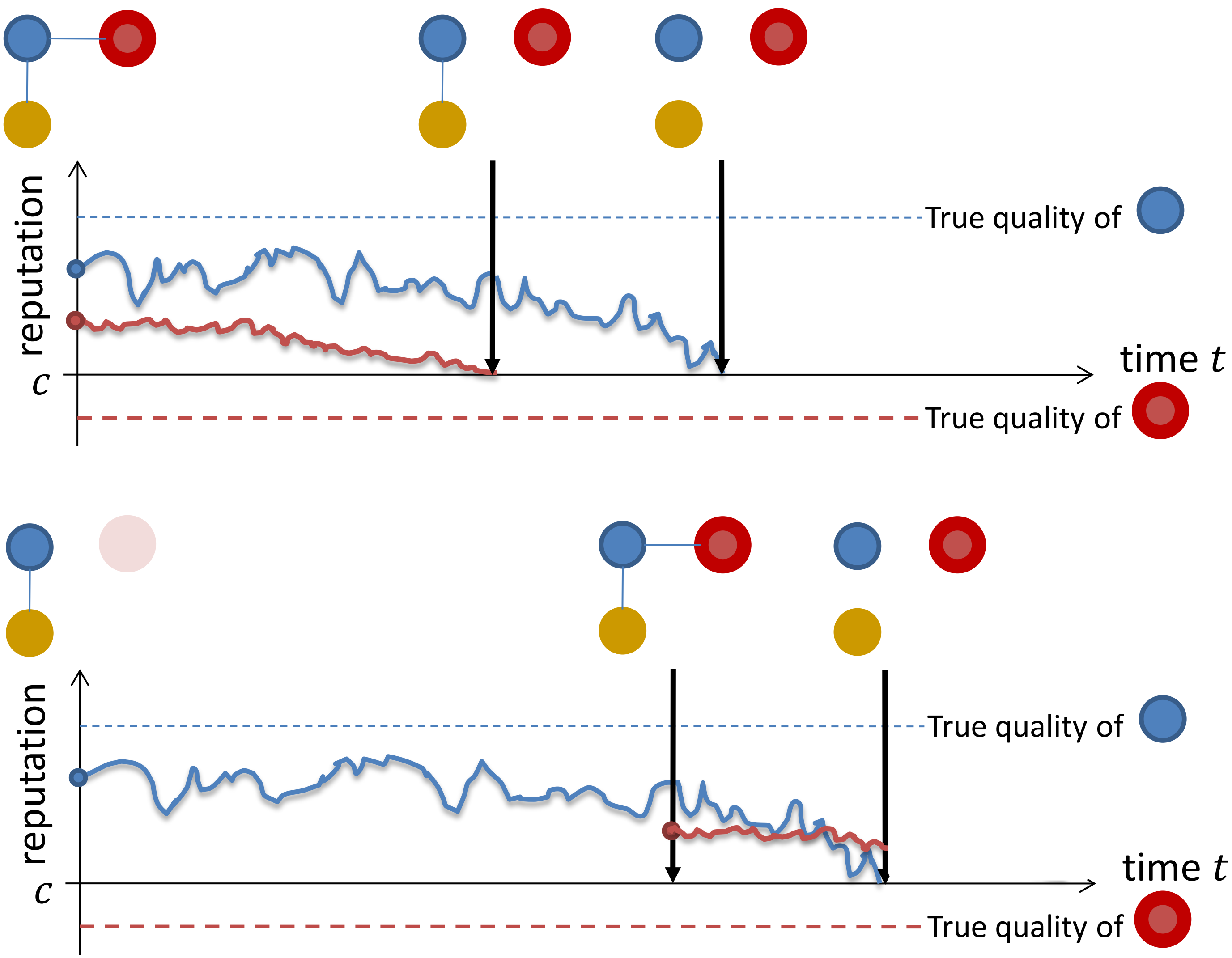
Theorem. The ex ante social welfare can be computed as follows

$$W = E_{\hat{\varepsilon}} \sum_i \left(\sum_{j: g_{ij}^0 = 1, t_j = \infty} \frac{e^{\boxed{-\rho S_{ij}(t)}} - e^{\boxed{-\rho F_{ij}(t)}}}{\rho} \frac{\mu_j}{P(S_j)} \right)$$

Delaying Entry Can Improve Welfare

- By allowing agents to enter later, social welfare can be improved in certain settings
- Agents can have more time to cement their reputations without getting ostracized from the network as quickly

Delaying Entry Can Improve Welfare



Blue agent receives and produces benefits for longer!

What is accomplished

- The first model of endogenous network evolution with incomplete information and learning
  - Rigorous characterization of learning and network co-evolution
  - Understanding emergent behaviors of strategic agents
- Guiding network formation
  - Planning initial configuration
  - Encouraging experimentation
  - Deciding “entry” times of agents
- Limitations: agents are myopic; no new links formed

Foresighted agents
(joint work with Yangbo Song, Economics, UCLA)

Different model

- Heterogeneous agents
- Actions other than connect/disconnect
- Endogeneity of "states" – history
  - Proper link with repeated games
- Private and public knowledge
- Predictions are very different
  - Foresight leads to different networks and configurations
  - Sustainability of a richer set of networks in equilibrium
- Actions matter! Not just the connections!

Model: network formation + game

- N agents
- Time t = 0, 1, 2, …
- In each period t:
  - Network formation phase: links form/break; formation requires bilateral consent; breaking does not
  - Action phase: each agent plays a (possibly different) game with each person to whom she is (directly or indirectly) connected
  - Monitoring phase: agents monitor their opponents' actions with a certain technology

Model: network formation + game

- Agent i's one-period payoff:

$$u_i(\bar{\theta}, \mathbf{g}, \bar{e}) = \sum_{j : i \overset{\mathbf{g}}{\leftrightarrow} j} \delta^{d_{ij}-1} f(\theta_i, \theta_j, e_{ij}, e_{ji}) - \sum_{j : ij \in \mathbf{g}} c$$

network

distance between i, j

types of i, j

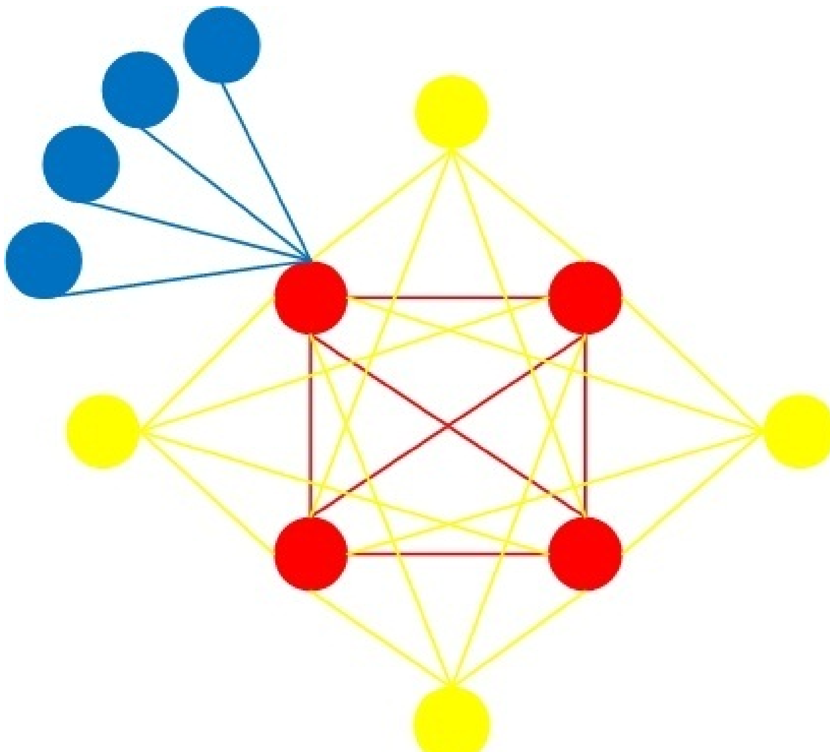
actions of i, j

link maintenance cost

- Agents discount the future by factor γ per period
- In equilibrium, an agent maximizes her discounted sum of payoffs (given strategies of others)

Main results: efficient network

- An efficient network has a core-periphery structure



Main features:

1. Large clustering coefficient

   -- one's neighbors are likely to be linked

2. Large triangle/agent ratio

   -- well-connected agents are linked

3. Large fraction of closed triangles

   -- strong indicator of a core

4. Short diameter

   -- agents are densely connected

Main results: equilibrium topology/action

- Perfect monitoring: folk theorem with simple strategies → full cooperation is sustainable; equilibrium strategies are proof against (many) *coalitional deviations*
- Imperfect costly monitoring: high connectivity degree cannot be sustained (too many friends to monitor); large diameter cannot be sustained (too far to punish effectively) → full cooperation may not be sustainable, fragmentation may occur
- Characterize how patience, type distribution and link maintenance cost affect set of sustainable networks + cooperation

Some implications

- At social optimum, "better" agents (higher types) should be more connected, but are NOT necessarily better off than others (benefits "extracted" from them - better agents are exploited)
- With foresight, social welfare may be higher than that predicted by previous theory
- The network is the structure along which information is transmitted and network evolves endogenously → information transmission evolves *endogenously*
- With limited monitoring, making a few close friends may be better than many casual ones

Comparison vs. network games

	Games played on fixed networks	**Our model**
Network formation	Exogenous	Endogenous
Role of network	Channel of interaction OR monitoring	Both
Efficient network characterization	No	Yes
Relation between sustainable network and time discount	No	Yes

Comparison vs. network formation

	Myopic network formation	**Our model**
Cooperation and punishment	No	Yes
Sustaining efficiency	Often impossible	Possible, depending on time discount and monitoring
Networks that persist over time	Few and simple	Many and complex
Interrelation between network and action	No	Yes

Conclusion

- Models of endogenous network formation
  - heterogeneous agents and heterogeneous information
    - Information gathering and dissemination
  - myopic agents, incomplete information
    - Learning and the network co-evolve
    - Probabilistic predictions about emerging networks
  - foresighted agents
    - Interaction of actions and information through the network
    - Many more networks emerge and are stable

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