Optical Flow and Dense Correspondence

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Overview

Optical Flow Estimation

Dense Elastic Shape Matching
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Dense Elastic Shape Matching
Variational Optical Flow

\[
\min_{u: \Omega \rightarrow \mathbb{R}^2} \int_{\Omega} |f_1(x) - f_2(x + u)| \, dx + J(u)
\]

Input video

Optical flow field
Variational optical flow estimation

\[ \min_{u: \Omega \rightarrow \mathbb{R}^2} \int_{\Omega} \left| f_1(x) - f_2(x + u) \right| \, dx + J(u) \]

Variational optical flow estimation

Horn, Schunck ‘81

Non-quadratic regularization

Nagel, Enkelmann ‘86, Herve, Shulman ‘89

Black, Anandan ‘93

Coarse to fine warping

Memin, Perez ‘98, Brox et al. ‘04, Zach et al. ’07

Adaptive regularization

Wedel et al. ’09, Werlberger et al. ‘10, …
Motion Layer Decomposition

Input video

Synthesized video

Schoenemann & Cremers TIP ’12
Solution by linearization & coarse-to-fine warping does not provide optimality guarantees.
Optical Flow as a Multilabel Problem

Optical flow: \[
\min_{u: \Omega \rightarrow \Gamma \subset \mathbb{R}^d} \int_{\Omega} |f_1(x) - f_2(x + u)| \, dx + J(u)
\]

Challenge: Thousands of labels cannot be handled in previous relaxations.

Goldluecke, Cremers ECCV ’10, Strekalovskiy et al. ICCV ’11
Large Label Spaces

Optical flow: \[
\min_{u: \Omega \rightarrow \mathbb{R}^d} \int_{\Omega} \left| f_1(x) - f_2(x + u) \right| dx + J(u)
\]

Challenge: Thousands of labels cannot be handled in previous relaxations.

Goldluecke, Cremers  *ECCV '10*,  Strekalovskiy et al.  *ICCV '11*
Convex Optical Flow

\[ \min_{u: \Omega \to \Gamma} E_{\text{data}}(u) + E_{\text{reg}}(u) = \min_{u: \Omega \to \Gamma} \int_{\Omega} \rho(x, u) \, dx + \sum_{i=1}^{d} J(u_i) \]

Introduce: \( v_i(x, \gamma_i) := \delta(u_i(x) - \gamma_i) \quad \forall \ i \in \{1, \ldots, d\}, \gamma_i \in \Lambda_i \)

\[ \min_{v_1, \ldots, v_d} \int_{\Omega \times \Gamma} \rho(x, \gamma) \prod_{i=1}^{d} v_i(x, \gamma_i) \, dx \, d\gamma = \min_{v_1, \ldots, v_d} \sup_{q \in Q} \left\{ \sum_{i=1}^{d} \int_{\Omega \times \Lambda_i} q_i v_i \, dx \, d\gamma_i \right\} \]

with: \( Q = \left\{ (q_i: \Omega \times \Lambda_i \to \mathbb{R})_{i=1}^{d} \mid \sum_{i=1}^{d} q_i(x, \gamma_i) \leq \rho(x, \gamma) \quad \forall x, \gamma \right\} \)

Goldluecke, Cremers ECCV ’10, Strekalovskiy et al. ICCV ’11
Convex Optical Flow

Experimental optimality bounds $\sim 3\% - 5\%$

Goldluecke, Cremers ECCV ’10, Strekalovskiy et al. ICCV ’11
Overview

Optical Flow Estimation

Dense Elastic Shape Matching

Windheuser et al., ICCV ‘11
Robustness to Articulation / Deformation
Robustness to Missing Parts
The 3D Shape Matching Problem

- Favor meaningful correspondences
- Allow for stretching, shrinking and bending
- Assure geometric consistency
- Optimal or near-optimal solutions
Some Related Work

Modeling:
- physics-based: Litke et al., SGP '05, Wirth et al., EMMCVPR '09
- Gromov-Hausdorff: Memoli, Sapiro, Found. Comp. Math. '05, Bronstein et al., PNAS '06
- Feature descriptors: Sun et al., SGP '09, Aubry et al., 4DMOD '11

Diffeomorphic matching:
- Kurtek et al., CVPR '10

Combinatorial point matching:
- Torresani et al., ECCV '08
- Zeng et al., CVPR '10
- Lipman, Daubechies, Adv. Math. '11

3D shape benchmarks:
- Sumner, Popovic '04, Vlasic et al. '08, Boyer et al. '11
The 3D Shape Matching Problem

Matching \( \varphi : (X \subset \mathbb{R}^3) \rightarrow (Y \subset \mathbb{R}^3) \)

Optimum \( \varphi^* = \arg \min_{\varphi \in \text{Diff}^+(X,Y)} E(\varphi) + E(\varphi^{-1}) \)

Thin shell energy \( E(\varphi) = E_{\text{bend}}(\varphi) + E_{\text{stretch}}(\varphi) \) [Koiter 1966]

Non-convex optimization problem!
Graph Representation of Matching

Matching: $\varphi: (X \subset \mathbb{R}^3) \rightarrow (Y \subset \mathbb{R}^3)$

Graph: $\Gamma_\varphi = \{(x, \varphi(x)) \mid x \in X\} \subset X \times Y$, $\pi_X: \Gamma_\varphi \rightarrow X$, $\pi_Y: \Gamma_\varphi \rightarrow Y$

Optimum: $\Gamma^* = \arg \min_\Gamma E(\Gamma)$ → 2D closed minimal surface in 4D

Polynomial-time solutions?
Planar Elastic Shape Matching

Matching \( \varphi : X \rightarrow Y \)

Solution: Shortest cyclic and monotonous path on a torus.

\[ \Gamma^* = \arg \min_{\Gamma} E(\Gamma) \]

Schmidt, Farin, Cremers, ICCV 2007: Fast shape matching in sub-cubic runtime
Shortest Path as Integer Linear Program

Integer Linear Program:

\[
\begin{align*}
\min_{\Gamma \in \{0,1\}^N} & \quad E^\top \Gamma \\
\text{subject to} & \quad \begin{pmatrix} \partial \\ \pi_X \\ \pi_Y \end{pmatrix} \cdot \Gamma = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.
\end{align*}
\]
Extension to 3D Shape Matching
Matching of Triangle Pairs

\[ \Gamma_i = \begin{cases} 
1, & \text{match} \\
0, & \text{no match} 
\end{cases} \]

Windheuser et al., ICCV 2011
Allow for Shrinking / Expansion

\[ \Gamma_j = \begin{cases} 
1, & \text{match} \\
0, & \text{no match} 
\end{cases} \]

*Windheuser et al., ICCV 2011*
Integer Linear Program

Indicator variable $\Gamma_i \in \{0, 1\}$ for each possible basic correspondence.

Assignment cost $E_i \in \mathbb{R}$ for each basic correspondence.

Determine best matching $\Gamma = (\Gamma_1, \ldots, \Gamma_N) \in \{0, 1\}^N$ by solving

$$\min_{\Gamma \in \{0,1\}^N} \sum_{i=1}^{N} \Gamma_i E_i$$

subject to

$$\begin{pmatrix} \delta \\ \pi_X \\ \pi_Y \end{pmatrix} \cdot \Gamma = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$ 

Solve relaxed problem & iteratively binarize variables until binary solution.
Computed Matching (3360 triangles)
Surface Matching (1504 triangles)
Surface Matching (2456 triangles)
Surface Matching (2440 triangles)
Surface Matching (3620 triangles)
Runtime Comparison

- Eckstein-Bertsekas
- Interior Point

Runtime in Seconds vs. Number of Faces
Surface Matching (6454 triangles)
Matching with Missing Parts
Comparison to Point Matching Methods

Bronstein et al. point matching

- inversion: 12/30, partial inversion: 8/30
- mean geodesic error: 0.079

Proposed surface matching

- Preserves orientation
- mean geodesic error: 0.03
Optical Flow Estimation

- variational methods
- fast coarse-to-fine algorithms
- near-optimal solutions by convex relaxation

Elastic 3D Shape Matching

- Dense matching via LP relaxation
- Stretching, shrinking and bending
- Requires no initialization