Laser Plasma simulation for direct drive, indirect drive and Fast ignition Fusion

J.C Adam
Ecole Polytechnique(France)

Overview of:

the different schemes for inertial fusion,

the challenges that must be faced in term of numerical modeling,

the tools that are currently being used and their potential extension

Brief presentation of some 2D and 3D results of the simulation of the interaction of ultrahigh intensity laser pulse with plasma at (almost) solid density

Collaborators: A. Héron and J.L. Vay

GENERAL SCHEME OF INERTIAL FUSION(1)

The basic principle is common to all the schemes:

The objective is to compress with the help of powerful laser a pellet (500µ to 1000µ initially, filled with a mixt of DT) down to few tens of micron (it could also be ion beams)

This represents of compression by a factor > 1000

This will create a hot spot at the center of the target that will start fusion reactions and ignite it

The range of physical processes involved in this compression process is obviously tremendous

No single code can describe all of them.We certainly need models but the basic physics is still often unknown

GENERAL SCHEME OF INERTIAL FUSION(2)

The length of the laser pulse involved is typically of the order of 1ns.

The energy of the pulse in the current scheme is 1-2 MJ (back in the 70 the projected energy needed was 10-20KJ …)

The front of the laser pulse first evaporates the exterior of the target creating a plasma that expands creating a corona
The final size of the corona is of the order of 1cm.

As the plasma expands the laser propagate up the so called critical density where the laser frequency is equal to the plasma frequency.

Energy is damped there creating a shock wave that compress the target by rocket effect.

GENERAL SCHEME OF INERTIAL FUSION(3)

The 3 different schemes (direct drive, indirect drive, fast ignition) differ by the problem they aim to solve.

The most basic scheme has just been described it is:

1 Direct drive

A key problem with this scheme is the necessary perfect spherical symmetry of the target and the one of the compression

If not perfect the ablation surface develops a Raleigh Taylor instability that inhibits the compression.

This imply sophisticated target preparation with roughness below 1λ (not 1μ) and complicated geometry of the illumination (NIF and LMJ have almost 200 beams)

GENERAL SCHEME OF INERTIAL FUSION(4)

2 Indirect drive

In order to alleviate these symmetries constraints it was imagined to use the following scheme:

Put the target into a Holrhaum, i.e. a cylindrical vessel of few tens of mm of diameter open at both end and covered with gold (inside)

Shoot the laser through the cylinder edges on the gold lining (again 200 beams)

It generates X rays with an isotropic distribution in space that compress the target by the rocket mechanism

Due to the short wavelength of X rays the coupling efficiency to the target is much better than with light

It is the current scheme for NIF to day

GENERAL SCHEME OF INERTIAL FUSION (5)

3 Fast ignition

The indirect drive method solves some problems but creates also new ones.

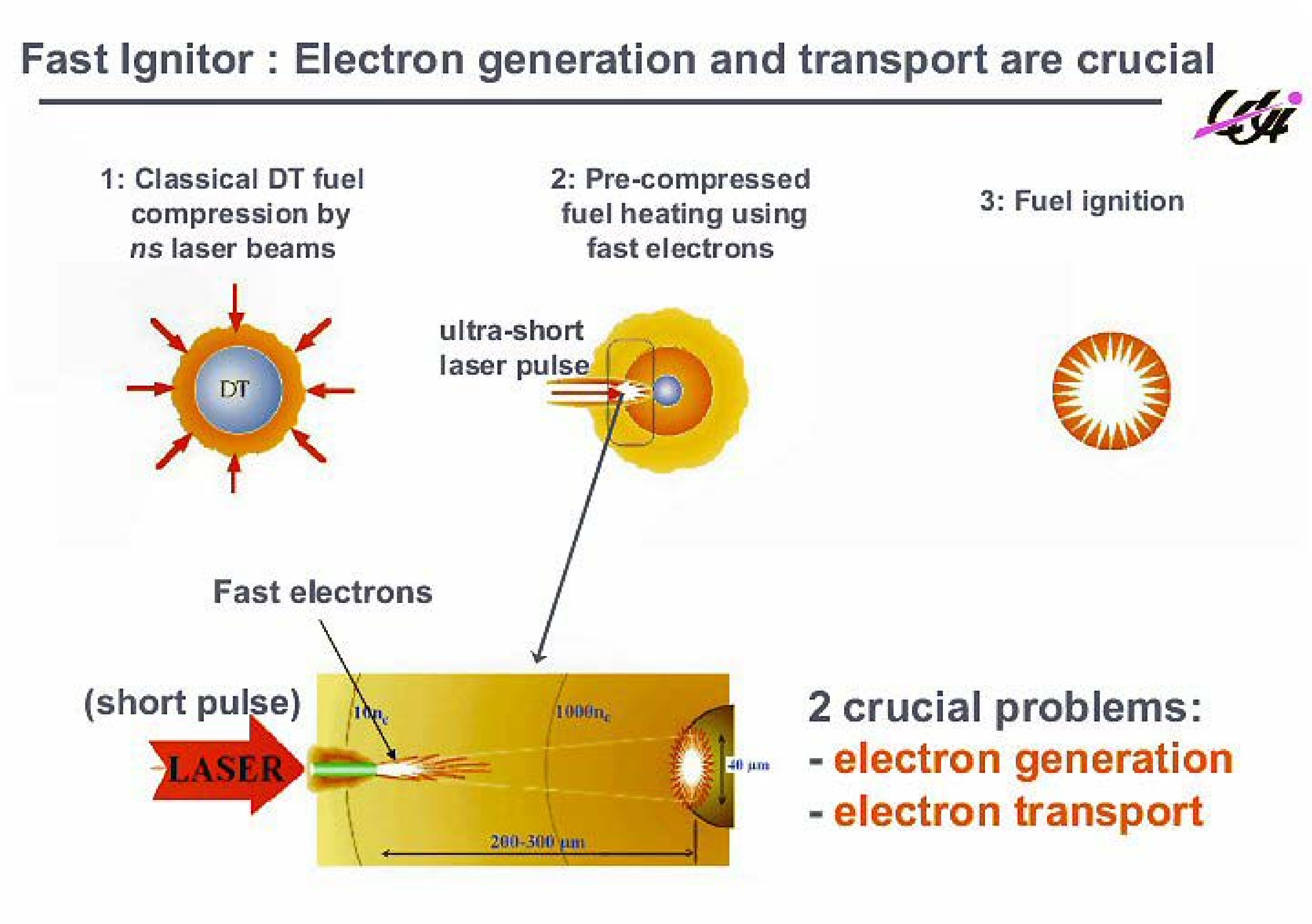
An alternative approach to the direct drive scheme was therefore proposed known as the fast ignitor scheme

To alleviate symmetry constraints the hot spot is not obtained by compression

At some intermediate level of compression an ultra intense ultra short laser pulse is shot on the target (10^{20}W/cm^2,1ps)

It creates a hot spot at the edge of the compress target that triggers ignition

A cartoon of Fast ignition



Fast Ignitor : Electron generation and transport are crucial

1: Classical DT fuel compression by *ns* laser beams

2: Pre-compressed fuel heating using fast electrons

3: Fuel ignition

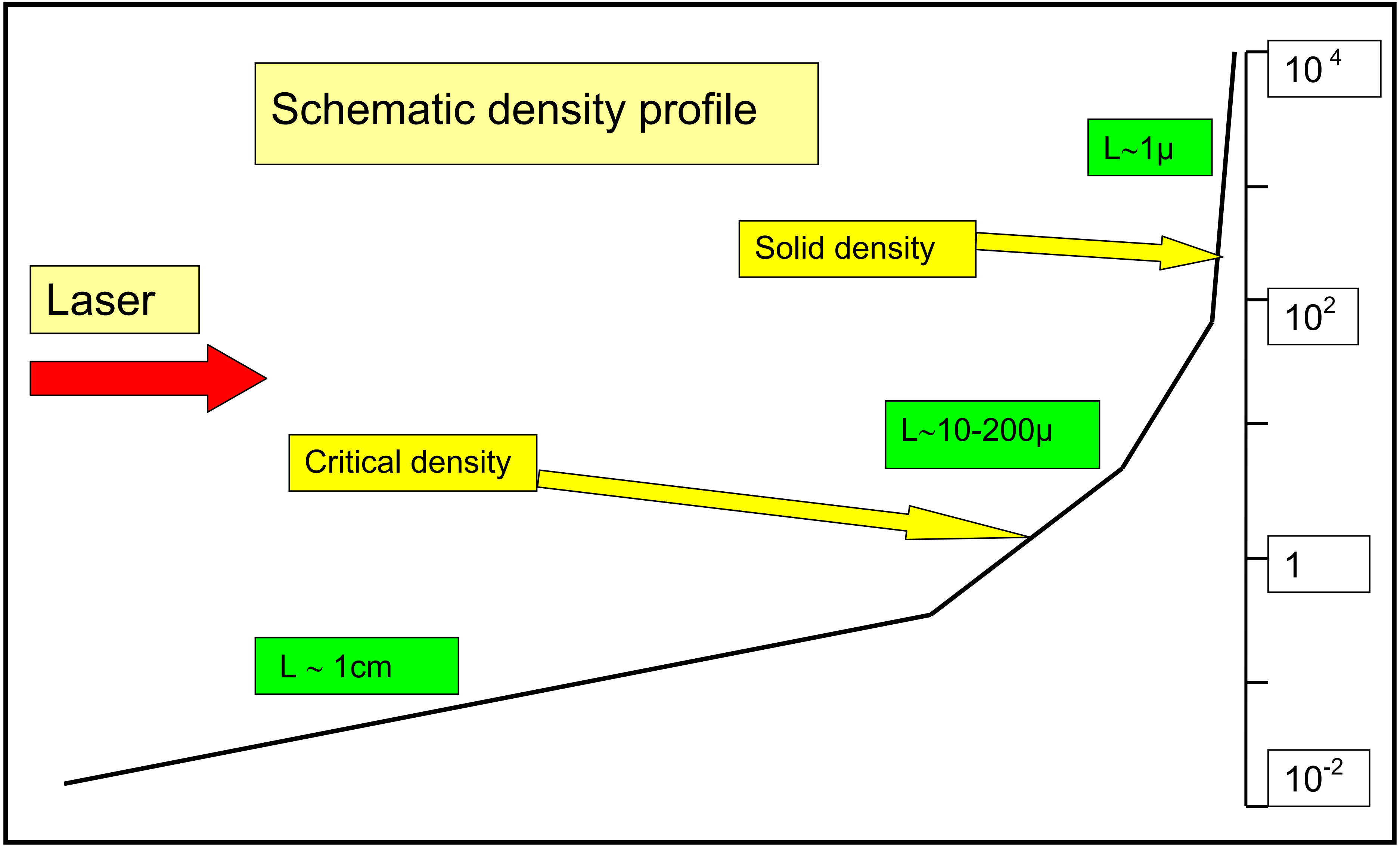
ultra-short laser pulse

Fast electrons

(short pulse)

2 crucial problems:
- electron generation
- electron transport

Schematic density profile



Laser

Solid density

L~1μ

Critical density

L~10-200μ

L ~ 1cm

10^4

10^2

1

10^{-2}

A Few orders of magnitude

$$\frac{\omega_p}{\omega_0} = \sqrt{\frac{n}{n_c}} \qquad \frac{\lambda_d}{\lambda_0} = \sqrt{\frac{n_c}{n}}$$

skin depth $\lambda_s \sim \frac{c}{\omega_p} \sim \sqrt{\frac{n_c}{n}}$

Assume a 1KeV plasma

n/n_c	10^{-2}	1	10^{2}
ω_p/ω_0	10^{-1}	1	10
λ_D/λ_0	10^{-1}	10^{-2}	10^{-3}
λ_s/λ_0		1	10^{-1}

GENERAL SCHEME OF INITIAL FUSION (6)

We will make a distinction between 3 regions

Above solid density: it is Lasnex and al Kingdom

It involves very complicated atomic and nuclear processes, radiative hydrodynamics .., it is hardly a plasma problem

Between solid density and critical density

It is a full kinetic problem, the physics of both electron and ion is important. It must be handled by full PIC code and/or (?) implicit PIC

Below critical density is a plasma physics domain that was quite overlooked in the initial design of FCI

It an essential ingredient: the physics there determines the amount of energy that will be effectively used to compress the core

It must described by kinetic theory

Some kind of quasi static model can be used (hybrid model)

Kinetics Problems related to inertial fusion(1)

Coupling of the main laser beam with the plasma. In particular propagation of laser Beam of High intensity (> 10^{14}W/cm^2) during 100ps-1000ps:

Role of parametric instabilities (electronic and ionic)
They corresponds to nonlinear coupling of the laser light with plasma waves(Raman) or ion acoustic waves(Brillouin).
It induces backscattering that can be anything between 10% and 50%

For the fast ignitor scheme propagation of the short laser pulse between the critical surface and the overdense plasma (role of relativistic electronic instabilities

Cloupling of the laser energy to the target in the region of high density.

Transport of fast electrons into the target

Kinetics Problems related to inertial fusion(2)

All the preceding problems have very different time scales and density scales.

Reproducible experiments are needed to understand the physics involved in all these processes.

Typically they are based on the study of high intensity laser pulse with solids.

Numerical models are needed to interpret those experiments.

PIC codes are the general tool to handle kinetic problem.

Electromagnetic explicit PIC code remains the most widely used, but they cannot resolve all the problems.

Tradeoff have to made which depend on the specific approximations that can be afford

Kinetic description of a relativistic plasma

The system of equations to solve is the Vlasov equation (one for each species)

$$\frac{\partial f}{\partial t} + \vec{p}\frac{\partial f}{\partial \vec{x}} + q + \left(\vec{E} + \vec{p} \times \vec{B}\right)\frac{\partial f}{\partial \vec{p}} = 0$$

coupled to Maxell and Poisson equation

As Bolzmann equation it describes an incompressible flow in phase space

Why a PIC code to solve Vlasov equation ?

Because in the absence of dynamic adaptive techniques to dicretize the phase space they are the best tool that we have to sample the phase space

Why not use a space and time discretization of the partial differential equations? (1)

Because of the dimensionality of the phase space which would be 6 in 3D which far too much for the largest computers to day. Even 2D problems are extremely limited

The solution tend to develop fine structure in phase space that are difficult to handle by a finite difference scheme as well with finite elements

Modern computers allow small 2D problems to be handle efficiently in the classical limit (i.e. non relativistic) using splitting techniques based on the fact that the characteristics of Vlasov equation are straight lines.

The relativistic problem is much more difficult to handle: characteristics are no longer straight lines. This implies costly iterative schemes

What are particle codes used in Plasma physics?(1)

The physical approach is to consider the way Vlasov equation is derived:

The most basic description of a plasma starts with Klimontovitch equations which are the equations of motion of all the physical particles coupled with Maxwell equations.

This describes any plasma collisionnal or not.

Under the assumption that there are enough particles in the Debye sphere one can show that binary collisions are negligible and that the interaction between particles occurs only through collective field. This is the so called mean field approximation

This approximation yields the Vlasov equation.

What are particle codes used in plasma physics ?(2)

A particle code integrates Klimontovitch equations using a set of macro particles which is obviously small as compared to the real number of particles in the Debye sphere

Binary interactions are not completely suppressed as they should be

This means that any numerical model is going to enhance fluctuations with respect to reality or in other words increase collisionnality

Techniques have been developed to reduce collisionnality.

First effect of the finite size particle is to suppress Coulomb divergence when $d < R$

What are particle codes used in Plasma physics? (3)

A key point in particles codes used to study laser plasma interaction is the introduction of a grid to compute the fields

The introduction of a grid speed up the computation, yields some smoothing of short wavelength but also introduces a lost of translation invariance which has negative consequences on code stability

The force depends not only of the distance between to particles but also of their position relative to the grid.

Wavelengths shorter than the mesh size are not resolved

This introduces so called aliasing effects which induce a numerical instability. The existence of this instability yields self heating of the plasma until the stability limit is reached

This limits the mesh size that can be used and consequently the time step.

How good are these codes ?(1)

The standard scheme uses linear interpolation .

It is a kind of optimal trade off between accuracy and cost. With this scheme the mesh size is theoretically limited to a few (<3) Debye length

The aliasing instability can be greatly reduced by filtering the short wavelengths poorly represented on the grid.

We use a forth order filter that was introduced in Waves by E.Lindman (Los Alamos°) and our experience shows that mesh size up to 10 Debye lengths can be used with simulation lasting several 1000 ω_{pe}^{-1} without any visible heating

Higher order spline interpolation?

Higher order splines are more expensive

They improve the stability of the model and coarser grids can be used which also imply potentially larger time steps.

Cubic splines combined with spatial filtering allow mesh size up 100 Debye lengths.(Abe 1986)

The possibility to use such a large mesh size obviously depends on the problem that is being handled.

The only importance criteria is cost effectiveness for a given physic to be solved

Time splitting (1)

The physics of plasma is inherently multi scale: ions are much heavier than electrons and it is useless to advance them with the same time step as electrons.

Schemes that sub-cycle the electrons motion have been developed and their stability established long time ago (1982)

The constraint is $\omega_{pe} dt_i < \pi$ which, for a mass ratio of 2000, generally implies that the time step for ion can be 5 to 10 times larger than for electrons which makes the cost of advancing the ions almost negligible

Time splitting(2)

The Courant condition for Maxwell equation is often unnecessarily restrictive. Particles can be advanced using a somewhat larger time step.

The stability condition here is $\omega_0 dt_e < \pi$ which in most cases means that the time step for particles can be 2 to 3 times larger for particles than for fields.

The combined use of both these splitting yields an improvement close to a factor 4 on the cost of a time step.

We have been running codes over more than 20 years without meeting any stability problem

Full PIC An example of application: Interaction of a laser at relativistic intensity with a strongly overdense plasma

A key problem in the fast ignition scheme if the transport of the energetic electron from into the dense plasma

The basic idea is that the relativistic particles self pinch due to the magnetic field that their beam will generate

To day there are numerous experimental and simulation evidences that this not true.

The main problem with simulations in this domain is the size of the system

Velocity of light is 300µm/ps

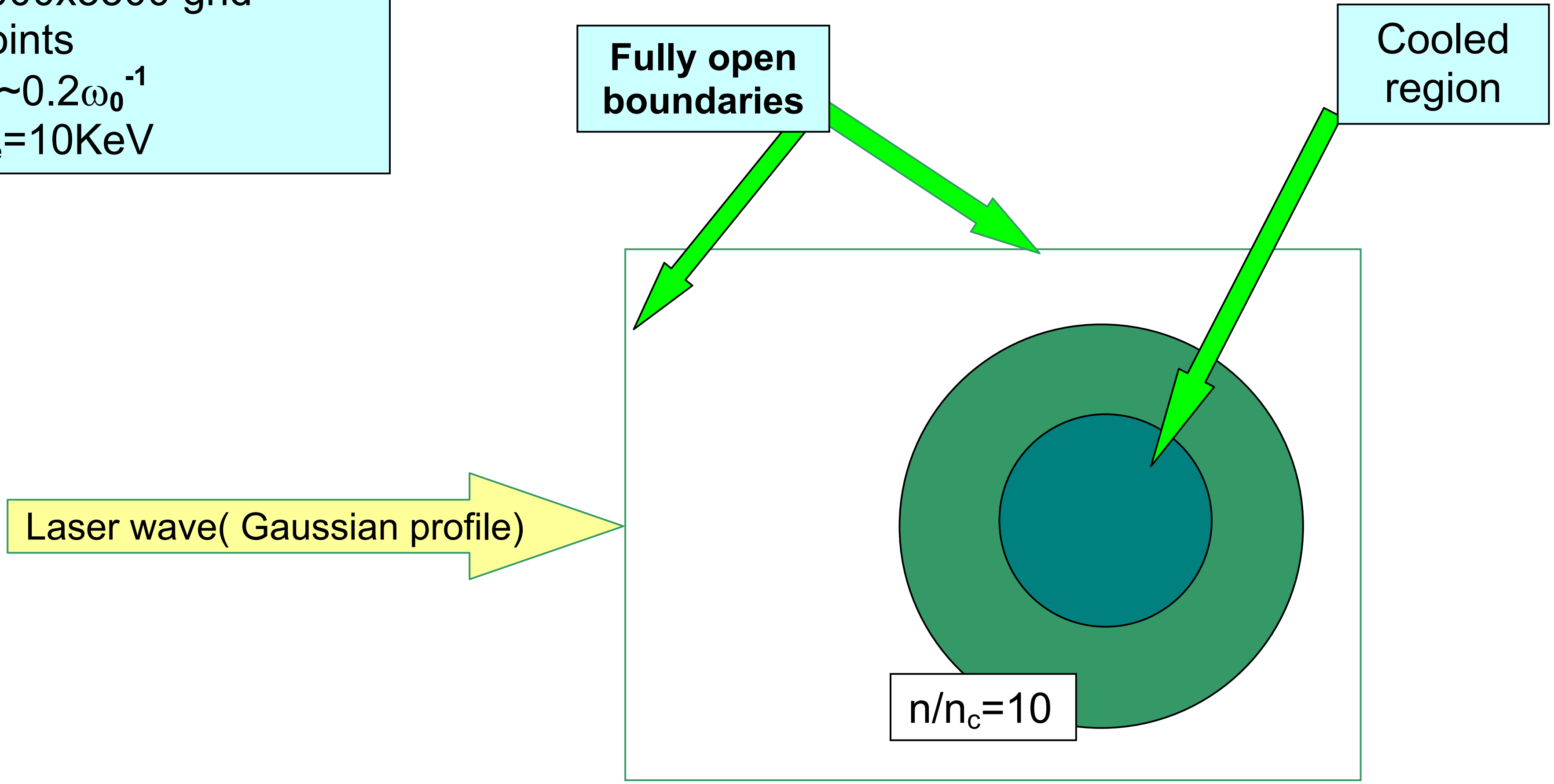
The role of boundary conditions is immediately important

A significant size of vacuum is needed

A big simulation

Relatively low resolution

150 x 180 μ
$dx=dy=0.2k_0^{-1} = 0.03\mu$
5000x5300 grid points
$dt \sim 0.2\omega_0^{-1}$
$T_e=10KeV$



Fully open boundaries

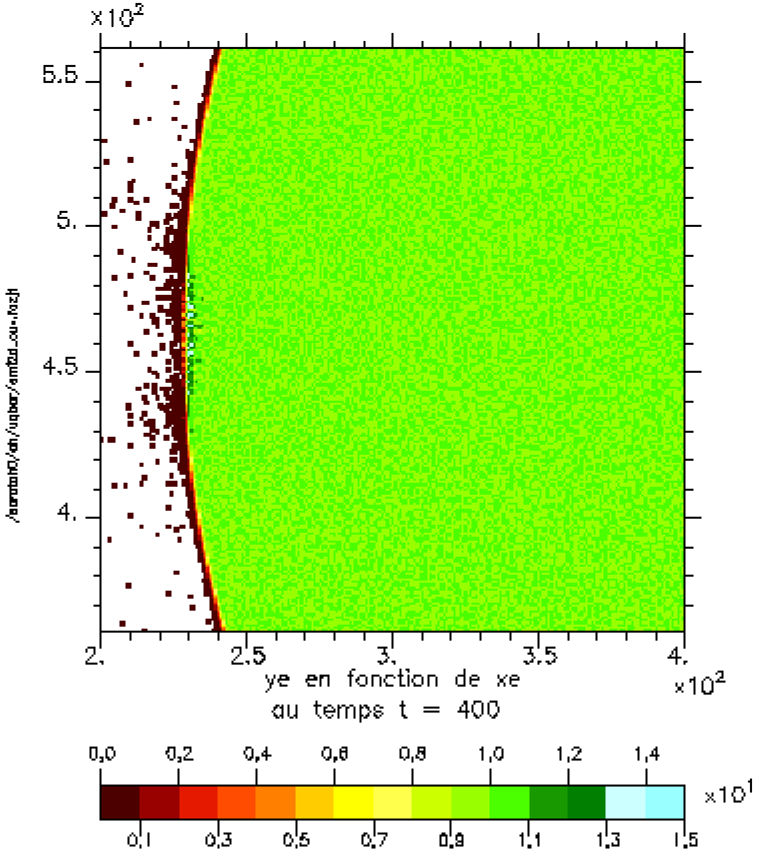
Cooled region

Laser wave(Gaussian profile)

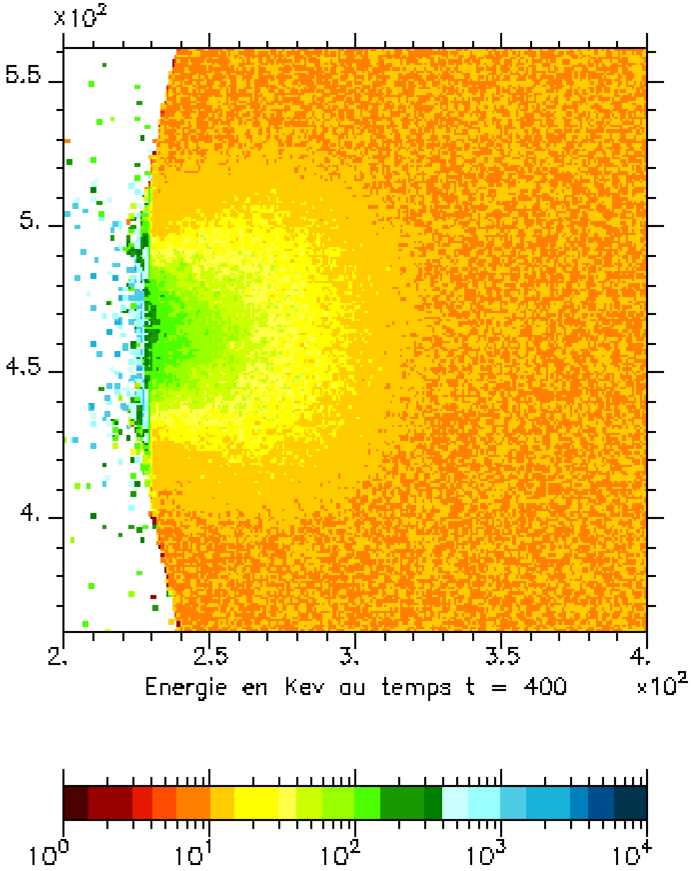
$n/n_c=10$

Typical results of that simulation
(100fs after the impact)
30x30µ

Electron density



Kinetic energy(KeV)

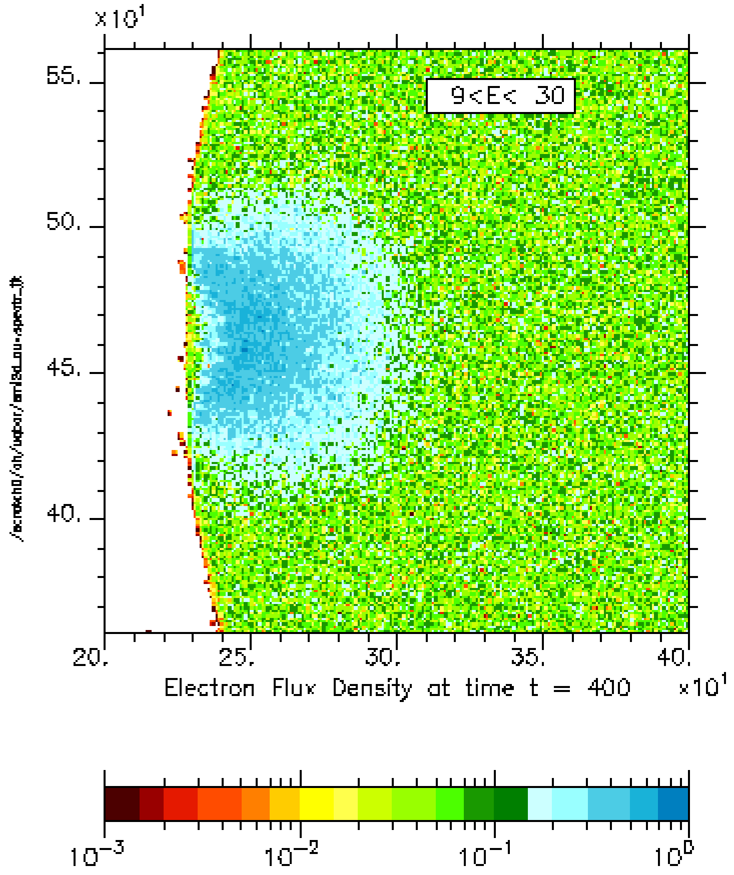


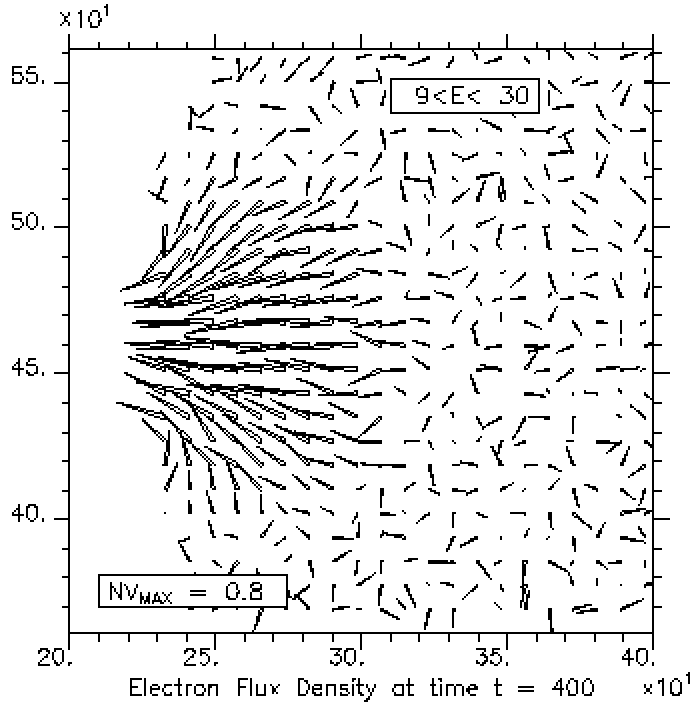
Typical results of a that simulation
(t=100fs)

9<E<30KeV

modulus of current density

current density

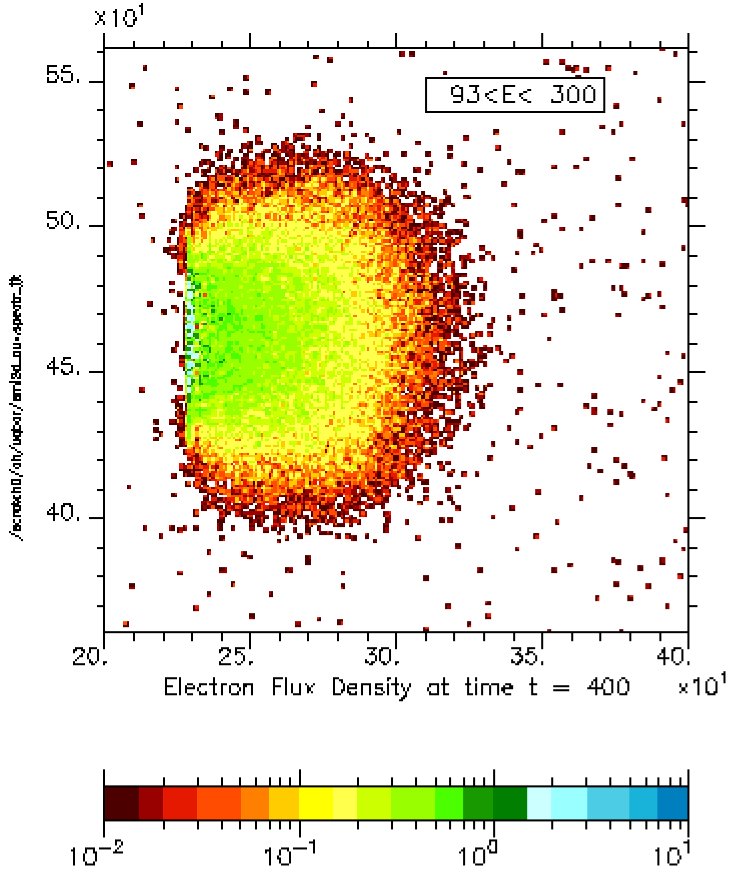




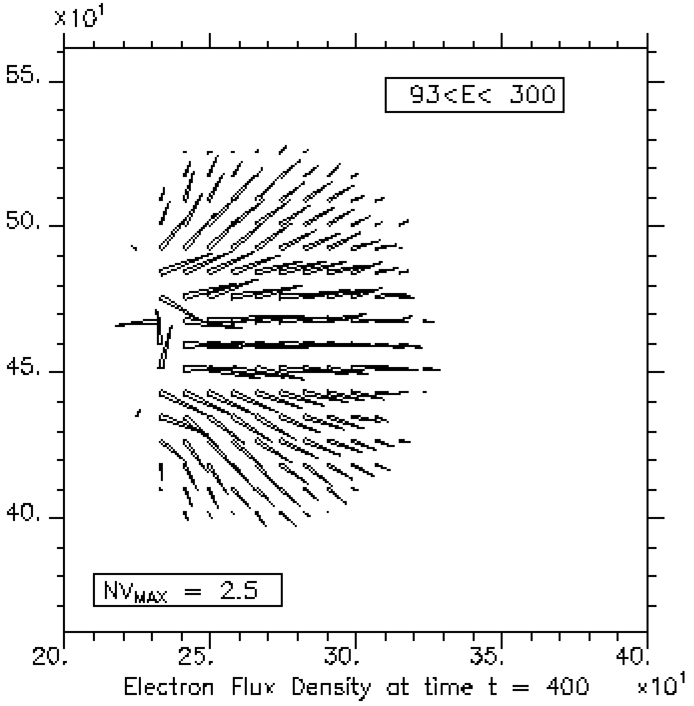
Typical results of a that simulation
(t=100fs)

93<E<300KeV

modulus of current density



current density



A very simple simulation system

2D Systems

90/180 points per laser wavelength

90/180 time steps per laser period

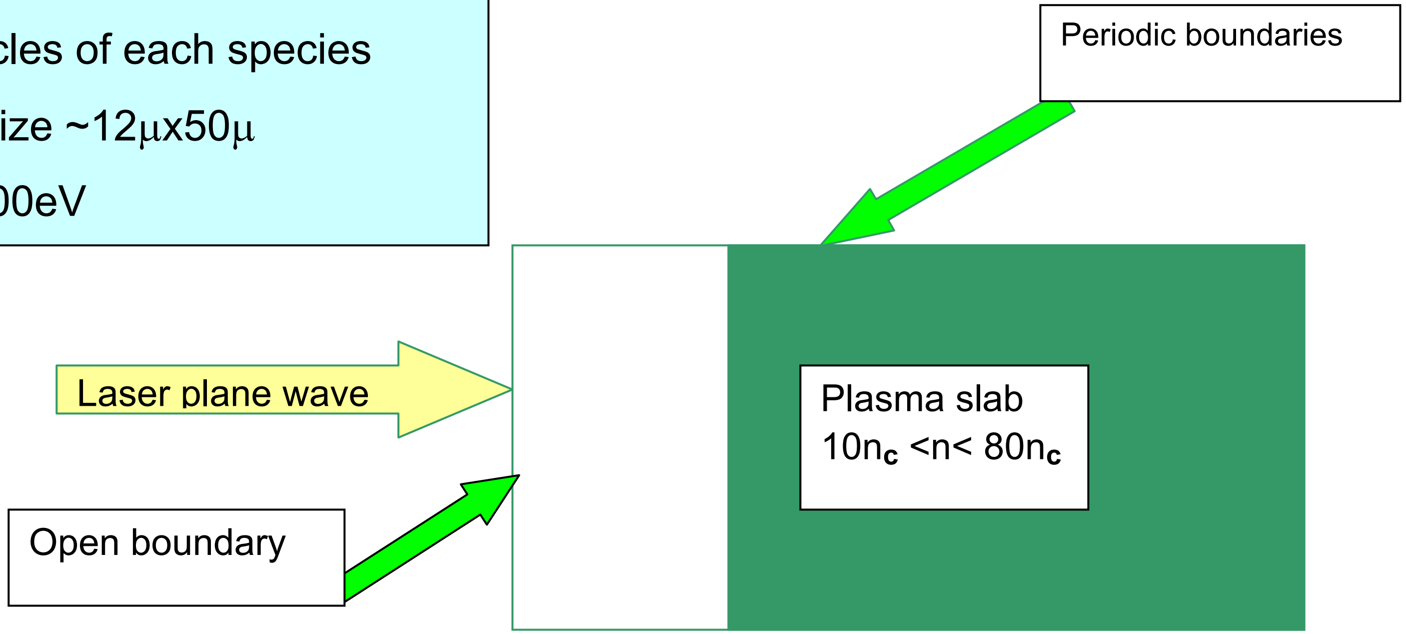
10-1000 particles/cells

~15 x 10^6 grid points

~200 x 10^6 particles of each species

simulation box size ~12μx50μ

250eV<Te<10000eV



Periodic boundaries

Laser plane wave

Plasma slab

$10n_c < n < 80n_c$

Open boundary

3D Systems are similar

They are rods 2λ x 2λ x 64λ

We used between 60 and 120 grid points per wave length

Up to 24 particles per cell

2 10^9 particles of each species

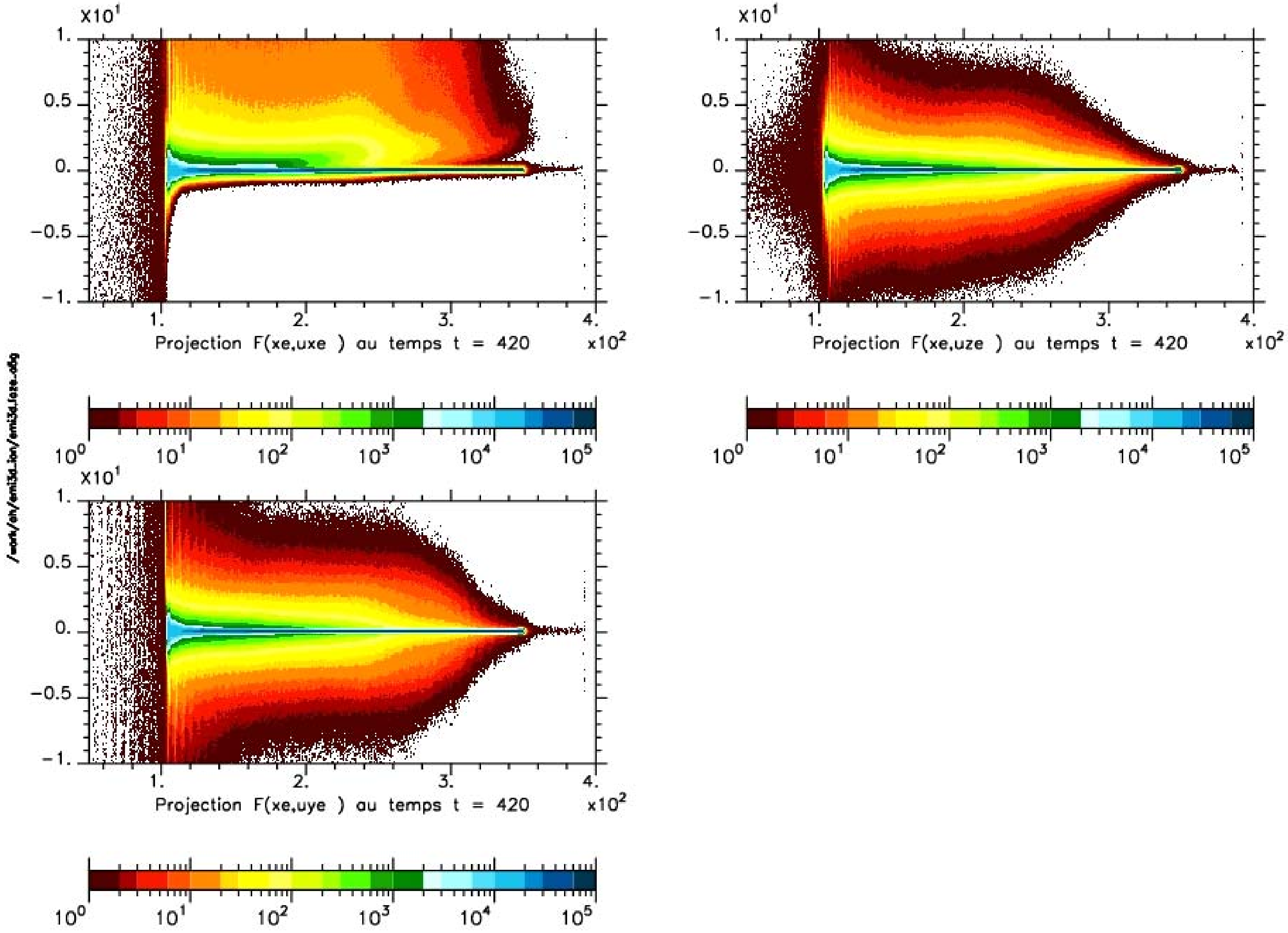
128Gbytes of memory

Phase Space in 3D

$n=80n_c$,1KeV,40µm plasma slab

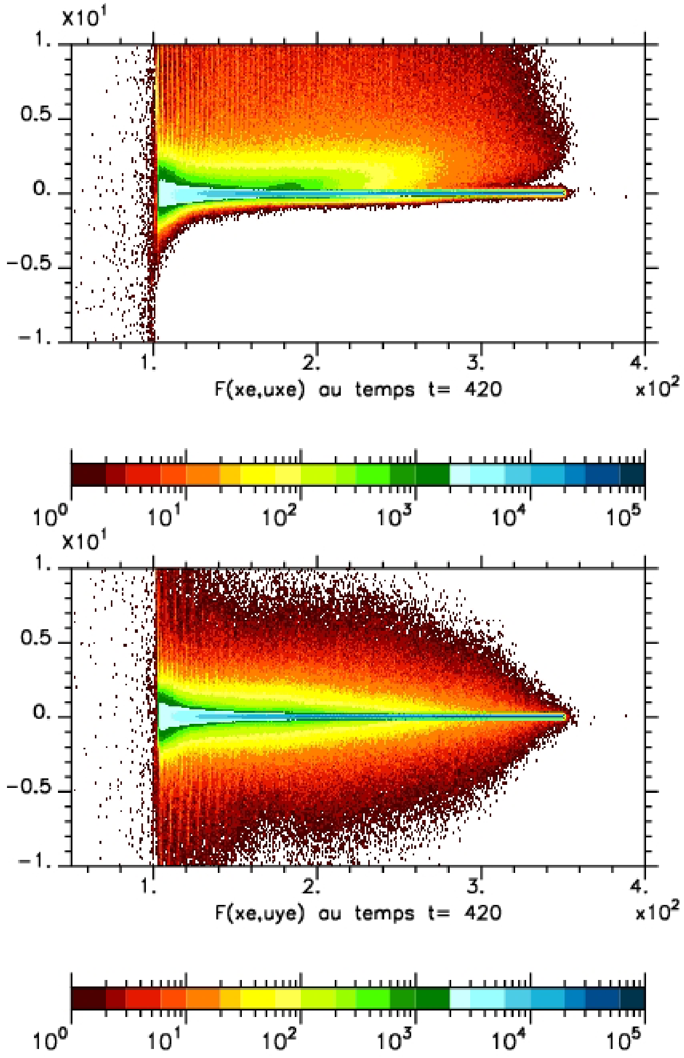
$\phi=10^{20}W/cm^2$ (v_{osc}/c ~9), maximum energy on y scale 4.5MeV

time=$420\omega_0^{-1}$ (150fs after impact)

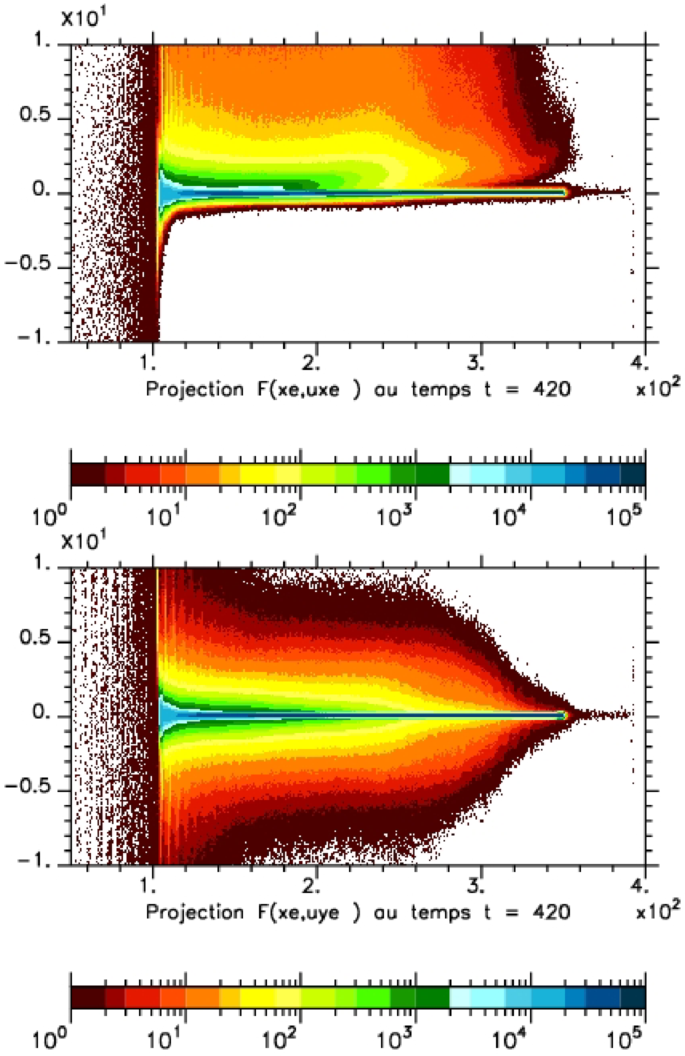


Comparison Phase Space in 2D and 3D

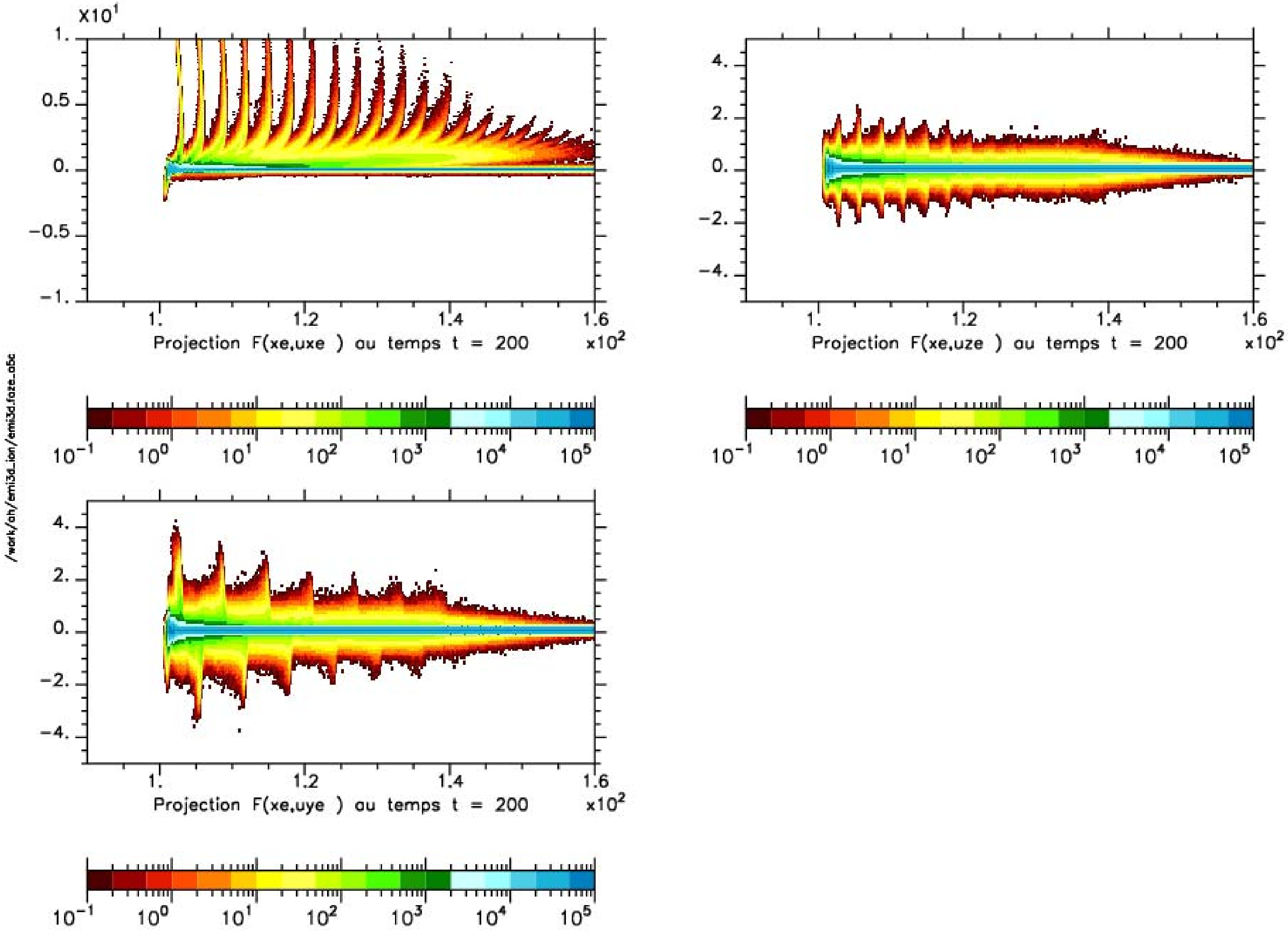
2D simulation



3D simulation



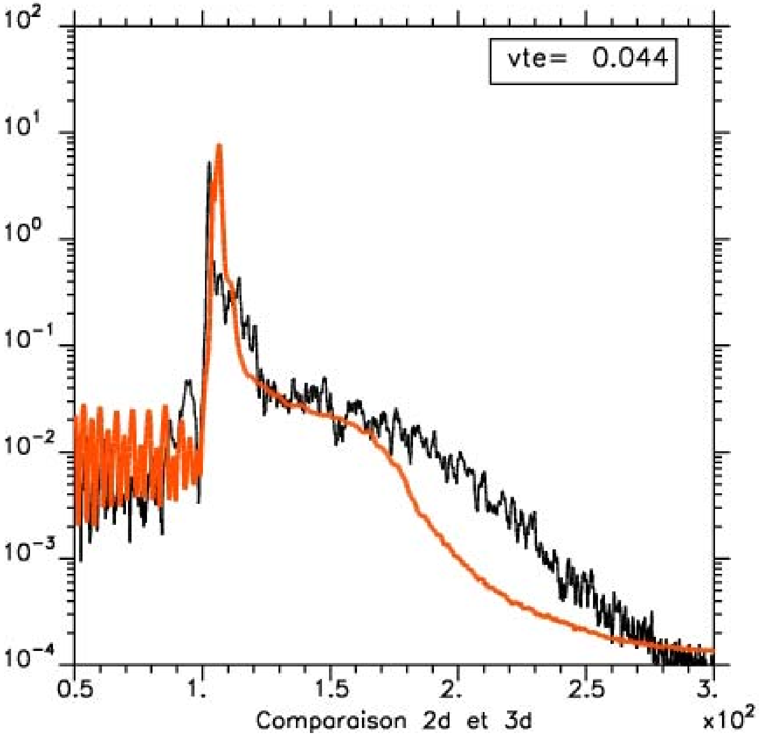
Blow up of the 3D phase space at an early time (25fs after impact)



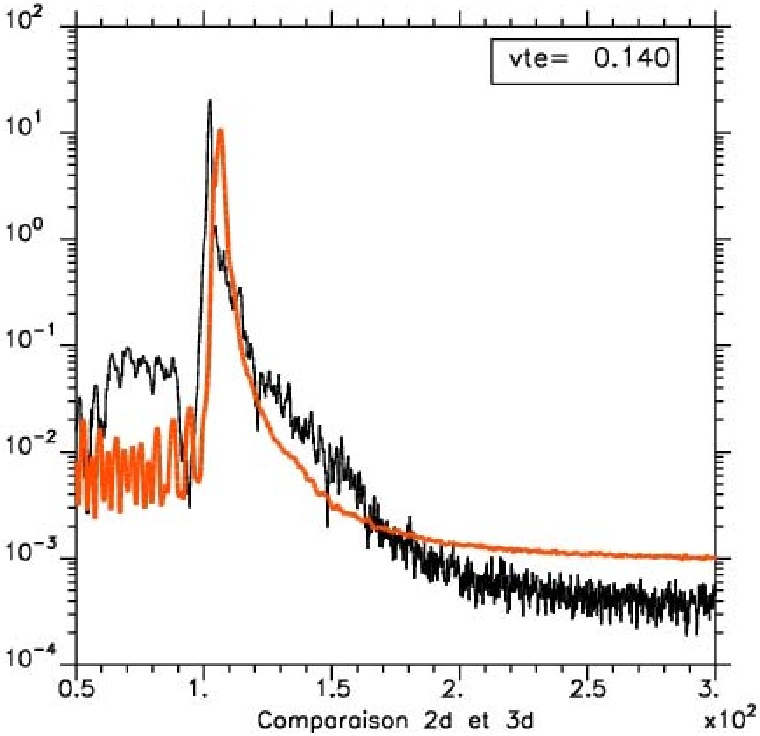
Comparison of the profiles of the low frequency magnetic field in 2D and 3D

2D simulation in black
3D simulation in orange
$|B^2|$ 1 is 100Mg^2 (10^4 T^2)

vte=0.044 ;



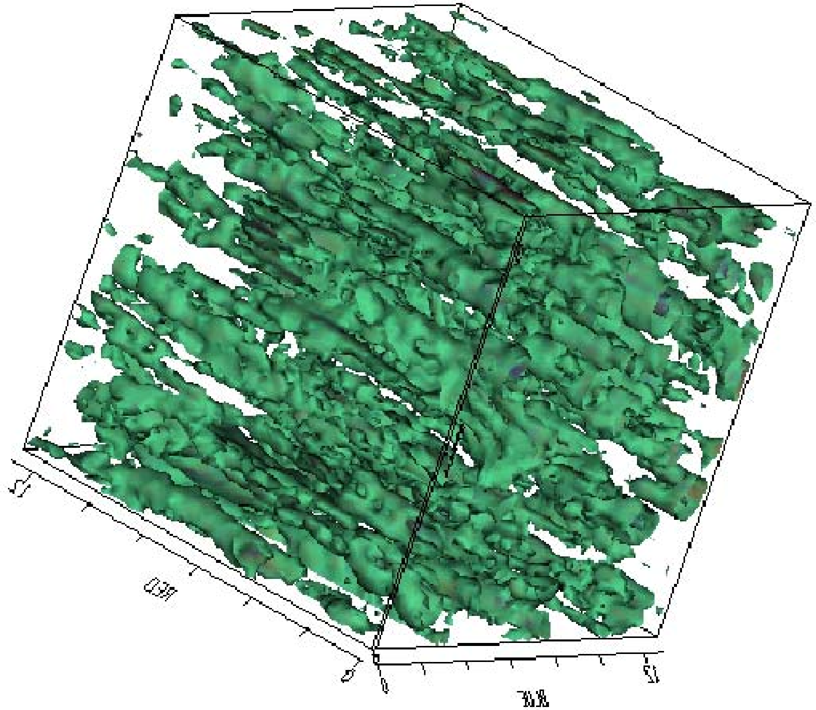
vte=0.140



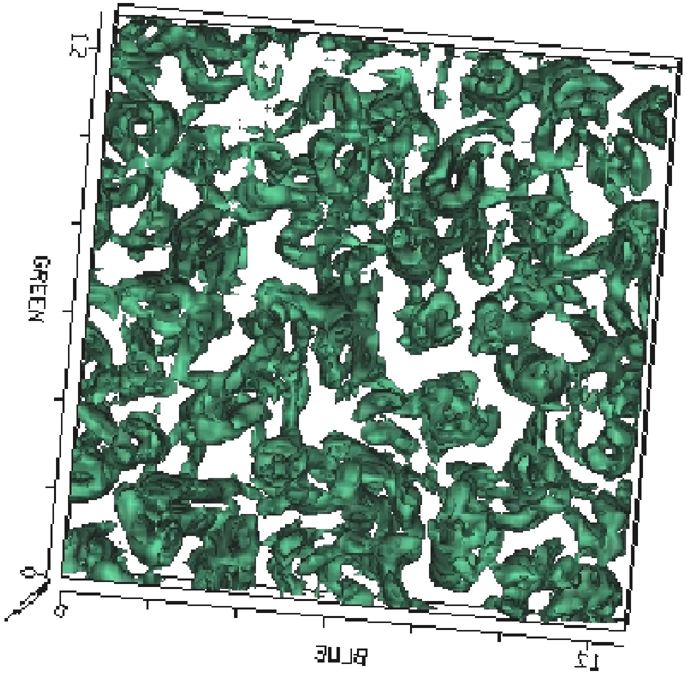
Structure of the 3D magnetic fied
The size of the cube is $2\lambda x\ 2\lambda x 2\lambda$

$n=80n_c$
$\phi=10^{20}w/cm^2$

Side View Isosurface 25Mg $x=130k_0^{-1}$



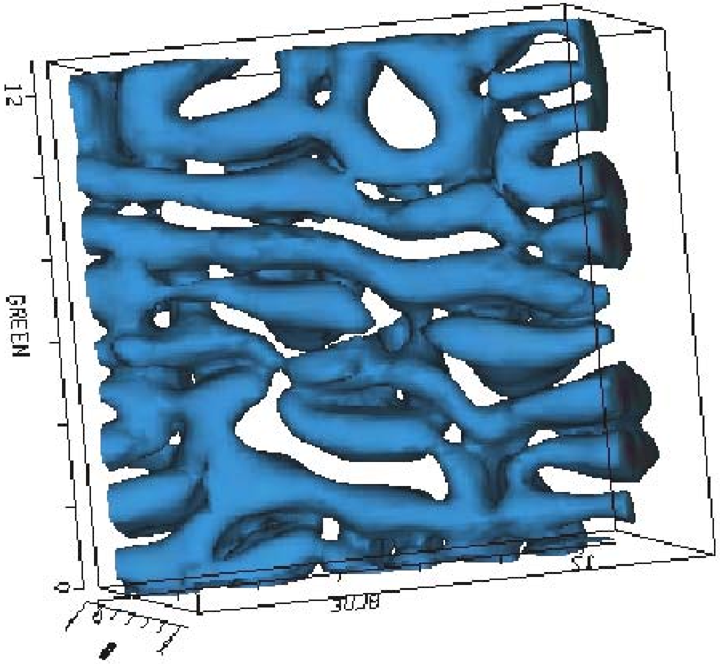
Front View(from inside)

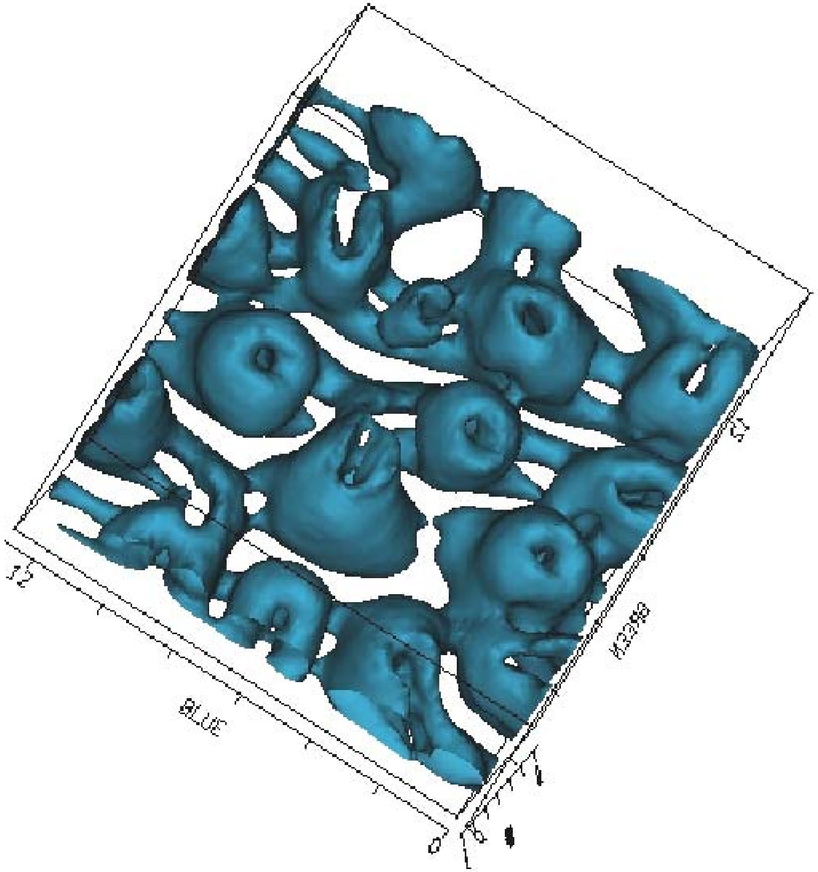


Structure of the 3D magnetic fied
The size of the cube is 2λx 2λx2λ

$n=10n_c$

$\phi=10^{19}$w/cm^2





A simpler model: no more laser

At each time step in the yellow region a given fraction of particles is replaced by particles with an arbitrary distribution function.

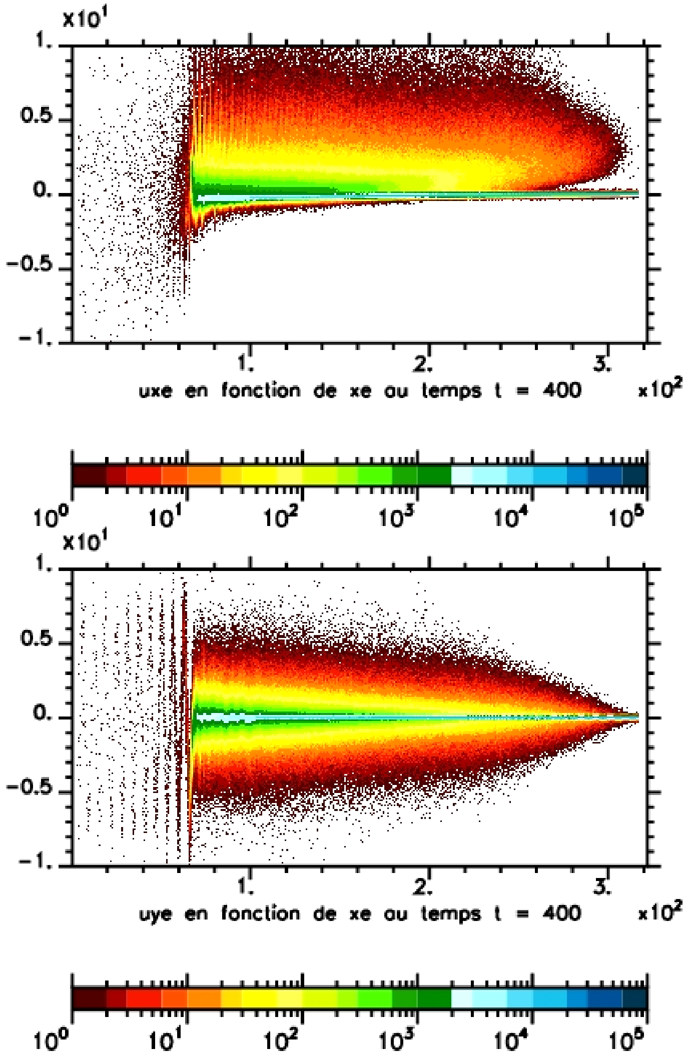
This allow full control of the injected distribution.

Typical injection distribution function is:

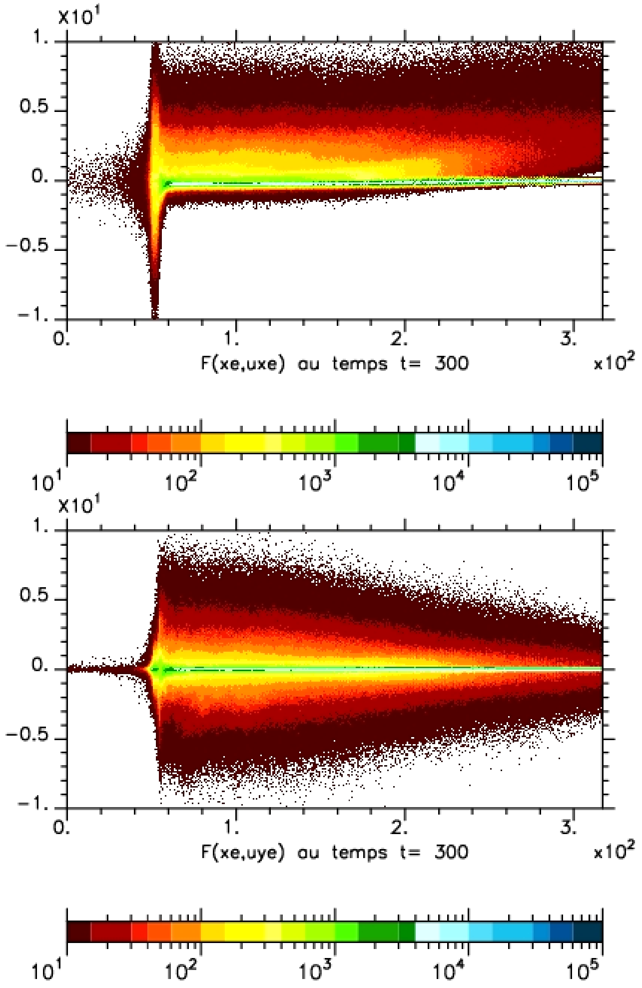
hot along the longitudinal direction (T_{hot} ~ 1 to 1.5Mev)
cold along the transverse direction(T_{Perp} ~1KeV)

Comparison of the simple model with the simpler one !

Laser,$\phi = 10^{19}$W/cm ^{2}



Hot beam, 1MeV, 5%



Conclusions in term of modelling

For this problem the main difficulty may not be the laser

The constraint on stability due to aliasing instability can be cured by adequate spatial filtering and use of higher order spline

The constraint imposed by the development of a very short wave length instability on a scale of a skin depth cannot be relaxed

Obviously MR would be nice but due to courant condition the time step still be determined by the smallest mesh size(See J.L Vay talk)

Underdense low frequency short wavelength simulations
Hybrid methods

Problem :how to study the propagation of an intense laser pulse through a long under-dense plasma on very long time scales as compare to the laser frequency (several tens of picoseconds)

This propagation excites a nonlinear instability known as Brillouin backscattering .It is a low frequency short wavelength instability

In these conditions it is believed that the electrons dynamic can be ignored.

Ions are handled like in any explicit PIC code

The EM field is handled through envelope equations. This avoids the resolution of the fast time scale

Hybrid methods(2)

One starts with the wave equations on the vector potential A and takes its spatial average over one laser period, this yields

$$-2i\omega_0 \frac{\partial}{\partial t} A_z - \omega_0^2 A_z - c^2 \nabla^2 A_z = -\omega_{pe}^2 A_z$$

Electrons are described like fluid in Bolzmann equilibrium with the ions

This removes high frequency oscillation and yields the following non linear Poisson equation:

$$-\nabla^2 \phi = 4\pi \Big(\sum_s Z_s n_s - n_0 e^{\frac{e\Phi}{T_e}}\Big)$$

Where $n_e = n_0 e^{\frac{e\Phi}{T_e}}$ and $\Phi(x,t) = \phi(x,t) + \frac{m_e}{2}(eA_z / m_e c)^2$

The second term in Φ is the ponderomotive potential.

It is due to the non linear response of the plasma to the laser impulsion.

Because it cannot be time centered the solution of these equations is obtained iteratively.

The time step can be up to 40 times lager than in a standard PIC

The mesh remains a fraction of the Debye length, which makes it difficult to simulate several millimetre of plasma in 2D

To my knowledge no 3D extension exits

Implicit codes

Another approach to the study of “low frequency” phenomena is the use of implicit codes

They come in different flavours

Moment methods (where fluid equations are used to get the field at time N+1)

Direct methods that obtain all the information needed from the particles

The issue related to implicit particles codes is that one cannot solve the nonlinear problem consisting of the full set of equations of motion of all the particles coupled with Maxwell equations at a given time step.

The field equations must be decoupled from the equations of motion of particles with the help of some predictor correction scheme.

In the moment case fluid equations are used to build the predictor. The difficulty is to find an adequate closure of the fluids equations.

The direct method avoid this problem by dealing only with particles

A. Frieman will describe this scheme in more details,I guess

However it is important to note that in the derivation of the model there is a long wavelength approximation. This implies it cannot be used to study low frequency short wavelength waves

Collisions(1)

Most PIC used to day are collisionless.

The study of the transport of electrons in dense matter requires the introduction of collision

The plasma in which they may play a significant role impose very severe constraints on the PIC model due to the smallness of the Debye length

They also require a high quality collisionless particle code:self heating due to numerical collisions must be much less than collisionnal relaxation

Collisions(2)

They can be handled in different way depending on their kind

Electron-ion can be handle on the basis of a Langevin equations, the main effect of a collision being to deflect the direction of the velocity without exchange of energy .

Electron –Electron collisions cannot be handled that way and the scheme generally use is due to Abe and is very similar to the one use in the DMSC method

Collisions(3)

Basically it consists of the following steps:

Sort the particles per cells (for simplicity let assume there an even number of particles in each cells which obviously not true)

Pair them randomly

Change to the centre of mass of each pair

Add a random vector computed on the basis of the collision cross section corresponding to their relative velocity in center of mass

Compute the new velocities of each particle

Go back to the reference frame

We have performed extensive check of this procedure and it does behave correctly despite it transforms an N^2 process in an N one!

Coupling of particles codes(1)

The numerical scheme use by the direct method scheme is derived from Gear scheme for stiff differential equations It introduces a third order damping term that scales as $(\omega_0 dt)^3/2$.

Even with a time step such that $\omega_0 dt \sim 0.1$ this scheme introduces an unacceptable damping if one needs to propagate a wave from the vacuum up to the overdense plasma.

In other word if one wants model the interaction of an intense laser beam with a cold plasma at solid density it will require the coupling of an explicit particle codes and an implicit code

JLVay have recently proposed a new scheme for AMR that can also be used to coupled an explicit and an implicit code(to be tried)

Coupling of particles codes(2)

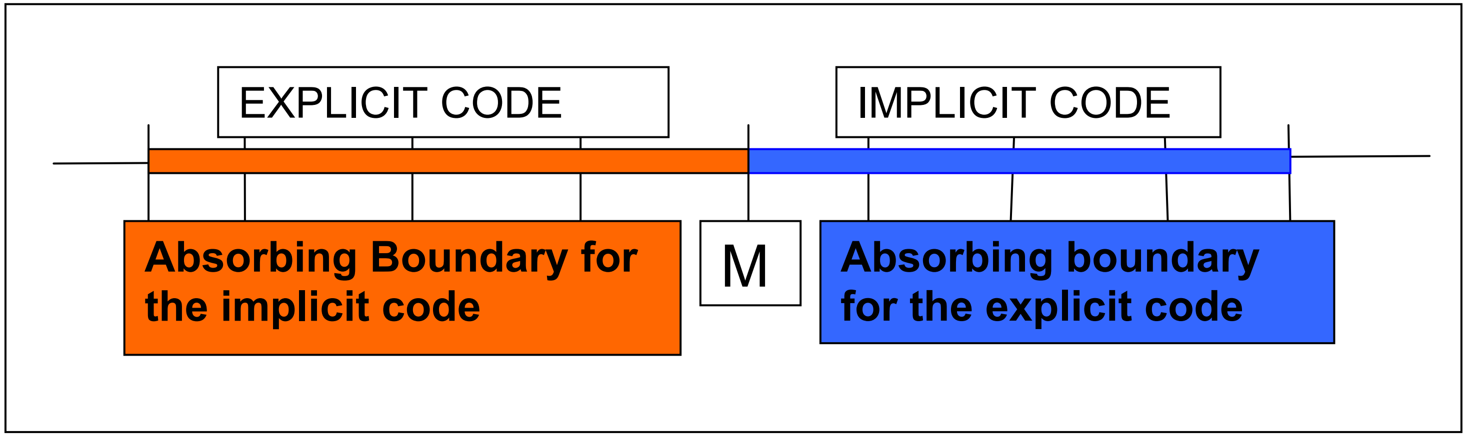
For simplicity it will described in 1D but it is immediately (almost) applicable to higher dimensions

The scheme uses J.L. Vay absorbing boundary conditions that extremely good PML conditions.

A key point in term of simplicity of the coupling is that the explicit scheme and the implicit scheme use the same staggered grid.

Consider a system that uses an explicit scheme on a given mesh on the left of point M and an implicit scheme on the right of M. Point M belong to both system

Coupling of particles codes(2)



The left system can be terminated on the right by absorbing boundary conditions that will suppress the outgoing wave of the explicit part.

Conversely the right system can be terminated on the left by absorbing boundary conditions that will kill the reflected waves propagating toward the source.

Particles moves freely through the boundary and gives the correct source terms in both region

Because mesh size can be different on both side of the boundary the same scheme can be applied to perform a mesh refinement using the same model on both side

Summary

Inertial fusion is an extremely complex multiscale problem that is considerably complicated by the absence of ordering

A full scale model can probably not rely on basic principles

Considerable progress need to be accomplish with the help of kinetic codes to built a reliable model

Very little has been accomplished yet in term of mesh refinement