Fragment Grammars: Productivity and Reuse in Language

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-ness

-ness

- *circuitousness*, *grandness*, *orderliness*, *pretentiousness*, *cheapness*, *coolness*, *warmness*, ...

-ness

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- Adj>N

-ness

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- Adj>N
- *grand* + -ness

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- Adj>N
- *grand* + -ness
- *pine-scentedness*

-ity

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- **pine-scentedity*

-ity

-ity

- But ...

-ity

- But ...
  - -ile/-al/-able/-ic/-(i)an

-ity

- But ...
  - -ile/-al/-able/-ic/-(i)an
  - *Bayesable*

-ity

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  - -ile/-al/-able/-ic/-(i)an
  - *Bayesable*
    - *Bayesability*

-ity

- But ...
  - -ile/-al/-able/-ic/-(i)an
  - *Bayesable*
    - *Bayesability*
  - *Coolity is not trying* (from *Huffington Post*)

-th

-th

- *warmth*, *width*, *truth*, *depth*, ...

-th

- *warmth*, *width*, *truth*, *depth*, ...
- Adj>N

-th

- *warmth*, *width*, *truth*, *depth*, ...
- Adj>N
- *heal/health*, *dead/death*, *young/youth*, *vile/filth*, *slow/sloth*

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- *roomth*, *greenth*

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Many enjoy the warmth, Vikings prefer the ***coolth***

Problem of Productivity

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- How are such differences in productivity represented by the adult language user?
- How are such differences learned by the child?

Outline

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1. The Proposal.

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2. Five Models of Productivity and Reuse.

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3. English Derivational Morphology

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4. Conclusion

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The Proposal

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Starting Computational System

W	$\longrightarrow$	N	
W	$\longrightarrow$	V	
W	$\longrightarrow$	Adj	
W	$\longrightarrow$	Adv	
N	$\longrightarrow$	Adj	*-ness*
N	$\longrightarrow$	Adj	*-ity*
N	$\longrightarrow$	*electro-*	N
N	$\longrightarrow$	*magnet*	
N	$\longrightarrow$	*dog*	
...			
V	$\longrightarrow$	N	*-ify*
V	$\longrightarrow$	Adj	*-ize*
V	$\longrightarrow$	*re-*	V
V	$\longrightarrow$	*agree*	
V	$\longrightarrow$	*count*	
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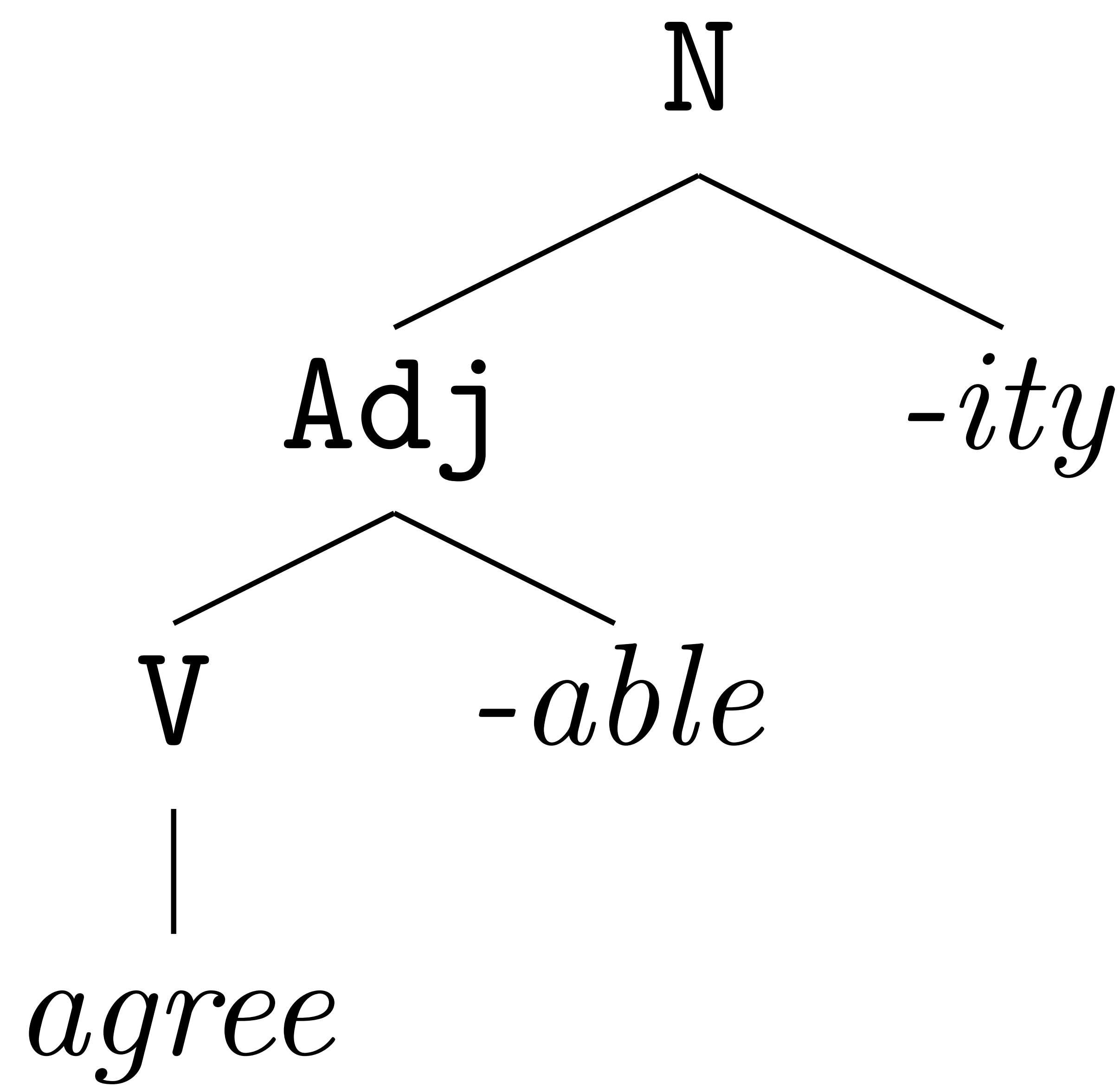
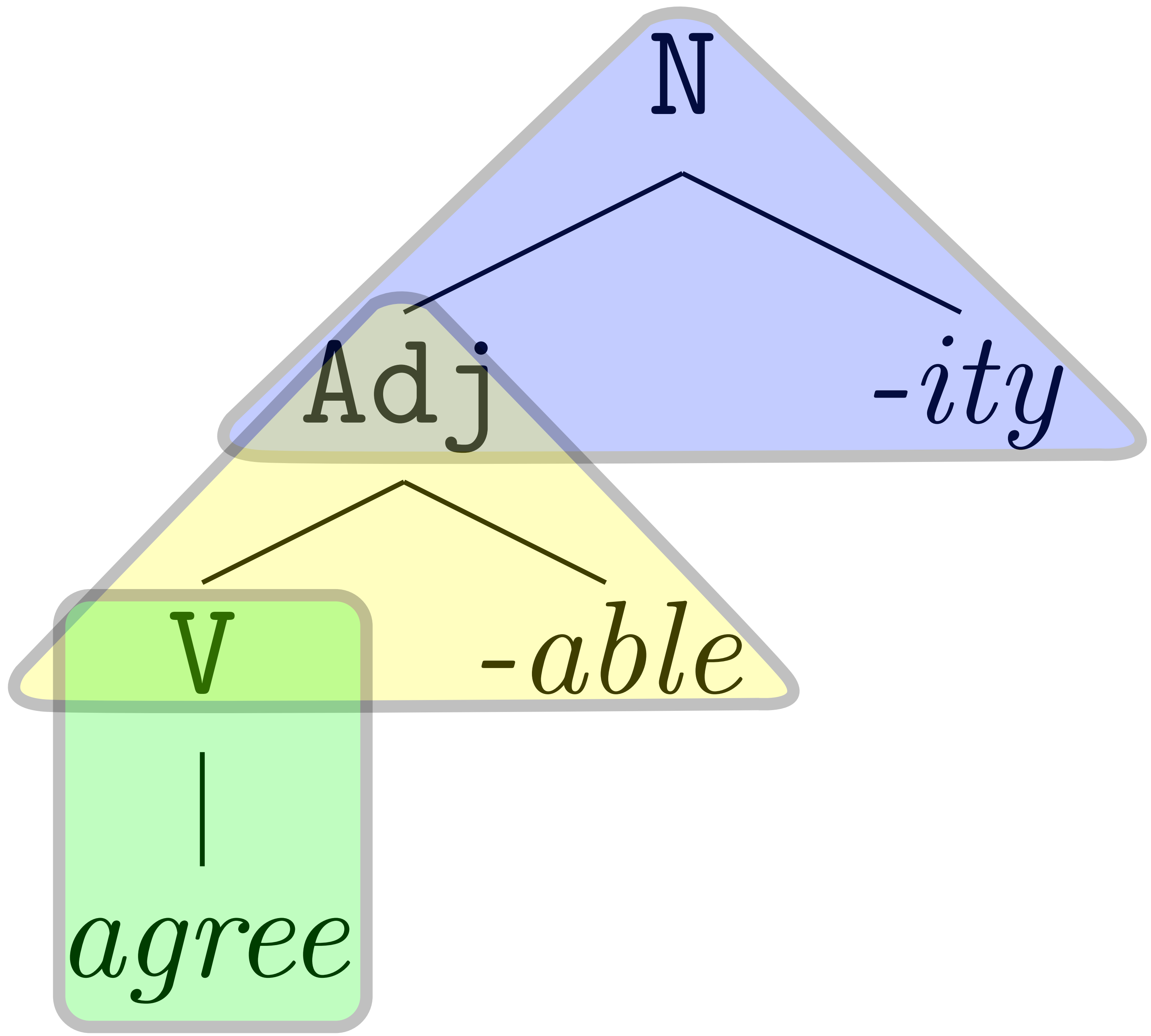
Adj	$\longrightarrow$	*dis-*	Adj
Adj	$\longrightarrow$	V	*-able*
Adj	$\longrightarrow$	N	*-ic*
Adj	$\longrightarrow$	N	*-al*
Adj	$\longrightarrow$	*tall*	
...			
Adv	$\longrightarrow$	Adj	*-ly*
Adv	$\longrightarrow$	*today*	
...			



Subcomputations



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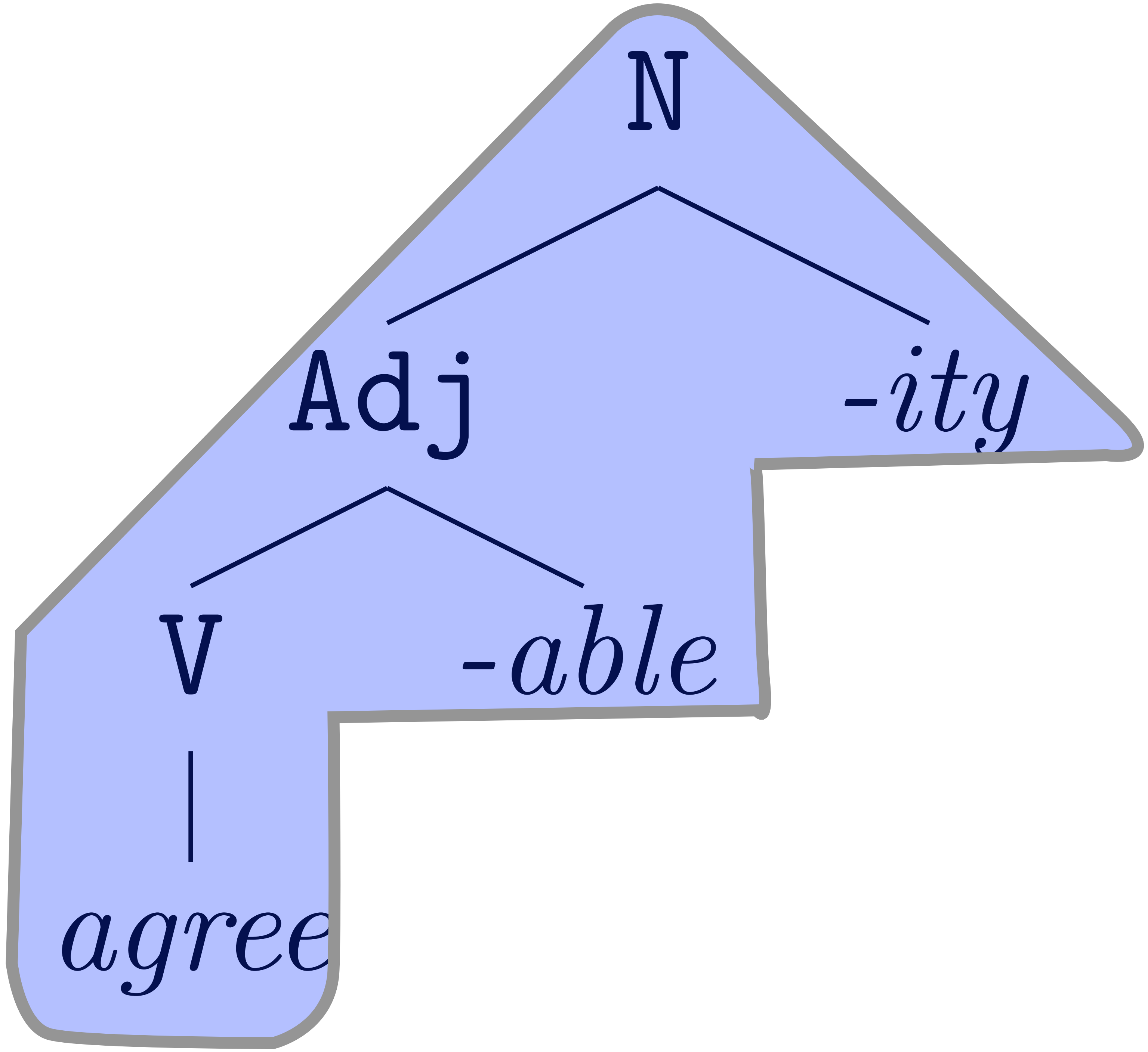

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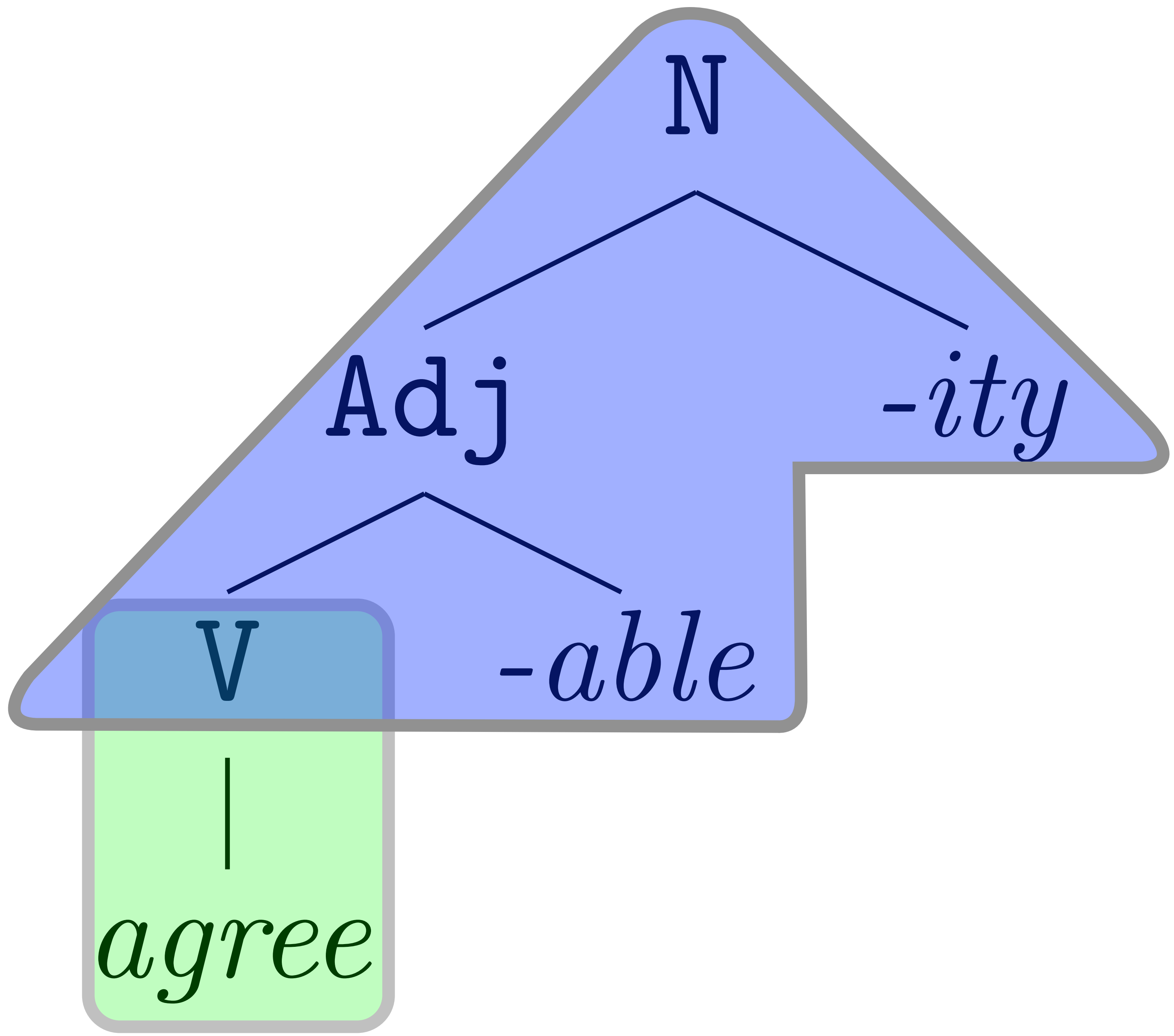

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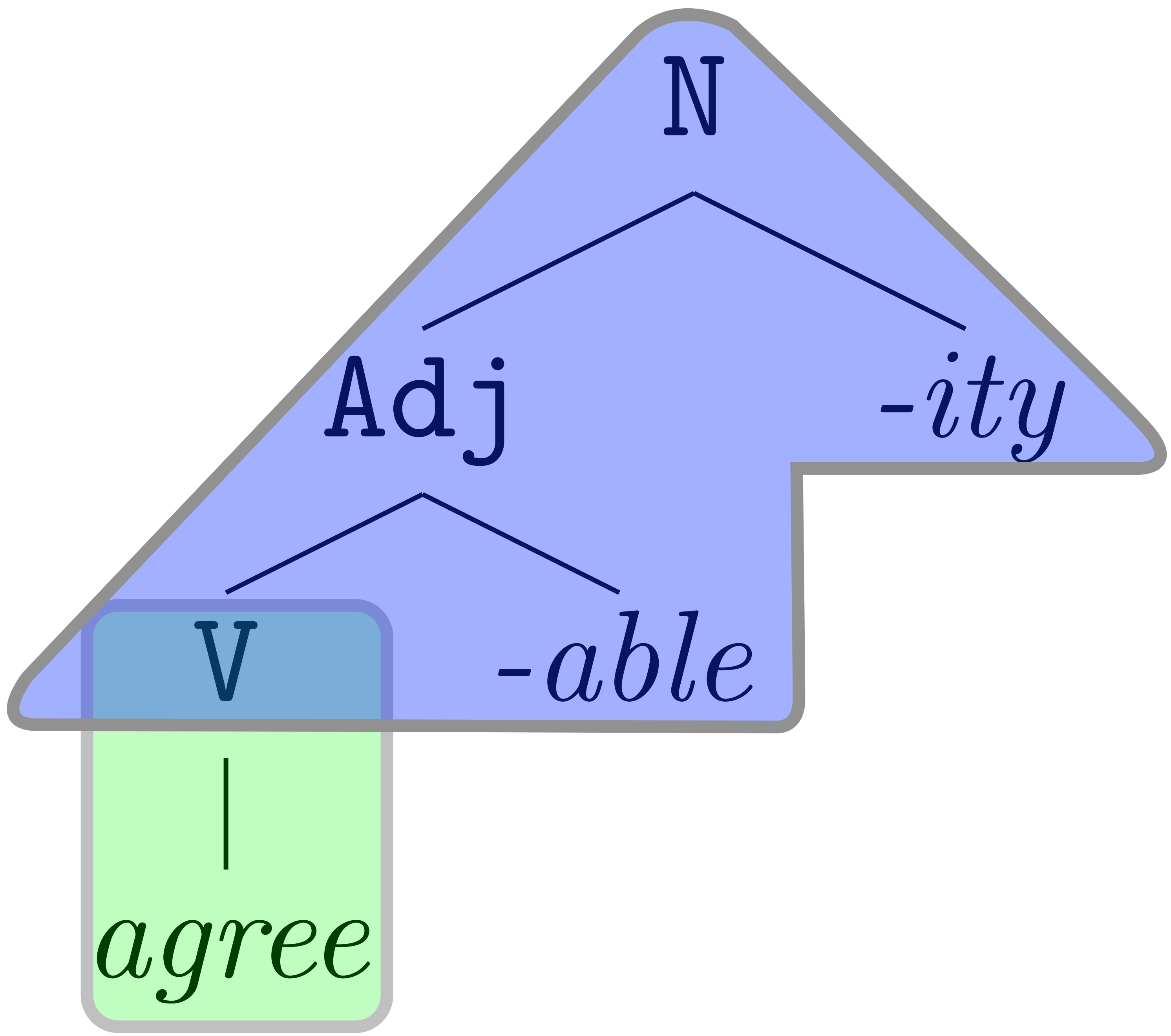
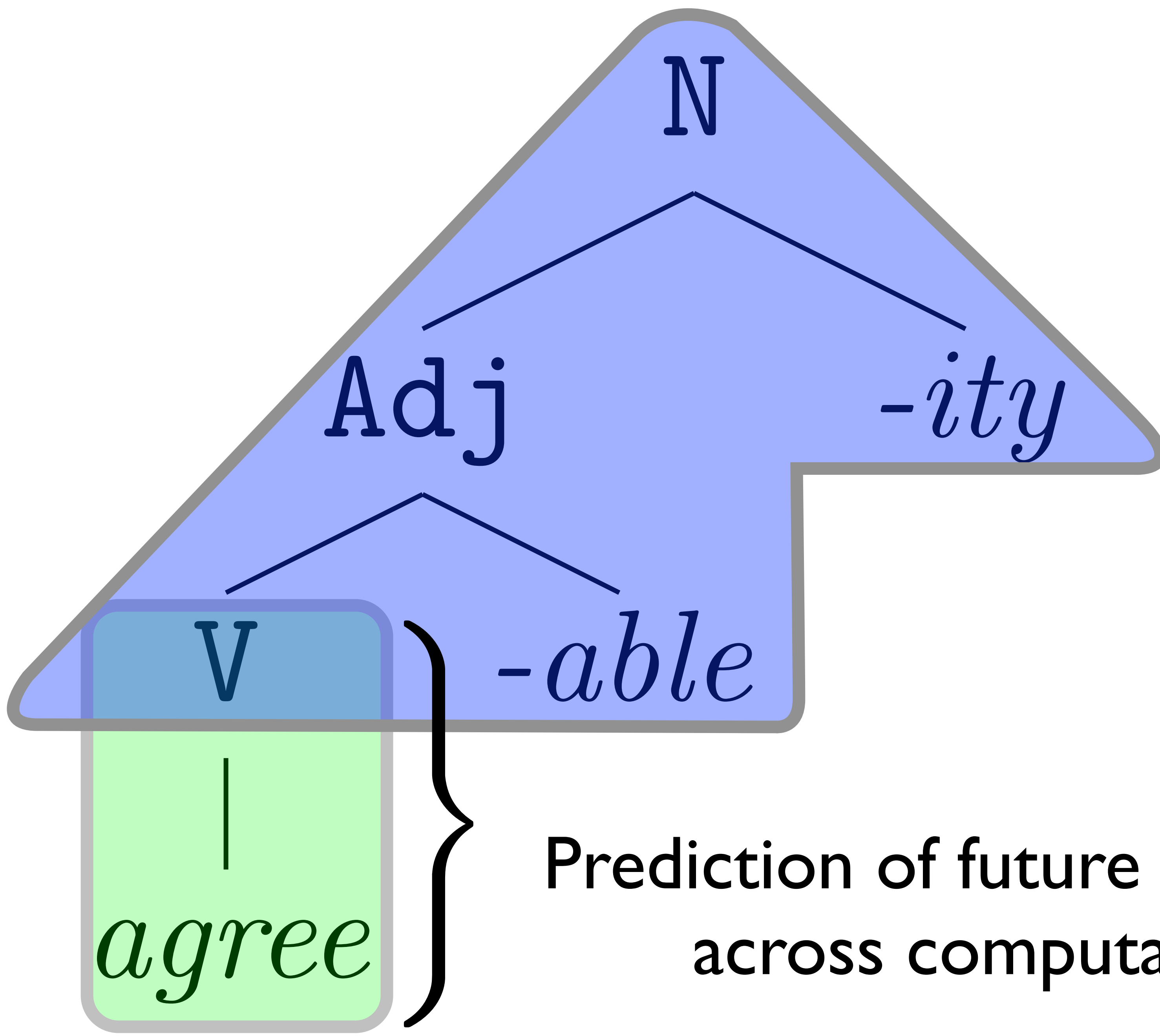
Bayesian Rational Analysis (Anderson, 1992)

- Find subcomputations which provide best explanation for the data.
- What *evidence* is available to the learner?
    - Which patterns give rise to productivity, which patterns imply reuse?

Subcomputations as Predictions

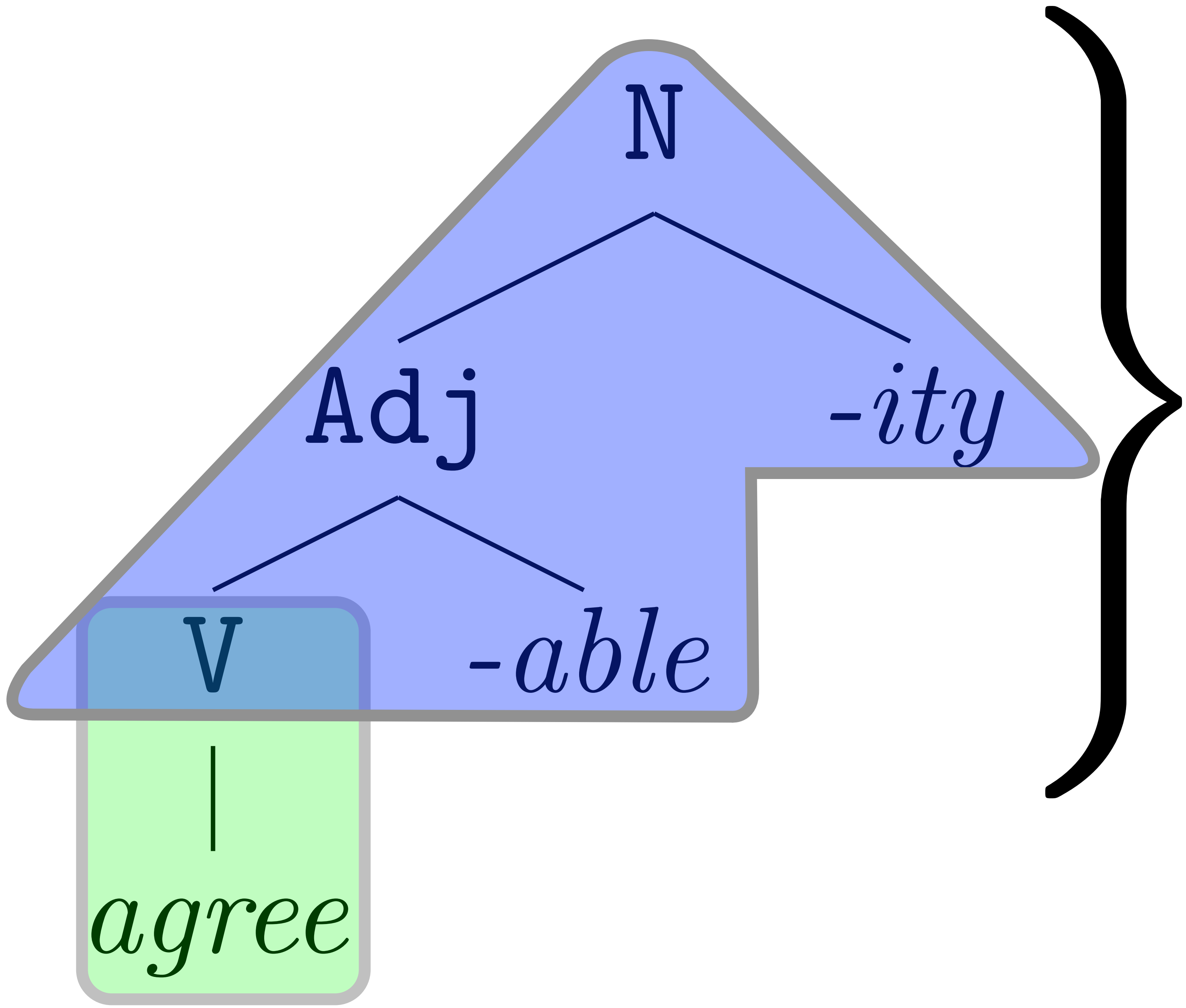


Subcomputations as Predictions



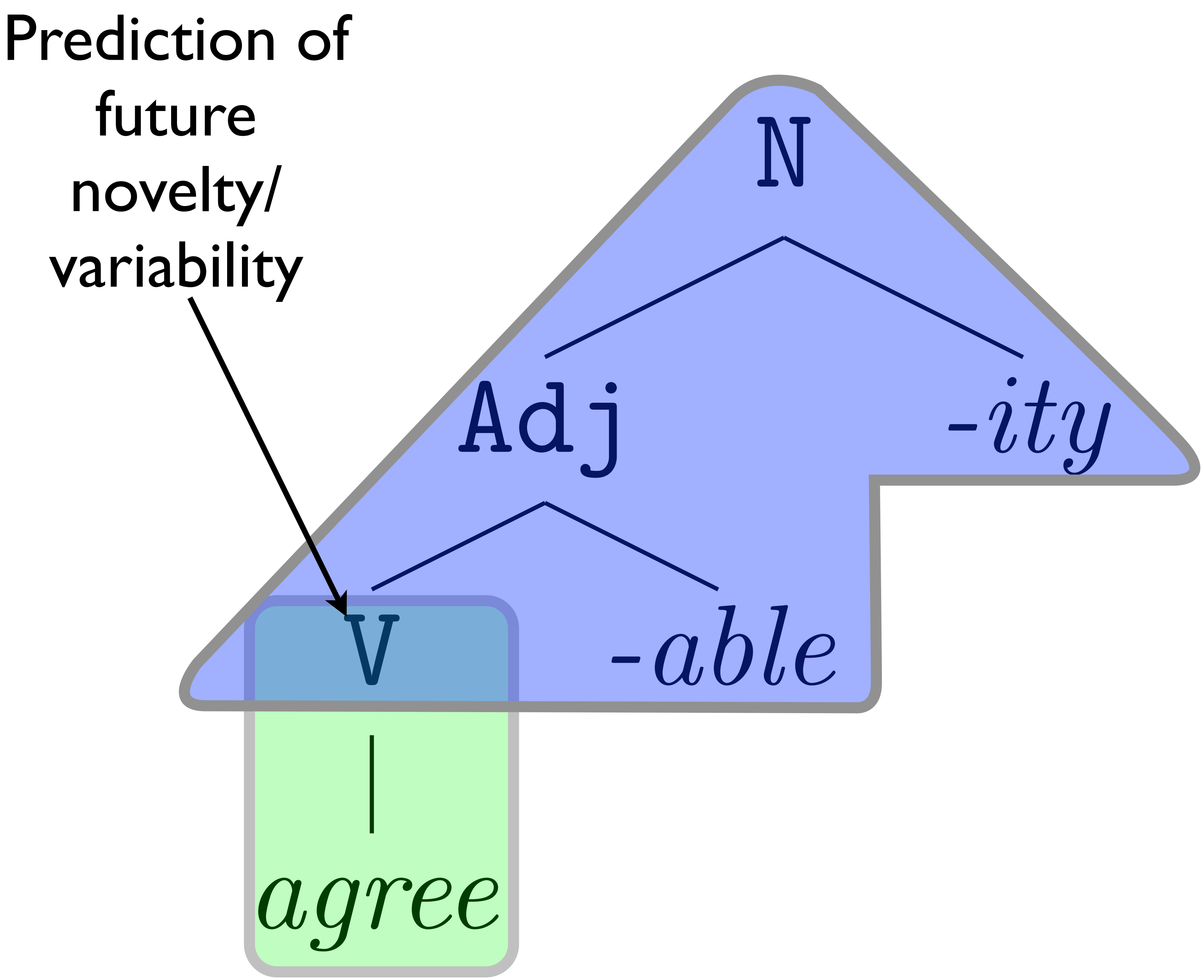
Prediction of future reusability
across computations

Subcomputations as Predictions



Prediction of future reusability of <u>*combination*</u>

Subcomputations as Predictions



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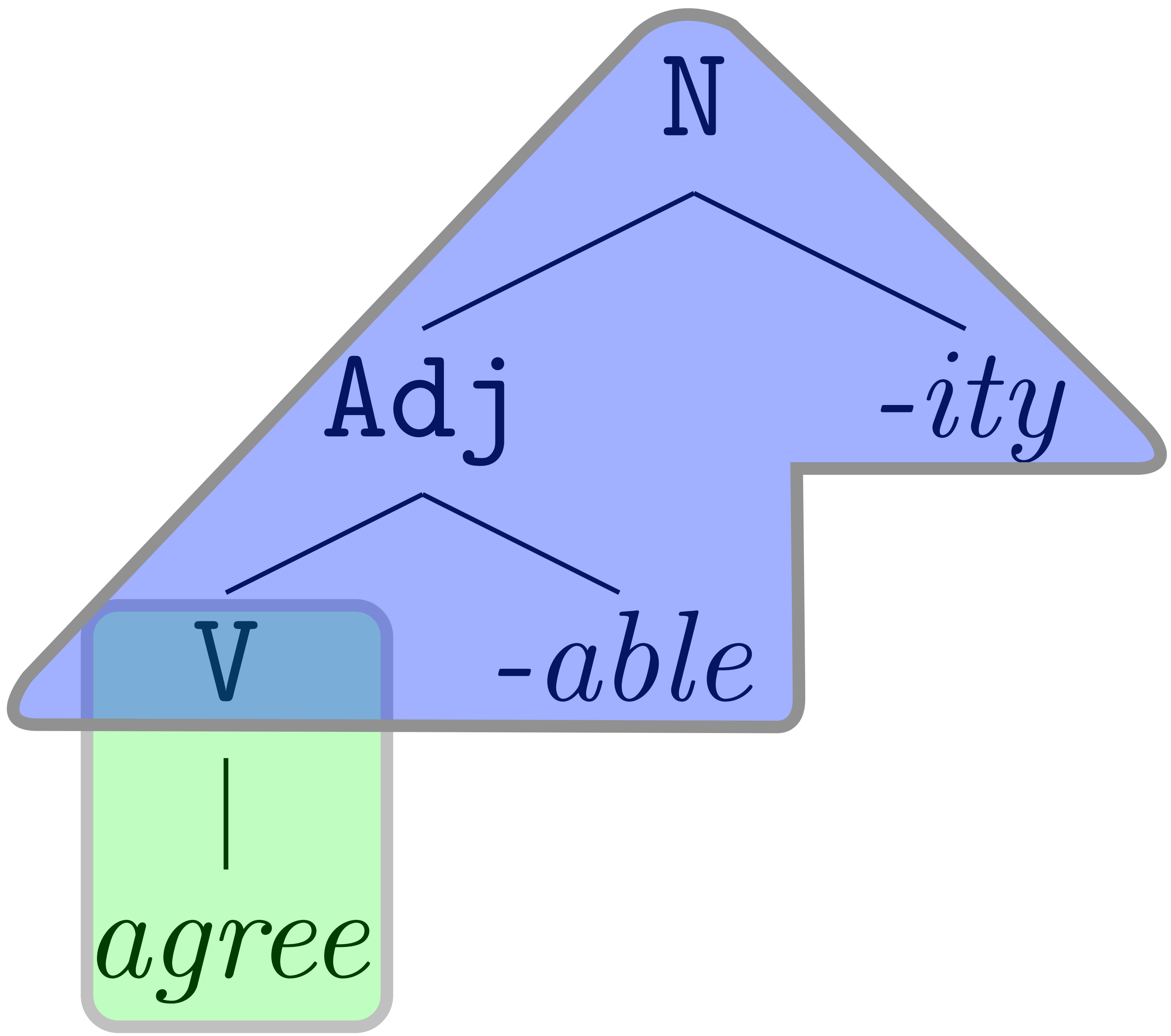

Tradeoff between productivity and reuse

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The Formal Model:
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- Generalization of *Adaptor Grammars* (Johnson et al., 2007).
- Bayesian non-parametric distributions (*Pitman-Yor*).
- Notion of *compiling* subcomputations via tools from probabilistic programming (Church language; Goodman et al., 2008).
  - Stochastic memoization (Johnson et al., 2007) of stochastically lazy/eager programs.

Fragment Grammars via Probabilistic Programming *(Church)*

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- Alternative to more standard mathematical formalization (see, O'Donnell, 2011).
- Highlights relationship between formalisms (PCFGs, Adaptor Grammars, Fragment Grammars).
- Cross fertilization of ideas from the theory of programming languages.
- Caveat: Church inference algorithms do not work well for these models.

Fragment Grammars via Probabilistic Programming

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1. Stochastic computation via `unfold`

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Context Free Grammars

W	⟶	N	
W	⟶	V	
W	⟶	Adj	
W	⟶	Adv	
N	⟶	Adj	*-ness*
N	⟶	Adj	*-ity*
N	⟶	*electro-*	N
N	⟶	*magnet*	
N	⟶	*dog*	
...			
V	⟶	N	*-ify*
V	⟶	Adj	*-ize*
V	⟶	*re-*	V
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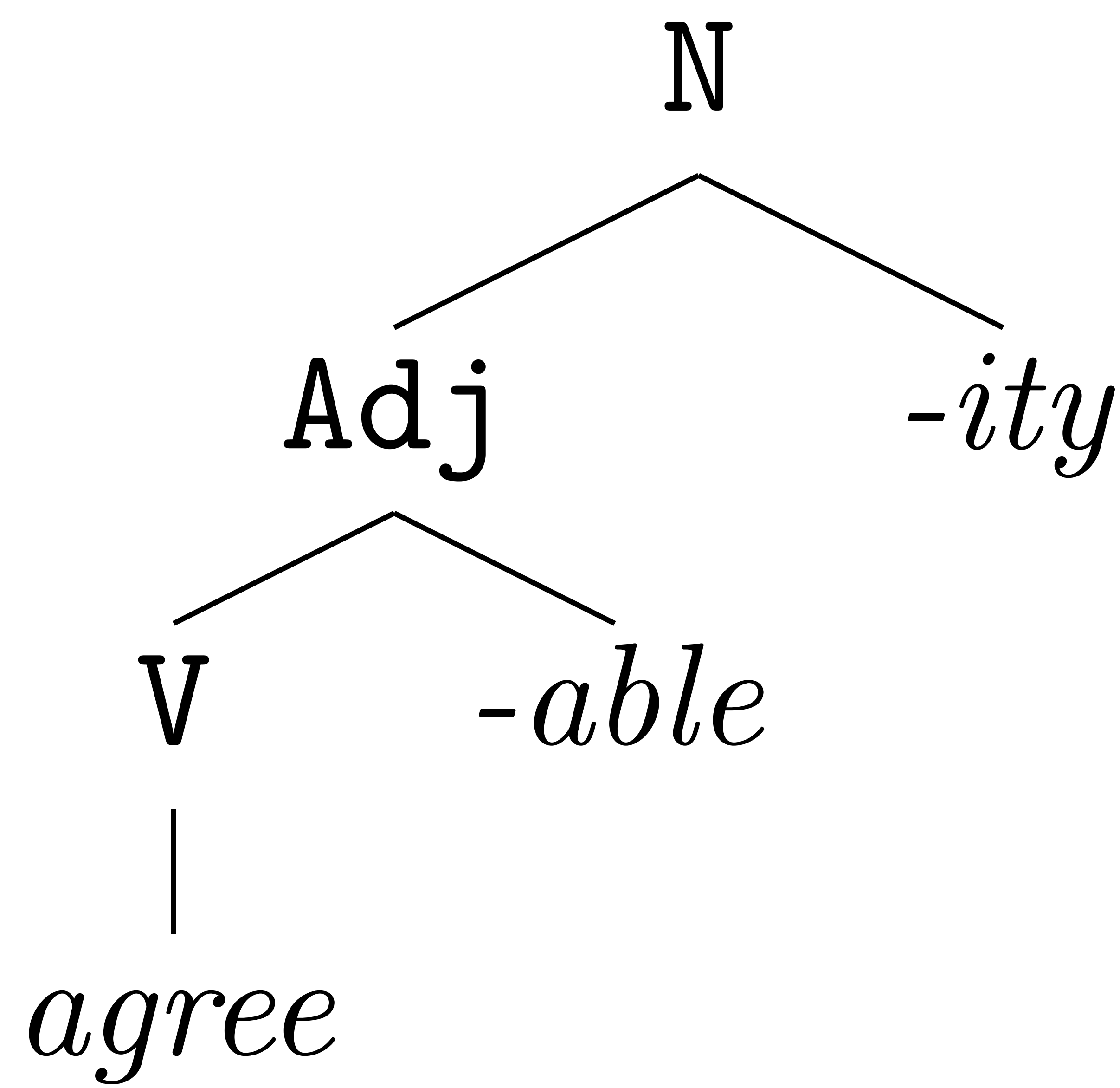
Adj	⟶	*tall*	
...			
Adv	⟶	Adj	*-ly*
Adv	⟶	*today*	
...			



Declarative Knowledge of Constituent Structure

p_{W_1}	W	$\longrightarrow$	N	
p_{W_2}	W	$\longrightarrow$	V	
p_{W_3}	W	$\longrightarrow$	Adj	
p_{W_4}	W	$\longrightarrow$	Adv	
p_{N_1}	N	$\longrightarrow$	Adj	*-ness*
p_{N_2}	N	$\longrightarrow$	Adj	*-ity*
p_{N_3}	N	$\longrightarrow$	*electro-*	N
p_{N_4}	N	$\longrightarrow$	*magnet*	
p_{N_5}	N	$\longrightarrow$	*dog*	
	...			
p_{V_1}	V	$\longrightarrow$	N	*-ify*
p_{V_2}	V	$\longrightarrow$	Adj	*-ize*
p_{V_3}	V	$\longrightarrow$	*re-*	V
p_{V_4}	V	$\longrightarrow$	*agree*	
p_{V_5}	V	$\longrightarrow$	*count*	
	...			
p_{Adj_1}	Adj	$\longrightarrow$	*dis-*	Adj
p_{Adj_2}	Adj	$\longrightarrow$	V	*-able*
p_{Adj_3}	Adj	$\longrightarrow$	N	*-ic*
p_{Adj_4}	Adj	$\longrightarrow$	N	*-al*
p_{Adj_5}	Adj	$\longrightarrow$	*tall*	
	...			
p_{Adv_1}	Adv	$\longrightarrow$	Adj	*-ly*
p_{Adv_2}	Adv	$\longrightarrow$	*today*	
	...			

Declarative Knowledge of Constituent Structure

```
(define sample-rhs
  (lambda (nonterminal)
    (case nonterminal
      (('W) (multinomial (list (list 'N) (list 'V) (list 'Adj) (list 'Adv) ... )
                         (list p_W1 p_W2 p_W3 p_W4 ...)))
      (('N) (multinomial (list (list 'Adj 'ness) (list 'Adj 'ity) (list 'electro 'N) (list 'magnet) (list 'dog) ...)
                         (list p_N1 p_N2 p_N3 p_N4 p_N5 ...)))
      (('V) (multinomial (list (list 'N 'ify) (list 'Adj 'ize) (list 're 'V) (list 'agree) (list 'count) ...)
                         (list p_V1 p_V2 p_V3 p_V4 p_V5 ...)))
      (('Adj) (multinomial (list (list 'dis 'Adj) (list 'V 'able) (list 'N 'ic) (list 'N 'al) (list 'tall) ...)
                           (list p_Adj1 p_Adj2 p_Adj3 p_Adj4 p_Adj5 ...)))
      (('Adv) (multinomial (list (list 'Adj 'ly) (list 'today) ...)
                                 (list p_W1 p_W2 ...))))))
```

Fundamental Recursive Computation: `unfold`

```
(define unfold
 (lambda (symbol)
  (if (terminal? symbol)
   symbol
   (map unfold (sample-rhs symbol)))))
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```

Choose a right-hand side for
symbol:
N → Adj *-ity*

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```

Recursively apply `unfold` to each symbol on right-hand side

Computation Trace

```
(unfold 'N)
```

Computation Trace

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```

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      (if (terminal? symbol)
         symbol
         (map unfold (sample-rhs symbol)))))
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Computation Trace

(unfold ‘N)

(sample-rhs ‘N)

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(define unfold
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Computation Trace

```
(unfold 'N)
     |
(sample-rhs 'N)
```

Computation Trace

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(unfold 'N)
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N → Adj *-ity*

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(unfold 'N)
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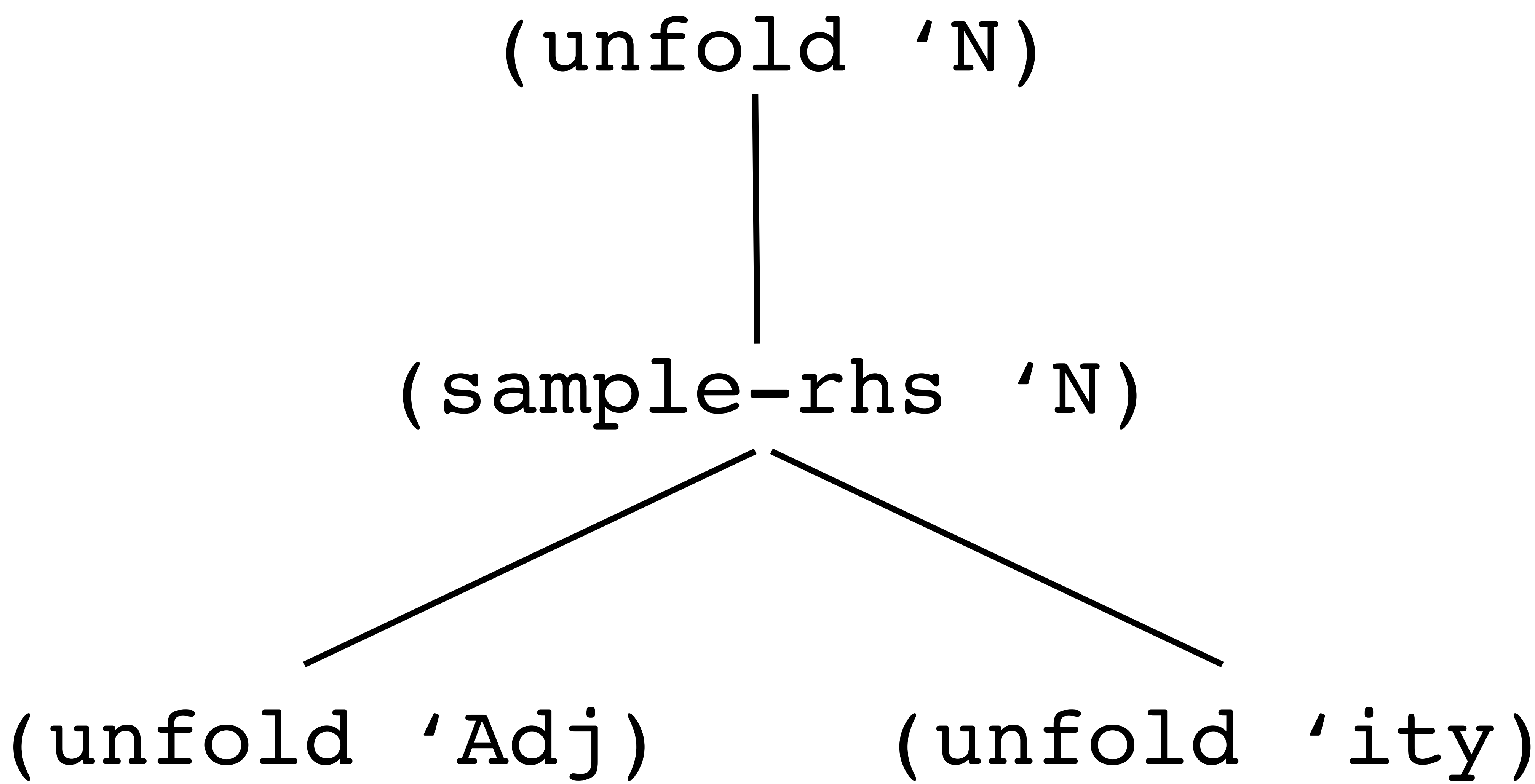
(sample-rhs 'N)
```

Computation Trace



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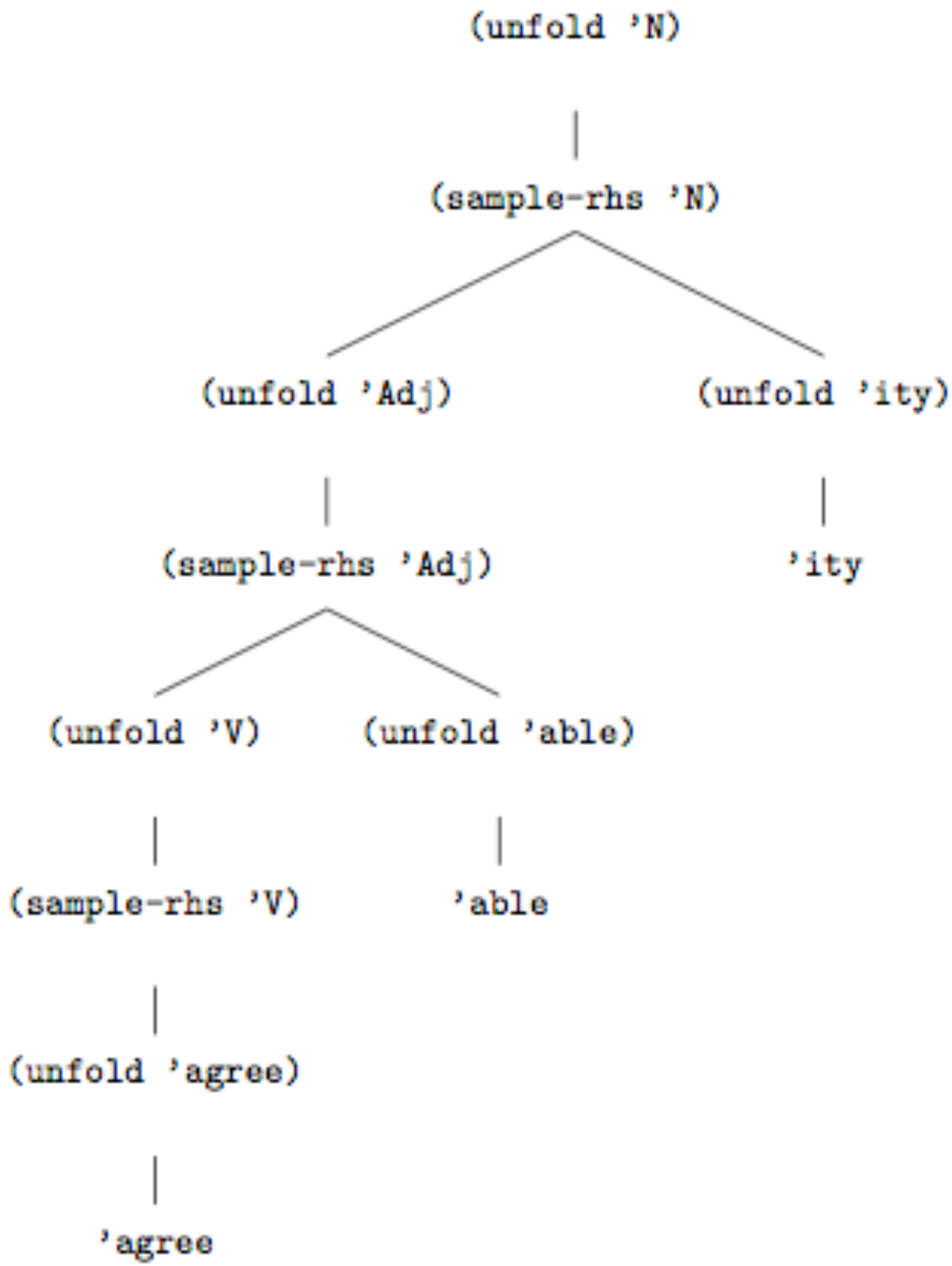
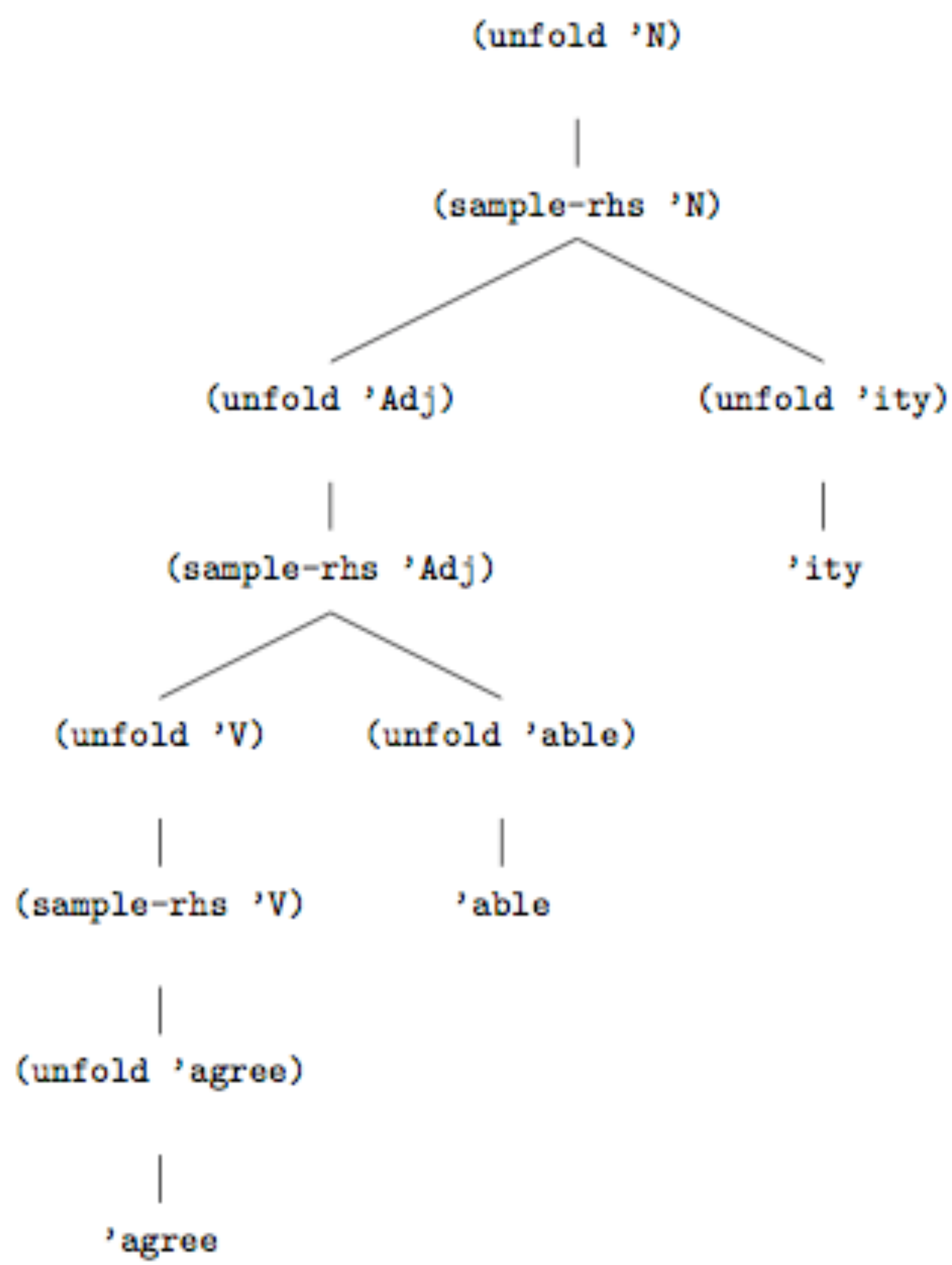
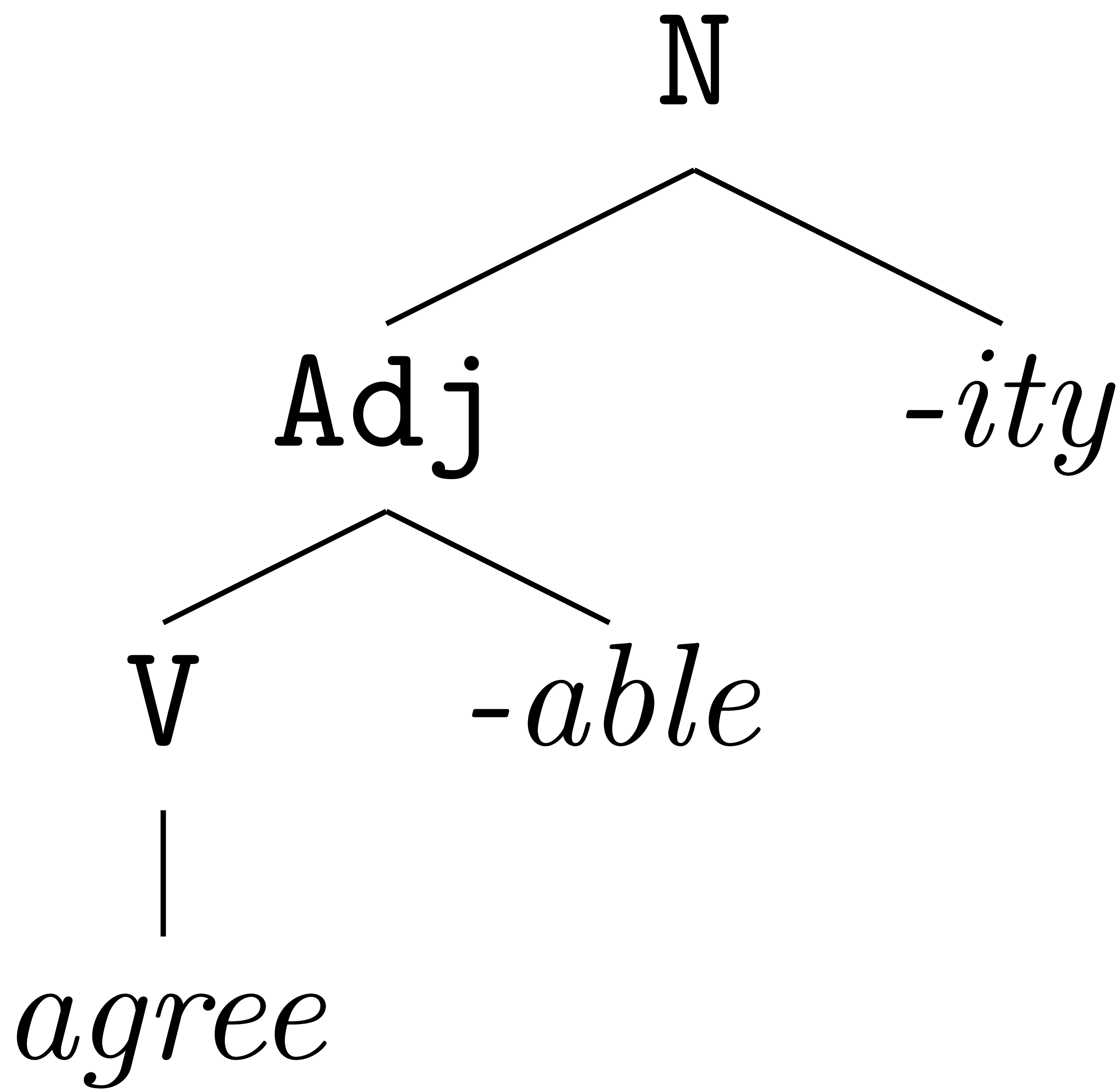
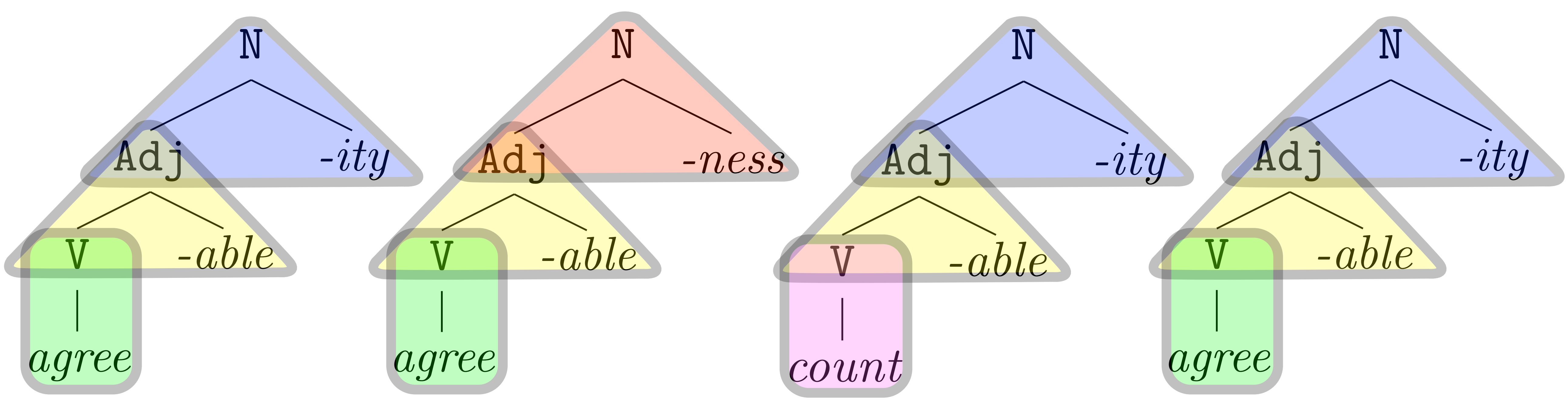


Trace as Tree





Reusability for PCFGs



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2. Stochastic reuse via memoization
3. Partial computations via stochastic laziness

Memoization

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- Higher-order function: `mem`

Reuse through Memoization

```
(define eye-color
  (lambda (person)
    (if (flip 0.5) ‘blue ‘brown)))
```

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    (eye-color ‘bob)  => ‘blue
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    (eye-color ‘bob)  => ‘brown
                         ...
```

Reuse through Memoization

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(define eye-color
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Anywhere in the program where (**eye-color** 'bob) is used, we will *reuse* same value.

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(eye-color ‘bob)  => ‘blue
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Reuse through Memoization

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Anywhere in the program where `(eye-color ‘bob)` is used, we will *reuse* same value.

```
(eye-color ‘bob)  => ‘blue
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```

Reuse through Memoization

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(eye-color 'bob)   => 'blue
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(eye-color 'bob)   => 'blue
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                   ...
```

Stochastic Reusability

- Deterministic memoization always returns same value after first call, but sometimes we want to **probabilistically** favor reuse.

Stochastic Reusability

```
(define location
  (lambda (person)
   (sample-location-in-world)))
```

Stochastic Reusability

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(define location
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(location 'bob)  => 'UCLA
```

Stochastic Reusability

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(define location
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(location ‘bob)  => ‘UCLA
(location ‘bob)  => ‘Antarctica
```

Stochastic Reusability

```
(define location
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   (sample-location-in-world)))
```

```
(location 'bob)  => 'UCLA
(location 'bob)  => 'Antarctica
(location 'bob)  => 'London
```

Stochastic Reusability

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(define location
  (lambda (person)
   (sample-location-in-world)))


(location 'bob)  => 'UCLA
(location 'bob)  => 'Antarctica
(location 'bob)  => 'London
(location 'bob)  => 'Thailand
```

Stochastic Reusability

```
(define location
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(location 'bob)  => 'UCLA
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(location 'bob)  => 'London
(location 'bob)  => 'Thailand
                ...
```

Stochastic Reusability

```
(define location
  (stochastic-mem (lambda (person)
   (sample-location-in-world))))
```

Stochastic Reusability

```
(define location
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 (location 'bob)  => 'home
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```

```
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(location 'bob)  => 'office
```

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(define location
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Stochastic Memoization

(Goodman et al., 2008; Johnson et al., 2007)

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Stochastic Memoization

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- Adaptor Grammars: Anything that can be computed can be stored and reused *probabilistically*.
- Memoization distribution: *Pitman-Yor Processes* (Pitman & Yor, 1995).
- Stochastic memoization + PCFGs = Adaptor Grammars.

Pitman-Yor Process

Pitman-Yor Process

- Generalization of the Chinese Restaurant Process

Pitman-Yor Process

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- Two parameters:

Pitman-Yor Process

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- Two parameters:
  - $a \in [0,1]$

Pitman-Yor Process

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- Two parameters:
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  - $b > -a$

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Probability of Reuse

$$\frac{y_i - a}{N + b}$$

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y_i: Total number of observations of value i

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$$\frac{a \cdot K + b}{N + b}$$

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Properties of PYPs

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Properties of PYPs

- Rich get richer, concentrates distribution on a few values.
- Prefers fewer customers/tables/tables-per-customer.
- Prefers to generate novel values proportional to how often novelty has been generated in the past.

Adaptor Grammars

(Johnson et al., 2007)

```
(define adapted-unfold
  (PYMem a b
    (lambda (symbol)
      (if (terminal? symbol)
        symbol
        (map unfold (sample-rhs symbol))))))
```

Properties of Adaptor Grammars

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- Reuse previous computations (subtrees).

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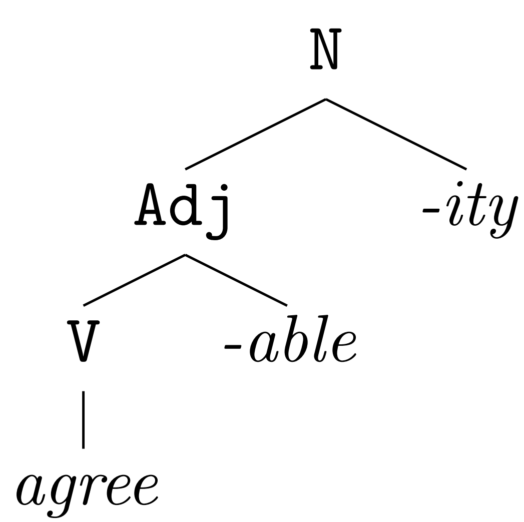

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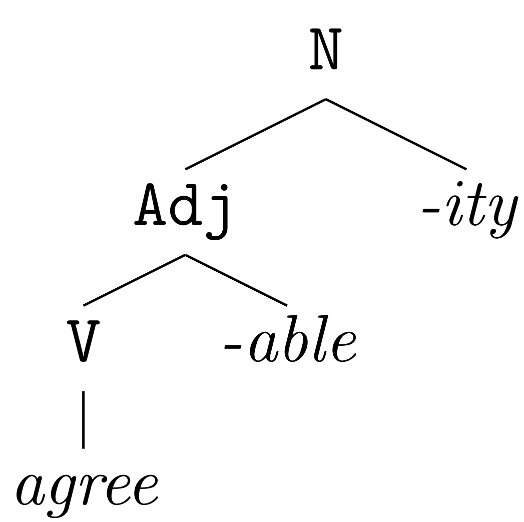
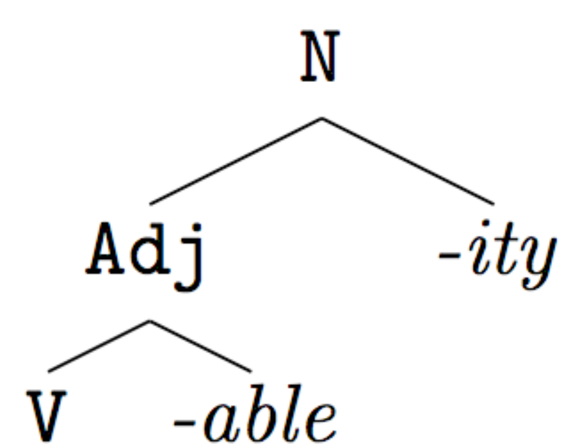
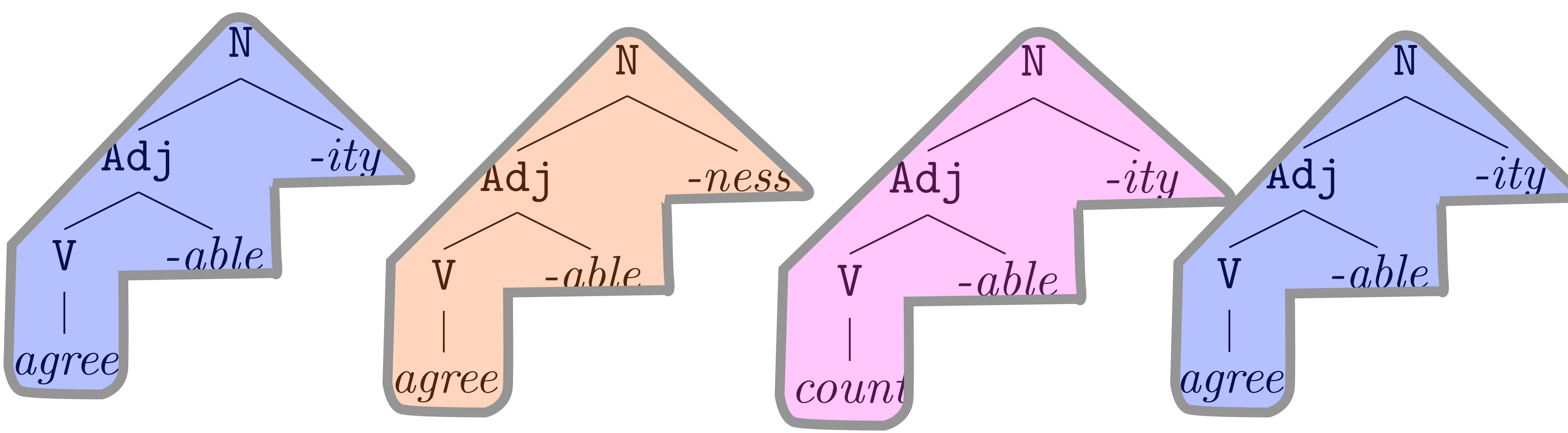


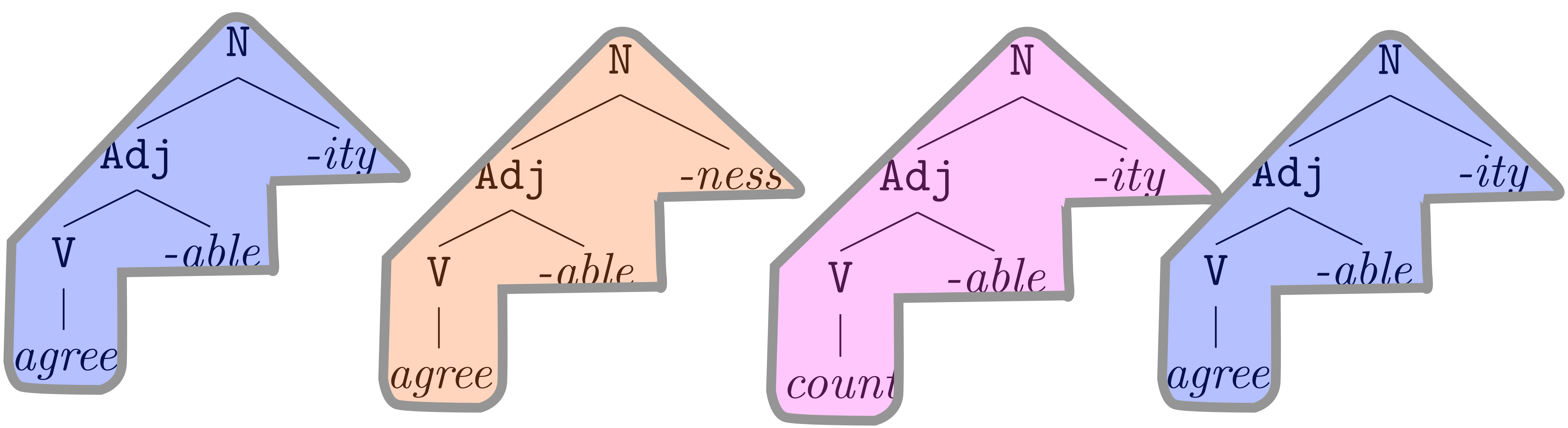


Reusability for Adaptor Grammars



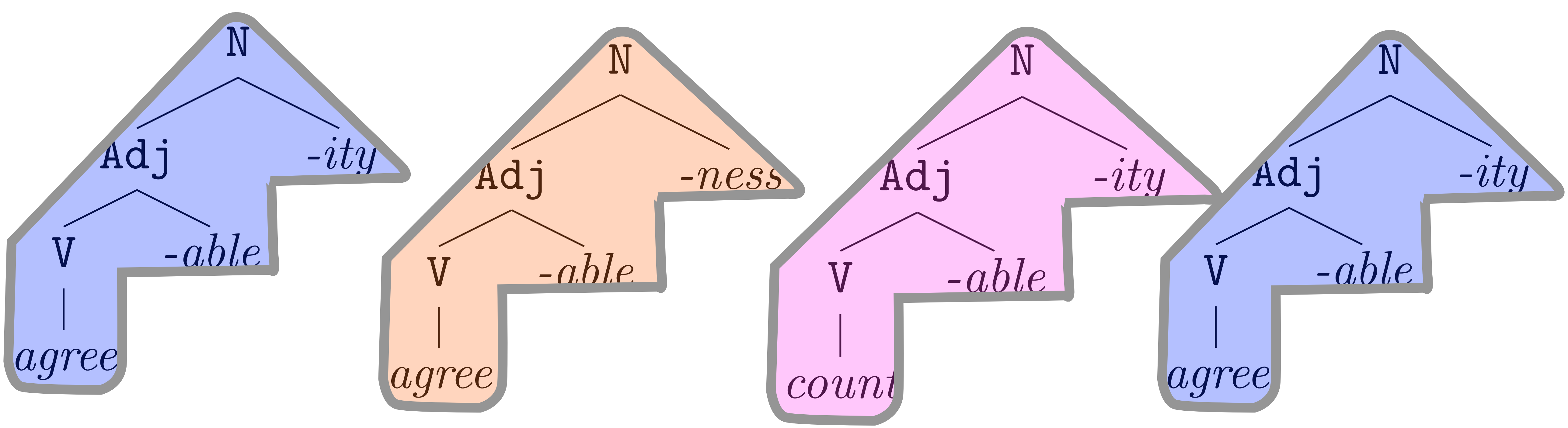
Reusability for Adaptor Grammars

1. Always possible to use base grammar.



Reusability for Adaptor Grammars

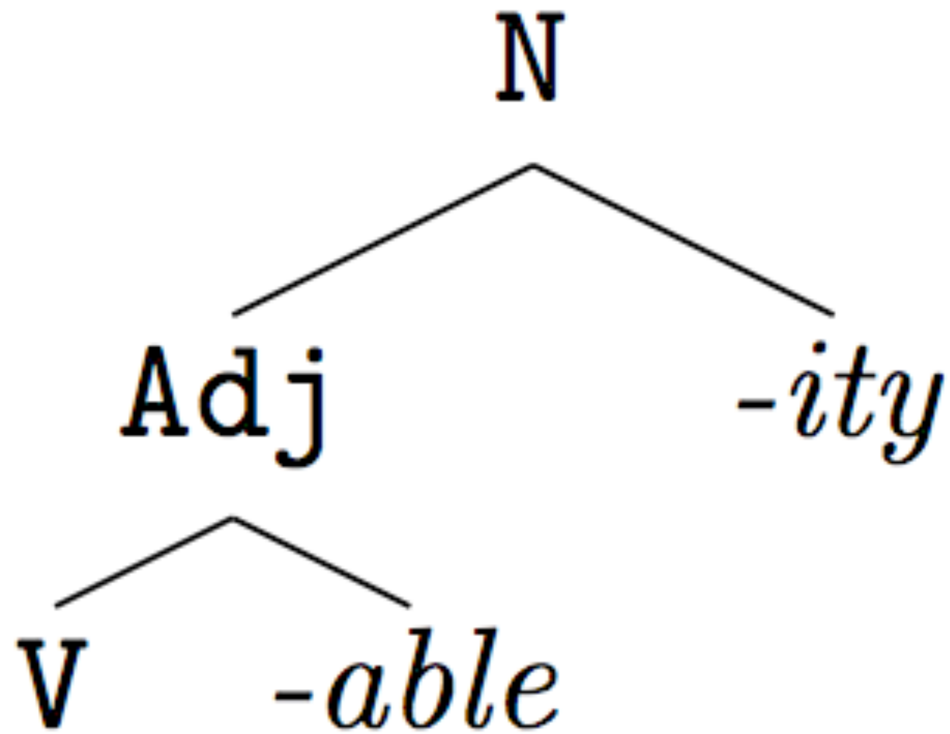
1. Always possible to use base grammar.
2. Fully recursive.



Fragment Grammars via Probabilistic Programming

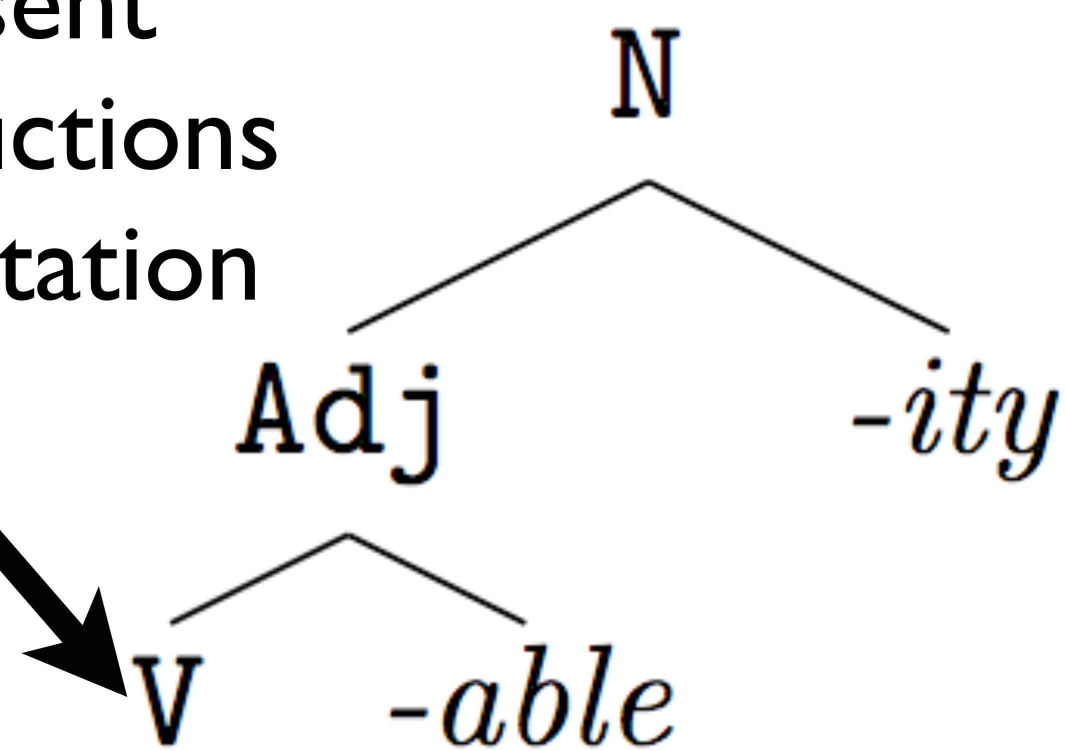
1. Stochastic computation via `unfold`
2. Stochastic reuse via memoization
3. Partial computations via stochastic laziness

Goal: Represent Partial Computations



Goal: Represent Partial Computations

Variables represent "delayed" instructions for later computation



Lazy and Eager Evaluation

Lazy and Eager Evaluation

- Eager Evaluation: Do as much work as early as possible.

Lazy and Eager Evaluation

- Eager Evaluation: Do as much work as early as possible.
- Lazy Evaluation: Delay work until it is absolutely necessary to continue computation.

Example

```
(define add3
 (lambda (x y z)
   (+ x y z)))
```

Eager Evaluation

```
(add3 (+ 1 2 3) (* 2 4) (- 3 1))
```

Eager Evaluation

```
(add3 (+ 1 2 3) (* 2 4) (- 3 1))
```

Eager Evaluation

```
(add3 6 (* 2 4) (- 3 1))
```

Eager Evaluation

```
(add3 6 (* 2 4) (- 3 1))
```

Eager Evaluation

```
(add3 6 8 (- 3 1))
```

Eager Evaluation

```
(add3 6 8 (- 3 1))
```

Eager Evaluation

```
(add3 6 8 2)
```

Eager Evaluation

```
(define add3
  (lambda (x y z)
    (+ x y z)))
```

```
(add3 6 8 2)
```

Eager Evaluation

```
(define add3

  (lambda (x y z)

    (+ x y z)))
```

```
(+ 6 8 2)
   x y z
```

Eager Evaluation

```
16
```

Lazy Evaluation

```
(add3 (+ 1 2 3) (* 2 4) (- 3 1))
```

Lazy Evaluation

```
(define add3
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```

Lazy Evaluation

```
(define add3
  (lambda (x y z)
    (+ x y z)))
```

```
(+ (+ 1 2 3) (* 2 4) (- 3 1))
```

x y z

Lazy Evaluation

```
(define add3
  (lambda (x y z)
    (+ x y z)))
```

```
(+ (+ 1 2 3) (* 2 4) (- 3 1))
   x         y       z
```

Argument expressions are delayed until their values are needed by another computation.

Lazy Evaluation

```
(+ (+ 1 2 3) (* 2 4) (- 3 1))
```

Primitive + procedure forces evaluation of arguments.

Lazy Evaluation

```
(+ (+ 1 2 3) (* 2 4) (- 3 1))
```

Lazy Evaluation

```
(+ 16 (* 2 4) (- 3 1))
```

Lazy Evaluation

```
(+ 16 (* 2 4) (- 3 1))
```

Lazy Evaluation

```
(+ 16 8 (- 3 1))
```

Lazy Evaluation

```
(+ 16 8 (- 3 1))
```

Lazy Evaluation

```
(+ 16 8 2)
```

Lazy Evaluation

```
16
```

λ-calculus: Order of Evaluation

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- *Applicative order* (eager evaluation): evaluate arguments first, then apply function.

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- *Normal order* (lazy evaluation): copy arguments into procedure, only evaluate when needed.

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- *Church-Rosser theorem*: Order doesn't matter for deterministic λ-calculus.

λ-calculus: Order of Evaluation

- *Applicative order* (eager evaluation): evaluate arguments first, then apply function.
- *Normal order* (lazy evaluation): copy arguments into procedure, only evaluate when needed.
- *Church-Rosser theorem*: Order doesn't matter for deterministic λ-calculus.
- Does matter for Ψλ-calculus!

Ψλ-calculus: Order of Evaluation

```
(define same?
  (lambda (x)
    (equal? x x)))
```

Ψλ-calculus: Order of Evaluation

```
(define same?
 (lambda (x)
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```

Ψλ-calculus: Order of Evaluation

```
(define same?
  (lambda (x)
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```

```
(same? (flip))
```

Ψλ-calculus: Order of Evaluation

```
(define same?
  (lambda (x)
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```

`(same? (flip))` —eager→ P(`true`) = 1

Ψλ-calculus: Order of Evaluation

```
(define same?
  (lambda (x)
    (equal? x x)))
```

`(same? (flip))`

- eager → P(true) = 1
- lazy → P(true) = 1/2

Tradeoff

- Laziness allows you to delay computation and, thus, **preserve randomness** and variability until the last possible moment.
- Eagerness allows you to determine random choices early in computation and, thus, **share** choices across different parts of a program.

Random Evaluation Order

Random Evaluation Order

- Idea: Stochastically mix lazy and eager evaluation in Ψλ-calculus.

Random Evaluation Order

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- Ultimately allow **learning** of which computations should be performed in advance and which should be delayed.

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Random Evaluation Order

- Idea: Stochastically mix lazy and eager evaluation in Ψλ-calculus.
- Ultimately allow **learning** of which computations should be performed in advance and which should be delayed.
- Assume eager evaluation strategy and add `delay` primitive.
- Apply to `unfold` (can be applied fully generally).

Stochastic Lazy `unfold`

```
(define stochastic-lazy-unfold
  (lambda (symbol)
    (if (terminal? symbol)
      symbol
      (map delay-or-unfold (sample-rhs symbol)))))
```

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Stochastic Lazy `unfold`

```
(define delay-or-unfold
  (lambda (symbol)
    (if (flip)
      (delay (stochastic-lazy-unfold symbol))
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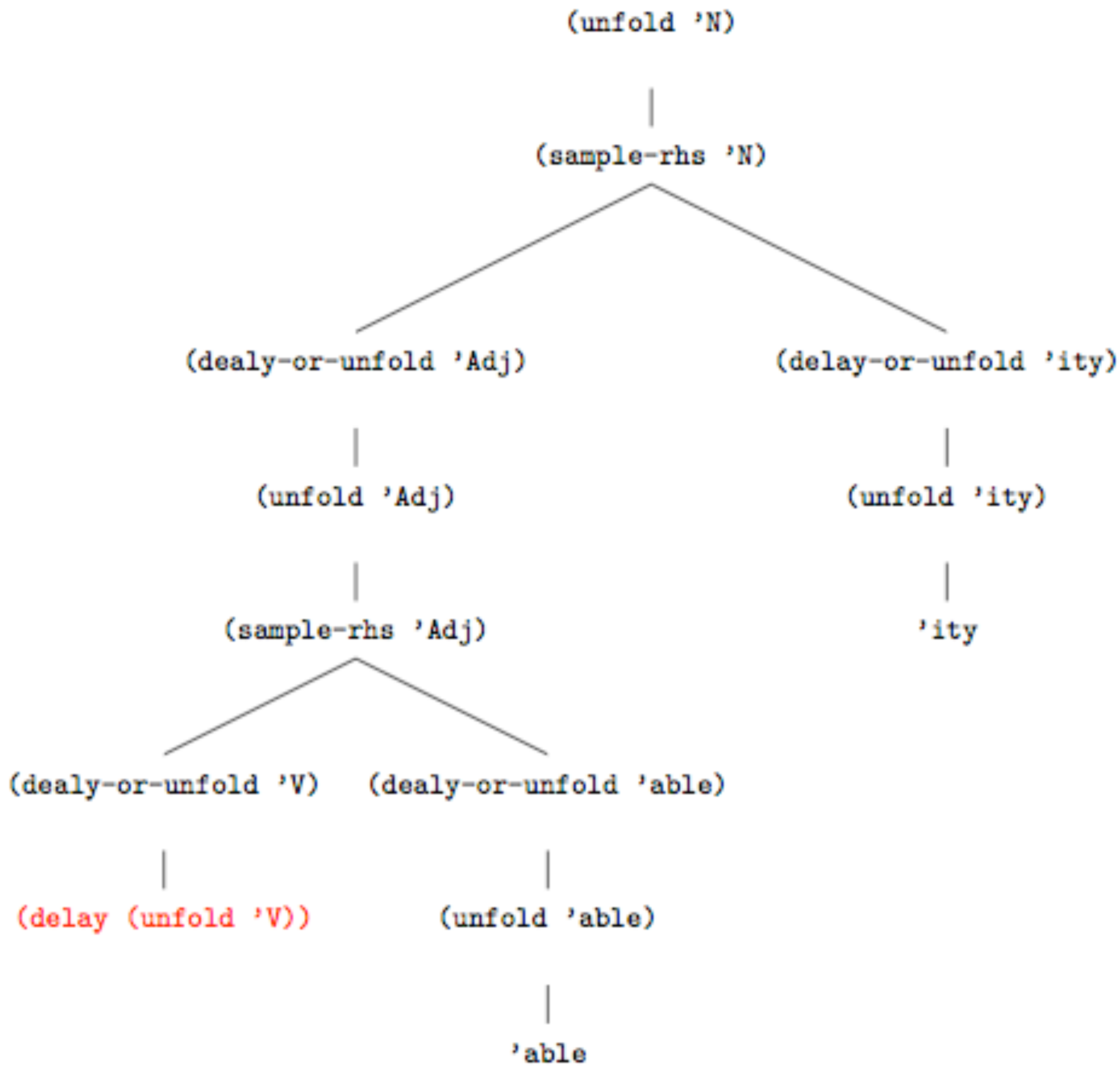
Computation Trace with Delay



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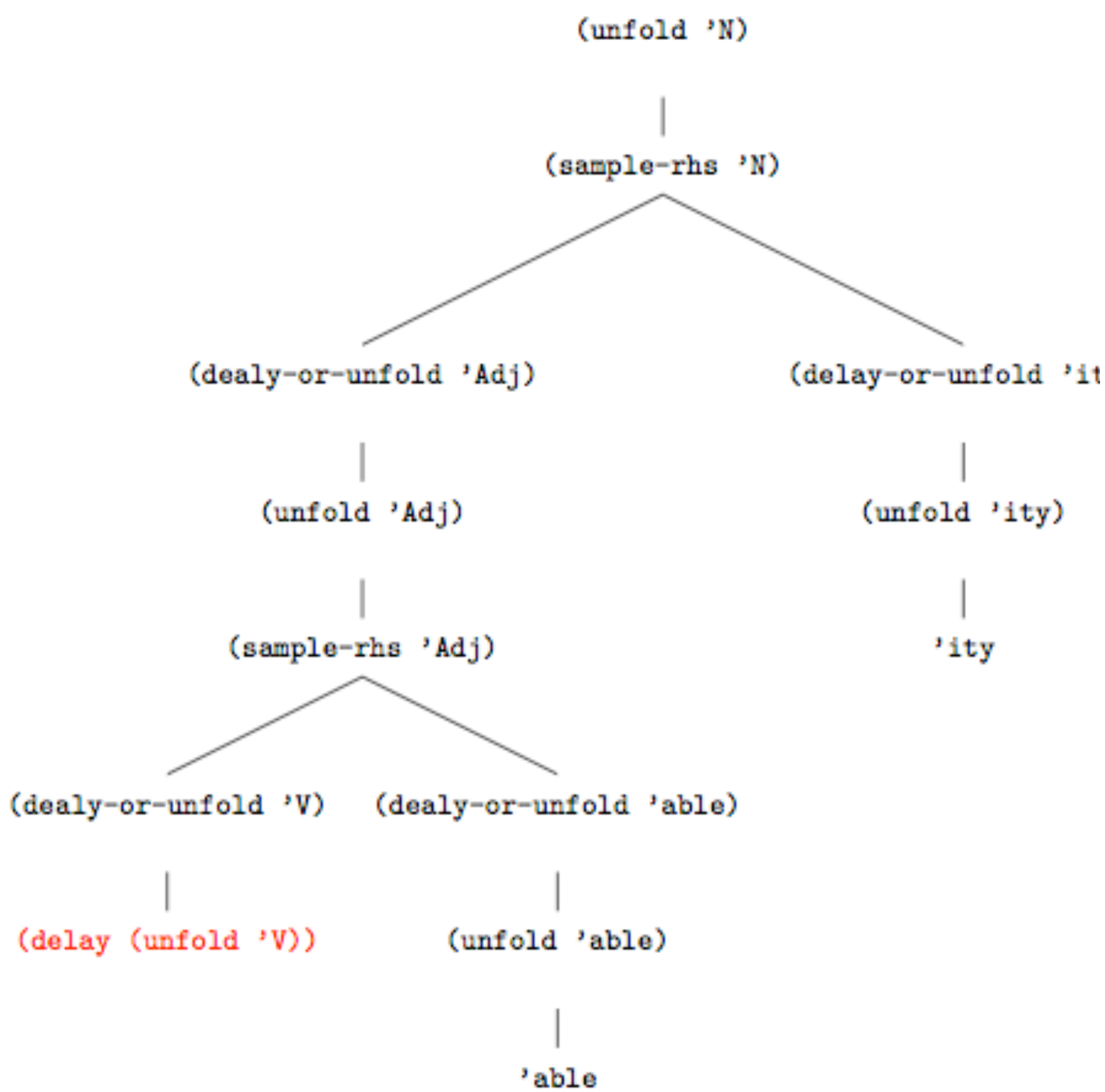

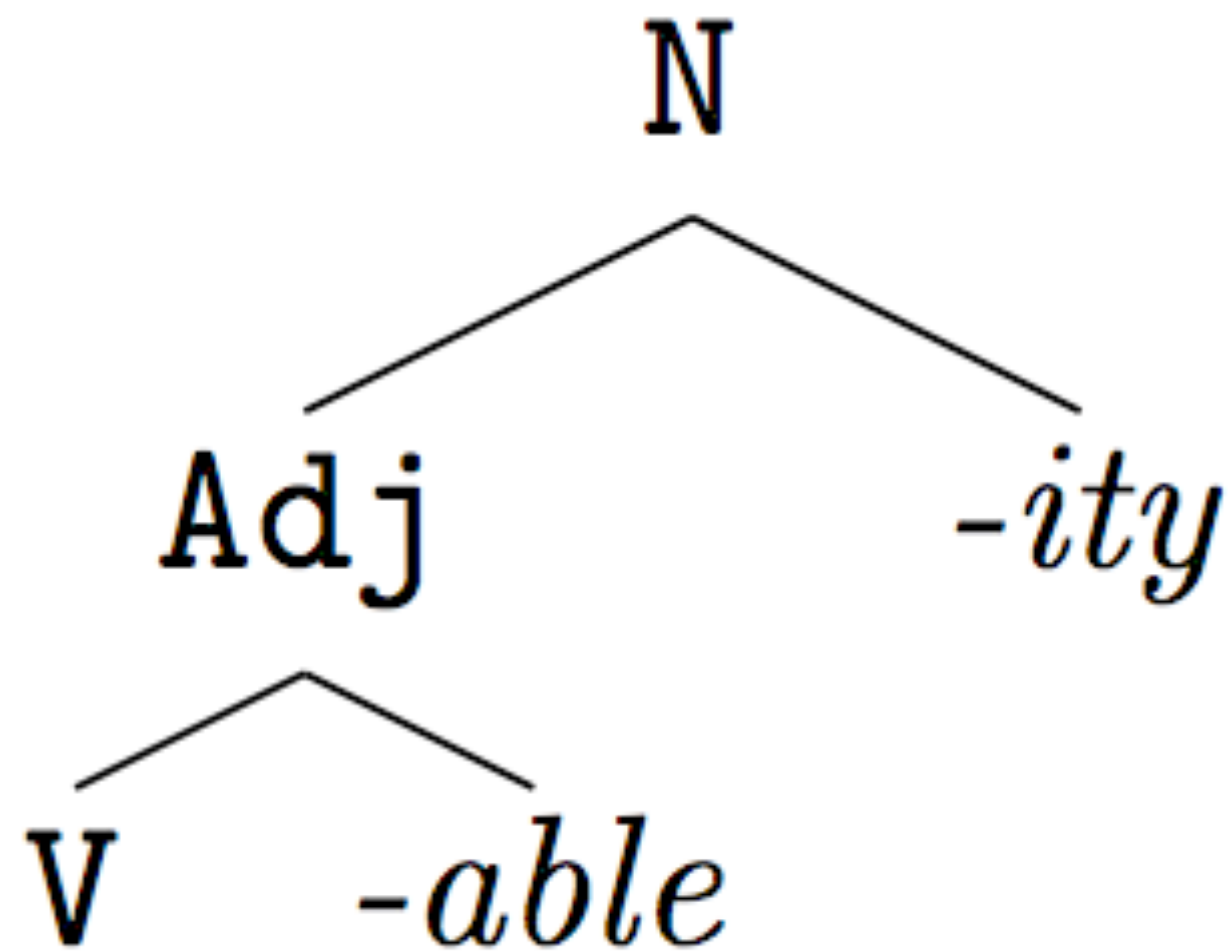

Reusing Delayed Computations

Reusing Delayed Computations

- Need to be able to reuse partial evaluations.

Reusing Delayed Computations

- Need to be able to reuse partial evaluations.
- Memoize stochastically lazy unfold.

Fragment Grammars

```
(define stochastic-lazy-unfold
  (lambda (symbol)
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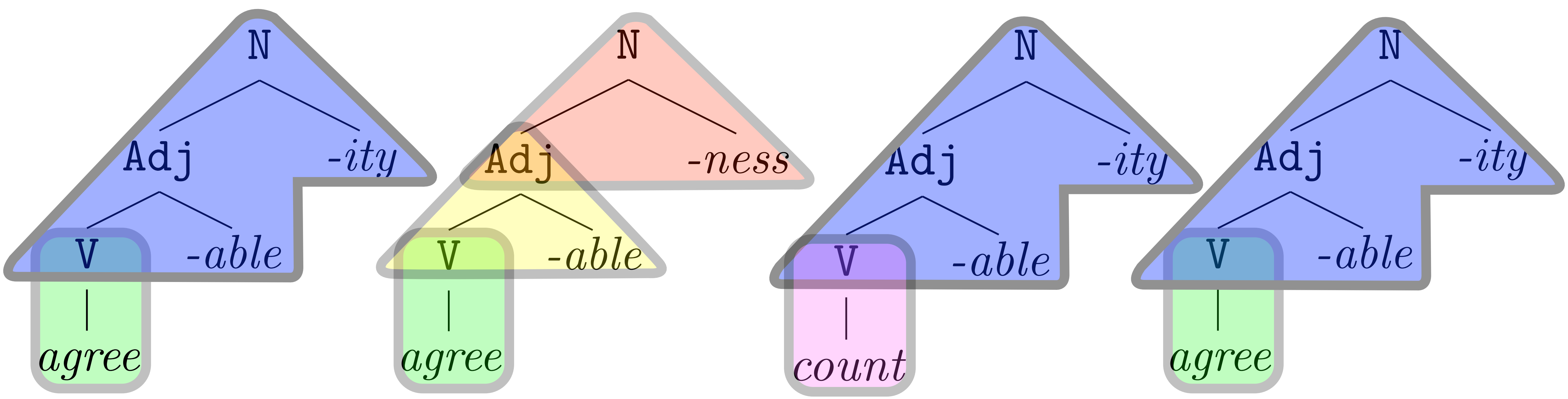
      symbol
      (map delay-or-unfold (sample-rhs symbol)))))

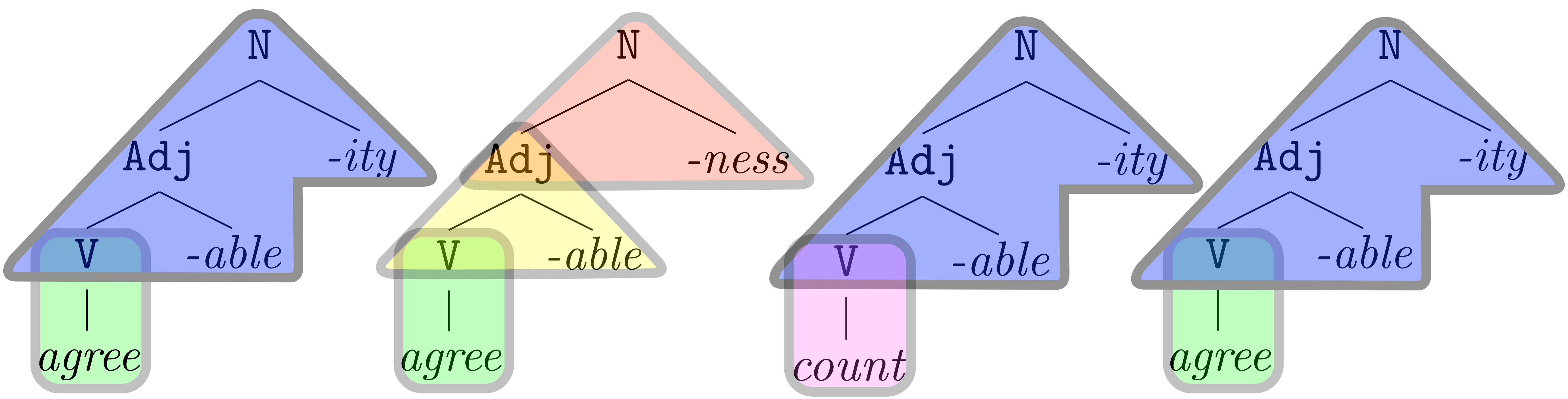
(define delay-or-unfold
  (PYMem a b (lambda (symbol)
    (if (flip)
      (delay (stochastic-lazy-unfold symbol))
      (stochastic-lazy-unfold symbol)))))
```

Fragment Grammar
Reusable Computations



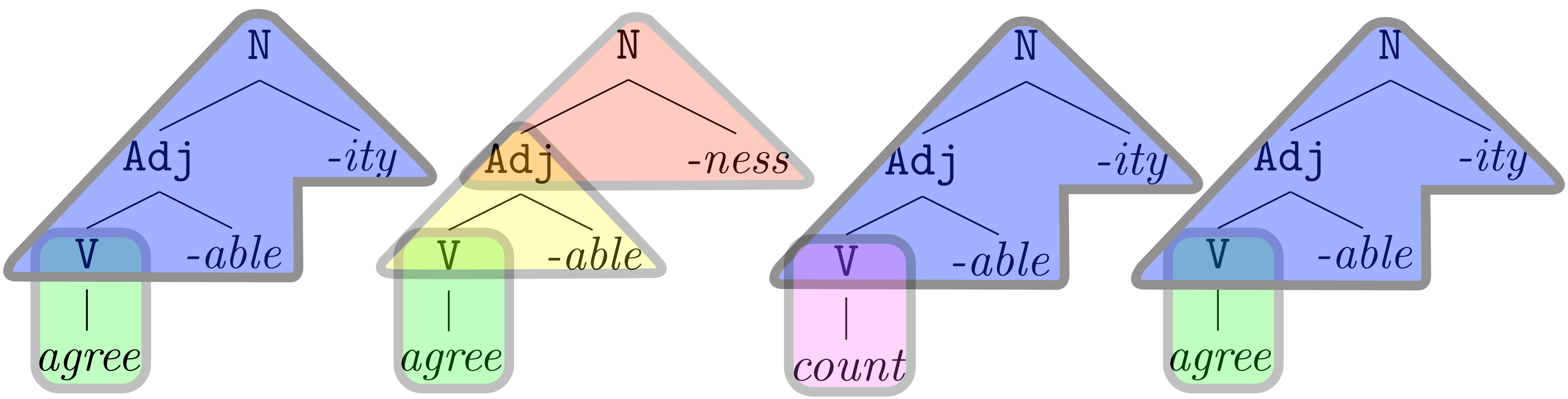
Fragment Grammar Reusable Computations

1. Always possible to use base grammar.



Fragment Grammar
Reusable Computations

1. Always possible to use base grammar.
2. Fully recursive.



Outline

1. The Proposal.
2. Five Models of Productivity and Reuse.
3. English Derivational Morphology
4. Conclusion

Five Models

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- 4 approaches to productivity and reuse.

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- Capture historical proposals from the literature.

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- State-of-the-art probabilistic models.

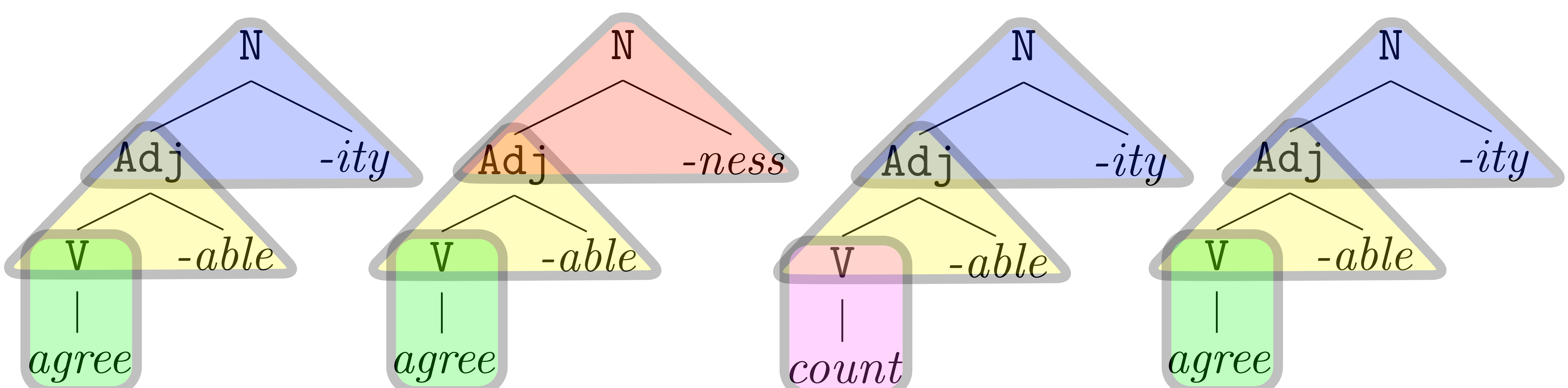
Five Models

- 4 approaches to productivity and reuse.
- Capture historical proposals from the literature.
- State-of-the-art probabilistic models.
    - Allow for variability and learning.

MDPCFG

Multinomial-Dirichlet Context-Free Grammars
(Full-Parsing)

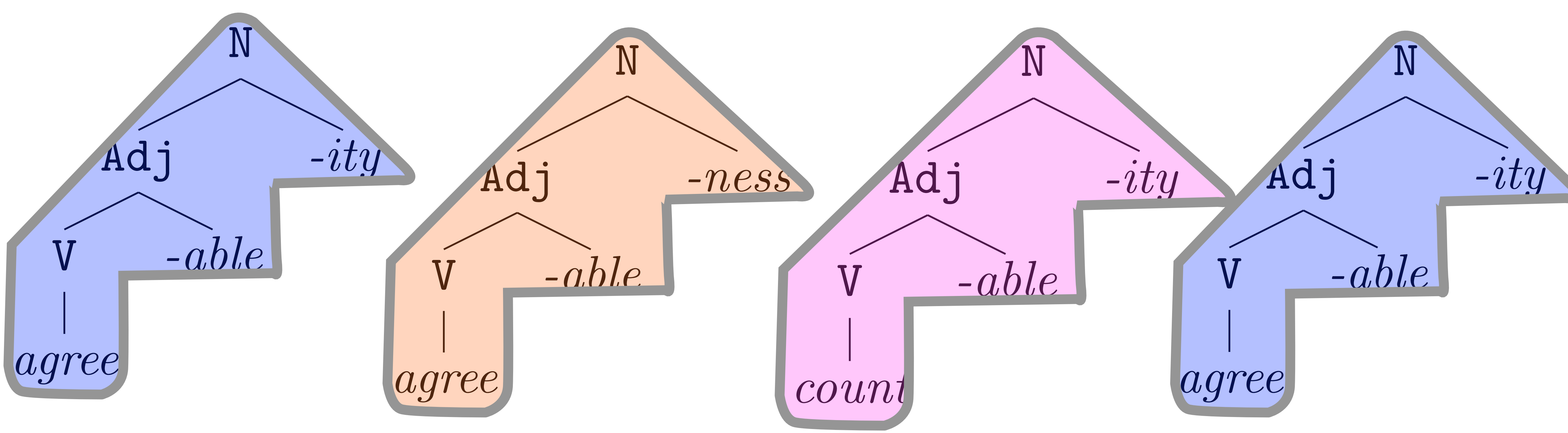
- All generalizations are productive
- Formalization: *Multinomial-Dirichlet Probabilistic Context-free Grammar* (MDPCFG; Johnson, et al. 2007a)



MAG

MAP Adaptor Grammars
(Full-entry)

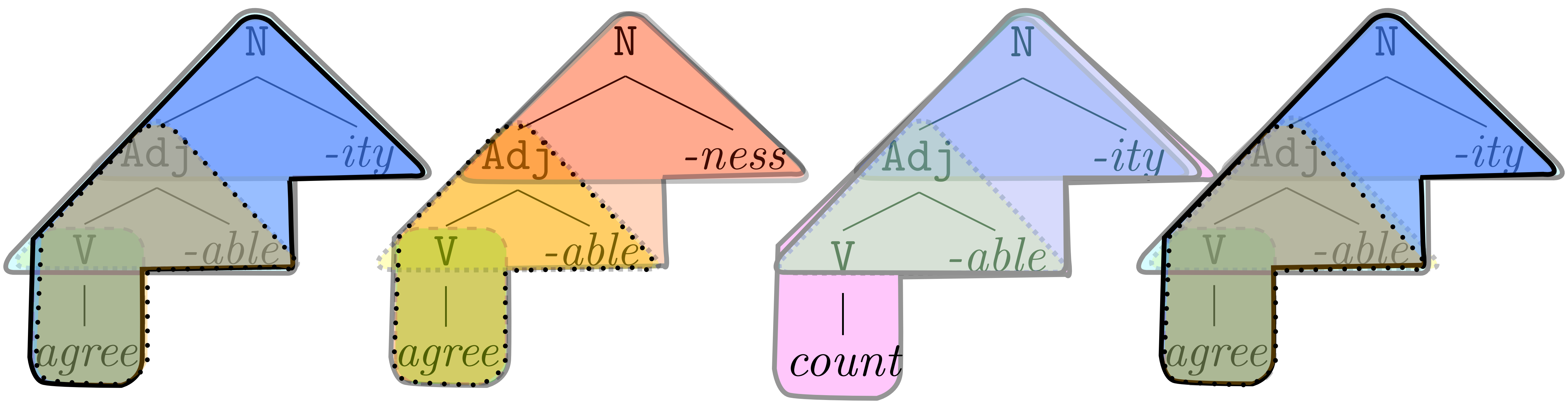
- Store whole form after *first* use.
- Formalization: *Adaptor Grammars* (AG; Johnson, et al. 2007).
- Always possible to compute productively with small probability; Fully recursive.
- Formalizes classic lexicalist theories (e.g., Jackendoff, 1975).



DOP1/GDMN

Data-Oriented Parsing
(Exemplar-based)

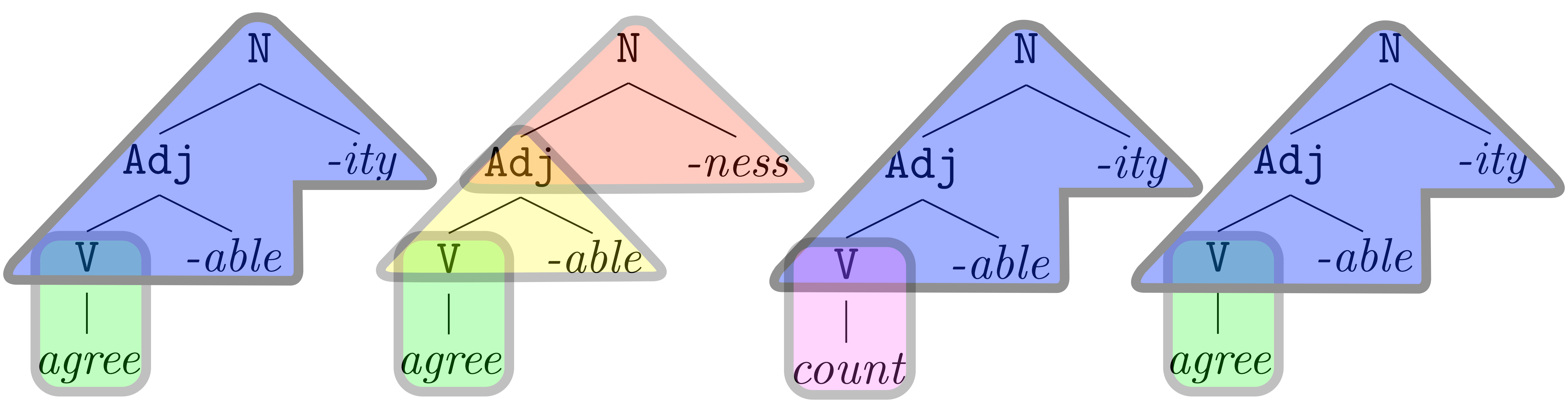
- Store *all* generalizations consistent with input
- Formalization: *Data-Oriented Parsing 1* (DOP1; Bod, 1998), *Data-Oriented Parsing: Goodman Estimator* (GDMN; Goodman, 2003)
- Recently proposed as models of syntax (e.g., Snider, 2009; Bod, 2009)



FG

Fragment Grammars
(Inference-based)

- Store *best* set of subcomputations for explaining the data.
- Formalization: *Fragment Grammars* (FG; O'Donnell, et al. 2009)
- Generalization of *Adaptor Grammars*



Outline

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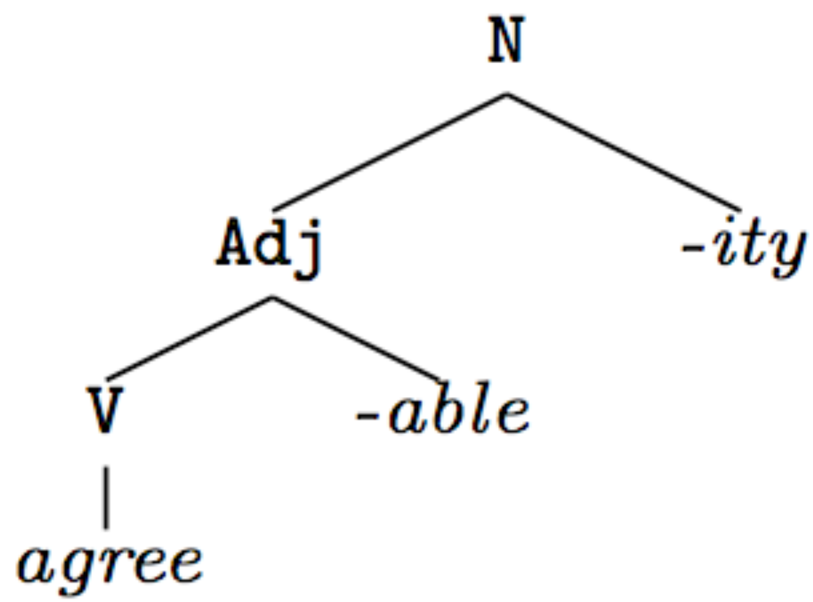
English Derivational Morphology

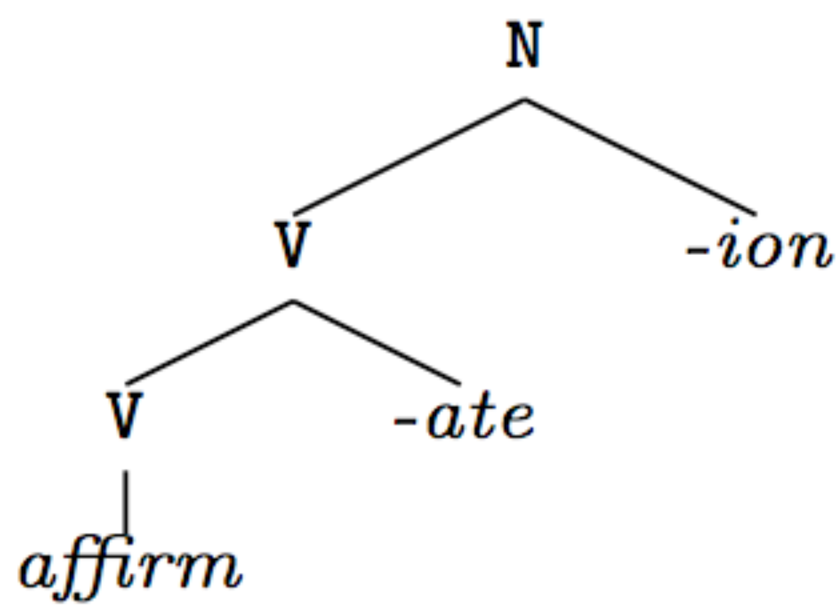
Productive	+ness *(goodness)*, +ly *(quickly)*
Semi-productive	+ity *(ability)*, +or *(operator)*
Unproductive	+th *(width)*

Simulations

- Words from CELEX.
- Extensive heuristic parsing/hand correction.
- Input format.
  - No phonology or semantics.

Derivational Inputs





English Derivational Morphology

Productive	+ness *(goodness)*, +ly *(quickly)*
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Unproductive	+th *(width)*

1. Individual suffix productivity differences (-ness/-ity/-th).
2. Suffix sequences.

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2. Suffix combinations.

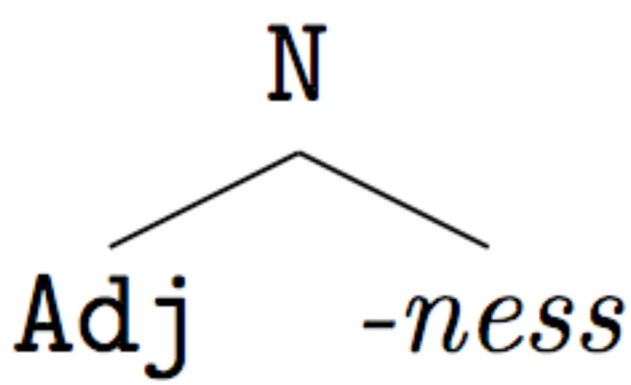
Productivity

- No gold-standard dataset or measure.
  - E.g., Large databases of *wug*-tests or naturalness judgments.
- Analyses.

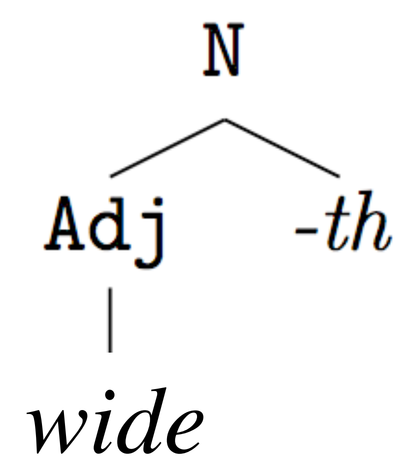
1. Convergence with other theoretical measures.

How is Productivity Represented?

- Relative probability of fragments with or without variables.



V.



Baayen's Corpus-Based Measures

- Baayen's $\mathcal{P}$ / $\mathcal{P}^*$ (e.g., Baayen, 1992)
  - $\mathcal{P}$: Prob(NOVEL | SUFFIX) i.e. rate of growth of forms with suffix
  - $\mathcal{P}^*$: Prob(SUFFIX | NOVEL) i.e. rate of growth of vocabulary due to suffix

Productivity Correlations

($\mathcal{P}/\mathcal{P}^*$ values from Hay & Baayen, 2002)

Model	FG (Inference-based)	MDPCFG (Full-parsing)	MAG (Full-listing)	DOP1 (Exemplar-based)	GDMN (Exemplar-based)
$\mathcal{P}$	**0.907**	-0.0003	0.692	0.346	0.143
$\mathcal{P}^*$	**0.662**	0.480	0.568	0.402	0.500

English Derivational Morphology

Productive	+ness *(goodness)*, +ly *(quickly)*
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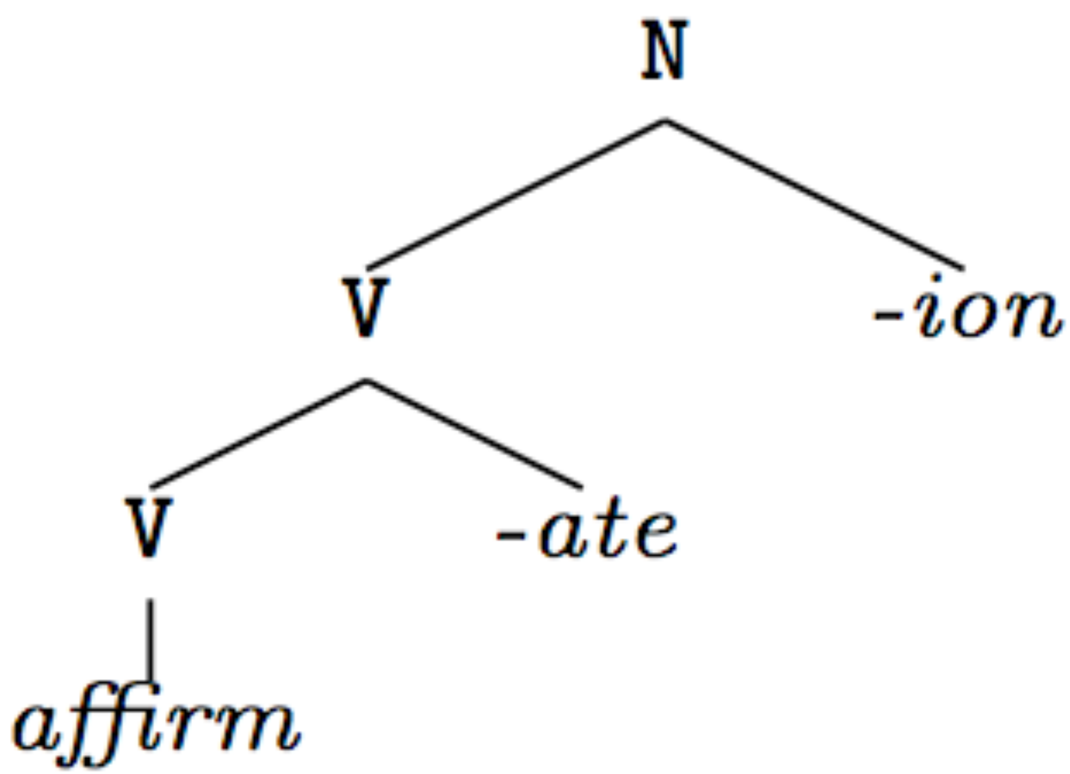
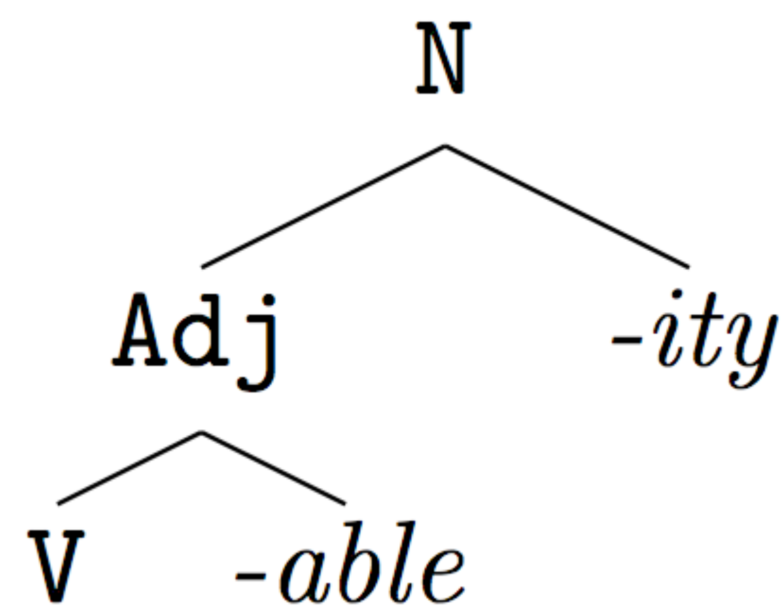
Generalizable Combinations

Frozen Combinations



Generalizable Combinations



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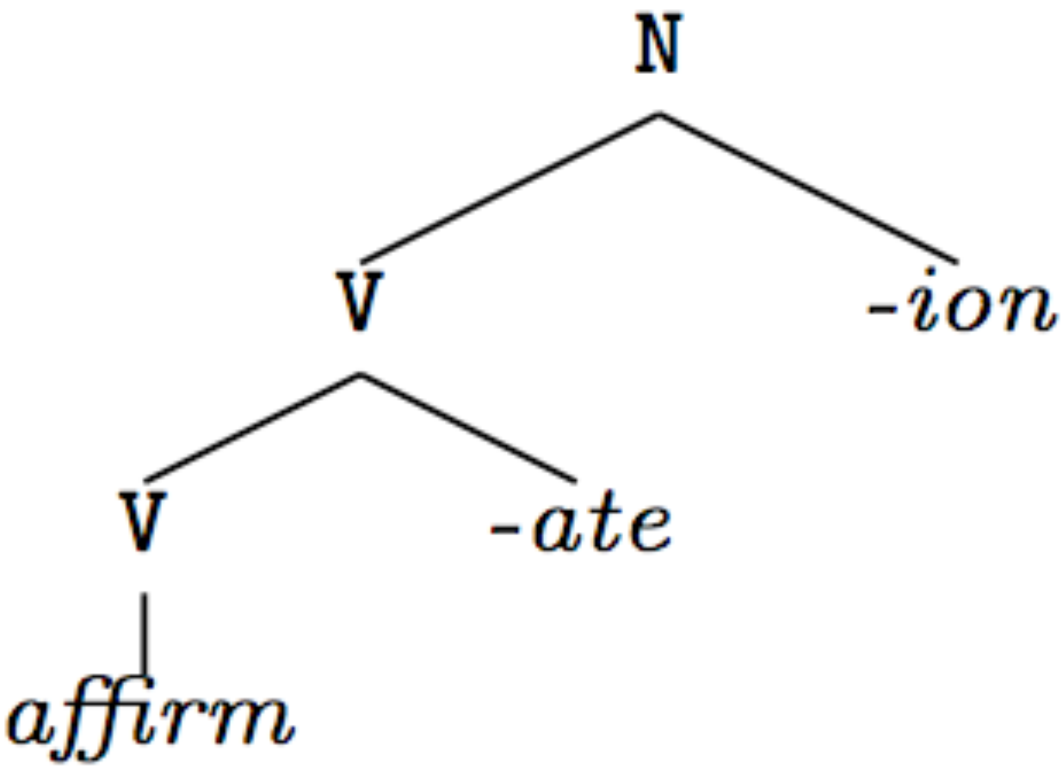
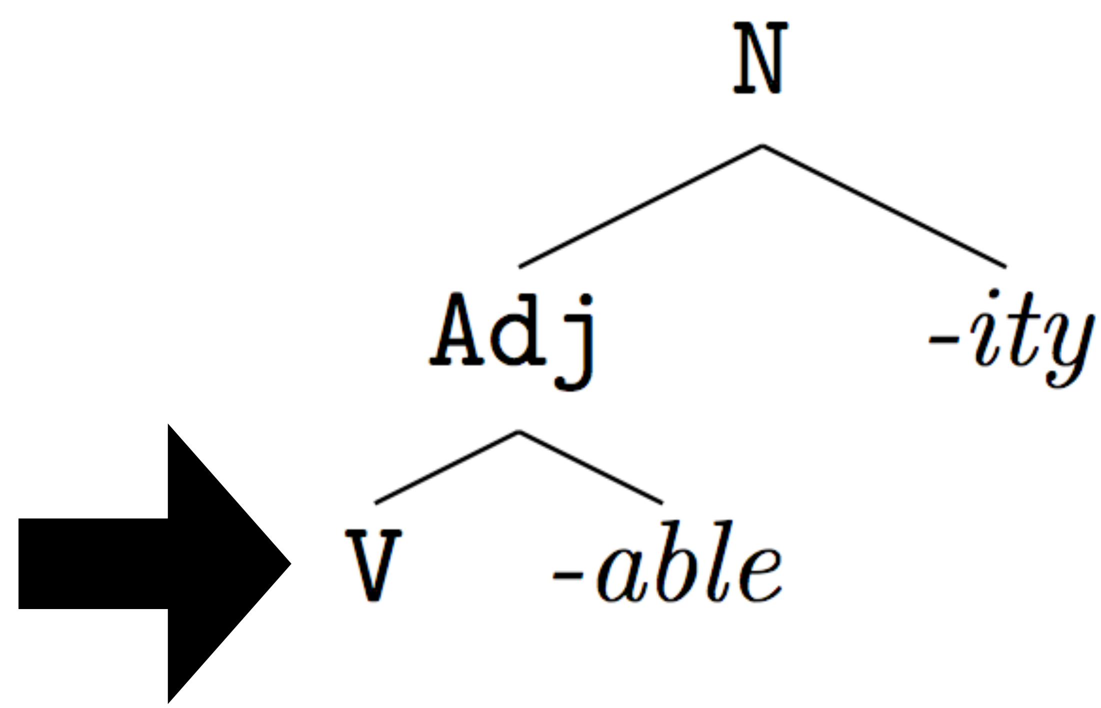


Generalizable Combinations



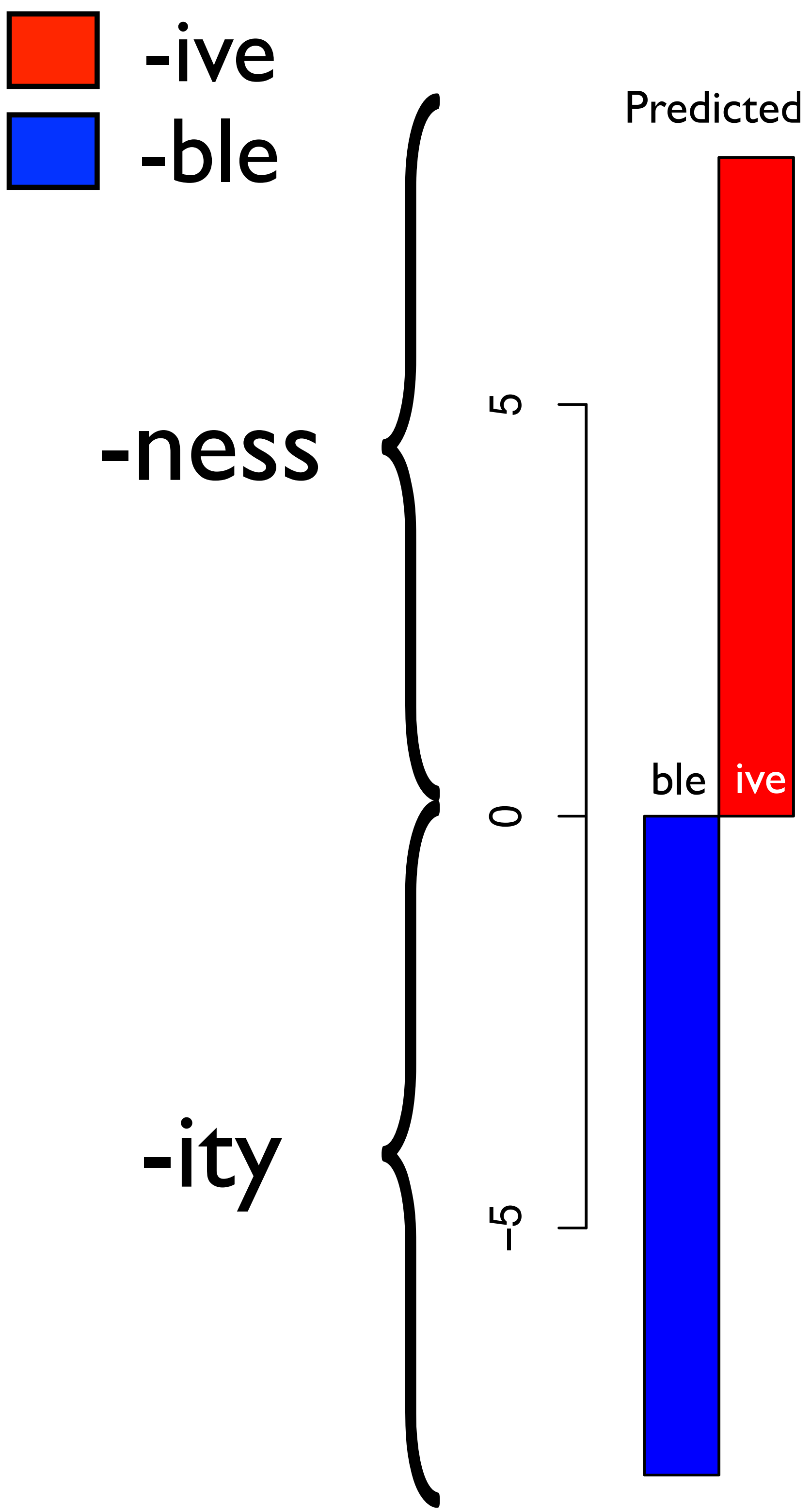
-ity v. -ness

- -ness more productive than -ity.
- -ity more productive than -ness after: -ile, -able, -(i)an, -ic.
  (Anshen & Aronoff, 1981; Aronoff & Schvaneveldt, 1978; Cutler, 1980)

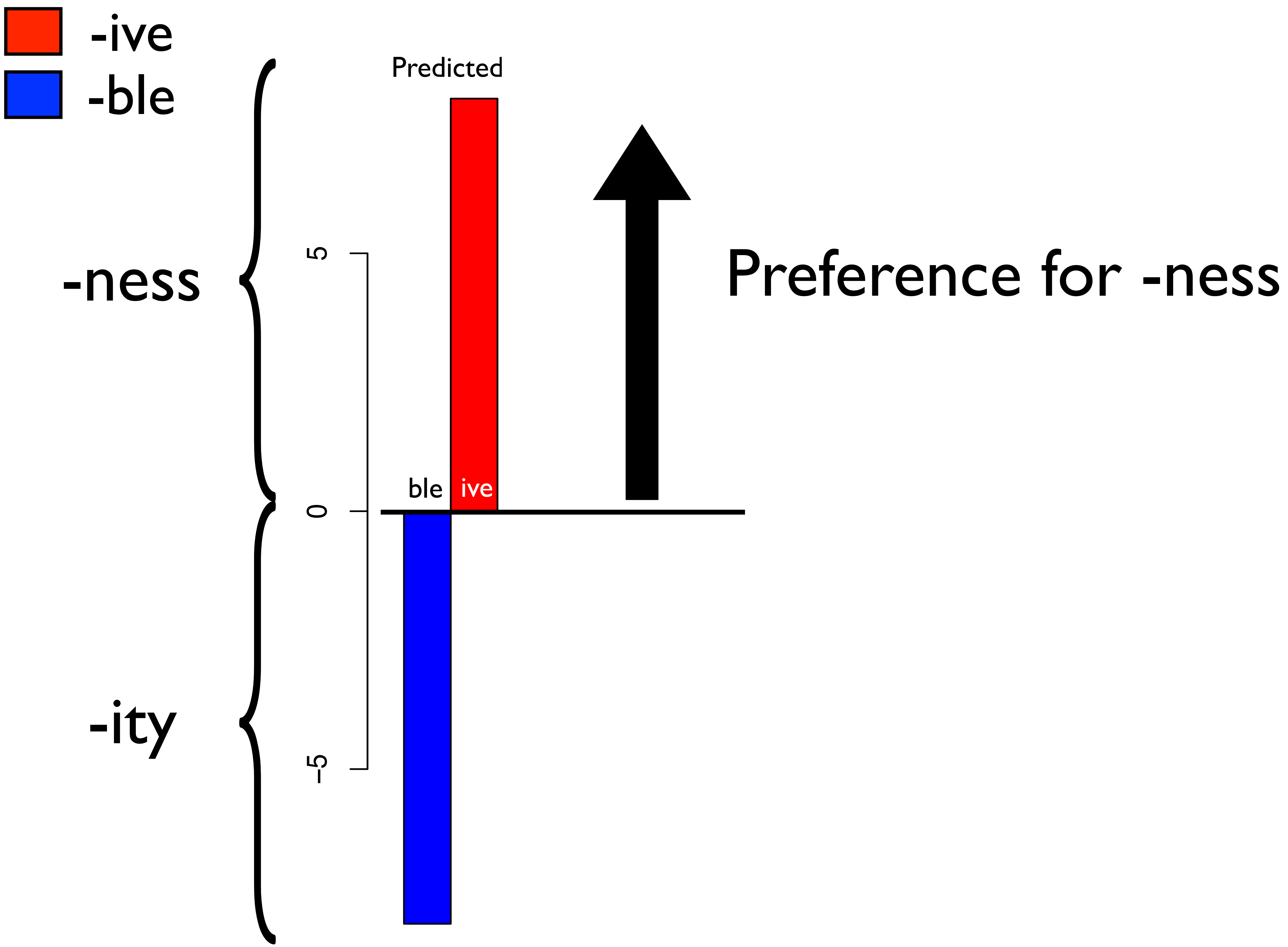
Two Frequent Combinations: -ivity v. -bility

- -ive + -ity: ***-ivity*** (e.g., selectivity).
  - Speaker prefer to use -ness with novel words (Aronoff & Schvaneveldt, 1978).
  - depulsiveness > depulsivity.
- -ble + -ity: ***-bility*** (e.g., sensibility).
  - Speakers prefer to use -ity with novel words (Anshen & Aronoff, 1981).
  - remortibility > remortibleness.

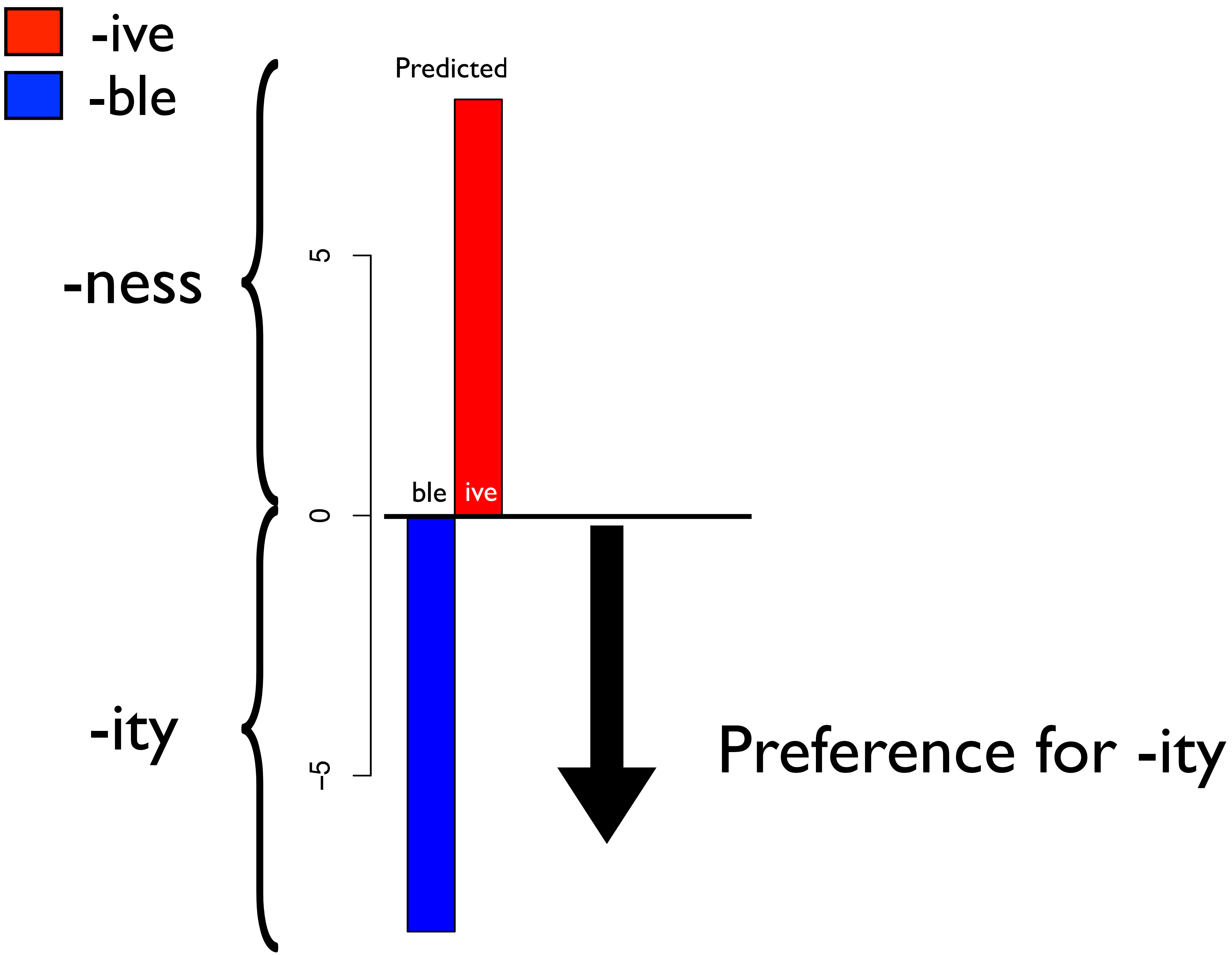
-ivity v. -bility



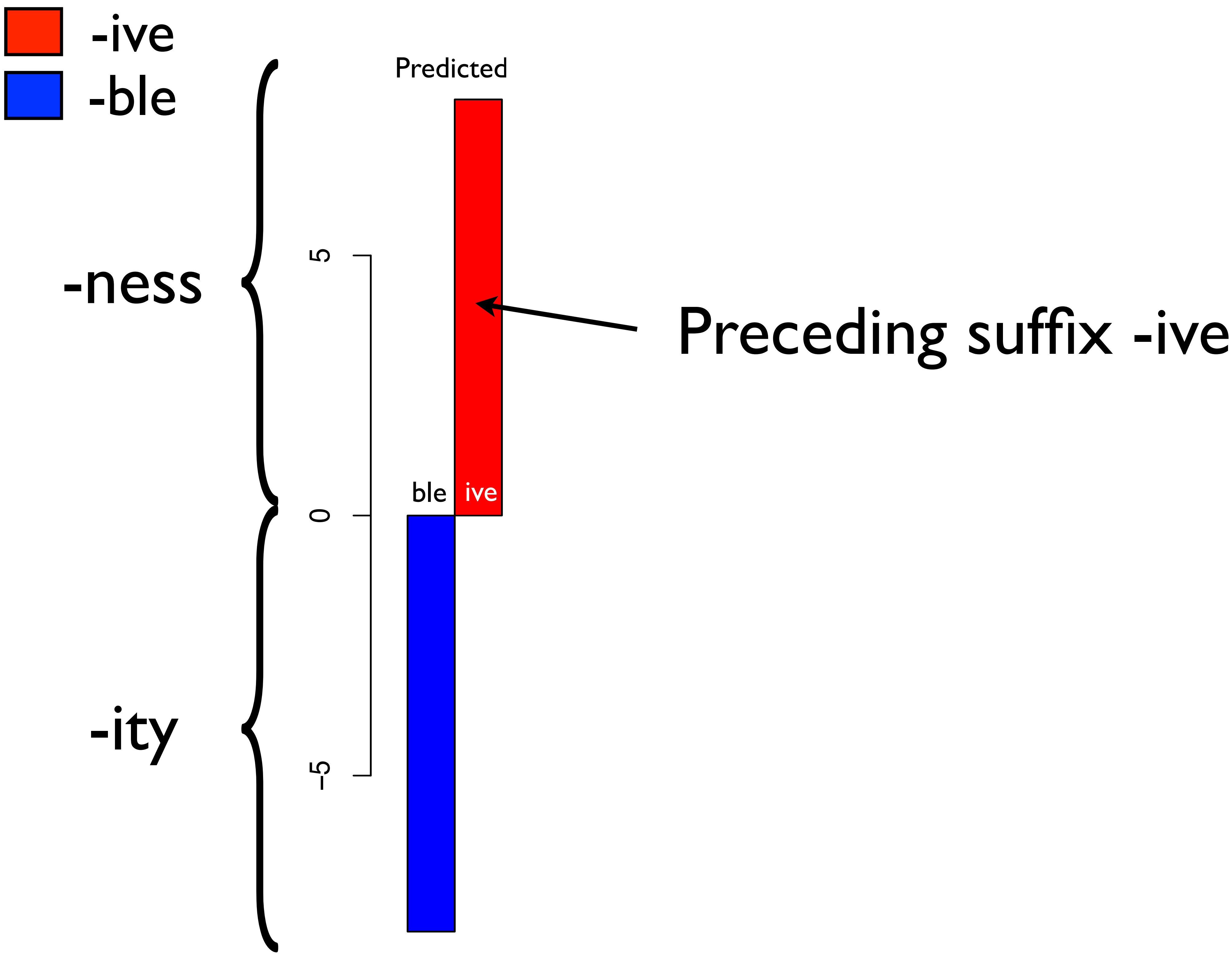
-ivity v. -bility



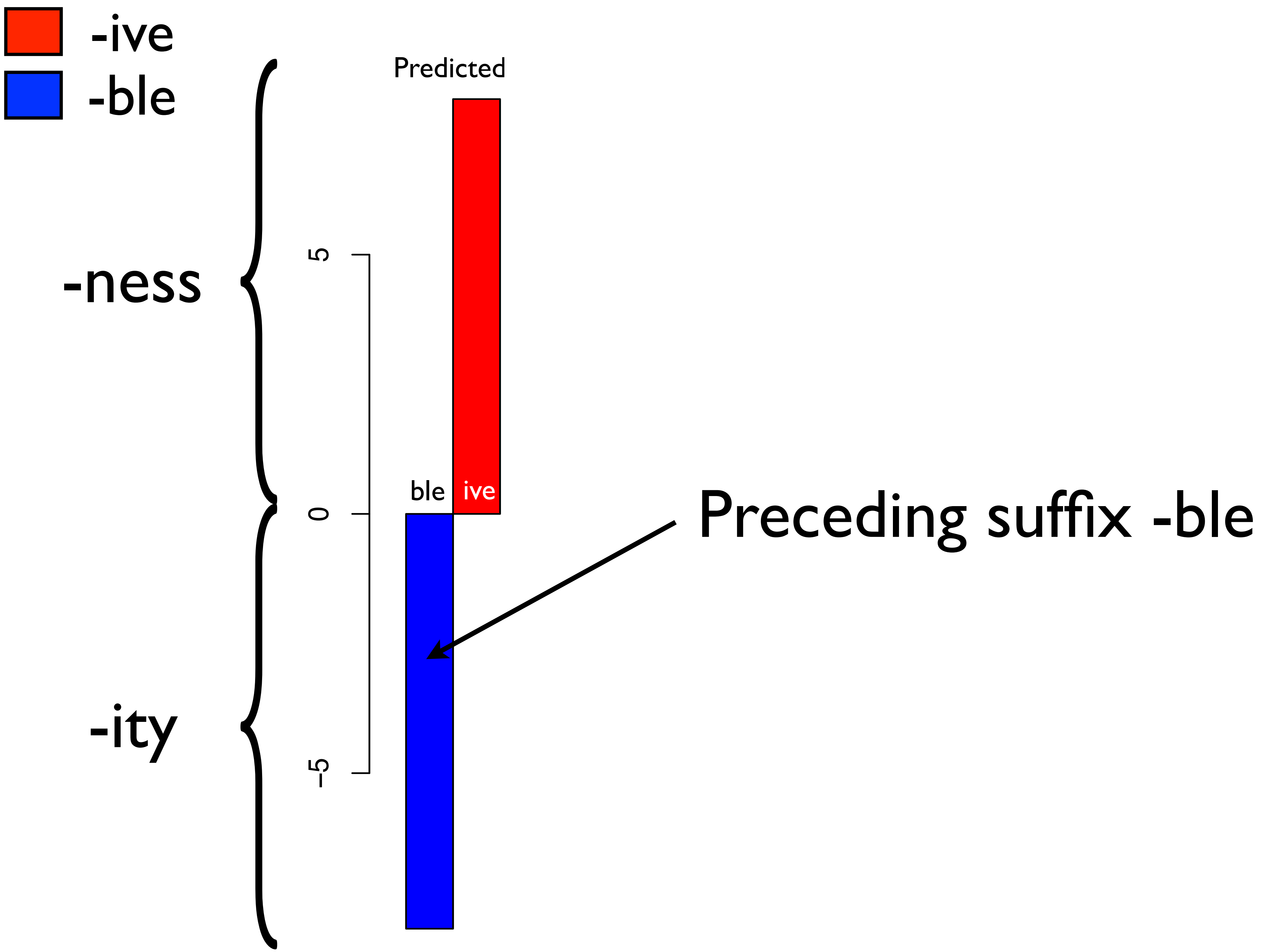
-ivity v. -bility



-ivity v. -bility

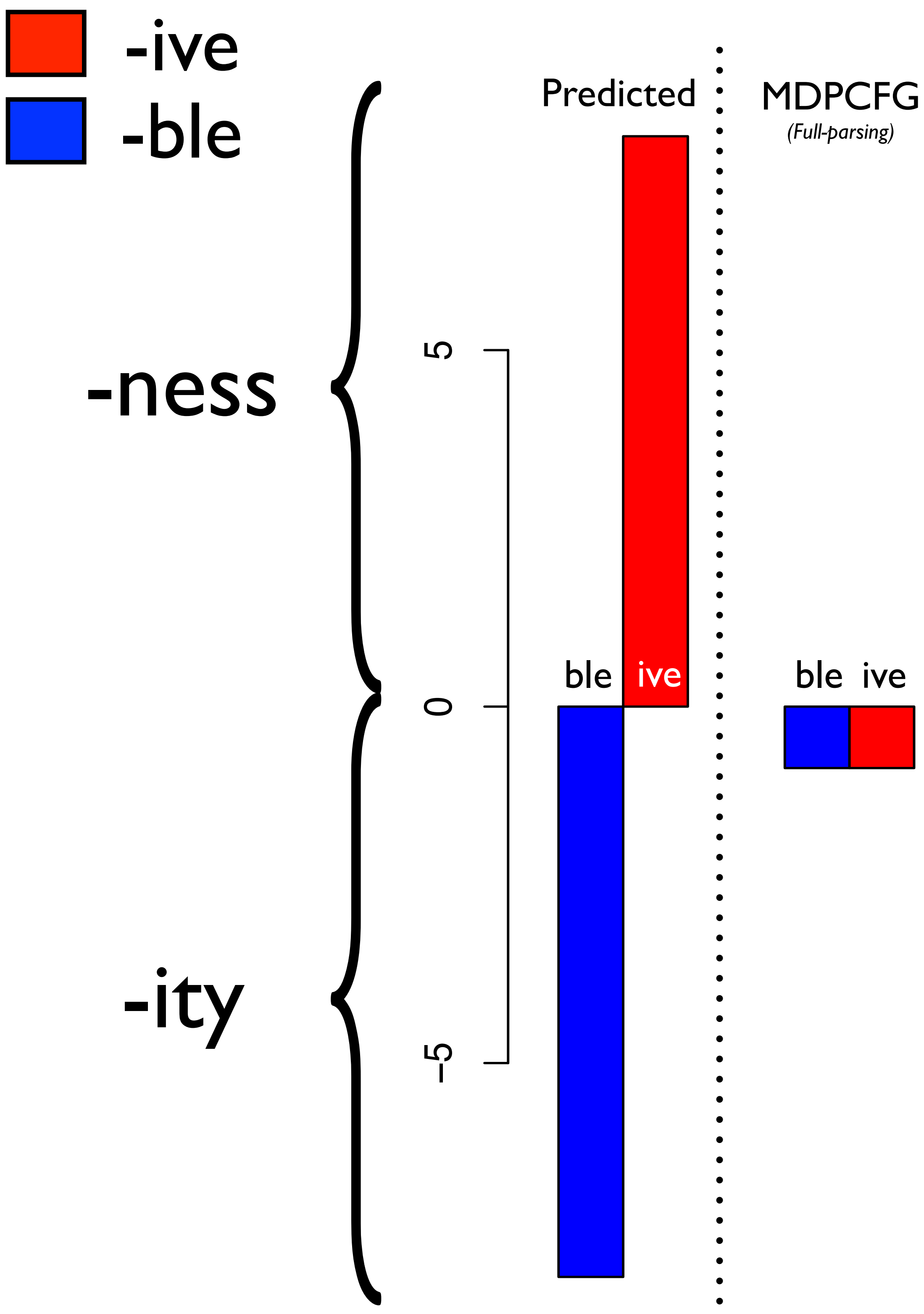


-ivity v. -bility



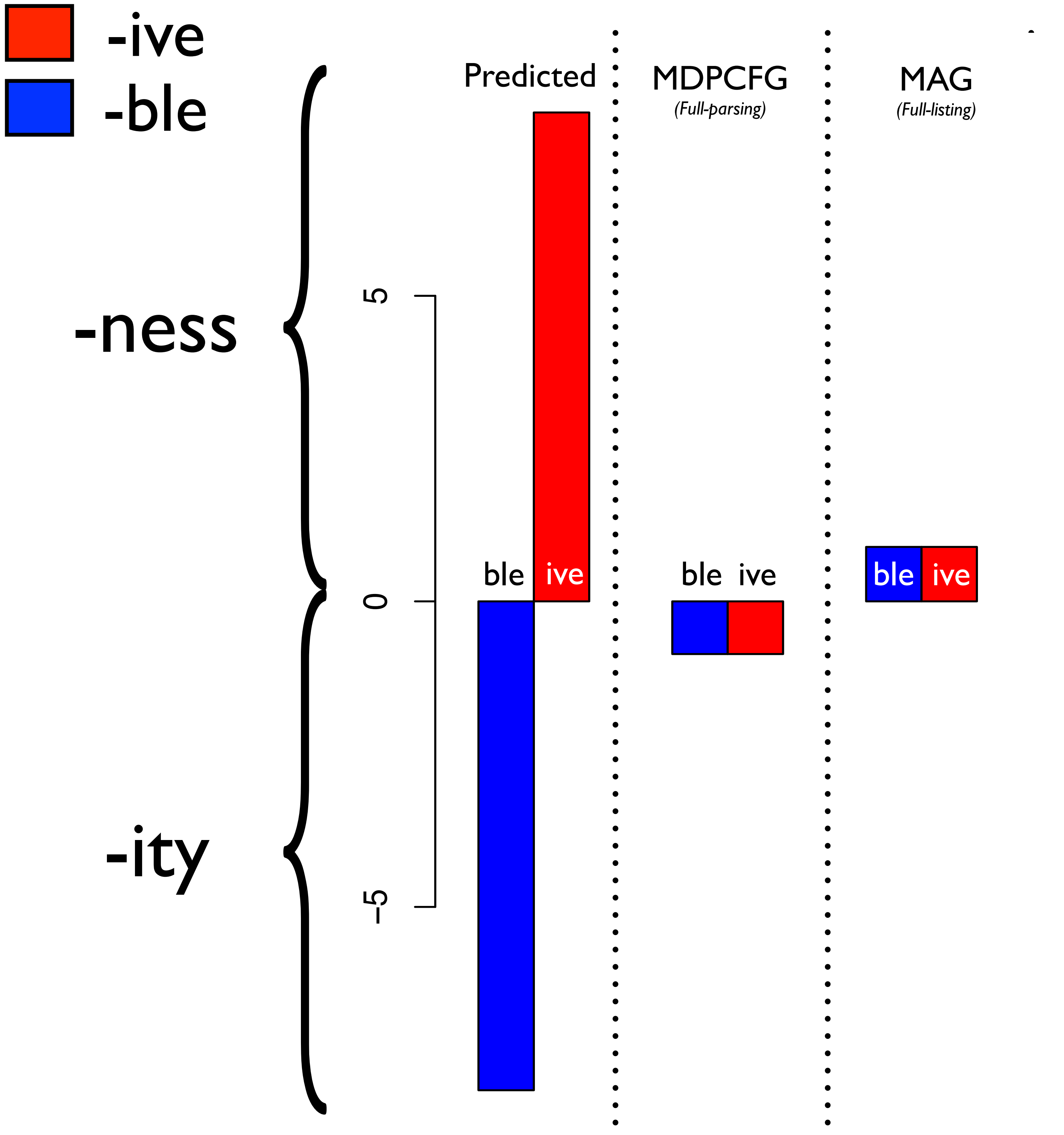
MDPCFG

(Full-parsing)



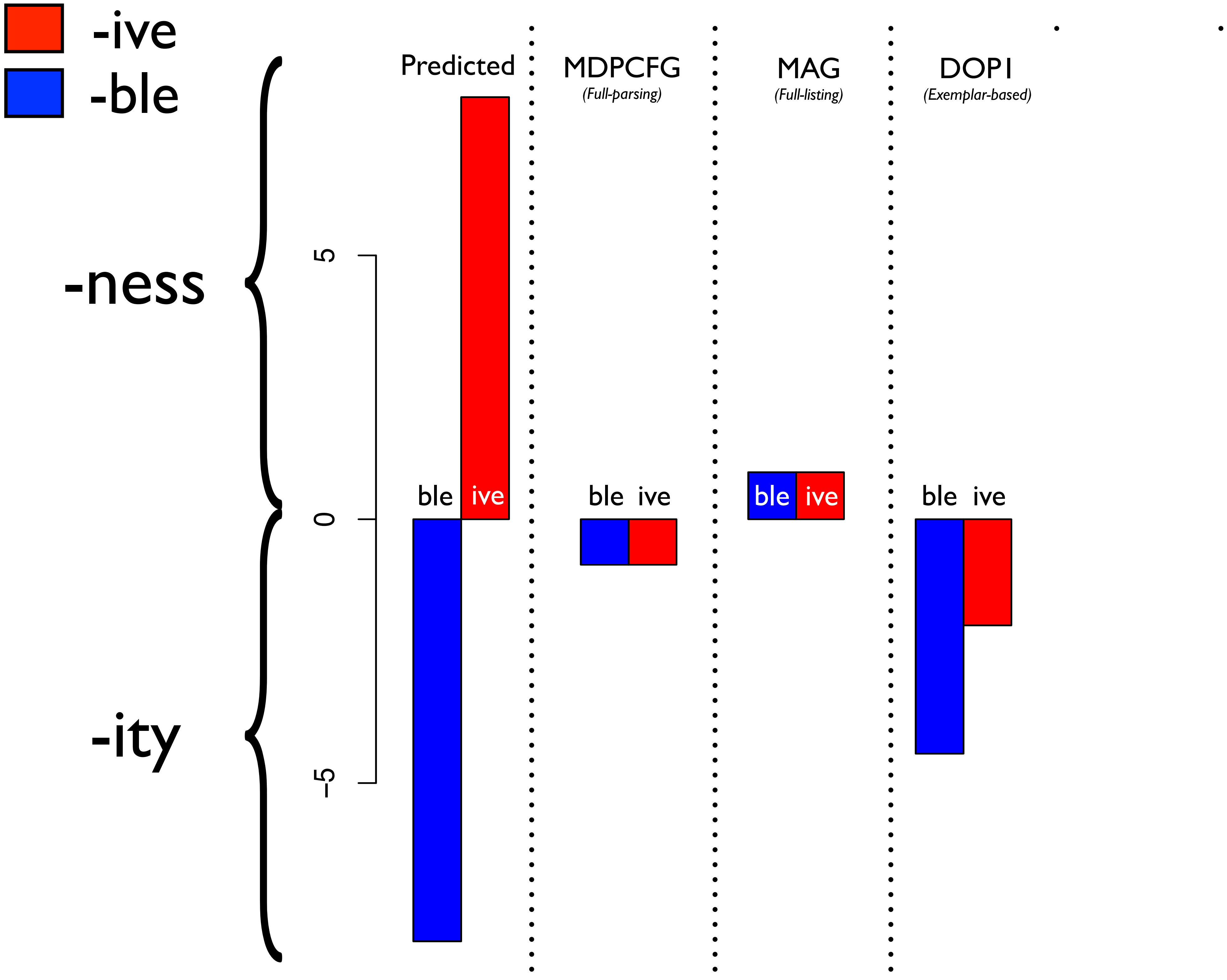
MAG

(Full-listing)



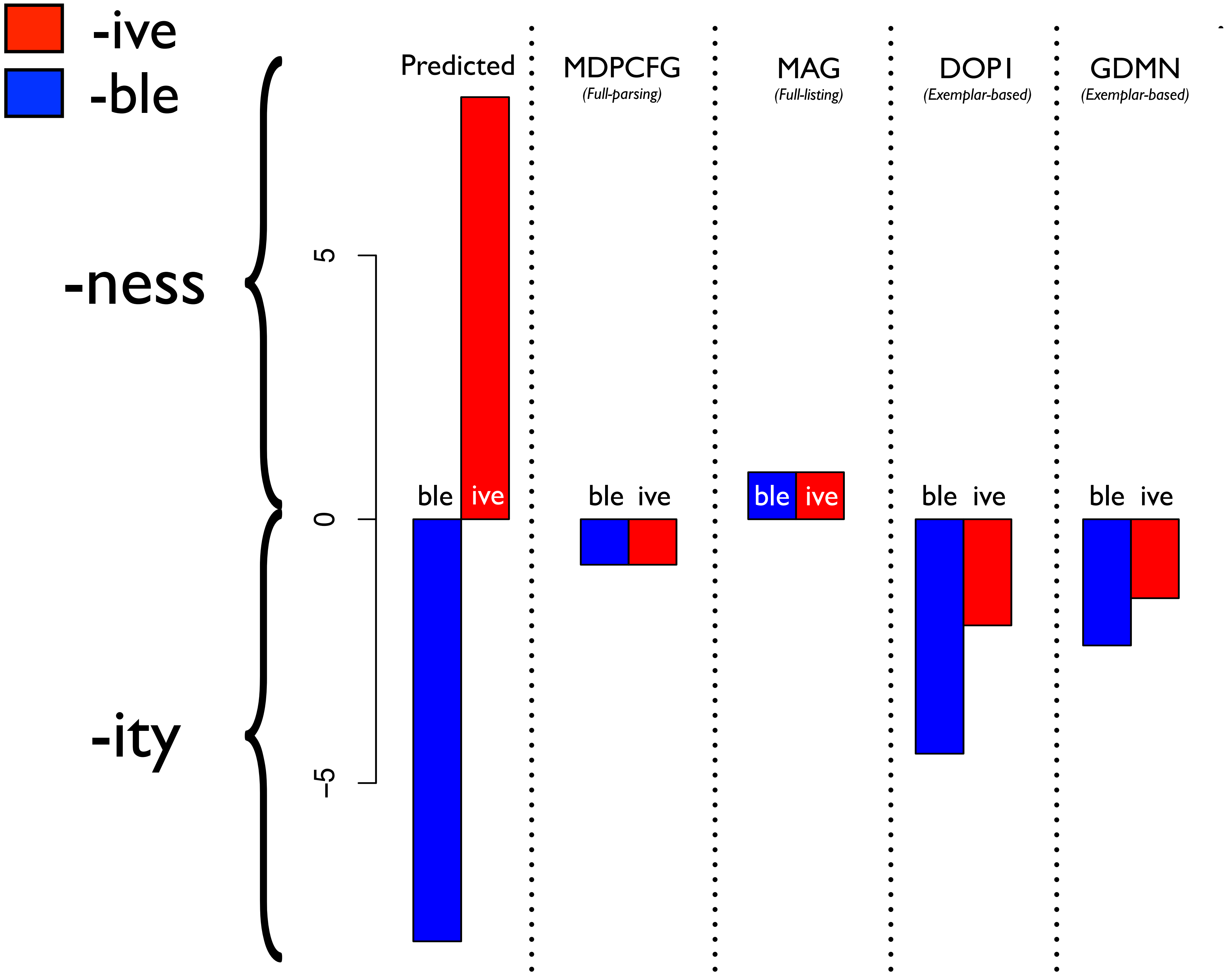
DOP1

(Exemplar-based)



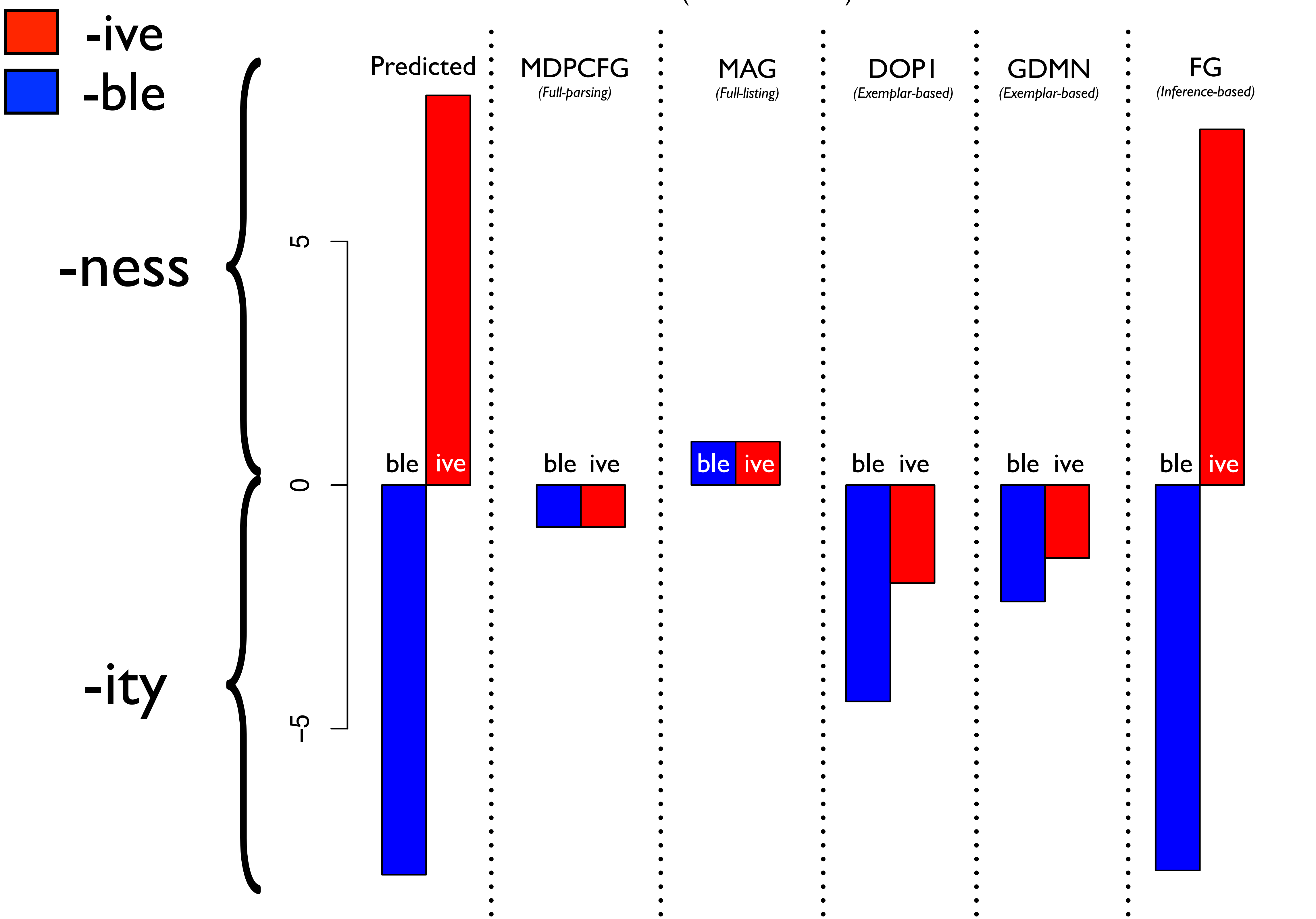
GDMN

(Exemplar-based)



FG

(Inference-based)



Discussion

- Inference-based approach able to correctly ignore high token frequency of -ivity because it balances a **tradeoff**.
- Other models use type or token frequencies.

Outline

1. The Proposal.
2. Five Models of Productivity and Reuse.
3. Empirical Evaluation

   The English Past Tense

   English Derivational Morphology
4. Conclusion

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- View productivity and reuse as an inference.

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  - Future work...

Thanks!