# From natural scene statistics to models of neural coding and representation (part 2)

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REDWOOD CENTER
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# Today’s talk

Group Sparse Coding

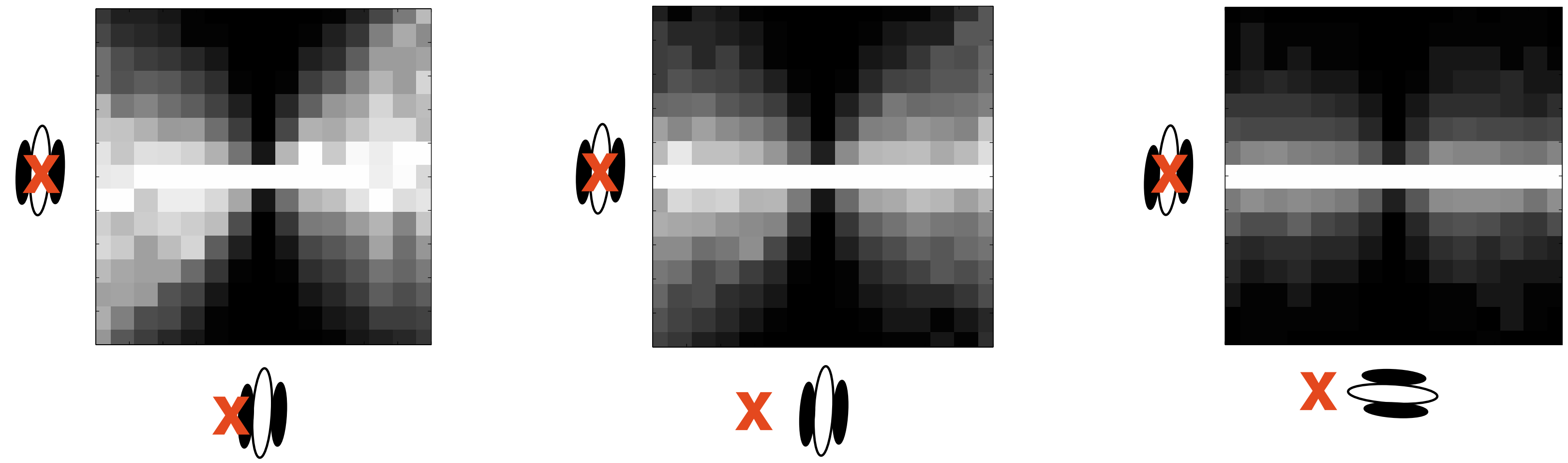
Hierarchical Models

# Group sparse coding

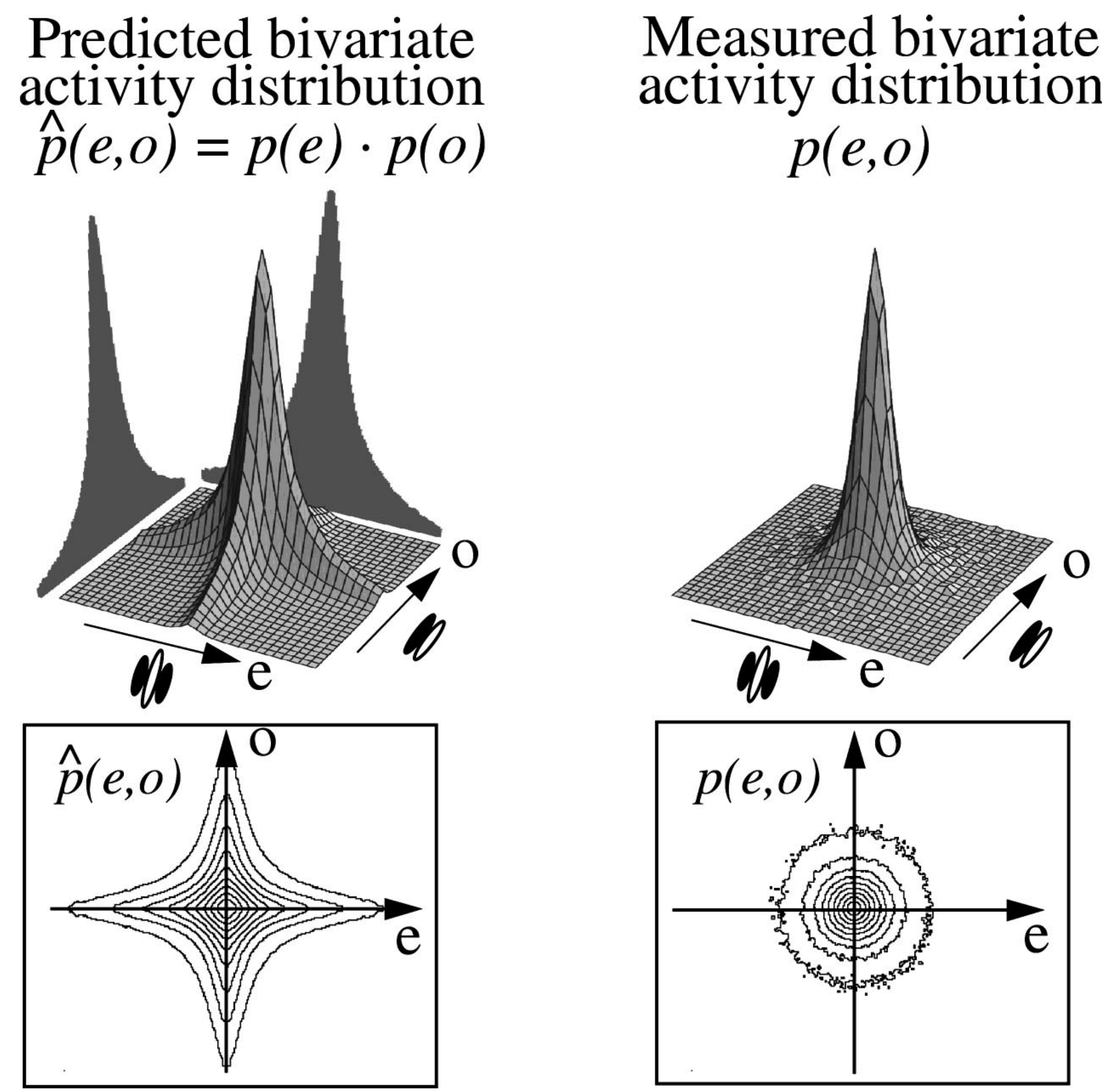
Sparse components exhibit statistical dependencies.

How to model this structure?

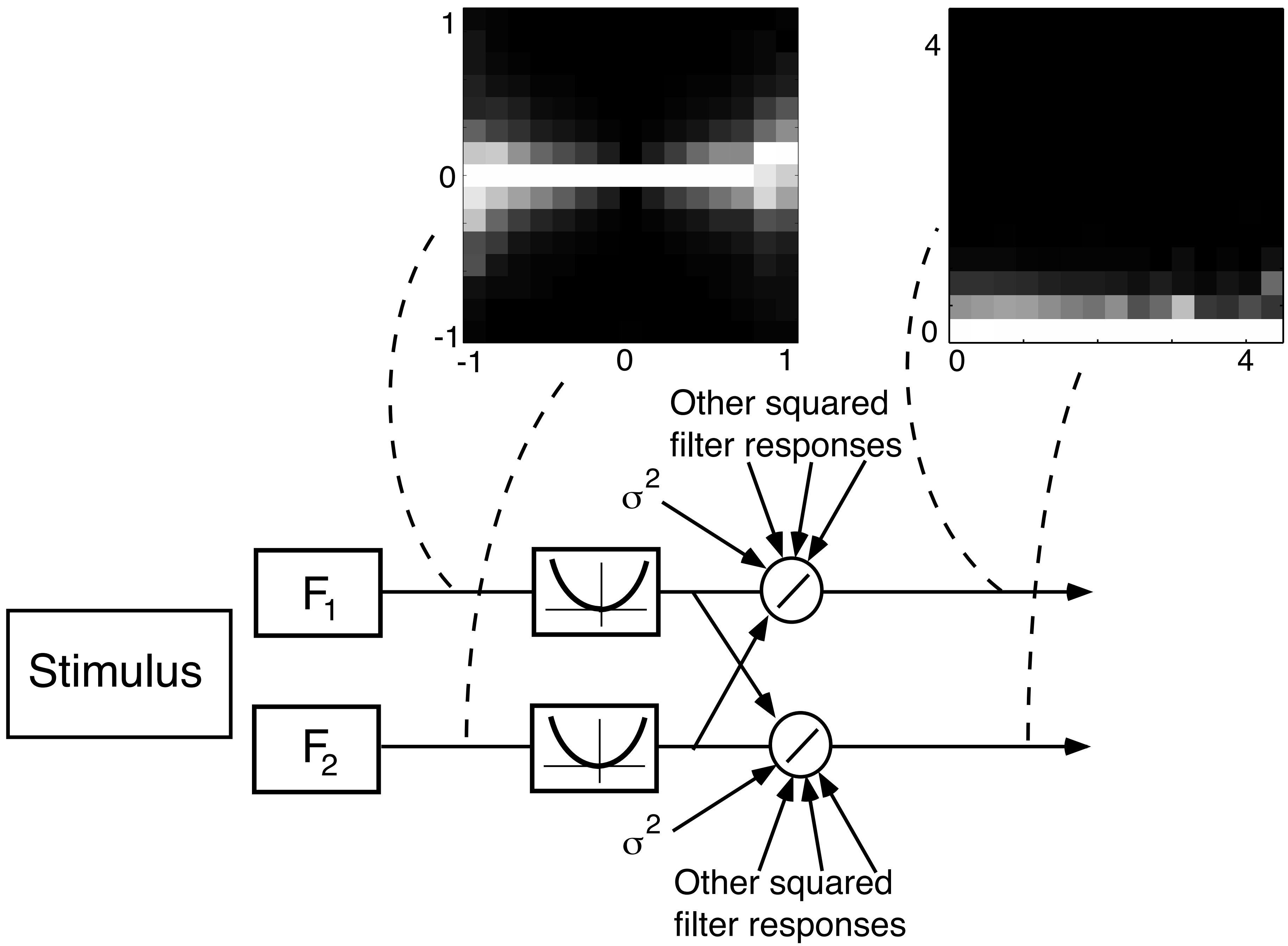
Simoncelli (1997)
‘bow ties’



Zetzsche (2000)
circularly symmetric
distributions

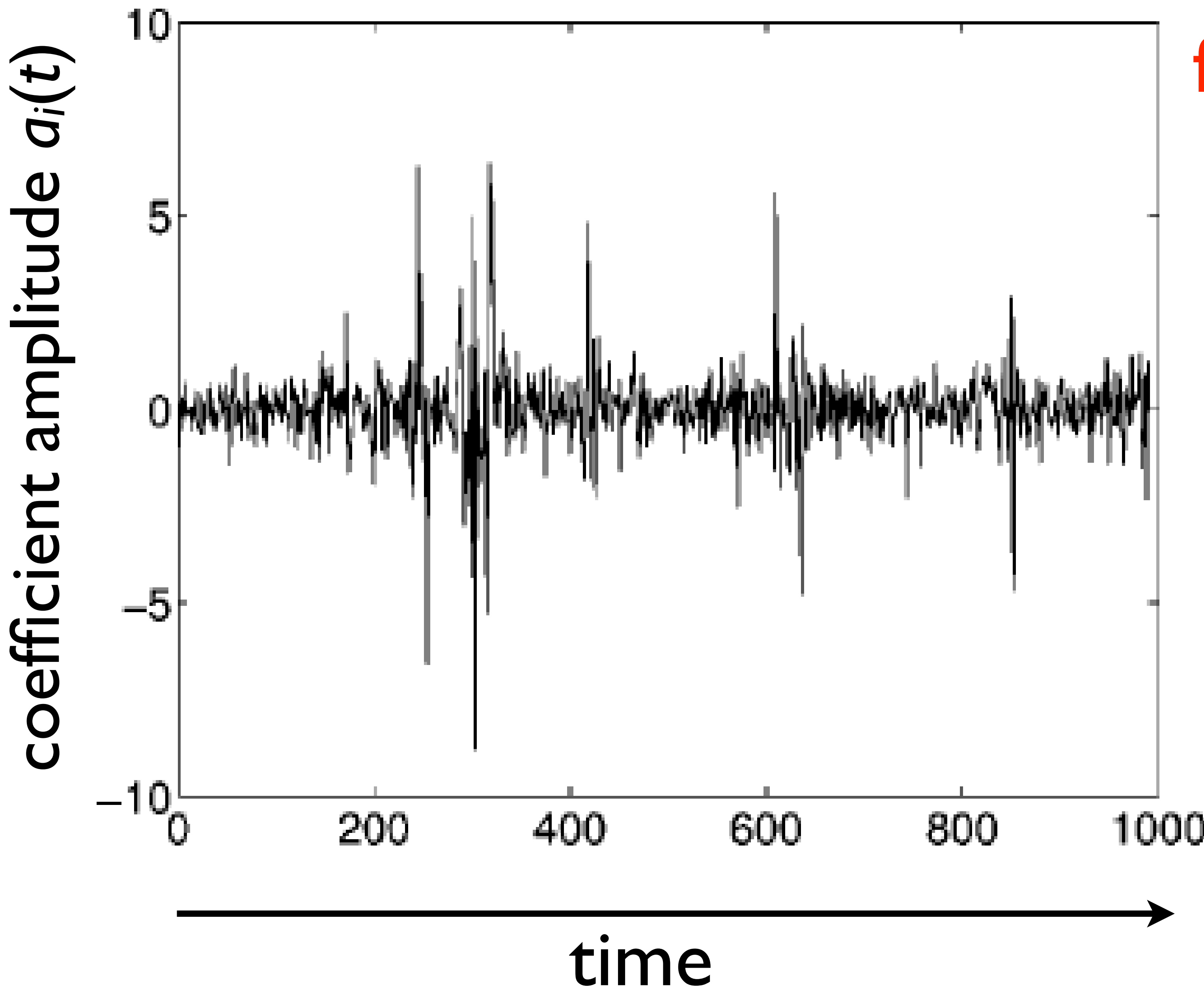


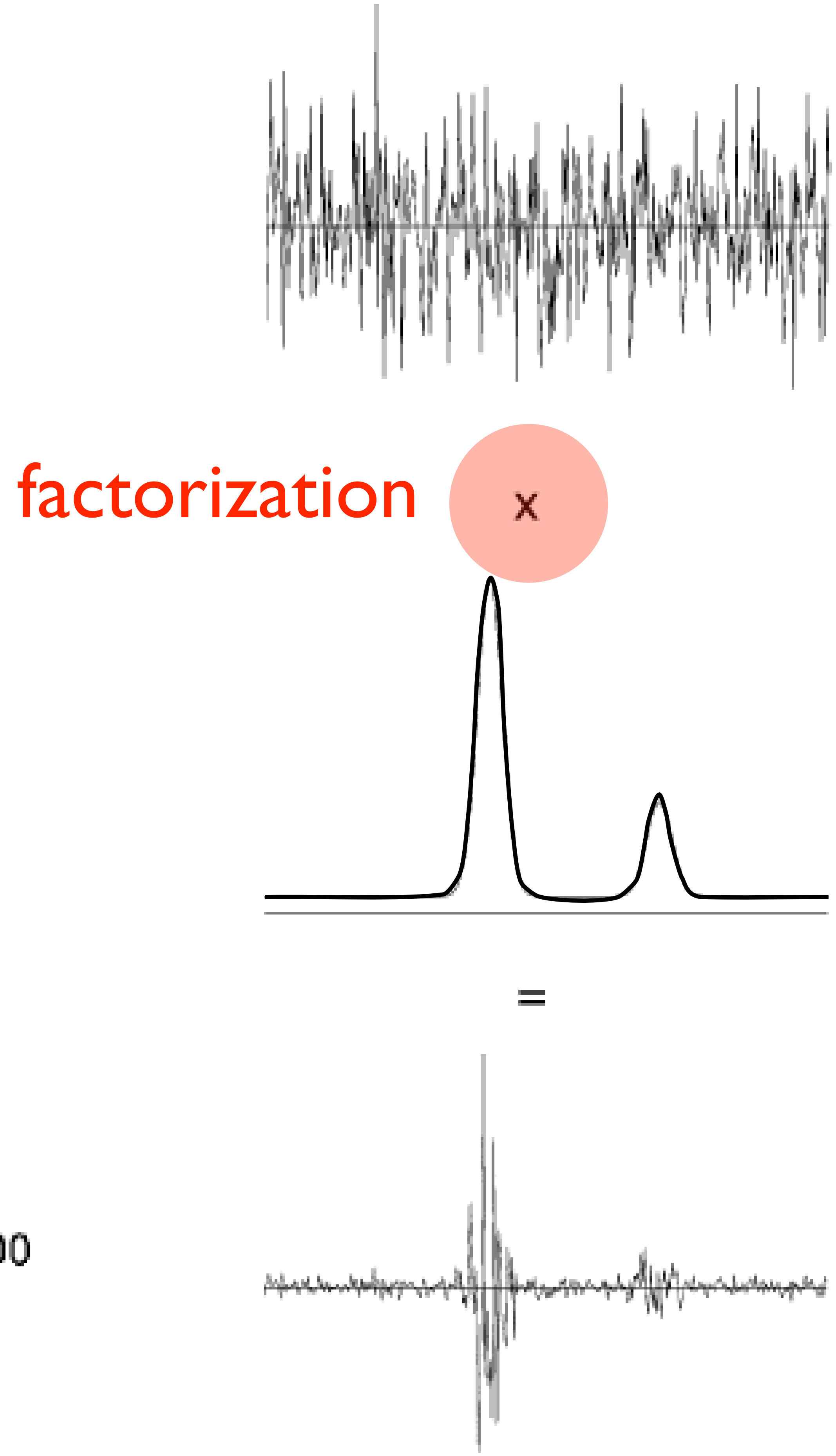
# Divisive normalization
(Schwartz & Simoncelli 2001)



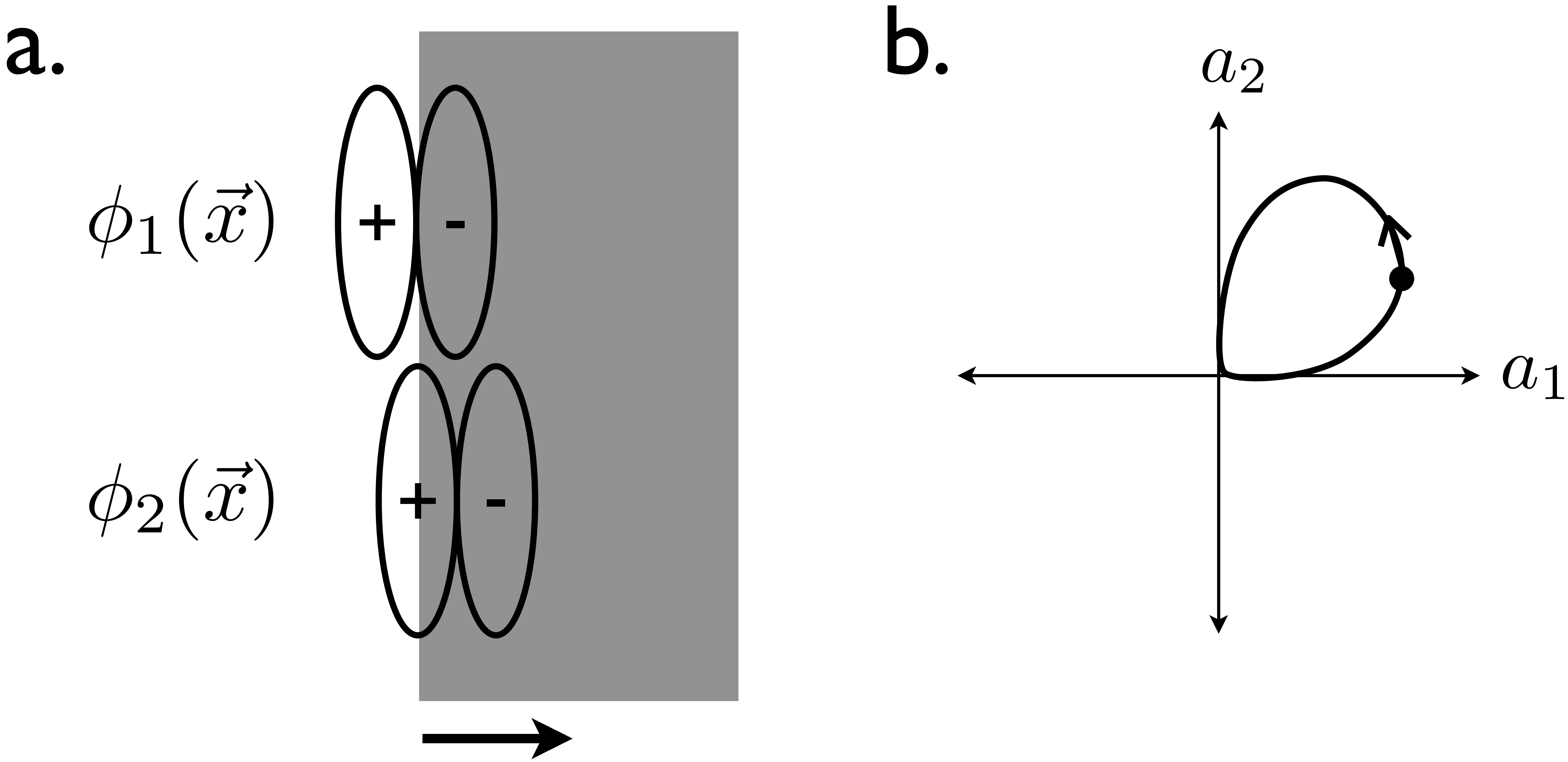
## Bubbles

Hyvarinen et al. (2003) JOSA *20*



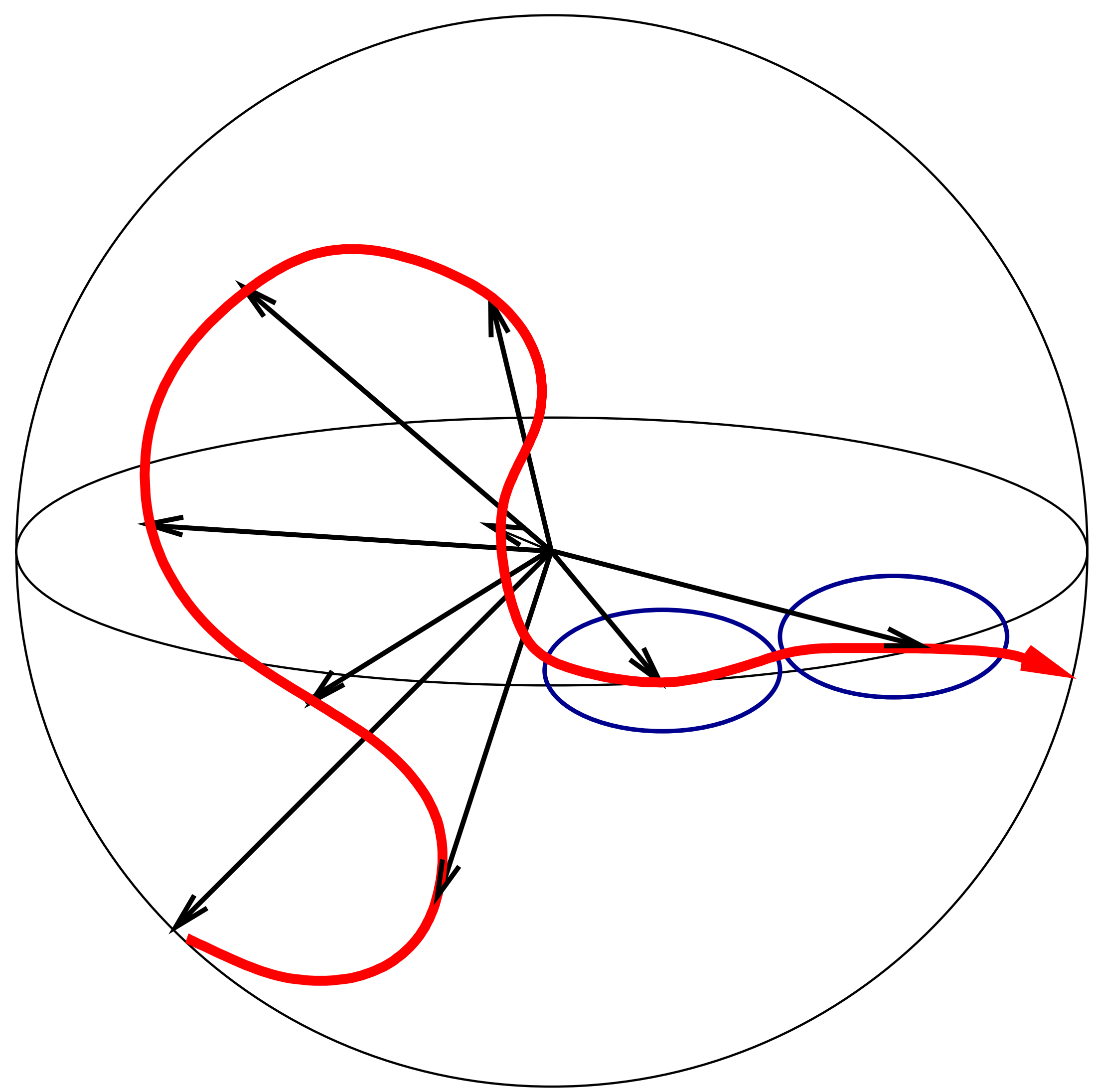



# Statistical dependencies among coefficients arise due to interpolation of image features occurring at different positions along a continuum



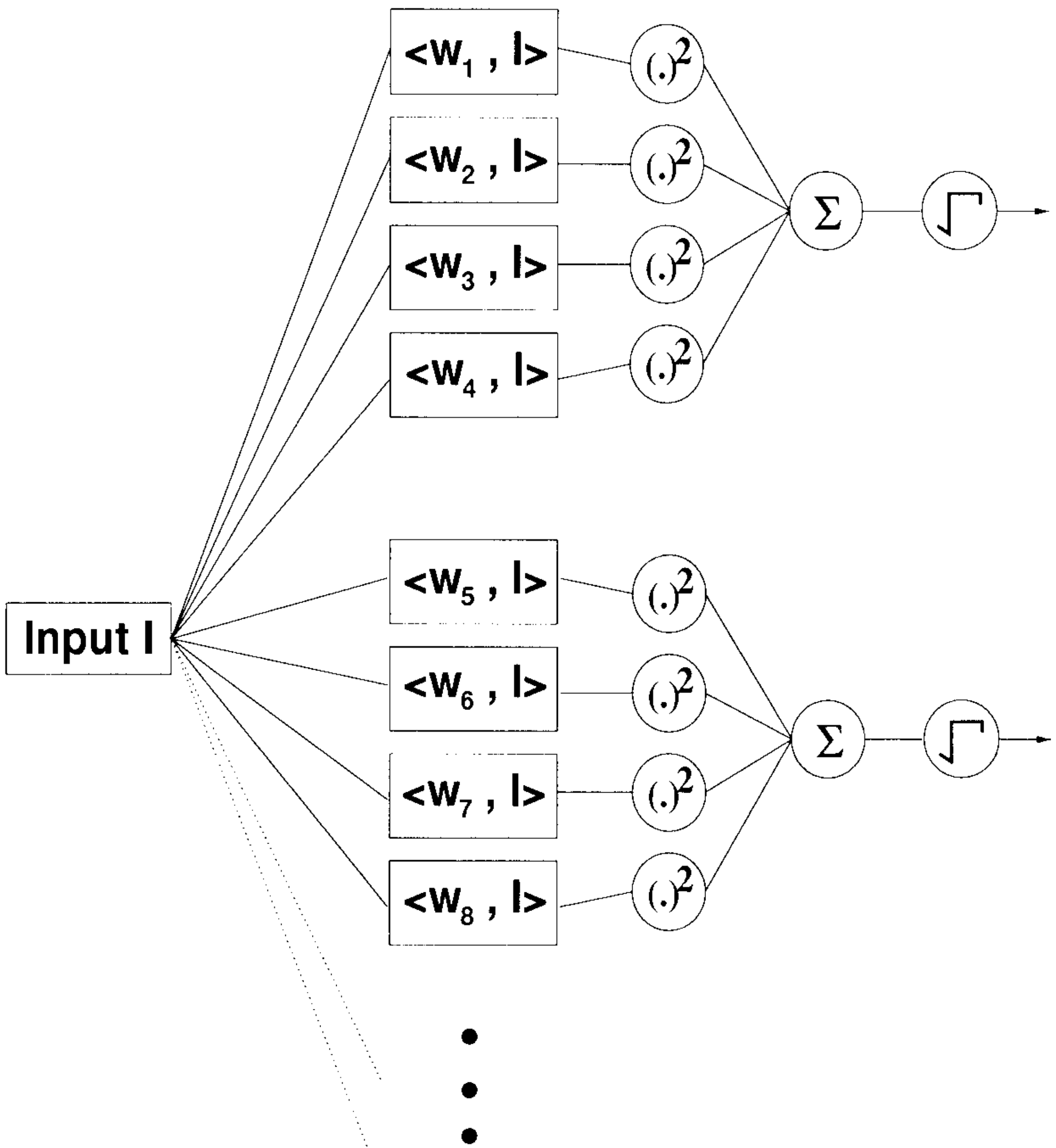
Sparse representations tile the manifold of natural images in such a way that data points along the manifold are spanned by a small number of basis vectors.

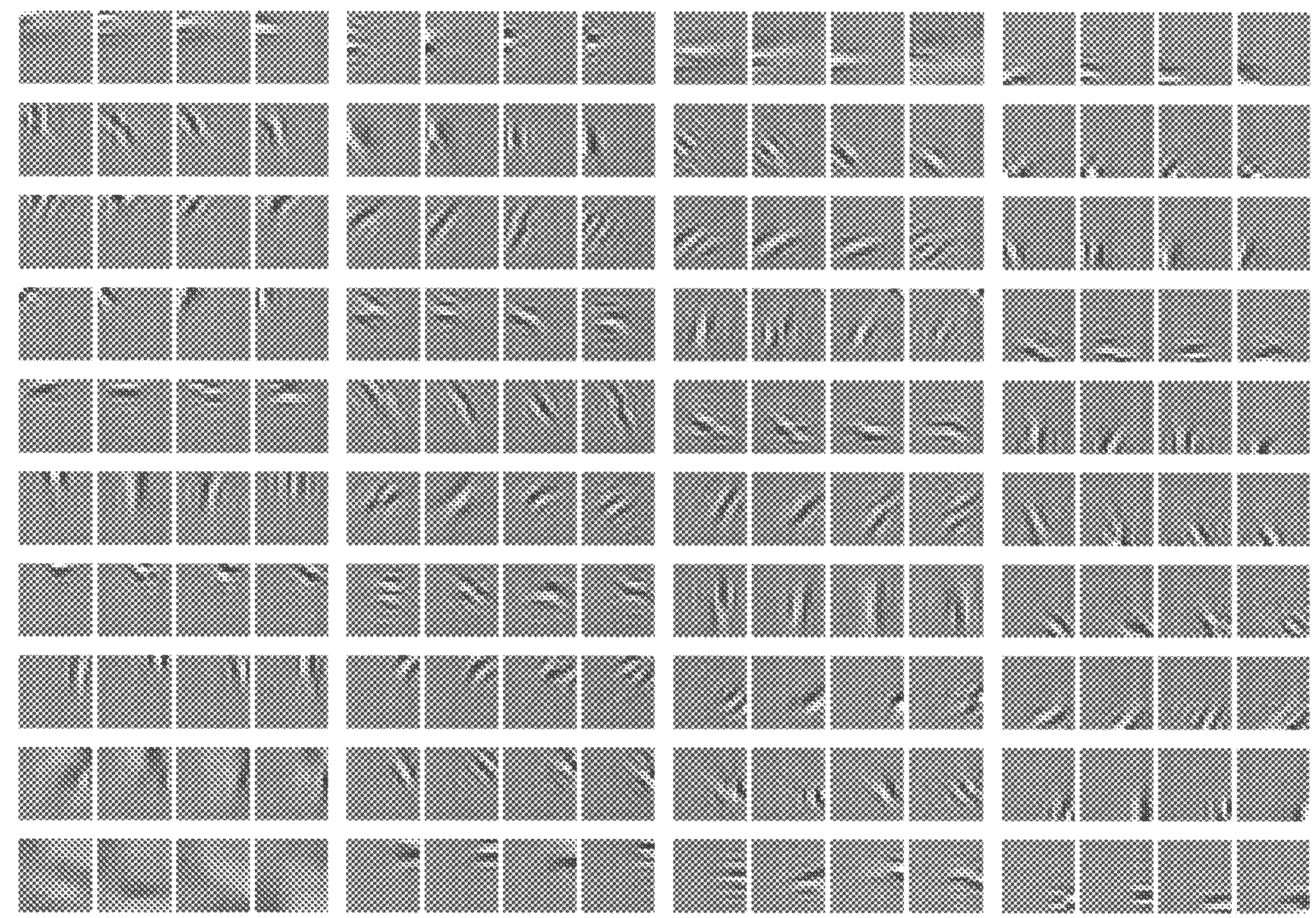
Coefficients are interpolation weights within linear subspaces spanned by basis vectors along the manifold.



# Subspace ICA model

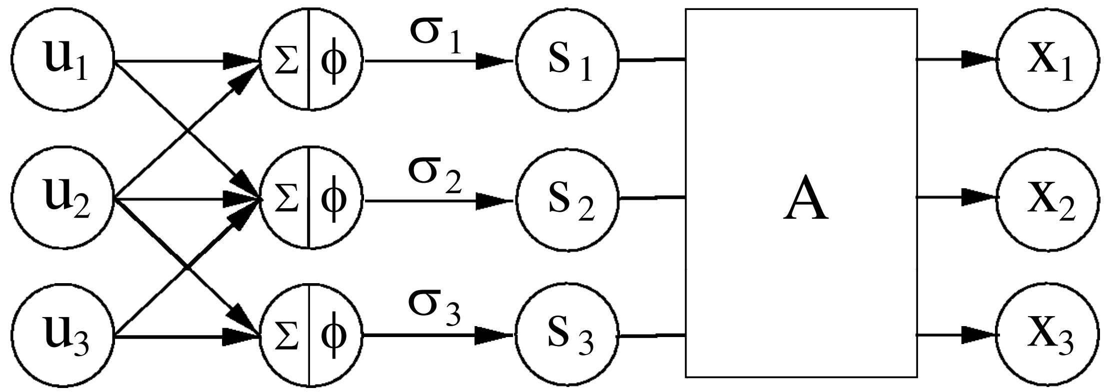
Hyvarinen & Hoyer (2000)

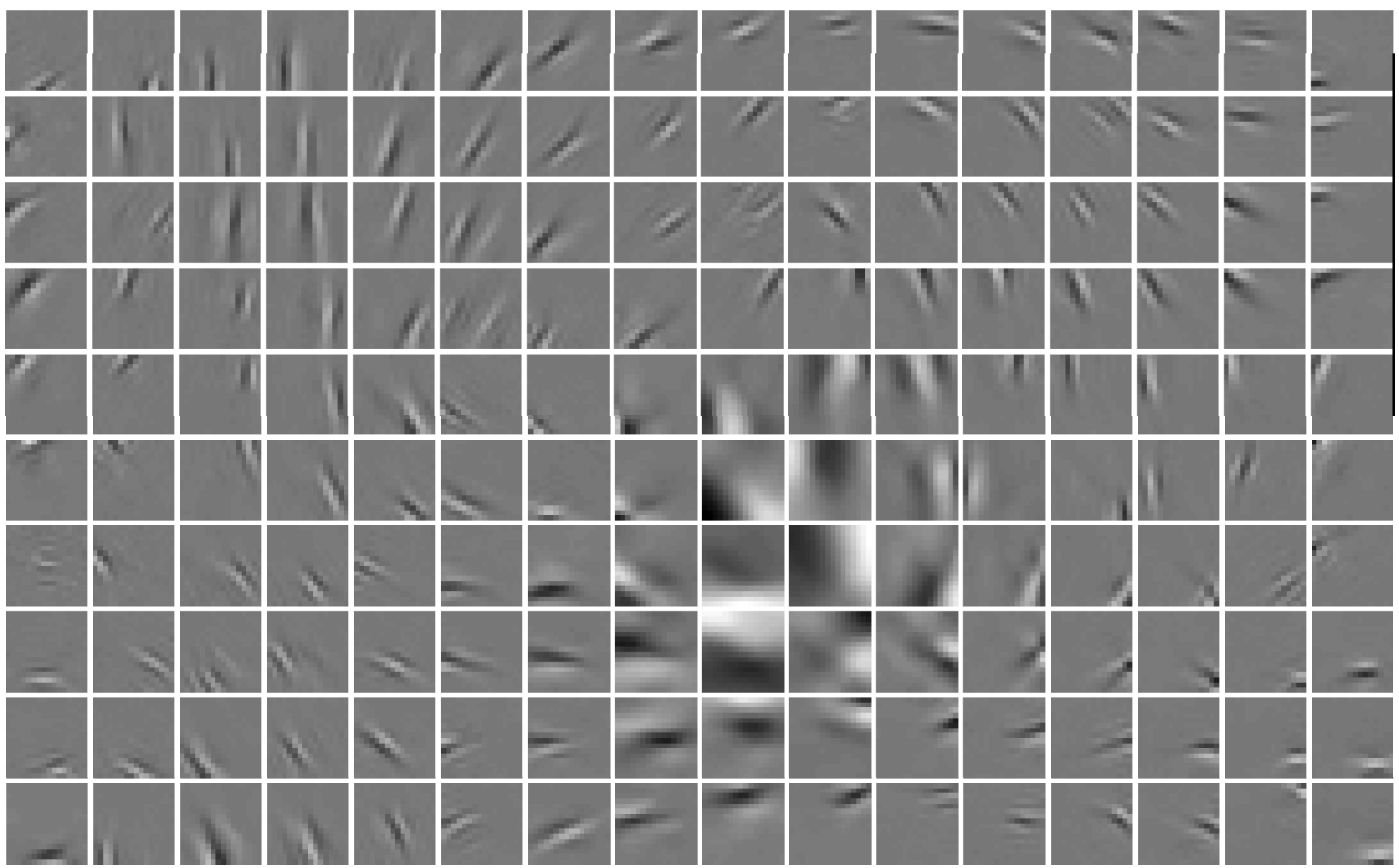


# Topographic ICA model

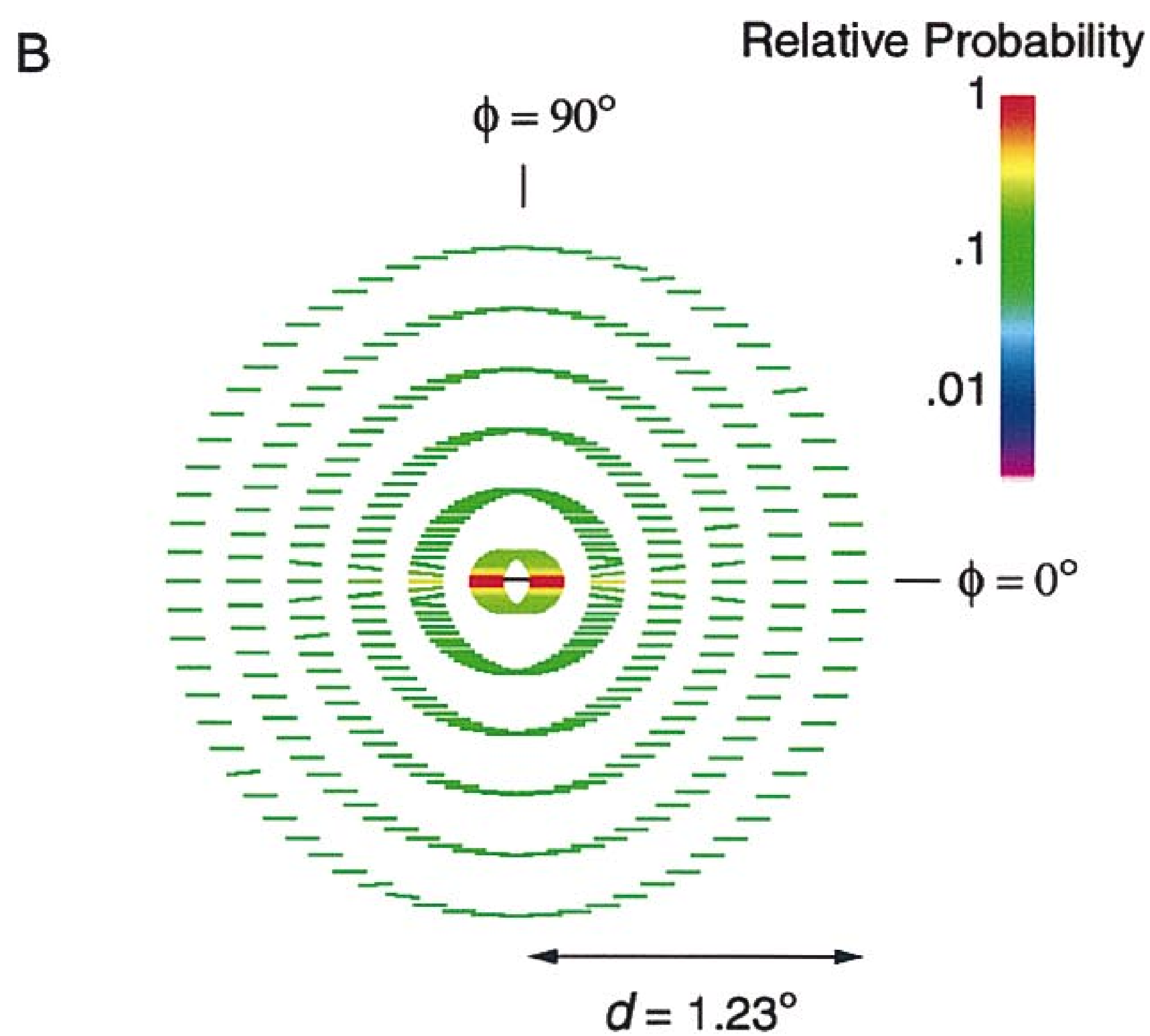
Hyvarinen, Hoyer & Inki (2001)

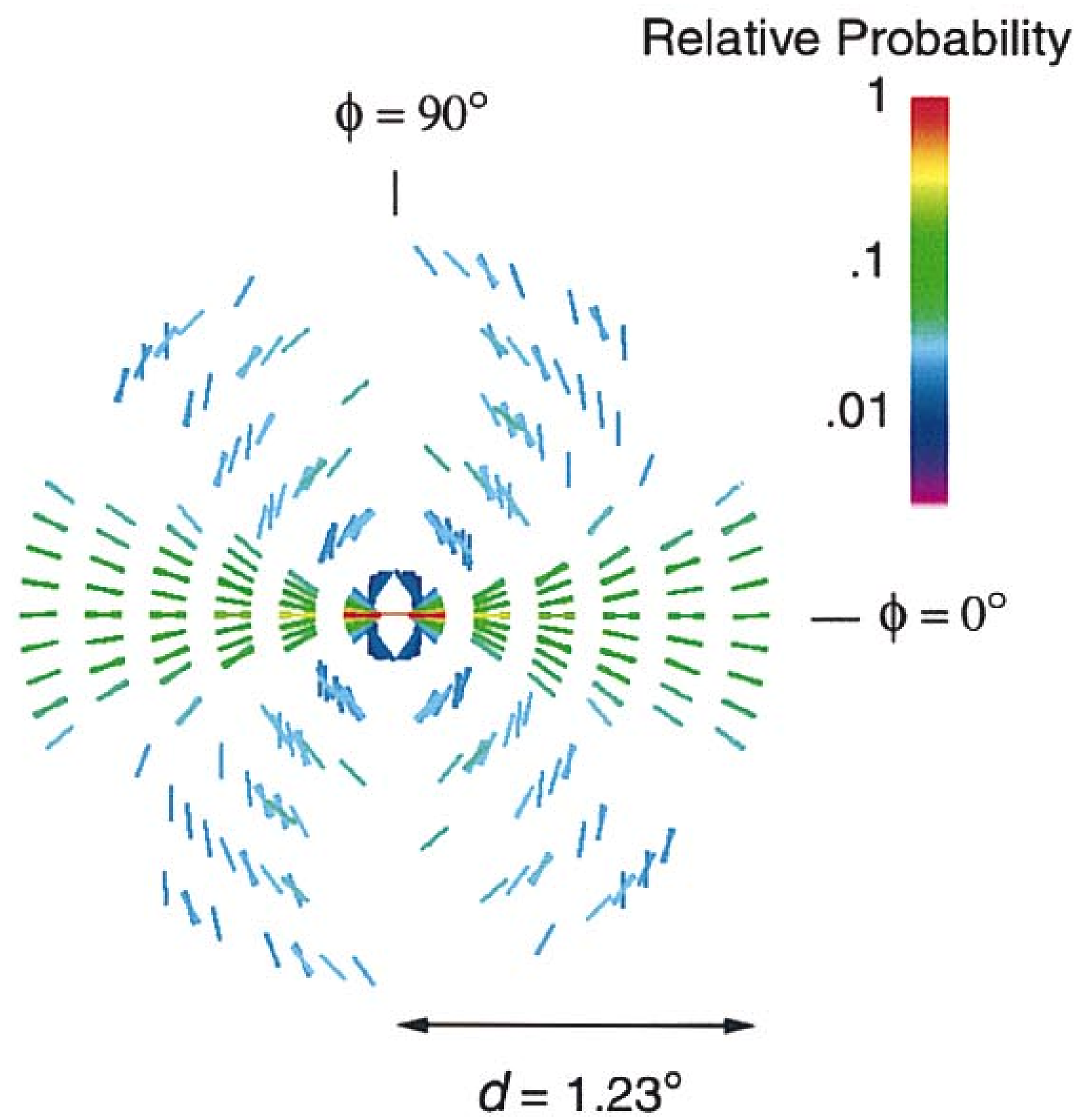


# ‘association field’ statistics

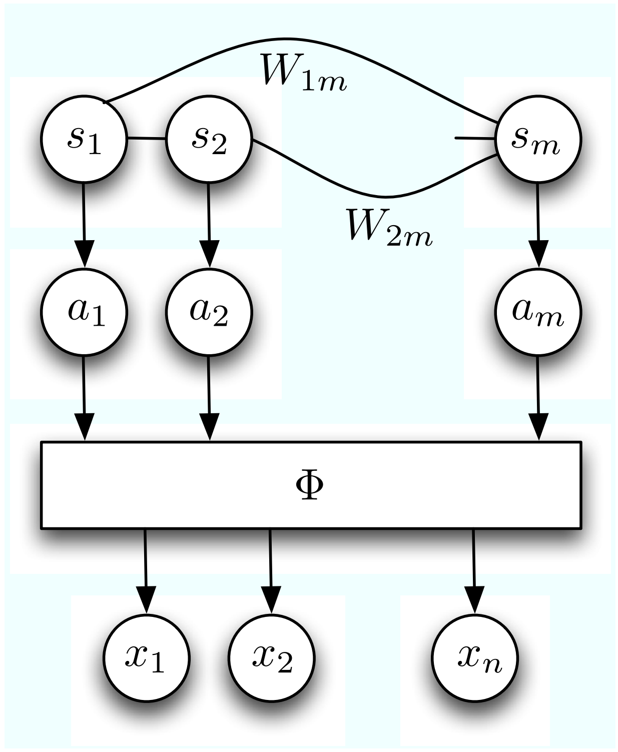
Geisler (2001)

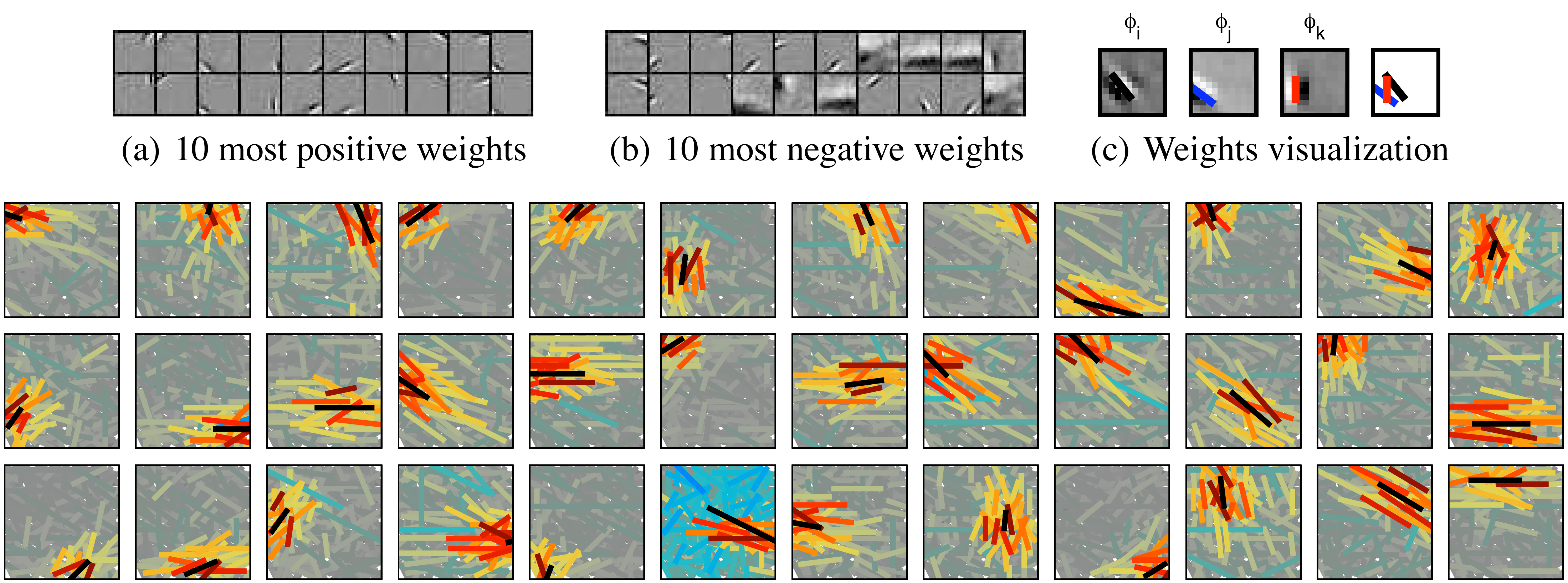


# Modeling dependencies with horizontal connections

Garrigues & Olshausen (2008)




(a) 10 most positive weights (b) 10 most negative weights (c) Weights visualization

(d) Association fields

# Hierarchical models

Cortex has hierarchical structure.

Scenes have compositional structure.

How to learn from images?

# Multiple paths to recognition
(Ledoux et al. 1994)

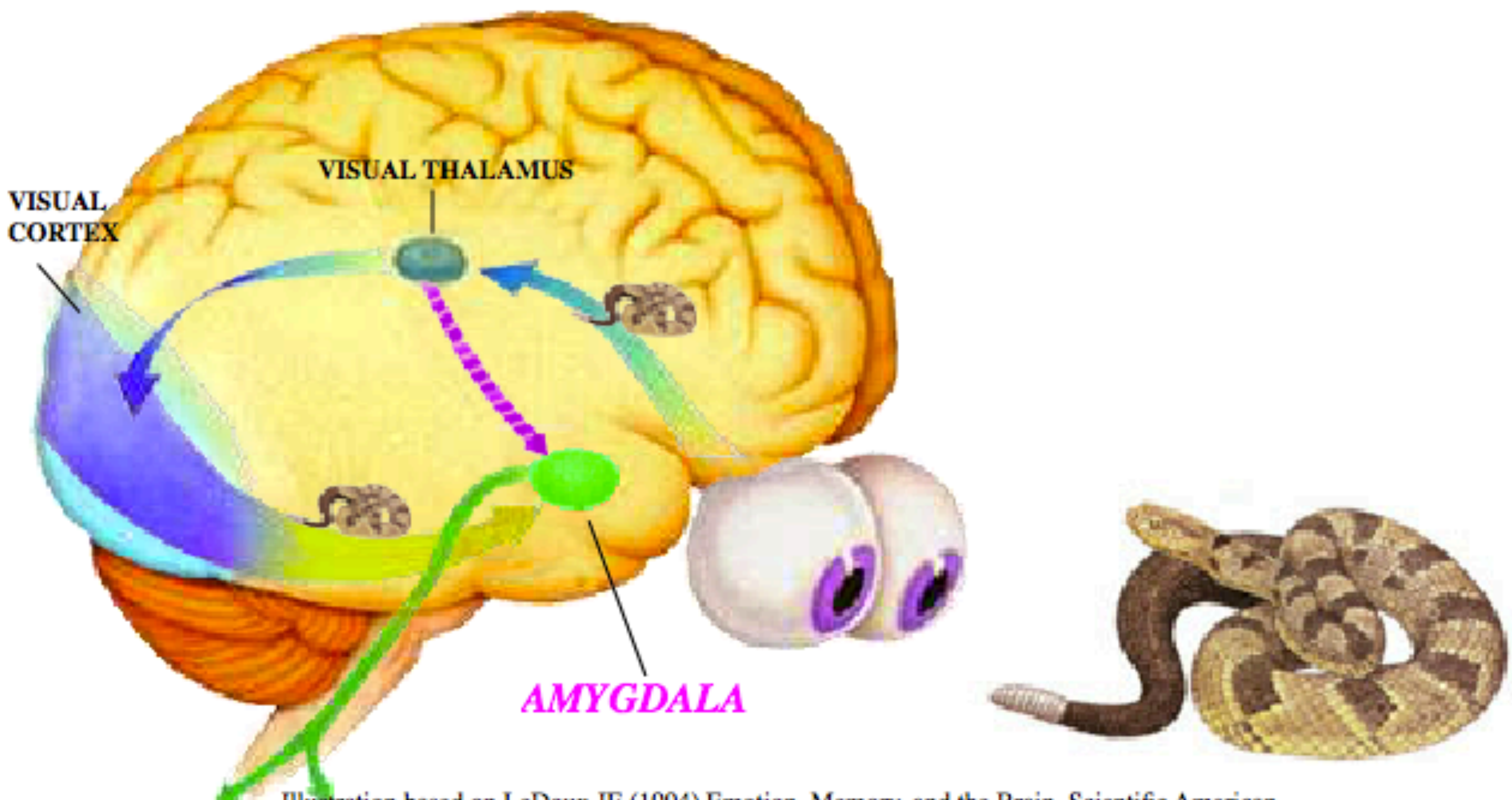
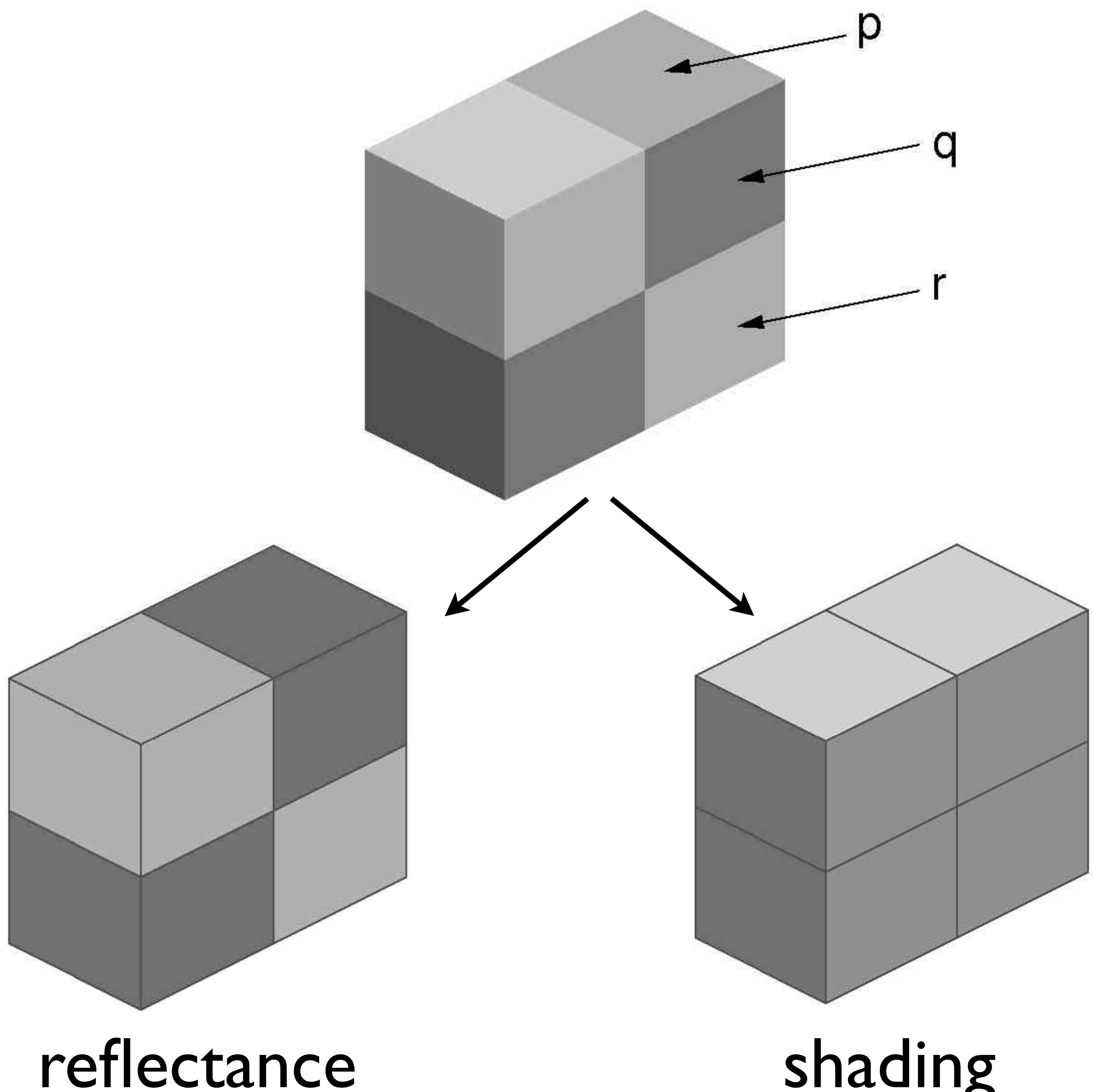


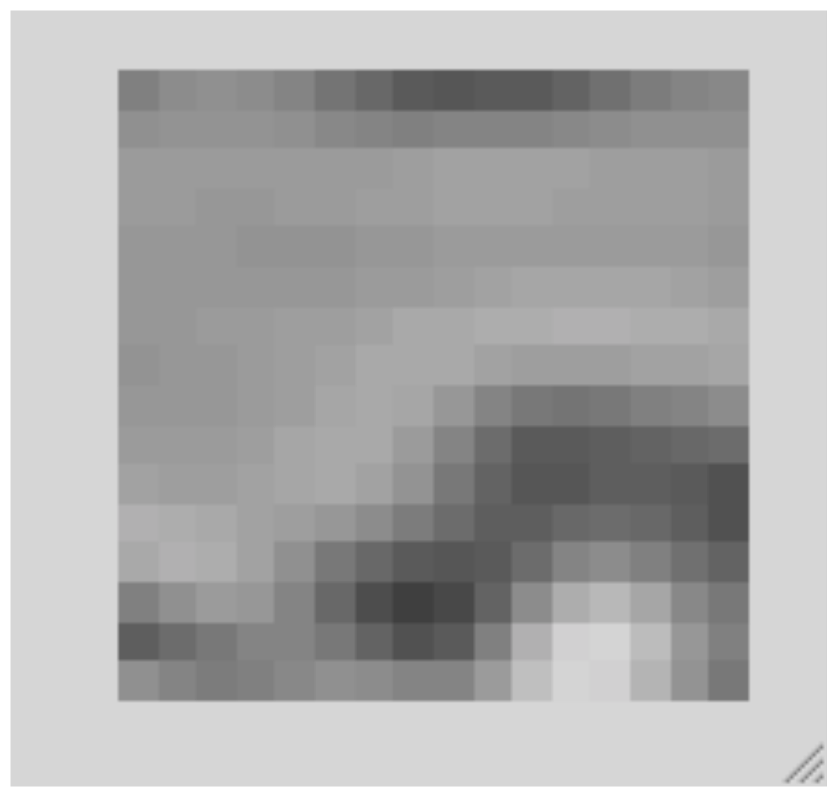

Illustration based on LeDoux JE (1994) Emotion, Memory, and the Brain. Scientific American.

# What do these edges mean?




(Adelson, 2000)

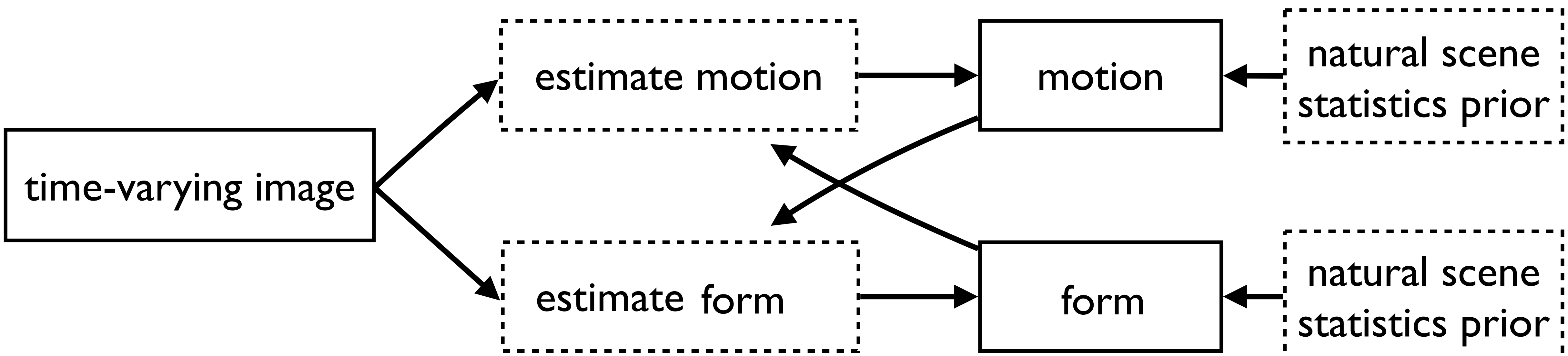
# How to factorize time-varying images into *form* and *motion*?



# Traditional motion estimation

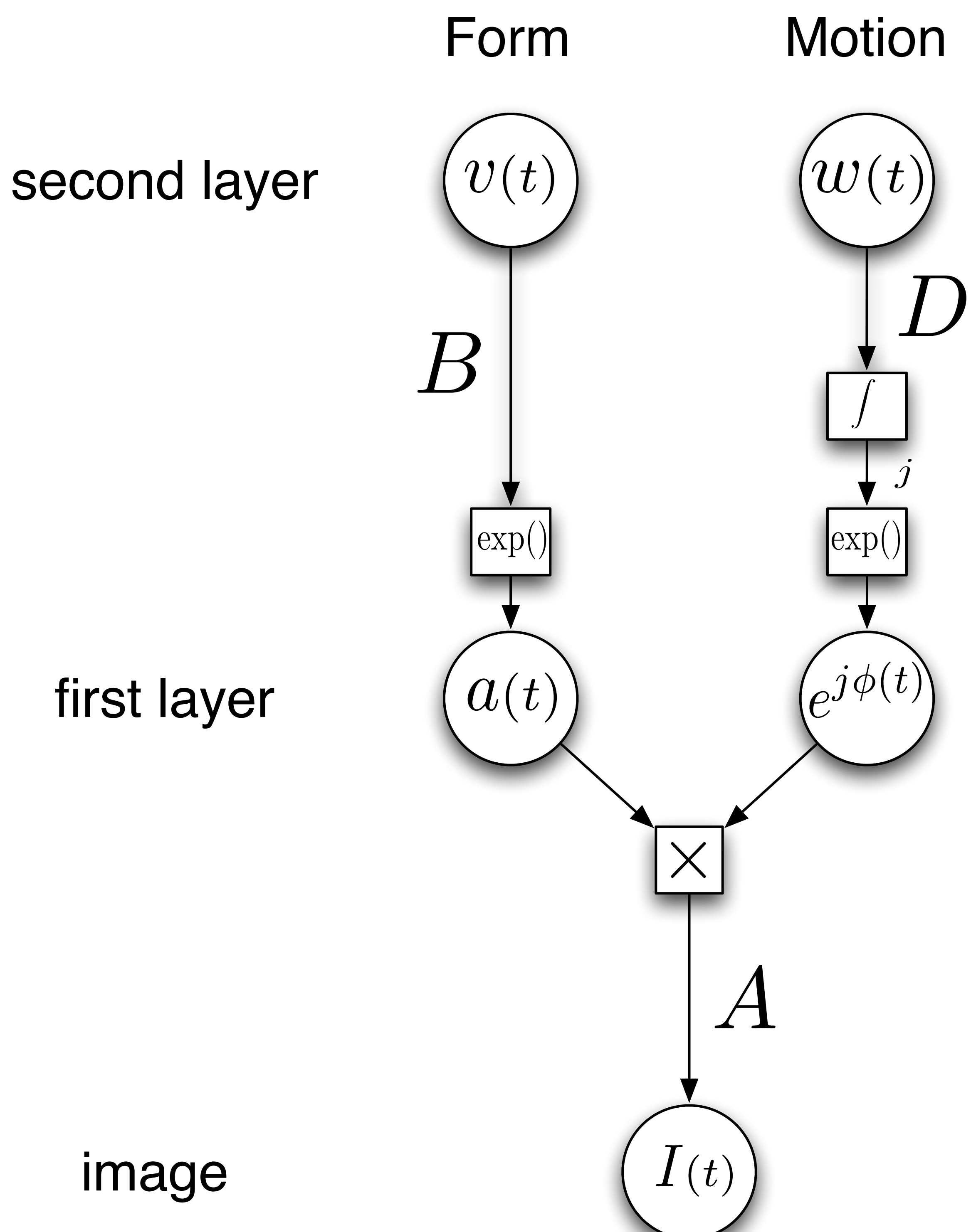

$$\hat{v} = \arg\min_{v} ||I(x, t + \Delta t) - I(x - v\Delta t, t)||^2$$

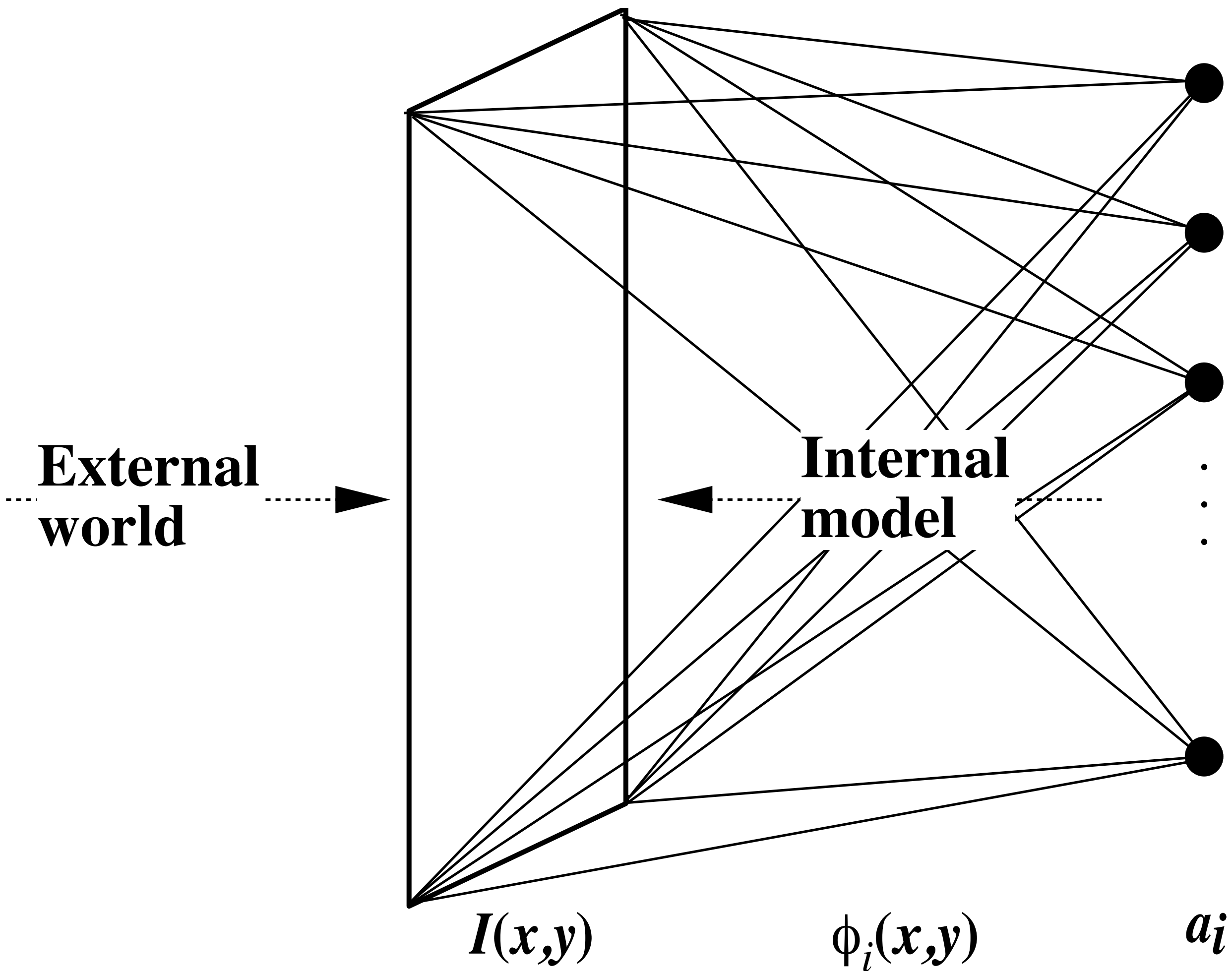
# Motion *and* form must be estimated simultaneously



# Factorization of form and motion

Cadieu & Olshausen (2012), *Neural Computation, 24*






# Sparse coding image model

$$I(x,y) = \sum_i a_i \phi_i(x,y) + \epsilon(x,y)$$

image

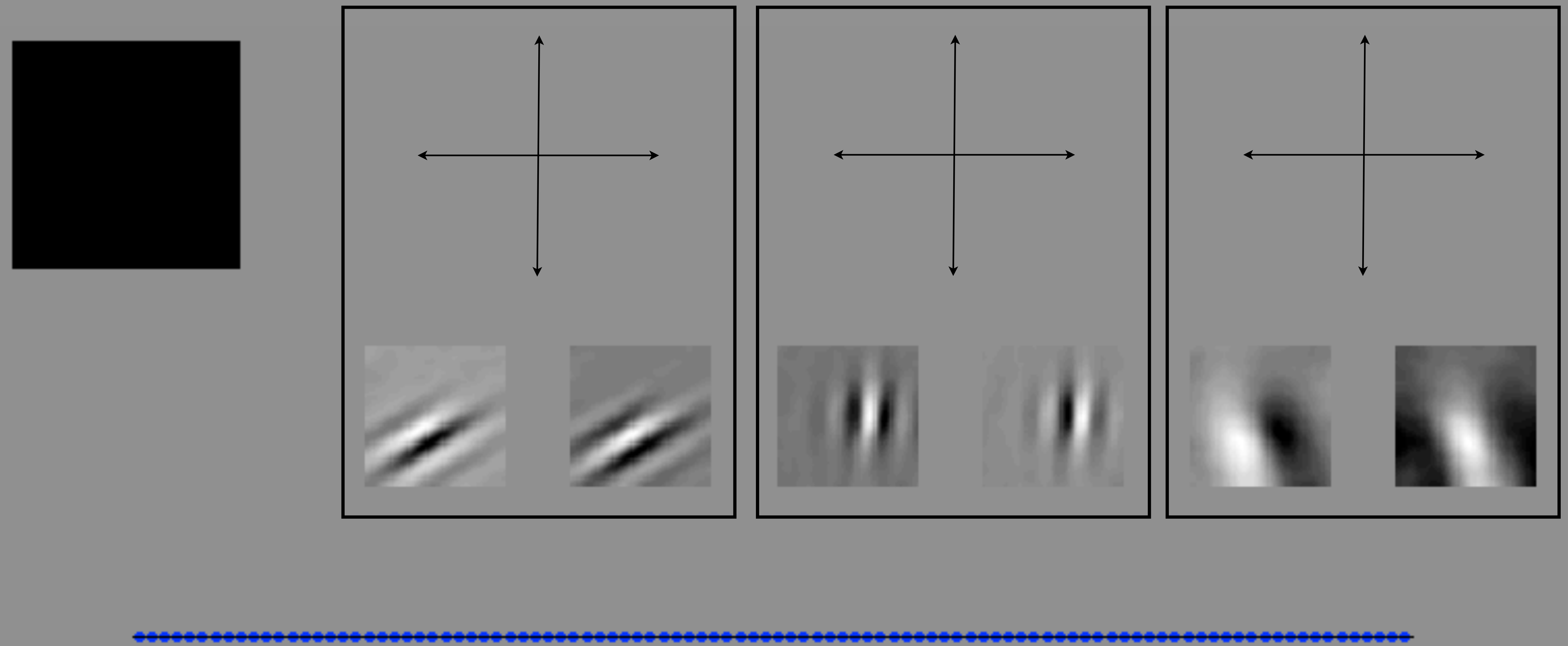
neural activities (sparse)

features

other stuff

# Amplitudes change slowly (presence of edge)

# Phases precess (movement of edge)



# Factorization of sparse components into *invariant* and *variant* parts

$$\mathbf{I}(t) = \Phi\,\mathbf{a}(t) + \epsilon(t)$$

$$a_i(t) = \boxed{\sigma_i(t)} \times \boxed{u_i(t)}$$

Invariant part: Presence, amplitude (contrast) of feature

Variant part: Interpolation weight, *relative* contribution of feature

See also:

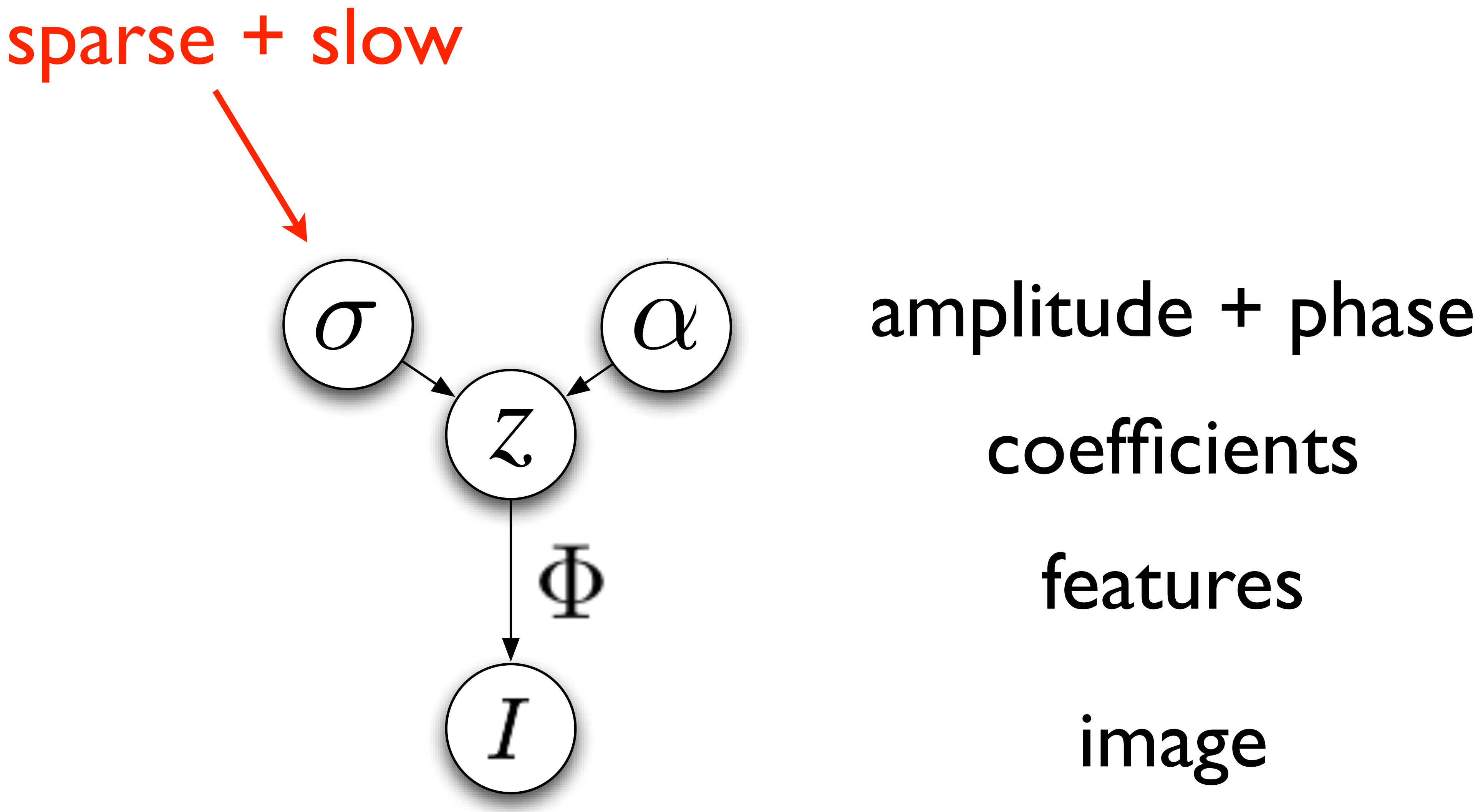
Wainwright & Simoncelli (2002)
Hyvarinen & Hoyer (2002)
Karklin & Lewicki (2003)
Schwartz & Simoncelli (2004)
Osindero & Hinton (2005)
Berkes, Turner & Sahani (2009)
Zou, Ng & Yu (2011)

# Amplitude and phase decomposition via complex basis functions

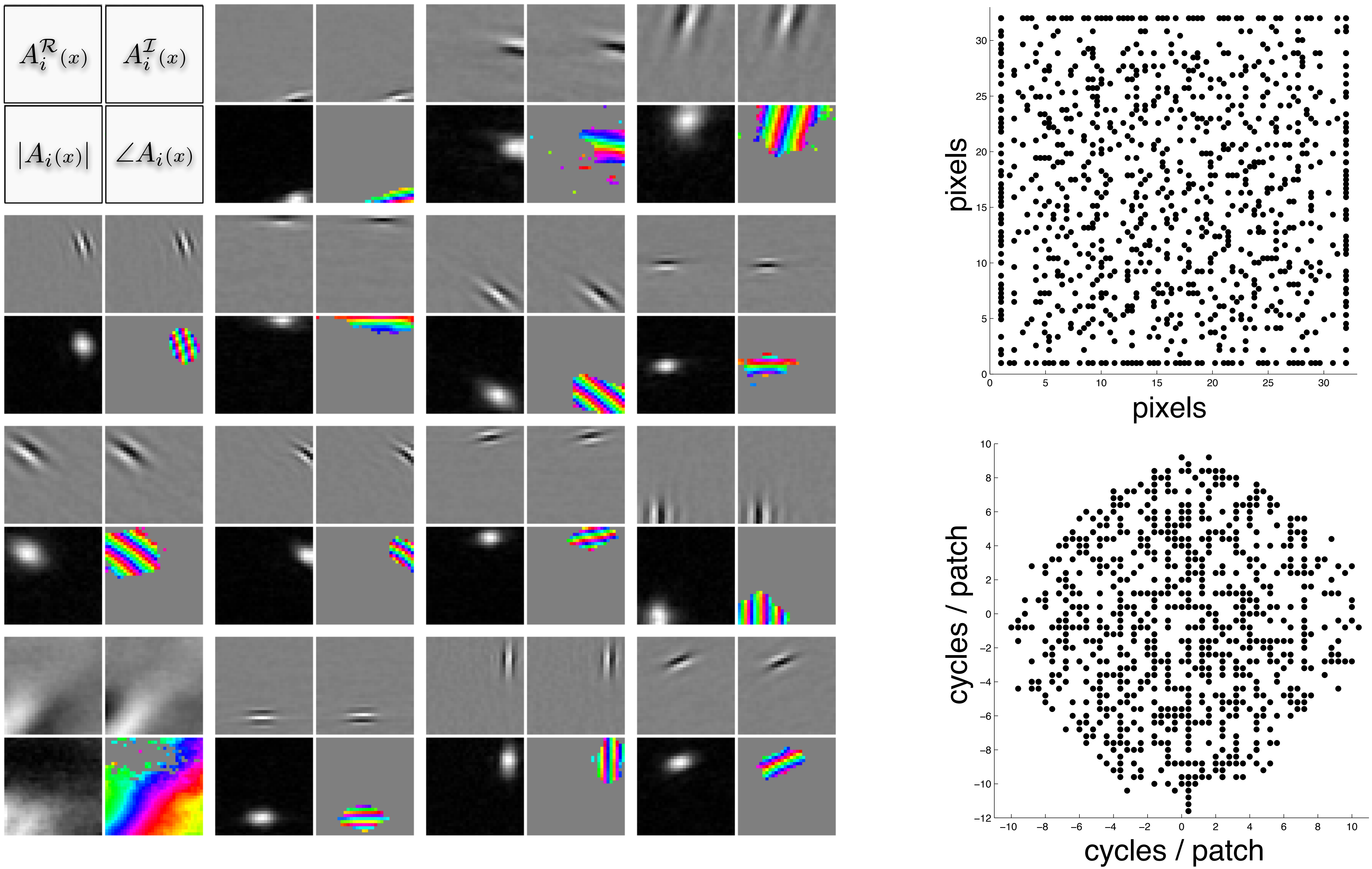
$$
\begin{aligned}
I(x,y,t) &= \Re\left\{\sum_i z_i^*(t)\,\phi_i(x,y)\right\} \\
&= \sum_i \sigma_i(t)\left[\cos\alpha_i(t)\,\phi_i^R(x,y) + \sin\alpha_i(t)\,\phi_i^I(x,y)\right]
\end{aligned}
$$

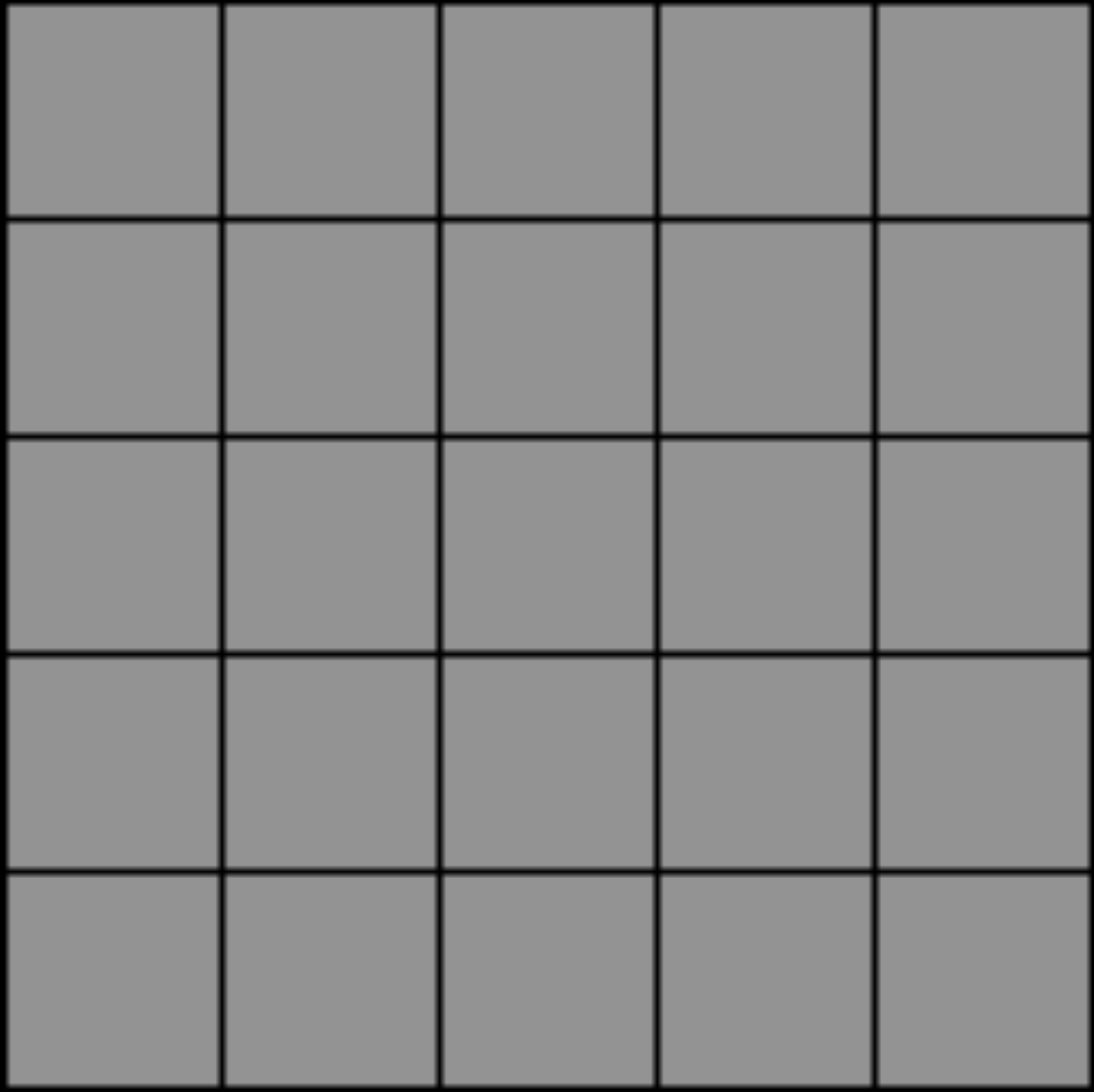
$$
\begin{aligned}
z_i(t) &= \sigma_i(t)\,e^{j\,\alpha_i(t)} \\
\phi_i(x,y) &= \phi_i^R(x,y) + j\,\phi_i^I(x,y)
\end{aligned}
$$

# Complex basis function model

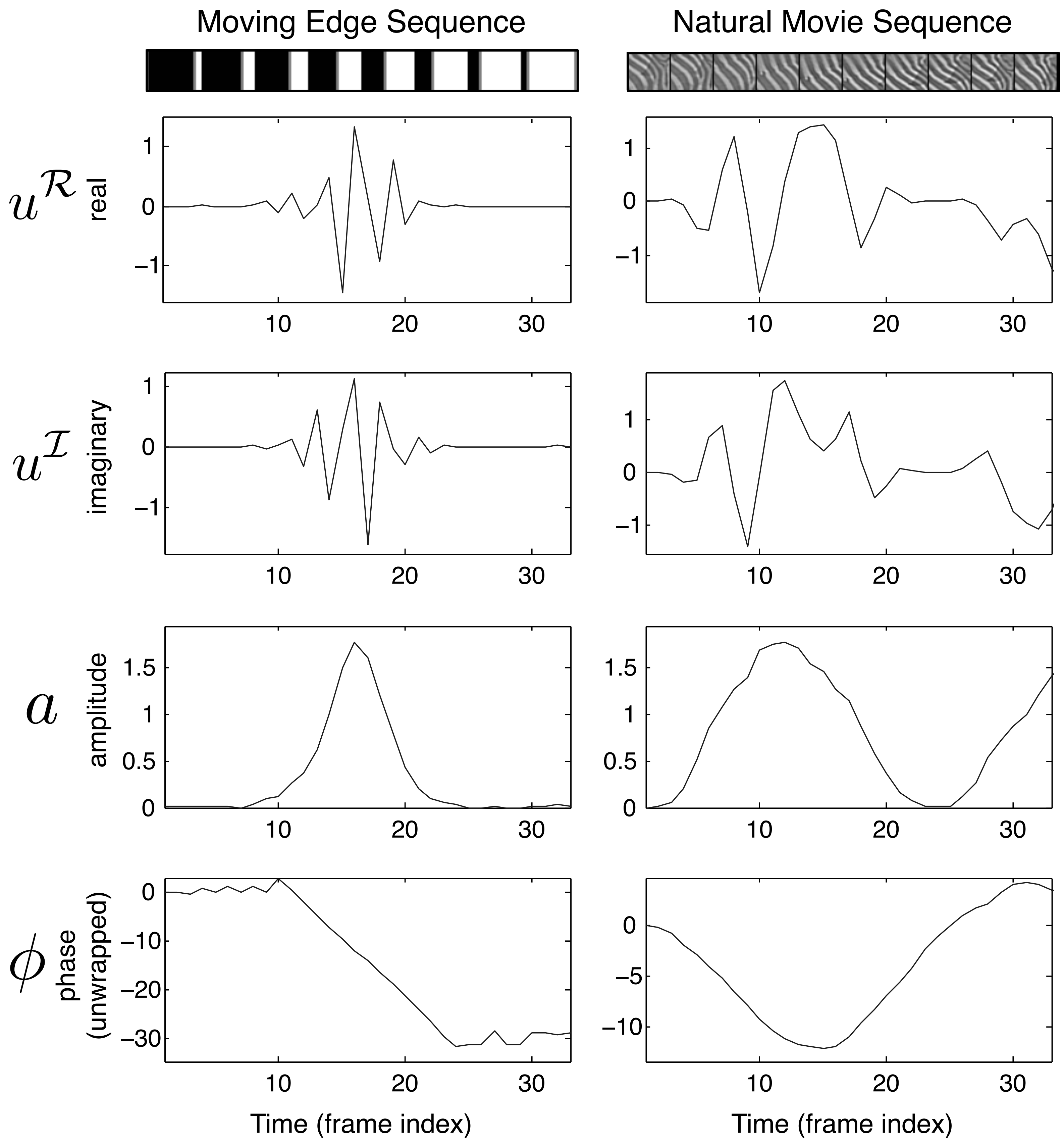


# Learned complex basis functions

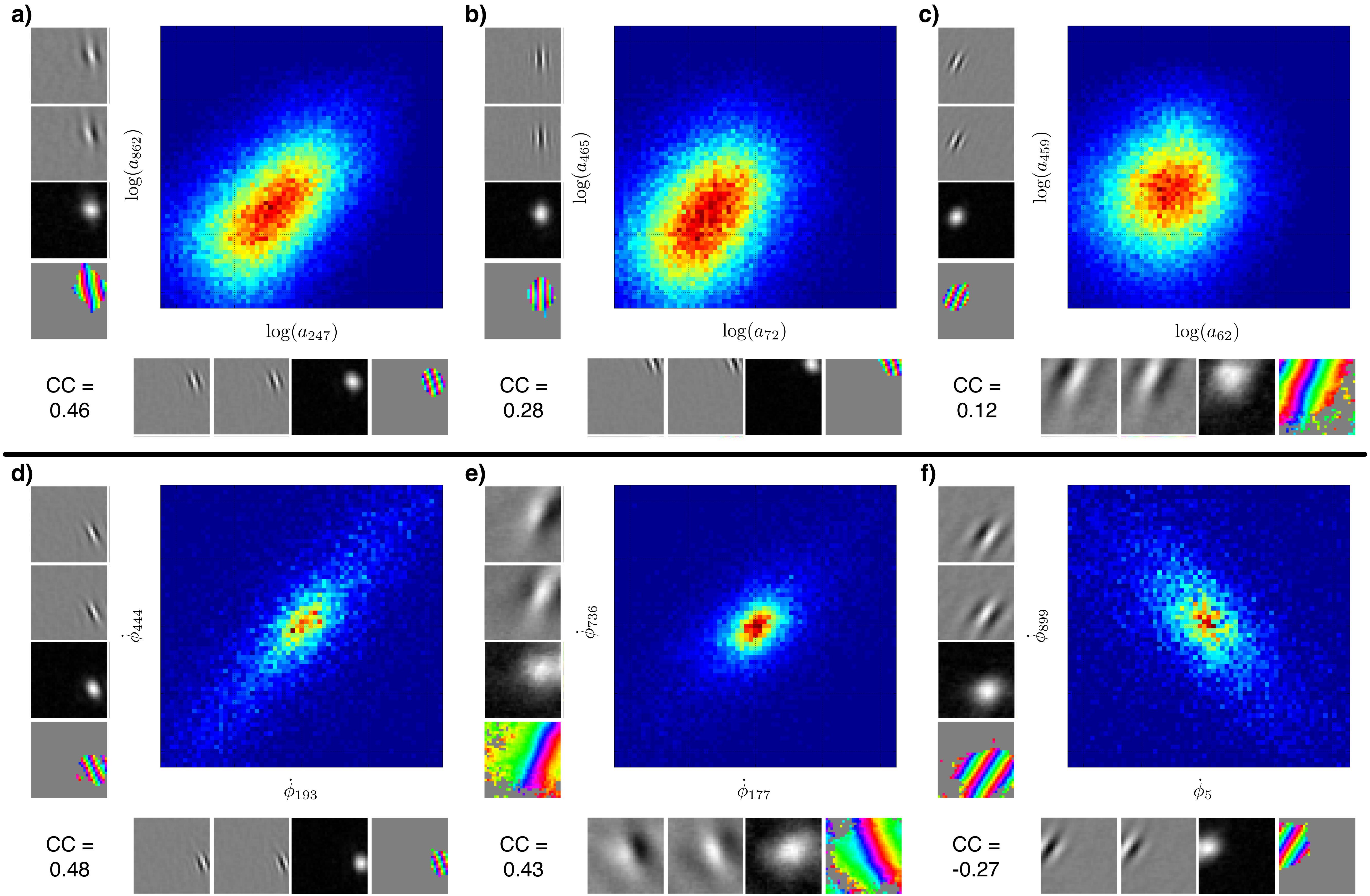




# Amplitude and phase *linearize* statistical dependencies



# Amplitude and phase *linearize* statistical dependencies



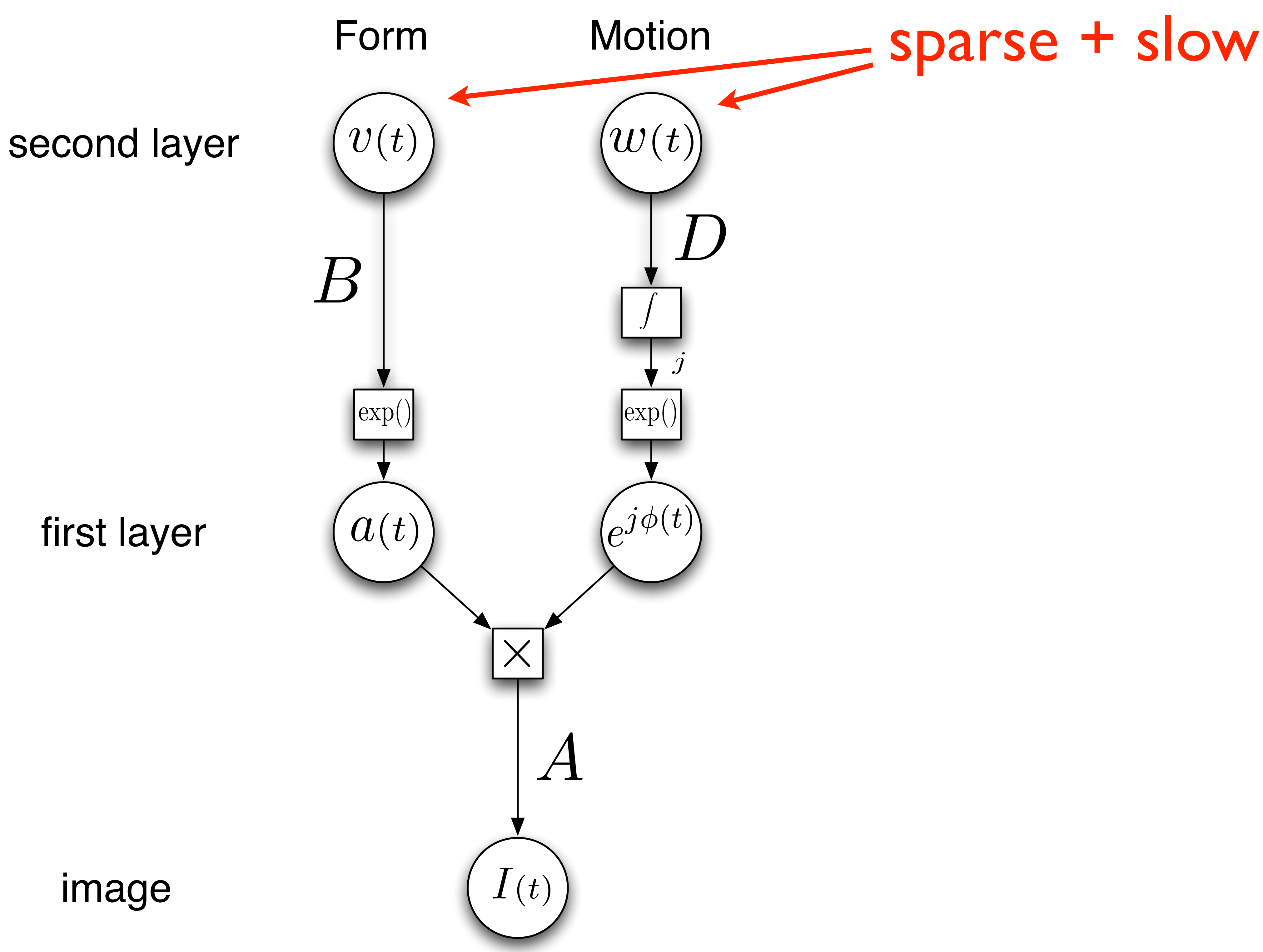
$\dot{\phi}$ may be computed from normalized simple cell responses

$$\frac{d}{dt}\phi(t) = \frac{d}{dt}\arctan\left(\frac{u_s}{u_c}\right) = \frac{\dot{u}_s u_c - u_s \dot{u}_c}{u_c^2 + u_s^2} = \frac{u_s u_c^{t-1} - u_s^{t-1} u_c}{u_c^2 + u_s^2}$$

(see Eero Simoncelli thesis)

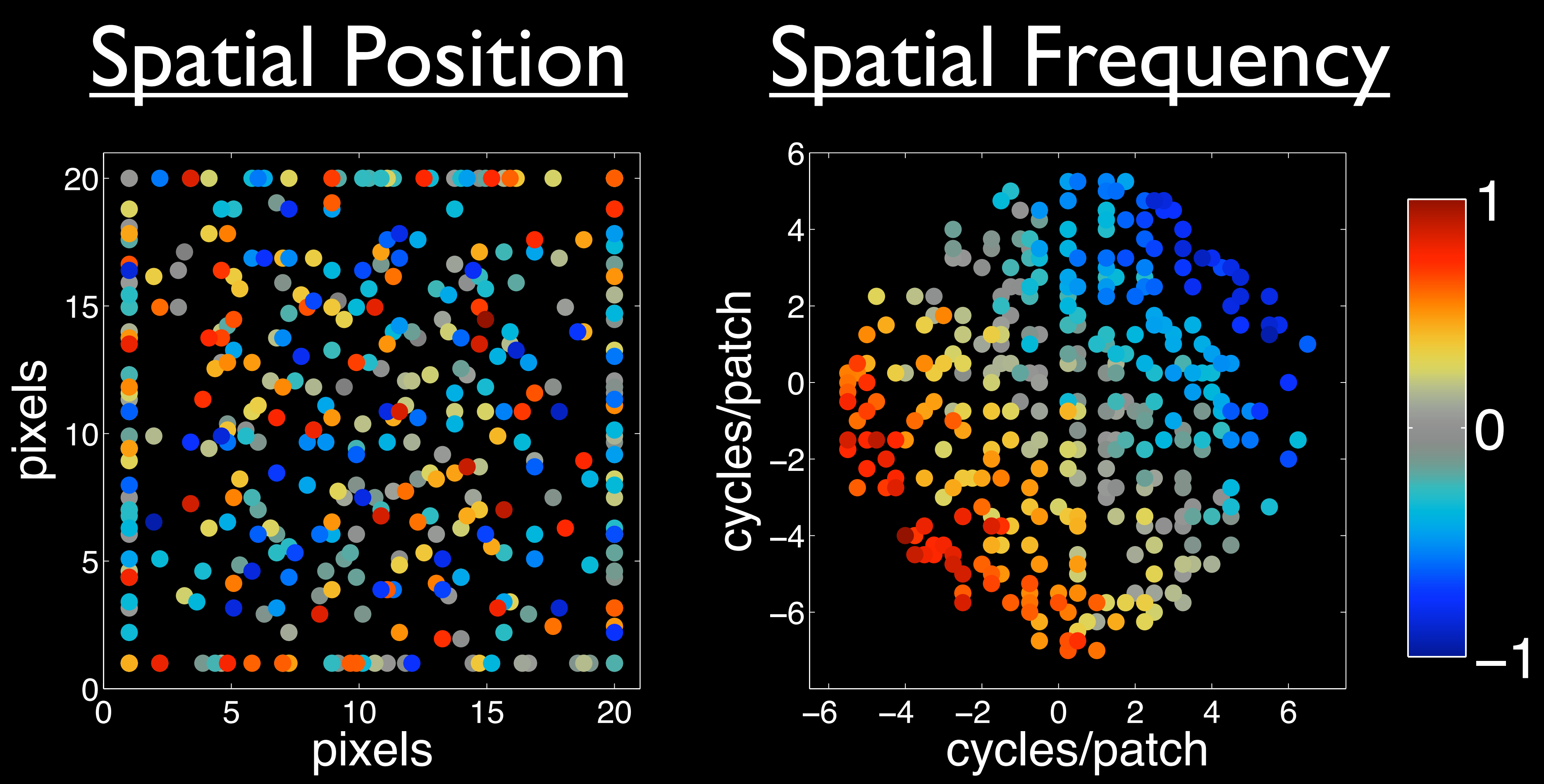
# Factorization of form and motion

Cadieu & Olshausen (2012), *Neural Computation, 24*



# Visualizing Second-layer Weights

$D_{10}$



# Learned form components *B*

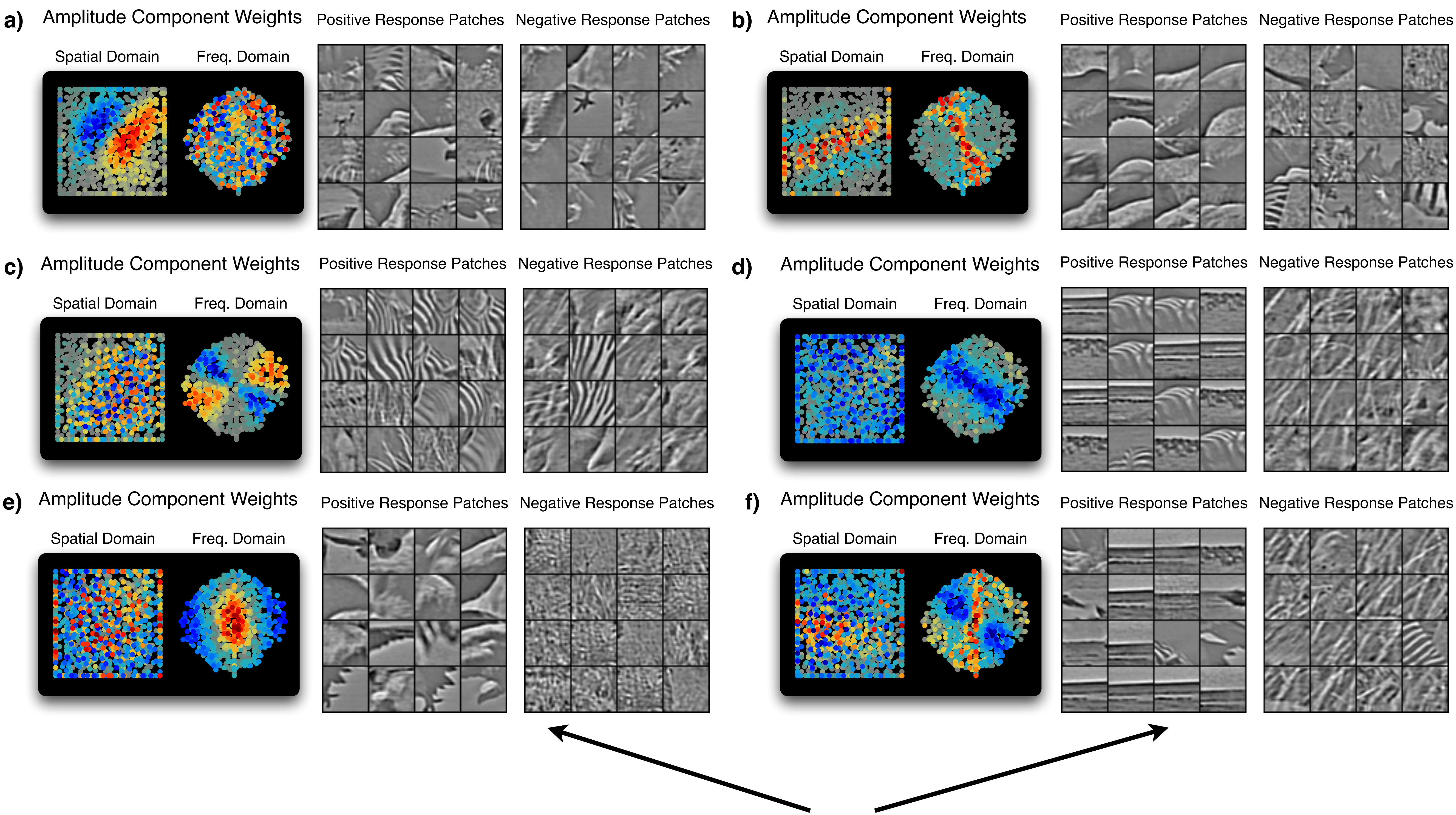
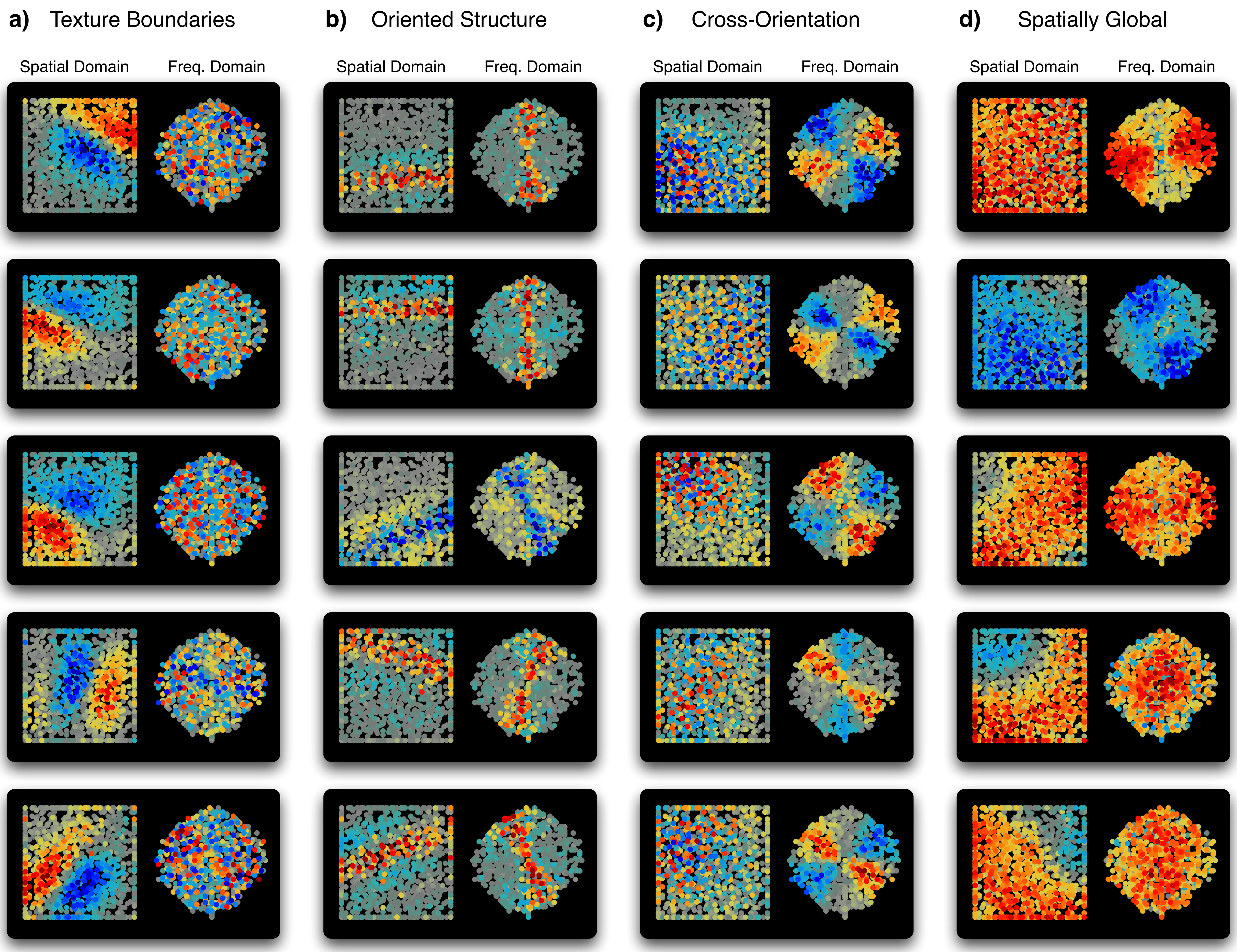
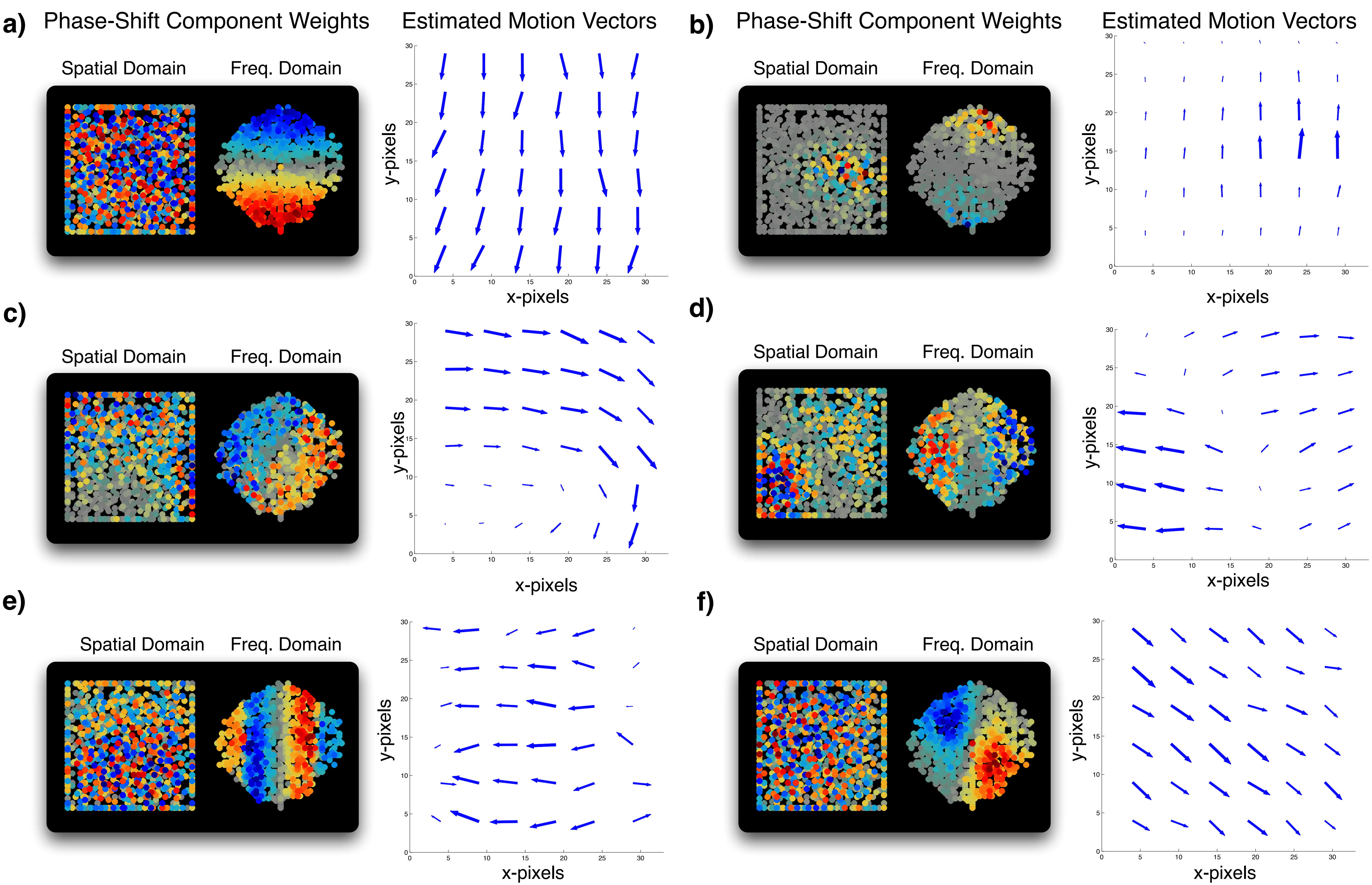



image patches yielding maximum response

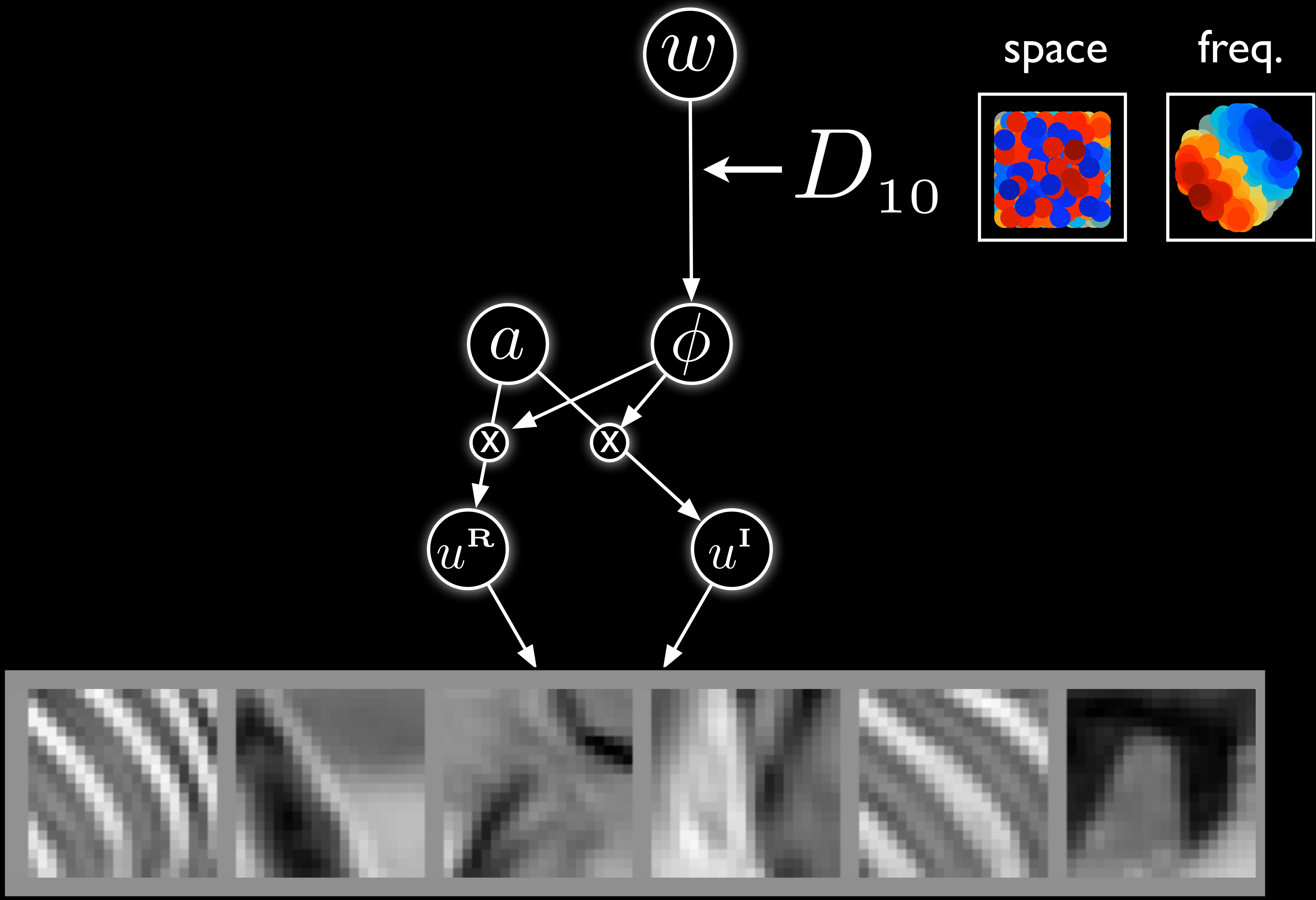
# Learned form components $B$



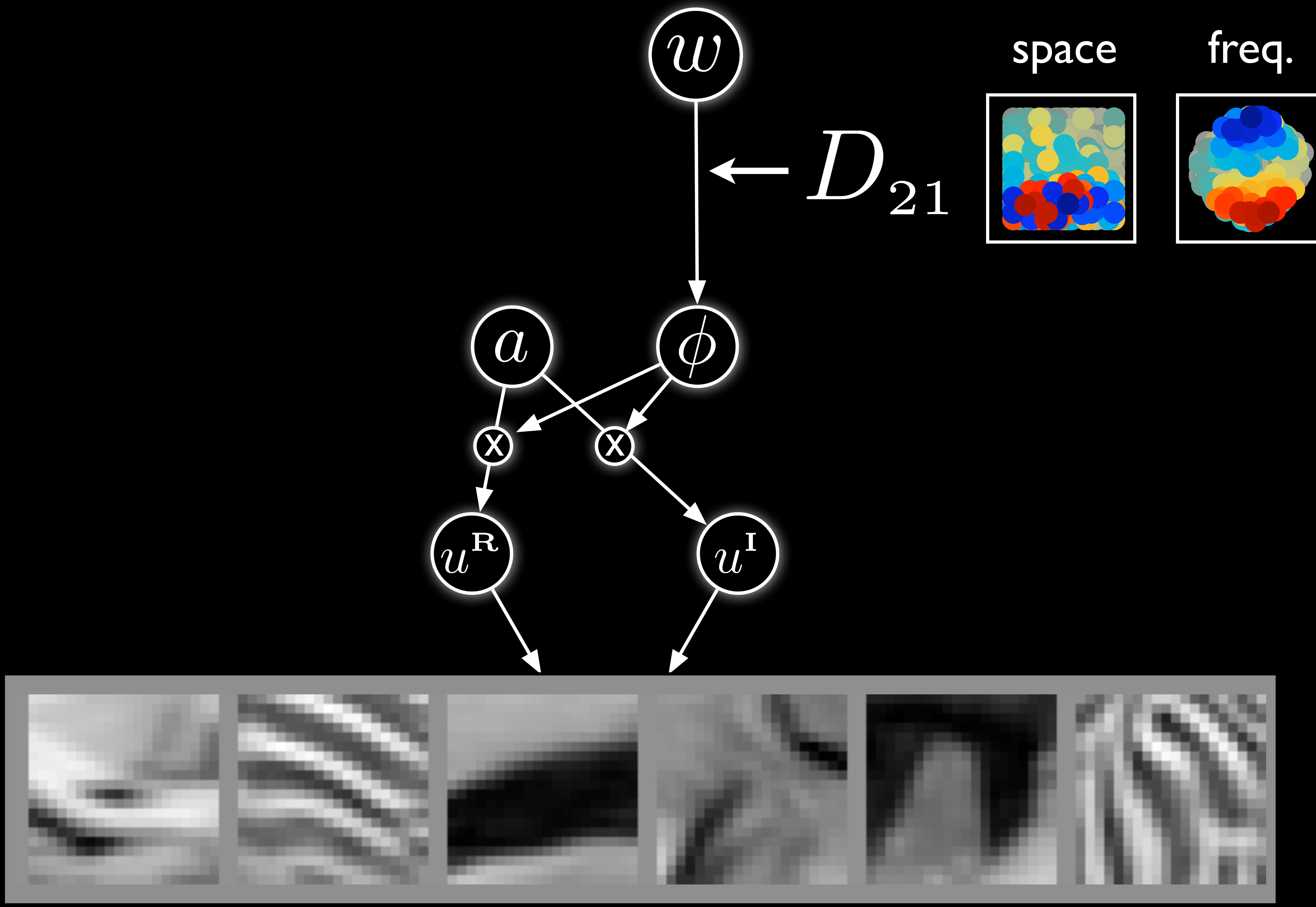
# Learned motion components *D*



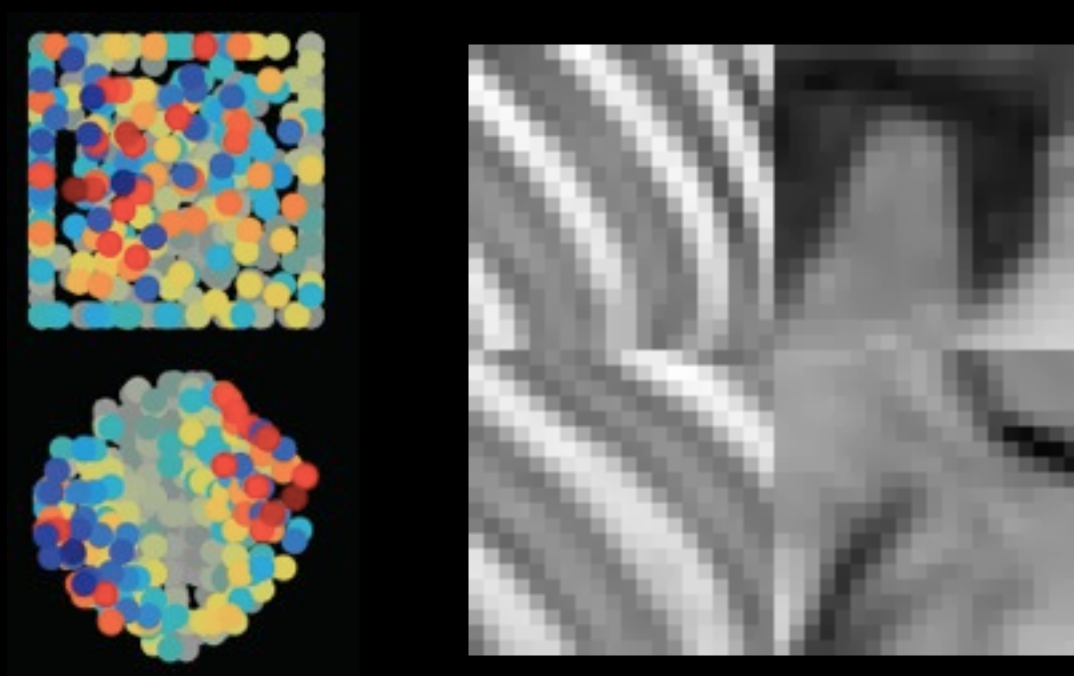
# Learned motion component

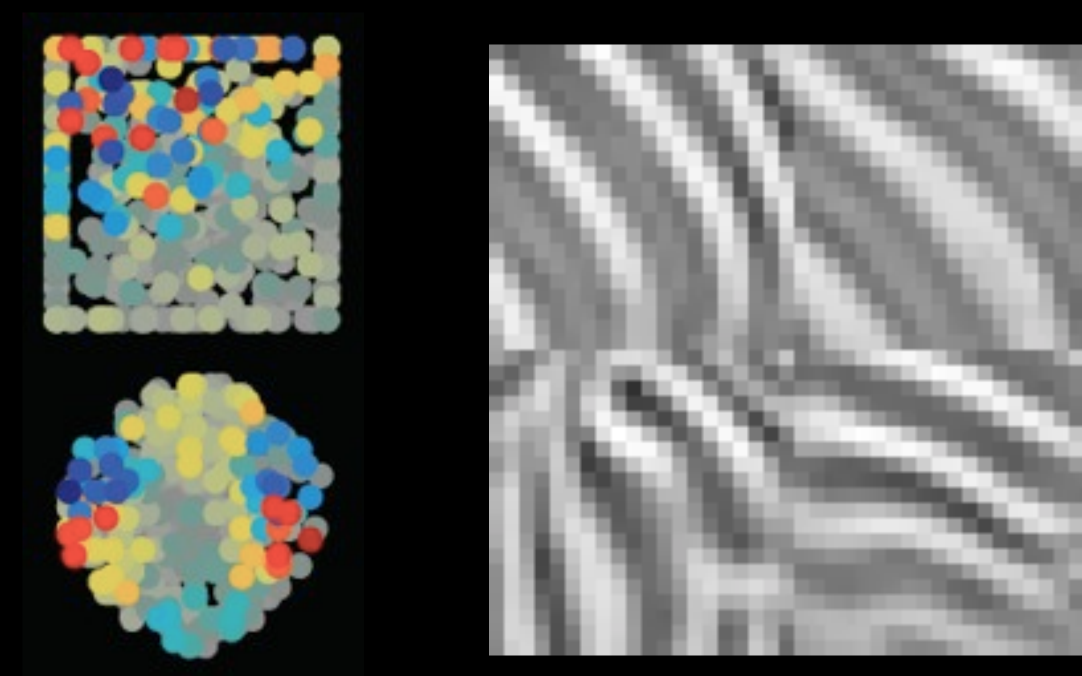


# Learned motion component

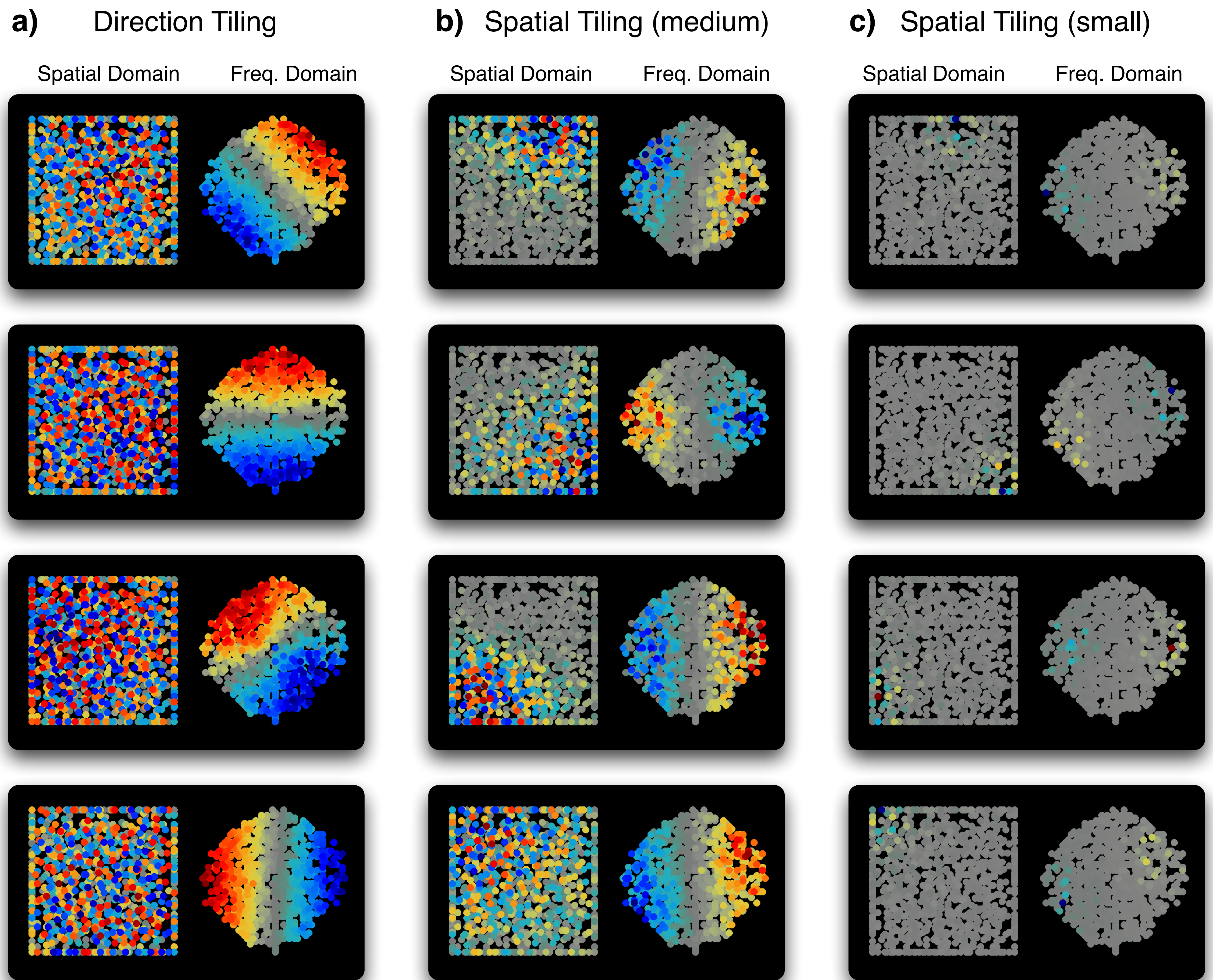


# Other examples



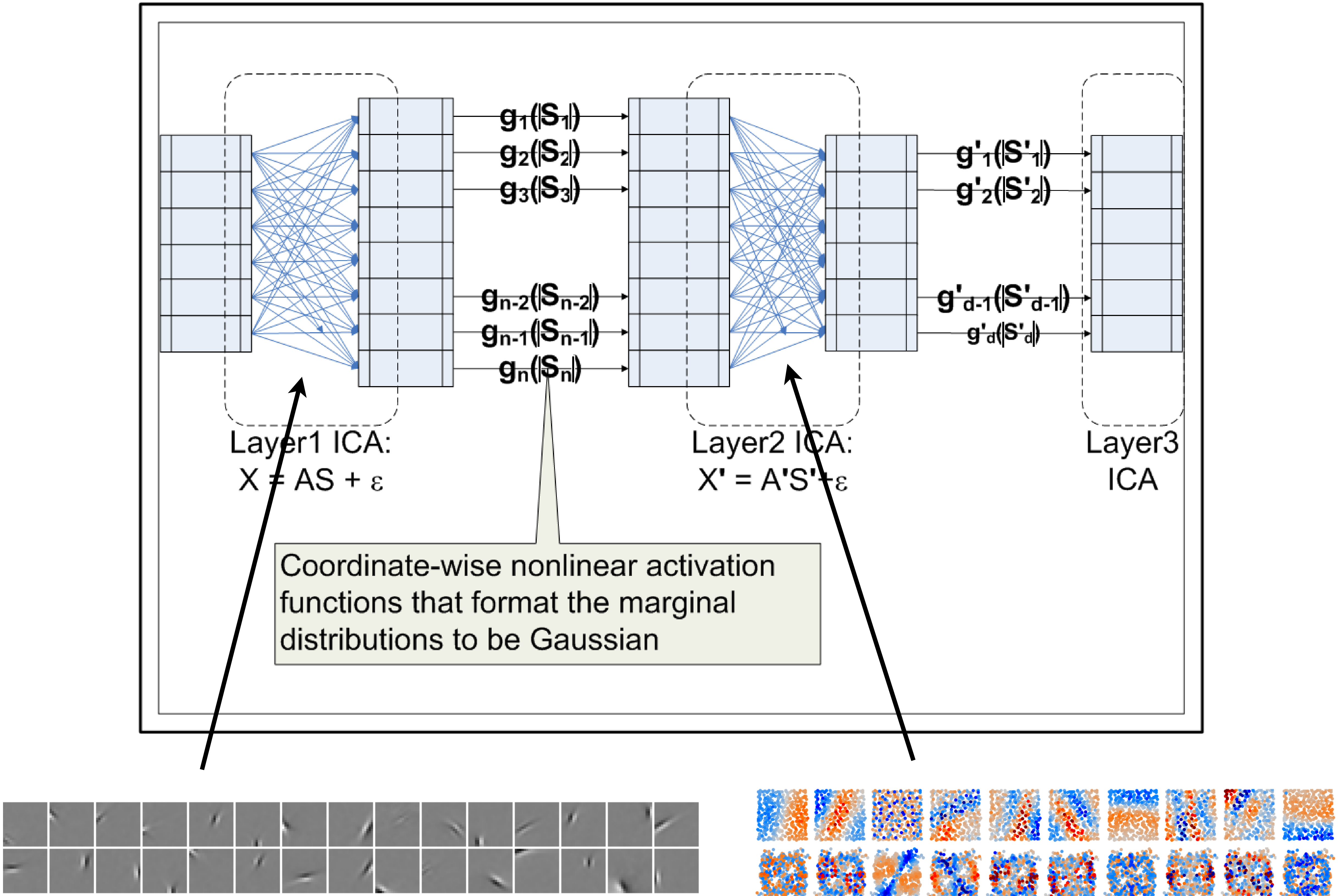


# Learned motion components *D*

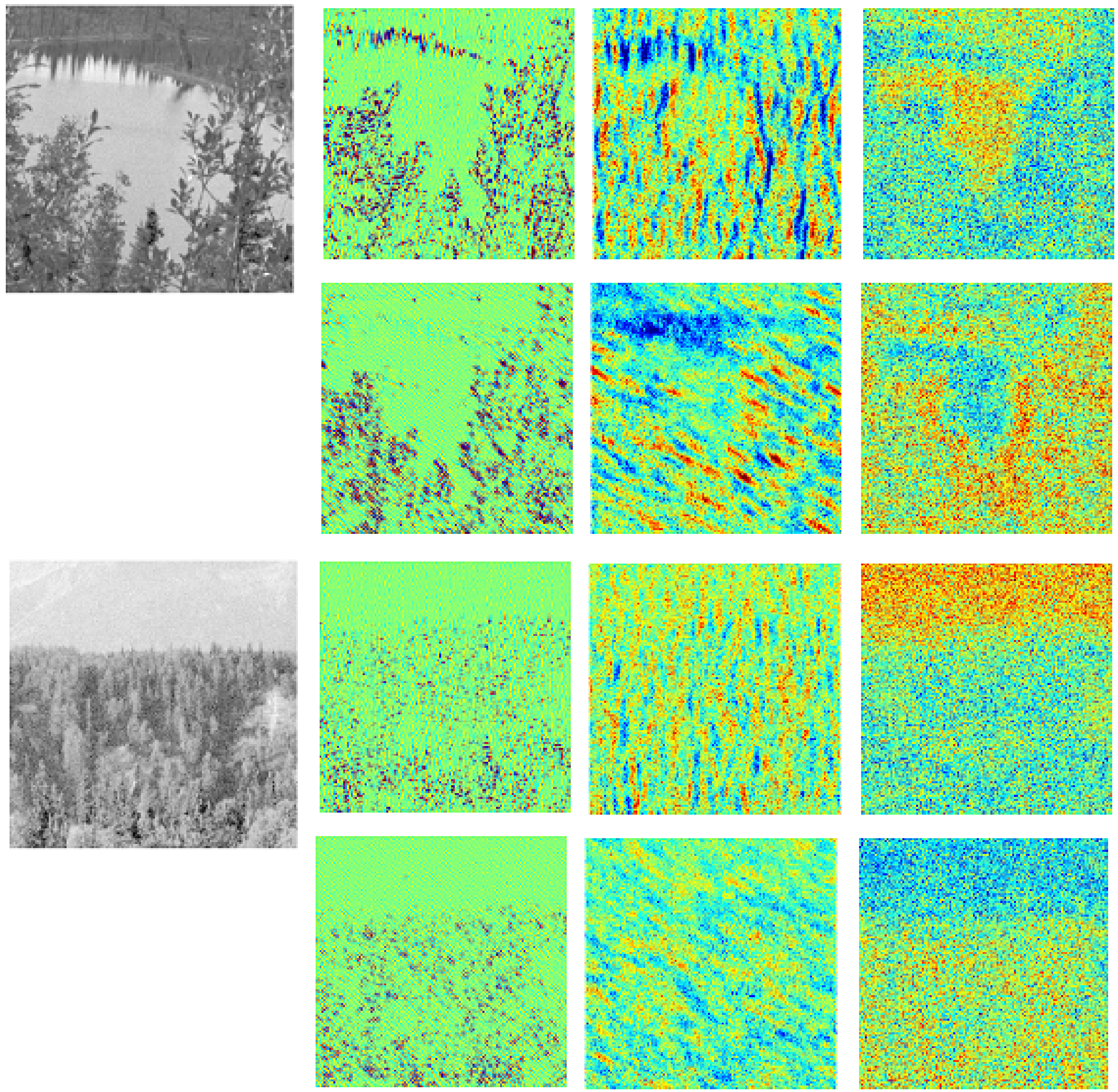


# Recursive ICA

Shan, Zhang & Cottrell (2007)



# Recursive ICA



| statistics | coding strategy | neurobiological substrate |
| --- | --- | --- |
| contrast histogram | histogram equalization | photoreceptors/ bipolar cells |
| autocorrelation function | whitening | retina/LGN |
| sparse components | localized, oriented, bandpass feature decomposition | V1 ‘simple cells’ |
| amplitude components | texture coding | V1/V2 ‘complex cells’ |
| phase components | motion coding | MT |