Tensor Voting:
A Computational Framework for Segmentation and Grouping

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Overview

- Introduction
  - issues
  - difficulties
  - examples
- Salient feature inference
  - tensor voting
  - 2D system
  - 3-D system
- Applications
  - dental, medical
  - shape from shading
  - shape from stereo
  - optical flow
- Perspectives

Problem Statement

- Given sparse input data, generate
  - Regions, curves, junctions in 2-D
  - Volumes, surfaces, curves, junctions in 3-D
  - N-*k* varieties in N-D
- Without models
- In the presence of outlier noise

Issues

- Representation: need to express
  - Geometric entities (points, curves, surfaces,…)
  - Handle arbitrary topology
  - Uncertainty
  - Constraints (continuity, discontinuities)
- Computational model:
  - Explicit vs. implicit optimization
  - Initial conditions, convergence, …

Constraints

Matter is cohesive

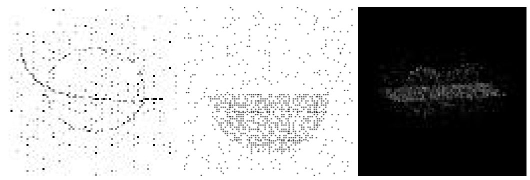
⇓

Smoothness

But not everywhere!

How can we implement it?

Examples



2-D curves 2-D regions 3-D surfaces

Approaches

- Regularization
  - ill-posed problem cast into a scalar, functional optimization
  - iterative
  - issues: choice of optimization functional, initialization, convergence...
- Consistent labeling
  - either a smooth feature, discontinuity, or outlier
  - relaxation (discrete, continuous, stochastic)
  - iterative

Approaches

- Clustering
  - formation of compact groups
  - issue: initialization
- Robust methods
  - parametric model fitting from noisy data set
  - need to specify what we look for
  - iterative
- Artificial neural networks
  - scalars only (Grossberg-Mingolla)

Computational Approach

Grouping/Matching

⇩

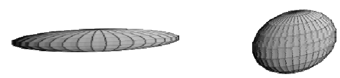
Optimization

⇩

Constraints/Evaluation Criteria

Tensor Voting

- Representation: 2nd order symmetric **Tensor**
  - shape: orientation certainty



  - **size:** feature saliency

Tensor Voting

- Constraint Representation: Voting fields
  - tensor fields
  - encode smoothness criteria
- Communication: **Voting**
  - non-iterative
  - no initialization

Our approach in a nutshell

- Each input site propagates its information in a neighborhood
- Each site collects the information cast there
- Salient features correspond to local extrema

Properties of Tensor Voting

- Linear
- Non-Iterative
- Extract all features *simultaneously*
- 1 parameter (scale)
- Objective thresholds
- Efficient
  - O(1) for parallel computation

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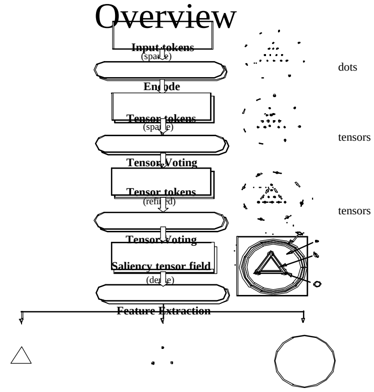
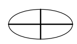


2-D Tensor Voting

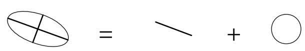
- Representation: 2-D **Tensor**
- Constraints: 2-D **Voting fields**
- Data communication: **Voting**

2-D Tensor

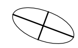
- ∘ and — are extreme cases of a tensor



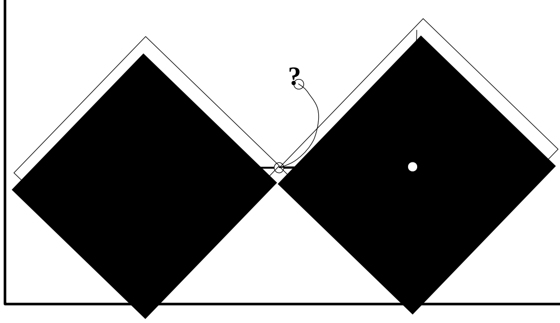
- Conversely, any tensor can be expressed as



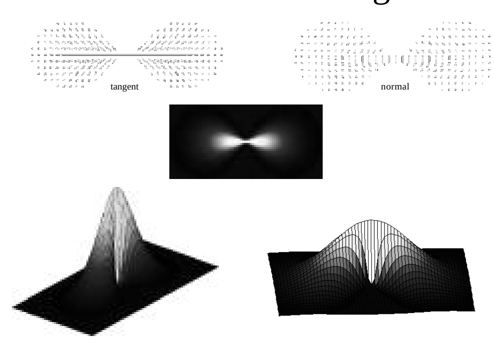
Decomposition & Interpretation

- in 2-D  is 3 numbers λ_{max}, λ_{min}, θ
- λ_{min} represents **orientation uncertainty**
- $(\lambda_{max} - \lambda_{min})$ represents **orientation saliency**

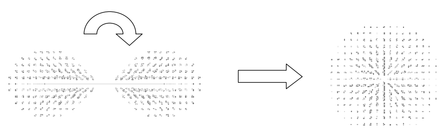
Design of Voting Field



Fundamental Stick Voting Field



2-D Ball Field



2D Voting Fields

Each input site **propagates** its **information** in a **neighborhood**

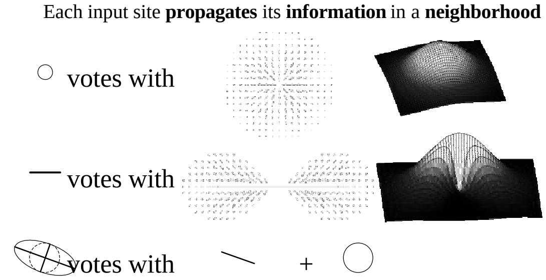
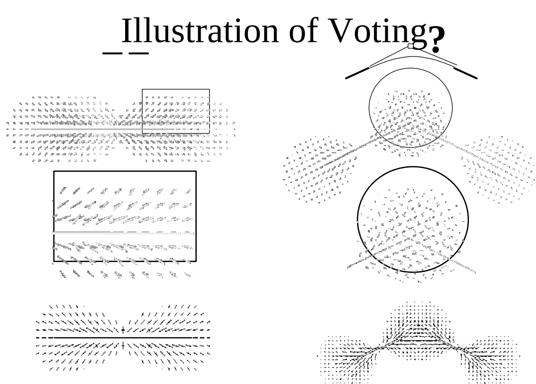


Illustration of Voting



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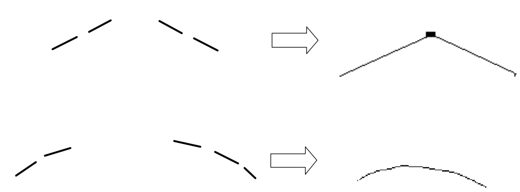
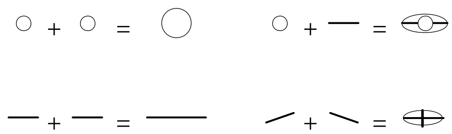


Vote Collection

Each site **collects** the information cast there

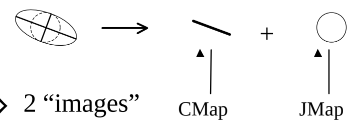
By tensor **addition** :



Vote Interpretation

Salient features correspond to local extrema

At each site

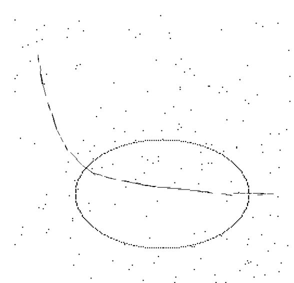


⇒ 2 “images”

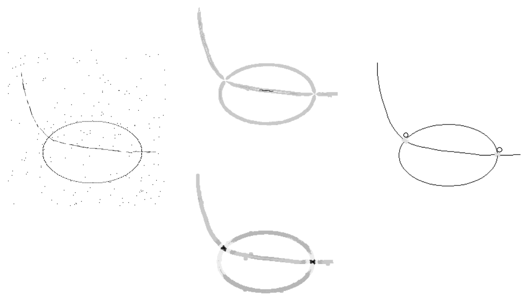
Feature Extraction

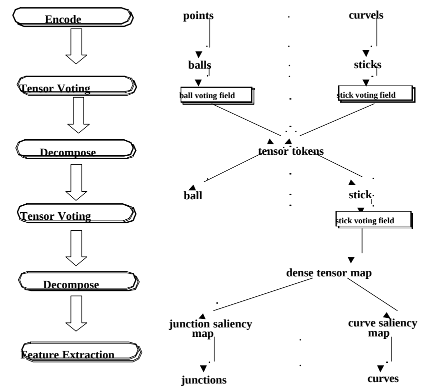
- **Curves** are local maxima of Cmap
- **Junctions** are local maxima of Jmap
- performed by a local marching process

2-D Feature Inference



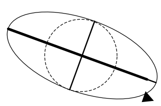
2-D Feature Inference



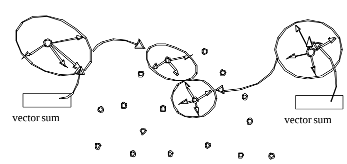


2-D Region Inference

- vote with ball voting field
- collect vector sum to infer polarity



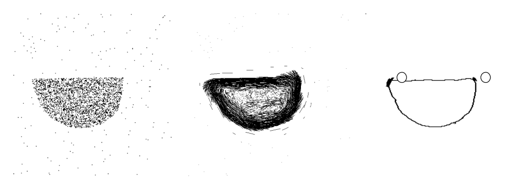
2-D Region Inference



2-D Region Inference

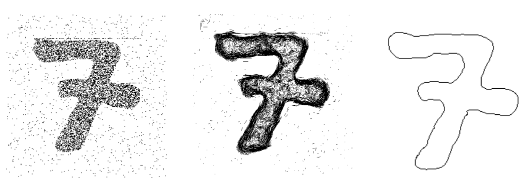
- Produces Boundariness Map
- Local extrema are boundary elements
- Regular (2^{nd} order tensor)voting generates boundary curve and junctions.

Results



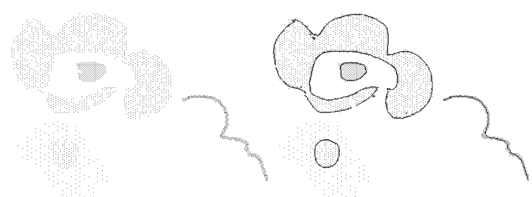
input points | region boundariness | curve and junctions

Results

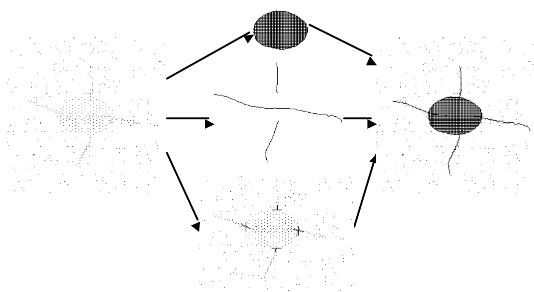


input points | region boundariness | curve

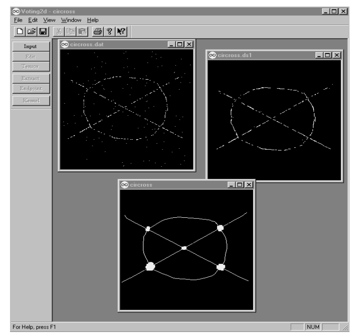
Results



Results



2-D system demo

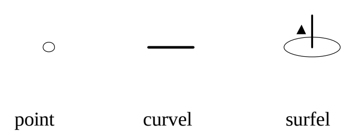


3-D Tensor Voting

- Representation: 3-D Tensor
- Constraints: 3-D Voting fields
- Data communication: Voting

3-D Tensor

Input may consist of



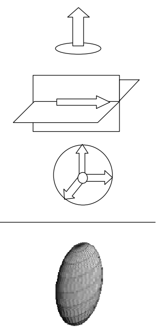
3-D Tensor = Ellipsoid

SMOOTH SURFACE

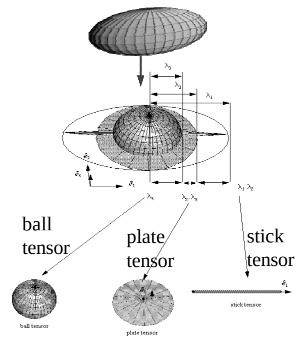
\+ CURVE JUNCTION

\+ POINT JUNCTION

= ELLIPSOID (TENSOR)



Decomposition



3 eigenvalues (l_{max} l_{mid} l_{min})

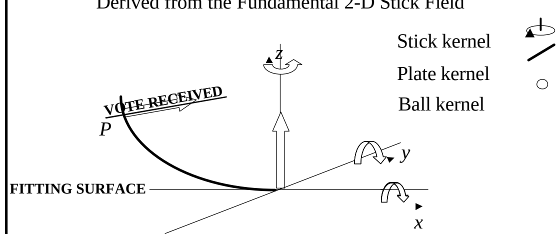
3 eigenvectors ($\mathbf{V}_{max}$ $\mathbf{V}_{mid}$ $\mathbf{V}_{min}$)

Interpretation

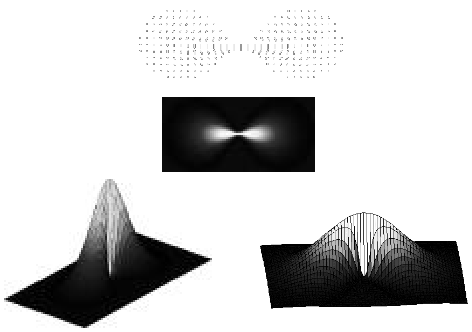
	Saliency	Direction
Surface	$l_{max} - l_{mid}$	**V_{max} = normal**
Curve	$l_{mid} - l_{min}$	**V_{min} = tangent**
Junction	l_{min}	**arbitrary**

3-D Voting Fields

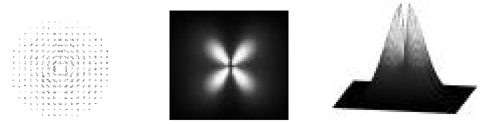
Derived from the Fundamental 2-D Stick Field



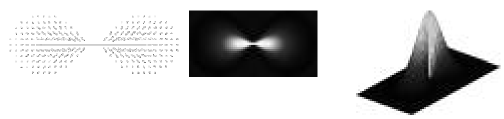
3-D Stick Voting Field



3-D Plate Voting Field



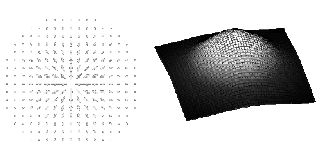
$\lambda_{max} - \lambda_{mid}$

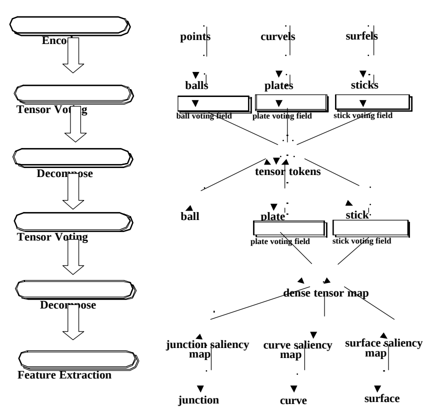


$\lambda_{mid} - \lambda_{min}$

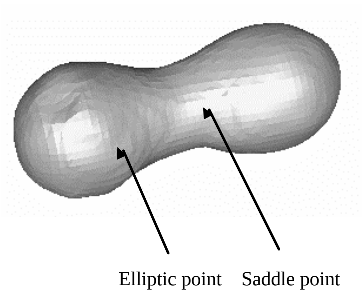
3-D Ball Voting Field

- Isotropic tensor field

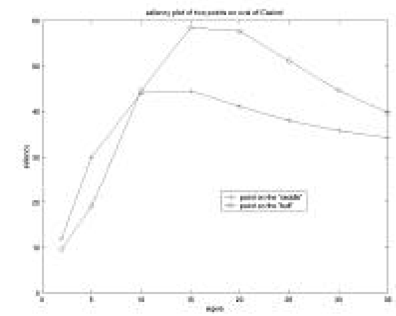




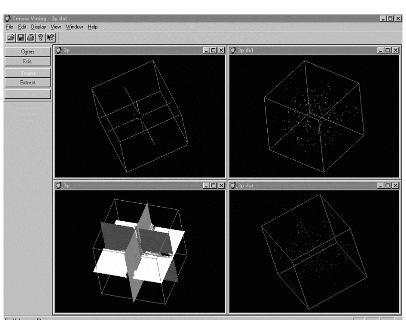
Issue of Scale



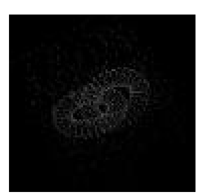
Scale

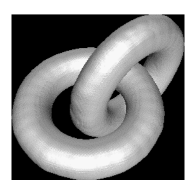


3-D System

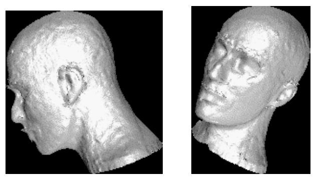


3-D Feature Inference



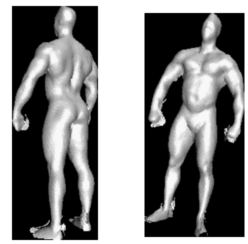


Results



noisy input two views of output surface

Results



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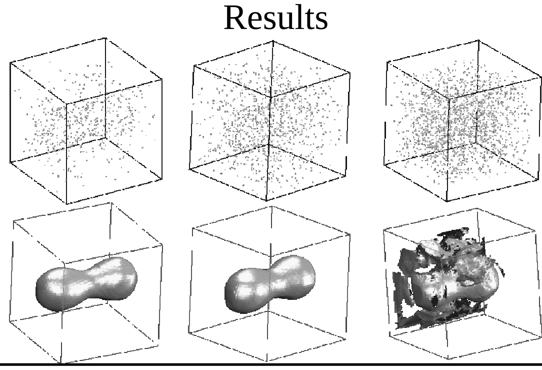

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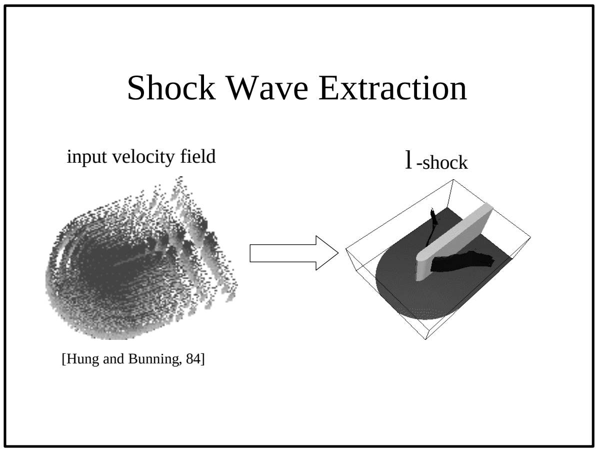
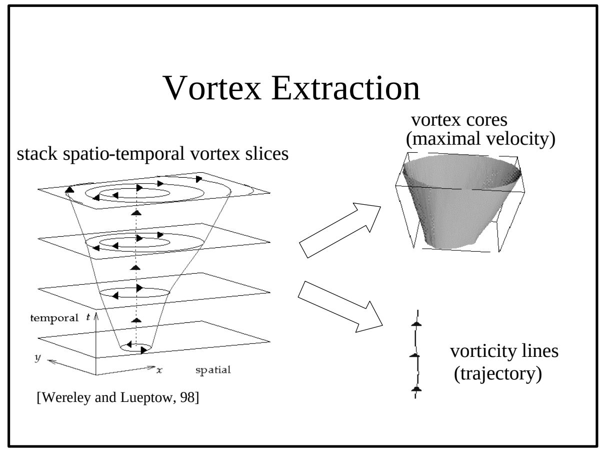
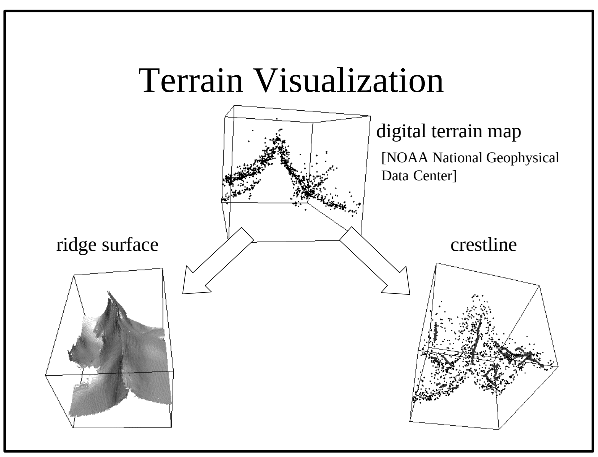
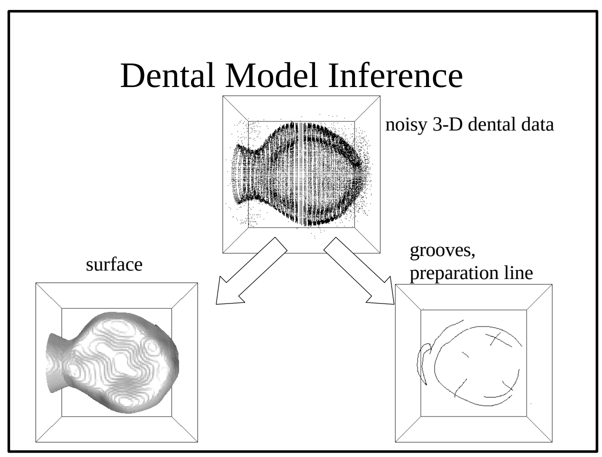

Shock Wave Extraction
input velocity field
l-shock
[Hung and Bunning, 84]

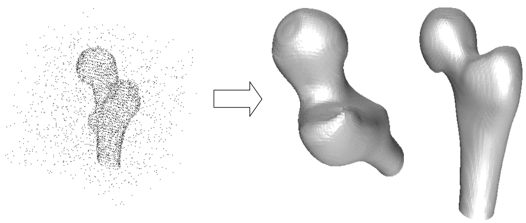

Vortex Extraction
stack spatio-temporal vortex slices
vortex cores
(maximal velocity)
temporal t
y
x
spatial
vorticity lines
(trajectory)
[Wereley and Lueptow, 98]


Terrain Visualization
digital terrain map
[NOAA National Geophysical Data Center]
ridge surface
crestline


Dental Model Inference
noisy 3-D dental data
surface
grooves,
preparation line

Femur reconstruction

- Reconstructing the proximal bone of lower limb



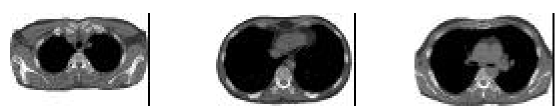
MRI/CT Segmentation (with C.-K. Tang)

- Intensity thresholding
- Vote for bounding surfaces
- Multiple scales
- Vote for surfaces

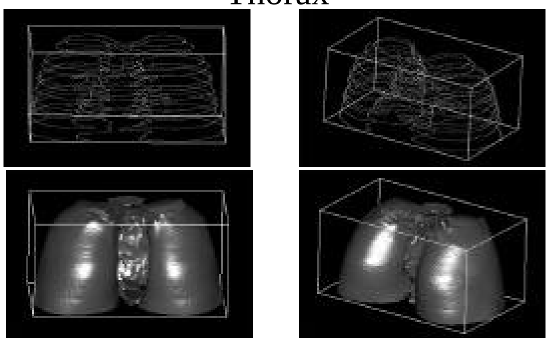
Data Sets

- Thorax
  12 CT slices
  (courtesy of Washington Health Science Ctr)
- Brain
  McGill Brainweb

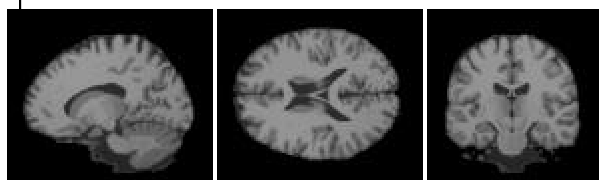
Thorax



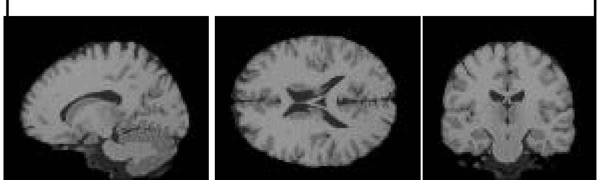
Thorax



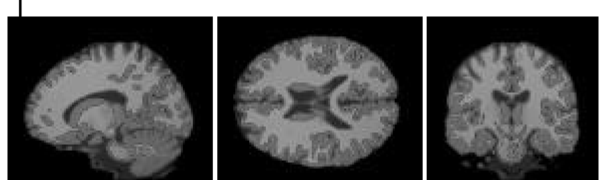
CSF Boundary



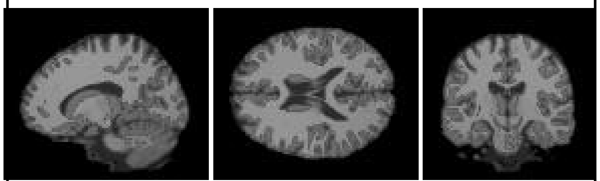
Gray Matter Boundary



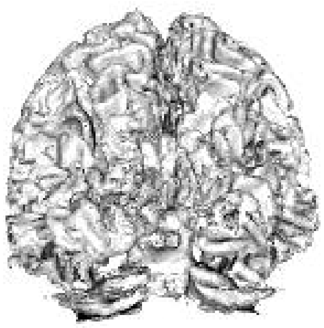
White Matter Boundary



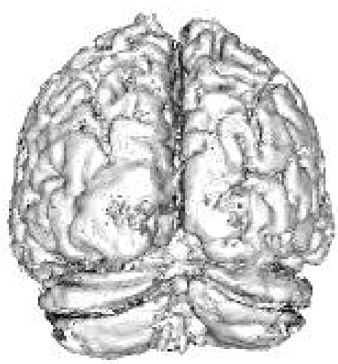
Cortex Boundary



Inner Cortex Surface



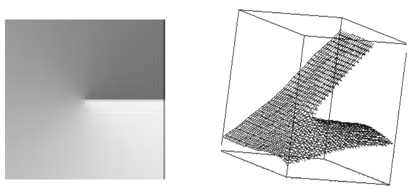
Outer Cortex Surface



Low-level vision

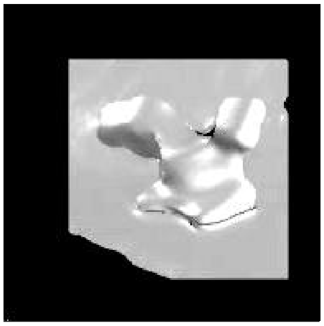
- Shape from shading
- Stereo
- Motion

Results



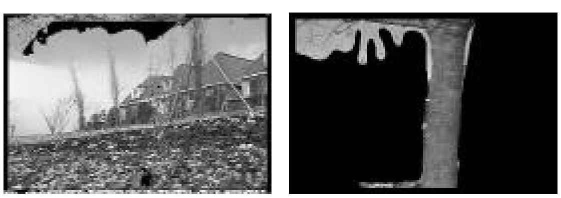
Renault Pair

Results



Flower Garden Sequence

Flower Garden Sequence Layers



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Conclusion

- Unified framework
- Applicable to many problems
- Non-iterative optimization
- Promising results
- Issues ...

Perspectives

- Stronger mathematical validation
- 1^{st} and 2^{nd} order Voting Integration
- Multiple scales Integration
- N-D extensions