A Decentralized Scheme for Efficient Coordination of Large-Scale PEV Charging

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Motivation

- The electrical power drawn by plug-in electric vehicle (PEV) chargers will (eventually?) begin to impact the grid.
- At the system-wide level, control objectives tend to focus on filling the overnight valley in background demand.
- At the distribution level, proposed control strategies address:
  - Transformer overloads
  - Loss minimization
  - Voltage degradation
  - Tap-change minimization
- Few control strategies also take into account the effects of charging on battery health.

Goals

- A decentralized approach to scheduling PEV charging that considers trade-offs between:
  - Energy price
  - Battery degradation
  - Distribution network effects
- The resulting collection of PEV charging strategies should be efficient (socially optimal).
- Convergence should only require a few iterations.

Formulation

- PEV population: $\mathcal{N} \equiv \{1, ..., N\}$.
- Horizon: $\mathcal{T} \equiv \{0, ..., T-1\}$.
- Admissible charging strategies:

$$u_{nt} \geq 0, \quad t \in \mathcal{T}$$
$$\|\boldsymbol{u}_n\|_1 \equiv \sum_{t \in \mathcal{T}} u_{nt} \leq \Gamma_n$$

  where Γ_n is the energy capacity of the n-th PEV.
- The set of admissible charging controls is denoted $\mathcal{U}_n$.

Demand charge

- Distribution-level impacts are largely a consequence of coincident high charger power demand u_{nt}.
- Undesirable effects can be minimized by encouraging lower power levels,

$$Cost_{demand,nt} = g_{demand,nt}(u_{nt})$$

where $g_{demand,nt}(\cdot)$ is a strictly increasing function.

Battery degradation cost

Experimentation with $\mathrm{LiFePO_4}$ lithium-ion batteries gave a degradation model:

$$\mathfrak{d}_{cell}(I,V) = \beta_1 + \beta_2 I + \beta_3 V + \beta_4 I^2 + \beta_5 V^2 + \beta_6 IV + \beta_7 V^3$$

relating energy capacity loss per second (in Amp$\times$Hour$\times$Sec^{-1}) to charging current I and voltage V.

- Degradation cost:

$$\mathfrak{g}_{cell}(I,V) = P_{cell}\Delta T V \mathfrak{d}_{cell}(I,V)$$

  where P_{cell} is the price ($/Wh) of battery cell capacity.
- Over the useable state of charge (SoC) range, $V \approx V_{nom}$.
- Battery degradation cost can be expressed as:

$$\begin{aligned} Cost_{degrad,nt} &= g_{cell,n}(u_{nt}) = M_n \mathfrak{g}_{cell}\left(\frac{10^3 u_{nt}}{M_n V_{nom}}, V_{nom}\right) \\ &= a_n u_{nt}^2 + b_n u_{nt} + c_n \end{aligned}$$

Centralized formulation

System cost:

$$J(\boldsymbol{u}) \triangleq \sum_{t\in\mathcal{T}} \left\{ c\left(d_t + \sum_{n\in\mathcal{N}} u_{nt}\right) + \sum_{n\in\mathcal{N}} g_{nt}(u_{nt}) \right\} - \sum_{n\in\mathcal{N}} \left\{ h_n\left(\|\boldsymbol{u}_n\|_1\right) \right\}$$

where:

- $\boldsymbol{u}_n \in \mathcal{U}_n$ for all $n \in \mathcal{N}$.
- $c(\cdot)$ gives the generation cost with respect to the total demand $d_t + \sum_{n\in\mathcal{N}} u_{nt}$, and d_t denotes the aggregate inelastic base demand at time t.
- $g_{nt}(u_{nt}) = g_{demand,nt}(u_{nt}) + g_{cell,n}(u_{nt})$ captures the demand charge and battery degradation cost of the n-th PEV.
- $h_n\left(\|\boldsymbol{u}_n\|_1\right)$ denotes the benefit function of the n-th PEV with respect to the total energy delivered over the charging horizon, with:

$$h_n\left(\|\boldsymbol{u}_n\|_1\right) = -\delta_n(\|\boldsymbol{u}_n\|_1 - \Gamma_n)^2$$

Centralized formulation (continued)

- Utility function of the n-th PEV:

$$v_n(\boldsymbol{u}_n) \triangleq h_n(\|\boldsymbol{u}_n\|_1) - \sum_{t\in\mathcal{T}} g_{nt}(u_{nt})$$

- The system cost $J(\boldsymbol{u})$ can be rewritten:

$$J(\boldsymbol{u}) = \sum_{t\in\mathcal{T}} c\left(d_t + \sum_{n\in\mathcal{N}} u_{nt}\right) - \sum_{n\in\mathcal{N}} v_n(\boldsymbol{u}_n)$$

- Assume the electricity generation cost can be approximated by:

$$c(y_t) = \frac{1}{2}\mathrm{a}y_t^2 + \mathrm{b}y_t + \mathrm{c}$$

- Marginal generation cost:

$$p(y_t) \triangleq c'(y_t) = \mathrm{a}y_t + \mathrm{b}$$

Centralized optimization

Optimization problem:

$$\min_{\mathbf{u}\in\mathcal{U}} J(\boldsymbol{u})$$

Assumptions:

(A1) $c(\cdot)$ is monotonically increasing, strictly convex and differentiable.

(A2) $g_{nt}(\cdot)$, for all $n \in \mathcal{N}$, $t \in \mathcal{T}$, is monotonically increasing, strictly convex and differentiable.

Optimization solution

- The efficient (socially optimal) charging behavior is unique and can be characterized by the associated KKT conditions:

$$\frac{\partial}{\partial u_{nt}} J(\boldsymbol{u}) \geq 0, \quad u_{nt} \geq 0, \quad \frac{\partial}{\partial u_{nt}} J(\boldsymbol{u}) u_{nt} = 0$$

for all $n \in \mathcal{N}$ and $t \in \mathcal{T}_n$, where:

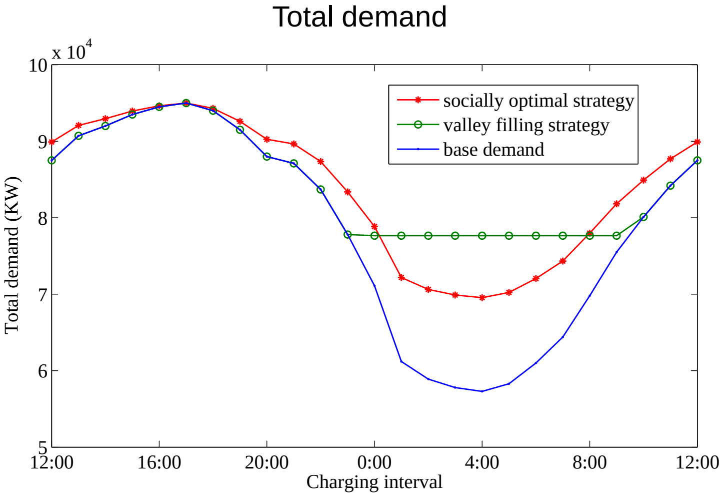
$$\frac{\partial}{\partial u_{nt}} J(\boldsymbol{u}) = c'\big(d_t + \sum_{n \in \mathcal{N}} u_{nt}\big) - \frac{\partial}{\partial u_{nt}} v_n(\boldsymbol{u}_n)$$

- The optimal charging behavior $\boldsymbol{u}^{**}$ is uniquely specified by:

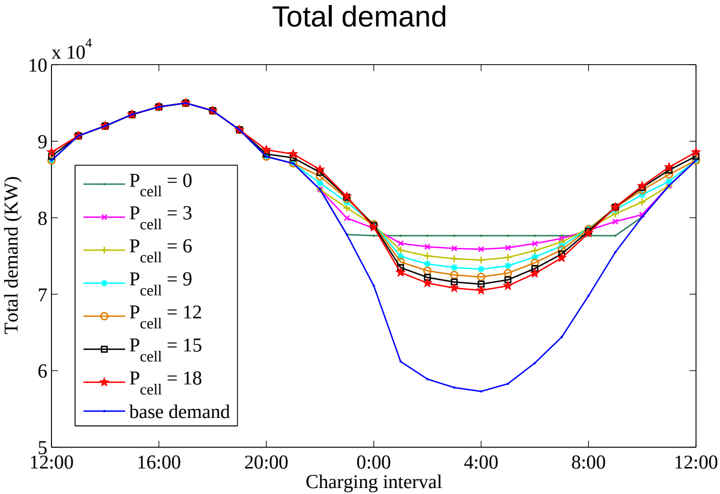
$$p_t^{**} \begin{cases} = \frac{\partial}{\partial u_{nt}} v_n(\boldsymbol{u}_n^{**}), & \text{when } u_{nt}^{**} > 0, \\ \geq \frac{\partial}{\partial u_{nt}} v_n(\boldsymbol{u}_n^{**}), & \text{when } u_{nt}^{**} = 0, \end{cases}$$

where $p_t^{**} = c'\big(d_t + \sum_{n \in \mathcal{N}} u_{nt}^{**}\big)$.

Example

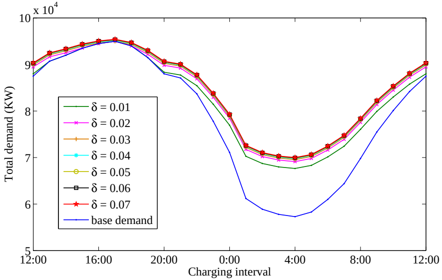

Total demand
x 10^4
10
9
8
7
6
5
Total demand (KW)
12:00
16:00
20:00
0:00
4:00
8:00
12:00
Charging interval
socially optimal strategy
valley filling strategy
base demand

Example - varying P_{cell}

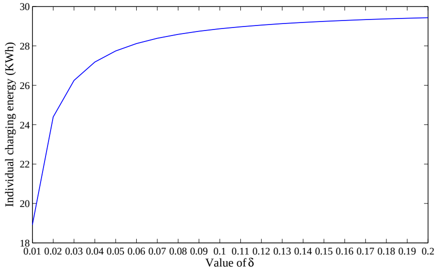


Example - varying terminal penalty, δ_n

Total demand



Delivered energy



Decentralized charging coordination

(S1) Each PEV autonomously determines its optimal charging strategy with respect to a given electricity price profile $\boldsymbol{p} \equiv (p_t, t \in \mathcal{T})$. This optimal strategy takes into account the trade-off between the electricity cost and local (demand and battery degradation) costs over the entire charging horizon.

(S2) The electricity price profile $\boldsymbol{p}$ is updated to reflect the latest charging strategies determined by the PEV population in (S1).

(S3) Steps (S1) and (S2) are repeated until the change in the price profile at (S2) is negligible.

Using an appropriate individual cost function and price update mechanism, (S1)-(S3) is convergent and achieves the socially optimal (centralized) solution.

Individual cost function

$$J_n(\boldsymbol{u}_n;\boldsymbol{p}) \triangleq \sum_{t\in\mathcal{T}} p_t u_{nt} - v_n(\boldsymbol{u}_n) = \sum_{t\in\mathcal{T}} \Big\{ p_t u_{nt} + g_{nt}(u_{nt}) \Big\} - h_n\Big(\sum_{t\in\mathcal{T}} u_{nt}\Big)$$

- Cost is composed of energy cost, local (demand and battery degradation) cost, and the benefit derived from the total delivered energy.
- The optimal charging strategy of the n-th PEV, with respect to $\boldsymbol{p}$:

$$\boldsymbol{u}_n^*(\boldsymbol{p}) = \operatorname*{argmin}_{\boldsymbol{u}_n\in\mathcal{U}_n} J_n(\boldsymbol{u}_n;\boldsymbol{p})$$

- This optimal response has the form:

$$u_{nt}(\boldsymbol{p}, A_n) = \max\big\{0, [g'_{nt}]^{-1}(A_n - p_t)\big\}, \quad t\in\mathcal{T}$$

  for some A_n, where g'_{nt} is the derivative of g_{nt}, and $[g'_{nt}]^{-1}$ denotes the corresponding inverse function.

Optimal form

- Define: $\mathcal{U}_n(\omega) \triangleq \left\{ \boldsymbol{u}_n \in \mathcal{U}_n;\ \text{s.t. } \|\boldsymbol{u}_n\|_1 = \omega \right\}$.
- Minimizing $J_n(\mathbf{u_n}; \mathbf{p})$ over $\mathcal{U}_n(\omega)$ is equivalent to minimizing:

$$F_n(\boldsymbol{u}_n; \boldsymbol{p}) \triangleq \sum_{t \in \mathcal{T}} \left\{ p_t u_{nt} + g_{nt}(u_{nt}) \right\}$$

  - This is just $J_n(\mathbf{u_n}; \mathbf{p})$ with $h_n(\cdot)$ excluded.
- Define the Lagrangian function,

$$L_n(\boldsymbol{u}_n, A_n; \boldsymbol{p}) \triangleq F_n(\boldsymbol{u}_n; \boldsymbol{p}) + A_n(\omega - \|\boldsymbol{u}_n\|_1)$$

- KKT conditions:
  - (i) $\frac{\partial L_n}{\partial A_n} = 0$.
  - (ii) $\frac{\partial L_n}{\partial u_{nt}} \geq 0$, $u_{nt} \geq 0$, with complementary slackness.
- Condition (ii) gives:

$$p_t + g'_{nt}(u_{nt}) - A_n \begin{cases} = 0, & \text{when } u_{nt} > 0 \\ \geq 0, & \text{otherwise} \end{cases}$$

Determining A_n

Define the function:

$$\mathcal{A}_n(\boldsymbol{p}, \cdot) : [0, \Gamma_n] \to [A_n^-(\boldsymbol{p}), A_n^+(\boldsymbol{p})]$$

such that

$$\mathcal{A}_n(\boldsymbol{p}, \omega) = A_n \quad \Longleftrightarrow \quad \|\boldsymbol{u}_n(\boldsymbol{p}, A_n)\|_1 = \omega$$

which implies

$$\|\boldsymbol{u}_n(\boldsymbol{p}, \mathcal{A}_n(\boldsymbol{p}, \omega))\|_1 = \omega$$

- $\mathcal{A}_n(\boldsymbol{p}, \omega)$ is strictly increasing with ω.

Determining A_n (continued)

Theorem: Assume $h_n(\omega)$ is continuously differentiable, increasing and concave on $0 \leq \omega \leq \Gamma_n$. Define,

$$f_n(\boldsymbol{p}, \omega) \triangleq \mathcal{A}_n(\boldsymbol{p}, \omega) - h_n'(\omega)$$

and

$$A_n^*(\boldsymbol{p}) = \begin{cases} \mathcal{A}_n(\boldsymbol{p}, \Gamma_n), & \text{if } f_n(\boldsymbol{p}, \Gamma_n) \leq 0 \\ \mathcal{A}_n(\boldsymbol{p}, 0), & \text{if } f_n(\boldsymbol{p}, 0) \geq 0 \\ \mathcal{A}_n(\boldsymbol{p}, \omega^*), & \text{if } f_n(\boldsymbol{p}, \omega^*) = 0 \end{cases}$$

where $0 < \omega^* < \Gamma_n$. Then the charging strategy $\boldsymbol{u}_n(\boldsymbol{p}, A_n^*(\boldsymbol{p}))$ uniquely minimizes the individual PEV cost function for a given $\boldsymbol{p}$, i.e. $\boldsymbol{u}_n^*(\boldsymbol{p}) = \boldsymbol{u}_n(\boldsymbol{p}, A_n^*(\boldsymbol{p}))$.

Price profile update mechanism

- Let

$$p_t^+(\boldsymbol{p}) = p_t + \eta\Big(c'\big(d_t + \sum_{n\in\mathcal{N}} u_{nt}^*(\boldsymbol{p})\big) - p_t\Big), \quad t \in \mathcal{T}$$

  where $\eta > 0$ is a fixed parameter, and $\boldsymbol{u}_n^*(\boldsymbol{p})$ is the optimal charging strategy for the n-th PEV with respect to $\boldsymbol{p}$.
- Define ν_{nt} as the Lipschitz constant for the function $[g'_{nt}]^{-1}(\cdot)$ over the interval $[g'_{nt}(0), g'_{nt}(\Gamma_n)]$, and:

$$\nu = \max_{n\in\mathcal{N}, t\in\mathcal{T}} \nu_{nt}$$

- Define κ as the Lipschitz constant for $c'(\cdot)$ over the typical range in the total demand.
- Then assuming the terminal valuation function h_n is increasing and strictly concave:

$$\|\boldsymbol{u}_n^*(\boldsymbol{p}) - \boldsymbol{u}_n^*(\boldsymbol{\varrho})\|_1 \leq 2\nu\|\boldsymbol{p} - \boldsymbol{\varrho}\|_1$$

Convergence of the algorithm

Theorem: Suppose $\alpha \equiv |1-\eta| + 2N\kappa\nu\eta < 1$ and consider any initial charging price $\boldsymbol{p}^{(0)}$. Then the decentralized algorithm converges to the efficient (centralized) solution $\boldsymbol{u}^{**}$. Moreover, for any $\varepsilon > 0$, the system converges to a price profile $\boldsymbol{p}$, such that $\|\boldsymbol{p} - \boldsymbol{p}^{**}\|_1 \le \varepsilon$, in $K(\varepsilon)$ iterations, with

$$K(\varepsilon) = \left\lceil \frac{1}{\ln(\alpha)} \Big(\ln(\varepsilon) - \ln(T) - \ln(\varrho_{max}) \Big) \right\rceil$$

where ϱ_{max} denotes the maximum possible price, and $\lceil x \rceil$ represents the minimal integer value larger than or equal to x.

Remarks

- In practice, convergence takes many fewer iterations than the upper bound.
- The upper bound on the iteration count, $K(\varepsilon)$, is on the order of $O(|\ln(\varepsilon)|)$, and is independent of the size of the PEV population.
- The proof establishes that

$$\|\boldsymbol{p}^{+} - \boldsymbol{\varrho}^{+}\|_1 < \|\boldsymbol{p} - \boldsymbol{\varrho}\|_1$$

  so the price update operator $\boldsymbol{p}^{+}(\boldsymbol{p})$ is a contraction map.
- By the contraction mapping theorem, the price $\boldsymbol{p}^{(k)}$ converges to a unique price profile $\boldsymbol{p}^{*}$ from any initial price profile $\boldsymbol{p}^{(0)}$, such that $\boldsymbol{p}^{+}(\boldsymbol{p}^{*}) = \boldsymbol{p}^{*}$ and so:

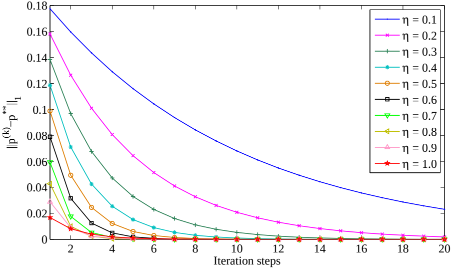
$$c'\Big(d_t + \sum_{n \in \mathcal{N}} u_{nt}^{*}(\boldsymbol{p}^{*})\Big) = p_t^{*}, \quad \text{for all } t \in \mathcal{T}$$

- The price converges to the generation marginal cost.

Illustration - convergence

Evolution of $\|\boldsymbol{p}^{(k)} - \boldsymbol{p}^{**}\|_1$ for various values of the price update parameter η.

- Convergence is guaranteed for $0 < \eta < 1.017$.



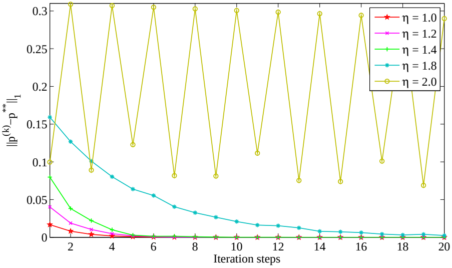
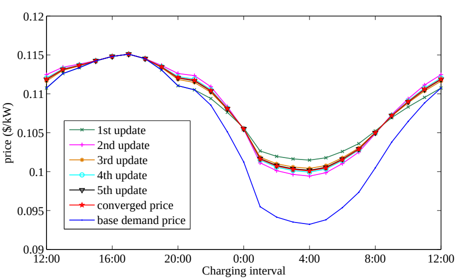


Illustration - algorithm updates

Price update parameter $\eta = 1$.

Price



Total demand

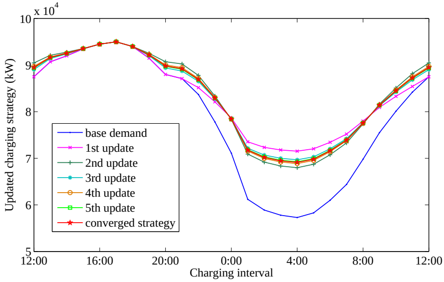
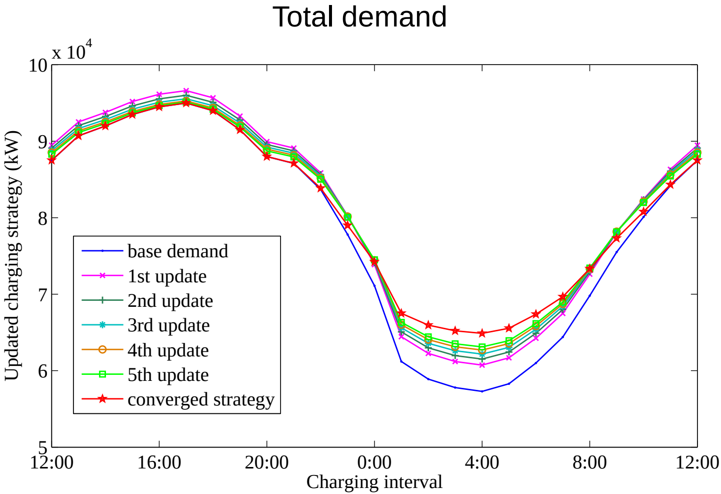


Illustration - heterogeneous population



Conclusions

- The proposed price-based strategy for coordinating the charging of a large population of PEVs:
  - Establishes a trade-off between the cost of energy and battery degradation.
  - Incorporates a charge that penalizes high demand, thereby assisting in mitigating high coincident charging on local distribution networks.
- An iterative decentralized algorithm:
  - Converges to the unique efficient collection of charging strategies.
  - At convergence, the price profile coincides with the generator marginal cost.