GEOMETRY PROCESSING ON VOXEL OBJECTS

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Digital geometry model in one slide

Topology and geometry processing of objects defined on regular lattices

- **Digital Objects** = subsets of $\mathbb{Z}^d$
- **Digital Topology** = adjacency relationship induced by the lattice
- **Digital surfaces** = cells of a Cartesian cubical complex

- **Geometrical predicates** = integer only computations
  ⇒ strong arithmetical results when doing geometry on grids
Why voxel objects?

- Widely used in geometric modeling and rendering to represent complex and interactive scenes
- Nice mathematical modeling framework

$64K^3$ voxel grid!

Material Sciences Applications

Non-invasive snow micro-tomographic images

Up to $2048^3$
Outline

- Curvature tensor estimation
- Laplace-Beltrami operator on digital surfaces
- Variational geometry processing: Ambrosio-Tortorelli functional

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CURVATURE TENSOR ESTIMATION
Digitization model

Given $M \subset \mathbb{R}^d$, its digitization at gridstep $h$ is

$$G_h(M) = \left( \frac{1}{h} \cdot M \right) \cap \mathbb{Z}^d$$

Example: $h^2 |G_h(M)|$ converges to the measure of $M$ as $h \to 0$

[Gauss, Dirichlet, Huxley...]
Multigrid convergence of a local estimator

Given a digitization process $D$, a local discrete geometric estimator $\hat{E}$ of some geometric quantity $E$ is multigrid convergent for the family of shapes $\mathcal{X}$ if and only if, for any $M \in \mathcal{X}$, there exists a grid step $h_M > 0$ such that the estimate $\hat{E}(D_M(h), \hat{x}, h)$ is defined for all $\hat{x} \in \partial(D_M(h))_h$ with $0 < h < h_M$, and for any $x \in \partial X$,

$$\forall \hat{x} \in \partial(D_M(h))_h \text{ with } \|\hat{x} - x\|_{\infty} \leq h, \quad |\hat{E}(D_M(h), \hat{x}, h) - E(M, x)| \leq \tau_{M,x}(h),$$

where $\tau_{M,x} : \mathbb{R}^+ \setminus \{0\} \rightarrow \mathbb{R}^+$ has null limit at 0. The convergence is uniform for $M$ when every $\tau_{M,x}$ is bounded from above by a function $\tau_M$ independent of $x \in \partial M$ with null limit at 0.
Digital/Continuous mapping

Let $M$ be a compact domain of $\mathbb{R}^d$ such that $\partial X$ has positive reach greater than $R$. Let $\partial_h M := \partial [G_h(M))]_h$. Then for any $0 < h < 2R/\sqrt{2}$,

- The Hausdorff between $\partial M$ and $\partial_h M$ is bounded by $\sqrt{d}h/2$
- For $d = 2$, there exists an homeomorphism between $\partial X$ and $\partial_h X$
- For $d \geq 3$, no homeomorphism, but
  - Projection operator $\xi : \partial_h M \rightarrow \partial M$ is surjective
  - Area of non-injective parts of $\xi$ the tends to zero
Digitization as an Hausdorff sampling of the continuous object

Can we estimate the curvature tensor on digital surfaces with multigrid convergence properties?
Huge literature on differential quantity estimators

- **Meshes**
  - Local estimators (1- or 2-rings) [Surazhsky et al. 2003][Gatke, Grimm 2006]
  - Gauss-Bonnet formula based estimators [Xu 2006]
  - Normal cycles [Cohen-Steiner, Morvan 2006]

- **Point Clouds**
  - Jet-Fitting approaches [Cazals, Pouget 2005]
  - Voronoi cell covariance measure (VCM) [Alliez et al. 2007][Merigot et al. 2011][Guel et al. 2014]

- **Generic framework**
  - Varifold approaches [Buet 2014][Buet et al. 2015]

→ accuracy depends on the mesh/point cloud quality
→ incompatible constraints on convergence theorem w.r.t. digital surface
Main contributions: Integral Invariant approach

If $M$ is compact and $\partial M$ has positive reach $\rho$ and $C^3$-continuity then

- Mean curvature and principal curvatures estimations converge in $O(h^{\frac{1}{3}})\) |
- Normal vector field estimation converges in $O(h^{\frac{2}{3}})$ |
- Principal Curvature directions converge in $\frac{1}{|k_1(M,x) - k_2(M,x)|}O(h^{\frac{1}{3}})$
Overall proof scheme

\[ \kappa(M, x) := \frac{3\pi}{2R} - \frac{3A_R(M, x)}{R^2} + O(R) \quad \text{[Pottmann et al. 2007]} \]

\[ A_R(M, x) \to \text{Area} \left( B_{R/2}(x/h) \cap G_h(M) \right) \]

\[ \hat{\kappa}^R(G_h(M), x, h) \quad \text{[Pottmann et al. 2007]} \]

\[ \hat{\kappa}^R(G_h(M), \hat{x}, h) \to \kappa(M, x) \]
Multigrid convergence of the digital curvature estimator

Let $M$ be a convex shape in $\mathbb{R}^2$ with $C^3$ bounded positive curvature boundary.

$$\forall x \in \partial M, \forall \hat{x} \in \partial[G_h(M)], \|\hat{x} - x\|_{\infty} \leq h \Rightarrow$$

$$|\hat{k}^R(G_h(M), \hat{x}, h) - \kappa(M, x)| = O(R)$$

$$+ O\left(\frac{h^\beta}{R^{1+\beta}}\right)$$

$$+ O\left(\frac{h^{\alpha'}}{R^2}\right) + O\left(h^{\alpha'}\right) + O\left(\frac{h^{2\alpha'}}{R^2}\right)$$

$\rightarrow$ Setting $R = kh^{\alpha}$, we select $\alpha$ to minimize all errors.

$$|\hat{k}^R(G_h(M), \hat{x}, h) - \kappa(M, x)| \leq O\left(h^{\frac{1}{3}}\right) \quad \text{setting} \quad R = kh^{\frac{1}{3}}$$
Curvature tensor on digital surfaces

\[ \lambda_1^R(G_h(M), x, h) := \frac{6}{\pi R^6} (\hat{\lambda}_2 - 3 \hat{\lambda}_1) + \frac{8}{5R} \]
\[ \lambda_2^R(G_h(M), x, h) := \frac{6}{\pi R^6} (\hat{\lambda}_1 - 3 \hat{\lambda}_2) + \frac{8}{5R} \]
\[ \hat{w}_1^R(G_h(M), x, h) := \hat{v}_1 \]
\[ \hat{w}_2^R(G_h(M), x, h) := \hat{v}_2 \]
\[ \hat{n}^R(G_h(M), x, h) := \hat{v}_3 \]

\( \{\hat{v}_i, \hat{\lambda}_i\} \) are the eigenvalues/eigenvectors of the covariance matrix of \( B_r(x) \cap M \)

\[ \left| \lambda_i^R(G_h(M), \hat{x}, h) - \kappa_i(M, x) \right| \leq O \left( h^{\frac{3}{2}} \right) \] setting \( R = k h^{\frac{1}{2}} \)

\[ \exists h_M \in \mathbb{R}^+, \forall h \in \mathbb{R}, 0 < h < h_M, \]
\[ \forall x \in \partial M, \forall \hat{x} \in \partial (G_h(M))_h \text{ avec } \|\hat{x} - x\|_\infty \leq h, \]
\[ \|\hat{w}_1^R(G_h(M), \hat{x}, h) - w_1(M, x)\| \leq \frac{1}{|\kappa_1(M, x) - \kappa_2(M, x)|} O(h^{\frac{3}{2}}), \]
\[ \|\hat{w}_2^R(G_h(M), \hat{x}, h) - w_2(M, x)\| \leq \frac{1}{|\kappa_1(M, x) - \kappa_2(M, x)|} O(h^{\frac{3}{2}}), \]
\[ \|\hat{n}^R(G_h(M), \hat{x}, h) - n(M, x)\| \leq O(h^{\frac{3}{2}}). \]
In summary

- Robust, Efficient implementation (convolutions)
- Parametrized by a integration radius $R$ or a grid step $h$
- Proof relies on digital/continuous relationships and geometrical moment estimation

Multigrid convergent curvature tensor estimation
LAPLACE-BELTRAMI OPERATOR
Motivations

\[ \Delta u = \nabla \cdot \nabla u \]

Key operator for many geometry processing tasks
Discretization of the Laplace-Beltrami operator

Many discretization scheme for triangular meshes, polygonal meshes, point clouds...

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(update of “Discrete Laplace operators: No free lunch” [Wardetzky et al., 2007])

Strong consistency of the operator:

\[
\lim_{\epsilon \to 0} ||\Delta_\epsilon v - \Delta v||_{L^\infty} = \lim_{\epsilon \to 0} \sup_{x \in \partial M} |(\Delta_\epsilon v)(x) - (\Delta v)(x)| = 0, \quad \forall v \in C^2(\partial M).
\]
What about Digital Surfaces?

- No Laplace-Beltrami operator with strong consistency property exists on digital surfaces.
- Anisotropic nature of digital surfaces may lead to geometrical inconsistencies.
Heat equation based Laplace-Beltrami operator on meshes

\[ \Delta g(x, t) = \frac{\partial}{\partial t} g(x, t), \quad u = g(\bullet, 0) \rightarrow \Delta g(x, t) = \lim_{t \to 0} \frac{1}{t} \int_{\partial M} p(t, x, y)(u(y) - u(x))dy \]

\[ (\mathcal{L}_t, u)(x) := \frac{1}{t(4\pi t)^\frac{d}{2}} \int_{y \in \partial M} e^{-\frac{||y-x||^2}{4t}} (u(y) - u(x))dy, \]

As \( t \to 0 \), \((\mathcal{L}_t, u)\) converges to \( \Delta u \).

\[ (\mathcal{L}_{MESH} u)(p) := \frac{1}{4\pi t^2} \sum_{f \in F} A_f \sum_{q \in V(f)} e^{-\frac{||p-q||^2}{4t}} (u(q) - u(p)) \]

If the mesh is a nice triangulation of a smooth manifold, \((\mathcal{L}_{MESH} u)\) converges to \((\mathcal{L}_t u)\) \((t \approx 1/\text{density})\).

(multigrid) Digital Surfaces are not nice triangulations!
Convolution based Laplace-Beltrami operator on Digital Surfaces

\[ (L_h \tilde{u})(s) := \frac{1}{t_h(4\pi t_h)^\frac{d}{2}} \sum_{r \in S} e^{-\frac{||r-s||^2}{4t_h}} [\tilde{u}(r) - \tilde{u}(s)]\mu(r) \]

As \( h \to 0 \), \((L_h \tilde{u})\) converges to \( \Delta u \).

(\( \tilde{u} \) is the extension of \( u \) from \( M \) to \( \mathbb{R}^3 \) along the normal vectors)

Main result [Caissard, C., Lachaud 18]
Measure of a surface element \( \mu(s) := \mathbf{n}_s \cdot \mathbf{n}_s^e \)
Convolution based Laplace-Beltrami operator on Digital Surfaces

\[(L_h \tilde{u})(s) := \frac{1}{t_h(4\pi t_h)^{\frac{d}{2}}} \sum_{r \in S} e^{-\frac{||r-s||^2}{4t_h}} [\tilde{u}(r) - \tilde{u}(s)]\mu(r)\]

As \(h \to 0\), \((L_h \tilde{u})\) converges to \(\Delta u\).

(\(\tilde{u}\) is the extension of \(u\) from \(M\) to \(\mathbb{R}^3\) along the normal vectors)

Main result [Caissard, C., Lachaud 18]
Sketch of the proof

\[ |(\Delta u)(\xi(s)) - (L_h \tilde{u})(s)| \leq |(\Delta u)(\xi(s)) - (L_1 u)(\xi(s))| + |(L_1 u)(\xi(s)) - (L \tilde{u})(s)| + |(L \tilde{u})(s) - (L_h \tilde{u})(s)| \]
Sketch of the proof

\[
\begin{aligned}
| (\Delta u)(\xi(s)) - (L_h \tilde{u})(s) | & \leq | (\Delta u)(\xi(s)) - (L_1 u)(\xi(s)) | \\
& + | (L_1 u)(\xi(s)) - (L \tilde{u})(s) | \\
& + | (L \tilde{u})(s) - (L_h \tilde{u})(s) | \\
\end{aligned}
\]

[Belkin et al]  
Projection error  
Digital integration error
Sketch of the proof

\[
\left| (\Delta u)(\xi(s)) - (L_h \tilde{u})(s) \right| \leq \frac{1}{2} \left| (\Delta u)(\xi(s)) - (\mathcal{L}_i u)(\xi(s)) \right| + \left| (\mathcal{L}_i u)(\xi(s)) - (\mathcal{L}_i \tilde{u})(s) \right| + \left| (\mathcal{L}_i \tilde{u})(s) - (L_h \tilde{u})(s) \right|
\]

[Belkin et al] \hspace{5cm} \text{Projection error} \hspace{5cm} \text{Digital integration error}

**Lemma**

Let \( s \in \partial_h M \), a function \( u \in C^2(\partial M) \) and its extension \( \tilde{u} \). For \( t_h = h^\alpha, 0 < \alpha \leq \frac{2}{2+d} \) and \( h \leq h_{\max} \) with \( h_{\max} \) the minimum between \( \text{Diam}(\partial M), K_3(d, \alpha, \text{Diam}(\partial M)) \) and \( R\sqrt{d + 1} \), we have

\[
\left| (\mathcal{L}_i u)(\xi(s)) - (\mathcal{L}_i \tilde{u})(s) \right| \leq \text{Area}(\partial M) \| \nabla u \|_\infty \left[ K_1(d) h^{1-(1+\frac{d}{2})} + K_2(d) h^{2-\frac{3d}{2}} \right]
\]

with

\[
K_1(d) := \frac{\sqrt{d + 1}}{2^{d-1} e \pi^{\frac{d}{2}}} \quad \text{and} \quad K_2(d) := \frac{3(d + 1)}{2^{d+\frac{5}{2}} \sqrt{e \pi^{\frac{d}{2}}}}.
\]

Technical proof using the regularity of \( u \) and Hausdorff distance between \( \partial M \) and \( \partial_h M \).
Sketch of the proof

\[
| (\Delta u)(\xi(s)) - (L_h \tilde{u})(s) | \leq \\
\begin{align*}
(\Delta u)(\xi(s)) - (L'_1 u)(\xi(s)) & + (L'_1 u)(\xi(s)) - (\tilde{L} u)(s) & + (\tilde{L} u)(s) - (L_h \tilde{u})(s)
\end{align*}
\]

[Belkin et al]  
Projection error  
Digital integration error
Sketch of the proof

\[
\left| (\Delta u)(\xi(s)) - (L_h \tilde{u})(s) \right| \leq \left| (\Delta u)(\xi(s)) - (L_1 u)(\xi(s)) \right| + \left| (L_1 u)(\xi(s)) - (L \tilde{u})(s) \right| + \left| (L \tilde{u})(s) - (L_h \tilde{u})(s) \right|
\]

[Belkin et al]

Projection error

Digital integration error

Let \( M \) be a compact domain whose boundary has positive reach \( R \). For \( h \leq \frac{R}{\sqrt{d+1}} \), the digital integral is multigrid convergent toward the integral over \( \partial M \). More precisely, for any measurable function \( f : \mathbb{R}^{d+1} \rightarrow \mathbb{R} \), one gets

\[
\left| \int_{\partial M} f(x) d\mathcal{H} - \text{DI}_h(f, M_h, \mathbf{n}) \right| \leq 2^{d+3}(d + 1)^{\frac{3}{2}} \text{Area}(\partial M) \left( \text{Lip}(f) \sqrt{d + 1} \right) h
\]

\[
+ \|f\|_{\infty} \cdot \|\mathbf{n} - \mathbf{n}_{\text{surf}}\|_{\ell_1},
\]

\( \text{(DI}_h(f, M_h, \mathbf{n}) \approx \text{summation of } f \text{ evaluated at each surfel } s \text{ and weighted by } \mu(s) \)

\[
\Rightarrow \text{We need a multigrid convergent normal vector estimation}
\]

Remaining steps: We set \( f(x) := \frac{1}{\Omega} e^{-\frac{|r-x|^2}{2\Omega^2}} (\tilde{u}(x) - \tilde{u}(s)) \) and we derive bounds for \( \text{Lip}(f) \) and \( \|f\|_{\infty} \).
Let $s$ be a surfel in $\partial_h M$, a function $u \in C^2(\partial M)$ and its extension $\tilde{u}$. Let $t_h = h^\alpha$ and let the convergence speed of the normal estimator be in $O(h^\beta)$. Let $h_0$ be the minimum between $\text{Diam}(\partial M), R/\sqrt{d+1}$ and $K_3(d, \alpha, \text{Diam}(\partial M))$. For $0 < h \leq h_0$ we have

$$\lim_{h \to 0} |(\Delta u)(\xi(s)) - (L\tilde{u})(s)| = 0$$

if $0 < \alpha < \min\left(\frac{2}{d+2}, \frac{2\beta}{d+1}\right)$.
In summary

**Laplace-Beltrami operator on Digital Surfaces**

- *Strong consistency* thanks to a multigrid convergent normal vector field
- Discrete operator is not as sparse as the cotangent one
- Efficient implementation (convolutions on compact support)
PIECEWISE SMOOTH RECONSTRUCTION
Problem statement

⇒ Piecewise smooth reconstruction of the normal vector field
Ambrosio-Tortorelli functional

\[
\mathcal{AT}_\varepsilon(u, v) = \alpha \int_M |u - g|^2 \, dx + \int_M |v \nabla u|^2 \, dx + \lambda \int_M \varepsilon |\nabla v|^2 + \frac{1}{4\varepsilon} |1 - v|^2 \, dx
\]

attachment term\hspace{2cm} smoothness term\hspace{2cm} discontinuities length

- Two functions to optimize: \( u \) (scalar map) and \( v \) (feature scalar map, \( M \to [0, 1] \))
- \( v \approx 1 \) on smooth parts, \( v \approx 0 \) near features
- Quadratic terms
- the AT functional \( \Gamma \) converges to Mumford-Shah's functional \( \mathcal{AT}_\varepsilon \xrightarrow{\varepsilon \to 0} \mathcal{MS} \)
- (integration domain does not change, no Hausdorff measure)
Ambrosio-Tortorelli functional

\[ \mathcal{A}_\varepsilon(u, \nu) = \alpha \int_M |u - g|^2 \, dx + \int_M |\nu \nabla u|^2 \, dx + \lambda \int_M \varepsilon |\nabla \nu|^2 + \frac{1}{4\varepsilon} |1 - \nu|^2 \, dx \]

- \( \varepsilon \) -- thickness of the feature set ([distance])
- \( \alpha \) -- attachment coefficient to control the smoothing strength ([area\(^{-1}\)])
- \( \lambda \) -- proportional to the length of discontinuities ([distance\(^{-1}\)])
Discretization

"à la" Discrete Exterior Calculus [Hirani, Desbrun, Grady...]:

\( \mathcal{M} \) is a cellular complex (\( \mathcal{M}' \) its dual), \( \sigma^k \) are \( k \)-cells of \( \mathcal{M} \) (resp. \( \sigma'^k \) of \( \mathcal{M}' \))

- \( k \)-forms are vectors of \(|\{\sigma^k\}|\) scalars
- Linear operators are matrices
- e.g.
  - \( d_k \) (exterior derivative) maps primal \( k \)-forms to primal \((k + 1)\)-forms
  - wedge product \( \alpha \wedge \beta \) maps \( k \)-forms and \( l \)-forms to \((k + l)\)-forms
  - Hodge-star \( \ast_k \) operator to maps primal \( k \)-forms to dual \( k \)-forms
  - ...

Curvature Tensor Estimation
Laplace-Beltrami
Piecewise smooth reconstruction
DGTal
Discretization (bis)

\[ \mathcal{A}\mathcal{T}_e(u, v) = \alpha \int_M |u - g|^2 \, dx + \int_M |v \nabla u|^2 \, dx + \lambda \int_M \epsilon |\nabla v|^2 + \frac{1}{4\epsilon} |1 - v|^2 \, dx \]

attachment term \hspace{1cm} smoothness term \hspace{1cm} discontinuities length

\( v \) is a primal 0-form, \( u \) is a triple of dual 0-forms \((u_1, u_2, u_3)\) and we discretize \( \mathcal{A}\mathcal{T}_e \)
Discretization (ter)

\[ \mathcal{AT}_e^d(u, v) := \alpha \sum_{i=1}^{3} \langle u_i - g_i, u_i - g_i \rangle_0 + \sum_{i=1}^{3} \langle v \wedge d_0 u_i, v \wedge d_0 u_i \rangle_1 + \lambda \epsilon \langle d_0 v, d_0 v \rangle_1 + \frac{\lambda}{4\epsilon} \langle 1 - v, 1 - v \rangle_0 \]

- if \( \gamma \) is a primal 0-form and \( \beta \) a dual 1-form, then \( \gamma \wedge \beta = \text{diag}(\beta) M \gamma \) with \( M := \frac{1}{2} |d_0| \)
- \( A \) is the matrix form of \( d_0 \)
- \( B \) is the matrix form of \( d_0 \)
- \( u_i, v, g \) are column vectors containing associated \( k \)-form scalars
- \( S_i \) is a diagonal matrix encoding the Hodge star \( \star_i \)

\[ \mathcal{AT}_e^d(u, v) = \alpha (u - g)^T S_0 (u - g) + u^T B^T \text{Diag}(M v) S_1 \text{Diag}(M v) B u + \lambda \epsilon \langle v, v \rangle_1 + \frac{\lambda}{4\epsilon} \langle 1 - v, 1 - v \rangle_0 \]
Optimization

\[ \min_{u,v} \mathcal{A}T_e \iff (\nabla_u \mathcal{A}T_e = 0) \land (\nabla_v \mathcal{A}T_e = 0) \]

For a given \( \epsilon \):

\[
\nabla_u \mathcal{A}T_e[u,v] = 0 \iff \begin{bmatrix} \alpha S_0^- - B^T \text{Diag}(Mv)S_1 \text{Diag}(Mv)B \end{bmatrix} u = \alpha S_0 g
\]

\[
\nabla_v \mathcal{A}T_e[u,v] = 0 \iff \begin{bmatrix} \lambda \overline{S} + \lambda \epsilon A^T S_1 A + M^T \text{Diag}(Bu)S_1 \text{Diag}(Bu)M \end{bmatrix} v = \frac{\lambda}{4} \epsilon S_0
\]

\( \Rightarrow \) only linear system solves on sparse matrices!
\( \lambda \) parameter
ε parameter
Noise level w.r.t. $\alpha$ parameter
In summary

- Anisotropic normal vector field regularization with feature selection
- Sharp features
- Parameters make sense :)
- Variational problem discretization using a combinatorial representation of the digital surface

Chicken/egg problem: measure of quads $\mu(s)$ used in the DEC operators
Ambrosio-Tortorelli on meshes

Conclusion

- Nice geometrical model with many interactions (arithmetic's, theory of words, computational geometry, discrete mathematics...)
- Very specific discrete/continuous properties
- Related to various areas (image processing, material sciences, geometrical modeling, rendering...) data