# Structure Prediction for 3D Scene Understanding 

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## Which problems we will be looking at?

- 3D Layout estimation
- 3D object detection


## A little bit about structure prediction

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- If you know how to do inference you will know how to do learning! Where does the complication come from?


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First task: 3D indoor scene understanding

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- Orientation maps [Leet el al 09], geometric context [Hoiem et al. 05]

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## Integral Images

- We are interested in computing the sum of some features inside a rectangle, and we want to vary the rectangle
- How can we do this efficiently?
- Compute the sum area table, also called integral image

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| :--- | :--- | :--- | :--- | :--- |
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## Generalization to 3D

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- This is called integral geometry [Schwing et al. 12a]
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- We can now write the problem in terms of potentials of order at most 2

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and $r$ only contains sets of 2 random variables

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- Life is a bit more complicated than what I showed you as I was varying the parameterization to make you understand easily
- Good news is that it still depends on pairwise potentials (which are accumulators) but there is quite a few more
- Some of this $r$ share the same weights, as they come from the integral geometry.
- If they are not shared then they do not represent the same problem
- This speed ups the message passing inference by a few orders of magnitude


## What are the implications?

- We can now write the problem in terms of potentials of order at most 2

$$
E\left(y_{1}, \cdots, y_{4}\right)=\sum_{r} \mathbf{w}_{r}^{T}\left(\mathbf{y}_{r}, \mathbf{x}\right)
$$

and $r$ only contains sets of 2 random variables

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## Exact Inference?

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- Message passing will not give the optimal
- What other algorithms do you know that give the optimal solution?
- Let's look at branch and bound


## Branch and Bound

```
Algorithm 1 branch and bound (BB) inference
    put pair \((\bar{f}(\mathcal{Y}), \mathcal{Y})\) into queue and set \(\hat{\mathcal{Y}}=\mathcal{Y}\)
    repeat
        split \(\hat{\mathcal{Y}}=\hat{\mathcal{Y}}_{1} \times \hat{\mathcal{Y}}_{2}\) with \(\hat{\mathcal{Y}}_{1} \cap \hat{\mathcal{Y}}_{2}=\emptyset\)
        put pair \(\left(\bar{f}\left(\hat{\mathcal{Y}}_{1}\right), \hat{\mathcal{Y}}_{1}\right)\) into queue
        put pair \(\left(\bar{f}\left(\hat{\mathcal{Y}}_{2}\right), \hat{\mathcal{Y}}_{2}\right)\) into queue
        retrieve \(\hat{\mathcal{Y}}\) having highest score
    until \(|\hat{\mathcal{Y}}|=1\)
```

We have to define:
(1) A parameterization that defines sets of hypothesis.
(2) A scoring function $f$
(3) Tight bounds on the scoring function that can be computed very efficiently

## Parameterization of the Problem

- Layout with 4 variables $y_{i} \in \mathcal{Y}, i \in\{1, \ldots, 4\}$ [Lee et al. 09]
- How do we define $\mathcal{Y}$ ?
- Is this problem continuous or discrete?

- We parameterize the sets by intervals of minimum and maximum angles

$$
\left\{\left[y_{1}^{\min }, y_{1}^{\max }\right], \cdots,\left[y_{4}^{\min }, y_{4}^{\max }\right]\right\}
$$

- Why intervals?
- We have defined already the scoring function. What about the bounds?


## Properties of the Bounds

Derive bounds $\bar{f}$ for the original scoring function $\mathbf{w}^{\top} \phi(\mathbf{y}, \mathbf{x})$ that satisfy:
(1) The bound of the interval $\hat{\mathcal{Y}}$ has to upper-bound the true cost of each hypothesis $y \in \hat{\mathcal{Y}}$,

$$
\forall y \in \hat{\mathcal{Y}}, \quad \bar{f}(\hat{\mathcal{Y}}) \geq \mathbf{w}^{\top} \phi(\mathbf{y}, \mathbf{x})
$$

(2) The bound has to be exact for every single hypothesis,

$$
\forall y \in \mathcal{Y}, \quad \bar{f}(y)=\mathbf{w}^{\top} \phi(\mathbf{y}, \mathbf{x}) .
$$

Can we define this for our problem?

## Intuitions from 2D

Let's look at the 2D case again

- We want to compute the bounding box that maximizes a scoring function
- Let's try to do this with branch and bound
- We define an interval as the max and min of the $x$ and $y$ axis of the rectangle

- The scoring function sums features in the rectangle defined by the BBox

$$
E\left(y_{1}, \cdots, y_{4}\right)=\sum_{i \in B B o x(\mathbf{y})} f_{i}(\mathbf{x})
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$$
E\left(y_{1}, \cdots, y_{4}\right)=\sum_{i \in \operatorname{BBox}(\mathbf{y})} f_{i}(\mathbf{x})
$$

## Branch and Bound for BBox prediction

- The scoring function sums features in the rectangle defined by the BBox

$$
E\left(y_{1}, \cdots, y_{4}\right)=\sum_{i \in B B o x(\mathbf{y})} f_{i}(\mathbf{x})
$$

- Some features are positive and some are negative
- Trick: Divide the space into negative and positive features

$$
E\left(y_{1}, \cdots, y_{4}\right)=\underbrace{\sum_{i \in B B o x(\mathbf{y})} f_{i}^{+}(\mathbf{x})}_{f^{+}(\mathbf{y}, \mathbf{x})}+\underbrace{\sum_{i \in B B o x(\mathbf{y})} f_{i}^{-}(\mathbf{x})}_{f^{-}(\mathbf{y}, \mathbf{x})}
$$

$\Rightarrow$ show an illustration

- Bound the positive and negative independently

$$
\operatorname{bound}(E(\overline{\mathcal{Y}}))=\bar{f}^{+}(\overline{\mathcal{Y}}, \mathbf{x})+\bar{f}^{-}(\overline{\mathcal{Y}}, \mathbf{x})
$$

## Bounding the functions

- Energy was defined as

$$
E\left(y_{1}, \cdots, y_{4}\right)=\underbrace{\sum_{i \in B B o x(y)} f_{i}^{+}(\mathbf{x})}_{f^{+}(\mathbf{y}, \mathbf{x})}+\underbrace{\sum_{i \in B B o x(\mathbf{y})} f_{i}^{-}(\mathbf{x})}_{f^{-}(\mathbf{y}, \mathbf{x})}
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- These bounds are very simple? What are they?


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- How can we compute them very fast?


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- How many integral images do we need?


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- How many integral images do we need?


## Algorithm for 2D BBox [Lampert et al. 06]

```
Algorithm 1 Efficient Subwindow Search
Require: image \(x\)
Require: quality bounding function \(\hat{f}\) (see Sect.III)
Ensure: \(\left(t_{\text {opt }}, b_{\text {opt }}, l_{\text {opt }}, r_{\text {opt }}\right)=\operatorname{argmax}_{y \in \mathcal{Y}} f(y)\)
    initialize \(P\) as empty priority queue
    set \([T, B, L, R]=[1, n] \times[1, n] \times[1, m] \times[1, m]\)
    repeat
        split \([T, B, L, R] \rightarrow\left[T_{1}, B_{1}, L_{1}, R_{1}\right] \dot{\cup}\left[T_{2}, B_{2}, L_{2}, R_{2}\right]\)
        push \(\left(\left[T_{1}, B_{1}, L_{1}, R_{1}\right] ; \hat{f}\left(\left[T_{1}, B_{1}, L_{1}, R_{1}\right]\right)\right.\) onto \(P\)
        push ( \(\left[T_{2}, B_{2}, L_{2}, R_{2}\right] ; \hat{f}\left(\left[T_{2}, B_{2}, L_{2}, R_{2}\right]\right)\) onto \(P\)
        retrieve top state \([T, B, L, R]\) from \(P\)
    until \([T, B, L, R]\) consists of only one rectangle
    set \(\left(t_{\mathrm{opt}}, b_{\mathrm{opt}}, l_{\mathrm{opt}}, r_{\mathrm{opt}}\right)=[T, B, L, R]\)
```

- How do we split?

- When do we terminate?


## 3D layout estimation

- Let's go back to our problem

- We parameterize the sets by intervals of minimum and maximum angles

$$
\left\{\left[y_{1}^{\min }, y_{1}^{\max }\right], \cdots,\left[y_{4}^{\min }, y_{4}^{\max }\right]\right\}
$$

- The scoring function sums features over the faces

$$
E\left(y_{1}, \cdots, y_{4}\right)=\sum_{r} \mathbf{w}_{r}^{T} \phi\left(\mathbf{y}_{r}, \mathbf{x}\right)=\sum_{\alpha} f_{\alpha}(\mathbf{y}, \mathbf{x})
$$

with $\alpha=\{$ floor, left_w, right_w, ceiling, front_w $\}$

- What about the bounds?


## Bounds for 3D layout

- The scoring function sums features over the faces

$$
E\left(y_{1}, \cdots, y_{4}\right)=\sum_{r} \mathbf{w}_{r}^{T} \phi\left(\mathbf{y}_{r}, \mathbf{x}\right)=\sum_{\alpha} f_{\alpha}(\mathbf{y}, \mathbf{x})
$$

with $\alpha=\{$ floor, left_w, right_w, ceiling, front_w $\}$

- Let's bound each "face" $\alpha$ separately
- Recall where the features come from

original image

orientation map

geometric context
- Some features are positive, some are negative. Why? How do I know which ones are positive/negative?


## Deriving bounds

- Inference can be then done by

$$
E\left(y_{1}, \cdots, y_{4}\right)=\sum_{\alpha} f_{\alpha}^{+}(x, y)+f_{\alpha}^{-}(x, y)
$$

- We can bound each of this terms separately

$$
\operatorname{bound}(E(\hat{\mathcal{Y}}, \mathbf{x}))=\sum_{\alpha \in \mathcal{F}} \bar{f}_{\alpha}^{+}(\hat{\mathcal{Y}}, \mathbf{x})+\bar{f}_{\alpha}^{-}(\hat{\mathcal{Y}}, \mathbf{x})
$$

- We construct bounds by computing the max positive and min negative contribution of the score within the set $\hat{\mathcal{Y}}$ for each face $\alpha \in \mathcal{F}$.

$$
\bar{f}_{\text {front-wall }}(\hat{\mathcal{Y}})=f_{\text {front-wall }}^{+}\left(x, y_{\text {up }}\right)+f_{\text {front-wall }}^{-}\left(x, y_{\text {low }}\right),
$$




## Efficient bounds

- How can we compute the bounds efficiently?


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- How can we compute the bounds efficiently?

- What's the complexity?
- How many evaluations?


## Efficient bounds

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## Results

> [A. Schwing and R. Urtasun, ECCV12]

Table: Pixel classification error in the layout dataset of [Hedau et al. 09].

|  | OM | GC | OM + GC | Other | Time |
| :---: | :---: | :---: | :---: | :---: | :---: |
| [Hoiem07] | - | 28.9 | - | - | - |
| Hedau09] $(\mathrm{a})$ | - | 26.5 | - | - | - |
| [Hedau09] (b) | - | 21.2 | - | - | $10-30 \mathrm{~min}$ |
| [Wang10] | 22.2 | - | - | - |  |
| [Lee10] | 24.7 | 22.7 | 18.6 | - | - |
| [delPero11] | - | - | - | 16.3 | 12 min |
| Ours | $\mathbf{1 8 . 6}$ | $\mathbf{1 5 . 4}$ | $\mathbf{1 3 . 6}$ | - | 0.007 s |

Table: Pixel classification error in the bedroom data set [Hedau et al. 10].

|  | [delPero11] | [Hoiem07] | [Hedau09](a) | Ours |
| :---: | :---: | :---: | :---: | :---: |
| w/o box | 29.59 | 23.04 | 22.94 | $\mathbf{1 6 . 4 6}$ |

- Takes on average 0.007 s for exact solution over $50^{4}$ possibilities !
- It's 6 orders of magnitude faster than the state-of-the-art!


## Qualitative Results



Let's try to detect objects in 3D

## 3D Object Detection

- Task: Given an image (e.g., rgb, rgbd, video), detect the 3D objects present in the scene


Figure: Image from [Jia et al. 13]

## Contextual Models for 3D Object Detection

- Simple approach: Imagine you were able "somehow" to get candidate 3D bounding boxes
- Task: identify the object labels labels (e.g., bed, table) as well as which ones are outliers
- Objects are not independent!
- This is however the assumption of most object detectors (both 2D and 3D)
- Can we create a model which reasons about multiple objects?



## Contextual Models for 3D Object Detection

- What would be the random variables?
- For each bounding box, $y_{i} \in\{0,1\}$ saying whether it is correct or not, or $y_{i} \in\{0,1, \cdots, C\}$
- When to use which parameterization?
- We can then write the energy

$$
E\left(y_{1}, \cdots, y_{n}\right)=\sum_{r} \mathbf{w}^{T} \phi_{r}(\mathbf{y}, \mathbf{x})
$$

- What would you encode in the potentials?
- What's the underlying graph?


## Our prior knowledge about the problem

If we have 3D blocks, then physics can be used to constrained object location, orientation, size, etc

- Stability: blocks are put such that they don't fell [Gupta et al. 10, Jia et al. 13]

- Support: a cup is on the table, but a table is not on a cup [Silberman et al. 12, Jia et al. 13]
- Semantic coherence: co-occurance of objects [Jia et al. 13, Lin et al. 13]


## Our prior knowledge about the problem

If we have 3D blocks, then physics can be used to constrained object location, orientation, size, etc

- Location: where objects appear with respect to the room [Hedau?] and each other [Lin et al. 13]



## Our prior knowledge about the problem

If we have 3D blocks, then physics can be used to constrained object location, orientation, size, etc

- Size coherence: Relative scale of objects [Lin et al. 13]
- High level context: type of scene, e.g., a cow can't be in a living room [Lin et al. 13]
- Layout: objects do not penetrate the room layout [Lee10, Hedau 10, Schwing12a, delPero12]
- more?


## Inference

- All these things mentioned are pairwise potentials (i.e., relations between two objects)
- If those are sub modular, use graph cuts!
- If not message passing
- In any case, you can do inference in ms [Lin et al. 13]
- Use standard methods for learning, e.g., CRF log loss or structured SVMs


## Results



