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## First-order probabilistic models of human cognition

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## Why logic + probability?

- Two complementary formalisms:
  - Logic: a framework for knowledge representation.
  - Probability: a framework for inductive inference.
- Why do we need to integrate them?
  - To account for our knowledge about the structure of the world
    - its form and content.
    - how it is used and acquired.
  - To capture linguistic meaning and use.

## Why logic + probability?

- To capture deep inductive biases

*F*: form

↓

*S*: structure

↓

*D*: data

Tree with species at leaf nodes

	F1	F2	F3	F4
mouse	○	○	○	○
squirrel	○	○	○	○
chimp	○	○	○	○
gorilla	○	○	○	○

## Outline

- The traditional debate in cognitive science: logic *versus* probability
  - The case of connectionism
  - Examples: knowledge about biology, social relations, language, visual objects
- Models of human reasoning that integrate probabilistic inference and logical representations
  - Ecological reasoning
  - Causal reasoning
- Other directions
  - Social reasoning, physical reasoning

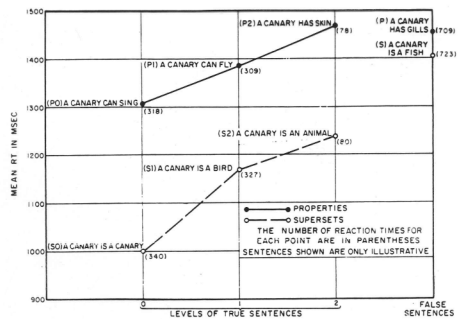
## Motivations for connectionism

- Build models with more neural plausibility.
- Overcome the problems of symbolic representations.
  - Inference is too rigid and brittle.
  - No general way to learn new representations.

## Semantic networks (Quillian, 1968)

- Useful for compression (memory)
- Useful for predicting properties of new objects.

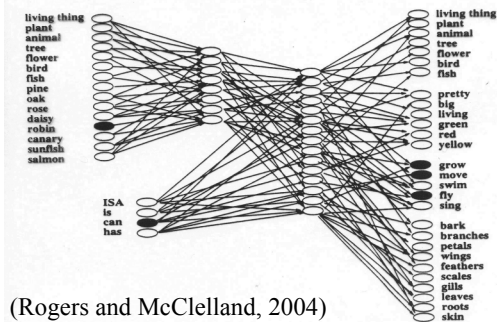
## Reaction time tests of hierarchy



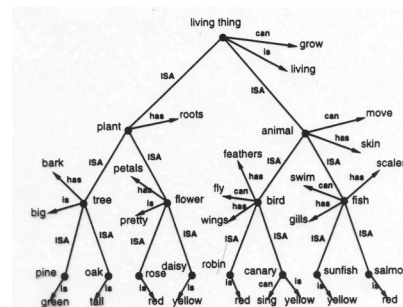
## Problems

- Typicality effects.
  - “canary is a bird” faster than “chicken is a bird”.
- Violations of hierarchy for atypical items.
  - “chicken is an animal” faster than “chicken is a bird.”
- How could this knowledge representation be learned in an unsupervised way?

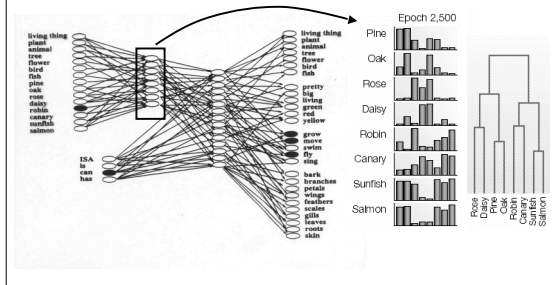
## An alternative architecture



## Training set



## Learned distributed representation

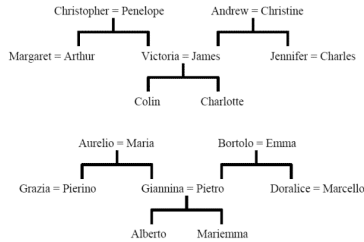


## Problems

- Does not actually capture generalization behavior very well.
  - Correlations:  $r < 0.7$  on basic property tasks.
  - Inductive bias is too weak
- Missing crucial abstract knowledge about the domain.
  - e.g.,  $ISA(x,y) \leq ISA(x,z) \& ISA(z,y)$
- Can we combine the best of logical representation and statistical learning and inference?

## Learning family relationships (Hinton, 1986)

- A family tree structure:

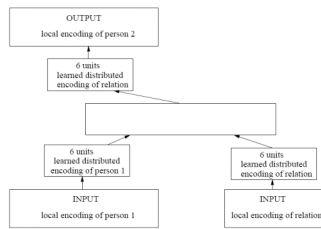


## The family relations dataset

father(Christopher, Arthur)		
father(Christopher, Victoria)		
father(Andrew, James)		
father(Andrew, Jennifer)		
father(James, Colin)		
father(James, Charlotte)		
mother(Penelope, Arthur)		
mother(Penelope, Victoria)		
mother(Christine, James)		
mother(Christine, Jennifer)		
mother(Victoria, Colin)		
mother(Victoria, Charlotte)		
husband(Christopher, Penelope)		
husband(Andrew, Christine)		
husband(Arthur, Margaret)		
husband(James, Victoria)		
husband(Charles, Jennifer)		
wife(Penelope, Christopher)		
wife(Christine, Andrew)		
wife(Margaret, Arthur)		
wife(Victoria, James)		
wife(Jennifer, Charles)		
son(Arthur, Christopher)		
son(Arthur, Penelope)		
son(James, Andrew)		
son(James, Christine)		
son(Colin, Victoria)		
son(Colin, James)		
daughter(Victoria, Christopher)		
daughter(Victoria, Penelope)		
daughter(Jennifer, Andrew)		
daughter(Jennifer, Christine)		
daughter(Charlotte, Victoria)		
daughter(Charlotte, James)		
brother(Arthur, Victoria)		
brother(James, Jennifer)		
brother(Colin, Charlotte)		
sister(Victoria, Arthur)		
sister(Jennifer, James)		
sister(Charlotte, Colin)		
uncle(Arthur, Colin)		
uncle(Charles, Colin)		
uncle(Arthur, Charlotte)		
uncle(Charles, Charlotte)		
aunt(Jennifer, Colin)		
aunt(Margaret, Colin)		
aunt(Jennifer, Charlotte)		
aunt(Margaret, Charlotte)		
nephew(Colin, Arthur)		
nephew(Colin, Jennifer)		
nephew(Colin, Margaret)		
nephew(Colin, Charles)		
niece(Charlotte, Arthur)		
niece(Charlotte, Jennifer)		
niece(Charlotte, Margaret)		
niece(Charlotte, Charles)		

## Learning family relationships (Hinton, 1986)

- Network architecture:



## Learning family relationships (Hinton, 1986)

- 112 possible facts of the form:
  - <person1, relation, person2>
  - <Christopher, father-of, Victoria>
  - <Colin, son-of, Victoria>
  - <Jennifer, aunt-of, Colin> . . .
- Trained on 108 examples, network usually generalizes well to the other 4.
  - Doesn't work well with less training.

## Linear Relational Embedding (Paccanaro and Hinton, 2002)

generate  $P^*$ ,  $Q^*$  is proportional to  $\exp(-\|P^* - Q^*\|^2) \exp(-\|Q^* - \mathbf{x}\|^2)$  as a suitable discriminative goodness function is:

$$G = \sum_{\mathbf{x}} \frac{1}{K} \log \frac{\exp(-\|P^* - Q^*\|^2) \exp(-\|Q^* - \mathbf{x}\|^2)}{\sum_{\mathbf{z}} \exp(-\|P^* - Q^*\|^2) \exp(-\|Q^* - \mathbf{z}\|^2)} \quad (1)$$

where  $K_{\mathbf{x}}$  is the number of triples in  $\mathcal{D}$  having the first two terms equal to the ones of  $\mathbf{x}$ , but differing in the third term. To understand why we need to introduce this factor, let us consider a set of  $K$  triples such having the same first two terms  $\mathbf{a}$  and  $\mathbf{b}$ , but differing in the third term, which we shall call  $\mathbf{z}_i$  with  $i = 1, \dots, K$ . We would like our system to assign equal probability to each of the correct answers, and therefore the discrete probability distribution that we want to approximate can be written as:

$$P_{\mathbf{x}} = \frac{1}{K} \sum_{i=1}^K \delta(\mathbf{z}_i - \mathbf{x}) \quad (2)$$

where  $\delta$  is the discrete delta function and  $\mathbf{x}$  ranges over the vectors in  $\mathcal{V}$ . Our system implements the discrete distribution:

$$Q_{\mathbf{x}} = \frac{1}{Z} \exp(-\|P^* - \mathbf{a}\|^2) \exp(-\|\mathbf{b} - \mathbf{x}\|^2) \quad (3)$$

where

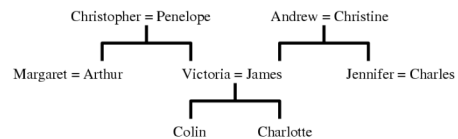
$$Z = \sum_{\mathbf{z}} \exp(-\|P^* - \mathbf{a}\|^2) \exp(-\|\mathbf{b} - \mathbf{z}\|^2) \quad (4)$$

is the normalization factor. The Kullback-Leibler divergence between  $P_{\mathbf{x}}$  and  $Q_{\mathbf{x}}$  can be written as:

$$KL(P_{\mathbf{x}} \| Q_{\mathbf{x}}) = \sum_{\mathbf{z}} P_{\mathbf{z}} \log \frac{P_{\mathbf{z}}}{Q_{\mathbf{z}}} = \sum_{\mathbf{z}} \frac{1}{K} \log \frac{\delta(\mathbf{z}_i - \mathbf{z})}{\frac{1}{Z} \exp(-\|P^* - \mathbf{a}\|^2) \exp(-\|\mathbf{b} - \mathbf{z}\|^2)} \quad (5)$$

- Relatively minor improvement, from 4 to 8 or 12 generalization trials....

## A more intuitive representation



- Relations: spouse ("="), parent (solid line)
- Attribute: male or female (type of name)
- Define other relations in terms of basic relations spouse, parent, and the attributes male, female.

### Minimal family description

spouse(Christopher, Penelope)  
 spouse(Andrew, Christine)  
 spouse(Arthur, Margaret)  
 spouse(James, Victoria)  
 spouse(Charles, Jennifer)  
  
 female(Penelope)  
 female(Christine)  
 female(Margaret)  
 female(Victoria)  
 female(Jennifer)  
 female(Charlotte)  
 NOT female(Colin)

### Abstract theory of kinship

spouse(x,y) <=> spouse(y,x)  
 NOT female(x) <=> spouse(x,y) AND female(y)  
 parent(x,y) <=> spouse(x,z) AND parent(z,y)  
  
 father(x,y) <=> parent(x,y) AND NOT female(x)  
 mother(x,y) <=> parent(x,y) AND female(x)  
 husband(x,y) <=> spouse(x,y) AND NOT female(x)  
 wife(x,y) <=> spouse(x,y) AND female(x)  
 son(x,y) <=> parent(y,x) AND NOT female(x)  
 daughter(x,y) <=> parent(y,x) AND female(x)  
 sibling(x,y) <=> parent(z,x) AND parent(z,y) AND ~(x=y)  
 brother(x,y) <=> sibling(x,y) AND NOT female(x)  
 sister(x,y) <=> sibling(x,y) AND female(x)  
 uncle(x,y) <=> (parent(z,y) AND brother(x,z))  
 OR (aunt(z,y) AND spouse(x,z))  
 aunt(x,y) <=> (parent(z,y) AND sister(x,z)) OR  
 OR (uncle(z,y) AND spouse(x,z))  
 nephew(x,y) <=> (parent(z,x) AND sibling(y,z) AND NOT female(x))  
 OR (nephew(x,z) AND spouse(y,z))  
 niece(x,y) <=> (parent(z,x) AND sibling(y,z) AND female(x))  
 OR (niece(x,z) AND spouse(y,z))

parent(Penelope, Arthur)  
 parent(Penelope, Victoria)  
 parent(Christine, James)  
 parent(Christine, Jennifer)  
 parent(Victoria, Colin)  
 parent(Victoria, Charlotte)

### Properties of this representation

- Useful for compression (memory)

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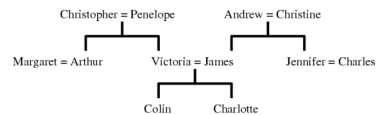
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 OR (niece(x,z) AND spouse(y,z))

parent(Penelope, Arthur)  
 parent(Penelope, Victoria)  
 parent(Christine, James)  
 parent(Christine, Jennifer)  
 parent(Victoria, Colin)  
 parent(Victoria, Charlotte)

### Properties of this representation

- Useful for compression (memory)
- Useful for predicting unknown relations
  - Margaret is Arthur's wife. What else do we know about her?



### Reasoning about kinship

- Problem: Consider a new person, Boris.
  - Is the mother of Boris's father his grandmother?
  - Is the mother of Boris's sister his mother?
  - Is the daughter of Boris's sister his grandfather?
  - Is the son of Boris's sister his son?

### Reasoning about kinship

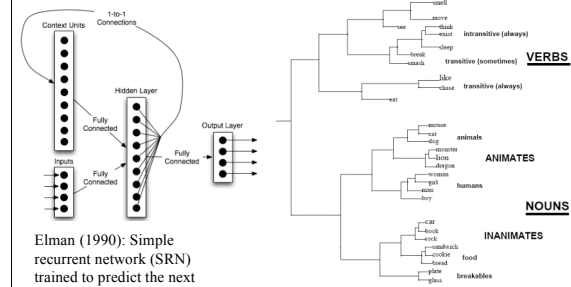
- Problem: Consider a new person, Boris.
  - Is the mother of Boris's father his grandmother?
  - Is the mother of Boris's sister his mother?
  - Is the daughter of Boris's sister his grandfather?
  - Is the son of Boris's sister his son? (Note: Boris and his family were stranded on a desert island when he was a young boy.)
- What this tells us about human knowledge
  - Depends on abstract knowledge about relations.
  - Abstractions must be probabilistic.
  - Knowledge representation and efficient inference on the scale of common-sense reasoning is not going to be easy.

## Language

INPUT	OUTPUT
000000000000000000000000000000110	(woman) 00000000000000000000000110000
000000000000000000000000000000000	(smash) 000000000000000000000000000000000
000000000000000000000000000000000	(skate) 000000000000000000000000000000000
000000000000000000000000000000000	(cat) 000000000000000000000000000000000
000000000000000000000000000000000	(cat) 000000000000000000000000000000000
000000000000000000000000000000000	(move) 000000000000000000000000000000000
000000000000000000000000000000000	(man) 000000000000000000000000000000000
000000000000000000000000000000000	(man) 000000000000000000000000000000000
000000000000000000000000000000000	(break) 000000000000000000000000000000000
000000000000000000000000000000000	(break) 000000000000000000000000000000000
000000000000000000000000000000000	(cat) 000000000000000000000000000000000
000000000000000000000000000000000	(cat) 000000000000000000000000000000000
000000000000000000000000000000000	(boy) 000000000000000000000000000000000
000000000000000000000000000000000	(boy) 000000000000000000000000000000000
000000000000000000000000000000000	(move) 000000000000000000000000000000000
000000000000000000000000000000000	(move) 000000000000000000000000000000000
000000000000000000000000000000000	(girl) 000000000000000000000000000000000
000000000000000000000000000000000	(girl) 000000000000000000000000000000000
000000000000000000000000000000000	(eat) 000000000000000000000000000000000
000000000000000000000000000000000	(eat) 000000000000000000000000000000000
000000000000000000000000000000000	(read) 000000000000000000000000000000000
000000000000000000000000000000000	(read) 000000000000000000000000000000000
000000000000000000000000000000000	(dog) 000000000000000000000000000000000
000000000000000000000000000000000	(dog) 000000000000000000000000000000000
000000000000000000000000000000000	(move) 000000000000000000000000000000000
000000000000000000000000000000000	(move) 000000000000000000000000000000000
000000000000000000000000000000000	(mouse) 000000000000000000000000000000000
000000000000000000000000000000000	(mouse) 000000000000000000000000000000000
000000000000000000000000000000000	(mouse) 000000000000000000000000000000000
000000000000000000000000000000000	(mouse) 000000000000000000000000000000000
000000000000000000000000000000000	(move) 000000000000000000000000000000000
000000000000000000000000000000000	(move) 000000000000000000000000000000000
000000000000000000000000000000000	(bock) 000000000000000000000000000000000
000000000000000000000000000000000	(bock) 000000000000000000000000000000000
000000000000000000000000000000000	(tom) 000000000000000000000000000000000

Elman (1990): Simple recurrent network (SRN) trained to predict the next word in a sentence.

## Language



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Hierarchical clustering on hidden layer activation vectors.

## Semantics with predicate logic

- Bill loves Mary.
  - loves(Bill, Mary)
- Bill thinks that John loves Mary.
  - thinks(Bill, loves(John, Mary))
- Bill thinks that all guys love Mary.
  - thinks(Bill, f.a. x guy(x) loves(x, Mary))
- Mary knows that Bill thinks all guys love her.
  - knows(Mary, thinks(Bill, f.a. x guy(x) loves(x, Mary)))
- Bill is afraid that Mary knows that he thinks all guys love her.
  - afraid(Bill, knows(Mary, thinks(Bill, f.a. x guy(x) loves(x, Mary))))
- Mary wonders if Bill realizes that she knows he thinks all guys love her....

## Semantics with predicate logic

If a burkle tumps that one of its gazzers will glip one of its rupples, then the burkle will prin that gazzar.

## Outline

- The traditional debate in cognitive science: logic *versus* probability
  - The case of connectionism
  - Examples: knowledge about biology, social relations, language, visual objects
- Models of human reasoning that integrate probabilistic inference and logical representations
  - Ecological reasoning (Shafto, Kemp, et al.)
  - Causal reasoning
- Other directions
  - Social reasoning, physical reasoning

## Property induction

Gorillas have T9 hormones.  
Seals have T9 hormones.  
Squirrels have T9 hormones.  
Horses have T9 hormones.

Gorillas have T9 hormones.  
Seals have T9 hormones.  
Squirrels have T9 hormones.  
Flies have T9 hormones.

“Similarity”, “Typicality”,  
“Diversity”

Gorillas have T9 hormones.  
Chimps have T9 hormones.  
Monkeys have T9 hormones.  
Baboons have T9 hormones.  
Horses have T9 hormones.

## Beyond similarity-based induction

- Reasoning based on dimensional thresholds: (Smith et al., 1993)

Poodles can bite through wire.  
 German shepherds can bite through wire.

---

Dobermans can bite through wire.  
 German shepherds can bite through wire.

- Reasoning based on causal relations: (Medin et al., 2004; Coley & Shafto, 2003)

Salmon carry E. Spirus bacteria.  
 Grizzly bears carry E. Spirus bacteria.

---

Grizzly bears carry E. Spirus bacteria.  
 Salmon carry E. Spirus bacteria.

## Different sources for priors

Chimps have T9 hormones.  
 Gorillas have T9 hormones.

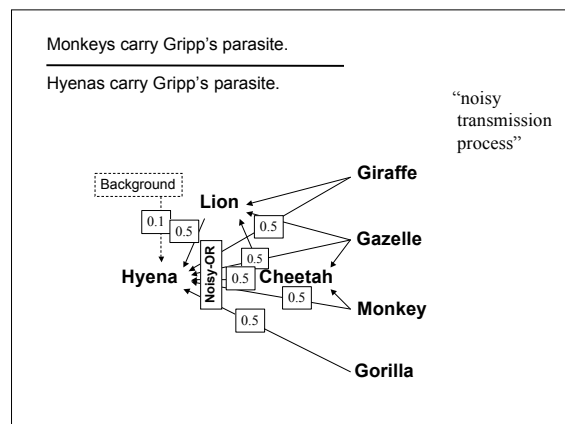
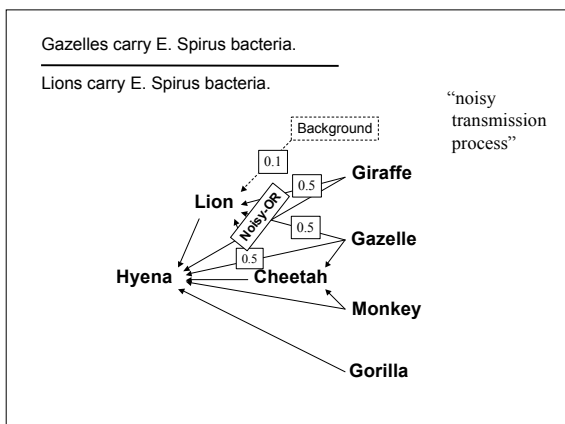
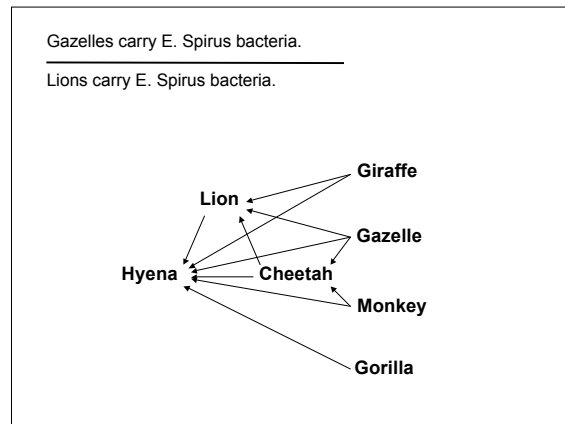
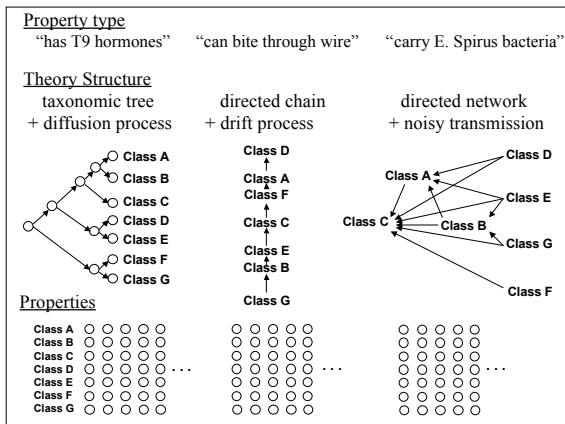
Taxonomic similarity

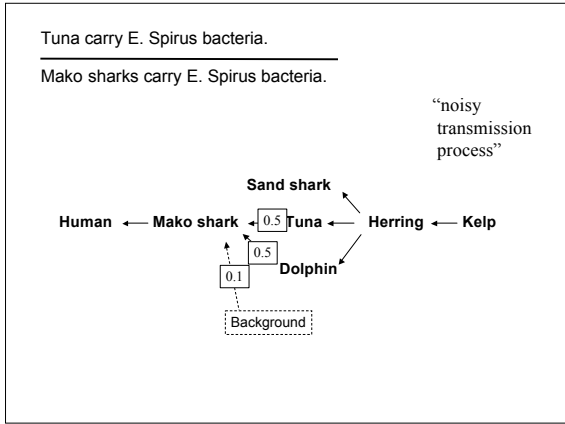
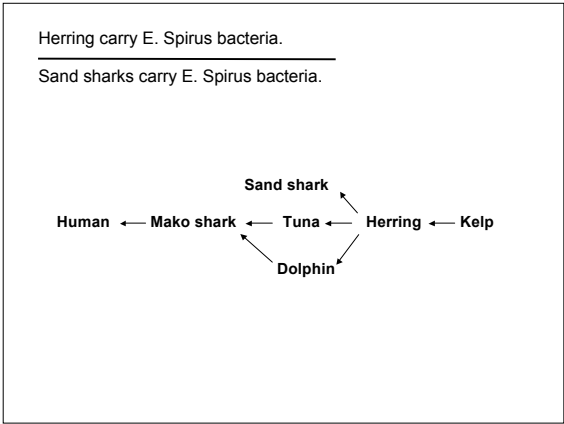
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Strength ordering

Salmon carry E. Spirus bacteria.  
 Grizzly bears carry E. Spirus bacteria.

Food web relations





### Theory as an RPM (BLOG syntax)

**Types:** Species, Disease

**Predicates:**  
Eats(species, species) non-random  
Has(species, disease) random

**Dependency statements:**  
Has(s, d) ~ NoisyORAggCPD[0.5 0.1]  
((Has(s', d) for species s': Eats(s, s')));

Theory (RPM)

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Structure

Data

Relational skeleton

### Reasoning with blank food webs

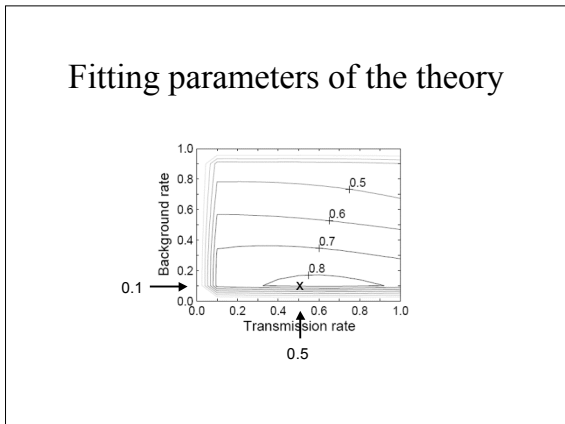
“Given that animal X has disease P, how likely is it that animal Y does?”

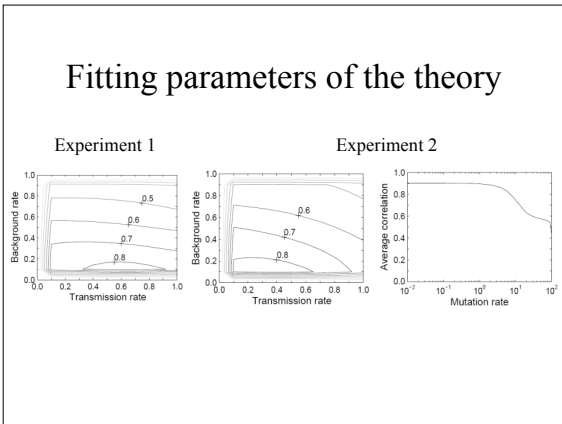
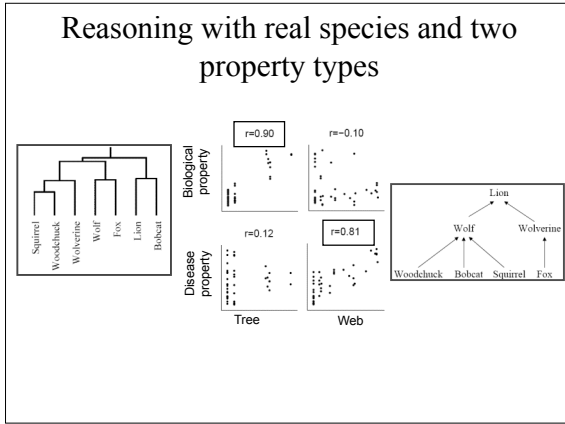
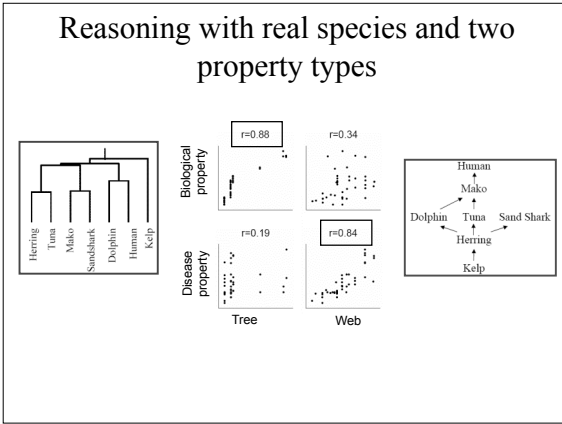
Human ratings

Model predictions:  $r=0.74$      $r=0.92$

Animal W  
Animal Z  
Animal V Animal Y Animal U  
Animal T  
Animal X

Animal D  
Animal A Animal F  
Animal E Animal C Animal G Animal B





### Summary: ecological reasoning

- We can write down simple intuitive theories of ecology as first-order probabilistic models (RPMs).
- Human property induction appears consistent with the use of these theories.
- Theories which fit human inference best are also most appropriate for the structure of the natural environment, both in qualitative structure and quantitative parameter values.
- No natural way of capturing this behavior with traditional modeling approaches based purely on logic or statistical learning.

### Outline

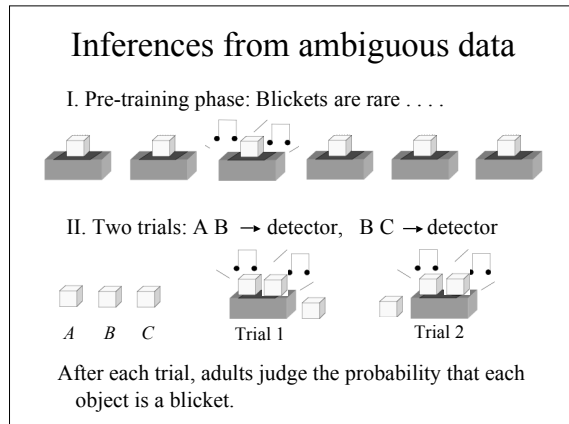
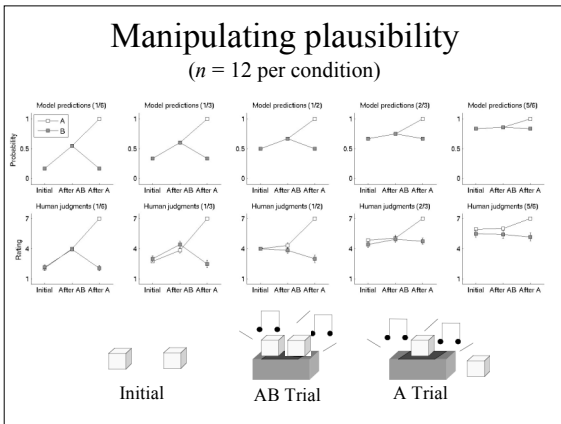
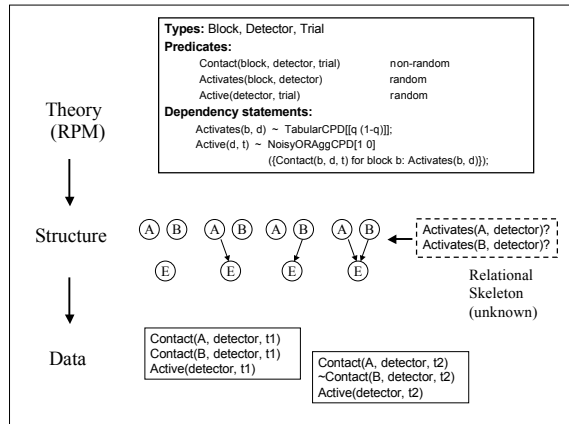
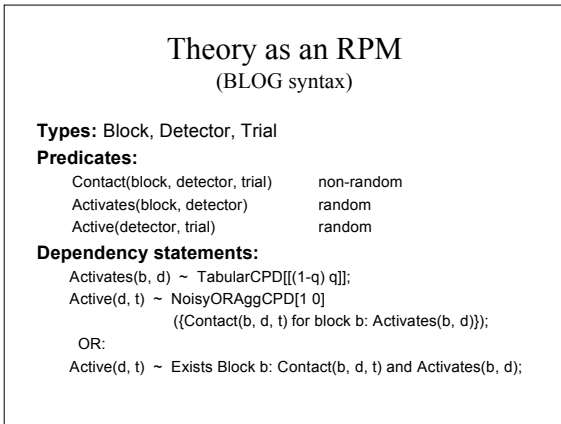
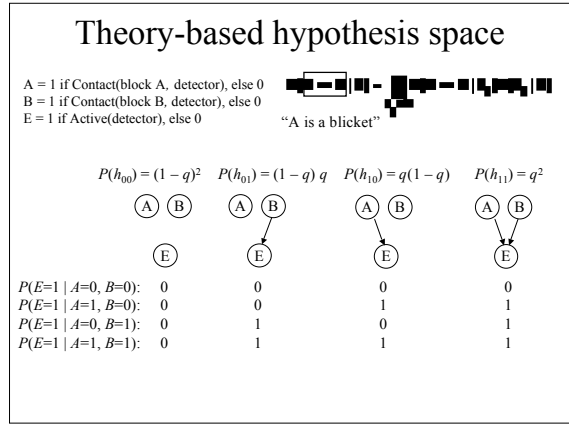
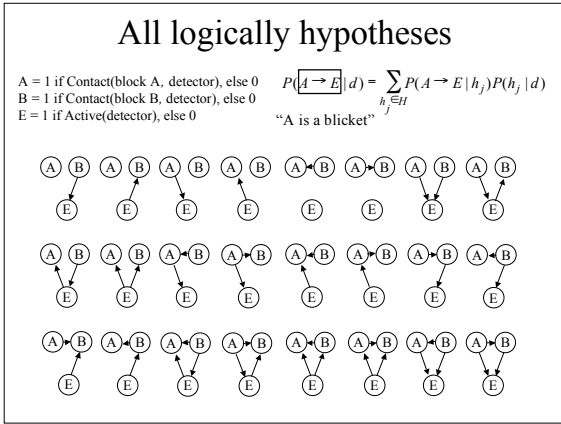
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### “Backwards blocking”

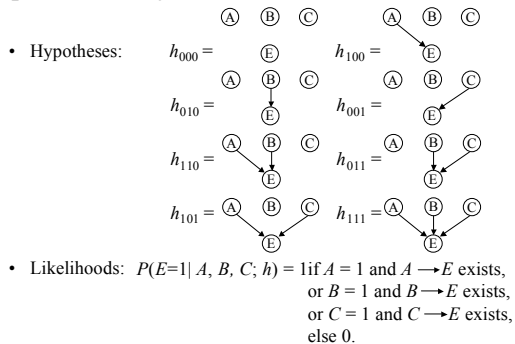
(Sobel, Tenenbaum & Gopnik, 2004)

- Two objects: A and B
- Trial 1: A B on detector – detector active
- Trial 2: A on detector – detector active
- 4-year-olds judge whether each object is a blicket
  - A: a blicket (100% say yes)
  - B: probably not a blicket (34% say yes)

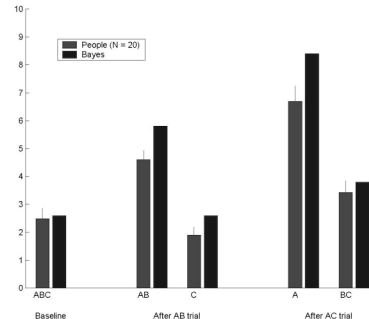




Same domain theory generates hypothesis space for 3 objects:

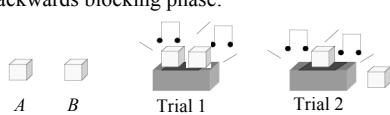


• “Rare” condition: First observe 12 objects on detector, of which 2 set it off.



### Manipulating the priors of 4-year-olds (Sobel, Tenenbaum & Gopnik, 2004)

- I. Pre-training phase: Blickets are rare.
- II. Backwards blocking phase:

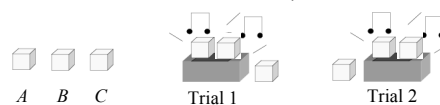


Rare condition: A: 100% say “a blicket”  
B: 25% say “a blicket”

Common condition: A: 100% say “a blicket”  
B: 81% say “a blicket”

### Ambiguous data with 4-year-olds

- I. Pre-training phase: Blickets are rare.
- II. Two trials: A B → detector, B C → detector



Final judgments:  
A: 87% say “a blicket”  
B or C: 56% say “a blicket”

### Outline

- The traditional debate in cognitive science: logic *versus* probability
  - The case of connectionism
  - Examples: knowledge about biology, social relations, language, visual objects
- Models of human reasoning that integrate probabilistic inference and logical representations
  - Ecological reasoning
  - Causal reasoning (Griffiths, Kemp, Goodman et al.)
- Other directions
  - Social reasoning, physical reasoning

### Propositional Theory (Deterministic)

- Scenario with students, courses, profs

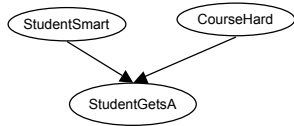
Dr. Pavlov teaches CS1 and CS120  
Matt takes CS1  
Judy takes CS1 and CS120

- Propositional theory

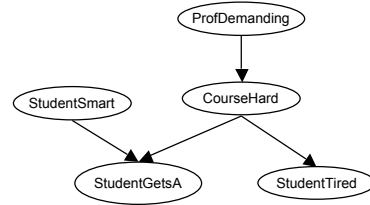
PavlovDemanding → CS1Hard      PavlovDemanding → CS120Hard  
CS1Hard → MattTired      ¬CS1Hard → MattGetsAlnCS1  
CS1Hard → JudyTired      ¬CS1Hard → JudyGetsAlnCS1  
CS120Hard → JudyTired      ¬CS120Hard → JudyGetsAlnCS120

MattSmart ∧ CS1Hard → MattGetsAlnCS1  
JudySmart ∧ CS1Hard → JudyGetsAlnCS1  
JudySmart ∧ CS120Hard → JudyGetsAlnCS120

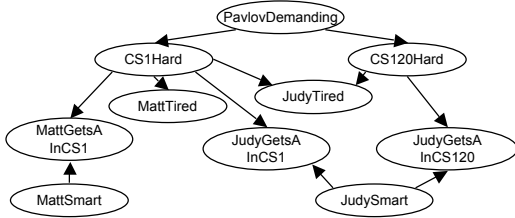
## Social Reasoning



## Social Reasoning



## Probabilistic Theory (Propositional)



- Specific to particular scenario (who takes what, etc.)
- No generalization of knowledge across objects

## First-Order Theory

### • General theory:

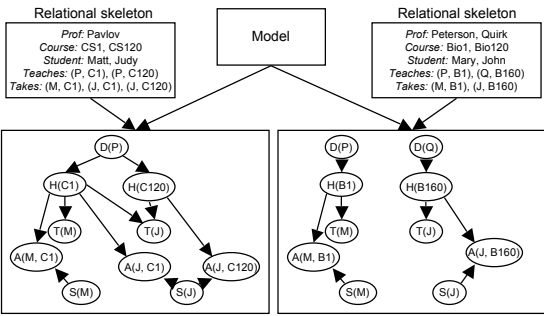
- $\forall p \forall c [\text{Teaches}(p, c) \wedge \text{Demanding}(p) \rightarrow \text{Hard}(c)]$
- $\forall s \forall c [\text{Takes}(s, c) \wedge \text{Hard}(c) \rightarrow \text{Tired}(s)]$
- $\forall s \forall c [\text{Takes}(s, c) \wedge \text{Easy}(c) \rightarrow \text{GetsA}(s, c)]$
- $\forall s \forall c [\text{Takes}(s, c) \wedge \text{Hard}(c) \wedge \text{Smart}(s) \rightarrow \text{GetsA}(s, c)]$

### • Relational skeleton:

Teaches(Pavlov, CS1)      Teaches(Pavlov, CS120)  
 Takes(Matt, CS1)  
 Takes(Judy, CS1)      Takes(Judy, CS120)

- Compact, generalizes across scenarios and objects
- But deterministic

## First-Order Probabilistic Model



## RPM (BLOG syntax)

**Types:** Professor, Course, Student

### **Predicates:**

TaughtBy(course) → professor      non-random  
 Takes(student, course)      non-random  
 Demanding(professor)      random  
 Smart(student)      random  
 Hard(course)      random  
 Tired(student)      random  
 GetsA(student, course)      random

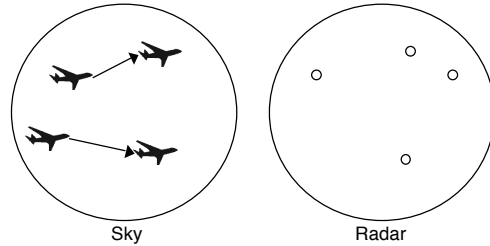
## RPM (BLOG syntax)

### Dependency statements:

```

Demanding(p) ~ TabularCPD[[0.8 0.2]];
Smart(s) ~ TabularCPD[[0.3 0.7]];
Hard(c) ~ TabularCPD[[0.6 0.4],
                    [0.1 0.9]]
                    (Demanding(TaughtBy(c)));
Tired(s) ~ CumGeomCPD[0.5]
          (#TRUE((Hard(c) for course c: Takes(s, c)));)
GetsA(s, c) ~ TabularCPD[[0.5 0.5],
                        [0.1 0.9],
                        [0.9 0.1],
                        [0.7 0.3]]
              (Hard(c), Smart(s));
    
```

## Generative Process for Aircraft Tracking



## BLOG Model for Aircraft Tracking

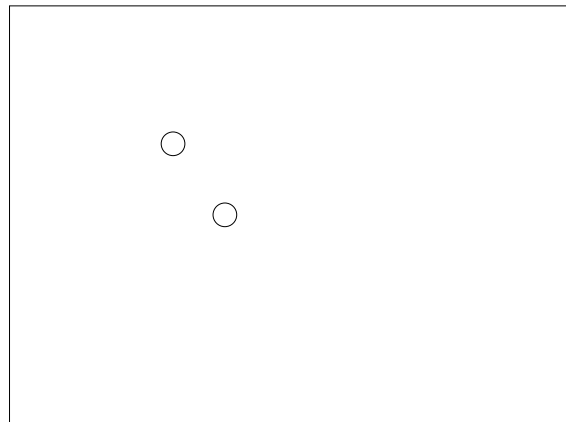
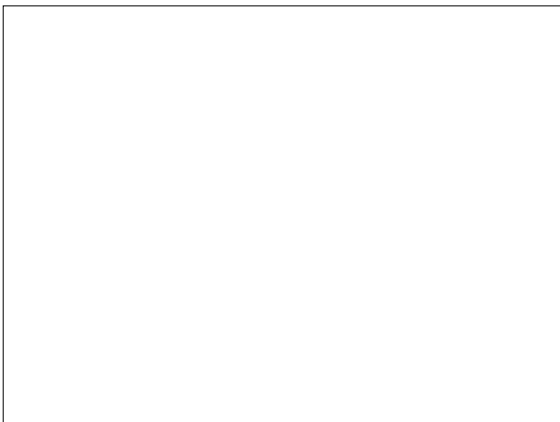
```

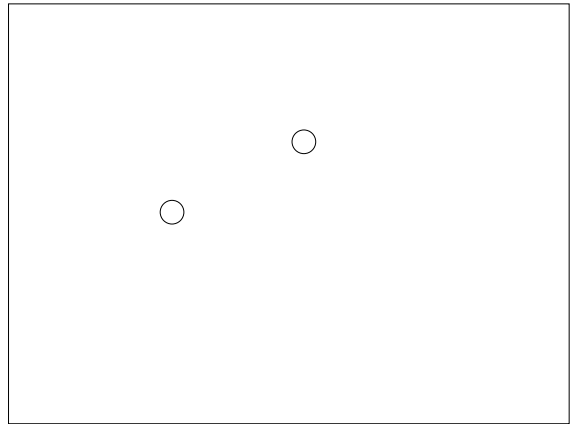
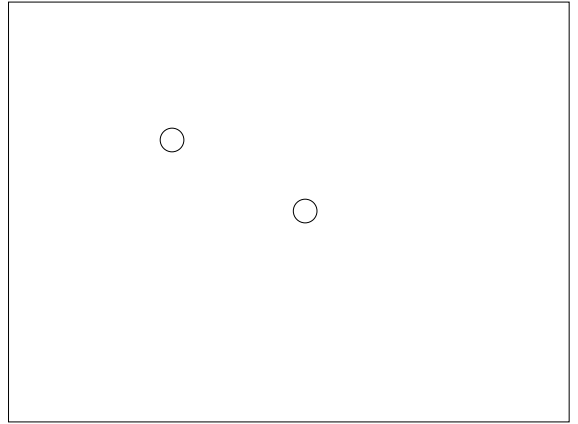
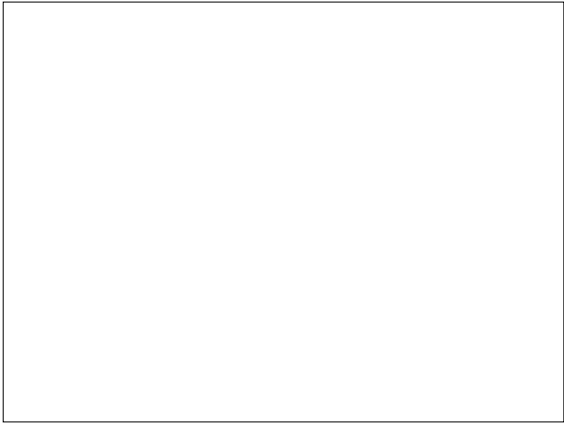
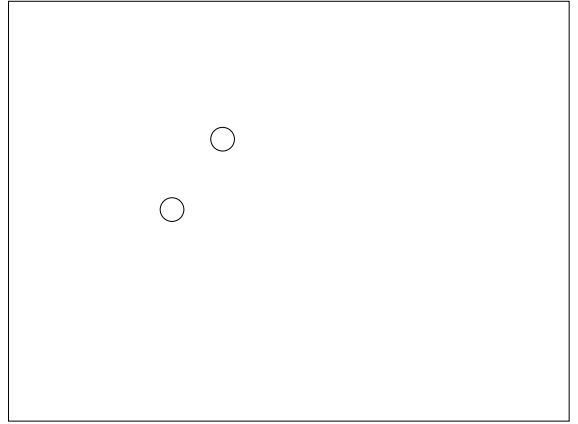
World ...
  #Aircraft ~ NumAircraftDistrib();
  State(a, t)
    if t = 0 then ~ InitState()
    else ~ StateTransition(State(a, Pred(t)));

Observations
  #Blip: (Source, Time) -> (a, t)
    ~ NumDetectionsDistrib(State(a, t));
  #Blip: (Time) -> (t)
    ~ NumFalseAlarmsDistrib();
  ApparentPos(x)
    if (Source(x) = null) then ~ FalseAlarmDistrib()
    else ~ ObsDistrib(State(Source(x), Time(x)));
    
```

## Apparent motion

- Visual system parses ambiguous experience into objects under several assumptions:
  - Objects typically do not disappear and appear spontaneously.
  - Objects typically follow “simple” space-time trajectories.

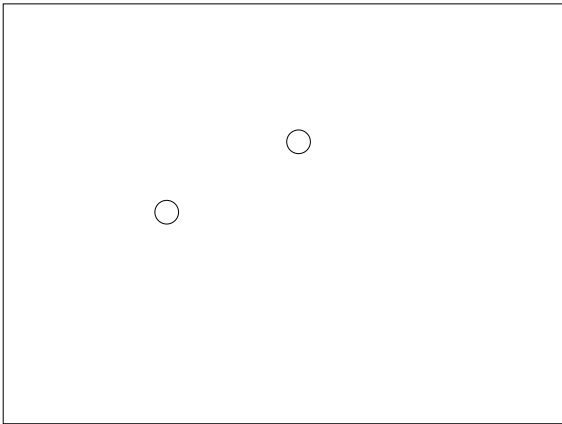
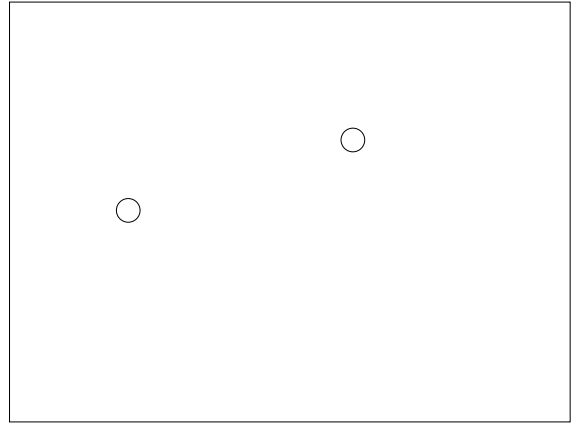
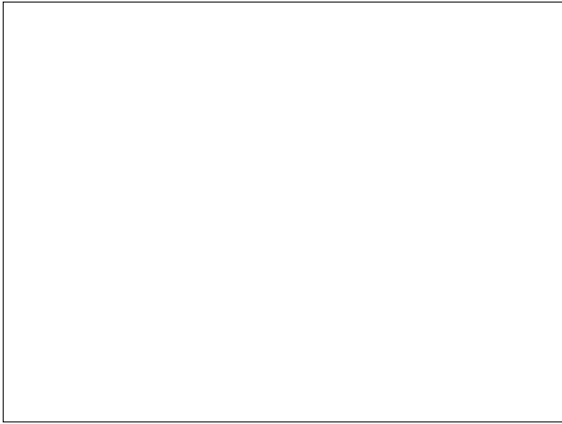
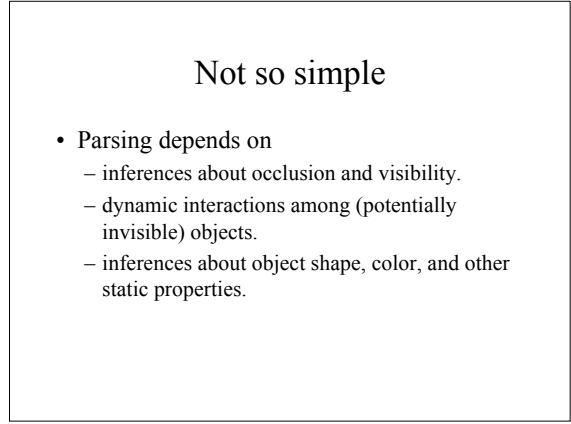


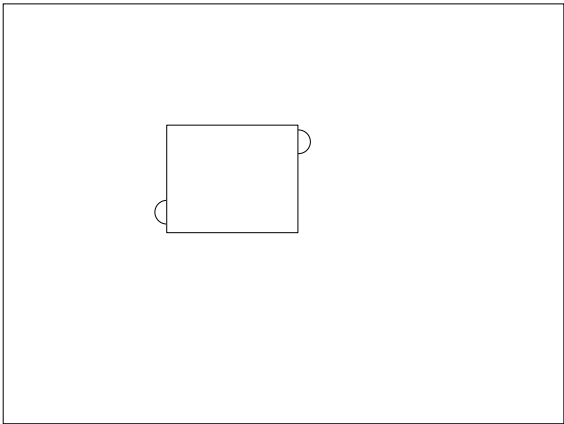
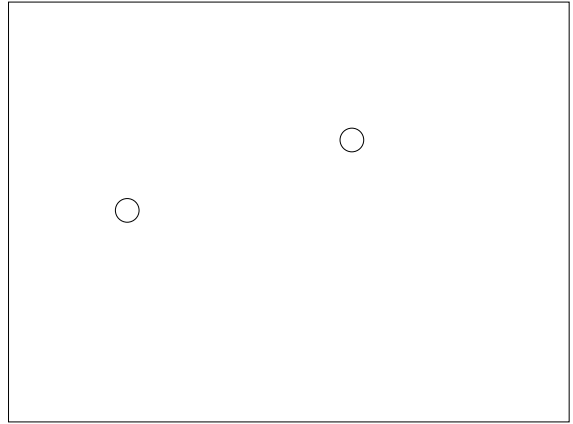
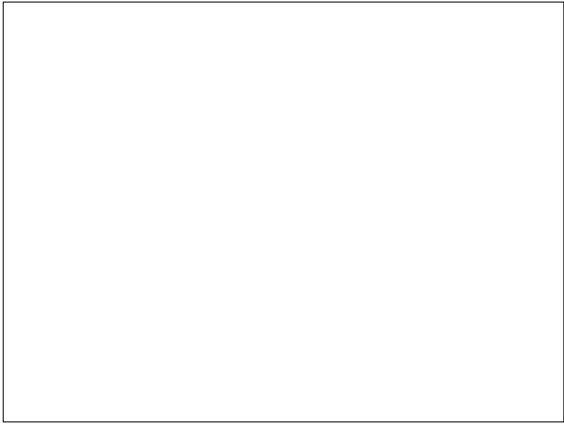
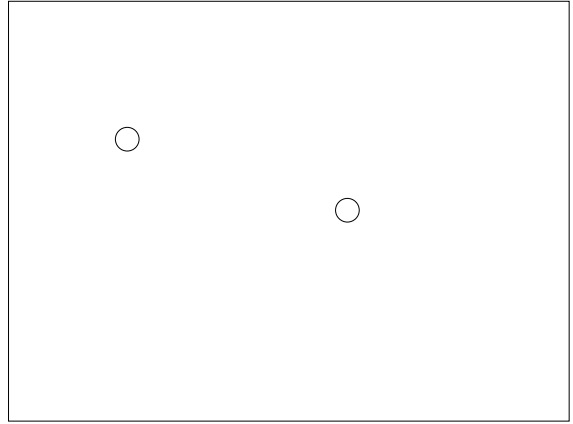
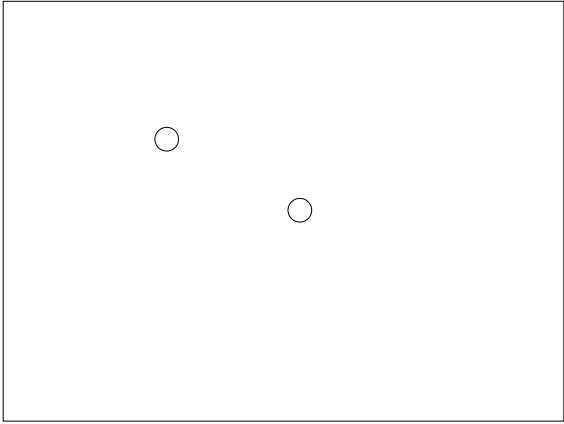


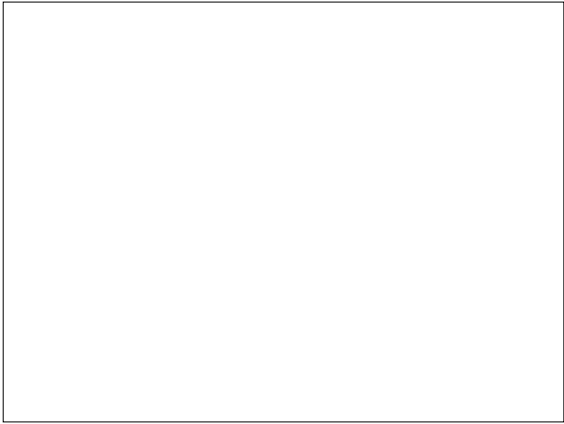
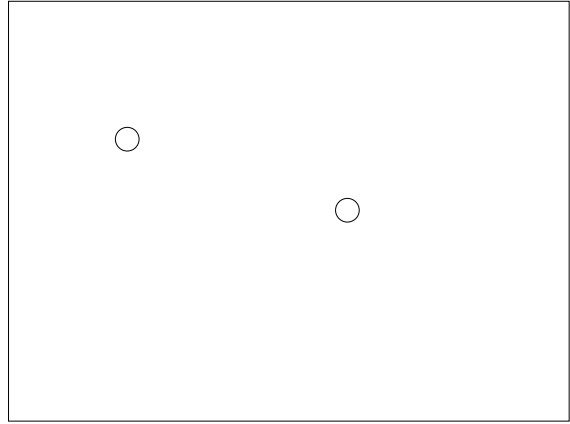
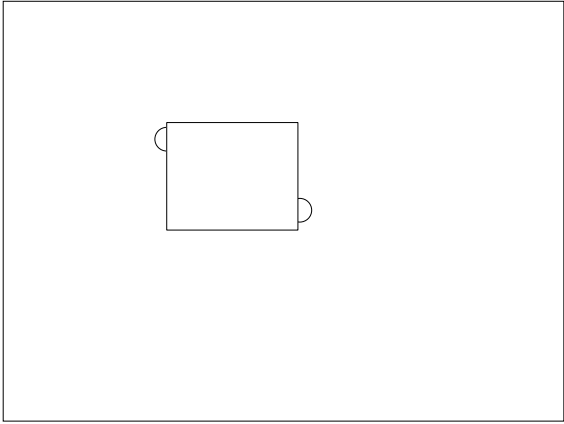


## Not so simple

- Parsing depends on
  - inferences about occlusion and visibility.
  - dynamic interactions among (potentially invisible) objects.
  - inferences about object shape, color, and other static properties.

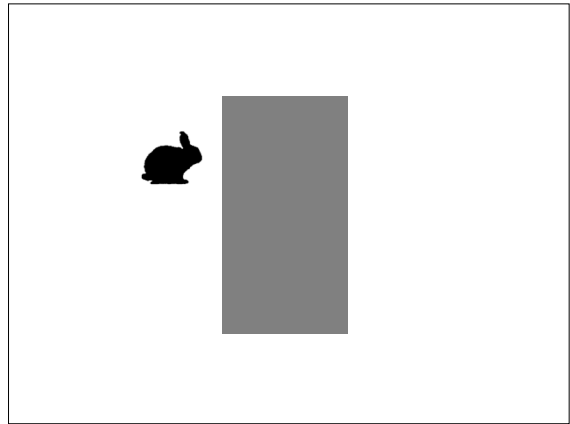




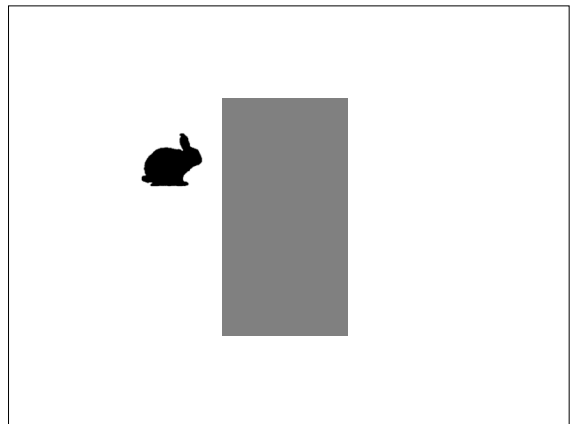
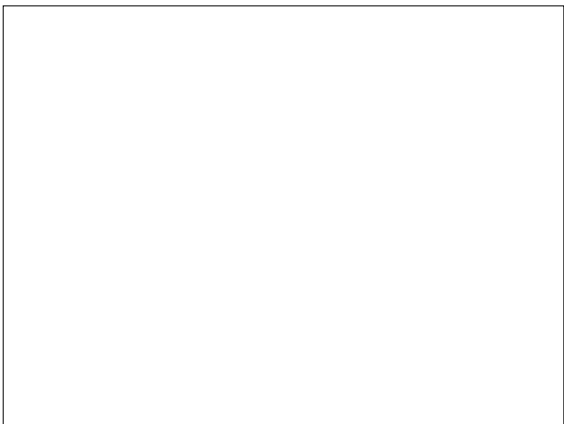
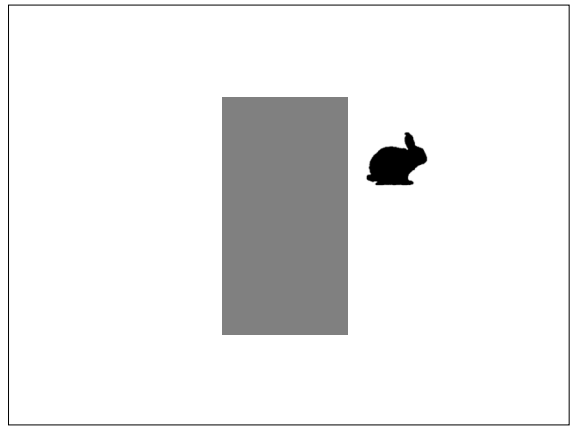
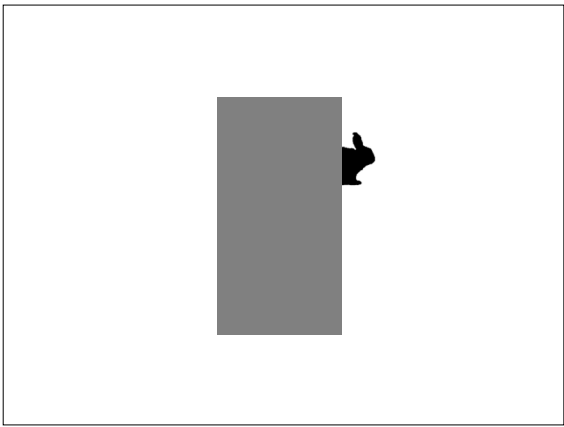
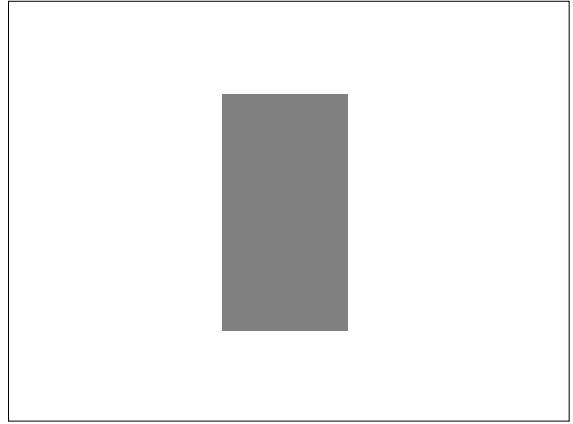
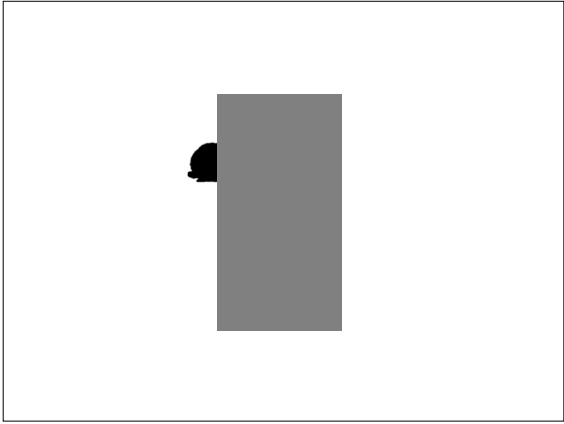


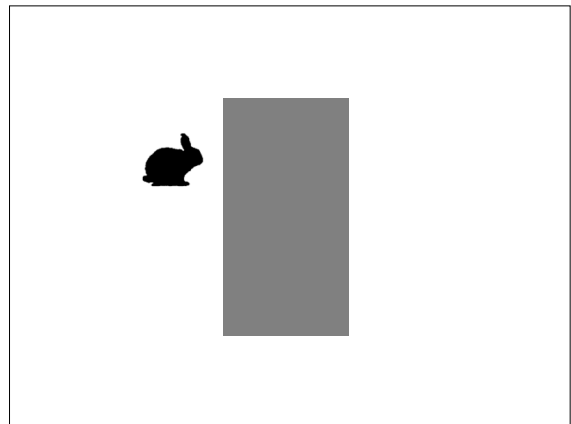
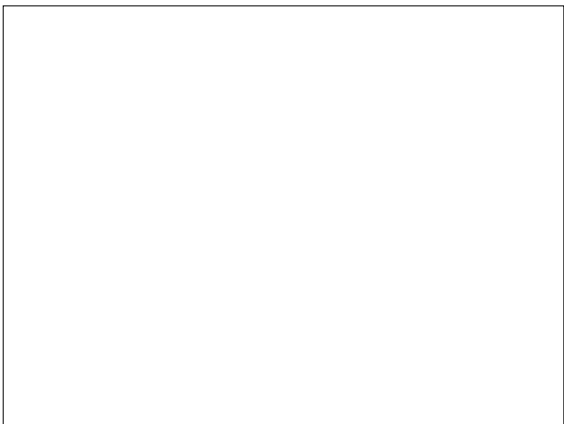
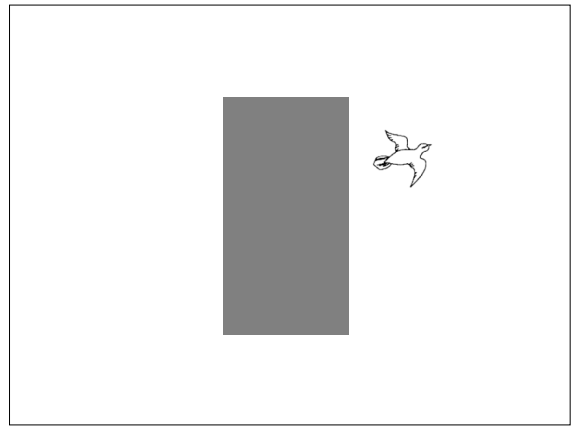
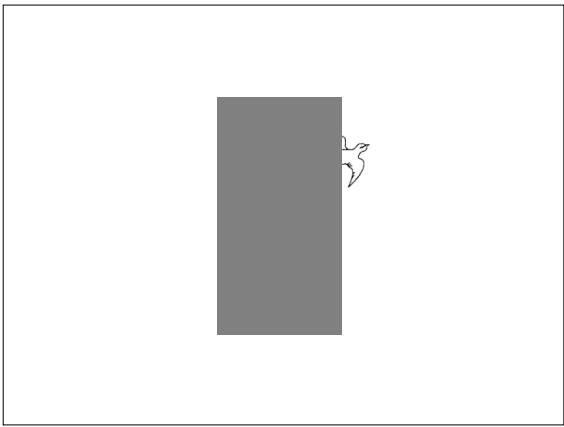
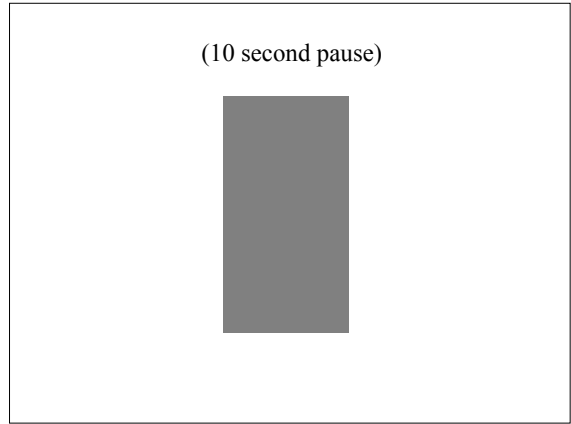
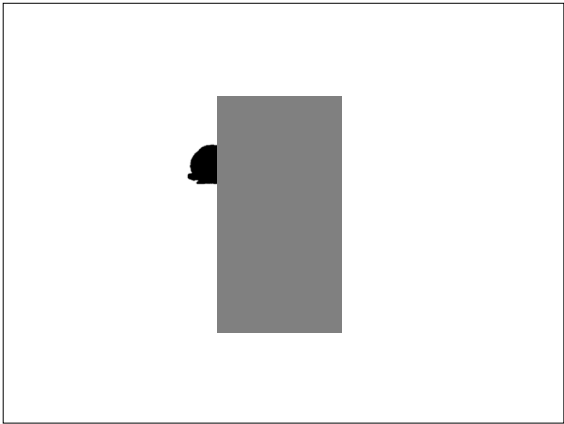
Not so simple

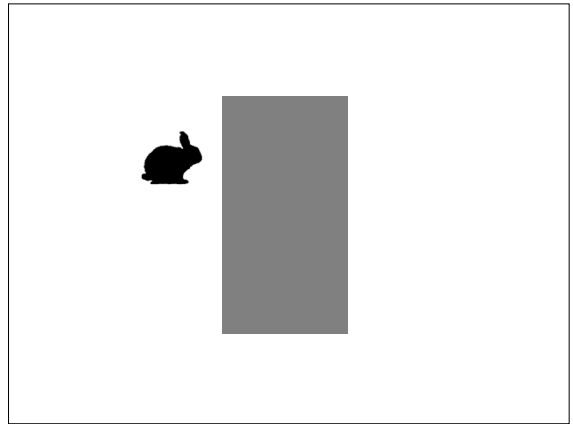
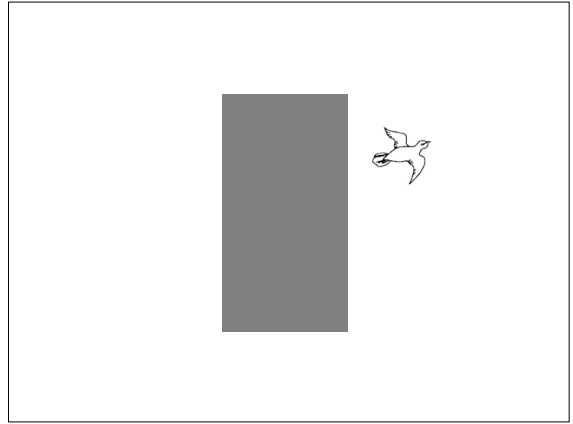
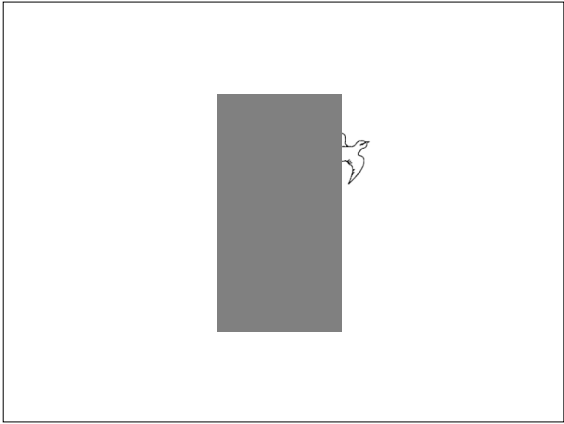
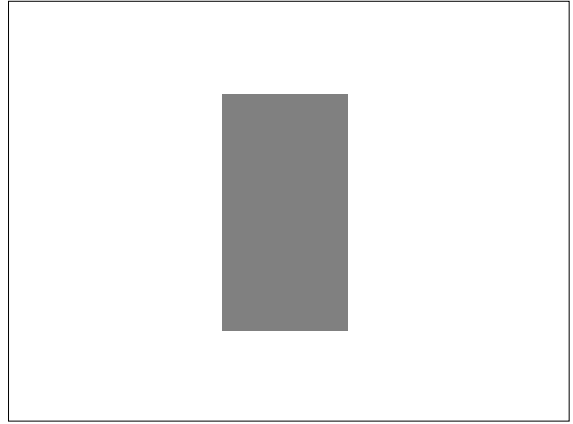
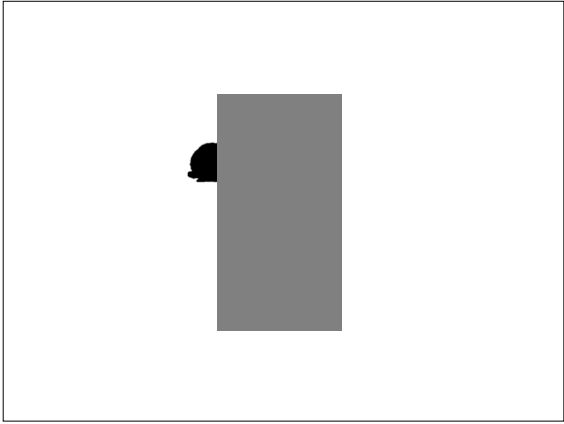
- Parsing depends on
  - inferences about occlusion and visibility.
  - dynamic interactions among (potentially invisible) objects.
  - inferences about object shape, color, and other static properties.
- Theory of objects must allow uncertainty in how many objects are in the scene.

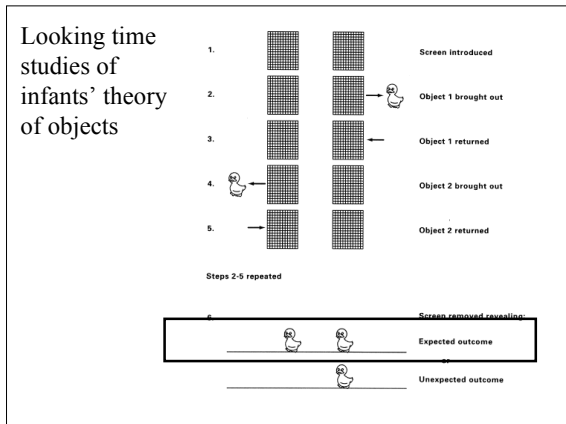
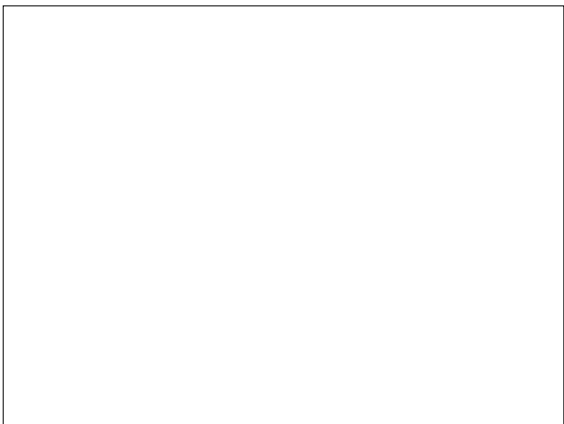
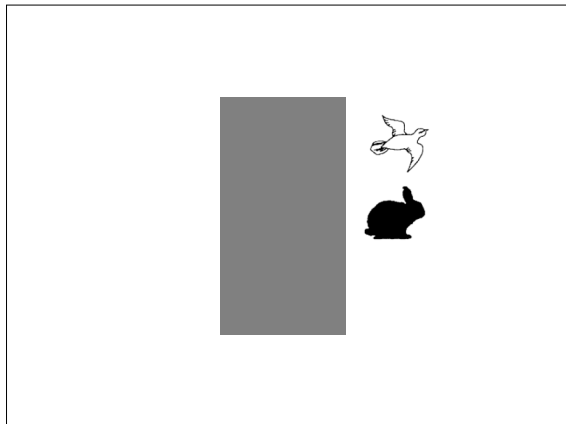
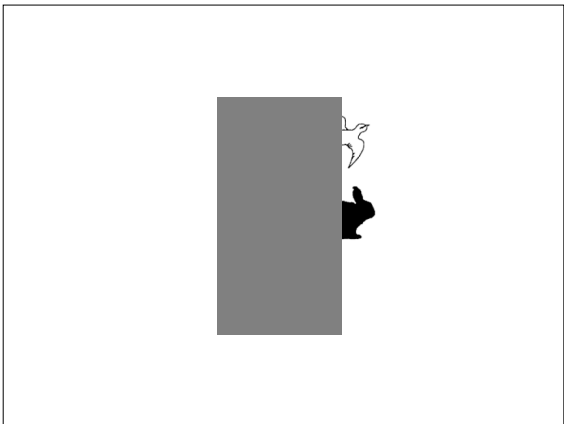
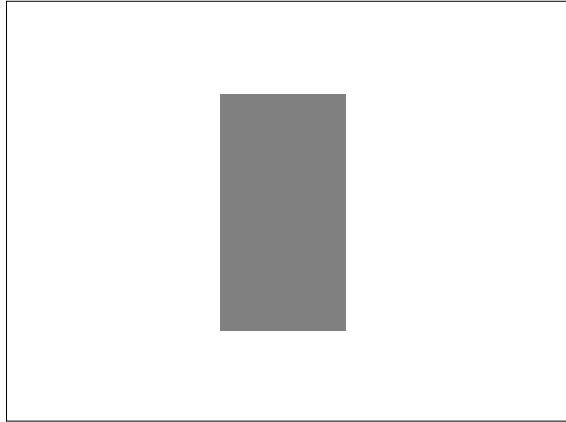
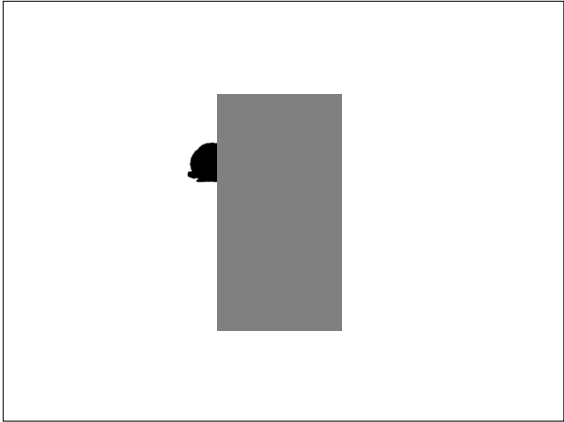


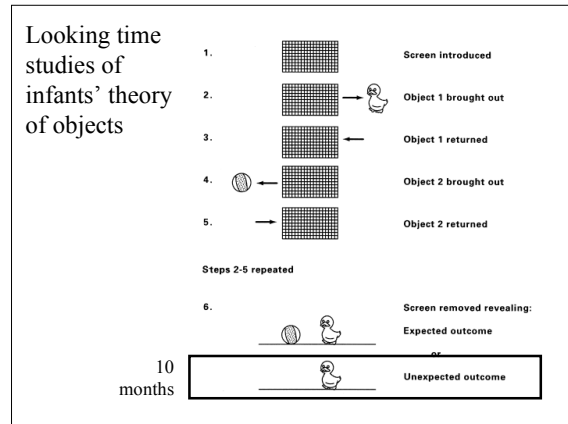
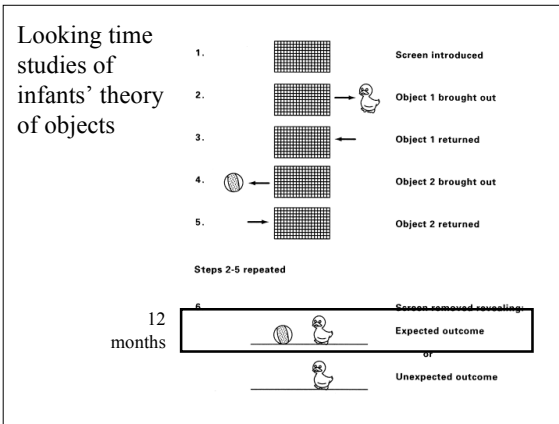












## Big open problems

- Use RPMs, BLOG, etc. to...
  - Formalize intuitive social or physical reasoning.
  - Explain human inferences in social or physical domains.
  - Explain the differences between children's theories at different ages.
  - Explain how children learn these theories.
  - Elucidate core representational capacities of human cognition.