D-BRANE COMPENDIUM

D-BRANES ARE A SOURCE OF R-R CHARGE...

ALSO A SOURCE OF CONFUSION FOR MANY MATHEMATICIANS AND PHYSICISTS.

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1. QM, DOF, & HILBERT SPACES IN STRING THY

- ONE <u>QM</u> PARTICLE ON $\mathbb{R}$: $\mathcal{H}_1 = L^2(\mathbb{R})$, ∞-DIM'L

  <u>Ex:</u> OSCILLATOR $\mathcal{H}_1$ HAS OCCUPATION # BASIS

  <u>Ex:</u> FREE PARTICLE: CLASSICAL DOF $\mathbb{R}$ LABELS EIGENSPACES

- <u>QFT</u>. # NOT CONSERVED

  MULTI-PARTICLE HILBERT SPACE: $\mathcal{H} = \otimes \mathcal{H}_1$

- <u>QM STRING</u>: HILB. SP. $V_1 = L^2(\text{loop space})$

  FOURIER DECOMP ON $S^1 \Rightarrow V_1 = \bigoplus_n \mathcal{H}_1^{(n)}$. FOCK SPACE

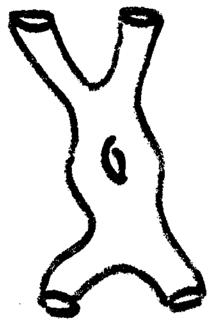
  (BIGGER IF TARGET $\mathbb{R}^d$ OR ADD FERMIONS

- SFT $V = \otimes V_1^*$ * SORT OF

  V UNWIELDY. SFT ACTION NOT WELL CONSTRUCTED.

  MORAL: <u>STATES</u> IN FULL STRING THY ARE <u>VECTORS</u> IN A (LARGE) <u>HILBERT SPACE</u>.

NEARBY DOF TO



CAPTURED BY WORLDSHEET THY $X: \Sigma \to M$

2. SUSY & BPS STATES

?∃ MORE THAN POINCARÉ $SO(3,1) \ltimes \mathbb{R}^{3,1}$ SYMMETRY ?

<u>NOT</u> AS LIE GROUP (ELSE TRIVIAL SCATTERING).

w/ <u>FERMIONS</u>, BUILD MINIMAL $(N=1)$ $\mathbb{Z}_2$-GRADED LIE ALG. EXT.

$$\{F,F\} = B$$

JACOBI: $[B_1,[B_2,Q]] - [B_2,[B_1,Q]] = [[B_1,B_2],Q]$

$\Rightarrow Q$ FERMIONIC LORENTZ REP (SPINOR) Q_α

$$\{Q^i_\alpha, Q^j_{\dot\beta}\} = \delta_{ij}\gamma^\mu_{\alpha\dot\beta} P_\mu \underset{N>1}{+} Z^{ij}_{\alpha\beta} \quad \text{(ANY DIM)}$$

REPS CONSTRUCTED BY "INDUCTION" FROM REST: $P=(m,0,0,0)$

FERMION OSC. ALG

$\gamma^0_{\alpha\dot\beta} = \delta_{\alpha\beta} \Rightarrow$ SUSY ALG $\longleftrightarrow$ FERMION OSC. ALG

$m \geq |Z|$

$N>1$: SOME OSC.S MAY ACT BY 0.

REPS HAVE DIM 2^{n-k}, $k \geq 0$

HARMONI REPS OF CHARGE Z

"BPS" $(k>0)$ $m = |Z|$

$$\mathcal{H} = \mathcal{H}_{BPS} \oplus \text{REST}$$

$$\text{sDIM}\, \mathcal{H}_{BPS} = \#\text{ BPS STATES}$$

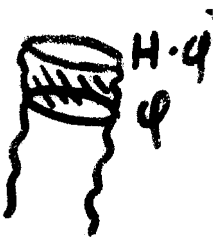
(KIND OF INVARIANT)

3. NONPERTURBATIVE STATES

TRADITIONALLY, QM DONE ON $\mathcal{H}$ W/ HAMILTONIAN, H

$$\hbar i \frac{d}{dt} v = H \cdot v$$

FEYNMAN: $\quad \text{PROB}(x \to y) = \left[\int_{\{x \overset{\gamma}{\rightsquigarrow} y\} =: \mathcal{P}} e^{iS(\gamma)/\hbar} \right]^2$



<u>EX:</u> 4D GAUGE THY

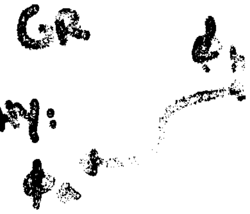
$$\mathcal{P} = \text{CONN}\left(\begin{array}{c} G \to P \\ \downarrow \\ M_4 \end{array} \right) = \bigoplus_k \mathcal{P}_k \longleftarrow \text{INST. \# OR TOP. TYPE}$$

$M_4 = M_3 \times \mathbb{R}_t$: TIME-IND CONFIGS DISCONNECTED: $\mathcal{H} = \bigoplus_\ell \mathcal{H}_\ell$

MONOPOLE CHARGE <u>CLASSICAL</u>

OPERATOR FORMALISM BASED ON VACUUM BLIND TO $\mathcal{H}_{\ell \neq}$

SOME NONPERTURBATIVE STATES:
- MONOPOLES IN YM
- BLACK HOLES IN GR
- KINKS IN LG THY:



<u>D-BRANES</u> ARE NONPERT. STATES OF FULL STRING (FIELD) THY.
INVISIBLE TO WORLDSHEET (CLOSED) STRING QM.
POLCHINSKI: BRANES CARRY RR CHARGE (CALCULATED FORCE)
NO PERT. STRING STATE DOES
<u>YET</u>: $\exists$ WORLDSHEET DESCRIPTION OF DOF NEAR D-BRANE

4. p-BRANES IN GRAVITY THY'S

(SOME) CLASSICAL STATES IN STRING THY CORRESPOND TO SUGRAV CONFIGURATIONS. CLASSICAL DESCRIPTION OF NONPERT STATES = NONTRIVIAL FIELD CONFIGS.

CLASSICAL MIN. ENERGY NONPERT STATE $\longleftrightarrow v \in \mathcal{H}_2$ w/ MIN E $\longleftrightarrow$ BPS ?

GR: $S_{EH} = \int R \Rightarrow R_{\mu\nu} = 0$

- MINKOWSKI
- T^d
- SCHWARZSCHILD

SUGRAV: GR + OTHER MATTER

$$S = S_{EH} + S_{MATTER}$$

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = T_{\mu\nu}$$

SOLUTIONS MAY HAVE <u>CHARGE</u>: $\int_{S^k_\infty} F \neq 0.$

THEY MAY BE SING. ON SUBMFLD S.T. $(p+1)$-DIM'L POINC INV $\Rightarrow$ p-BRANE ($p=0$ = BLACK HOLE)

<u>Exs</u>:

- $ds^2 = \left(1 + \frac{\varrho}{y^2}\right)\left[dy^2 + y^2 d\Omega_3^2\right] - dt^2 + \sum_{i=1}^{5} dx^i dx^i$

  $H_{NS} = Q\epsilon_3, \; Q = \varrho/\sqrt{2} \qquad e^{2\phi} = 1 + \varrho/y^2$ 5-BRANE MAG CHARGE, Q

- $ds^2 = (1 + \varrho/y)^{1/2}\left[dy^2 + y^2 d\Omega^2\right] + (1 + \varrho/y)^{-1/2}\left[-dt^2 + \sum_{i=1}^{6} dx^i dx^i\right]$

  $F_{RR} = Q\epsilon_2, \; Q = \varrho/2 \qquad e^{2\phi} = (1 + \varrho/y)^{-3/2}$

HAS QM DESC AS BOUNDARY QFT VACUUM. WORLDSHEET BQFT TREATS NEARBY DOF. Q-GRAV!

5. BRANES & THE WORLDSHEET

CONFORMAL SYMMETRY ALLOWS $\int [Dg_{\mu\nu}]$.

N=2 SCA ⟷ SPACETIME SUSY { RR FERMIONS $\{\psi_0^i, \psi_0^j\} = \delta^{ij}$ ⟷ BOSONS VIA SPECTRAL FLO

L&R ⇒ N=2 SPACETIME SUSY

BC's COUPLE L&R (e.g. $f(z)+g(\bar{z})=0$, $z \in \mathbb{R}$)

COUPLE L&R PRESERVING N=2 SCA: A: $G^+(z) = \bar{G}^+(\bar{z})$
B: $G^+(z) = \bar{G}^-(\bar{z})$

BC's DEFINE BCFT REPRESENTING NEARBY DOF TO SYMMETRY-PRESERVING SOL'N TO SUGRAV. BPS
(e.g. WILSON LINE VEVs SHIFT POSITION OF BRANE. -P)
IDENTIFYING SUGRAV SOL'N CORRESPONDING TO BC's IS A BIT OF AN ART.

Ω

$A \subset A_L \otimes A_R$

D-BRANES AS CFT STATES:

$a|B\rangle = \Omega(a)|B\rangle$
(+ COMPATIBILITY w/ MOD. -CARD

IN A GEOMETRIC MODEL, DEFINE DIRICHLET BC's FROM SUBMANIFOLDS*. A AND B TYPES YIELD DIFFERENT CONDITIONS ON SUBMANIFOLD (AS WELL AS SYMM PRESERV.)

IN SUGRAV, NEARBY FLUCTUATIONS CAPTURED BY ACTION DEPENDING ON POSITION* OF BRANE: DBI ~ AREA*.
CAN LOOK FOR BRANES BY REQUIRING SYMM OF DBI.

Ex: IN A σ-MODEL w/ $N=(2,2)$ (SO CY^+) AND $B=0$,
THESE CONDITIONS $\Rightarrow$ GEOMETRIC EQS FOR $C \subset M$.

A : C SLAG$^+$ + FLAT BUNDLE $\Leftarrow$ WORLDSHEET / WORLDVOL / SPACETIME

B : C cpx + NONLINEAR HYM

(CAN DO LG THY, TOO)

TOPOLOGICAL BRANES

COULD ASK TO PRESERVE $N=2$ SUSY ONLY, NOT $N=2$ SCA.
THEN TOPOLOGICAL TWIST $\Rightarrow$ BTFT.

AGAIN, TWO POSSIBILITIES: A, B.

IN σ-MODEL, A: COISOTROPIC$^+$
B: ELMT $D^b(M)$ (VIA RES. OF SHEAVES)

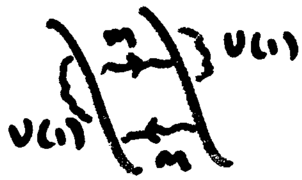
RMK: THESE CONDITIONS LESS GEOMETRIC,
DUE TO GREATER EQUIVALENCE IN TFT.

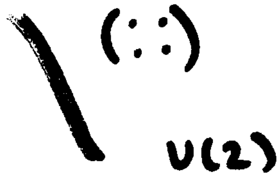
6. WHY ARE BRANES USEFUL?

- SUSY "BREAKING"

WE SAW THAT BRANES PRESERVE A FRACTION OF THE ORIGINAL SUSY. BRANES FILLING SPACETIME ⟹ MORE REALISTIC (LESS SUSY) THY.

$\oint_{\partial\Sigma} A$

2 BRANES 

COINCIDENT 

U(k) GAUGE GROUPS[+] FROM COINCIDENT BRANES

- GEOMETRIC ENGINEERING

ENHANCED GAUGE SYMMETRY WHEN BRANE STATE BECOMES MASSLESS (REVERSE HIGGS).

ADE SINGULARITIES (& GAUGE GRPS). FIBERING OVER $\mathbb{P}^1$ ALLOWS DECOUPLING LIMIT. ⟹ SW FROM GW.

- BLACK HOLE PHYSICS

S-V: $\frac{1}{4}$-BPS $\mathrm{II}/_{K3\times S^1}$ SUGRAV SOL'S w/ RR CHARGE Q. ($\frac{1}{2}$-BPS HAVE NO HORIZON) CONSTRUCT D-BRANE ($\frac{1}{2}$-BRS) THEN FIND $\frac{1}{2}$-BPS STATES OF WORLDVOLUME THY (MICRO!)

BRANE REDUCTION ⟼ σ-MODEL ON $\mathcal{M}_{K3\text{-INST}} \cong \mathrm{Sym}^{\cdot}(K3)$

COMPUTE # STATES AT LARGE Q

$O(Q)$: $\log \#$ = ENTROPY = $\frac{1}{4}$ AREA$_{\text{EVENT HORIZON}}$!

- DUALITY

MODULI SPACES OF BRANES CAN BE REVEALING

▶ 3-BRANE ON T^3 : $\tilde{T}^3$

▶ 0-BRANE ON $\tilde{M}$: $\tilde{M}$

CORRESPONDING ODD BRANE ON M : $\tilde{M}$

(FIBER/FAMILY DUALITY)

▶ $COASS \times \mathbb{R}^2$ M5-BRANE ON G_2 MFLD

K3-FIBERED G_2 $\leftrightarrow$ HET CY + BUNDLE

T^4-FIBERED G_2 $\leftrightarrow$ II "CY" + FLUX, DIL.

(FIBERWISE $M/K3 \leftrightarrow HET/T^3$

▶ INVARIANTS ^ BRANE REDUCTION $M/T^4 \leftrightarrow II/T^3$)

EQUALITY OF BRANE PHYSICS IN DUAL THEORIES AMOUNTS TO KONTSEVICH'S PROPOSAL

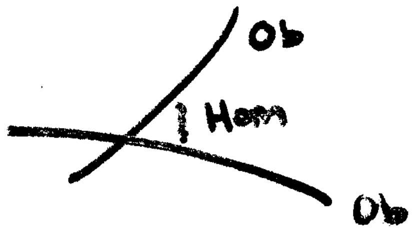
- AdS/CFT

LARGE N BRANES $\leftrightarrow$ U(N) GAUGE THY $\overset{\text{'t HOOFT}}{\longleftrightarrow}$ STRING THY

$\Downarrow$ $\quad$ $\|$? MALDACENA

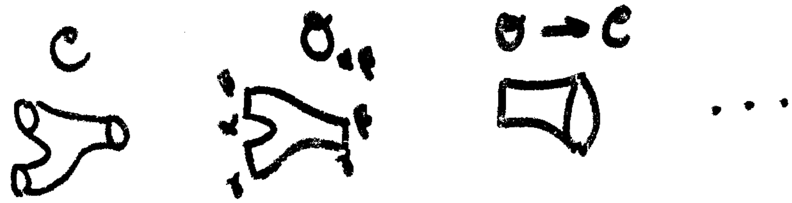
CURVED GEOM FROM BACK REACTION ON SPACE = AdS × SPHERE AT HORIZON

7. D-BRANES & TFT



COMPOSITION OF OPERATORS CREATING MASSLESS STATES ⇒ NONASSOCIATIVE PRODUCT. (FUKAYA)

MOORE & SEGAL CONSTRUCT AXIOMATICS



AND FIND RELATION TO K-THY AT THIS LEVEL.

RMK: S-W FIND OPERATOR ALG OF BQFT IN PRESENCE OF B-BRANE REDUCES TO NC DEFORMATION OF ALG OF F'S DEFINED BY MOYAL PRODUCT, IN A CERTAIN LIMIT.

MISCELLANEOUS QUESTIONS

- CAN BRANES END ON BRANES? YES. BQFT HAS NONPERT. STATES. PRES. SUSY ⇒ CONDS.

- BRANES CAN BE BLACK HOLES (Dϕ), MONOPOLES (CHARGE)

- ARE ALL BRANES D-BRANES? NO, AS WE SAW. D-BRANES GIVE BEST VIEW OF NONPERT. STR.

- WHY SO MANY DIFFERENT MATHEMATICAL DESCRIPTIONS?
  CAN'T PHYSICISTS MAKE UP THEIR MINDS?
  - PICK MODEL (σ-MODEL)
  - PICK BC (SUBMFLD + — MOST GENERAL ?)
  - FOR GIVEN <u>CLASS</u> OF BC'S, IMPOSE
    - WORLDSHEET SUSY AND/OR
    - CONFORMAL SYMMETRY AND/OR
    - OTHER SYMM.

  ⇒ EQS.

  ... ANYWAY, WHAT IS A MANIFOLD?

- WHAT ARE BRANE CHARGES?
  NAIVELY, COHOMOLOGY CLASSES. SHOULD LABEL COMPONENTS OF BRANE MODULI SPACE.
  B-BRANES IN σ-MODEL: K-THY (TACHYON ANNIHILATION)
  $0 \to E \to E \to 0$

- LAG/SLAG. WHAT TO BELIEVE?
  σ-MODEL A-BRANES. LAG PRESERVES SUSY.
  SLAG IS 1-LOOP APPROX OF BC PRESERVING SCA
  (BETTER AND BETTER APPROX AT LARGE RAD.)

STRING THY "IN PRESENCE OF BRANE"? THINK E&M NEAR ⊕

IF BRANE FILLS SPACE, THINK GR NEAR BLACK HOLE.

WHAT'S WITH THIS ORIENTIFOLDING? IF ORBIFOLD

GROUP ACTS DIFF'T ON L & R, "ORIENTIFOLD."

CAN LEAD TO SO GAUGE GROUPS.

CAN BRANES BE USED SO CAVALIERLY?

YES FOR QUALITATIVE BEHAVIOR.

NOT FOR QUANT., AS MASSIVE BRANE DISTORTS

BACKGROUND. TO REASON QUANT'LY, MUST

SHOW QUANTITY TOPOLOGICAL, OR FIND SMALL PAR.

DO NON-SUSY THY'S HAVE BRANES? YES, BCFT w/o SUSY.

WHAT IS THE PHASE? WHICH $N=1 \subset N=2$ PRESERVED. $(e^{i\theta}\Omega)$

IF BRANES ARE LIKE PARTICLES, CAN THEY ANNIHILATE IN PAIRS?

YES, BUT AS THEY ARE NONPERT, MUST APPEAL TO SFT.

ROLL TO VAC. w/o BRANES? YES (NUMERICS)