Learning to Solve Inverse Problems in Imaging

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Inverse problems in imaging

Observe: $y = X\beta + \varepsilon$

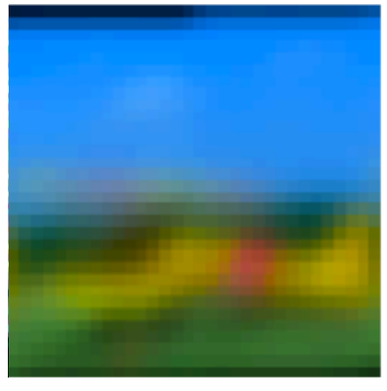
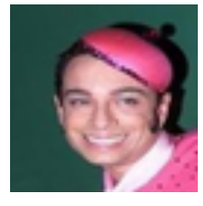
Goal: Recover β from y

- Inpainting
- Deblurring
- Superresolution
- Compressed Sensing
- MRI
- Radar

β



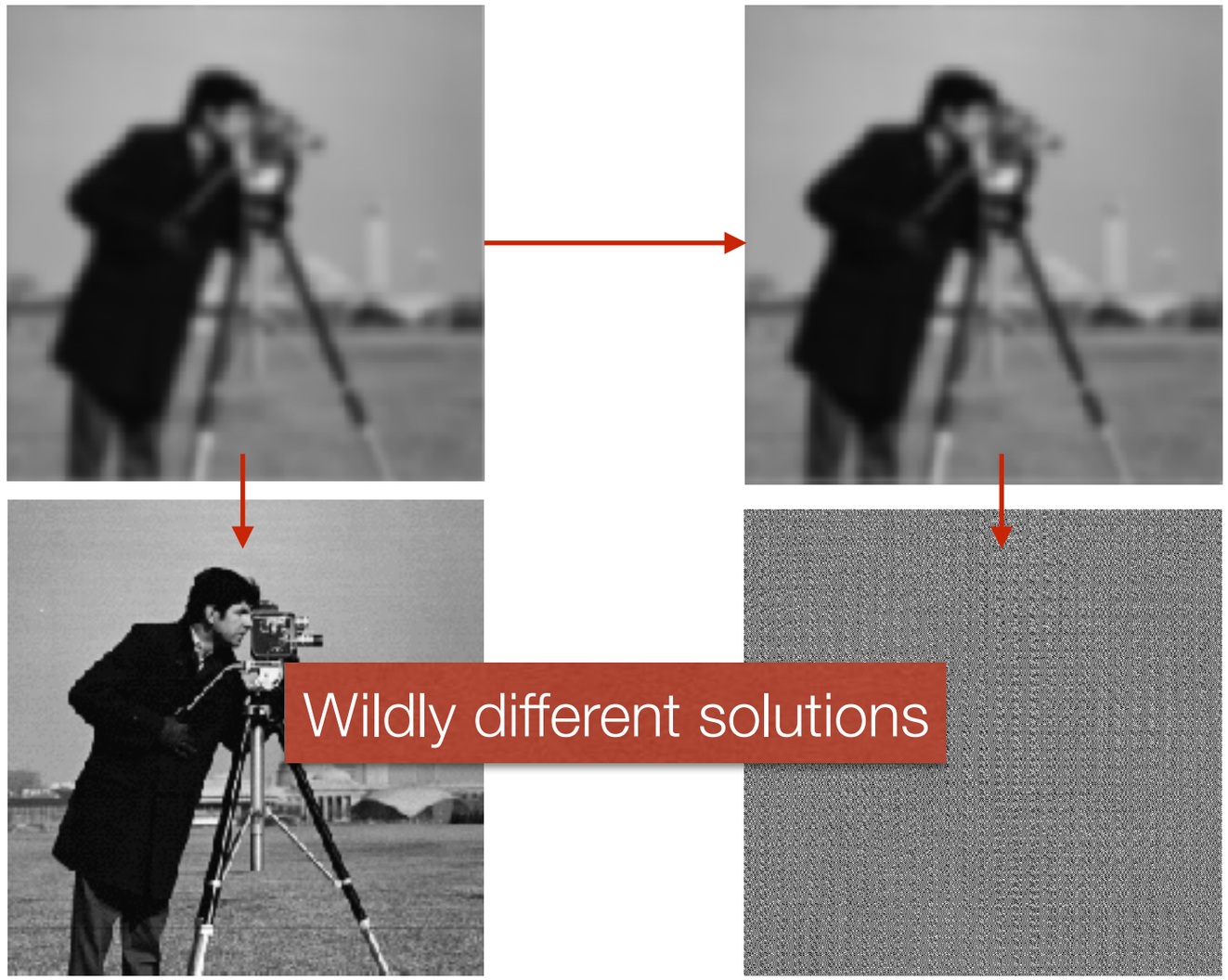
y

Classical approach: Tikhonov regularization (1943)

- Example: deblurring
- Least squares solution:

$$\hat{\beta} = (X^T X)^{-1} X^T y$$



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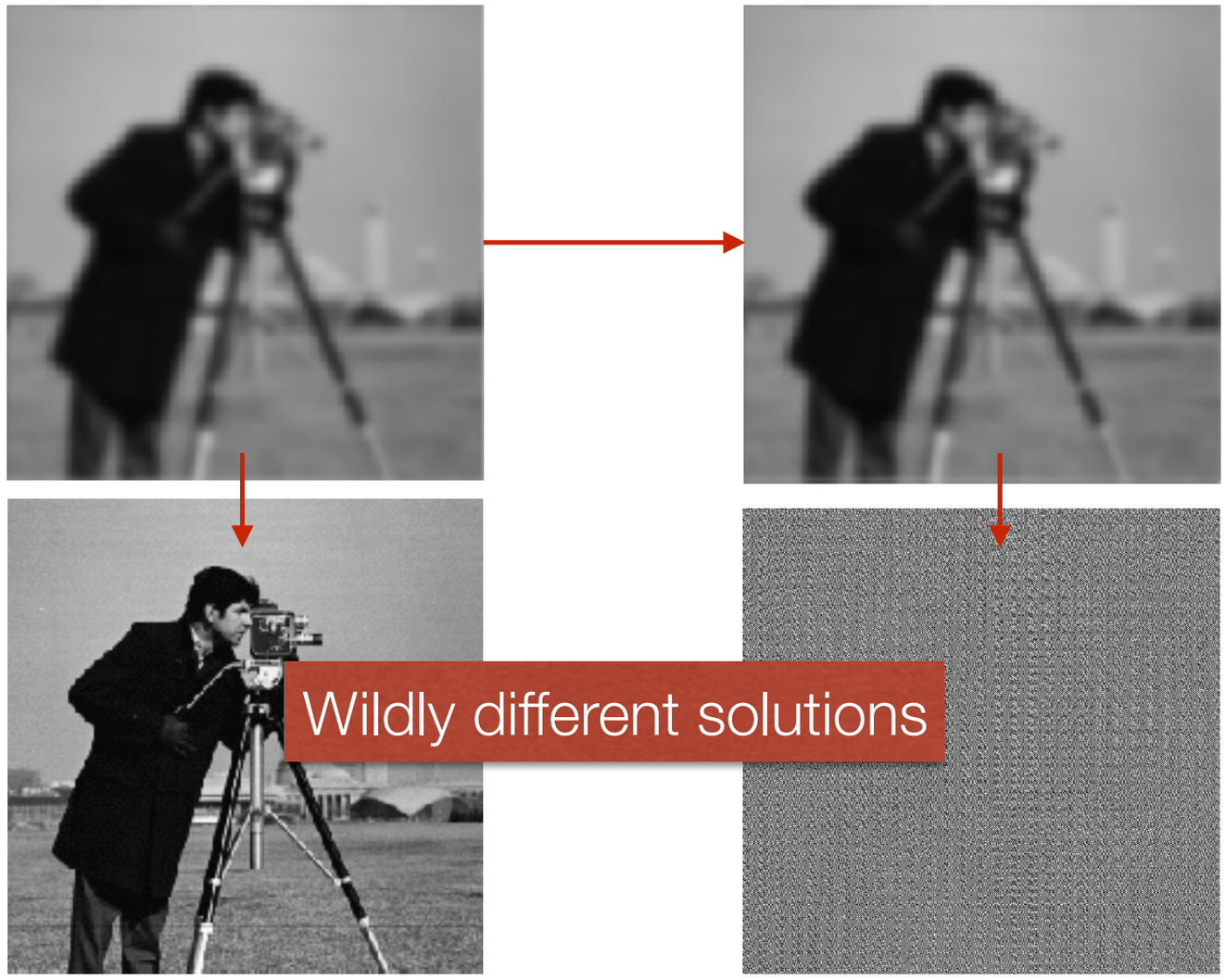
$$\hat{\beta} = (X^\top X)^{-1} X^\top y$$



- Tikhonov regularization (aka “ridge regression”)

$$\hat{\beta} = \arg\min_{\beta} \|y - X\beta\|_2^2 + \lambda\|\beta\|_2^2$$

$$= (X^\top X + \lambda I)^{-1} X^\top y$$

better conditioned; suppresses noise

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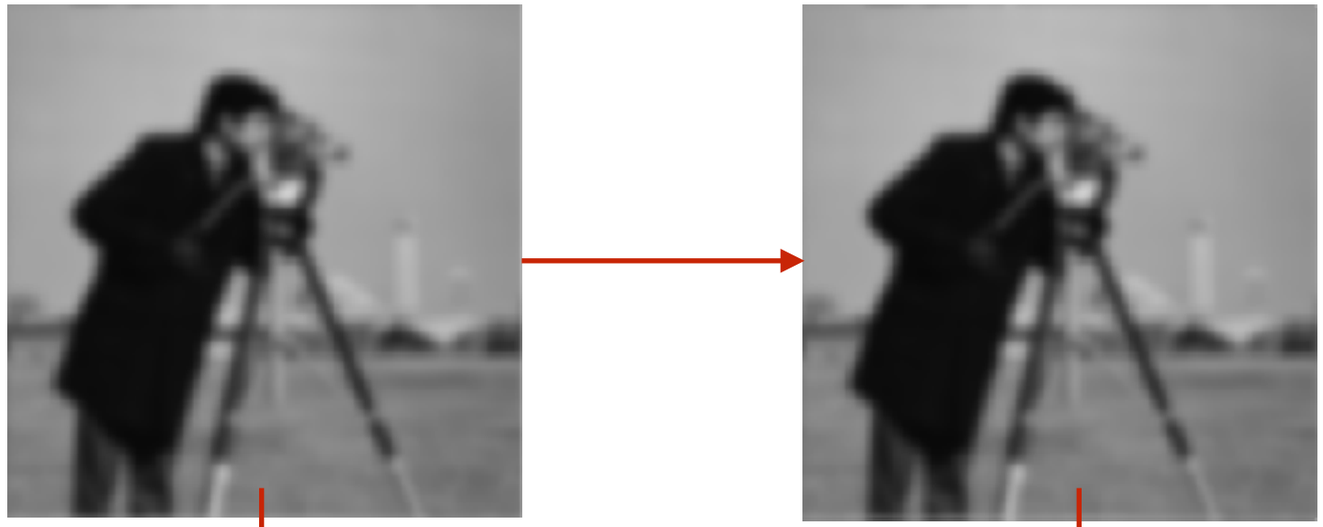

Tikhonov regularization

Wildly diffe

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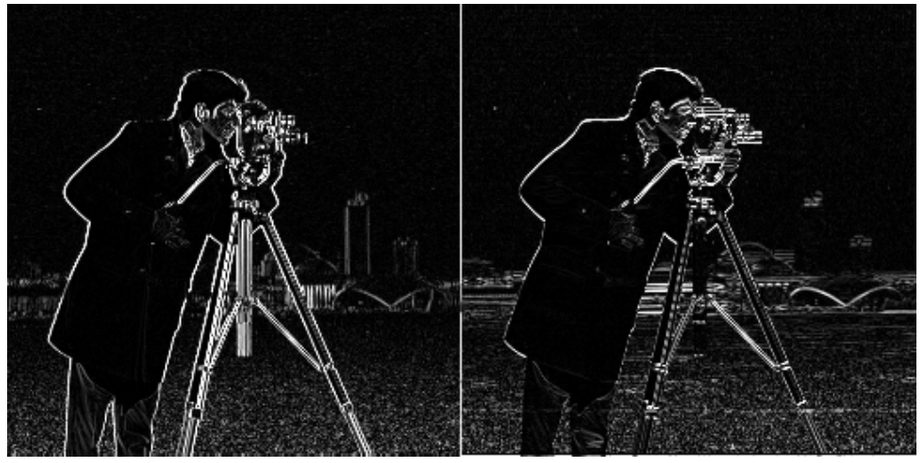
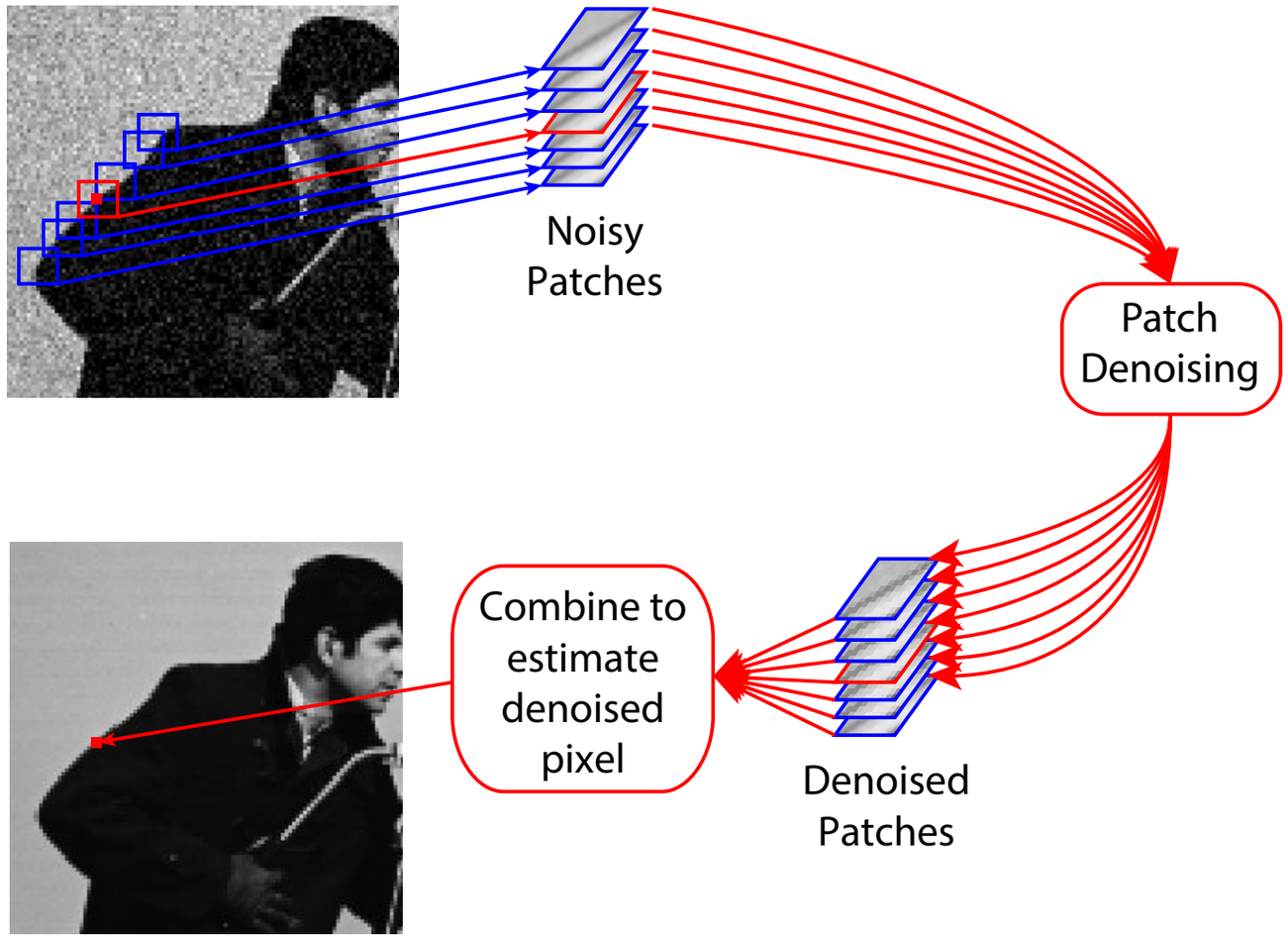
Geometric models of images

Total variation



(Wavelet) sparsity

Patch subspaces and manifolds

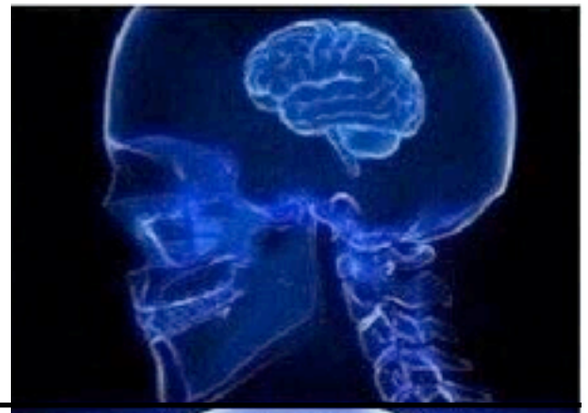


Regularization in inverse problems

Regularization in inverse problems

$$y \longrightarrow \boxed{\hat{\beta} = \arg\min_{\beta} ||y - X\beta||_2^2 + r(\beta)} \longrightarrow \hat{\beta}$$

Classical: r(β) is a pre-defined smoothness-promoting regularizer (e.g. Tikhinov or ridge estimation)



Geometric: r(β) reflects image geometry (e.g. sparsity, patch redundancy, total variation)

Learned: use training data to learn r(β)

Examples in recent literature

- Deep CNN's for signal recovery

  Dong, Loy, He, Tang, 2014
  Mousavi and Baraniuk, 2017
  Jin, McCann, Froustey, Unser, 2017
  Ye, Han, Cha, 2018

- Compressed sensing with GANs

  Bora, Jalal, Price, Dimakis, 2017

- Unrolled algorithms for solving inverse problems

  - Deep proximal gradient descent nets

    Chen, Yu, Pock, 2015
    Mardani et al, 2018

  - Deep ADMM nets

    Sun, Li, Xu, 2016
    Chang, Li, Poczos, Kumar, Sankaranarayanan, 2017

  - Deep half-quadratic splitting

    Zhang, Zuo, Gu, Zhang, 2017

  - Deep primal-dual nets

    Adler and Öktem, 2018

Classes of methods

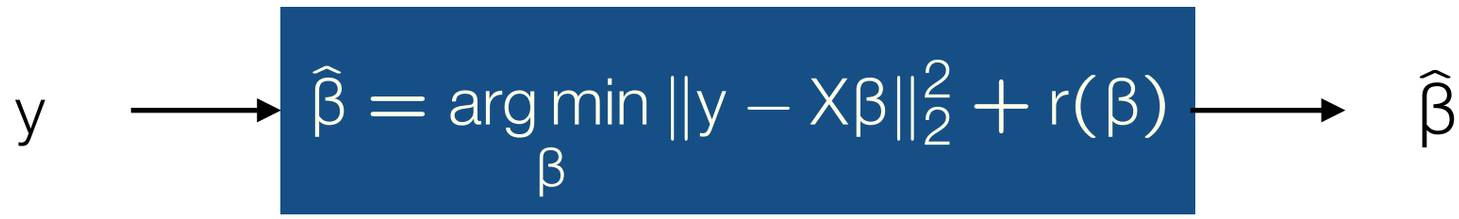
Model Agnostic
(Ignore X)

Decoupled
(First learn, then reconstruct)

Unrolled Optimization

Neumann Networks
(this talk!)

Deep proximal gradient



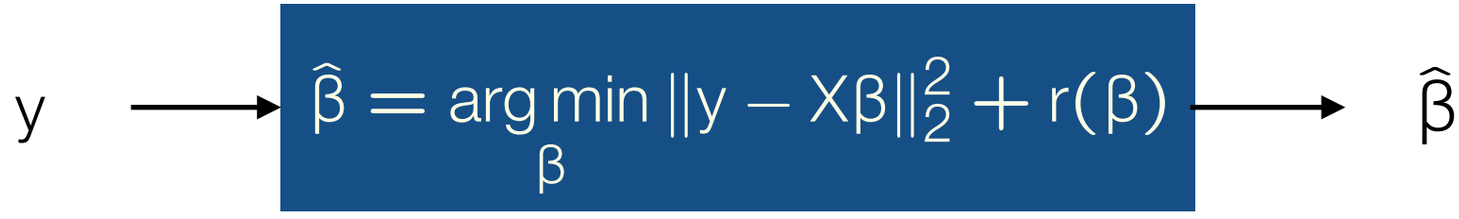
set $\hat{\beta}^{(1)}$ and stepsize $\eta > 0$

for $k = 1, 2, \ldots$

$$z^{(k)} = \hat{\beta}^{(k)} + \eta X^\top (y - X\hat{\beta}^{(k)}) \qquad \text{gradient descent}$$

$$\hat{\beta}^{(k+1)} = \arg\min_{\beta} \|z^{(k)} - \beta\|_2^2 + \eta r(\beta) \qquad \text{denoising}$$

Deep proximal gradient



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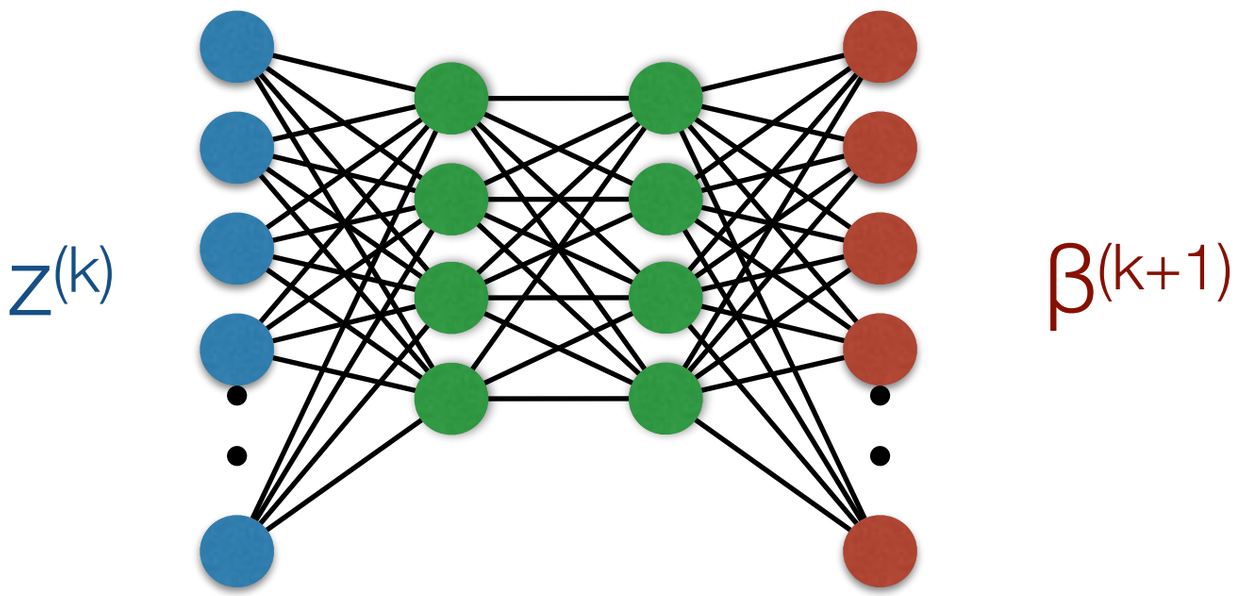
Replace with learned neural network



Chang, Li, Poczos, Kumar, & Sankaranarayanan 2017

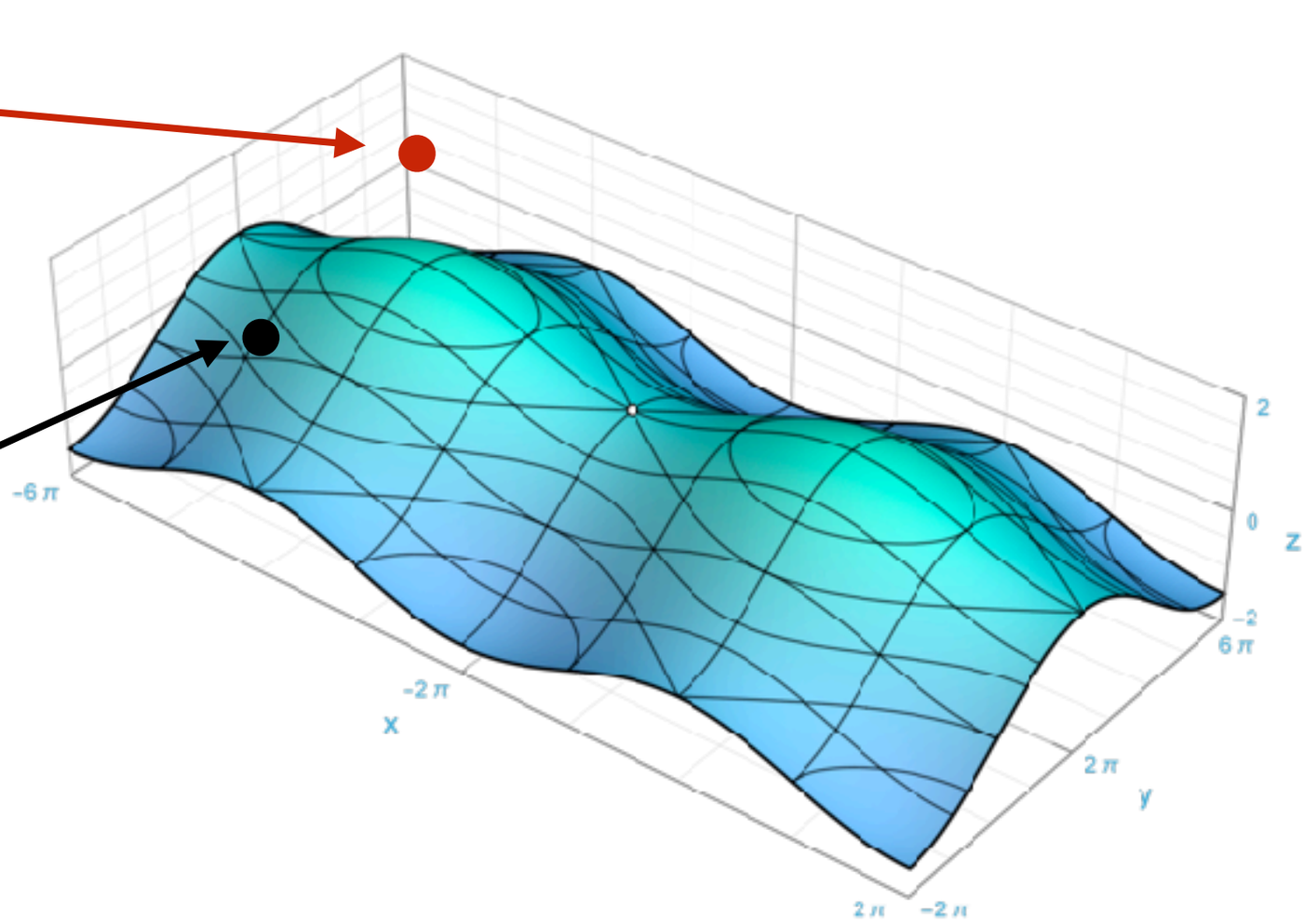
GANs for inverse problems

$$y \longrightarrow \boxed{\hat{\beta} = \arg\min_{\beta} \|y - X\beta\|_2^2 + r(\beta)} \longrightarrow \hat{\beta}$$

$$r(\beta) = \begin{cases} 0, & \beta \text{ on image manifold} \\ \infty, & \text{otherwise} \end{cases}$$

"Bad" image off manifold

"Good" image on manifold



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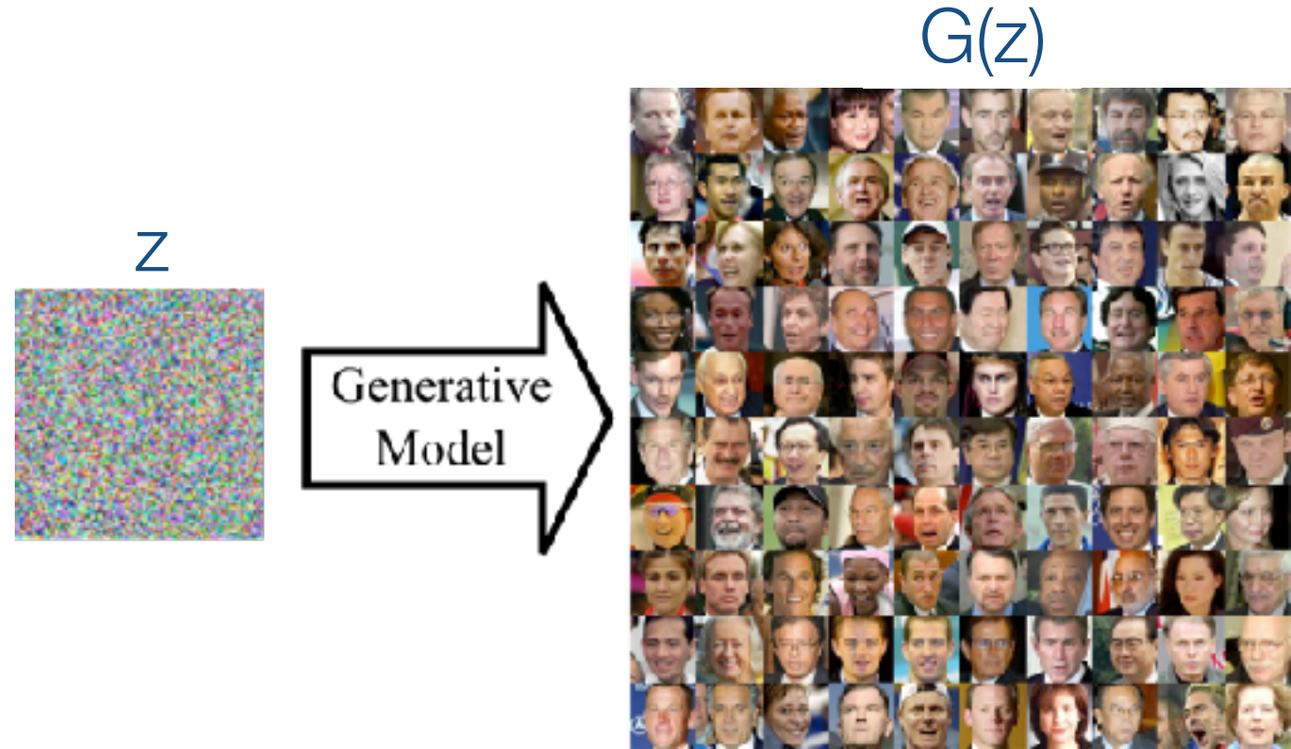
Learn generator G that outputs $\beta \in \mathbb{R}^d$ given $z \in \mathbb{R}^{d'}$ for $d' < d$

$$r(\beta) = \begin{cases} 0, & \beta \in \text{range}(G) \\ \infty, & \text{otherwise} \end{cases}$$



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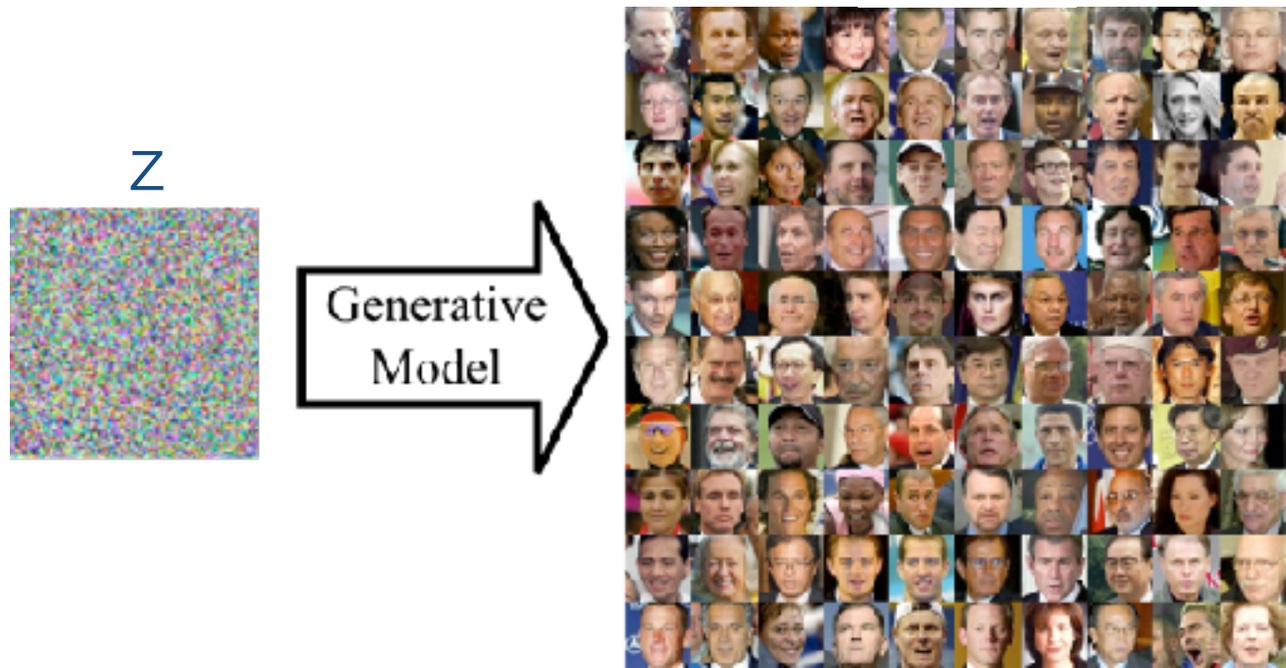
Choose $\beta \in \text{range}(G)$ that best fits data:

$$\hat{\beta} = \arg\min_{\beta \in \text{range}(G)} \|y - X\beta\|_2^2$$
$$= G(\hat{z})$$
$$\hat{z} = \arg\min_{z} \|y - XG(z)\|_2^2$$



Bora, Jalal, Price, Dimakis, 2017

How much training data?

Original
β

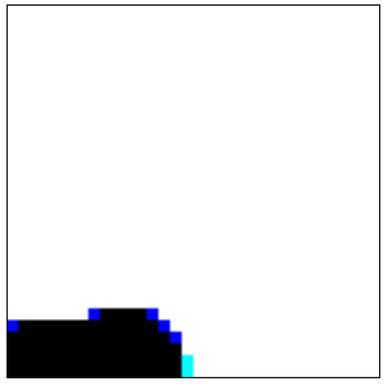
Observed
y

Reconstruction with convolutional neural network (CNN) trained with 80k samples

How much training data?

Original
β

Observed
y

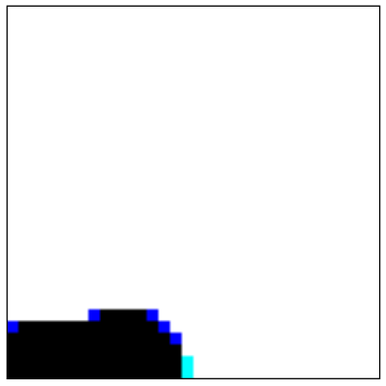


Reconstruction with convolutional neural network (CNN) trained with 2k samples

How much training data?

Original
β

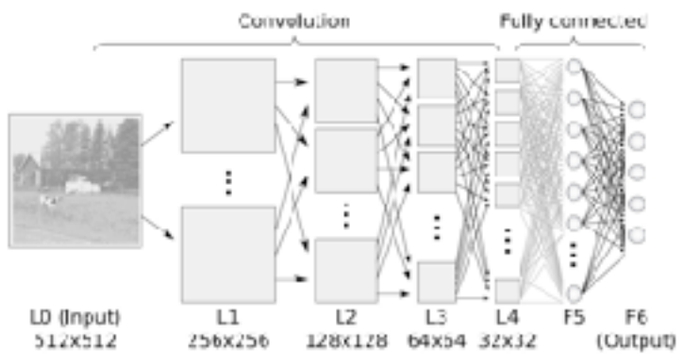
Observed
y



Reconstruction with convolutional neural network (CNN) trained with 2k samples

What people think he's referring to:

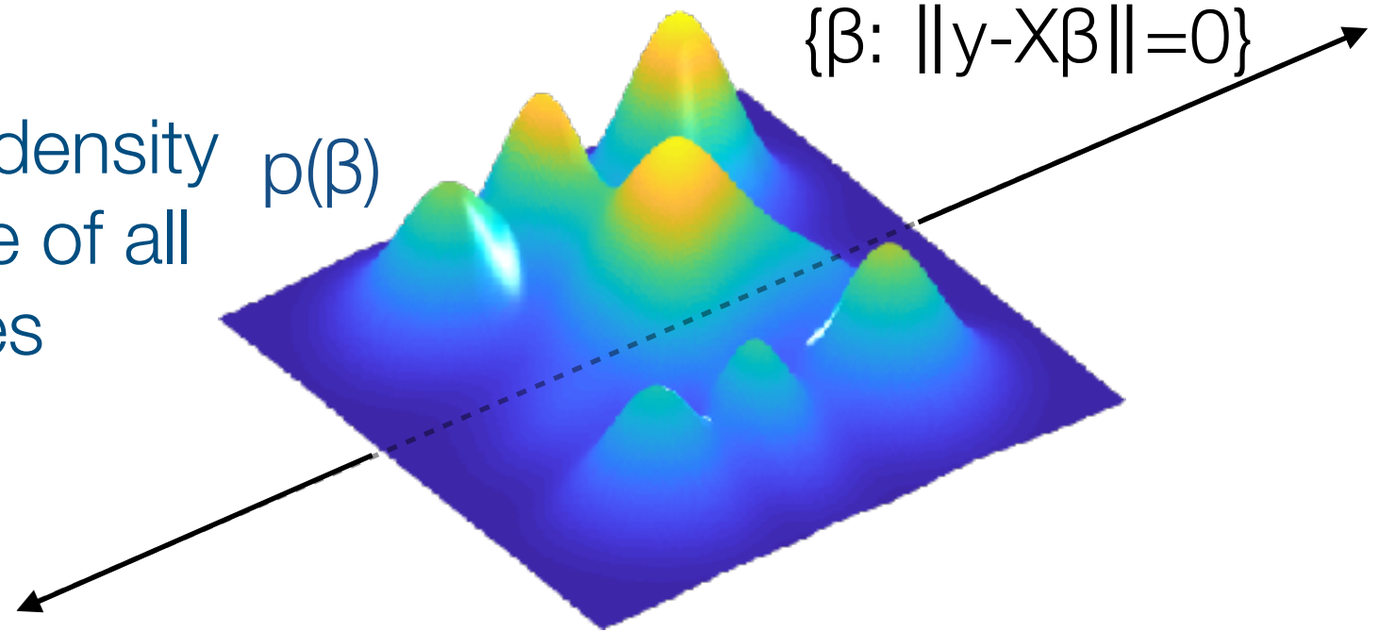
What he's actually referring to:



Learning a proximal operator or learning a generative model both implicitly require estimating p(β)

probability density over space of all images

p(β)



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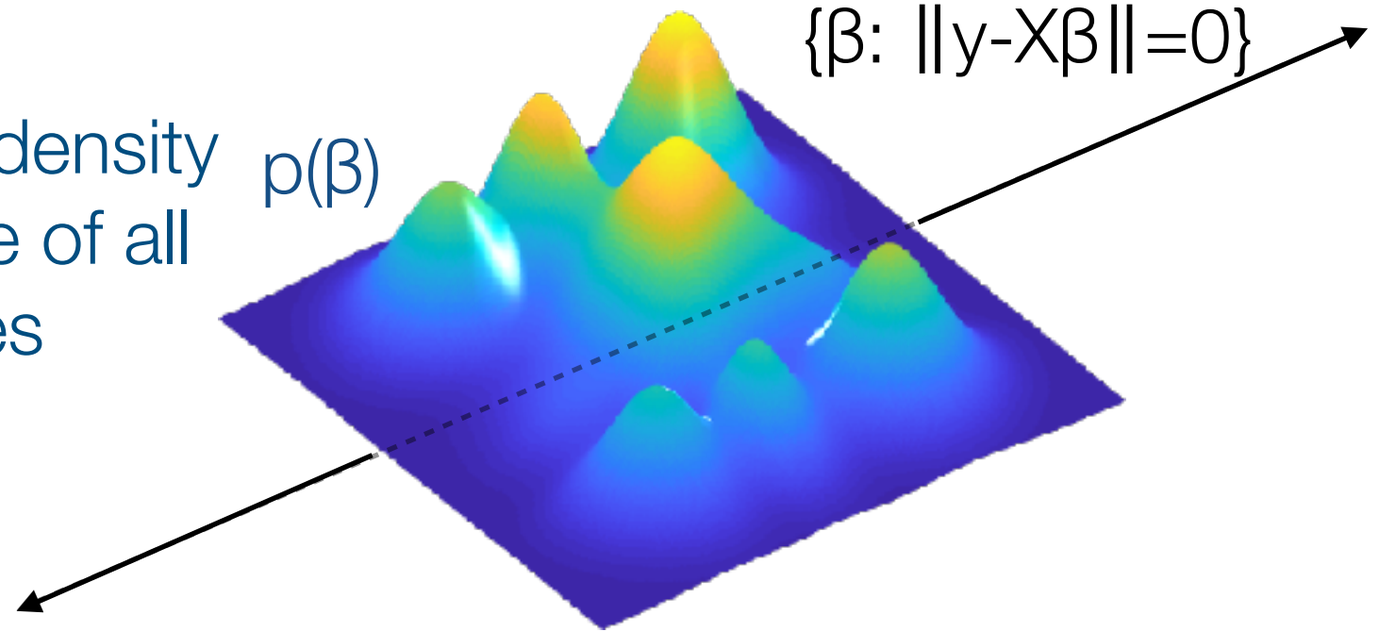


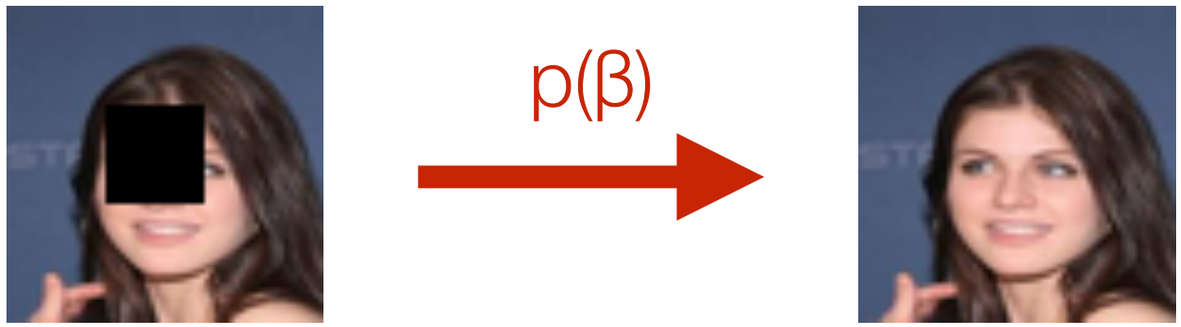
If $\beta \in \mathbb{R}^d$ and $p(\beta) \in \mathcal{B}_\alpha$ (Besov-α smooth functions), then the minimax rate for learning p(β)

$$\min_{\hat{p}} \max_{p \in \mathcal{B}_\alpha} \mathbb{E} \|\hat{p}(\beta) - p(\beta)\|_2 = \mathcal{O}\left(n^{-\frac{\alpha}{2\alpha+d}}\right)$$

No neural network can beat this rate!

Donoho, Johnstone, Kerkyacharian, and Picard 1996

Prior vs. conditional density estimation



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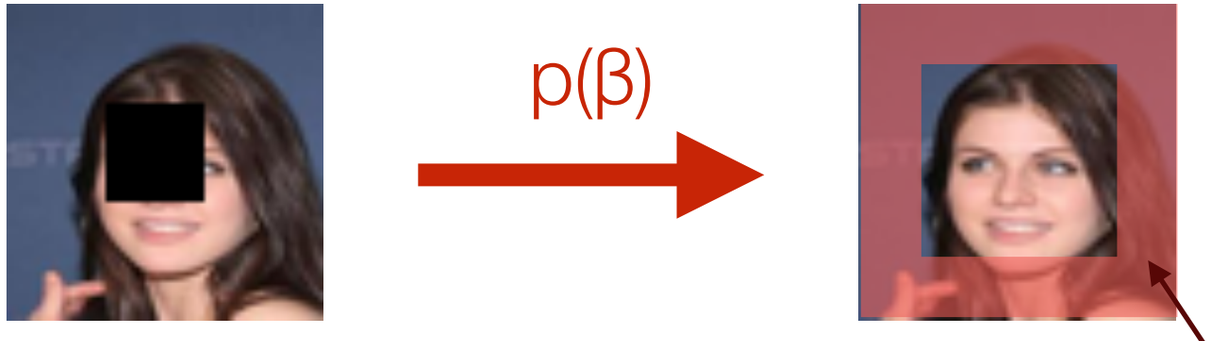
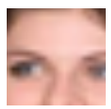


Prior vs. conditional density estimation

p(β)

pixels irrelevant to inpainting



$p(X_{\perp}\beta \mid X\beta)$

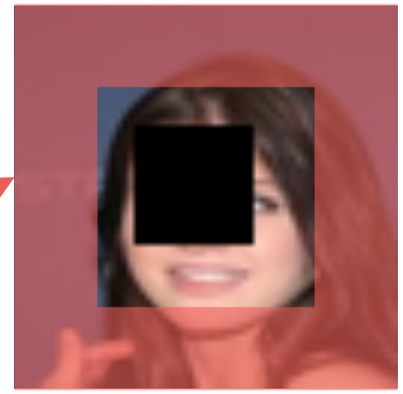
We need conditional density $p(X_{\perp}\beta \mid X\beta)$

Conditional density estimation

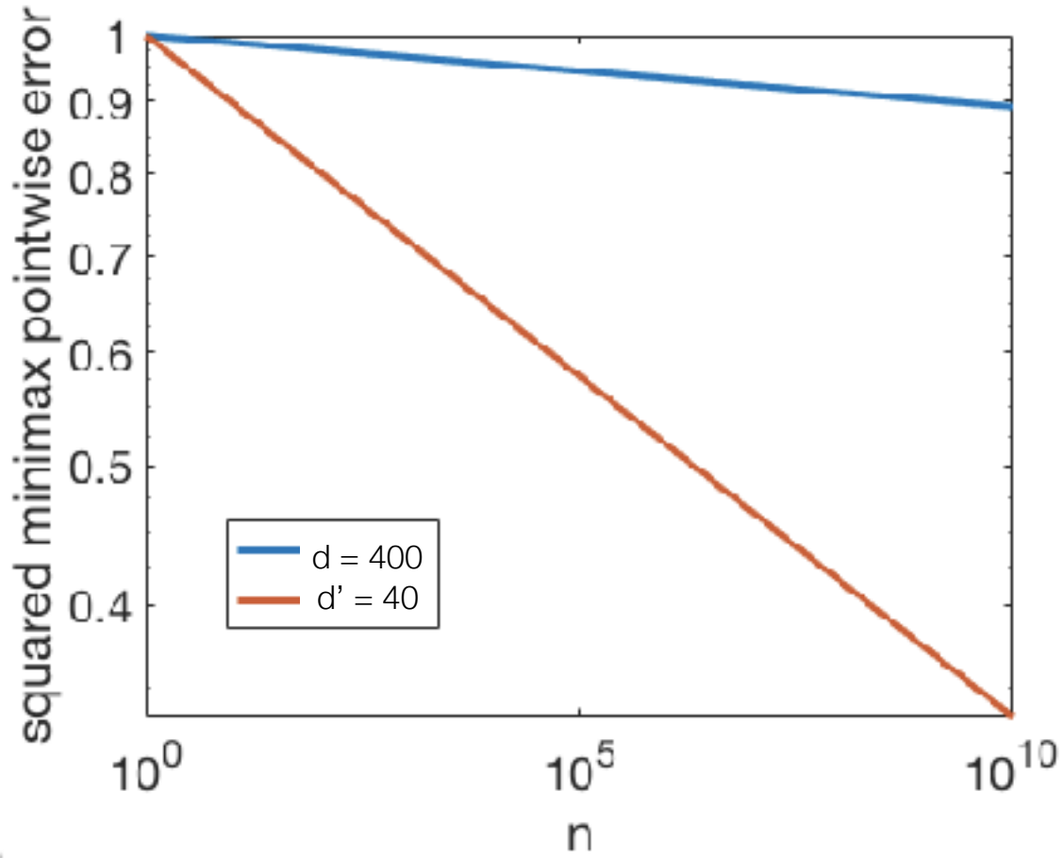
Conditional density [$p(X_\perp\beta \mid X\beta)$] estimation can be much easier than density [$p(\beta)$] estimation

If $X_\perp\beta$ only depends on d' elements in $X\beta$, then the minimax rate is

$$\min_{\hat{p}} \max_{p \in \mathcal{B}_\alpha} \mathbb{E} \| \hat{p}(X_\perp\beta | X\beta) - p(X_\perp\beta | X\beta) \|_2 = \mathcal{O}\left(n^{-\frac{\alpha}{2\alpha+d'}}\right)$$



Pixels irrelevant for inpainting



To reach a target squared pointwise error of ½:

- estimating $p(\beta)$ requires $n \approx 10^{60}$
- estimating $p(X_\perp\beta \mid X\beta)$ requires $n \approx 10^6$

Efromovich 2007
Bertin, Lacour, Rivoirard 2016
Nguyen 2018

Conditional density estimation

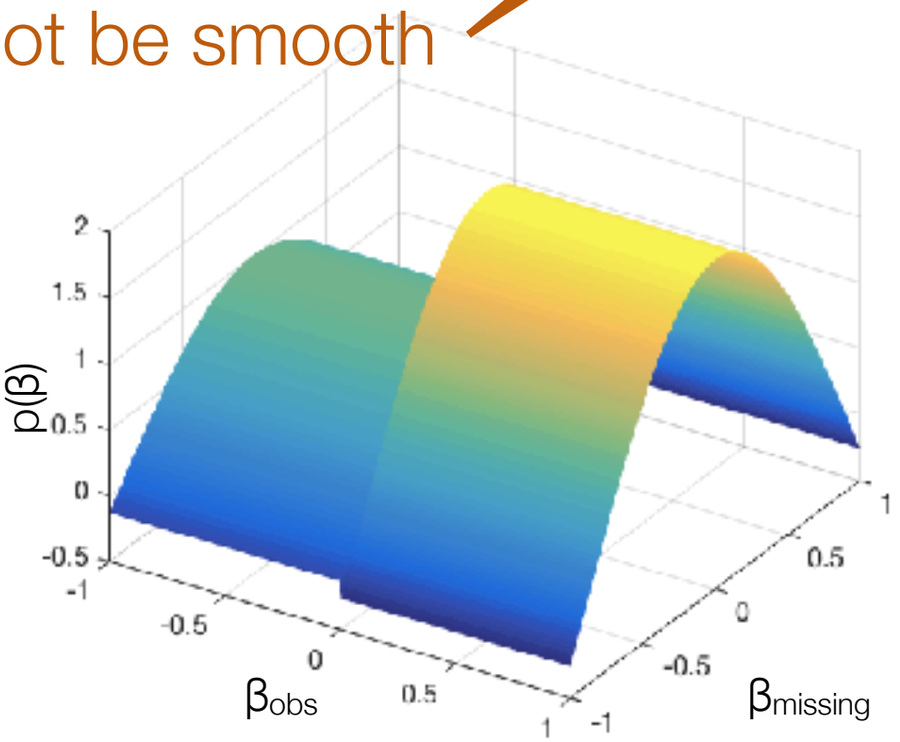
Conditional density [$p(X_{\perp}\beta \mid X\beta)$] estimation can be much easier than density [$p(\beta)$] estimation

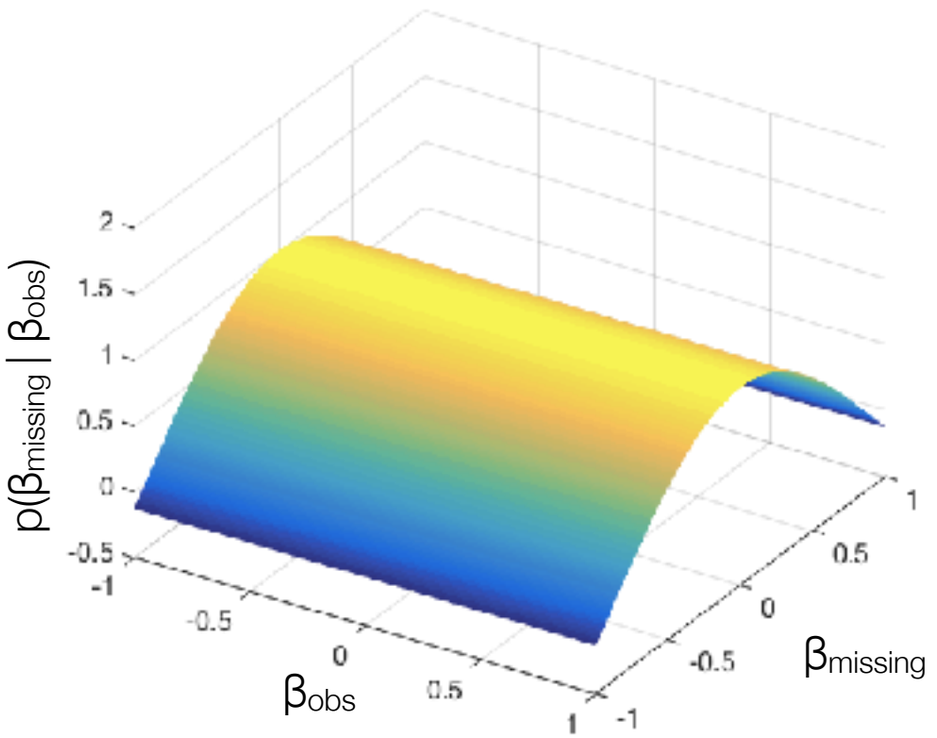
$$p(\beta_{missing}|\beta_{obs}) = \frac{p(\beta_{missing}, \beta_{obs})}{p(\beta_{obs})} = \frac{p(\beta)}{p(\beta_{obs})}$$

Can be smooth

Either may not be smooth

Smoother densities are easier to estimate





Implications for learning to regularize

Estimating conditional density $p(X_{\perp}\beta \mid X\beta)$ can require far fewer samples than estimating full density $p(\beta)$

X should be fully utilized in learning process

“Unrolled” gradient descent

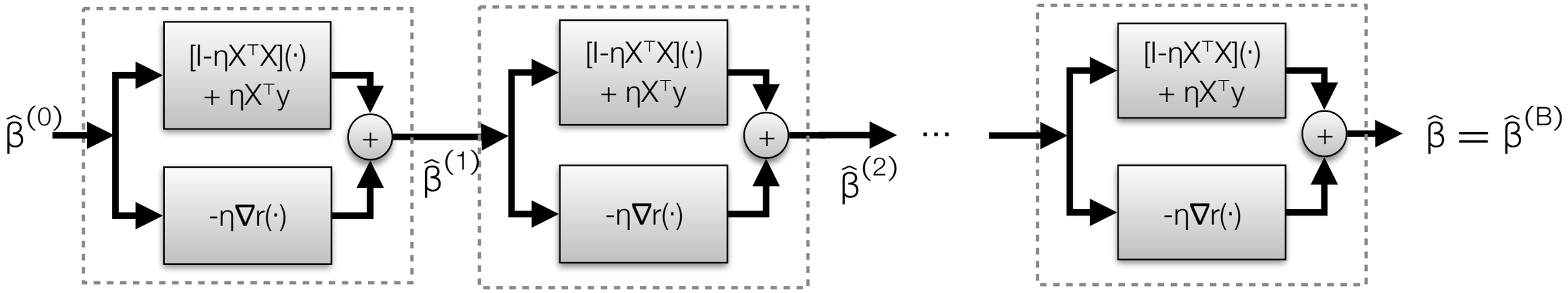
Assume $r(\beta)$ differentiable.

$$\hat{\beta} = \arg\min_{\beta} \|y - X\beta\|_2^2 + r(\beta)$$

set $\hat{\beta}^{(1)}$ and stepsize $\eta > 0$

for $k = 1, 2, \ldots$

$$\hat{\beta}^{(k+1)} = \hat{\beta}^{(k)} + \eta X^T(y - X\hat{\beta}^{(k)}) + \eta \nabla r(\hat{\beta}^{(k)})$$



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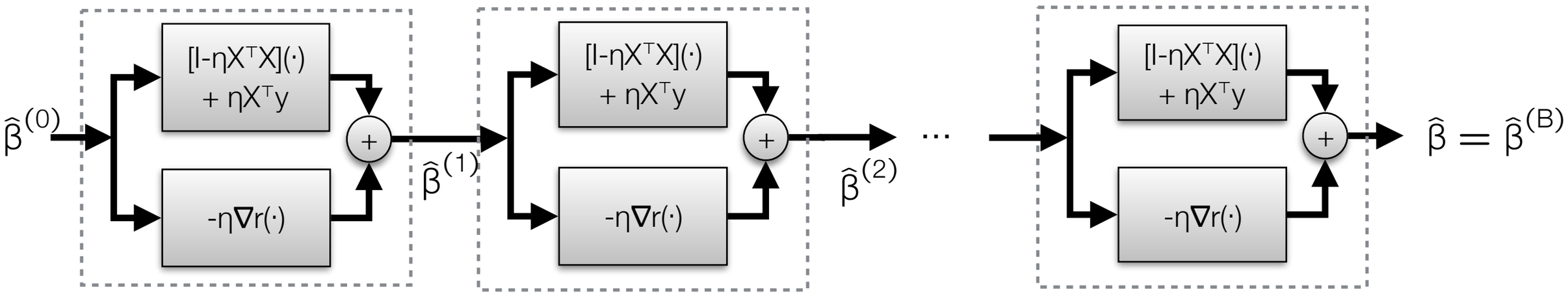
Replace with learned neural network



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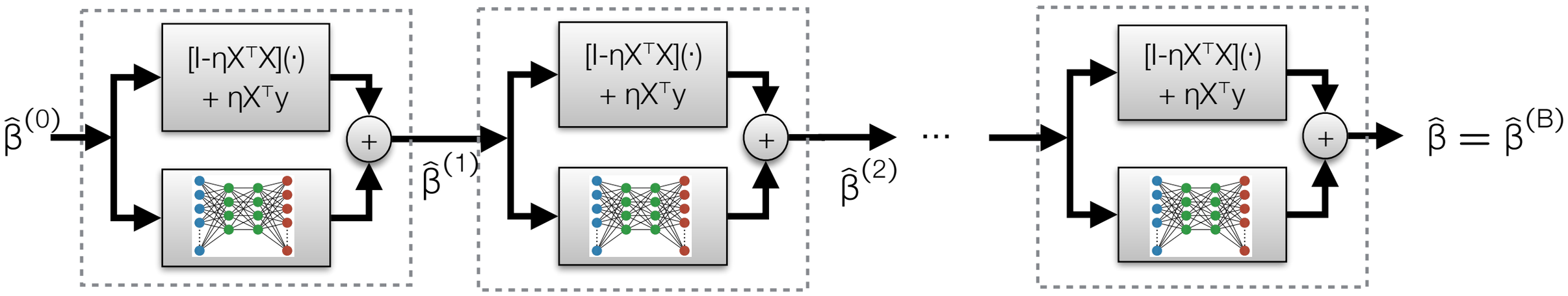

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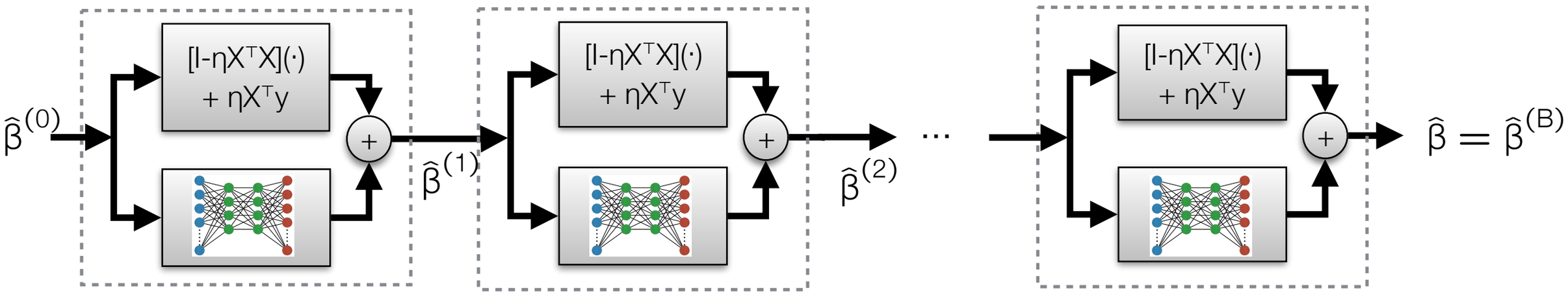
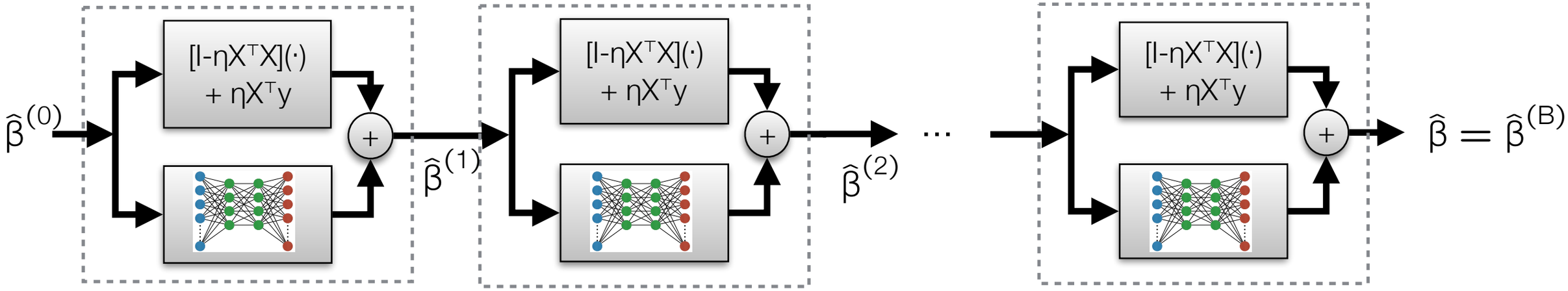


“Unrolled” optimization framework **trained end-to-end**

Beyond optimization

Unrolled methods so far originated in optimization — underlying theory does not apply here!



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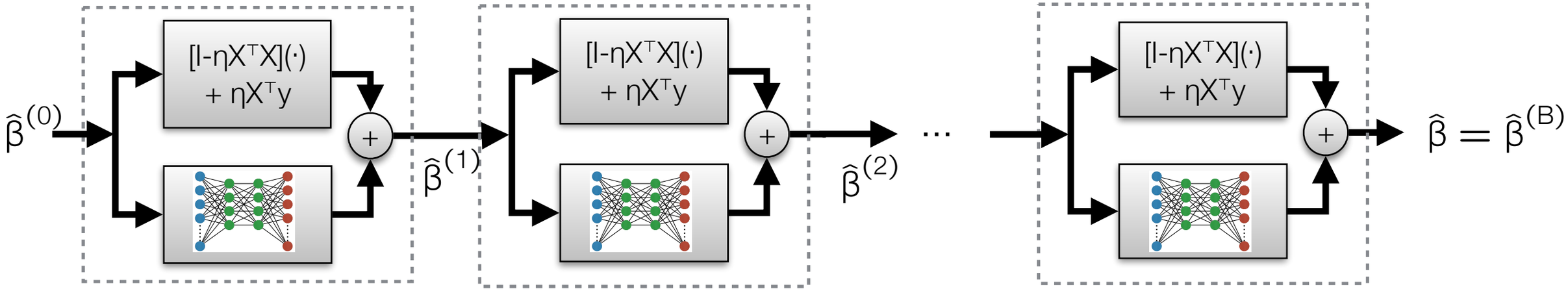
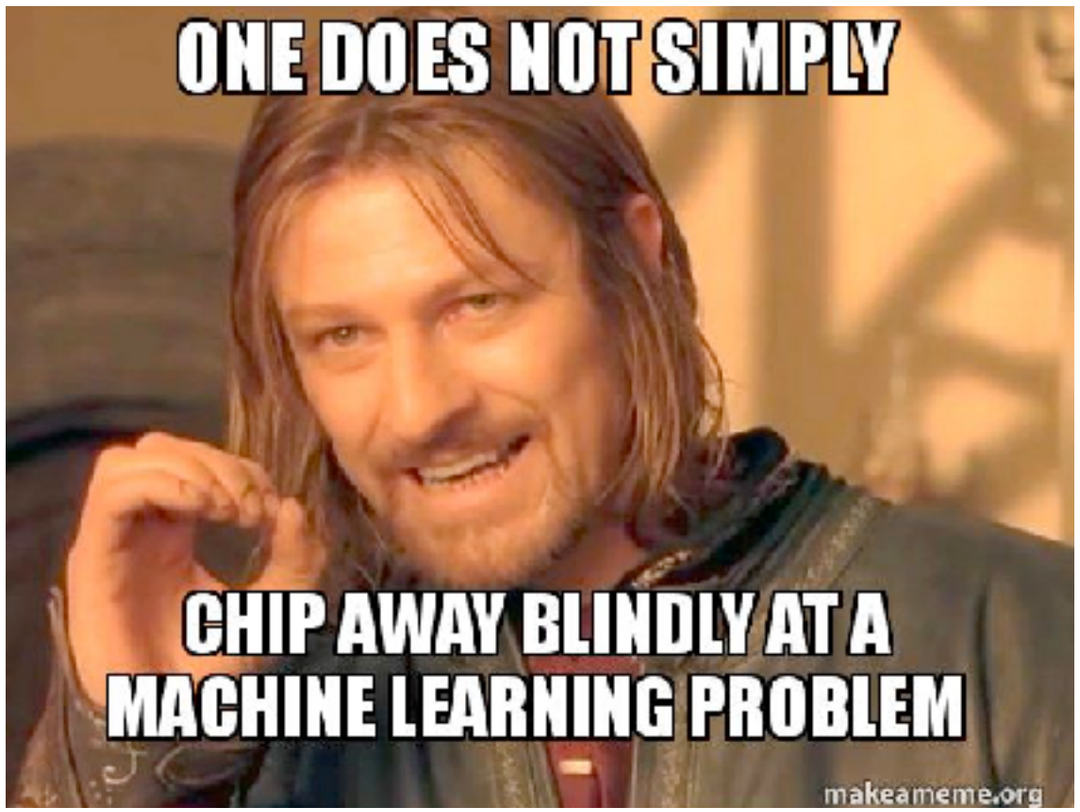




Neumann series

Assume r(β) differentiable.

$$\begin{aligned}\hat{\beta} &= \arg\min_{\beta} ||y - X\beta||_2^2 + r(\beta) \\ &= (X^\top X + \nabla r)^{-1} X^\top y \end{aligned} \tag{1}$$

Let A be a linear operator. Then the Neumann series is

$$(I - A)^{-1} = \sum_{k=0}^{\infty} A^k = I + A + A^2 + A^3 + \cdots \tag{2}$$

If A is contractive, we know higher-order terms are smaller.

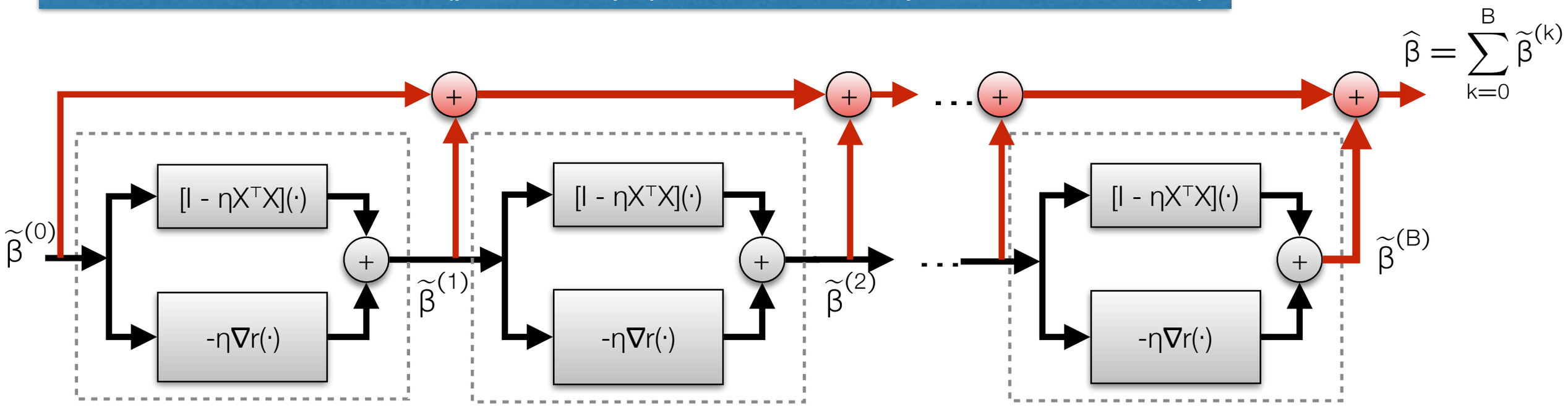
Can we estimate β by approximating (1) using (2)?
(e.g. $A = I - X^\top X + \nabla r$ if r is linear)

Neumann networks

Assume $r(\beta)$ differentiable.

$$\begin{aligned}\hat{\beta} &= \arg\min_{\beta} \|y - X\beta\|_2^2 + r(\beta) \\ &= (X^\top X + \nabla r)^{-1} X^\top y \\ &\approx \sum_{k=1}^{B} (I - \eta X^\top X - \eta \nabla r)^k \eta X^\top y\end{aligned}$$

Neumann network (parallel pipelines + skip connections):



Neumann networks

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$$= (X^\top X + \nabla r)^{-1} X^\top y$$

$$\approx \sum_{k=1}^{B} (I - \eta X^\top X - \boxed{\eta \nabla r})^k \eta X^\top y$$

Replace with learned neural network

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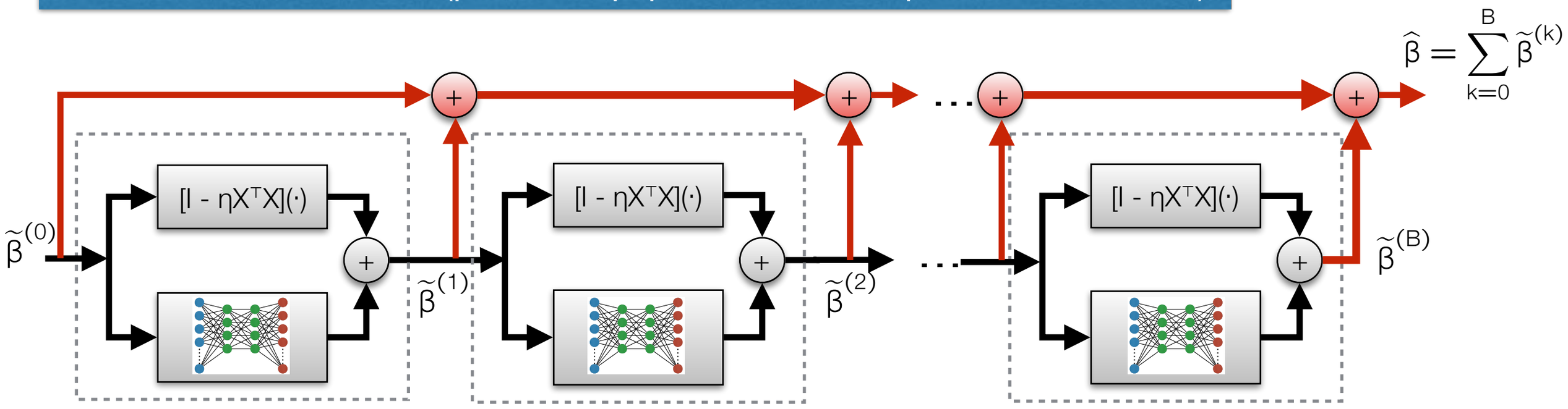
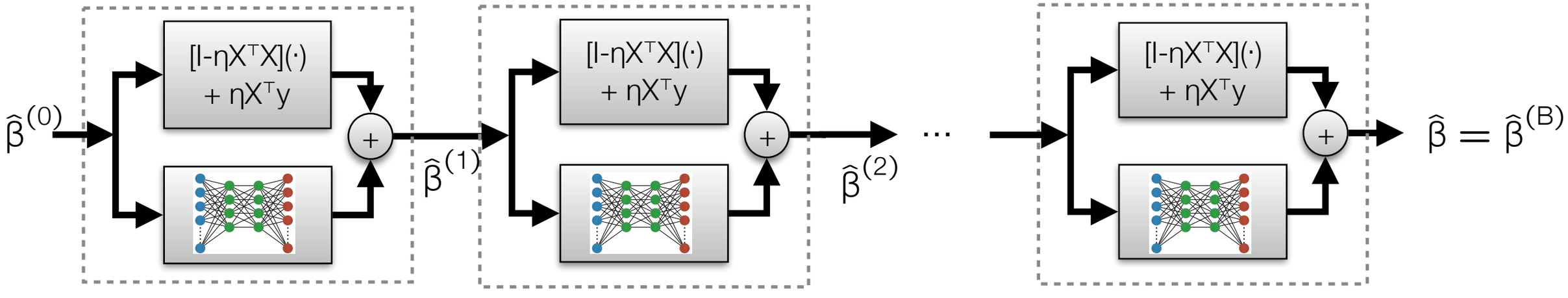
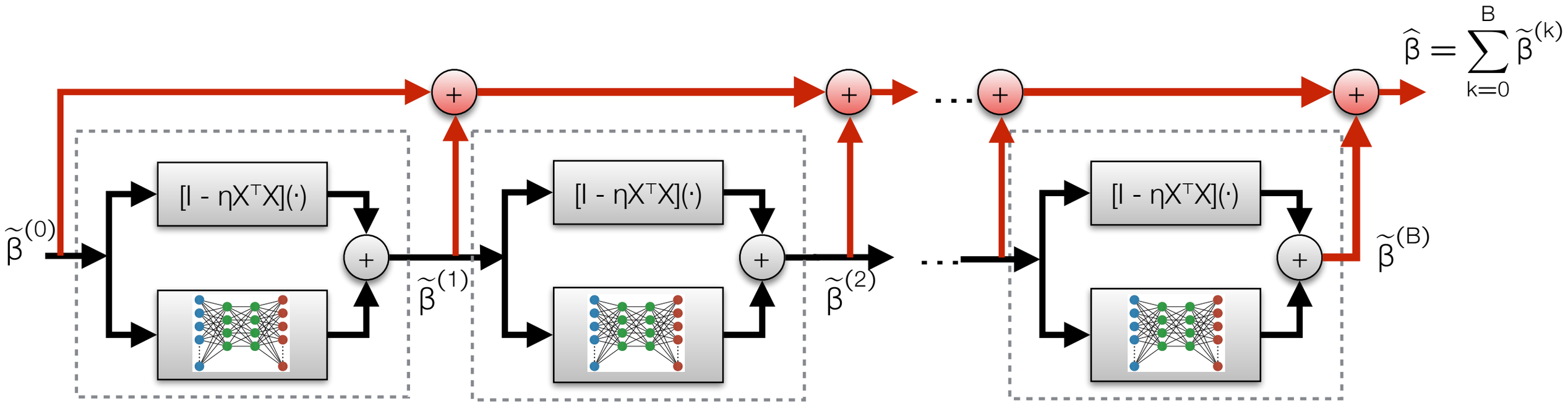


Comparison

Gradient descent network

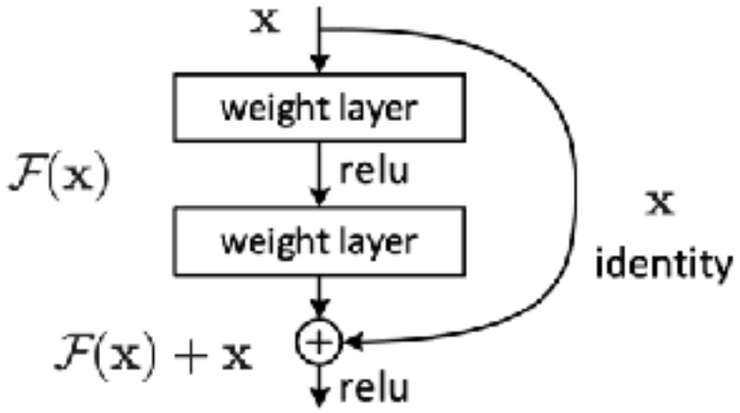


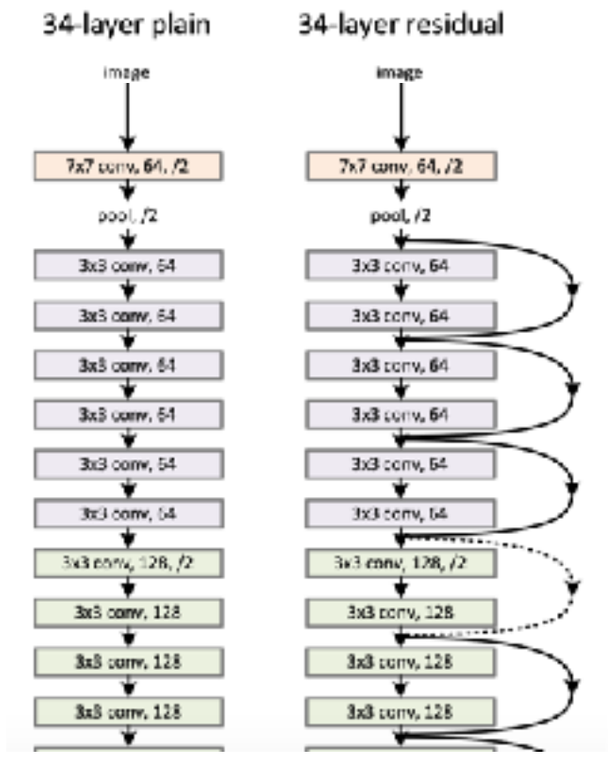
Neumann network (parallel pipelines + skip connections):



Skip connections in ResNets and DenseNets

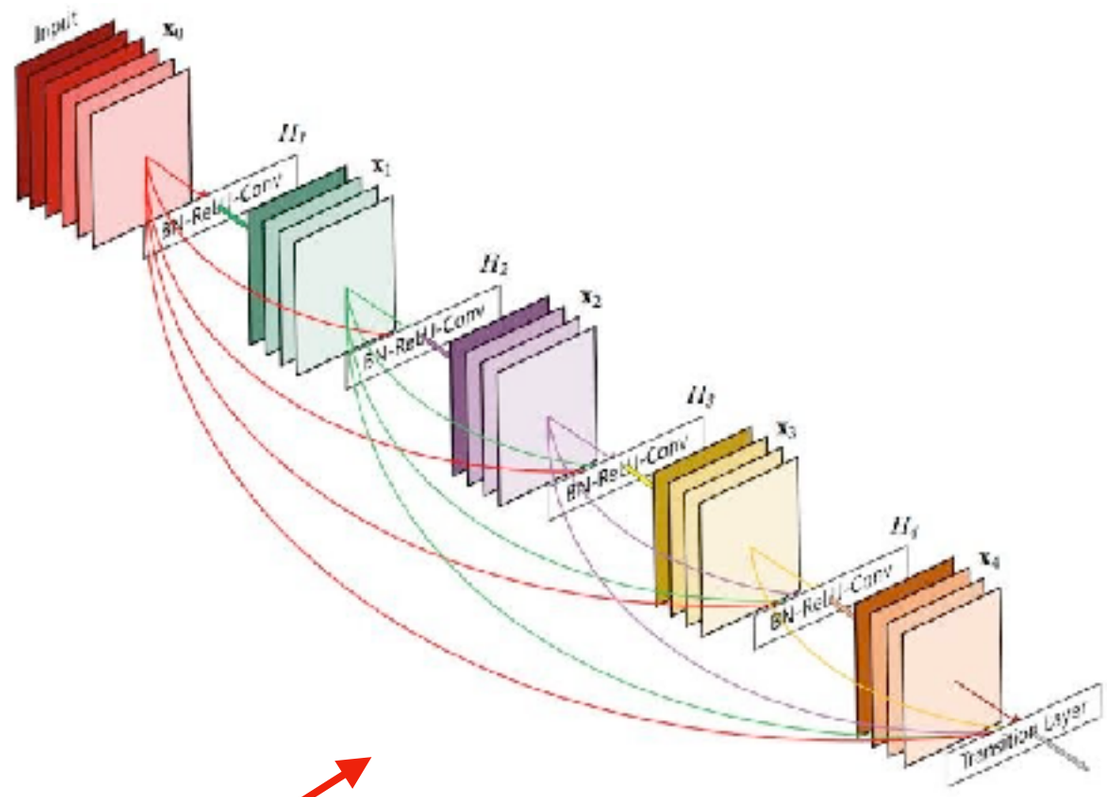
Residual Networks (ResNets)





He, Zhang, Ren, Sun 2015

Dense Convolutional Networks (DenseNets)



Huang, Liu, Van Der Maaten, & Weinberger 2017

skip connections

Experiments

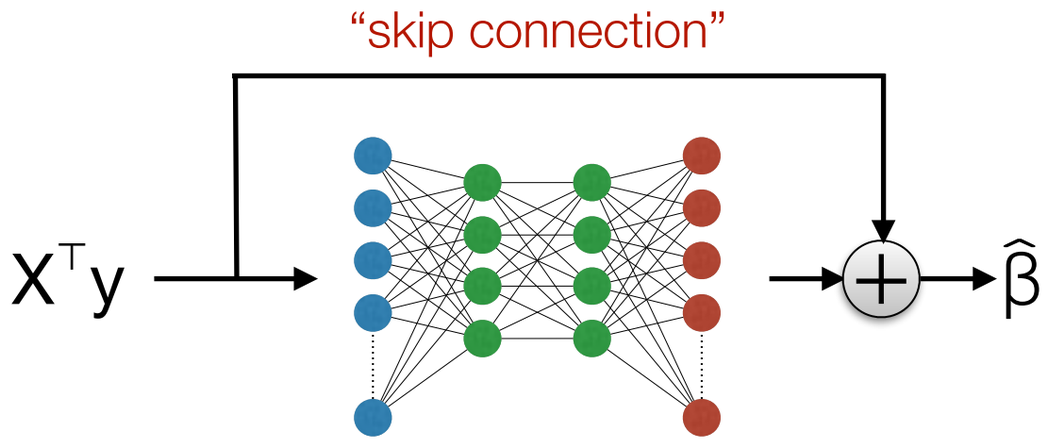
Comparison Methods

LASSO

$$\hat{\beta} = \arg\min_{\beta} \|y - X\beta\|_2^2 + \lambda \|\mathrm{DCT}(\beta)\|_1$$

discrete cosine transform
on 16x16 blocks

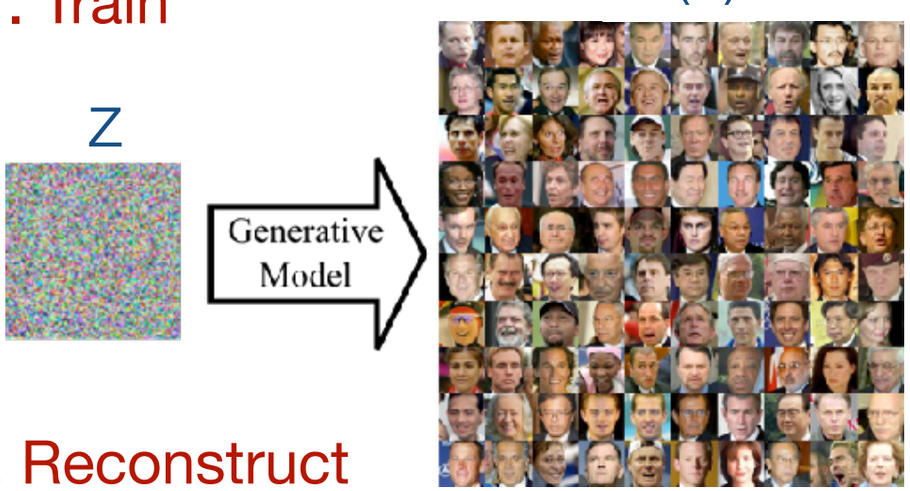
Residual Autoencoder



Mao, Shen, Yang, 2016

Design-agnostic GAN

1. Train

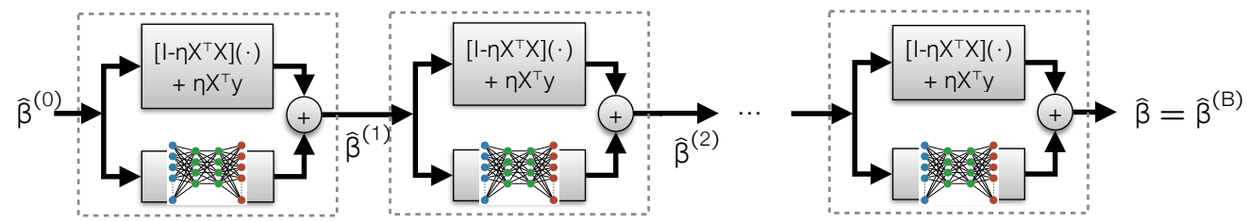


2. Reconstruct

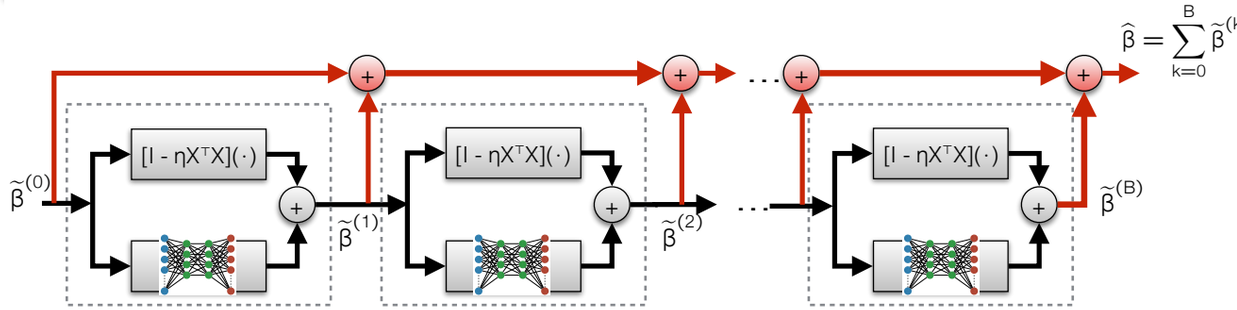
$$\hat{\beta} = \arg\min_{\beta \in \mathrm{range}(G)} \|y - X\beta\|_2^2$$

Bora, Jalal, Price, Dimakis, 2017

Unrolled Gradient Descent

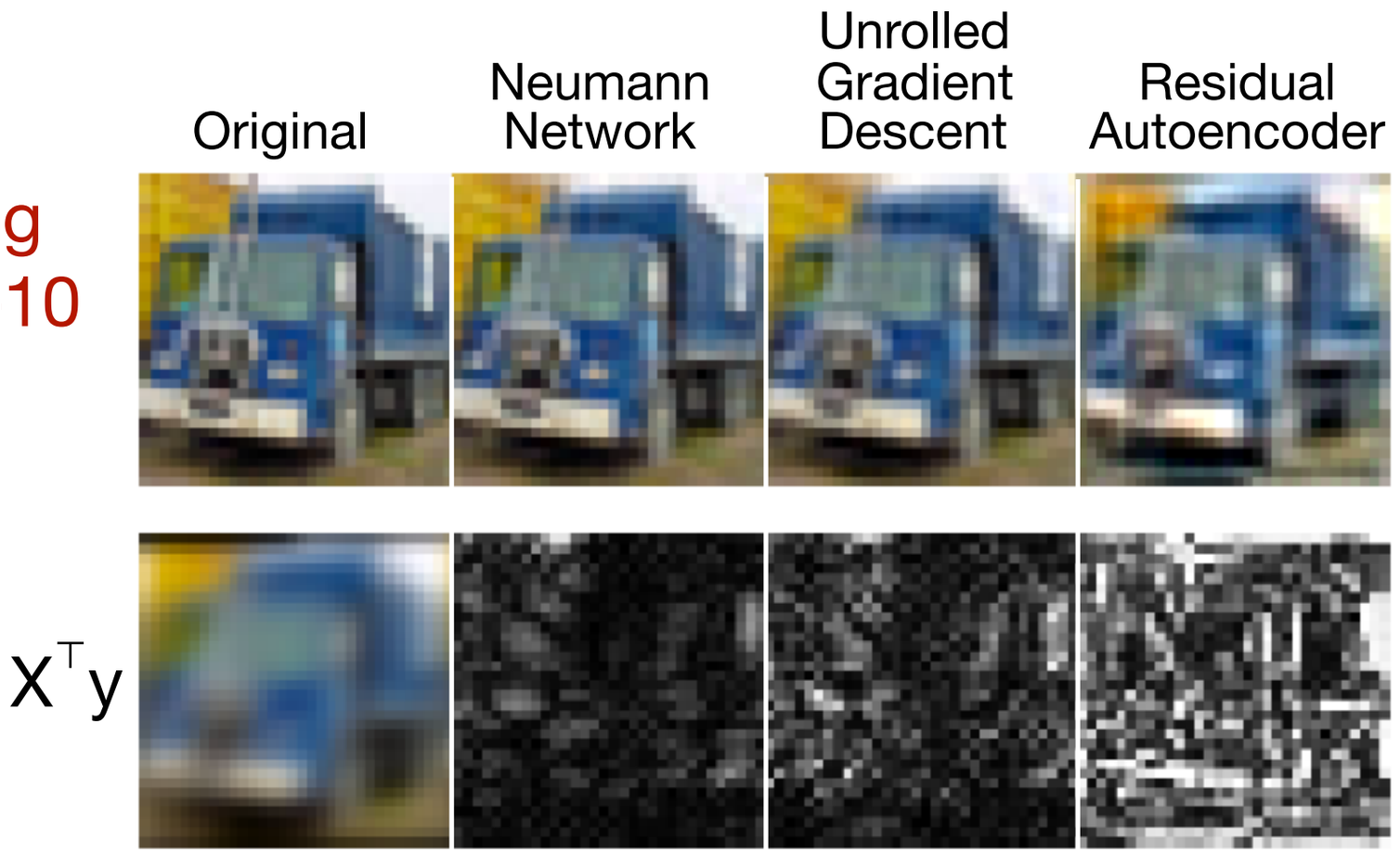


Neumann Network

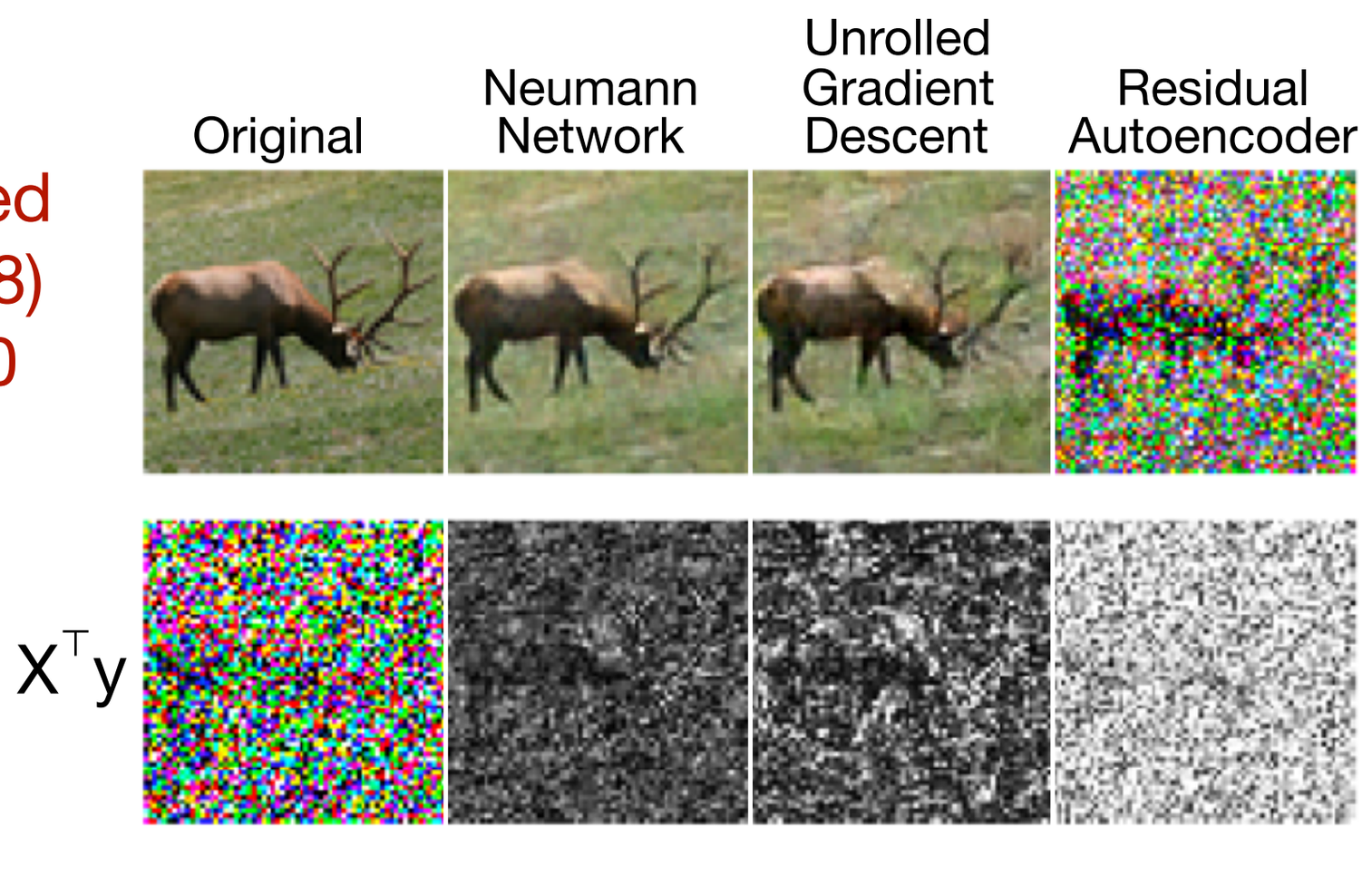


Examples

Deblurring on CIFAR-10



Compressed Sensing (x8) on STL-10

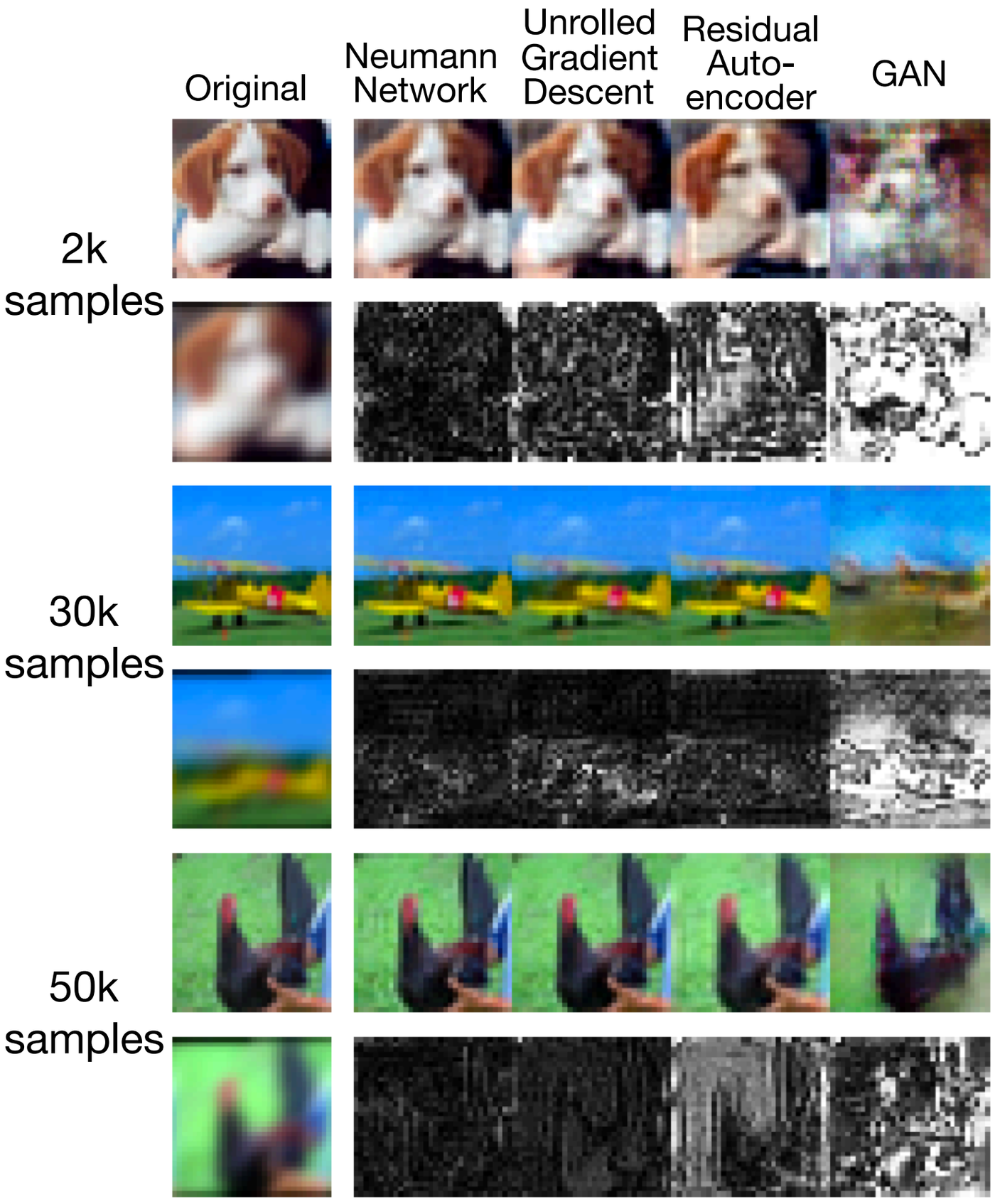


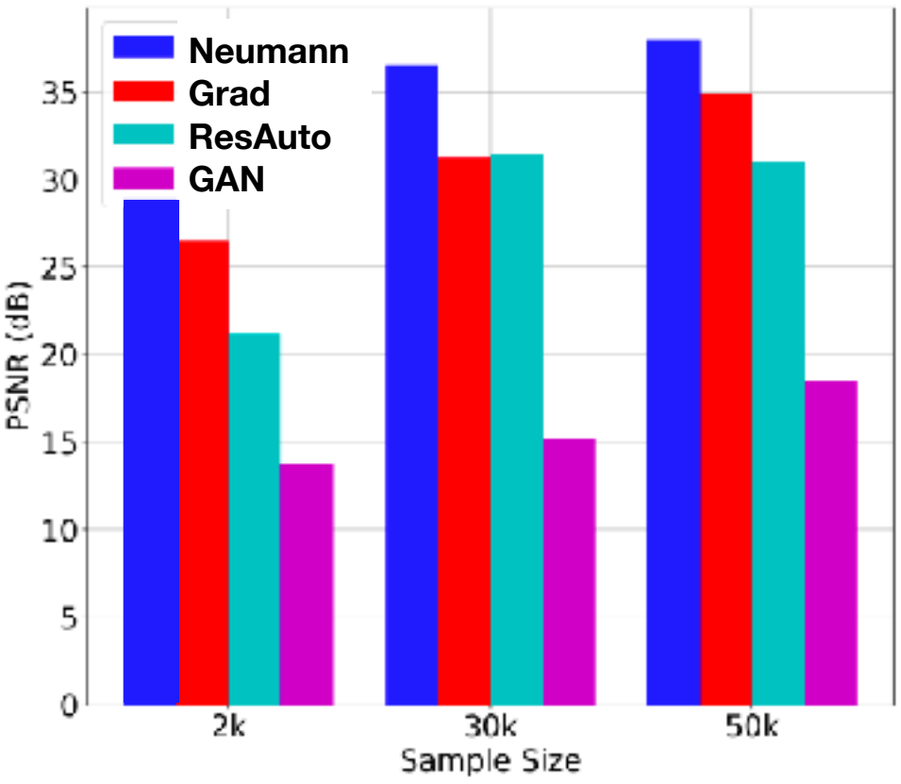
Summary of Results

		Inpaint	Deblur	CS2	CS8	SR4	SR10
CIFAR10	NN	28.20	**36.55**	33.83	**25.15**	24.48	**23.09**
	GDN	27.76	31.25	**34.99**	25.00	24.49	20.47
	ResAuto	**29.05**	31.04	18.51	9.29	**24.84**	21.92
	CSGM	17.88	15.20	17.99	19.33	16.87	16.66
	LASSO	19.34	23.70	22.74	16.37	20.03	19.93
CelebA	NN	**31.06**	**31.01**	**35.12**	**28.38**	**27.31**	23.57
	GDN	30.99	30.19	34.93	28.33	27.14	23.46
	ResAuto	29.66	25.65	19.41	9.16	25.62	**24.92**
	CSGM	17.75	15.68	17.99	18.21	18.11	17.88
	LASSO	15.99	14.82	24.37	17.61	16.56	22.74
STL10	NN	27.47	29.43	**31.98**	**26.65**	**24.88**	**21.80**
	GDN	**28.07**	**30.19**	31.11	26.19	**24.88**	21.46
	ResAuto	27.28	25.42	19.48	9.30	24.12	21.13
	CSGM	16.50	14.04	16.67	16.39	16.58	16.47
	LASSO	18.70	16.54	23.14	17.46	18.79	21.36

Table 1: PSNR comparison for the CIFAR, CelebA, and STL10 datasets respectively. Values reported are the median across a test set of size 256.

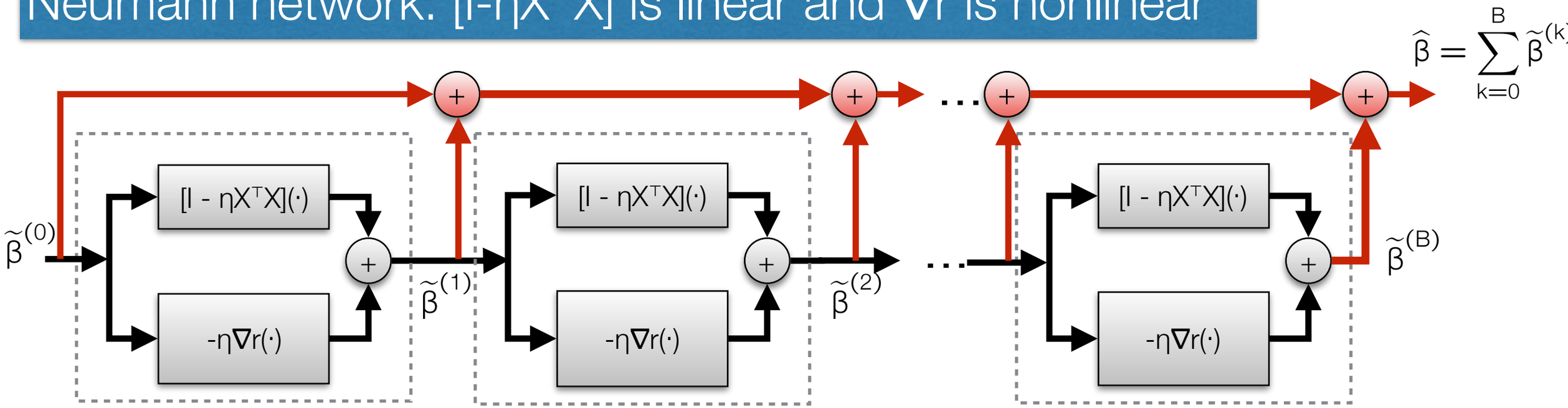
Sample Complexity





Preconditioning

Neumann network: [I-ηX^{T}X] is linear and ∇r is nonlinear

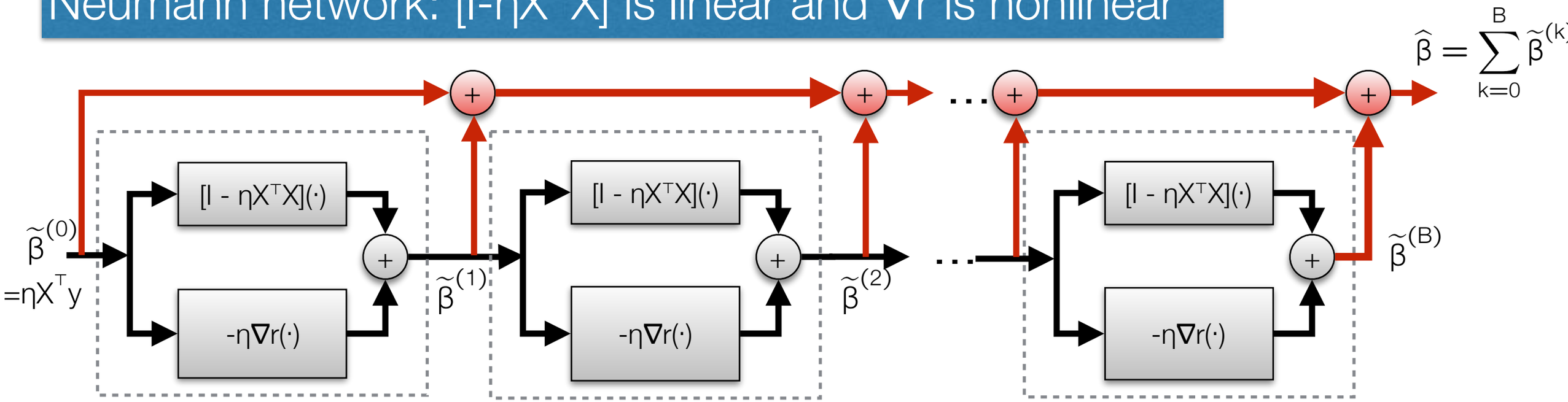


Preconditioned Neumann network:

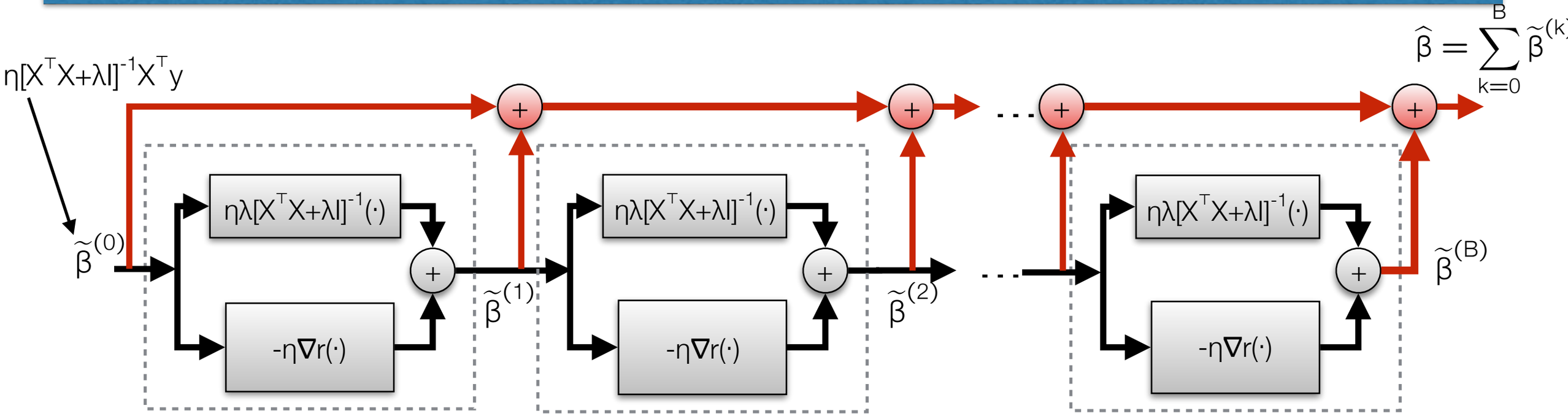
Instead of inputting $X^T y$, input Tikhinov estimate $(X^T X+\lambda I)^{-1} X^T y$ and adjust top blocks based on Neumann series expansion.

Preconditioning

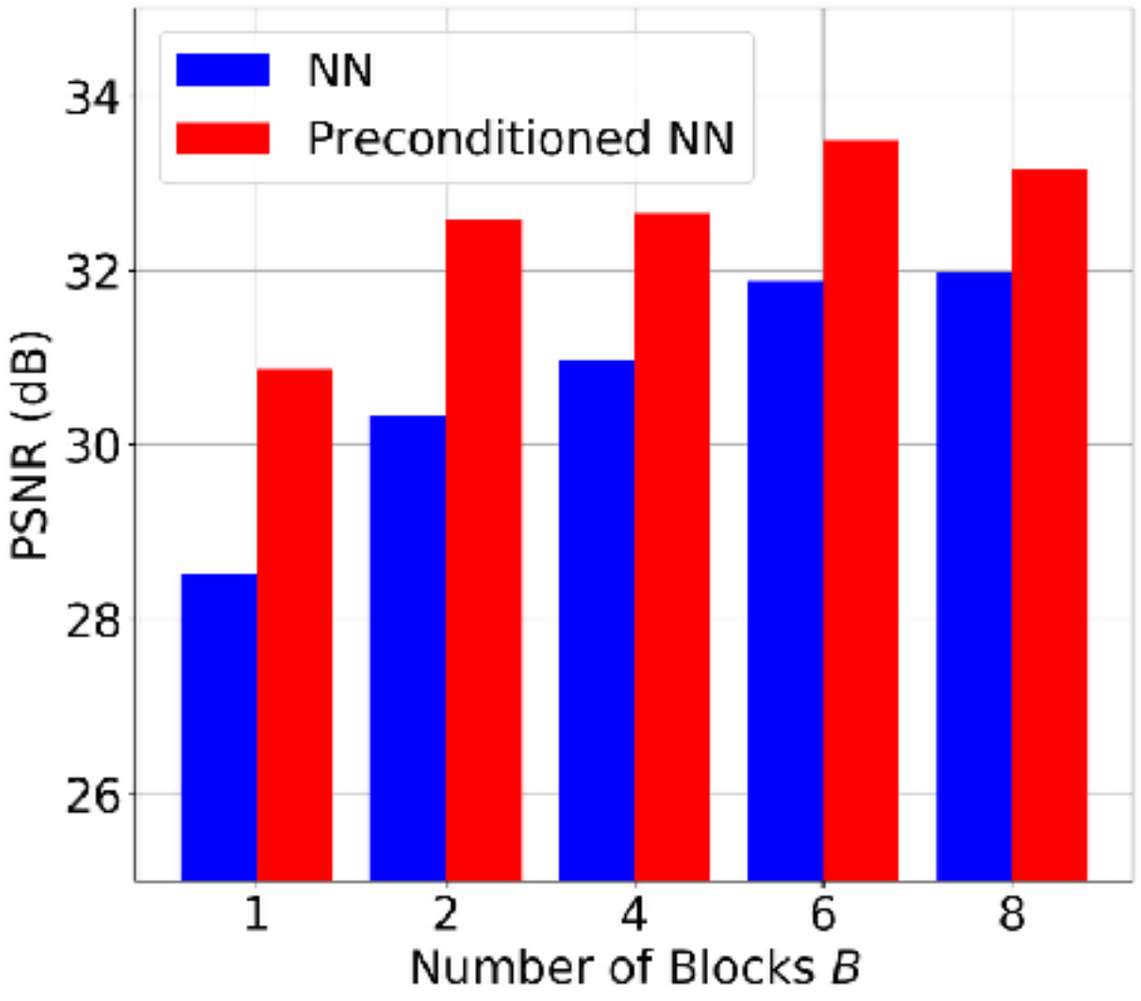
Neumann network: [I-ηX^{T}X] is linear and ∇r is nonlinear

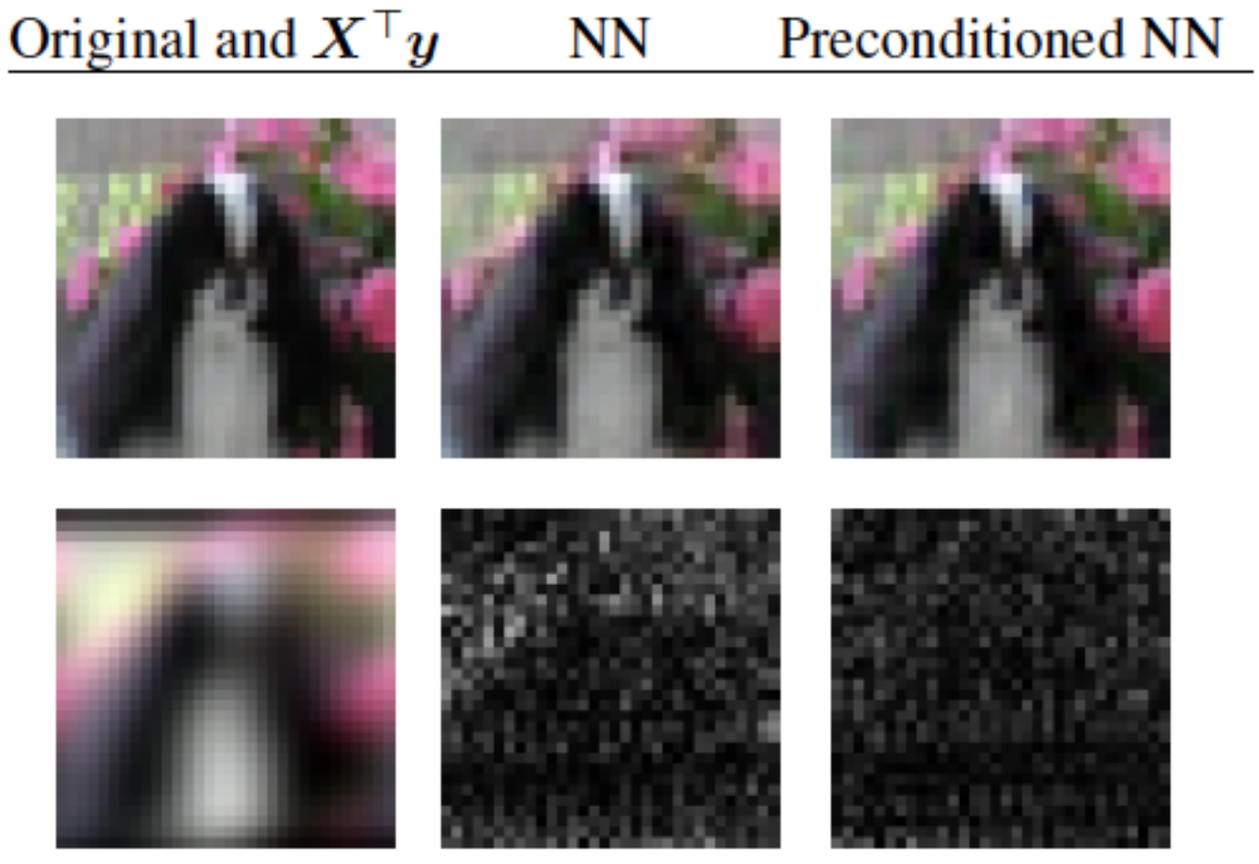


Preconditioned Neumann net: ηλ[I+λX^{T}X]$^{-1}$ is linear and ∇r nonlinear



Preconditioning





Theory

Neumann series for nonlinear operators?

If A is a *nonlinear* operator, Neumann series identity does not hold:

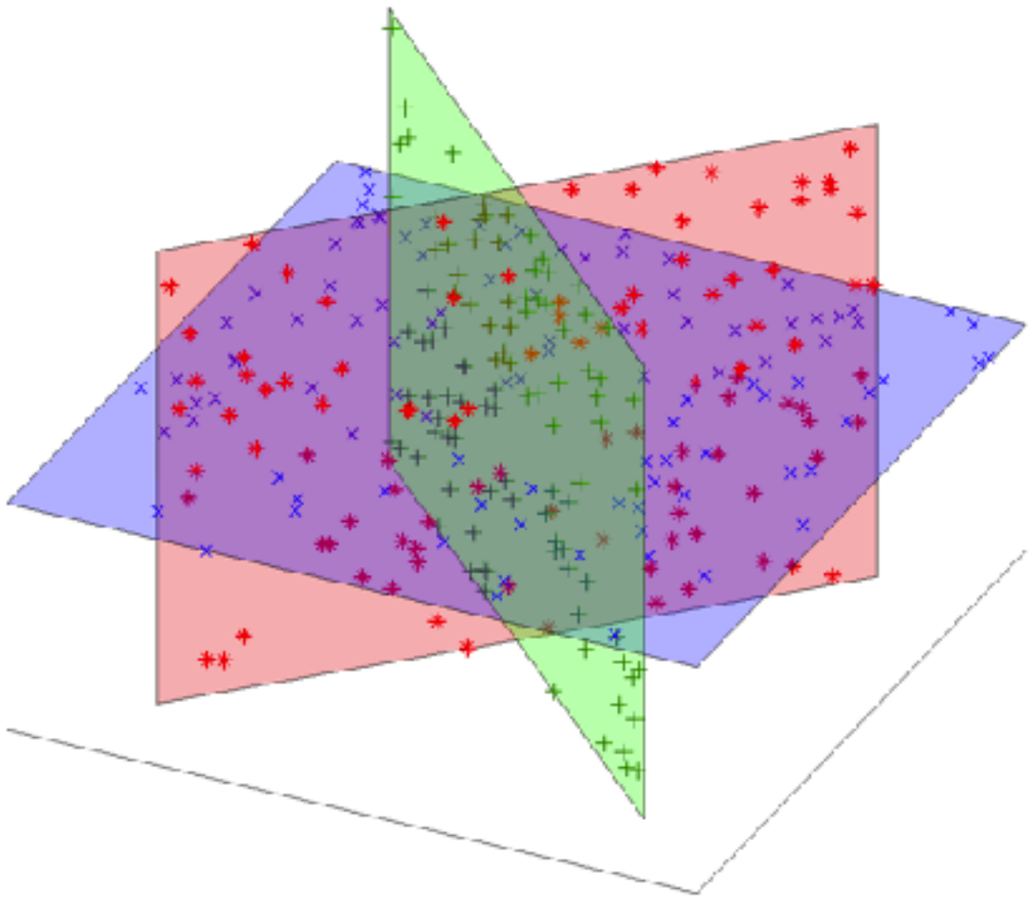
$$(I - A)^{-1} \neq \sum_{k=0}^{\infty} A^k$$

In our case, $A = I - \eta X^T X - \eta R$, where $R = \nabla r$ may be nonlinear

Can we justify Neumann net as an estimator beyond the linear setting?

Case Study: Union of Subspaces Models

Model images as belonging to a union of low-dimensional subspaces



Case Study: Union of Subspaces Models

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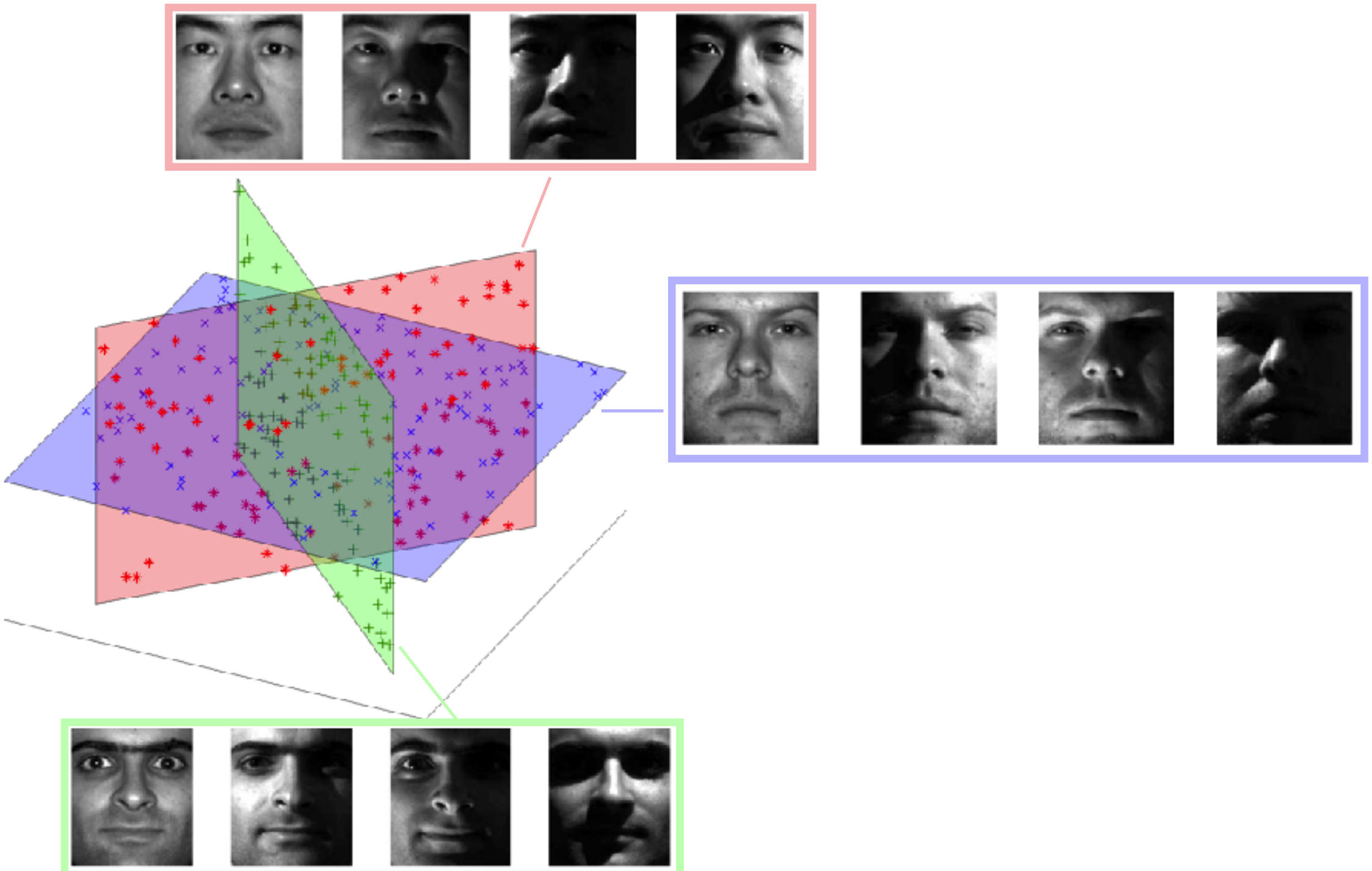
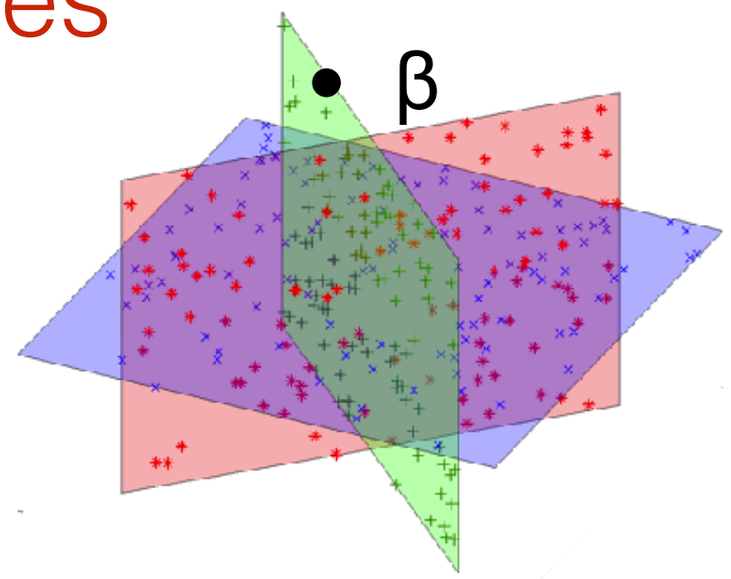


Images from: Extended Yale B dataset
& http://dhpark22.github.io/greedysc.html

Neumann nets and union of subspaces



For simplicity, assume:

- X has orthonormal rows
- measurements are noise-free: $y = X\beta \in \mathbb{R}^m$
- maximum subspace dimension < m/2
- the union of subspaces is “generic”

Lemma:

- Optimal “oracle” regularizer gradient R is piecewise linear in β

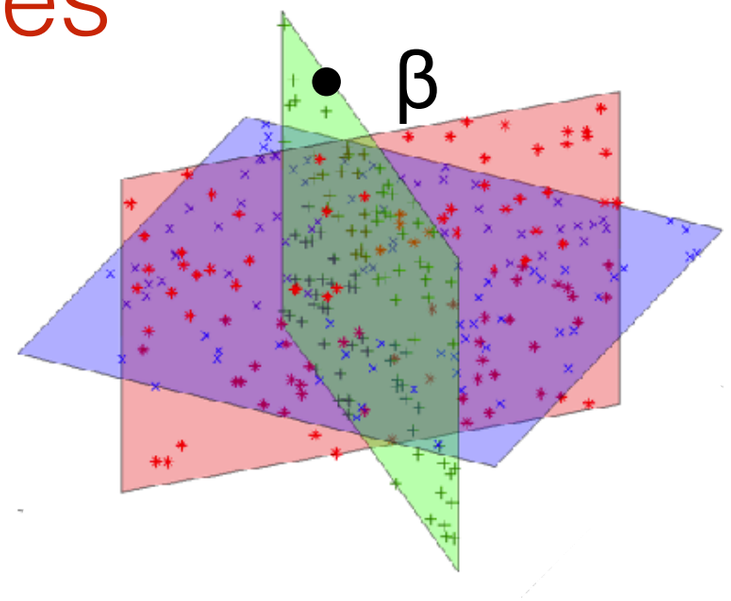
$$R^*(\beta) = \begin{cases} R_1\beta & \text{if } \beta \in S_1 \\ \vdots & \vdots \\ R_K\beta & \text{if } \beta \in S_K \end{cases}$$

S_k = set of points closer to subspace k than any other subspace

- Neumann network with ReLU activations can closely approximate this
- Outputs of all Neumann net blocks are in the same S_k for some k
  ⇒ for a fixed input, R behaves linearly

  ⇒ Neumann series foundation is justifiable and accurate

Neumann nets and union of subspaces



For simplicity, assume:

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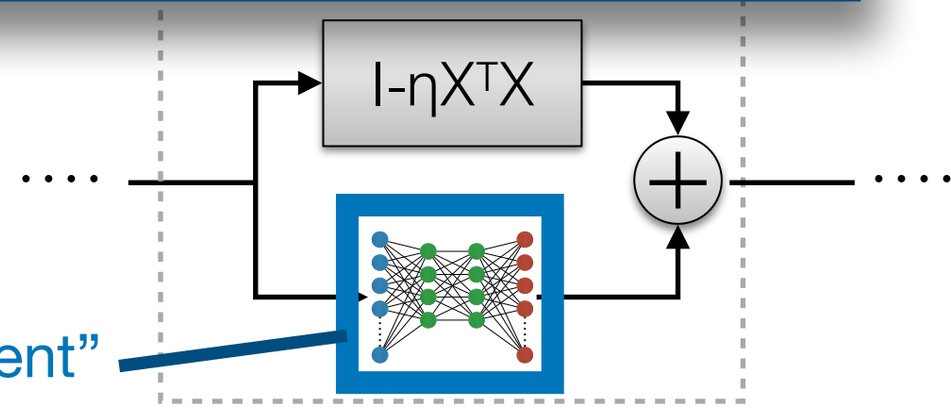
- maximum subspace dimension < m/2
- the union of subspaces is “generic”

Theorem (informal):

For a given step size $0 < \eta < 1$ and number of blocks B there exists a Neumann network estimator $\hat{\beta}(X\beta)$ with a piecewise linear learned component such that

$$\|\hat{\beta}(X\beta) - \beta\| \leq (1 - \eta)^{B+1} \|X\beta\|$$

for all β in the union of subspaces.



Neumann nets and union of subspaces

For simplicity, assume:

- X has orthonormal rows
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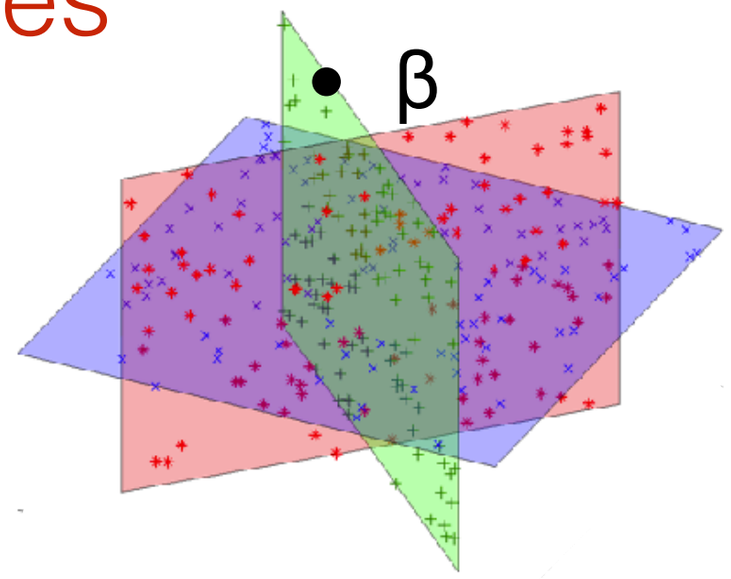

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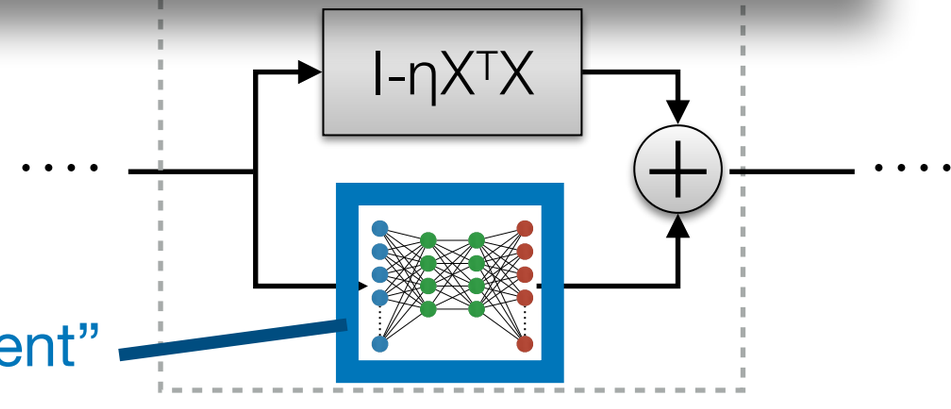
for all β in the union of subspaces.

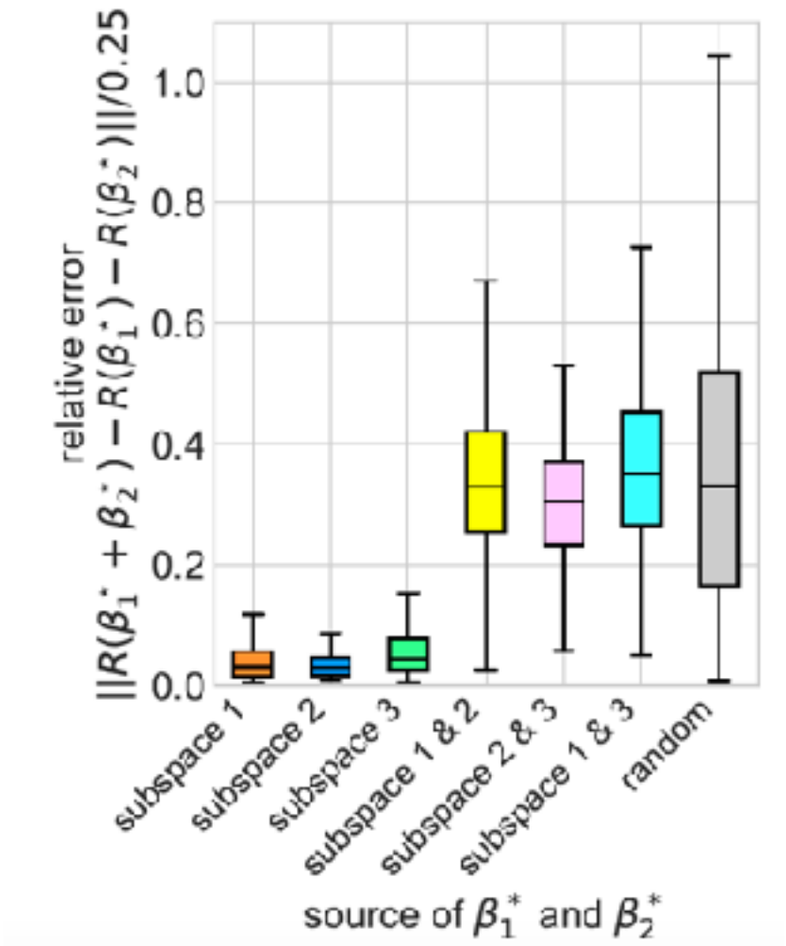
arbitrarily small reconstruction error



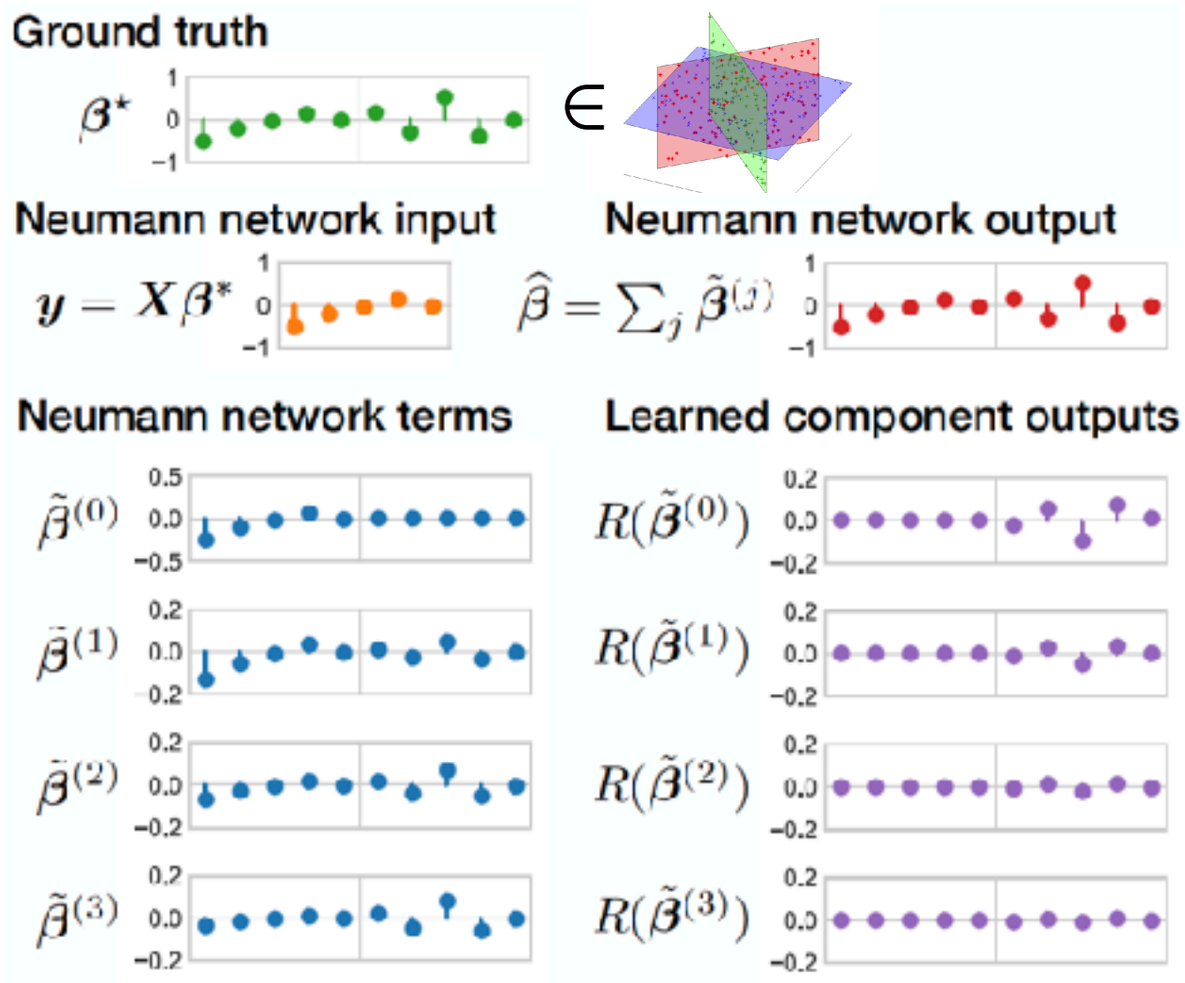
Empirical support for theory

Experiments on synthetic data show that when R is a deep ReLU network, the trained R behaves as the predicted R*

Test of Piecewise Linearity of R



R reflects union of subspaces structure



Outputs of all blocks in same subspace

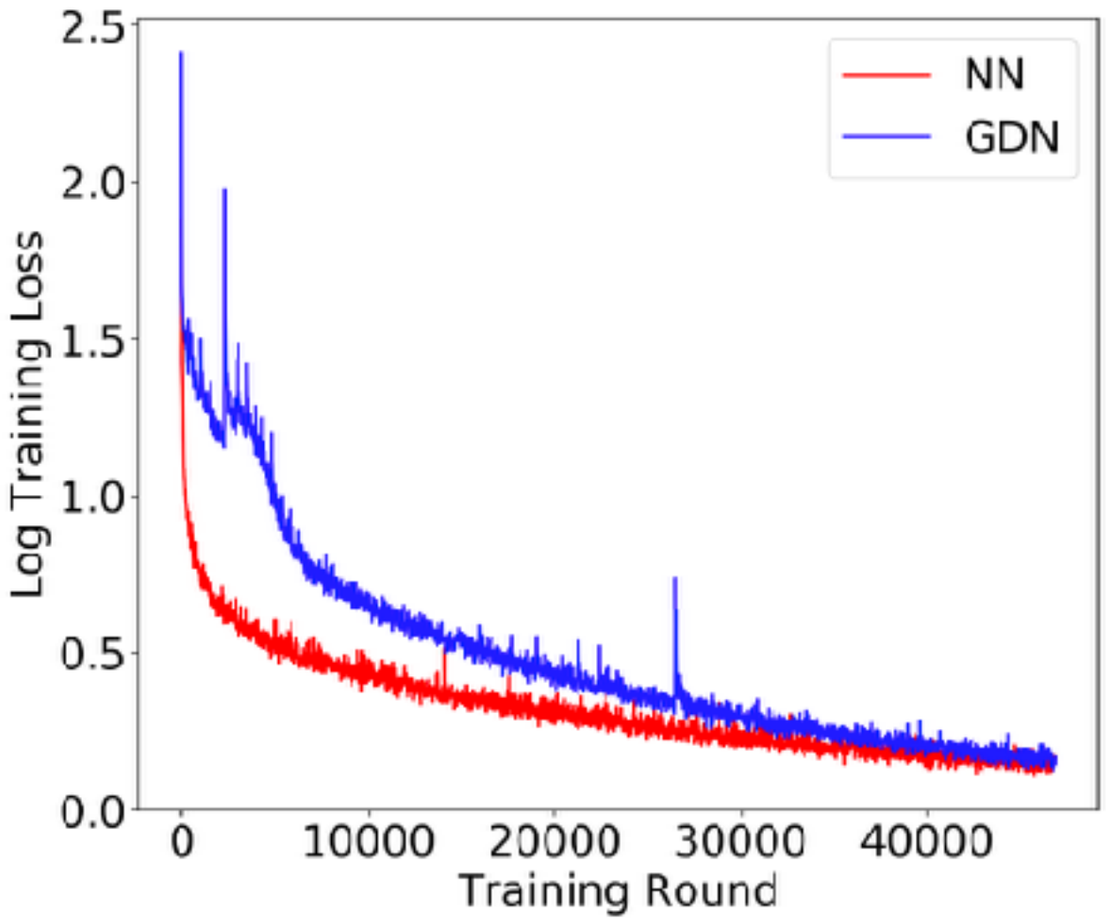
R only affects β in X's null space

Why do Neumann nets give a performance boost?

Hypothesis: friendlier optimization landscape

In our experience, gradient descent networks tended to be more sensitive to initialization and step size tuning.

Training curves for Neumann Network (NN)
and Unrolled Gradient Descent (GDN)
on CIFAR10 Deblurring



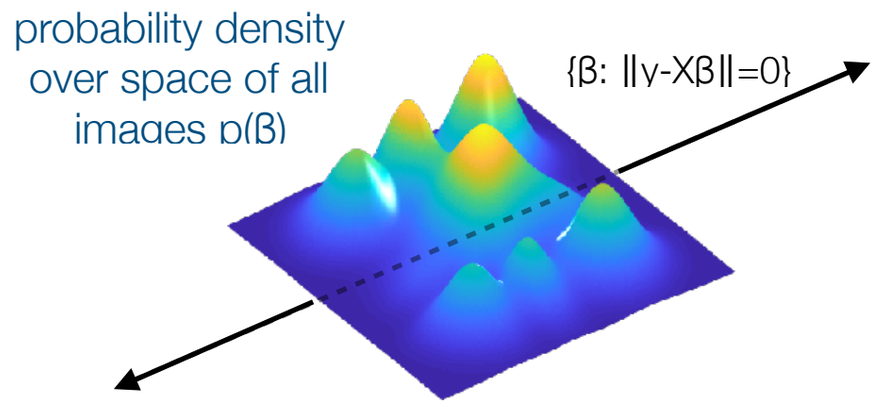
Conclusions

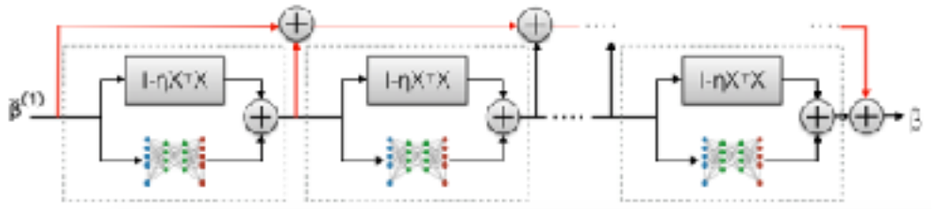
- Explicitly accounting for design (X) during training can dramatically reduce sample complexity.

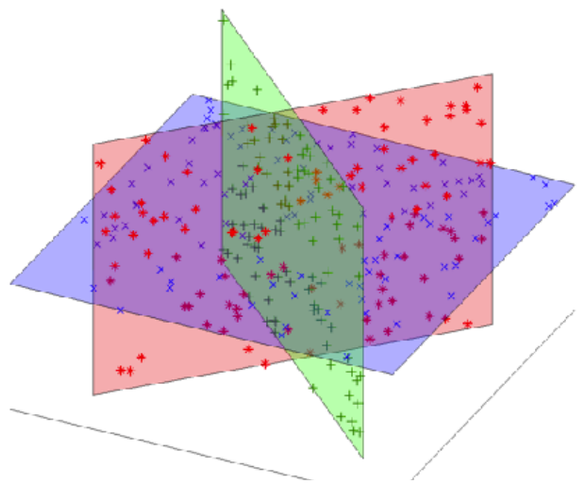
- Networks that include X in training, such as unrolling approaches and Neumann networks, perform well in the low-sample regime.

- Neumann networks are mathematically justified for union of subspaces.

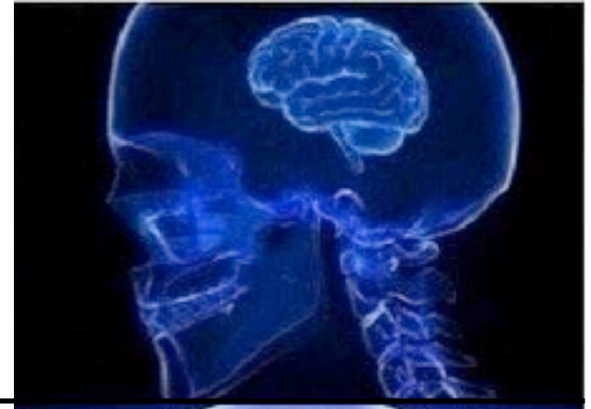
- Further benefits from Neumann networks, likely due to friendlier optimization landscape.







Classical: r(β) is a pre-defined smoothness-promoting regularizer (e.g. Tikhinov or ridge estimation)



Bayesian: r(β) = -log p(β)
Uses a prior distribution over space of β's (e.g. sparsity, patch redundancy, total variation)

Learned: use training data to learn r(β)

Next: using theory to guide network architecture design

arXiv:1901.03707 [pdf, other] cs.CV cs.LG stat.ML

Neumann Networks for Inverse Problems in Imaging

Authors: Davis Gilton, Greg Ongie, Rebecca Willett

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