A Decentralized Scheme for Efficient Coordination of Large-Scale PEV Charging

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The electrical power drawn by plug-in electric vehicle (PEV) chargers will (eventually?) begin to impact the grid.

At the system-wide level, control objectives tend to focus on filling the overnight valley in background demand.

At the distribution level, proposed control strategies address:

- Transformer overloads
- Loss minimization
- Voltage degradation
- Tap-change minimization

Few control strategies also take into account the effects of charging on battery health.
Goals

- A decentralized approach to scheduling PEV charging that considers trade-offs between:
  - Energy price
  - Battery degradation
  - Distribution network effects

- The resulting collection of PEV charging strategies should be efficient (socially optimal).

- Convergence should only require a few iterations.
Formulation

- PEV population: \( \mathcal{N} \equiv \{1, \ldots, N\} \).
- Horizon: \( \mathcal{T} \equiv \{0, \ldots, T - 1\} \).
- Admissible charging strategies:

\[
\begin{align*}
    u_{nt} &\geq 0, \quad t \in \mathcal{T} \\
    \|u_n\|_1 &\equiv \sum_{t \in \mathcal{T}} u_{nt} \leq \Gamma_n
\end{align*}
\]

where \( \Gamma_n \) is the energy capacity of the \( n \)-th PEV.

- The set of admissible charging controls is denoted \( \mathcal{U}_n \).
Demand charge

- Distribution-level impacts are largely a consequence of coincident high charger power demand $u_{nt}$.
- Undesirable effects can be minimized by encouraging lower power levels,

$$Cost_{\text{demand}, nt} = g_{\text{demand}, nt}(u_{nt})$$

where $g_{\text{demand}, nt}(\cdot)$ is a strictly increasing function.
Battery degradation cost

Experimentation with LiFePO$_4$ lithium-ion batteries gave a degradation model:

$$d_{\text{cell}}(I, V) = \beta_1 + \beta_2 I + \beta_3 V + \beta_4 I^2 + \beta_5 V^2 + \beta_6 IV + \beta_7 V^3$$

relating energy capacity loss per second (in Amp$\times$Hour$\times$Sec$^{-1}$) to charging current $I$ and voltage $V$.

- Degradation cost:
  $$g_{\text{cell}}(I, V) = P_{\text{cell}} \Delta TV d_{\text{cell}}(I, V)$$
  where $P_{\text{cell}}$ is the price ($$/Wh) of battery cell capacity.

- Over the useable state of charge (SoC) range, $V \approx V_{\text{nom}}$.

- Battery degradation cost can be expressed as:

  $$\text{Cost}_{\text{degrad}, nt} = g_{\text{cell}, n}(u_{nt}) = M_n g_{\text{cell}}\left(\frac{10^3 u_{nt}}{M_n V_{\text{nom}}}, V_{\text{nom}}\right)$$

  $$= a_n u_{nt}^2 + b_n u_{nt} + c_n$$
Centralized formulation

System cost:

$$J(u) \triangleq \sum_{t \in T} \left\{ c \left( d_t + \sum_{n \in \mathcal{N}} u_{nt} \right) + \sum_{n \in \mathcal{N}} g_{nt}(u_{nt}) \right\} - \sum_{n \in \mathcal{N}} \left\{ h_n(\|u_n\|_1) \right\}$$

where:

- $u_n \in \mathcal{U}_n$ for all $n \in \mathcal{N}$.
- $c(\cdot)$ gives the generation cost with respect to the total demand $d_t + \sum_{n \in \mathcal{N}} u_{nt}$, and $d_t$ denotes the aggregate inelastic base demand at time $t$.
- $g_{nt}(u_{nt}) = g_{\text{demand},nt}(u_{nt}) + g_{\text{cell},n}(u_{nt})$ captures the demand charge and battery degradation cost of the $n$-th PEV.
- $h_n(\|u_n\|_1)$ denotes the benefit function of the $n$-th PEV with respect to the total energy delivered over the charging horizon, with:

$$h_n(\|u_n\|_1) = -\delta_n(\|u_n\|_1 - \Gamma_n)^2$$
Centralized formulation (continued)

- Utility function of the $n$-th PEV:
  \[ v_n(u_n) \triangleq h_n(\|u_n\|_1) - \sum_{t \in T} g_{nt}(u_{nt}) \]

- The system cost $J(u)$ can be rewritten:
  \[ J(u) = \sum_{t \in T} c\left(d_t + \sum_{n \in \mathcal{N}} u_{nt}\right) - \sum_{n \in \mathcal{N}} v_n(u_n) \]

- Assume the electricity generation cost can be approximated by:
  \[ c(y_t) = \frac{1}{2}ay_t^2 + by_t + c \]

- Marginal generation cost:
  \[ p(y_t) \triangleq c'(y_t) = ay_t + b \]
Centralized optimization

Optimization problem:

$$\min_{u \in U} J(u)$$

Assumptions:

(A1) $c(\cdot)$ is monotonically increasing, strictly convex and differentiable.

(A2) $g_{nt}(\cdot)$, for all $n \in \mathcal{N}$, $t \in \mathcal{T}$, is monotonically increasing, strictly convex and differentiable.
Optimization solution

- The efficient (socially optimal) charging behavior is unique and can be characterized by the associated KKT conditions:

\[ \frac{\partial}{\partial u_{nt}} J(u) \geq 0, \quad u_{nt} \geq 0, \quad \frac{\partial}{\partial u_{nt}} J(u) u_{nt} = 0 \]

for all \( n \in \mathcal{N} \) and \( t \in \mathcal{T}_n \), where:

\[ \frac{\partial}{\partial u_{nt}} J(u) = c'(d_t + \sum_{n \in \mathcal{N}} u_{nt}) - \frac{\partial}{\partial u_{nt}} v_n(u_n) \]

- The optimal charging behavior \( u^{**} \) is uniquely specified by:

\[ p_t^{**} = \begin{cases} \frac{\partial}{\partial u_{nt}} v_n(u_{n}^{**}), & \text{when } u_{nt}^{**} > 0, \\ \geq \frac{\partial}{\partial u_{nt}} v_n(u_{n}^{**}), & \text{when } u_{nt}^{**} = 0, \end{cases} \]

where \( p_t^{**} = c'(d_t + \sum_{n \in \mathcal{N}} u_{nt}^{**}) \).
Example

Total demand

- socially optimal strategy
- valley filling strategy
- base demand

Charging interval

Total demand (KW)
Example - varying $P_{cell}$
Example - varying terminal penalty, $\delta_n$

![Total demand graph](image1)

![Delivered energy graph](image2)

- **Total demand**: Graph showing the total demand over a 24-hour period with different charging intervals.
- **Delivered energy**: Graph showing the delivered energy over a range of values for $\delta$.

**Key Points**:
- Centralized scheduling
- Decentralized scheduling
- Conclusions

<table>
<thead>
<tr>
<th>Charging Interval</th>
<th>Total Demand</th>
<th>Delivered Energy</th>
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<tr>
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<td>11</td>
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</table>
Decentralized charging coordination

(S1) Each PEV autonomously determines its optimal charging strategy with respect to a given electricity price profile \( p \equiv (p_t, t \in \mathcal{T}) \). This optimal strategy takes into account the trade-off between the electricity cost and local (demand and battery degradation) costs over the entire charging horizon.

(S2) The electricity price profile \( p \) is updated to reflect the latest charging strategies determined by the PEV population in (S1).

(S3) Steps (S1) and (S2) are repeated until the change in the price profile at (S2) is negligible.

Using an appropriate individual cost function and price update mechanism, (S1)-(S3) is convergent and achieves the socially optimal (centralized) solution.
Individual cost function

\[ J_n(u_n; p) \triangleq \sum_{t \in T} p_t u_{nt} - v_n(u_n) = \sum_{t \in T} \left\{ p_t u_{nt} + g_{nt}(u_{nt}) \right\} - h_n\left( \sum_{t \in T} u_{nt} \right) \]

- Cost is composed of energy cost, local (demand and battery degradation) cost, and the benefit derived from the total delivered energy.
- The optimal charging strategy of the \( n \)-th PEV, with respect to \( p \):
  \[ u^*_n(p) = \arg \min_{u_n \in U_n} J_n(u_n; p) \]
  This optimal response has the form:
  \[ u_{nt}(p, A_n) = \max \{ 0, [g'_{nt}]^{-1}(A_n - p_t) \}, \quad t \in T \]
  for some \( A_n \), where \( g'_{nt} \) is the derivative of \( g_{nt} \), and \([g'_{nt}]^{-1}\) denotes the corresponding inverse function.
Optimal form

- Define: $\mathcal{U}_n(\omega) \triangleq \left\{ u_n \in \mathcal{U}_n; \text{s.t.} \, \| u_n \|_1 = \omega \right\}$.
- Minimizing $J_n(u_n; p)$ over $\mathcal{U}_n(\omega)$ is equivalent to minimizing:
  $$F_n(u_n; p) \triangleq \sum_{t \in T} \left\{ p_t u_{nt} + g_{nt}(u_{nt}) \right\}$$

  This is just $J_n(u_n; p)$ with $h_n(\cdot)$ excluded.
- Define the Lagrangian function,
  $$L_n(u_n, A_n; p) \triangleq F_n(u_n; p) + A_n(\omega - \| u_n \|_1)$$
- KKT conditions:
  (i) $\frac{\partial L_n}{\partial A_n} = 0$.
  (ii) $\frac{\partial L_n}{\partial u_{nt}} \geq 0$, $u_{nt} \geq 0$, with complementary slackness.
- Condition (ii) gives:
  $$p_t + g'_{nt}(u_{nt}) - A_n \begin{cases} = 0, & \text{when } u_{nt} > 0 \\ \geq 0, & \text{otherwise} \end{cases}$$
Determining $A_n$

Define the function:

$$A_n(p, \cdot) : [0, \Gamma_n] \rightarrow [A_n^-(p), A_n^+(p)]$$

such that

$$A_n(p, \omega) = A_n \iff \|u_n(p, A_n)\|_1 = \omega$$

which implies

$$\|u_n(p, A_n(p, \omega))\|_1 = \omega$$

$A_n(p, \omega)$ is strictly increasing with $\omega$. 
Determining $A_n$ (continued)

**Theorem:** Assume $h_n(\omega)$ is continuously differentiable, increasing and concave on $0 \leq \omega \leq \Gamma_n$. Define,

$$f_n(p, \omega) \triangleq A_n(p, \omega) - h'_n(\omega)$$

and

$$A_n^*(p) = \begin{cases} A_n(p, \Gamma_n), & \text{if } f_n(p, \Gamma_n) \leq 0 \\ A_n(p, 0), & \text{if } f_n(p, 0) \geq 0 \\ A_n(p, \omega^*), & \text{if } f_n(p, \omega^*) = 0 \end{cases}$$

where $0 < \omega^* < \Gamma_n$. Then the charging strategy $u_n(p, A_n^*(p))$ uniquely minimizes the individual PEV cost function for a given $p$, i.e. $u_n^*(p) = u_n(p, A_n^*(p))$. 
Price profile update mechanism

- Let
  \[ p^+_t(p) = p_t + \eta \left( c'(d_t + \sum_{n \in \mathcal{N}} u^*_n(p)) - p_t \right), \quad t \in \mathcal{T} \]
  where \( \eta > 0 \) is a fixed parameter, and \( u^*_n(p) \) is the optimal charging strategy for the \( n \)-th PEV with respect to \( p \).

- Define \( \nu_{nt} \) as the Lipschitz constant for the function \( [g'_{nt}]^{-1}(\cdot) \) over the interval \( [g'_{nt}(0), g'_{nt}(\Gamma_n)] \), and:
  \[ \nu = \max_{n \in \mathcal{N}, t \in \mathcal{T}} \nu_{nt} \]

- Define \( \kappa \) as the Lipschitz constant for \( c'(\cdot) \) over the typical range in the total demand.

- Then assuming the terminal valuation function \( h_n \) is increasing and strictly concave:
  \[ \| u^*_n(p) - u^*_n(q) \|_1 \leq 2\nu \| p - q \|_1 \]
Convergence of the algorithm

**Theorem:** Suppose $\alpha \equiv |1 - \eta| + 2N\kappa\nu\eta < 1$ and consider any initial charging price $p^{(0)}$. Then the decentralized algorithm converges to the efficient (centralized) solution $u^{**}$. Moreover, for any $\varepsilon > 0$, the system converges to a price profile $p$, such that $\|p - p^{**}\|_1 \leq \varepsilon$, in $K(\varepsilon)$ iterations, with

$$K(\varepsilon) = \left\lceil \frac{1}{\ln(\alpha)} \left( \ln(\varepsilon) - \ln(T) - \ln(\rho_{max}) \right) \right\rceil$$

where $\rho_{max}$ denotes the maximum possible price, and $\lceil x \rceil$ represents the minimal integer value larger than or equal to $x$. 
Remarks

- In practice, convergence takes many fewer iterations than the upper bound.
- The upper bound on the iteration count, $K(\varepsilon)$, is on the order of $O(\|\ln(\varepsilon)\|)$, and is independent of the size of the PEV population.
- The proof establishes that
  \[ \|p^+ - \varepsilon^+\|_1 < \|p - \varepsilon\|_1 \]
  so the price update operator $p^+(p)$ is a contraction map.
- By the contraction mapping theorem, the price $p^{(k)}$ converges to a unique price profile $p^*$ from any initial price profile $p^{(0)}$, such that $p^+(p^*) = p^*$ and so:
  \[ c'(d_t + \sum_{n \in \mathcal{N}} u_{nt}^*(p^*)) = p_t^*, \quad \text{for all } t \in \mathcal{T} \]
- The price converges to the generation marginal cost.
Illustration - convergence

Evolution of $\|p^{(k)} - p^{**}\|_1$ for various values of the price update parameter $\eta$.

- Convergence is guaranteed for $0 < \eta < 1.017$. 
Illustration - algorithm updates

Price update parameter $\eta = 1$.

Price

Total demand
Illustration - heterogeneous population

Total demand

Updated charging strategy (kW)

Charging interval

12:00 16:00 20:00 0:00 4:00 8:00 12:00

base demand
1st update
2nd update
3rd update
4th update
5th update
converged strategy

5 6 7 8

x 10^4
Conclusions

- The proposed price-based strategy for coordinating the charging of a large population of PEVs:
  - Establishes a trade-off between the cost of energy and battery degradation.
  - Incorporates a charge that penalizes high demand, thereby assisting in mitigating high coincident charging on local distribution networks.

- An iterative decentralized algorithm:
  - Converges to the unique efficient collection of charging strategies.
  - At convergence, the price profile coincides with the generator marginal cost.