D-BRANE COMPRENDIUM

D-BRANES ARE A SOURCE OF R-R CHARGE...
ALSO A SOURCE OF CONFUSION FOR MANY MATHEMATICIANS
AND PHYSICISTS.

UCLA - IPAM 4/03

ERIC ZASLOW
1. QM, DOF, & HILBERT SPACES IN STRING THY

- One QM particle on $\mathbb{R}$: $\mathcal{H}_1 = L^2(\mathbb{R})$, do. dim.

  - Oscillator $\mathcal{H}_1$ has occupation & basis
  - Free particle: classical DOF $\mathbb{R}$ labels eigenspaces

- QFT: $\mu$ not conserved

Multi-particle Hilbert space: $\mathcal{H} = \bigotimes \mathcal{H}_1$

- QM string: Hilb. sp. $V_1 = L^2(\text{loop space})$

  Fourier decomps on $S^1 \rightarrow V_1 = \bigoplus \mathcal{H}_1^{(n)}$. Fock space

  (Billing if target $\mathbb{R}^d$ or add fermions)

- SFT $V = \bigotimes V_1$,

  * sort of

  $V$ unwieldy. SFT action not well constructed.

  Moral: States in full string thy are vectors

  in a (large) Hilbert space.

---

Nearby DOF to

\[ \xymatrix{ \bigotimes \ar@{-}[r] & \bigotimes \ar@{-}[r] & \bigotimes } \]

Captured by worldsheet thy $X: \Sigma \rightarrow M$
2. SUSY & BPS STATES

?? more than Poincaré $SO(3,1) \times R^3,1$ symmetry?

NOT AS LIE GROUP (ELSE TRIVIAL SCATTERING).

w/ FERMIONS, BUILD MINIMAL ($N=1$) $Z_2$-GRADED LIE ALG. EXT.

$\{F,F\} = B$

JACOBI: $[B_1, [B_2, Q]] - [B_2, [B_1, Q]] = [[B_1, B_2], Q]$

$\Rightarrow Q$ FERMIONIC LORENTZ REP (SPINOR) $Q \chi$

$\{Q_\alpha, Q_\beta\} = \delta_{\alpha \beta} \gamma^\mu P_\mu + Z^{ij}_\alpha \gamma^i \gamma^j (\text{ANY DIM})$

REPS CONSTRUCTED BY "INDUCTION" FROM REST: $P = (m, 0, 0, 0)$

$\gamma^0 \phi = \delta_{\chi \phi}$ $\Rightarrow$ SUSY ALG $\leftrightarrow$ FERMION OSC. ALG

$N>1$: SOME OSCS MAY ACT BY 0. $m \geq |Z_1|$

REPS HAVE DIM $2^{n-k}, k \geq 0$

"BPS" ($k>0$) $m = |Z_1|$

$\mathcal{H} = \mathcal{H}_{\text{BPS}} \oplus \text{REST}$

$s\dim \mathcal{H}_{\text{BPS}} = \# \text{BPS STATES}$

(KIND OF INVARIANT)
3. Nonperturbative States

Traditionally, QM done on \( H = \text{Hamiltonian} \),

\[
\text{He} \frac{d}{dt} \psi = H \psi
\]

Feynman: \( \text{Prob}(x \rightarrow y) = \int \int e^{iS(y)/\hbar} \frac{\hbar \cdot \Psi}{\Psi(0)} \)

\( \{x \sim y\} = \psi \)

EX: 4D Gauge Thy

\( \Phi = \text{Conn}(G \rightarrow P) = \bigoplus_{k} \Phi_{k} \leftarrow \text{Inst. # or Top. Type} \)

\( M_{4} = \mathbb{R} \times \mathbb{R} \): Time-Ind. Conf. Disconnected:

\( \mathcal{N} = \bigoplus \mathcal{N}_{k} \)

Monopole Charge Classical

Operator Formalism Based on Vacuum Blind to \( \mathcal{N} \)

Some Nonperturbative States:
- Monopoles in YM
- Black Holes in GR
- Kinks in LG Thy

\( D \)-Branes are nonperturbative states of full string (field) thy.

Invisible to Worldsheet (Closed) String QM.

Polchinski: Branes Carry RR Charge (Calculated)

No Pert. String State Does

Yet: 3 Worldsheet Description of DOF Near D-Branes
4. P-BRANES IN GRAVITY THY'S

(1.4) CLASSICAL STATES IN STRING THY CORRESPOND TO SUGRAV CONFIGURATIONS. CLASSICAL DESCRIPTION OF NONPERT STATES = NONTRIVIAL FIELD CONFIG.

CLASSICAL MIN. ENERGY NONPERT \( \leftrightarrow \) \( v \in H \) \( \leftrightarrow \) MIN \( E \leftrightarrow \) BPS STATE

GR: \( S_{EH} = \int R \Rightarrow R_{\mu\nu} = 0 \) \( \rightarrow \) MINKOWSKI

\( \rightarrow \) SCHWARZSCHILD

SUGRAV: GR + OTHER MATTER

\( S = S_{EH} + S_{MATTER} \)

\( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu} \)

Solutions may have charge: \( \int F \neq 0 \).

They may be sing. on submanif \( S^2 \).

S.T. \((p+1)\)-DIM'L POINC INV \( \Rightarrow \) P-BRANE

\((p=0 = BLACK HOLE)\)

Exs:

\( ds^2 = \left(1 + \frac{k^2}{y^2}\right)[dy^2 + y^2d\Omega^2] - dt^2 + \sum_{i=1}^L dx_i dx_i \)

\( H_{IH} = QE_2 \), \( Q = e_2 \) \( e^{2\phi} = 1 + e_2^2 \) 5-BRANE H \( \Rightarrow \) CHARGE, (Q

\( ds^2 = (1 + e_2 y)^2 [dy^2 + y^2d\Omega^2] + (1 + e_2 y)^2 [-dt^2 + \sum_{i=1}^L dx_i dx_i] \)

\( F_{RR} = QE_2 \), \( Q = e_2 \) \( e^{2\phi} = (1 + e_2 y)^{-3/2} \)

\( \uparrow \) HAS QM DEC. AS BOUNDARY QFT VACUUM. WORLDHEAT BUTT TREATS NEARBY DOF. Q - GRAV !
5. BRANES & THE WORLDSHEET

Conformal symmetry allows $\int [Dg_{\mu\nu}]$.

$N=2$ SCA $\leftrightarrow$ Spacetime Susy \[
\begin{align*}
\{ \psi_i^\alpha, \phi^a \} &= \delta^a_i \\
L &\Rightarrow \text{bosons via spectral flow}
\end{align*}
\]

$N=2$ Spacetime Susy

BC's couple $L\&R$ (e.g. $f(z) + g(z) = 0, z \in \mathbb{R}$)

Couple $L \& R$ preserving $N=2$ SCA:

- $A$: $C^+(z) = \overline{C^+(\overline{z})}$
- $B$: $C_-(z) = \overline{C_-(\overline{z})}$

BC's define BCFT representing nearby DOF to symmetry-preserving sol'n to sugra. BPS (e.g. Wilson line vevs shift position of brane. - $p$)

Identifying sugra sol'n corresponding to BC's is a bit of an art.

$D$-BRANES
As CFT states:

$$\begin{bmatrix} \begin{array}{c} \alpha \cr \beta \end{array} \end{bmatrix} \rightarrow \begin{bmatrix} \alpha \cr \beta \end{bmatrix}$$

$$|\beta\rangle = \Omega(|\alpha\rangle |\beta\rangle)$$

(+ compatibility $\mod. -\text{mod}$)

In a geometric model, define dirichlet BC's from submanifolds. A and B types yield different conditions on submanifold (as well as symm preserve.)

In sugra, nearby fluctuations captured by action depending on position of brane: DBI $\sim$ area.

Can look for branes by requiring symm of DBI.
Ex: In a $G$-model $N=(2,2)$ (so $CY^+$) and $B=0$, these conditions $\Rightarrow$ geometric Eqs for $C \subset M$.

A: $C$ slag + flat bundle \( \text{worldsheet} \) \( \text{worldvol} \) \( \text{spacetime} \)
B: $C$ spin + nonlinear Hym
(cans ou lo thy, too)

**Topological Branes**

Could ask to preserve $N=2$ susy only, not $N=2$ sca.

Then topological twist $\Rightarrow$ BTF.

Again, two possibilities: A, B.

In $G$-model, A: coisotropic
B: flat $D^b(M)$ (via res. of sheaves)

RMN: These conditions less geometric,
due to greater equivalence in TFT.
6. Why are branes useful?

- SUSY 'BREAKING''

  We saw that branes preserve a fraction of the original SUSY. Branes filling spacetime \(\Rightarrow\) more realistic (less SUSY) thy.

\[
\begin{array}{c}
\text{2 branes} \quad \begin{array}{c}
\text{U(1)} \\
\text{U(1)}
\end{array} \quad \text{coincident} \quad \begin{array}{c}
\text{U(1)} \\
\text{U(2)}
\end{array}
\end{array}
\]

\(\text{U(K)}\) gauge groups from coincident branes

- GEOMETRIC ENGINEERING

  Enhanced gauge symmetry when brane state becomes massless (reverse Higgs).

  ADE singularities (4 gauge groups). Fiberizing over \(\Pi'\) allows decoupling limit. \(\Rightarrow\) SW from GW.

- BLACK HOLE PHYSICS

  S-V: \(1/4\)-BPS \(\Pi/k3\times S1\) sugra\(n\) sol's 'RR \(\nu\) RR charles Q.

  (\(1/2\)-BPS have no horizon) construct D-brane (\(1/2\)-BRS) then find \(1/4\)-BPS states of worldvolume thy (micro!)

  Brane reduction \(\Rightarrow\) G-model on \(M_{K3, \text{inst}} \cong \text{Sym}(K3)\)

  Compute * states at large Q

  \(\text{O}(Q): \log N = \text{entropy} = \frac{1}{4} \text{area \ event horizon}\)
Duality

Moduli spaces of branes can be revealing

- 3-brane on $T^3$: $T^3$
- O-brane on $\tilde{M}$: $\tilde{M}$

Corresponding odd brane on $M$: $\tilde{M}$

(Fiber/family duality)

- $\text{cos} \times R^2$ M5-brane on $G_2$ MPLD
  - $K3$-fibered $G_2$ $\leftrightarrow$ HET CY + bundle
  - $T^4$-fibered $G_2$ $\leftrightarrow$ CY" + flux, dil.

(Fibernise $M/K3$ $\leftrightarrow$ HET/$T^3$

- Invariants: brane reduction $M/T^4$ $\rightarrow$ II/$T^3$

Equality of brane physics in dual theories amounts to Kontsevich's proposal

AdS/CFT

\[ \text{Large } N \text{ branes } \leftrightarrow U(N) \text{ gauge th} \leftrightarrow \text{string th} \]

\[ \downarrow \]

Curved geom from $\Rightarrow$ AdS x Sphere at horizon

Back reaction on space

Hooft

? Maldacena
7. D-BRANES A TFT

Composition of operators creating massless states → Nonassociative product. (Fukaya)

Moore-Segal construct axiomatics

\[ C \quad \mathcal{O}_{\text{op}} \quad 0 \to C \]

... 

And find relation to K-thy at this level.

Remark: S-W find operator all of BQFT in presence of B-brane reduces to NC deformation of all of Fos defined by Joyal product, in a certain limit.
MISCELLANEOUS QUESTIONS

- CAN BRANES END ON BRANES? YES, SQFT HAS NONPERT. STATES. PRE-SUN & CONDS.
- BRANES CAN BE BLACK HOLES (DΦ), MONOPOLES (CHARGE)
- ARE ALL BRANES D-BRANES? NO, AS WE SAW. D-BRANES GIVE BEST VIEW OF NONPERT. STR.
- WHY SO MANY DIFFERENT MATHEMATICAL DESCRIPTIONS? CAN'T PHYSICISTS MAKE UP THEIR MINDS?
  - PICK MODEL (G-MODEL)
  - PICK BC (SUBMfld + MOST GENERAL?)
  - FOR GIVEN CLASS OF BC'S, IMPOSE
    - WORLDVIFT SUSY AND/OR
    - CONFORMAL SYMMETRY AND/OR
    - OTHER SYMM.
  ⇒ ERS ... ANYWAY, WHAT IS A MANIFOLD?

- WHAT ARE BRANE CHARACTERS?
  NAIVELY, COHOMOLOGY CLASSES. SHOULD LABEL COMPONENTS OF BRANE MODULI SPACE.
  B-BRANES IN G-MODEL: K-THY (TACHYON ANNIHILATION) 0 → E → E → 0

- LAG/SLAG. WHAT TO BELIEVE?
  G-MODEL A-BRANES. LAG PRESERVES SUSY.
  SLAG IS 1-LOOP APPROX OF BC PRESERVING SCA (BEETTER AND BETTER APPROX AT LARGE RAD.)
STRING THY "IN PRESENCE OF BRANE"? THINK E&M NEAR +

IF BRANE FILLS SPACE, THINK GR NEAR BLACK HOLE.

WHAT'S WITH THIS ORIENTIFOLDING? IF ORBIFOLD
GROUP ACTS DIFF'NT ON L & R, "ORIENTIFOLD"
CAN LEAD TO SO GAUGE GROUPS.

CAN BRANES BE USED SO CAVALIERLY?

YES FOR QUALITATIVE BEHAVIOR.

NOT FOR QUANT., AS MASSIVE BRANE DISTORTS
BACKGROUND. TO REASON QUANTITIVELY, MUST
SHOW QUANTITY TOPOLOGICAL, OR FIND SMALL PAR.

DO NON-SUSY THY'S HAVE BRANES? YES, BCFT % SUSY.

WHAT IS THE PHASE? WHICH N=1 C N=2 PRESERVED. (EIGS)

IF BRANES ARE LIKE PARTICLES, CAN THEY ANIMULATE IN PAIRS?

YES, BUT AS THEY ARE NONPEAT, MUST APPEAL TO SFT.
ROLL TO VAC. % BRANES? YES (NUMERICS)