The mechanism(s) of breakdown in traffic models

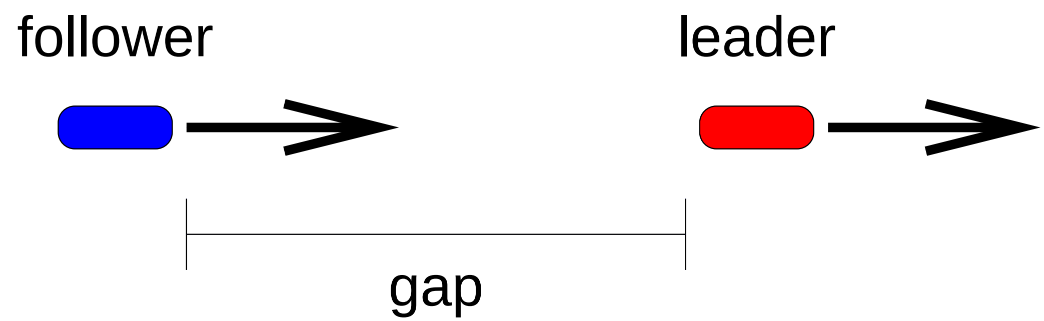
Joint work with Peter Wagner, Christopher Kayatz.

Also taking results from other people without specifically acknowledging them, in particular Krauss, Sugiyama, Bando, Kerner.

[[board]]

This is a simulation-based talk.

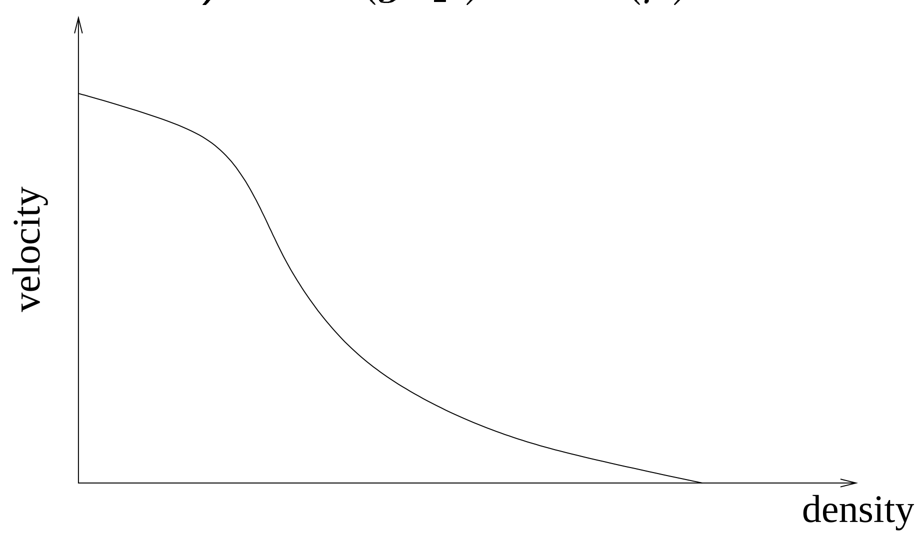
"Driving relation"



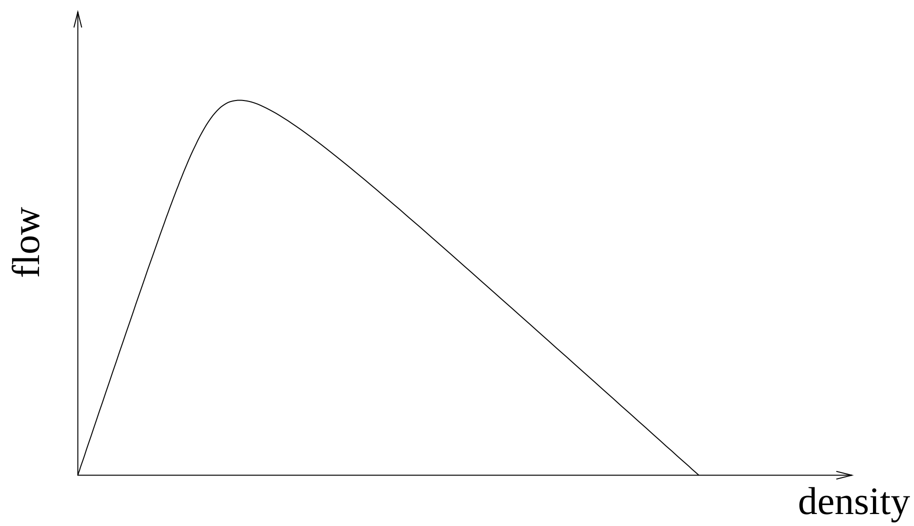
Leader drives with constant speed $\Rightarrow gap(v)$. [[$gap \approx \Delta x$, $gap \neq \Delta x$]]

Average out the noise. Deterministic until further notice.

Invert (assume monotonic) $\Rightarrow V(gap) \Rightarrow V(\rho)$.

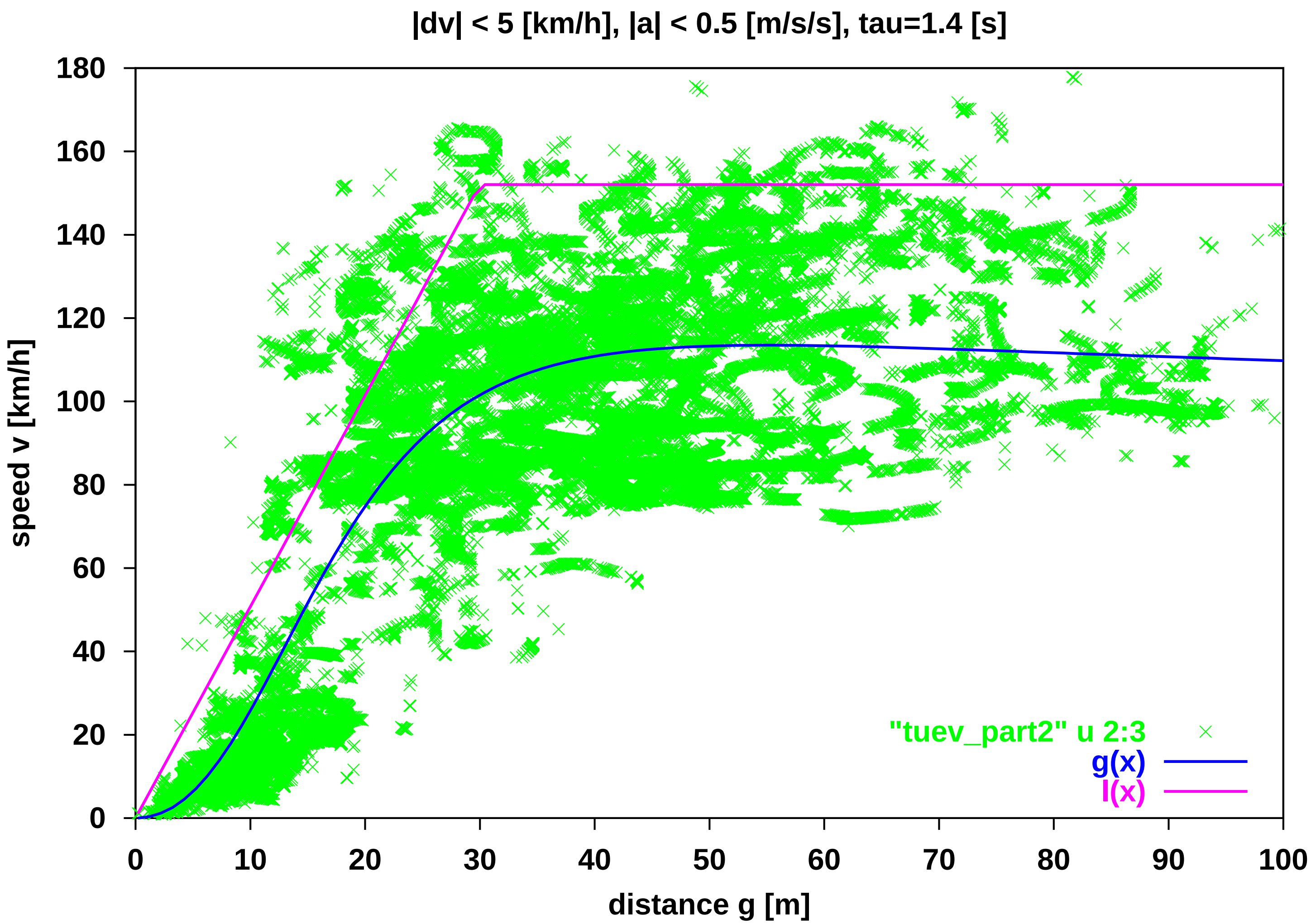


Because of $q = v\,\rho$, multiply by ρ and obtain



Without noise, this is the homogeneous solution!

In practice:



|dv| < 5 [km/h], |a| < 0.5 [m/s/s], tau=1.4 [s]
speed v [km/h]
0
20
40
60
80
100
120
140
160
180
0
10
20
30
40
50
60
70
80
90
100
distance g [m]
"tuev_part2" u 2:3
g(x)
l(x)

Stability of the homogeneous solution

Need model for dynamics:

- Simple gap-based controller:

$$v(t+\tau) \approx v(t) + \tau\,\gamma\Big(gap - G(v)\Big)$$

$\longleftrightarrow$

$$v(t+\tau) \approx v(t) + \tau\,\alpha\Big(V(gap) - v\Big)$$

$\longleftrightarrow$

$$a(t) = f(gap) = \alpha\Big(V(gap) - v\Big)\ .$$

- Higher order terms $a(t) = f(gap, \Delta v, ...)$.[a]

[a] Extreme case: "Traditional" car following model $a(t) \propto \Delta v$ Useless for simulations since arbitrary gap is homogeneous solution as long as $\Delta v = 0$. Need gap-term.

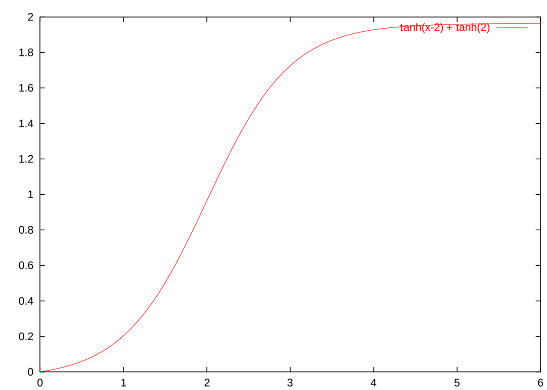
Example: "Optimal velocity" model

Sugiyama, Bando, et al

Linearly stable where $\left|\frac{dV}{d\rho}\right| < \frac{\alpha}{2}$.

Will use specifically

$$v(t+0.1) = v(t) + 0.1 \cdot \Big(\big(\tanh(gap-2) + \tanh(2) \big) - v(t) \Big) \; .$$

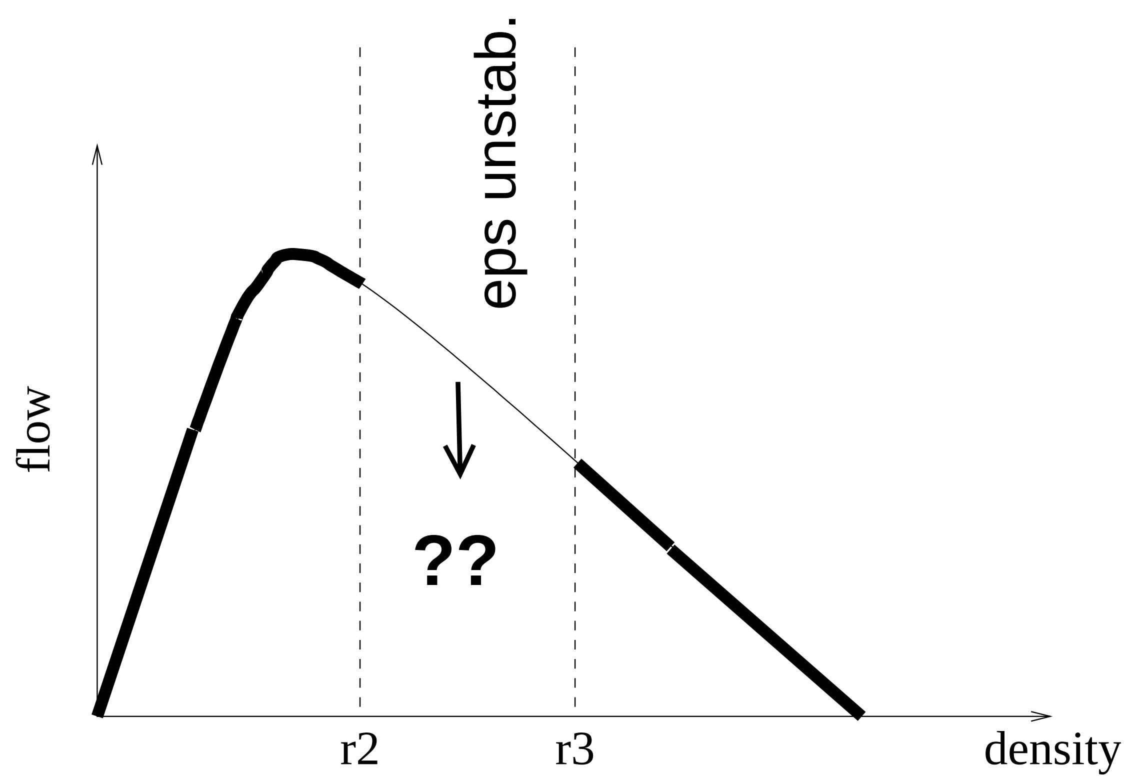


Linear instability of homogeneous solution

[[OVM1 hom unstab.]]

[[OVM2 hom stab.]]

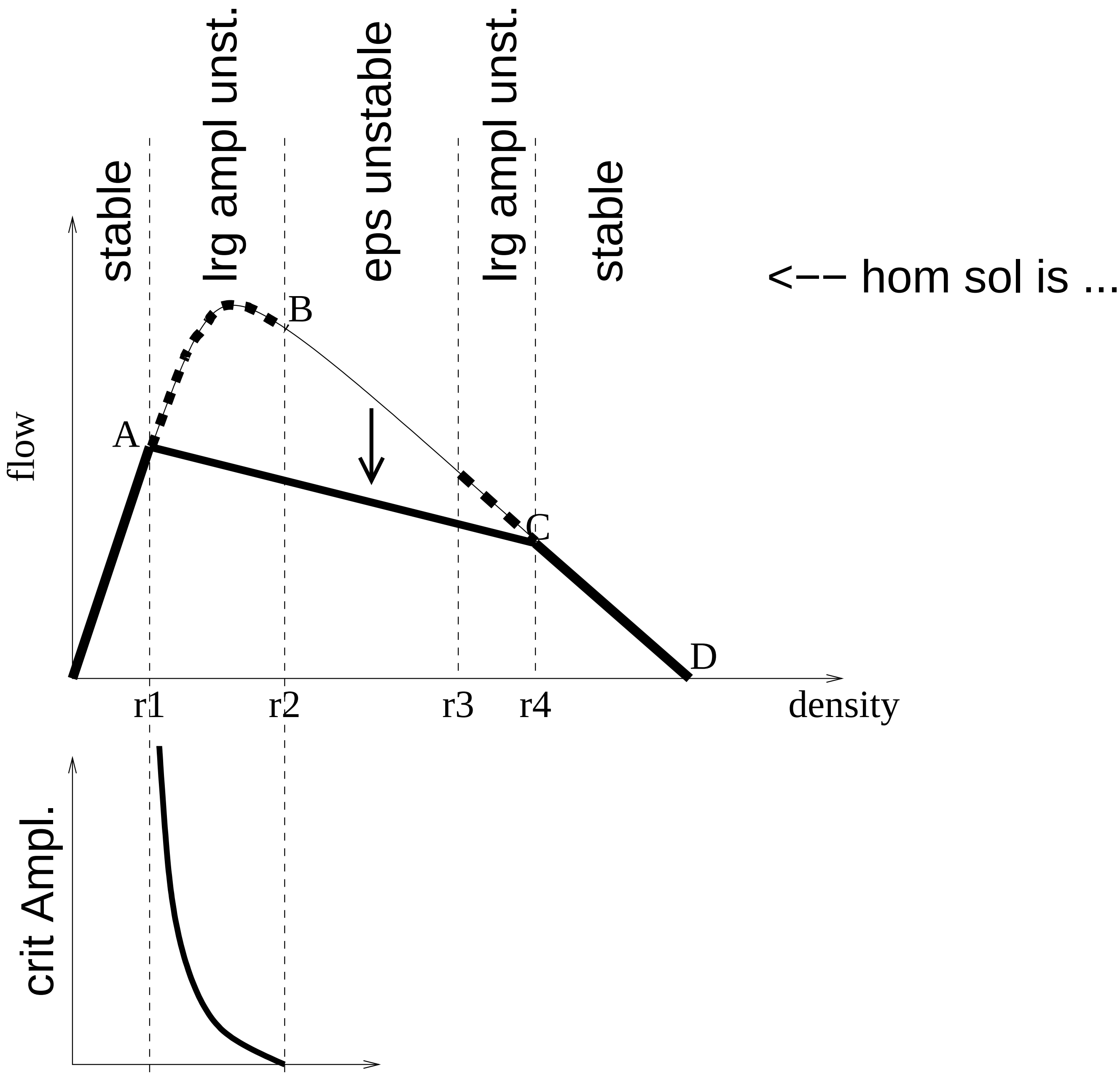
Linear stability analysis. Typically leads to ρ_2, ρ_3 where homogeneous solution is unstable in between:



Large amplitude instability

Linear stability analysis finds when hom sol unstable against ϵ-disturbance.
But what if we kick really hard (large amplitude)?

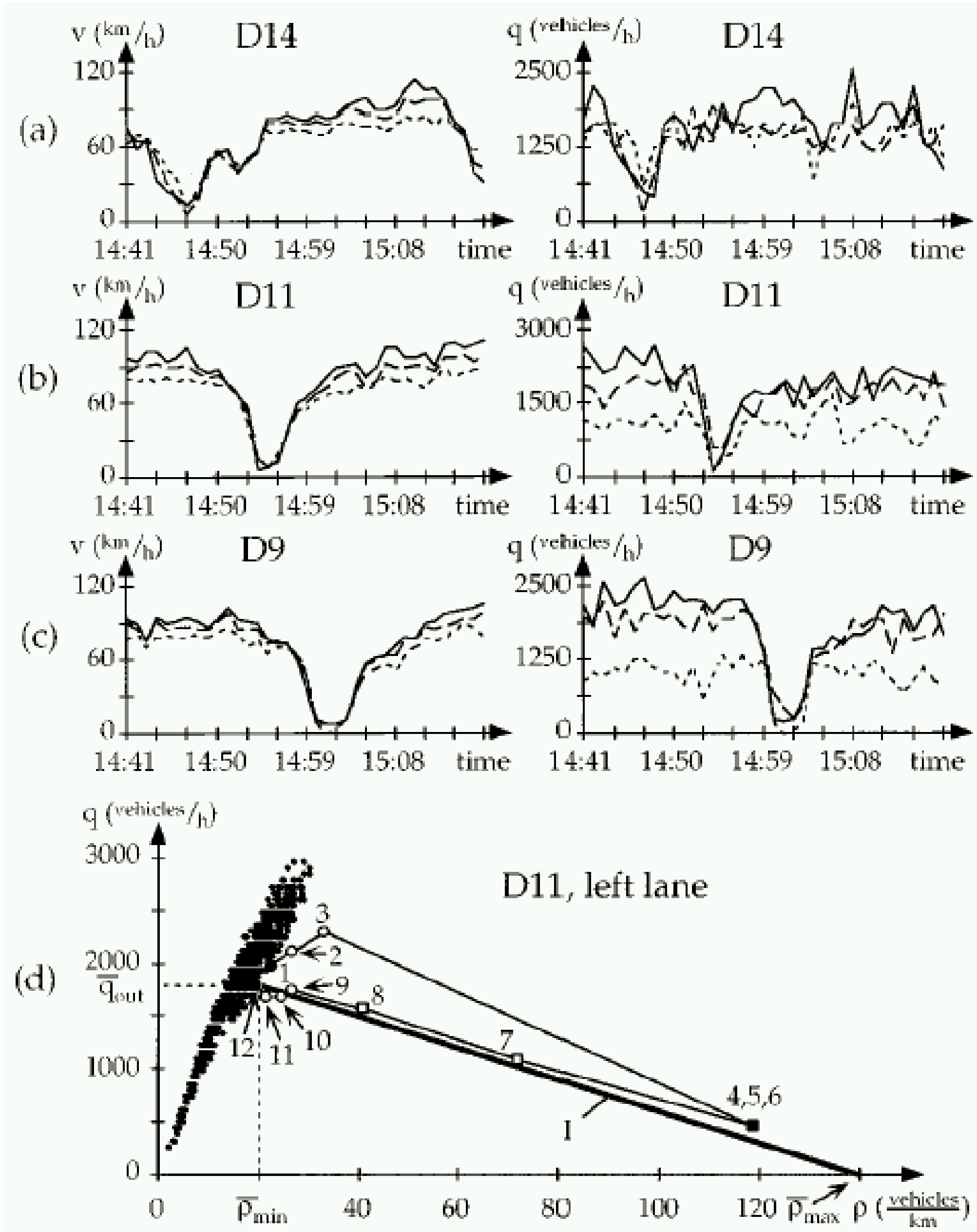
[[OVM3: lrg ampl instab.]]


stable
lrg ampl unst.
eps unstable
lrg ampl unst.
stable
<-- hom sol is ...
flow
A
B
C
D
r1
r2
r3
r4
density
crit Ampl.

Outflow from large jam

[[ovm4]]

“Outflow from jam” = disturbance with maximum amplitude.


(a)
v (km/h)
D14
q (vehicles/h)
D14
(b)
D11
D11
(c)
D9
D9
14:41
14:50
14:59
15:08
time
(d)
D11, left lane
q̄out
ρ̄min
ρ̄max
ρ (vehicles/km)
4,5,6
I

(Kerner)

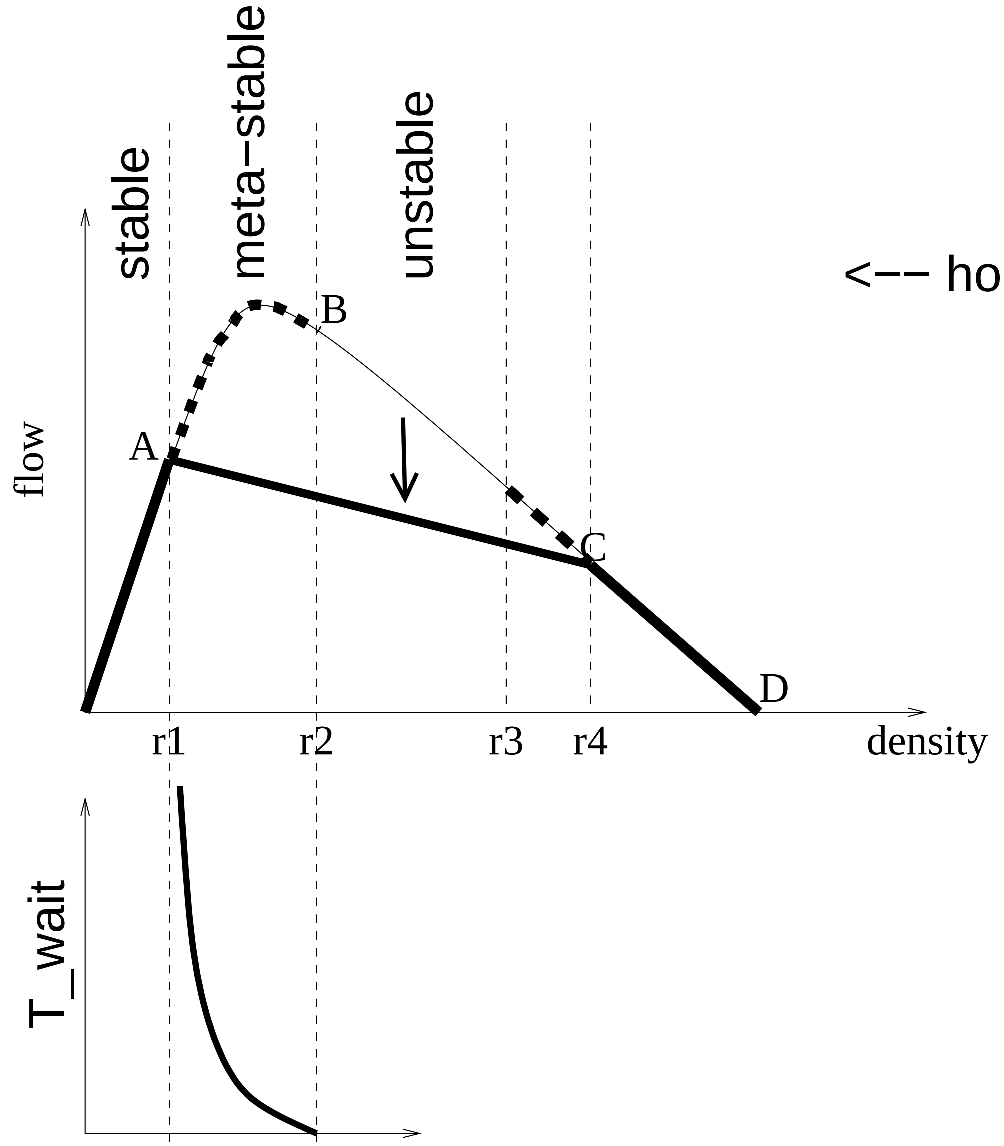
Now add noise

Conventional picture would be (we think):

- In the stable range, noise will not do anything.
- In the linear-unstable range, noise will immediately trigger the instability.
- In the large-amplitude-unstable range, noise will eventually add up in a way that the instability will be triggered.

  In consequence, critical disturbance size A_{bkdwn} should translate into waiting time T_{bkdwn}.

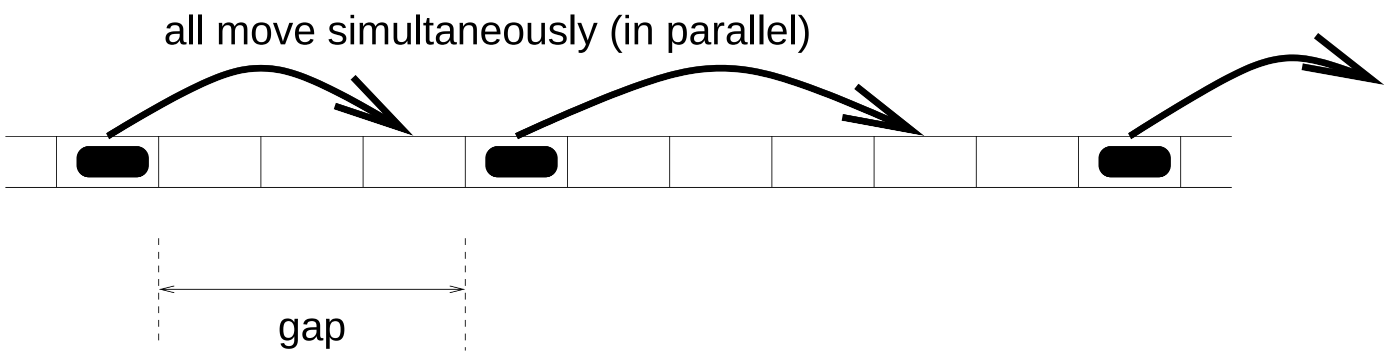
  In particular: $A_{bkdwn} \to 0 \longleftrightarrow T_{bkdwn} \to \infty$.



<−− hom sol is ...

Not systematically tested with OVM ... Use Cellular Automaton instead.

CA for traffic

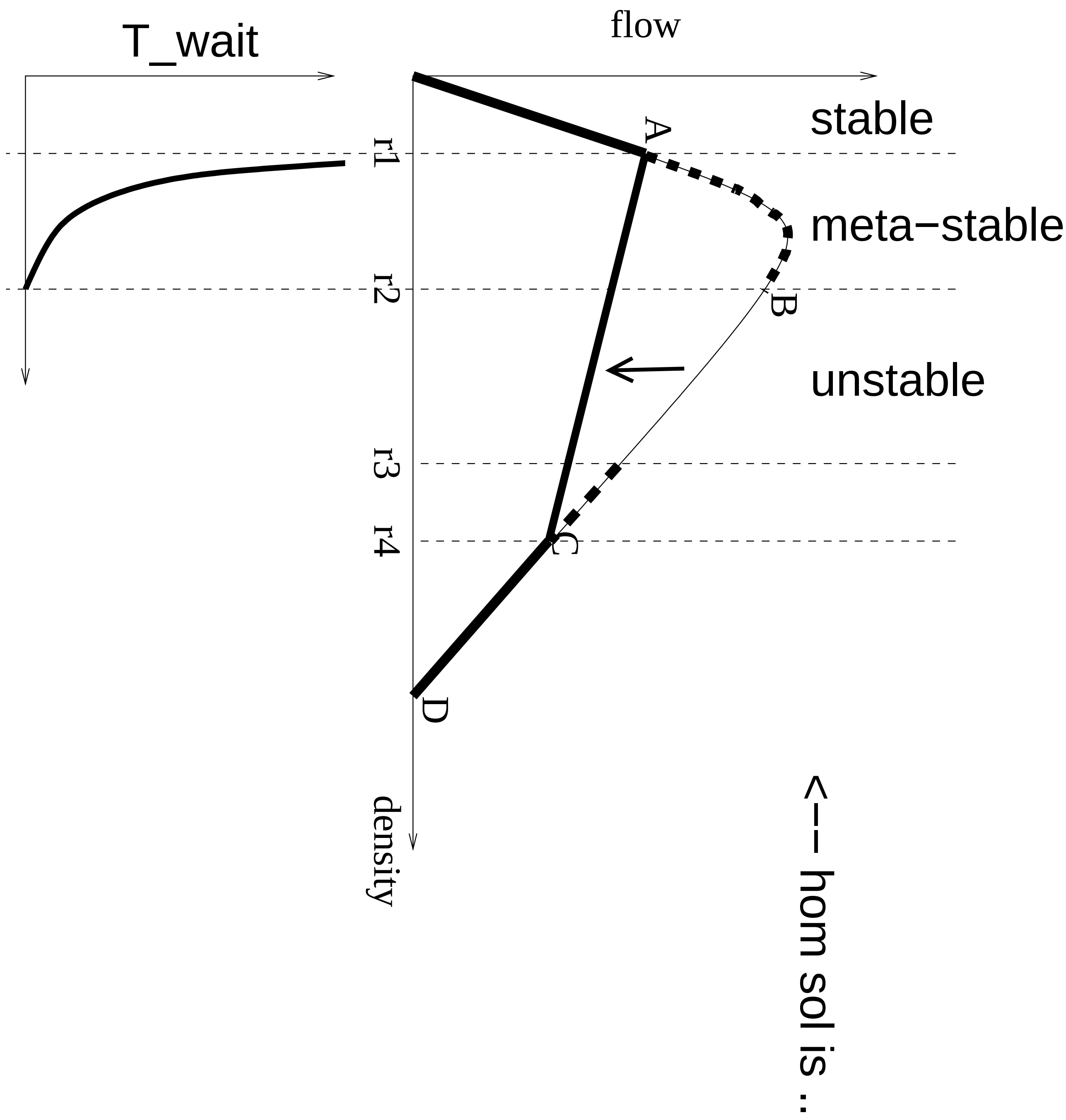


$\ell = 7.5m$, $\Delta t = 1sec$, $v_{max} = 5$. Rules:

1. if ($gap \geq 2$) then $v := \min[v+1, v_{max}, gap]$.
   Acceleration/slow to start, speed limit, car following.
2. ... else (i.e. if $gap \leq 1$) $v := \min[v_{max}, gap]$.
   Speed limit, car following, *no* acceleration.
3. with proba $p_{noise} \sim 0.2$ do $v := \max[0, v-1]$.
   Randomness

[[S2S1]]

[[S2S2: higher density]]


T_wait
flow
stable
meta−stable
unstable
r1
r2
r3
r4
A
B
C
D
density
<-- hom sol is ...

Consistent with field data.

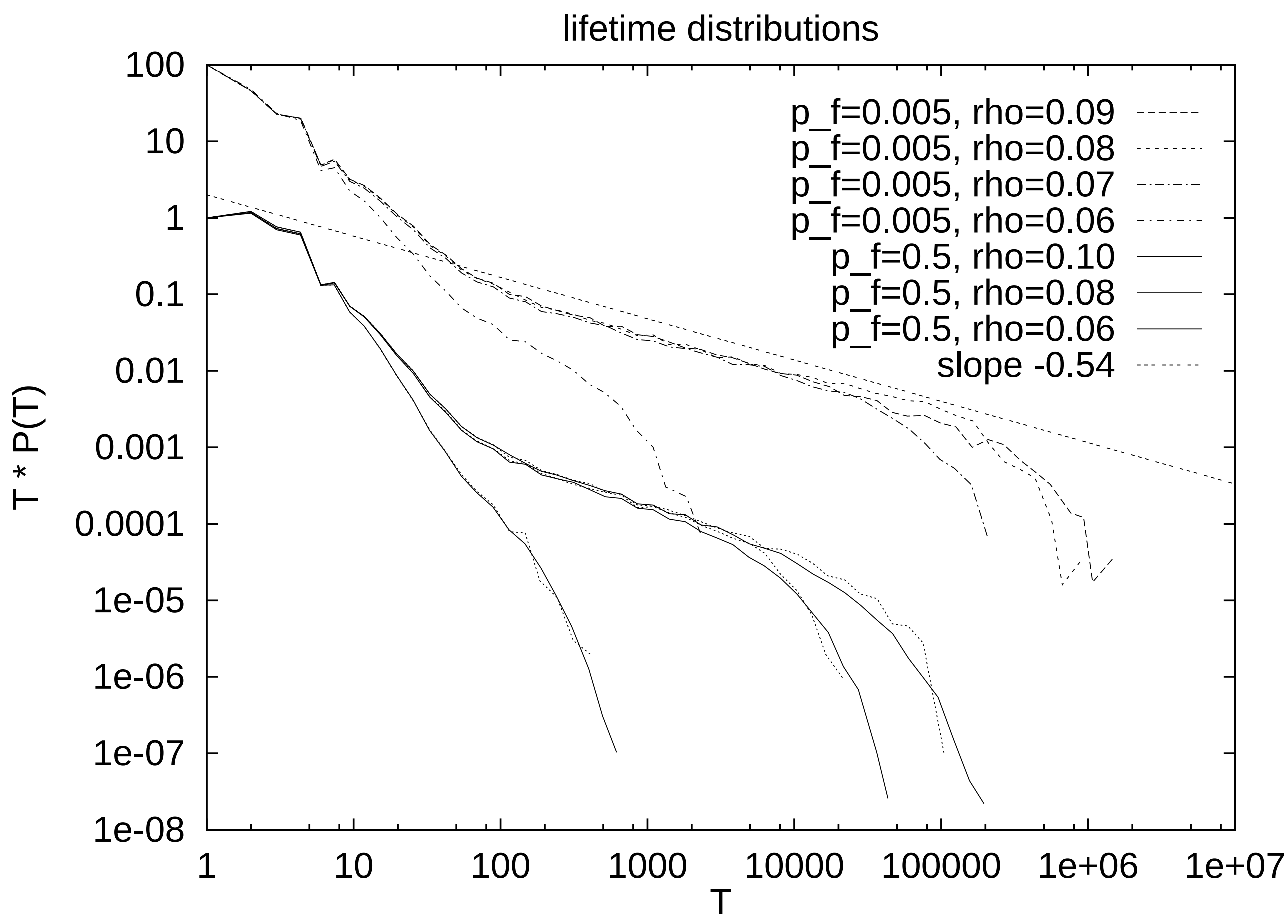
Interesting stuff ...

... happens near (slightly above) outflow density $\rho_{out} = \rho_1$.

[[stca1]]

Lifetime distribution of traffic jams

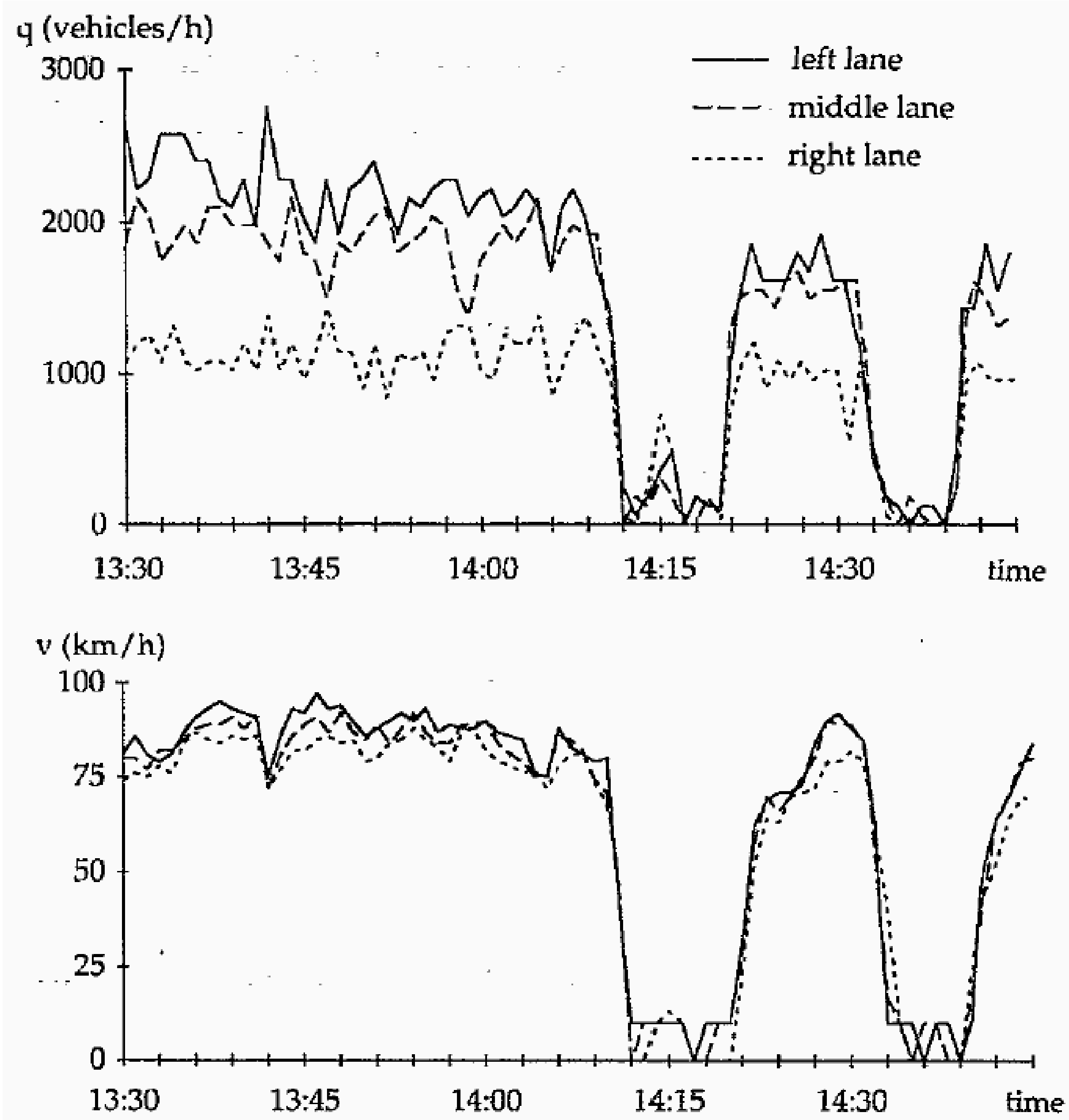
$P(T) =$ probability that traffic jam lives longer than T.

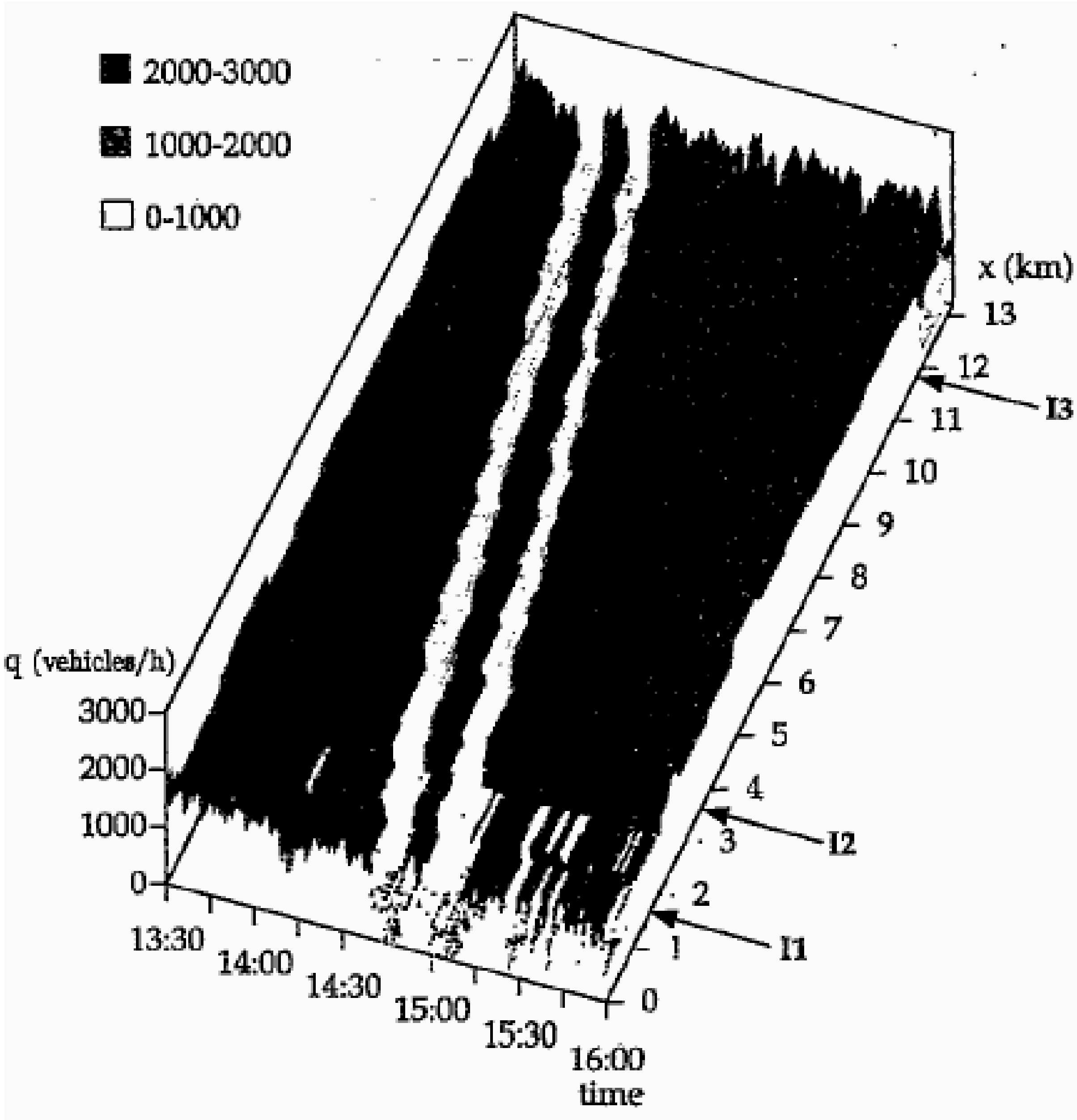


- Scaling law up to cut-off.

- Cut-off moves with noise at free driving, goes away if that noise $\rightarrow$ zero (cruise control limit).

- Internal structure of traffic jams is fractal (not shown).


q (vehicles/h)
3000
2000
1000
0
left lane
middle lane
right lane
13:30
13:45
14:00
14:15
14:30
time
v (km/h)
100
75
50
25
0
13:30
13:45
14:00
14:15
14:30
time

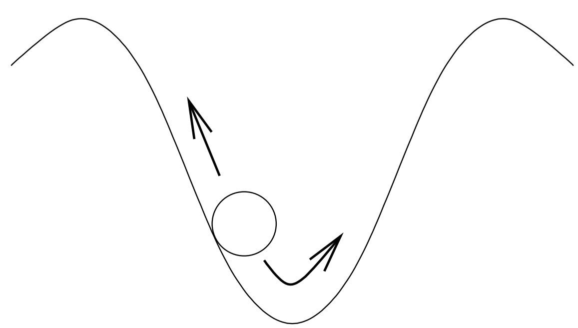

2000-3000
1000-2000
0-1000
q (vehicles/h)
3000
2000
1000
0
13:30
14:00
14:30
15:00
15:30
16:00
time
x (km)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
I1
I2
I3

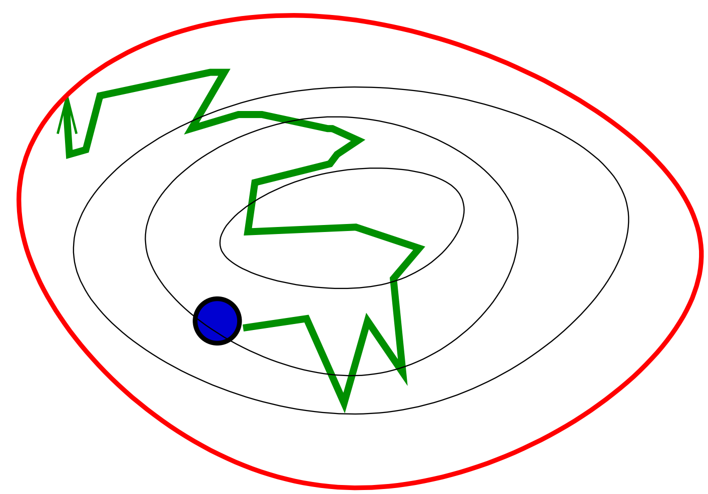
Yet another regime

We had: “In the large-amplitude-unstable range, noise will eventually add up in a way that the instability will be triggered.

In consequence: $A_{bkdwn} \longrightarrow T_{bkdwn}$.

In particular: $A_{bkdwn} \to 0 \;\longleftrightarrow\; T_{bkdwn} \to \infty$.”





This seems to be true for many models ...

... but ...

... there is at least one model where something else is going on.

Krauss model

Discrete time ($1 sec$), continuous space.

1. $v_{safe} := \alpha\, gap + (1-\alpha)\, v_{ahead}$ with $\alpha = \dfrac{2b}{v + v_{ahead} + 2b}$

   [[$b =$ braking capability; $b \to \infty$ results in $v_{safe} = gap$ as before; clean derivation of this possible but beyond this lecture]]

2. $v := \min[v + a, v_{max}, v_{safe}]$.

   Same structure as before. a acceleration.

3. $v := \max[0, v - \epsilon\, a\, \eta]$.

   η random number between zero and one, ϵ noise amplitude.

[[krauss1: s2s w/ immediate breakdown]]

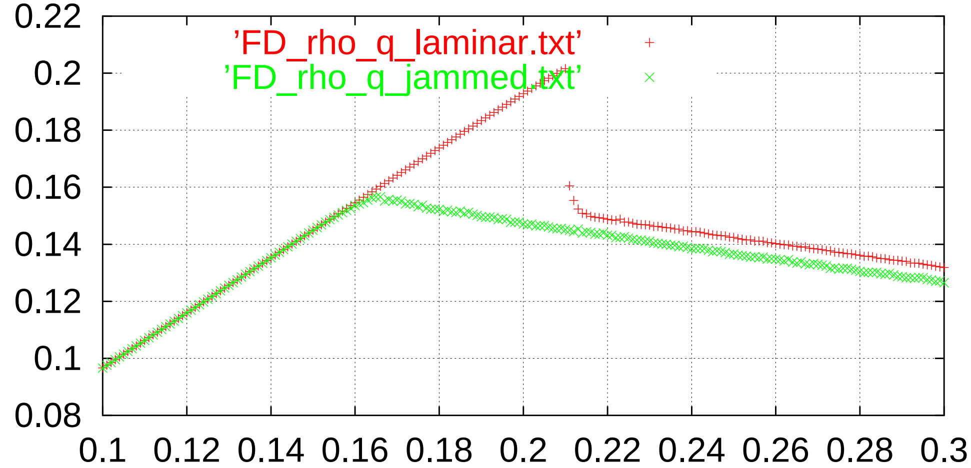
[[krauss2: s2s w/ later breakdown]]

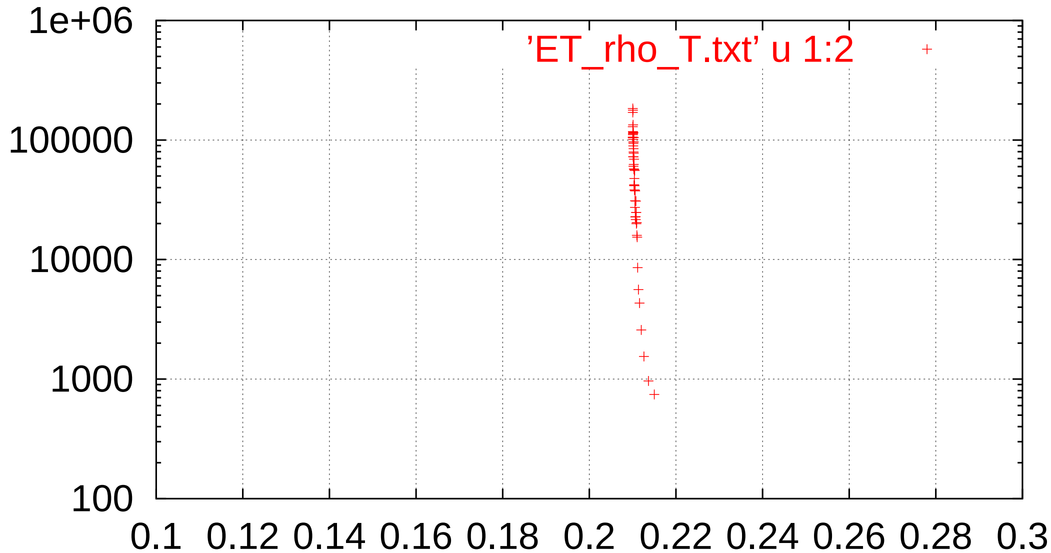
[[krauss3: “new” w/ very late breakdown]]

[[krauss4: “new” smaller density w/o breakdown]]

[[krauss5: “new” same density start w/ megajam]]

system size for demo much too small, have significant finite size effects





Waiting time divergence *not* at outflow density.

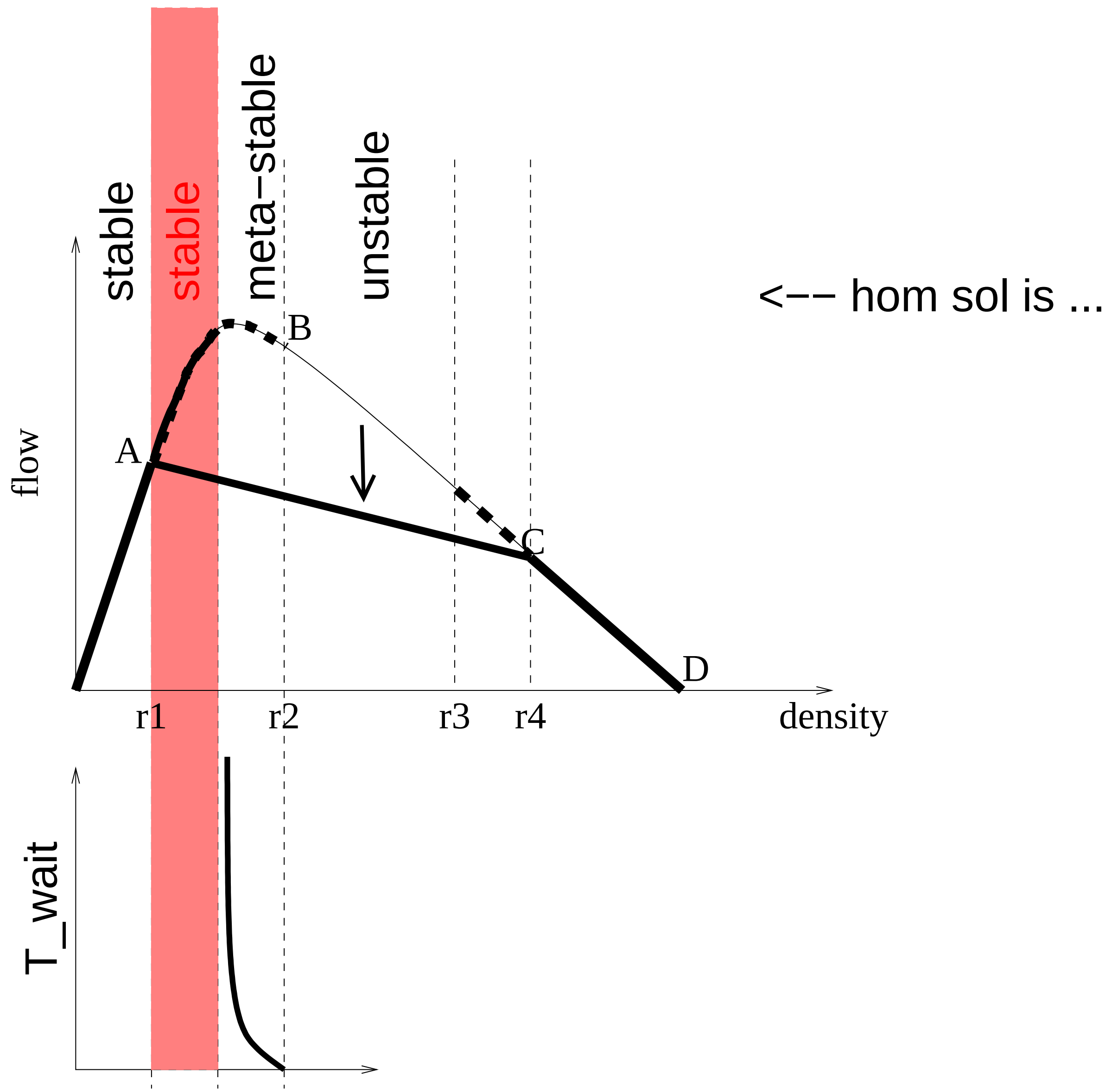
[[Even with simulations can do better than shown:

- Have theory that

$$T_{wait} \sim \exp\left(\frac{1}{\frac{1}{\rho_c} - \frac{1}{\rho}}\right) .$$

- Thus, show via simulations that divergence is indeed significantly away from $\rho_1 = \rho_{out}$.

In progress.]]



Have now truly bi-stable regime under noise.

Intuitive reason?

Noise is injected in a way that it cannot add up.

Parallel update!

Summary

[[drawing!!]]

Transition from 1-phase to 2-phase with increasing density.

Transition in "standard" model is close to critical but not exactly so.

Transition is exactly critical in cruise control limit.

(However, have nothing on "other side". Similar to $noise \to 0$ for slightly supercritical gas??)

New regime: For certain densities, both 1-phase and 2-phase solution are stable under noise.