⊛ Type II A Orientifolds with Intersecting D6-branes

$\Updownarrow$

M-theory on Compact Singular G_2 Holonomy Spaces

⊛ Employ Conformal field theory technique to quantize string theory

$\Downarrow$

calculate physical excitation (particles) & interactions among them

$\Downarrow$

Particle physics implications (Standard Model & Grand Unified Model) in $D=4$ & $N=1$ Supersymmetry

Outline

1 Essential steps in construct of Type IIA orientifolds w/ intersecting D6 bran

2 Relationship to M th on G_2

3 Specific Construction $(T^6/(Z_2 \times Z)$ orientifd,

$\downarrow$

SM GUT

4 CFT of couplings

W/ G. Shiu & A. Uranga hep-th/0107143
0107166
(original $T^6/(Z_2 \times Z_2)$ - orientifold w/ D6 :
3 family Standard Model, & 4-family GUT)
hep-th/0111179
(relationship to M-theory on G_2)

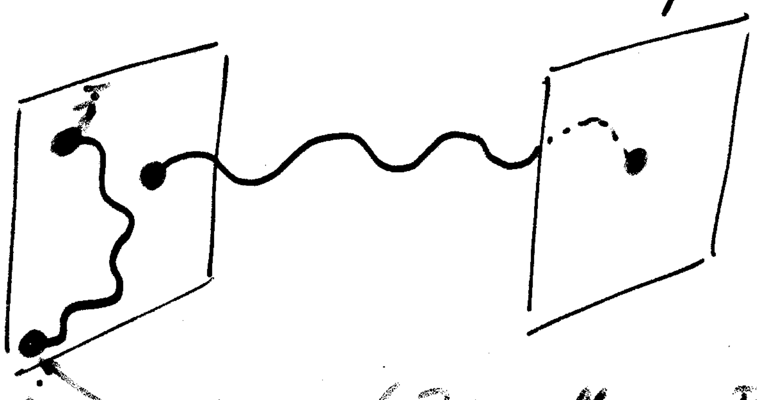
W/ G Shiu & I Papadimitriou
(systematic search for 3 family GUT) hep th/0212177

w/ I. Papadimitriou hep th/0303 97
(More 3-family SM s)

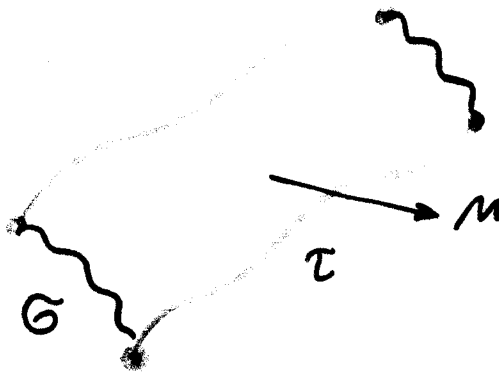
W/ I. Papadimitriou hep-th/0303083
(conformal field theory coupling calculation)

(A) D-brane - boundary of open strings

$D = 10$



charge (λ_{ij} - Chan-Paton factors)



$X^\mu(\sigma,\tau)$ - target space

world-sheet

Dp-brane $\quad$ $\mu = 1,\dots,p+1$ Neumann $\quad$ bound. cond.

$\mu = p+2 \dots 10$ Dirichlet $\quad$ $[X^\mu(\sigma=0),\ X^\mu(\sigma=\pi)]$

Quantization of string theory -
(Nambu Goto action w/ conformal invariance)
eqs. for X → free field theory w/ Neum. & Dirichlet b.c.

↓

2d conformal field theory techniques

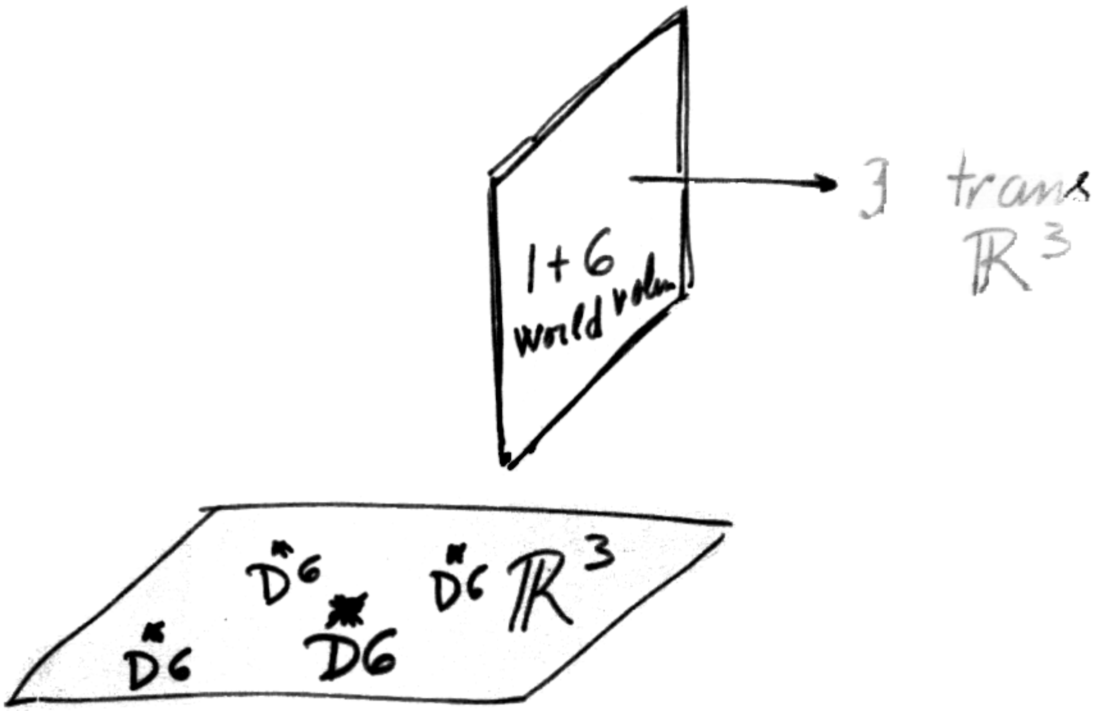
B) D p - brane source for $C_{(p+1)}$ potential

$dC_{(p+1)}$ $\quad F_{(p+2)}$ $\quad {}^*F_{(8-p)}$

(field strength)
Ramond-Ramond sector

D6 $\quad C_{(7)}$ $\quad F_{(8)} = {}^*F_{(2)}$

(magnetic source ${}^*F_{(2)}$)

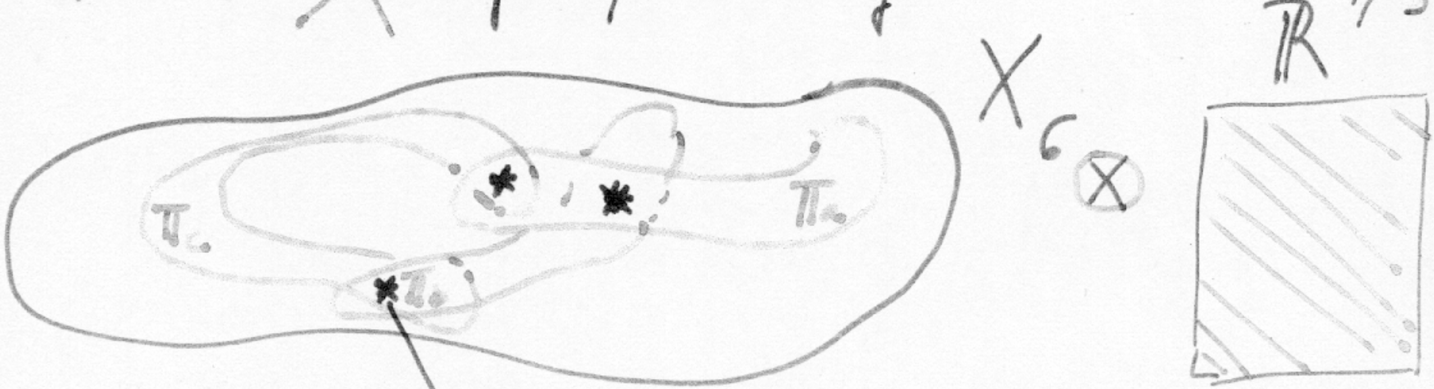


Type II A String-theory on

$\mathbb{R}^{1,3} \times X_6$ + D6-branes

(Calabi-Yau threefold)

$D=4$ $N=\cancel{2}\,1$ supersymmetry



intersection of 3-cycles

D6-branes wrap 3-cycles $\Pi_a \subset X_6$

Π_a - Special Lagr.

Spectrum: 4 dim.: gravity (closed string spectr

4 dim.: gauge degrees

N_a - # of branes wrapping Π_a $\quad \prod_a U(N_a)$

4 dim.: chiral superfields (MATTER

$I_{ab} = [\Pi_a]\cdot[\Pi_b]$ $\quad \sum I_{ab}\,(\square_a, \bar{\square}_b)$

(homol. class) × [hom a, b $\quad$ intersection number

Global consistency conditions

D6-brane – source for $C_{(7)}$

eqs for $C_{(7)}$ → Gauss law for charge conservation

$$\sum N_a [\Pi_a] = 0$$

Cannot satisfy unless

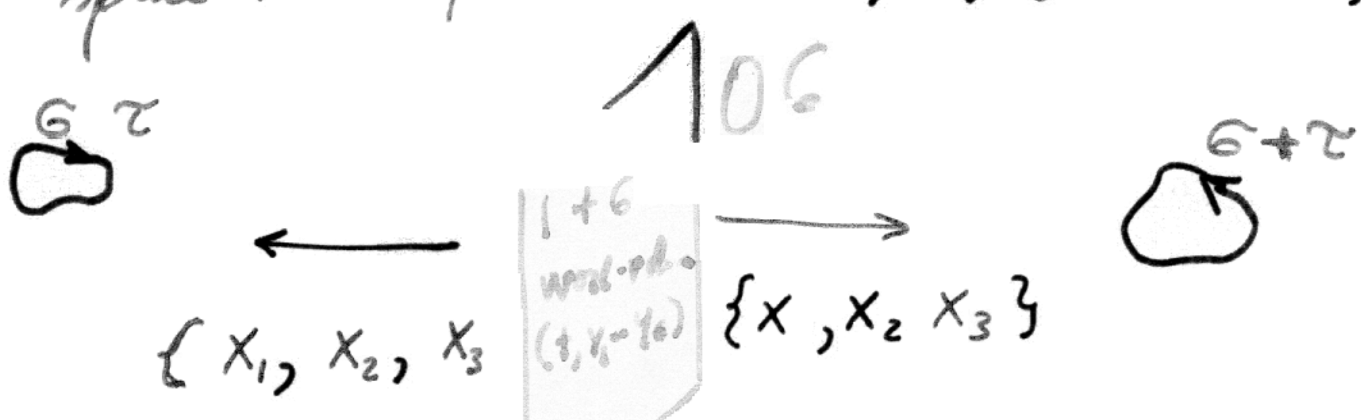
add anti-branes → ~~break~~ SUSY

add orientifold (O6) planes

ΩR sym str. action

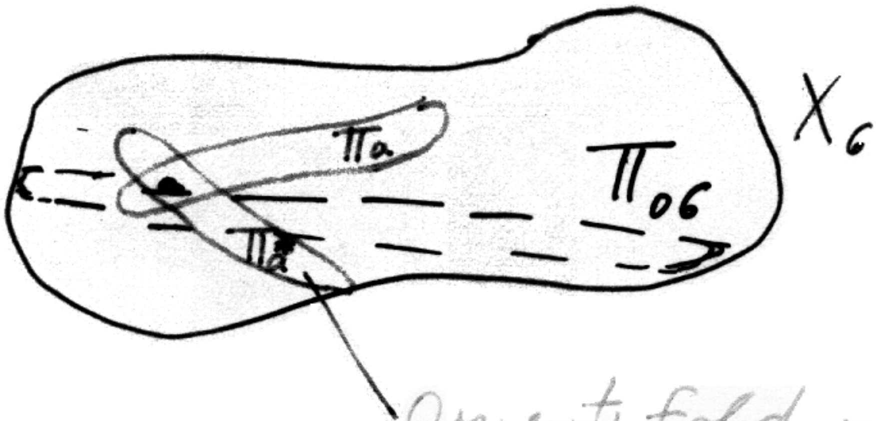
Ω world sheet parity projection $X^\mu(\sigma+\tau) \to X^\mu(\sigma-\tau)$

space time reflection $(x_1, x_2, x_3) \to (-x_1, x_2, -x_3,$



Negative tension (4) units of brane charge

Consistency condition (tadpole cancellation)



$$\sum_a N_a [\Pi_a] + \sum_a N_a [\Pi_a^*] \quad 4[\Pi_{O_6}] \equiv 0$$

global charge conservation

(subtleties for the spectrum

$I_{ab} + I_{a^*b^*}$ $\quad (\square_a \; \square_b)$

$I_{ab} + I_{ab}$ $\quad (\square_a \; \square_b)$

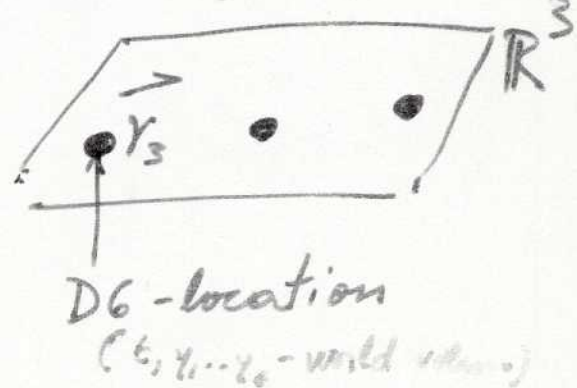
$I_{aa^*} + I_{aa}$ $\quad (\begin{smallmatrix}\square\\\square\end{smallmatrix}_a + \square\square_a$

Relation to M-theory on singular, compact G_2

$D=10$: Type IIA w/ D6-branes:

$$ds_{10}^2 = U^{-\frac{1}{2}}\left(-dt^2 + d\vec{y}_6^{\,2} + U\, d\vec{r}_3^{\,2}\right)$$

$$\vec{\nabla}_{r_3} \times \vec{A} = \nabla U \qquad (F_{\mu_1\mu_2})$$



$$e^{-2\phi} = U^{3/2} \sim g_s^{-2}$$

$$U = 1 + \sum_{I=1}^{N} \frac{4m}{|\vec{r} - \vec{r}_I|}$$

$$\text{K K reduction} \uparrow \downarrow \text{lift on } S^1$$

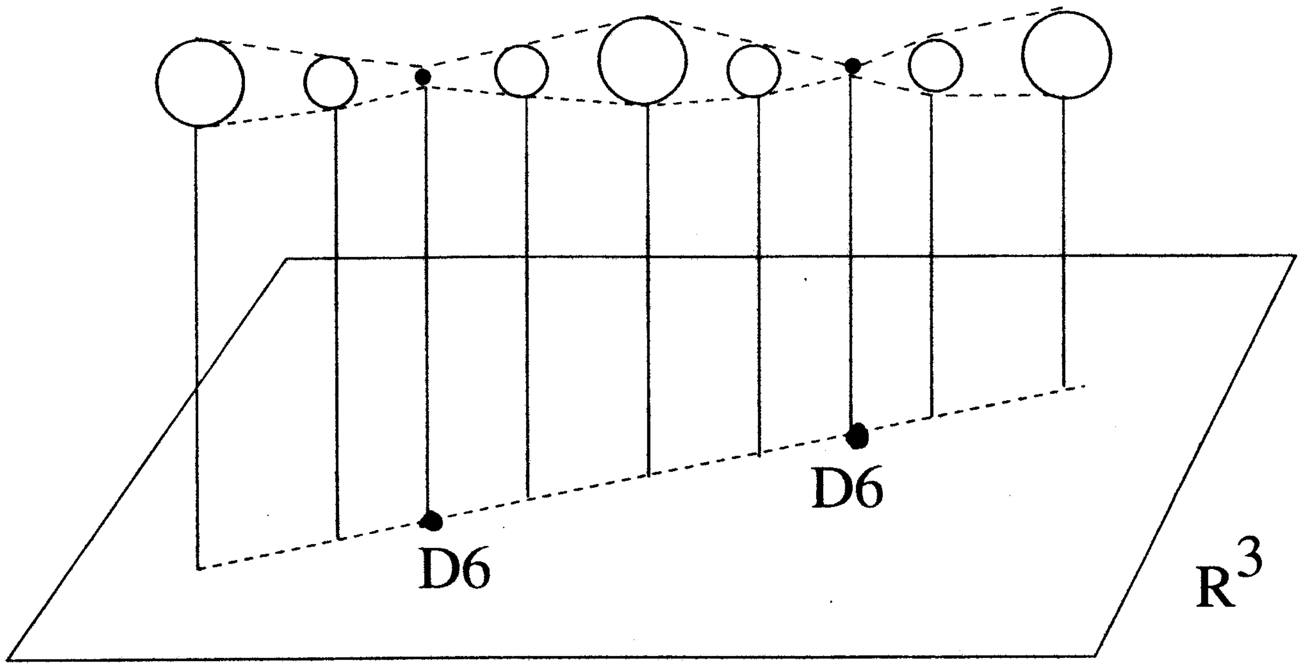
M-th ($D=11$ Supergr.) w/ metric-only (geometry!)

$$ds_{11}^2 = e^{-\frac{1}{6}\phi}\, ds_{10}^2 + \underbrace{e^{\frac{4}{3}\phi}}_{R^2_{S_1}}\left(d\theta + \vec{A}\, d\vec{r}_3\right)^2$$

$R^2_{S_1}$ - radius of circle

$$= \underbrace{\left(-dt^2 + dy_6^2\right)}_{\mathbb{R}^{(1,6)}} + \underbrace{ds_{TN}^2}_{\times\,(\text{Taub-Nut})}$$

co-dimension 4-singularity in M-theory



$$
\begin{aligned}
ds^2_{TN} &= U^{-1}\left(dx^4 + \vec{\omega}\cdot d\vec{r}\right)^2 + U\,d\vec{r}^{\,2} \\
U &= 1 + \sum_{I=1}^{N} \frac{4m}{|\vec{r}-\vec{r}_I|} \\
\vec{\nabla}\times\vec{\omega} &= -\nabla U
\end{aligned}
$$

$D=10$ Type IIA w/ O6-plane (negative tension)

$$\text{KK} \Big\Updownarrow \text{lift}$$

M-theory w/ metric only
$\mathbb{R}^{1,6} \times$ (Atiyah-Hitchin)

Seiberg & Witten

Type IIA on X_6 (CY_3)
w/ D6 branes wrapping 3 cycles $\Pi_a \subset X_6$
&
O6 plane wrapping 3 cycles $\Pi_{O6} \subset X_6$

$\Big\}$ $D=4$, $N=1$ SUSY

$$\text{KK reduction} \Big\Updownarrow \text{lift on } S^1$$

M-theory w/ metric only — pure geometry:
$\mathbb{R}^{1,3} \times$ (G_2 holonomy)

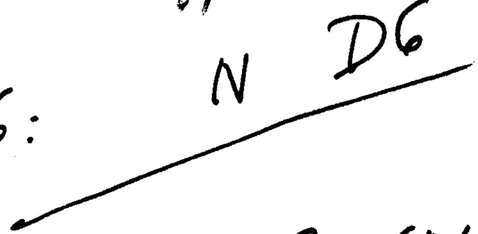
G_2: Hyperkähler fibered over each component of Q

$Q = \{ \cup \Pi_a \cup \Pi_{O6} \}$

Singularities of G_2

Acharya & Witten Witten
w/ Shiu & Uranga

Type IIA on X_6 $\longrightarrow$ M on G_2

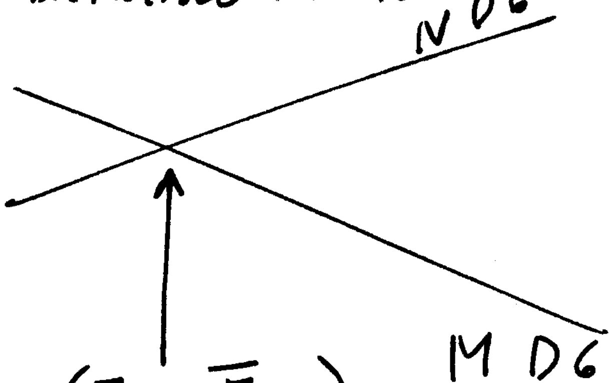
D6: 

$\longrightarrow$ co-dimension 4-sing.

A_{N-1}

$SU(N)$

Vector fields $\underline{\text{Adj}_N}$

Intersections 

$(\square_N, \bar{\square}_M)$
chiral fermions

$\longrightarrow$ codimension 7-sing
unfolding of Katz & Vafa

$SU(N+M)$ A_{N+M-1}

$\downarrow \text{Adj}_{N+M}$ $\qquad \downarrow$

$SU(N)\times SU(M)$ $A_N \times A_{M-1}$

$\text{Adj}_N,\ A_{N+M}$

$+$

$(\square_N, \bar{\square}_M)$

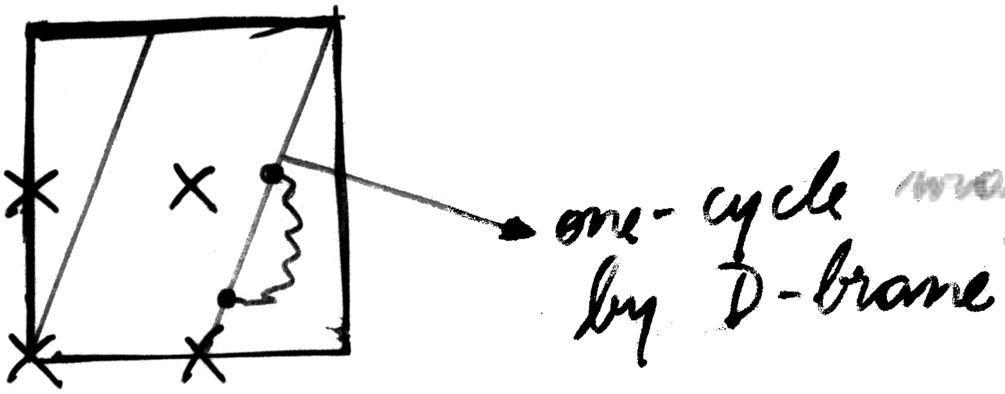
Explicit constructions

X_6 = orbifold $\left(T^6/(Z_N \times Z_M)\right)$

$\downarrow$

* Flat, compact spaces w/ isolated conical singularities w/ discrete $SU(3)$ holonomy

T^2/Z_2



* String theory - Free (w/ nontrivial boundary conditions)

$\downarrow$

quantize exactly (Conformal field th.)

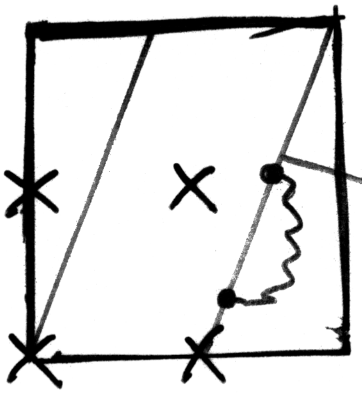
spectrum & couplings calculable

Explicit constructions

X_6 = orbifold $\left(T^6/(Z_N \times Z_M)\right)$

⊛ Flat, compact spaces w/ isolated conical singularities w/ discrete SU(3) holonomy

T^2/Z_2



one-cycle wrapped by D-brane

⊛ String theory – Free (w/ nontrivial boundary conditions)

quantize exactly (Conformal field th.)

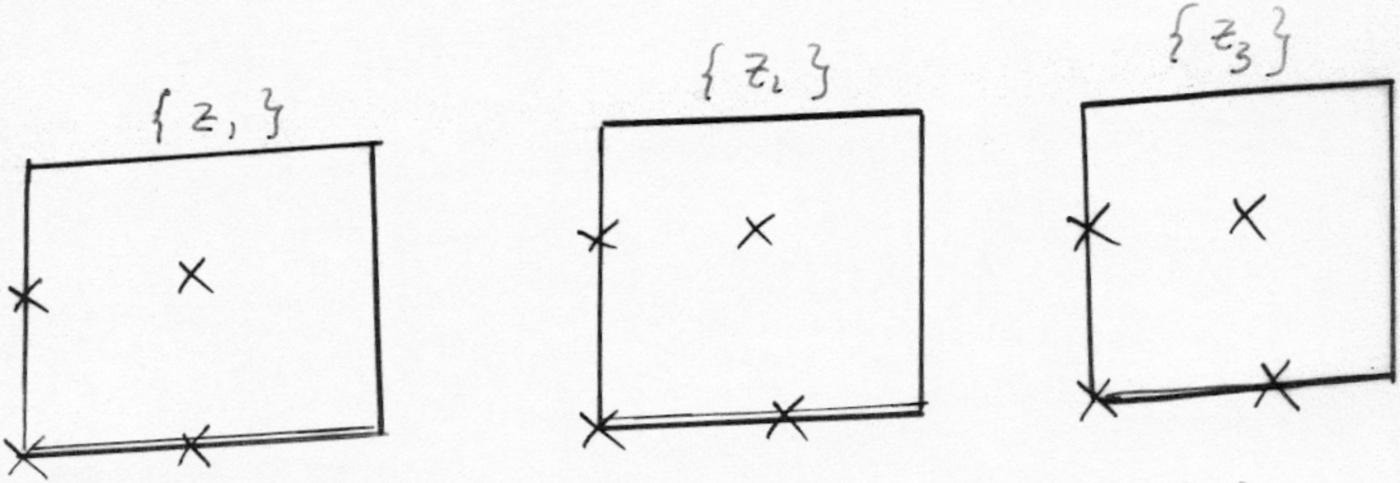
spectrum & couplings calculable

Type II A w/ intersecting D6-bra
W/ Shiu & Uranga'0

Model Highlights:

$T^6/(Z_2 \times Z_2)$ orbifold

$T^6 = T^2 \times T^2 \times T^2$



$(Z_2)_1 : \theta : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, +z_3)$

$(Z_2)_2 : \omega : (z_1, z_2, z_3) \rightarrow (-z_1, z_2, -z_3)$

$\Omega R :$ $(z_1, z_2, z_3) \rightarrow (\bar{z}_1, \bar{z}_2 \quad \bar{z}_2)$

worldsheet parity projection

Z_2 projection O_6-plane

Intersecting D6-branes:

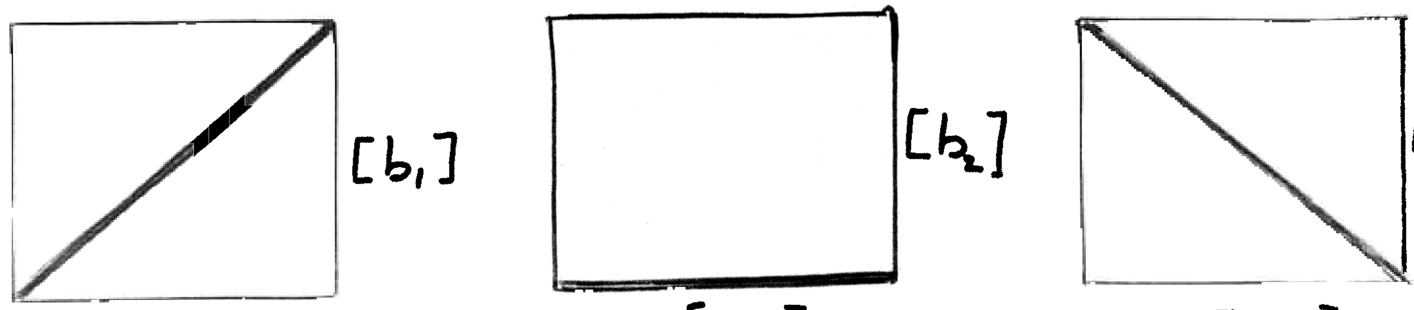
wrap 3-cycles of $T^6 = T^2 \otimes T^2 \otimes T^2$

$$[\Pi_a] \equiv \prod_{i=1}^{3} [\Pi_{a_i}]$$

homology class of 3-cycles

homology class of 1-cycle on each T^2

$T^2 \otimes T^2 \otimes T^2$



$[b_1]$ $[b_2]$ $[b_3]$

$[a_1]$ $[a_2]$ $[a_3]$

(n_a^i, m_a^i) : $(1,1)$ $(1,0)$ $(1,-1)$

wrapping numbers

$$[N_a, \{n_a^i, m_a^i\}]$$

$$[\Pi_{a_i}] = \prod_{i=1}^{3} (n_a^i [a_i] + m_a^i [b_i])$$

Orientifold image $\{m_a^i\} \rightarrow \{-m_a^i\}$

Consistency Conditions

(i) Cancellat ° Ramond-Ramond tadpoles

Blumenhagen et al.

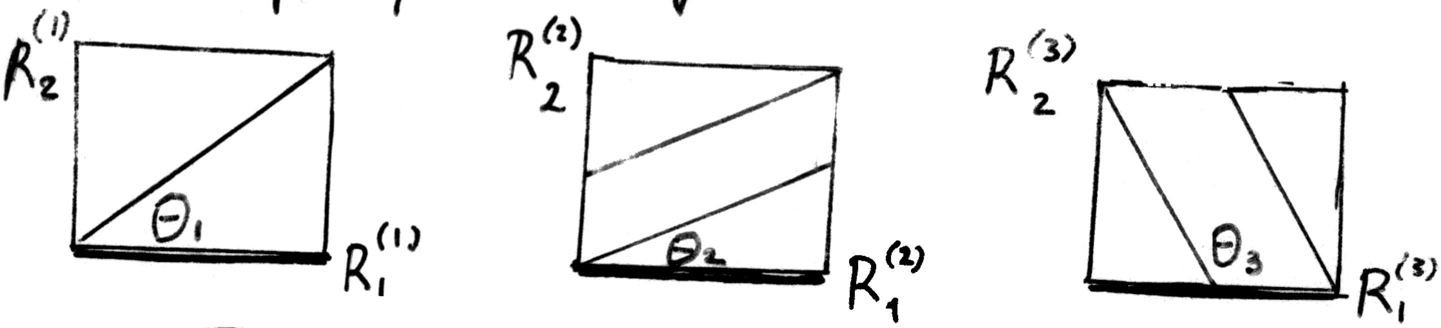
"D6-brane charge & orientifold image" =

= "Orientifold charge & $Z_2 \times Z_2$ images"

$$\sum_a N \left([\Pi_a] + [\Pi_{a'}]\right) = 4\left([\Pi_{O6}] + \sum_{i=\theta,\omega,\theta\omega} [\Pi_{O6}^{i}]\right)$$

$\sum_a N_a m_a^1 m_a^2 m_a^3 = 16 \qquad \sum_a N \quad ' \quad m_a^2 m_a^3 = 16, \ .$

(ii) Supersymmetry



$$\Theta_1 + \Theta_2 + \Theta_3 = 0 \ (\mathrm{mod}\ 2\pi)$$

$$\tan\left(\frac{m^1}{n^1}\chi_1\right) + \arctan\left(\frac{m^2}{m^2}\chi_2\right) + \arctan\left(\frac{m^3}{n^3}\chi_3\right)$$

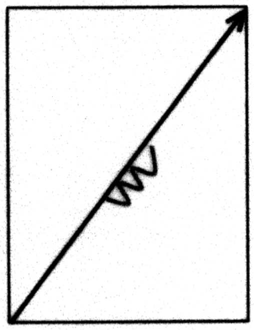
$$\chi_i \equiv \frac{R_2^{(i)}}{R_1^{(i)}}$$

original uct $\ = \{(\theta_1, -\theta_1, 0); (\theta_2, 0, -\theta_2); (0, \theta_3$

constraint on
$\chi_1 : \chi_2 : \chi_3$

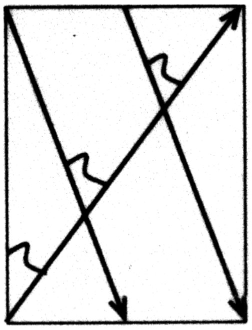
Open String Spectrum – Key ingredient

$$\left.\begin{array}{l} a: N=6 : U(3) \\ b: N=4 : U(2) \end{array}\right\} U(2)_L \times U(3)_c$$



$$U(N) \xrightarrow{\theta} U(N/2) \times U(N/2) \xrightarrow{\omega} U(N/2)$$

+ 3 Adjoint chiral multiplets

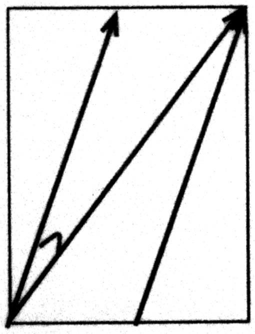


$$I_{ab} = \prod_{i=1}^{3} \left(n_a^i m_b^i - m_a^i n_b^i \right)$$

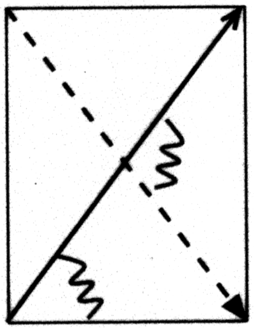
chiral multiplets in $(\square, \bar{\square})$

$$I_{ab} = 3 : \quad 3 \times (\underset{\sim}{3}, \underset{\sim}{\bar{2}})$$

(3 copies of left-handed quarks!)



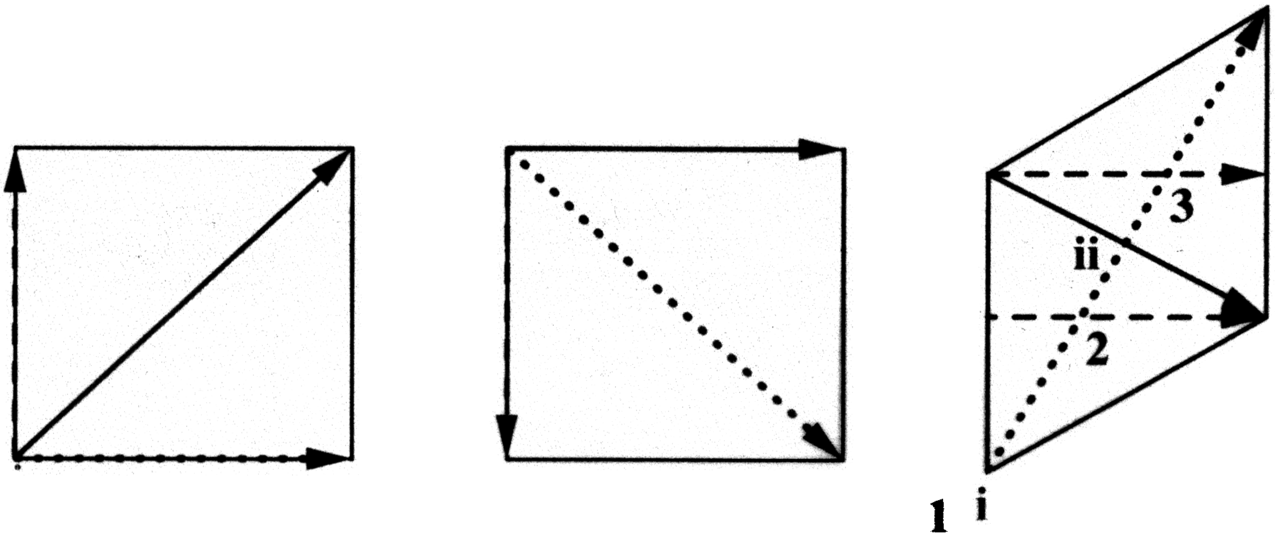
$I_{ab'}$ chiral multiplets in $(\square, \square)$



$$\frac{1}{2}\left(I_{aa'} - \frac{4}{2^k} I_{a,O6} \right) \square\square$$

$$\frac{1}{2}\left(I_{aa'} + \frac{4}{2^k} I_{a,O6} \right) \begin{array}{c}\square\\\square\end{array}$$

Building Blocks of the SM branes



$U(1)_8$ $(A_1$ -branes)

$U(2)_L$ $(B_1$ -branes)

$U(3)_C$ $(C'_1$ -branes)

(1, 2, 3)-Higgs Fields

(i, i i)-Left-Handed Quarks

Phenomenology

SM $\times$ $U(1)_{8-8'}$ $\times$ $U_{B-L-T_{3a}}$ $\times$ "Hidden Sectn"

STANDARD MODEL

"HIDDEN SECTOR"

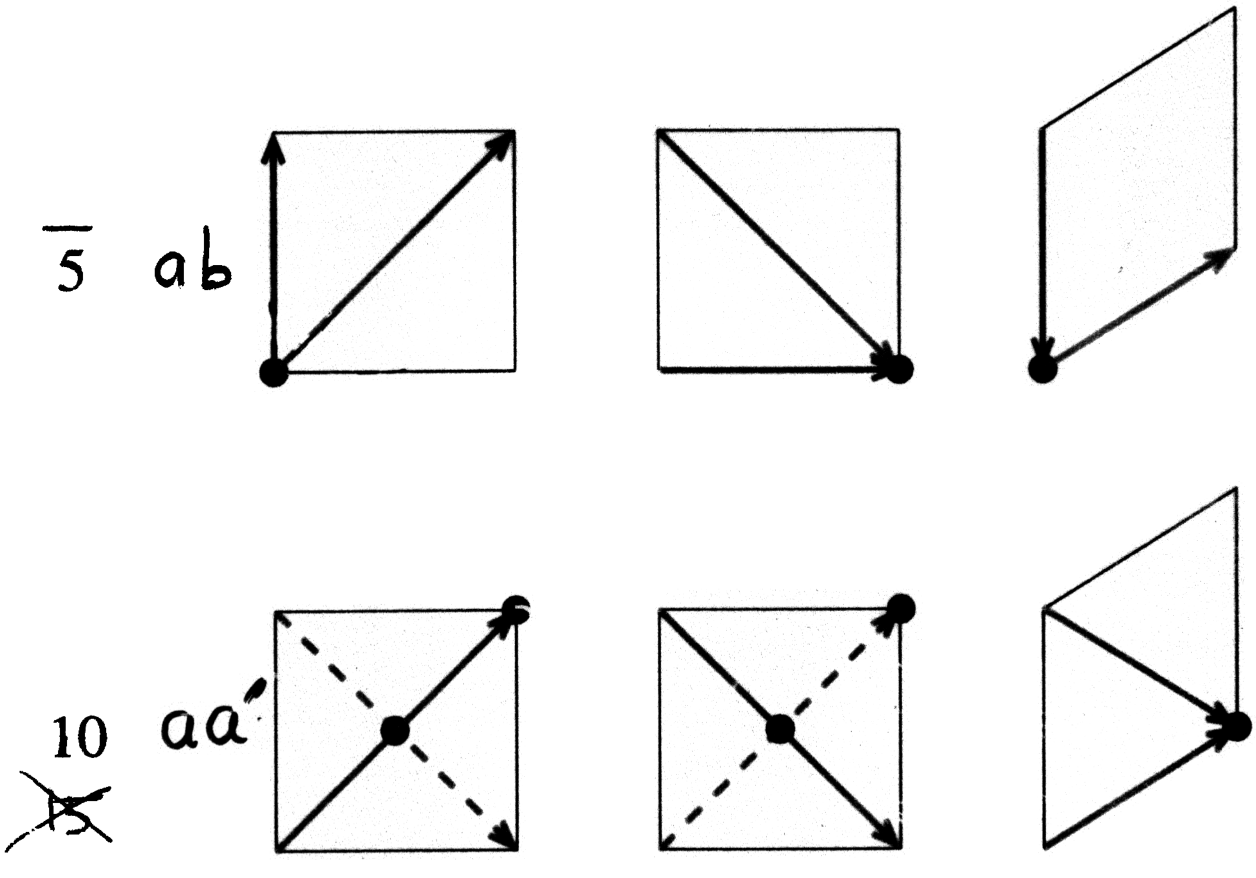
Sector	$U(3) \times U(2) \times USp(2) \times USp(2) \times USp(4)$	Q_3	Q_1	Q_2	Q_8	Q'_8	Q_Y	$Q_8 - Q'_8$	Field
	C L								
A_1B_1	$3 \times 2 \times (1, \bar{2}, 1, 1, 1)$	0	0	-1	± 1	0	$\pm\frac{1}{2}$	± 1	H_U, H_D
	$3 \times 2 \times (1, \bar{2}, 1, 1, 1)$	0	0	-1	0	± 1	$\pm\frac{1}{2}$	∓ 1	H_U, H_D
A_1C_1	$2 \times (\bar{3}, 1, 1, 1, 1)$	-1	0	0	± 1	0	$\frac{1}{3}, -\frac{2}{3}$	$1, -1$	U, D
	$2 \times (\bar{3}, 1, 1, 1, 1)$	-1	0	0	0	± 1	$\frac{1}{3}, -\frac{2}{3}$	$-1, 1$	U, D
	$2 \times (1, 1, 1, 1, 1)$	0	-1	0	± 1	0	1,0	$1, -1$	E, ν_R
	$2 \times (1, 1, 1, 1, 1)$	0	-1	0	0	± 1	1,0	$-1, 1$	E, ν_R
B_1C_1	$(3, \bar{2}, 1, 1, 1)$	1	0	-1	0	0	$\frac{1}{6}$	0	Q_L
	$(1, \bar{2}, 1, 1, 1)$	0	1	-1	0	0	$-\frac{1}{2}$	0	L
B_1C_2	$(1, 2, 1, 1, 4)$	0	0	1	0	0	0	0	
B_2C_1	$(3, 1, 2, 1, 1)$	1	0	0	0	0	$\frac{1}{6}$	0	3 families of quarks & lep
	$(1, 1, 2, 1, 1)$	0	1	0	0	0	$-\frac{1}{2}$	0	
$B_1C'_1$	$2 \times (3, 2, 1, 1, 1)$	1	0	1	0	0	$\frac{1}{6}$	0	Q_L
	$2 \times (1, 2, 1, 1, 1)$	0	1	1	0	0	$-\frac{1}{2}$	0	L
$B_1B'_1$	$2 \times (1, 1, 1, 1, 1)$	0	0	-2	0	0	0	0	
	$2 \times (1, 3, 1, 1, 1)$	0	0	2	0	0	0	0	

TABLE I. Chiral Spectrum of the open string sector in the three-family model. Notice that we have not included the aa sector, even though it is generically present in the model. As explained in the text, the non-chiral pieces in the ab, ab' and aa' sectors are generically not present.

$\chi_1 : \chi_2 : \chi_3 = 1:3:2$ (SUSY)

GUT Model

Consider an $SU(5)$ model. Chiral multiplets in the **10** representation can come from the intersection of a stack of branes with its orientifold image.



Therefore, all three angles θ_1, θ_2 and θ_3 are non-zero.

a : $SU(5)$

families $\{ 5$ & $10 \}$

Georgi-Glashow GUT model!

Couplings in open string sector
States at intersections
(Conformal field theory techniques)

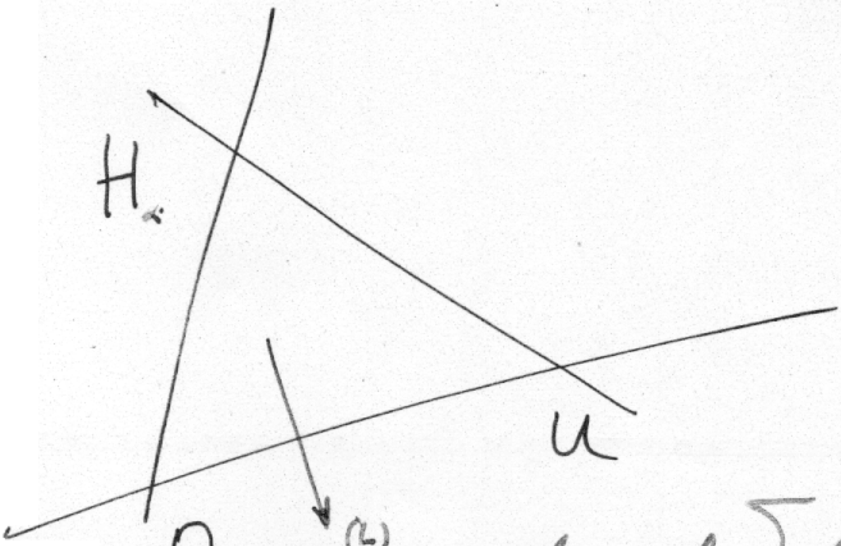
w/ Papadimitriou
hep-th/0303084

4 point amplitudes 4 field contact terms

* 3 point amplitudes Yukawa couplings

Determine classical ($A^L_{\alpha,\gamma}$) & quantum (h_0 p
exact result)

concrete application: D6-branes on T^6
w/ orientifold &
orbifold projection



$h \, H_\alpha Q_\gamma u$

$A^{(L)}_{\alpha,\gamma}$

area of triangle

$$h \simeq h_0 \sum_{\hat{L}} e^{-A^{(L)}/\alpha'}$$

$\hat{L}$ - lattice summation

Solution: $\langle \sigma_\nu(0)\, \sigma_{-\nu}(x)\, \sigma_\nu(1)\, \sigma_{-\nu}(\infty) \rangle \equiv$

$$Z_{qu}\; Z_{cl}$$

$$Z_{qu} = \text{const.}\, [x(1-x)]^{-\nu(1-\nu)} [F(x) F(1-x)]^{-\frac{1}{2}}$$

$$Z_{cl} = \sum_{m_1, m_2} e^{-S_{cl}(m_1, m_2)}$$

$$S_{cl} = \frac{\pi}{\alpha'} \sin \pi\nu \, F(x) F(1-x) \left[\left(\frac{m_1 L_1}{F(x)} \right)^2 + \left(\frac{m_2 L_2}{F(1-x)} \right)^2 \right]$$

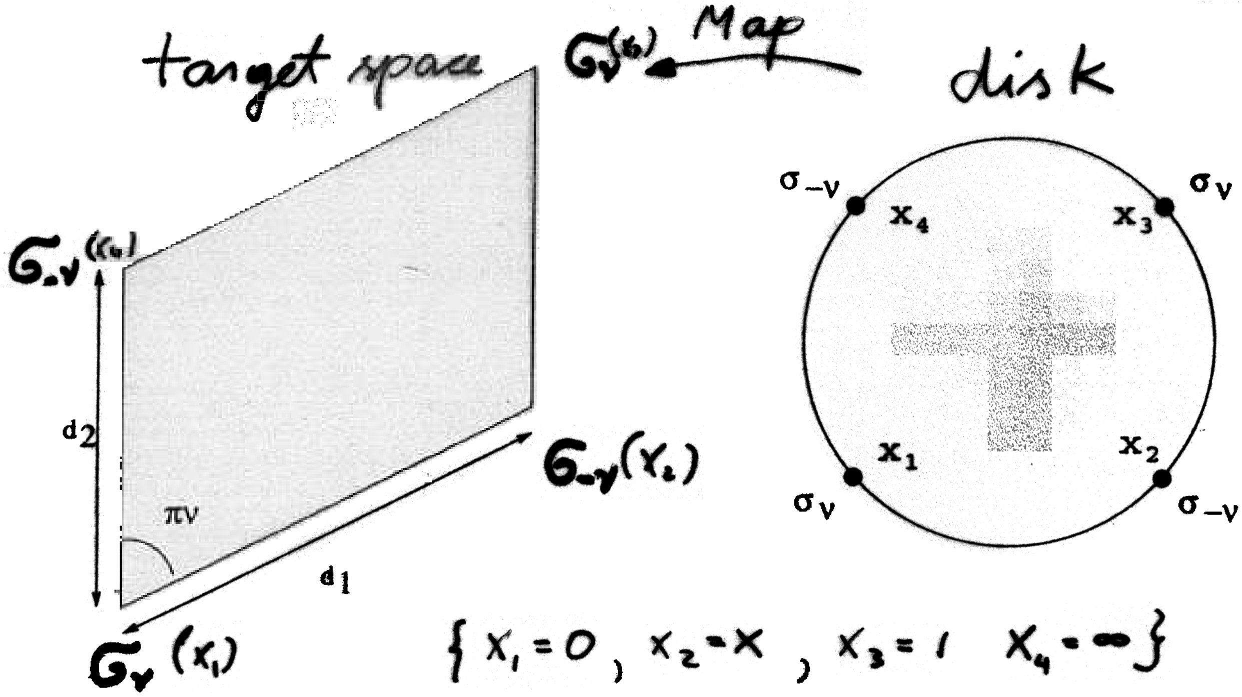
$$F(x) = \underset{\text{Euler Beta f.}}{B(\nu, 1-\nu)}\; \underset{\text{hypergeom. func.}}{F(\nu, 1-\nu; 1; x)}$$

$$\underset{\text{length of cycle}}{L_i} = \sqrt{(n_i R_1)^2 + (m_i R_2)^2} \qquad \sin \pi\nu = \frac{I_{12} R_1 R_2}{L_1 L_2} = \frac{I_{12}\, \chi}{\sqrt{n_1^2 + m_1^2 \chi^2}\sqrt{n_2^2 + m_2^2\chi^2}}$$

$$\underset{\text{intersection no.}}{I_{12}} = |n_1 m_2 - n_2 m_1| \qquad \chi \equiv \frac{R_2}{R_1}$$

$$\text{const.} = 2\pi \frac{\pi}{\sqrt{\sin \pi\nu}}$$ (factorizing on $x \to 1$; s-channel exchange of A_μ)

$$Z \equiv \langle \sigma_\nu(x_1)\, \sigma_{-\nu}(x_2)\, \sigma_\nu(x_3)\, \sigma_{-\nu}(x_4) \rangle$$



$\{x_1 = 0,\ x_2 = x,\ x_3 = 1,\ x_4 = \infty\}$

(a) (b)

FIG. 1: Target space: the intersection of two parallel branes separated by respective distances d_1 and d_2 and intersecting at angles $\pi\nu$ (Figure 1a). World-sheet: a disk diagram of the four twist fields located at $x_{1,2,3,4}$ (Figure 1b). The calculation involves a map from the world-sheet to target space.

$$X = X_{cl} + X_{qu}$$

$$Z = Z_{qu} \cdot Z_{cl}$$

Z_{qu}: via stress energy method

(Dixon, Friedan, Martinec & Shenker '87

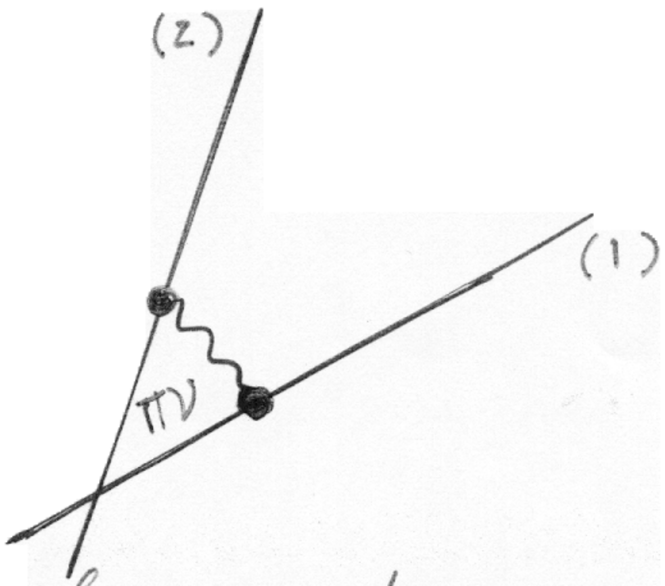
Z_{cl}: $\sum_i e^{-S_{cl\,i}}$

(X_i - classical solutions)

Highlights

Since D6 branes wrap product of 3 1 cycles (of each T^2) focus on correlation function Z_i associated w/ intersections of 1 cycles on T^2

full amplitude $Z \;\; \prod_{i=1}^{3} Z_i$



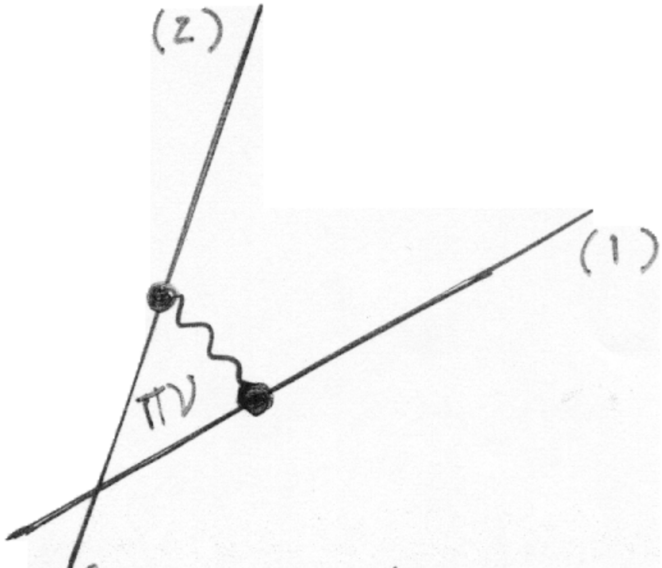
quantization of open string modes (ψ; X) at intersection — non integer moded (by ν)

Sector created by "twisted vacuum" ($|s_\nu\rangle$, $|\sigma_\nu\rangle$)

Highlights

Since D6 branes wrap product of 3 1 cycles (of each T^2) focus on correlation function Z_i associated w/ intersections of 1 cycles on T^2

full amplitude $Z \;\; \prod_{i=1}^{3} Z_i$



quantization of open string modes (ψ; X) at intersection — non integer moded (by ν)

Sector created by "twisted vacuum" ($|s_\nu\rangle$, $|\sigma_\nu\rangle$)

$$Z \equiv \langle \sigma_\nu(x_1)\,\sigma_{-\nu}(x_2)\,\sigma_{-\lambda}(x_3)\,\sigma_\lambda(x_4)\rangle$$

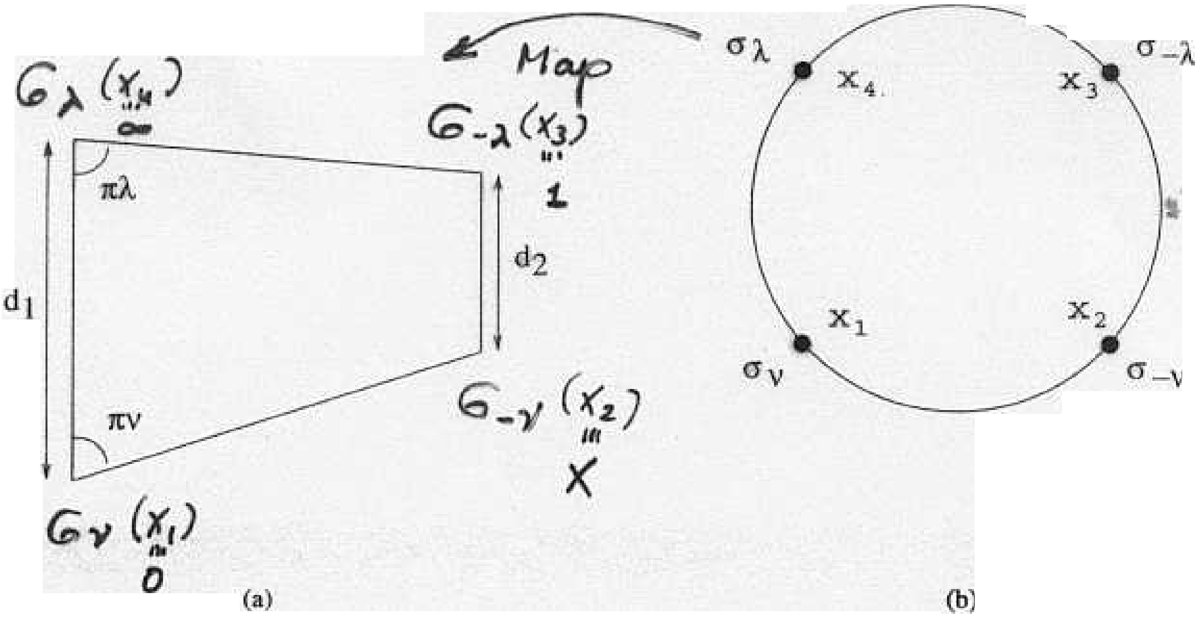


FIG. 2: Target space: the intersection of two branes intersecting respectively with the two parallel branes at angles $\pi\nu$ and $\pi\lambda$, respectively (Figure 2a). World-sheet: a disk diagram of the four twist fields located at $x_{1,2,3,4}$ (Figure 2b).The calculation involves a map from the world-sheet to target space, allowing for a factorization on three-point function.

$$Z = Z_{qu} \cdot Z_{cl}$$

Analogous technique (more complicated boundary conditions)

Z_{qu}

$Z_{cl} \sim e^{-S_{cl}}$ (S_{cl} regular as $x \to 1$ $(d_2 \to 0)$)

Factorization on:

$$\langle \sigma_\lambda(x_1)\,\sigma_{-\nu-\lambda}(x_3)\,\sigma_\lambda(x_4)\rangle$$

Yukawa coupling

Three point amplitude

$$Z_3 \quad \lim_{x\to 1} \prod \langle G_\nu 10 \; G_{-\nu}(x \; G_{\lambda_i}(\;) \; G_{\lambda_i}(\infty) \rangle$$

$$\frac{\sum_{i=1}^{3} A(m_i)}{2\pi\alpha'}$$

$$\circledast \quad 2\pi \prod_{i=1}^{3} \sqrt{\frac{4\pi B(\nu_i, 1-\nu_i)}{B(\nu_i,\lambda_i)\, B(\nu_i, 1-\nu_i-\lambda_i)}} \sum_{\{m\}} \ell$$

$(m$ area of triangle formed by 3 intersecting branes in i-th T^2

$S_{cl_i} \; A(m)$ studied in detail

Cremades Ibañez & Marchesano hep-th/0302105

(cf Abel & Owen hep-th/030 24)

includes both Z_{qu} & Z_{cl}

↓

further study

Outlook – "G_2 phenomenology"

(i) Systematize orientifold constructions within $T^6/(Z_2 \times Z_2)$ orbifolds

systematic SU(5) GUT: w/ Shiu & Papadimitriou

more standard models ... w/ Papadimitriou

other orbifolds (cycle recombination needed)

$T^6/(Z_4)$ Blumenhagen, Görlich & Ott

$T^6/(Z_4 \times Z_2)$ Honecker

other CY_3 ....

(ii) Couplings: CFT techniques *

Yukawa couplings

4-point couplings – proton decay in GUT (four-fermi couplings) Klebanov & Witten

⋮

M.C. (work in progress)

* Applications more general