Dynamic Cournot R&D Games in Commodity Production

Mike Ludkovski[1]
Joint work with Ronnie Sircar (Princeton)

[1] Dept of Statistics & Applied Probability UC Santa Barbara

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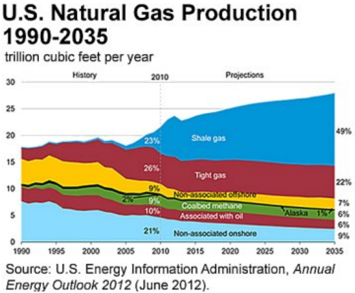
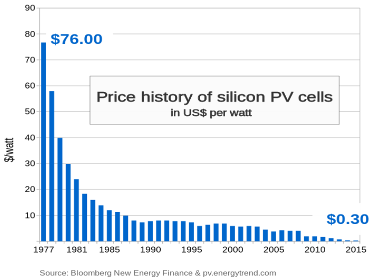
Structural Shifts in the Oil/Energy Markets

Shale gas/oil revolution

Ongoing advances in solar/wind/storage





OPEC Supply Wars:

HOME » FINANCE » NEWS BY SECTOR » ENERGY

Saudi Arabia increases oil output to crush US shale frackers

Saudi Aramco chief says kingdom is now pumping 9.8m barrels per day in bid to win oil price war with US shale frackers

TECH 5/05/2015 @ 1:03PM | 36,065 views

Why Tesla Batteries Are Cheap Enough To Prevent New Power Plants

Forbes, 5/1

Tesla's New Battery Doesn't Work That Well With Solar

Even Elon Musk's SolarCity, the biggest supplier in the U.S., isn't ready to install Tesla's home battery for daily users

Tesla's Powerwall may be the future but it's not a battery breakthrough

Mashable 5/1

Bloomberg 5/4

Resource and Commodities Markets

- These events can be classified as:
  - competitive effects: collusion vs oligopoly
  - fundamental value: competitive advantage through production costs
- These developments motivate study of long-term noncooperative dynamic game frameworks for energy markets

Supply-Side Competition

- Instantaneous: through production levels
- Cournot game: agents choose q_i
- Market clearing price solves $D(p) = Q = \sum_i q_i$
- Eg: Middle East oil crowding out Russian and Canadian supply
- Long-term: structural competition by optimizing profitability
- Try to lower *production costs*
- Eg: shale oil revolution

NYT Op-ED 1/27/2015

"the plunge in oil prices offers a sobering reminder of the power of markets over policy".

Research and Development

- Production costs are lowered through R&D, i.e. technological breakthroughs
- The latter have allowed extraction of many new fossil fuels:
  - Deep-sea offshore oil
  - Shale natural gas
  - Oil sands
- More broadly, advances in solar technology or efficient biomass conversion have allowed to diversify the portfolio of electricity supply sources.
- R&D: costly w/uncertain outcome
- Also: abrupt = "jump-like" and ongoing = multi-stage.
- We propose a simple stochastic model to capture these uncertainties and embed within a Cournot game framework

Game Model

- **Cournot** market: players control supply
- Production levels $q_t^i \geq 0$; production costs c_t^i
- Price is given by inverse demand curve $P = D^{-1}(Q)$ based on aggregate supply $Q := \sum_i q^i$
- Profit from production is $\pi := q_t^i \cdot (P(Q_t) - c_t^i)$
- Production cost can be lowered through R&D: control evolution of c_t^i
- R&D effort $a_t^i \geq 0$ with convex cost function $\mathcal{C}(a)$
- Players look at their total discounted profit:

$$R_i := \mathbb{E}\left[\int_0^{\infty} e^{-rt}\left\{(P(Q_t) - c_t^i)q_t^i - \mathcal{C}(a_t^i)\right\} dt\right].$$

- Look for closed-loop Markov Nash equilibrium

Endogenizing production costs

- Producers expend effort to **lower production costs**
- Typically (depends on the prudence of inverse-demand curve), lower costs $\to$ more production/more profit
- Also, as costs fall, may become dominant enough to convert to a monopoly
- Anticipation of these benefits is the key driver behind investing in R&D: there is dynamic interaction between R&D and production
- Market structure is endogenous

Interpretation I: R&D as Control

- Technology state is a controlled stochastic state
- Industrial Organization: optimizing R&D investment by a monopolist/central planner facing uncertainty
- Sustained effort w/uncertain outcome: Kamien and Schwartz (1978), Lafforgue (2008), numerous papers addressing climate change mitigation
- Instantaneous switch w/fluctuating benefit: real options
- We extend this single agent setting to include game effects

Interpretation II: Racing Game

- Agents race to capture first-mover advantage of lower costs
- Patent racing: Reinganum (1982), Judd (2003) multi-stage preemption games
- But: no intermediate profits, all about a single ultimate prize

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- But: no intermediate profits, all about a single ultimate prize
- Timing games: impact of technological change on strategic competition
- Fudenberg & Tirole (1985), Weeds (2002).
- Real option games: Azevedo and Paxson (2010)
- Deterministic differential game: Cellini & Lambertini (2009), ...
- But: 1-shot games – focus on coordination/preemption, no dynamics

Interpretation III: Dynamic Cournot game

- Cournot Games: Sircar et al. (2010–), Dasarathy and Sircar (2014)
- We endogenize production costs
- Links to literature on exhaustible resources (Hotelling, Pindyck, ...)
- In sum: first model to combine **Cournot** + **dynamic** + **stochastic R&D**

One-Shot Game w/Linear Demand

Static Cournot duopoly with production costs c^1 and c^2:

- Suppose inverse demand is linear $P(Q) = 1 - \sum_i q^i$: choke price is 1
- The respective profit is $R_1 := q^1(1-(q^1+q^2)-c^1)$ and $R_2 := q^2(1-(q^1+q^2)-c^2)$
- In equilibrium, actions satisfy

$$\frac{\partial R_1(\cdot; q^{2,*})}{\partial q^1} = 0 = \frac{\partial R_2(\cdot; q^{1,*})}{\partial q^2} \iff \begin{cases} 1 - 2q^{1,*} - q^{2,*} - c^1 = 0 \\ 1 - q^{1,*} - 2q^{2,*} - c^2 = 0 \end{cases}$$

- Interior eqm solution is $q^{i,*} = \frac{1+c^i-2c^{\bar{i}}}{3}$; Game value $v_i = (q^{i,*})^2 = \frac{(1+c^i-2c^{\bar{i}})^2}{9}$
- Analogous to a deterministic stationary cont-time game: $R_i = \int_0^\infty e^{-rt} q_t^i (1 - q_t^i - q_t^{\bar{i}} - c^i) dt$

Blockading

- Production rate must be non-negative
- If $c^i > \frac{1+c^{\bar{i}}}{2}$, producer i is blockaded and does not produce at all, $q_i^* = 0$. In that case other player has monopoly with $q_{\bar{i}}^* = \frac{1-c^{\bar{i}}}{2}$
- Blockading when c^i is large (close to 1) relative to $c^{\bar{i}}$. No blockading if $c^i < 0.5$

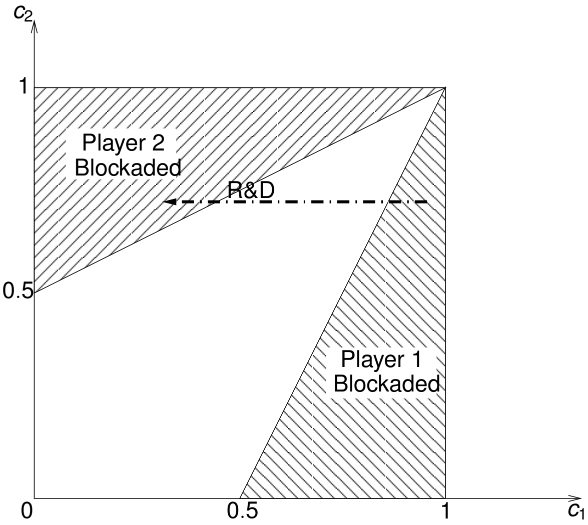

Figure: Fixed-cost Cournot game.

Technology Ladders

- Model technology as a discrete ladder: $c(1) > c(2) > ... \geq 0$
- If currently at n-th stage, a breakthrough moves the producer to $n+1$-st stage of technology
- $c(n) = \exp(-\mu n)$: efficiency improves proportionally by $\mu\%$
- $c(n) = (1-\mu n)_+$: absolute improvements in efficiency– eventually will reach zero costs

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- $c(n) = (1 - \mu n)_+$: absolute improvements in efficiency– eventually will reach zero costs
- effort $a_t \Rightarrow$ breakthroughs occur at rate λa_t
- N_t^i: point process for the technology advances of player i. Given (a_t^i), (N_t^i) is a Poisson process with controlled intensity $\lambda^i a_t^i$
- Dynamic production costs are $c_t^i = c(N_t^i)$
- R&D incurs costs $\mathcal{C}(a_t)$ per unit time; convex + increasing

Dynamic R&D

- Continuous-time strategies: q_t^i, a_t^i
- Only uncertainty is from arrivals counted by (N_t^i).
- Strategies therefore assumed to be in feedback form for (N_t^1, N_t^2)
- If $a_t^i \equiv 0$, the corresponding state (c^1, c^2) is absorbing

Piecewise Deterministic Property

Between R&D advances the game is stationary. Can be viewed as an array of coupled deterministic Cournot games

Constructing Nash Equilibrium

- Given initial technology stages (n_1, n_2), game values are denoted by $v_i(n_1, n_2)$
- τ^i is the time of first R&D success by player i – controlled by effort (a^i_t)
- Given (q^i, a^i), v_i's satisfy the recursions

$$v_1(n_1, n_2) = \mathbb{E}\Big[\int_0^{\tau^1 \wedge \tau^2} e^{-rs}\{q^1_s(P(Q_s) - c^1(n_1)) - \mathcal{C}(a^1_s)\}\, ds + e^{-r\tau^1 \wedge \tau^2}[1_{\{\tau^1 \le \tau^2\}} \cdot v_1(n_1 + 1, n_2) + 1_{\{\tau^1 > \tau^2\}} \cdot v_1(n_1, n_2 + 1)]\Big]$$

- By the **piecewise deterministic** property, under every Markov Nash equilibrium, $q^i_t \equiv q^i, a^i_t \equiv a^i$ are constant for $t \in [0, \tau^1 \wedge \tau^2]$
- So $\tau^1 \wedge \tau^2 \sim Exp(\lambda^1 a^1 + \lambda^2 a^2)$

Duopoly Game Values

- Using properties of Poisson arrival times, Nash equilibria can be constructed by

$$v_1(n_1, n_2) = \sup_{q \geq 0, a \geq 0} \frac{q(1 - q - q^{2,*} - c^1(n_1)) - \mathcal{C}(a) + \lambda^1 a v_1(n_1 + 1, n_2) + \lambda^2 a^{2,*} v_1(n_1, n_2 + 1)}{\lambda^1 a + \lambda^2 a^{2,*} + r}$$

- Similar equation for $v_2(n_1, n_2)$
- production rates: $q^{1,*} = \frac{1-2c^1(n_1)+c^2(n_2)}{3}$, similar for $q^{2,*} = \ldots$
- For $a^{1,*}, a^{2,*}$ the f.o.c's yield a system of two nonlinear equations characterizing the Nash equilibrium

Recursive Static Games

$$v_1(n_1, n_2+1)$$

$$\Big\downarrow \lambda^2 a^2$$

$$v_1(n_1, n_2) \xleftarrow{\lambda^1 a^1} v_1(n_1+1, n_2)$$

- Can solve recursively on a lattice
- Boundary condition is $v_i(N_1, N_2) = \frac{(1-2c^i(N_1)+c^j(N_2))^2}{9r}$; also when $n_1 = N_1$, no further R&D is possible for P1 (1-dim optim by P2)
- $a^{1,*}$ depends on $v_1(n_1+1, n_2) - v_1(n_1, n_2) > 0$ and $v_1(n_1, n_2+1) - v_1(n_1, n_2) < 0$
- $\mathcal{C}(a) = a^2/2 + \kappa a$:
  - Have a system of two coupled quadratic equations for $a^{i,*}$
  - if $\kappa > 0$ then R&D may be unprofitable, so $a^{i,*} = 0$ is possible
  - Analytic expressions to determine whether R&D is zero

First Example: Unilateral R&D

- Only P1 can innovate – cost profile is $(c_1(n), c_2)$
- Game value is

$$v_1(0) = \mathbb{E}_0\Big[\int_0^{\tau^1} e^{-\rho_1 s}(\pi(0) - \mathcal{C}(a_1(0)))\,ds + \int_{\tau^1}^{\tau^2} e^{-\rho_1 s}(\pi(1) - \mathcal{C}(a_1(1)))\,ds$$
$$+ \int_{\tau^1}^{\tau^2} e^{-\rho_1 s}(\pi(2) - \mathcal{C}(a_1(2)))\,ds + \ldots + \int_{\tau^N}^{\infty} e^{-\rho_1 s}\pi(\bar{n})ds\Big]$$

- NPV of R&D: $G_1(n) = v_1(n) - \int_0^\infty e^{-\rho_1 s}\pi_1(n)\,ds = v_1(n) - \frac{\pi_1(n)}{\rho_1}$
- Linked to immediate benefit $\propto \partial\pi_1/\partial c_1\Big|_{c_1=c_1(n)} = -2\frac{\ell(n)}{\ell(n)+1}q_1(n)$ – monopolist is most sensitive
- Level of R&D is further affected by: **expectation** of future profits (less future gains as get close to $c^1 = 0$), **shape** of the cost curve $n \mapsto c^1(n)$ (marginal efficiency of R&D), **current** revenue levels

Value of R&D

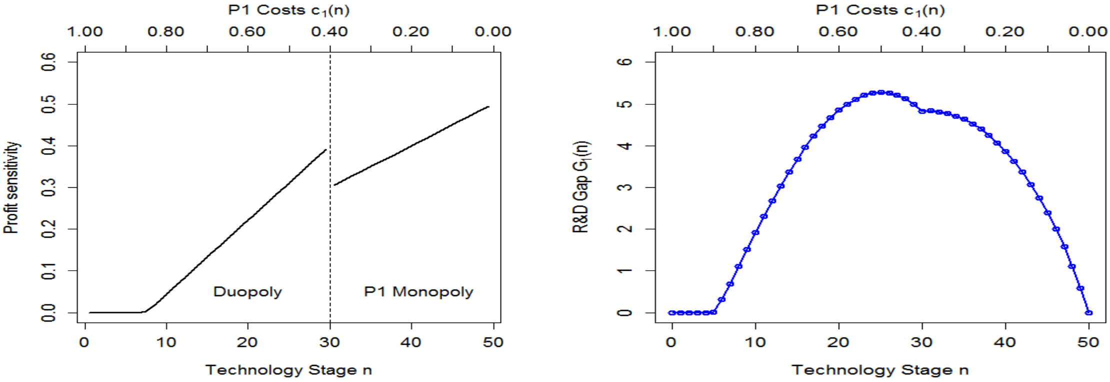


Figure: Left: sensitivity of instantaneous profit to production costs $\partial\pi(c_1, c_2)/\partial c_1$. P2 costs are fixed at $c_2 = 0.7$. Right: gap $G_1(n)$ between the NPV from optimal R&D and NPV from zero R&D. Linear monopoly model.

Dynamic Profiles: Unilateral R&D

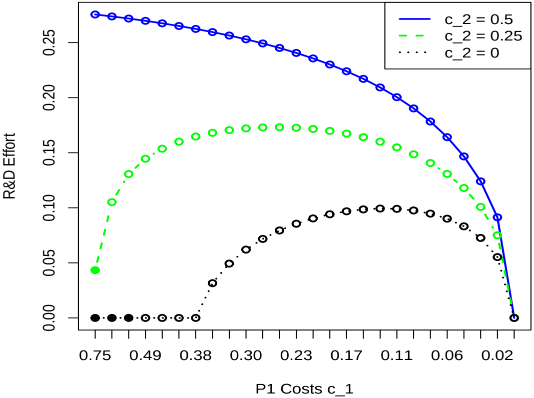


Figure: Effort Curves for Unilateral R&D in a Cournot Duopoly. Quadratic effort cost $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$. Here $c^1(n) = 0.75 - 1.5\sqrt{n}$ ($q^1(n)$ is linear). Filled points indicate stages where P1 is blockaded

Bilateral R&D – symmetric setting on a lattice

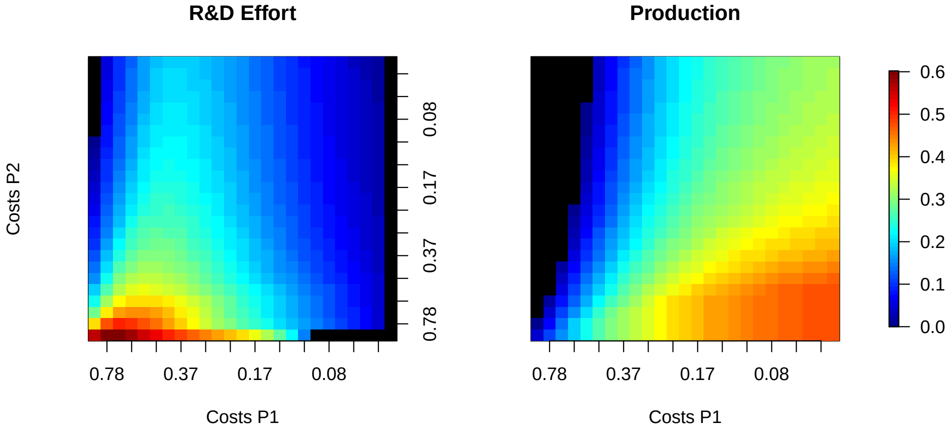


Figure: Right panel shows the effort $a^1(n_1, n_2)$ and left panel the production rate $q^1(n_1, n_2)$. Quadratic costs $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$. $c^i(n) = e^{-n/8}$. Black regions indicated blockading stages

Bilateral R&D

- R&D effort levels are asymmetric: put most effort when slightly ahead of competitor
- Therefore, player with lower costs tends to extend her advantage ("mean-aversion") – known also in patent racing models (Judd 2003)
- Competition is dynamically unstable (tends to collapse into a monopoly)
- $q = 0, a > 0$: **Optimism** about future profits can spur R&D even if blockaded right now
- $q > 0, a = 0$: c large: **pessimism** if too far from profitability
- $q > 0, a = 0$: c^i very low: **complacent** monopolist
- Outside input (subsidies) can spur endogenous advances both for very inefficient alternatives and for efficient monopolies

Distribution of (N_t^1, N_t^2)

$t = 2$

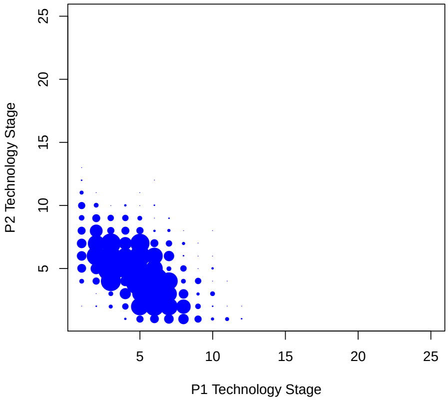


Figure: Distribution of (N_t^1, N_t^2). Costs are $c^i(n) = e^{-n/8}$. Quadratic effort curve $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$.

Distribution of (N_t^1, N_t^2)

$$t = 4$$

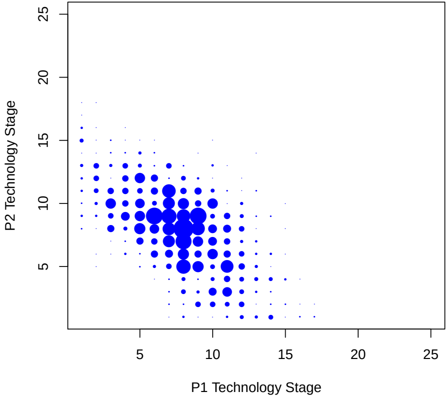


Figure: Distribution of (N_t^1, N_t^2). Costs are $c^i(n) = e^{-n/8}$. Quadratic effort curve $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$.

Distribution of (N_t^1, N_t^2)

$$t = 8$$

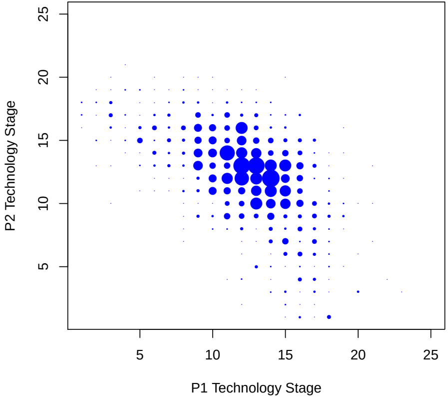


Figure: Distribution of (N_t^1, N_t^2). Costs are $c^i(n) = e^{-n/8}$. Quadratic effort curve $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$.

Distribution of (N_t^1, N_t^2)

$t = 15$

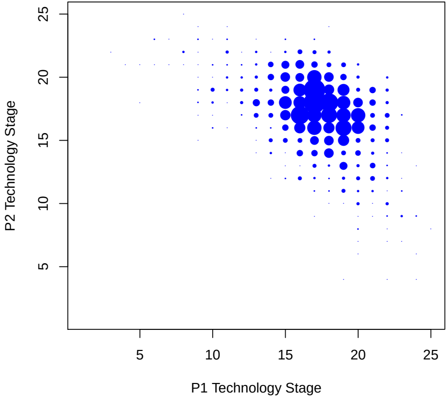


Figure: Distribution of (N_t^1, N_t^2). Costs are $c^i(n) = e^{-n/8}$. Quadratic effort curve $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$.

Distribution of (N_t^1, N_t^2)

$t = 25$

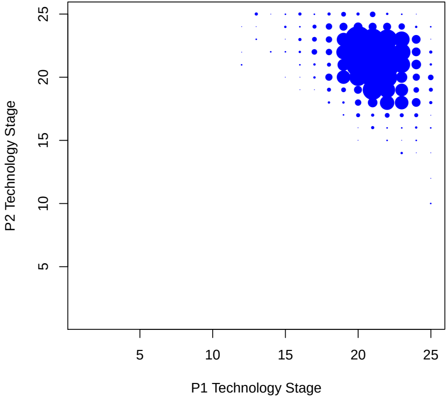


Figure: Distribution of (N_t^1, N_t^2). Costs are $c^i(n) = e^{-n/8}$. Quadratic effort curve $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$.

Distribution of (N_t^1, N_t^2)

$t = 40$

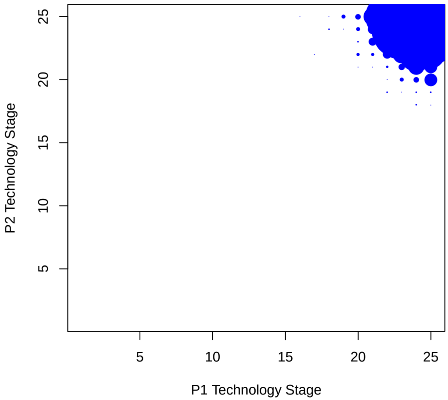


Figure: Distribution of (N_t^1, N_t^2). Costs are $c^i(n) = e^{-n/8}$. Quadratic effort curve $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5$, $r = 0.1$.

Sample Path of (N_t^1, N_t^2)

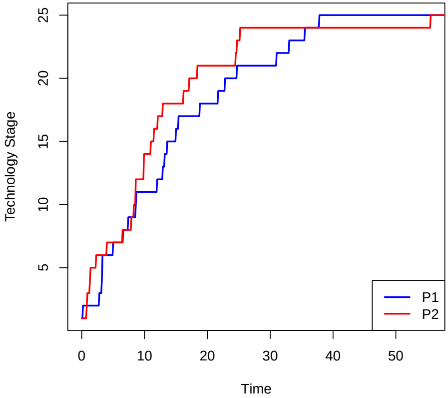


Figure: Sample path of (N_t^1, N_t^2). Costs are $c^i(n) = e^{-n/8}$. Quadratic effort curve $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$.

Impact of Uncertainty

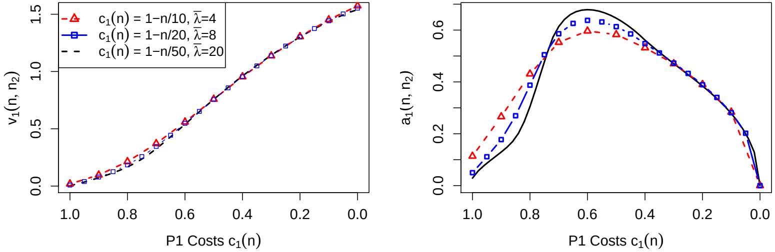
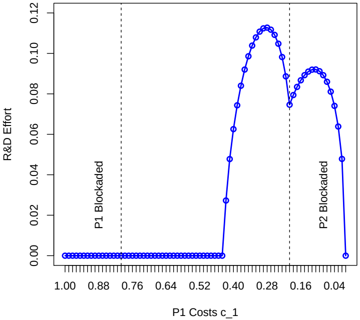


Figure: Comparison of game values $v_1(\cdot, n_2)$ and effort levels $a_1(\cdot, n_2)$. Bilateral symmetric R&D game with linear technology progress $c_1(n) = 1 - n/M$, $\bar{\lambda} = 0.4M$ for $M = 10, 20, 50$.

- Study **effect of uncertainty** by linearly scaling the R&D ladder $c(n)$ and rate of progress λ
- Take $c(n) = f(n/M), \lambda = \lambda M$ where $c \mapsto f(c)$ is cont R&D curve on $[0, 1]$.
- Impact is ambiguous: more uncertainty can spur/deter R&D investment!
- Extra benefit from very quick successes may outweigh less flexibility: $\left(\frac{M\bar{\lambda}a}{\rho+M\bar{\lambda}a}\right)^M$ is decreasing in M

Deterministic Limit

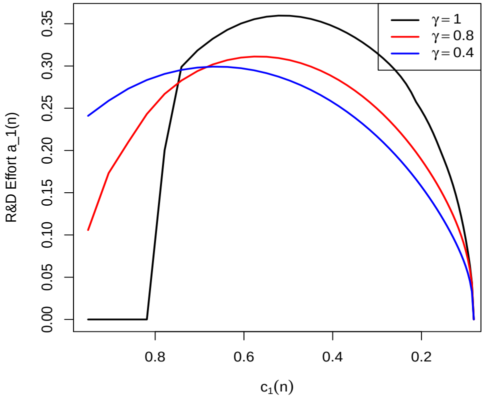
- As $M \to \infty$:
  $dc_1(t) = -\bar{\lambda} a(t)\, dt$
- Deterministic unilateral R&D:
- $$r\bar{g}_1(c) = \pi_1(c) + \frac{(\lambda \bar{g}_1'(c) - \kappa)_+^2}{2} = 0$$



- revenue $\pi_1(c)$ is piecewise (three phases): the ODE has 2 **fixed boundaries** and a free boundary.
- The effort level $a(c) \propto \bar{g}'(c)$ is non-smooth: kinks at $c \simeq 0.45$ and at $c = 0.2$.

Effect of Competition

- **Substitutability** fraction γ
- Demand is $P_i(q_i, Q_{-i}) = 1 - q_i - \gamma Q_{-i}$
- So far: perfectly substitutable goods ($\gamma = 1$)
- $\gamma = 0$: equivalent to a monopoly.
- $\gamma \downarrow$: $v \uparrow$, $q \uparrow$
- Impact on R&D is ambiguous



Extensions: R&D Complementary to Production

- Firm has **fixed labor supply** L. Allocate L between production and R&D: $a_t^i + q_t^i = L^i$
- Sharpens the trade-off between immediate revenue and future higher profits
- Cost of R&D is now implicit (quadratic if assume linear demand $P(Q)$)
- Will tend to decrease R&D over time
- May be optimal to voluntarily lower/suspend production to advance technology (e.g. to lock-in monopoly)

Extensions: spill-overs

- Spill-over effects: R&D by one player may have impact for the other one
- $\lambda_i(t) = \bar{\lambda} F(a_i(t), a_j(t), N_i(t), N_j(t))$
- Short-term spill-over: effort by j affects discovery rate of i
- Long-term spill-over: previous discoveries of j (ie $N_j(t)$) affects discovery rate of i
- Also possibly instantaneous spill-over: simultaneous jump in N_i and N_j
- Easy to incorporate since the basic recursive structure still holds

Extensions: Exhaustible Resources + R&D

- When considering competition between old and new energy (fossil fuels vs. renewables), exhaustible reserves play a crucial role
- X_t – level of reserves at date t; $dX_t = -q_t dt$ lowered through production
- Oil industry (P1): low production costs c^1, but also marginal cost of exhaustibility
- Renewables industry (P2): high current production costs $c^2(0)$; potential for R&D
- P1 chooses (q_t^1); P2 chooses (q_t^2) and (a_t^2). State is (x, n)
- Leads to a **system of nonlinear ODEs** in x, coupled through n
- Can allow P1 to also explore for new reserves

Extensions: Switching Technologies

- Consider two integrated producers who can each use **either** cheap fossil fuels, or expensive backstops (oil sands)
- Resources allocated between production and R&D (advancing backstop technology)
- Uncertainty in advances will spur earlier R&D investments as marginal value of cheap reserves rises
- Related to the model of Harris, Howison and Sircar (2010)

Conclusion

- Programme: **stochastic** framework for natural resource oligopolies
- Stochasticity + Repeated games + Endogenizing the market structure leads to numerous non-trivial phenomena

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THANK YOU!

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